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*Research article*

## **Referral coordination mechanism for the hierarchical system with Internet diagnosis platform**

**Jingyang Wang<sup>1</sup>, Xiaoyu Wang<sup>2</sup>, Miao Yu<sup>2</sup> and Dandan Yu<sup>3,\*</sup>**

<sup>1</sup> Information Science and Engineering School, Northeastern University, Shenyang, China

<sup>2</sup> School of Management, Shenyang Jianzhu University, Shenyang, China

<sup>3</sup> Information Center, The First Affiliated Hospital of Dalian Medical University, Dalian, China

\* **Correspondence:** Email: 182158874@qq.com.

**Abstract:** This paper explores referral coordination within a hierarchical healthcare system utilizing an Internet diagnosis platform. A reverse referral game model is established between a general hospital (GH) and community hospital(s) (CH). First, two system types are analyzed: a noncompetitive system consisting of a GH and a CH, and a competitive system consisting of a GH and multiple CHs. Using a queueing game framework, we derive optimal referral strategies under centralized decision-making. We further characterize optimal contracting mechanisms under two scenarios: an outsourcing arrangement where the GH sets prices to achieve perfect coordination, and a decentralized setting where both parties maximize individual profits (imperfect coordination). Comparing coordination efficiency across systems reveals that outsourcing mechanisms achieve perfect coordination in both configurations. Under imperfect coordination, while the noncompetitive system exhibits marginally higher coordination efficiency, the competitive system yields significantly higher treatment thresholds and GH profits. Thus, competition in referral systems with Internet diagnosis platforms enhances resource utilization and substantially increases GH profitability.

**Keywords:** coordination; mechanism design; hierarchical system; Internet diagnosis platform

**Mathematics Subject Classification:** 90B22, 91A80, 90B50

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### **1. Introduction**

In recent years, Internet diagnosis platforms have emerged as a novel medical service, garnering significant attention and recognition. These platforms adopt Internet technology and digital tools to provide online diagnosis, medical consultation, telemedicine, and related services to patients and doctors. Due to their convenience and efficiency, they offer patients an alternative to traditional in-person medical care while also introducing innovative approaches for the rational distribution and utilization of medical

resources. This advantage of Internet-based medical services became particularly apparent during the COVID-19 pandemic. At its peak, Internet-based medical services accounted for 13% of total outpatient visits, a substantial increase from virtually zero at the beginning of 2020 [1].

Hierarchical healthcare systems have been widely implemented in various settings. For example, in China's healthcare landscape, these hierarchical systems typically comprise general, primary, and community hospitals. The general hospitals (GHs) serve as high-level healthcare institutions which offer specialized services for patients with severe and emergency conditions, whereas primary and community hospitals (CHs) function as lower-level healthcare facilities, providing basic and routine medical services for patients with common and chronic diseases [2]. Under this system, different levels of healthcare organizations collaborate and coordinate their efforts to enhance the overall efficiency of the healthcare system [3]. Despite the widespread promotion of hierarchical systems, their effectiveness remains less than optimal. Issues such as difficult access to medical care in GHs and the underutilization of resources in CHs persist. Two primary factors contribute to these challenges: First, GHs possess more medical resources, advanced equipment, and skilled professionals, making them the preferred choice for numerous patients. Second, the absence of clear access control measures results in patients disproportionately utilizing the high-quality resources available at GHs. According to data from the National Health and Wellness Commission of the People's Republic of China in 2023, by the end of 2022, 45% of patients sought treatment at advanced hospitals, which represent less than 4% of the total number of healthcare institutions [4].

To address the shortcomings in hierarchical healthcare implementation, some GHs have begun integrating Internet diagnosis platforms into their systems. This integration aims to achieve several objectives, including providing direct prediagnosis services to patients, thereby alleviating congestion in offline GH medical services. Furthermore, patients with mild conditions identified online are referred to CHs (i.e., reverse referral), enhancing the utilization of medical resources. Reverse referrals are a distinctive feature of China's tiered healthcare system and may prove effective in mitigating the issue of GH overutilization while balancing healthcare resource distribution across tiers. However, in medical practice, CHs often hesitate to accommodate large numbers of downwardly referred patients due to inadequate diagnostic and treatment capacities [5]. Conversely, GHs, constrained by limited service capacity, tend to prioritize offline initial consultations for patients with higher return rates rather than revisits from online patients with mild ailments. This dynamic inevitably undermines the overall effectiveness of the system. Furthermore, to our knowledge, there is currently no research focusing on referral coordination within hierarchical systems featuring Internet diagnosis platforms.

Therefore, it is necessary to design referral mechanisms to coordinate GH and CHs. These mechanisms should induce GHs to offer appropriate outsourcing prices, incentivizing CHs to accept downwardly referred revisit patients from the online GH platform. This approach aims to create a mutually beneficial scenario where CHs can enhance resource utilization, and potentially increase revenue, and GHs can attend to more patients with complex diseases. The paper addresses the following questions: (1) How is best to determine the optimal criteria for referral? (2) Can outsourcing mechanisms be perfectly coordinated in both the noncompetitive system (NS) and the competitive system (CS)? (3) What is the impact of introducing competition on coordination efficiency?

In this paper, we examine the coordination of operational benefits through centralized and decentralized decision-making. First, centralized decision-making within the hierarchical healthcare system optimizes the overall utility of the system by determining the optimal treatment threshold and

referral strategy. Second, we propose the optimal contract form for GHs under service outsourcing agreements that achieve optimal system utility, motivating CHs to adopt the optimal referral threshold. We then analyze the equilibrium contract form for GHs and the equilibrium treatment threshold for CHs under a two-stage game equilibrium solution. Finally, numerical experiments explore the coordination of outsourced service contracts in NS and CS models, comparing coordination efficiencies. The model prioritizes profit maximization over cost minimization, reflecting the inherent conflict of interest between GHs and CHs that complicates referral processes.

The rest of this paper is organized as follows. Section 2 provides a review of the relevant literature. In Section 3, we describe our model and list the model assumptions. Section 4 describes the optimal treatment threshold in the centralized case and discusses perfect and imperfect coordination in the decentralized decision-making under two systems. Numerical analyzes are performed in Section 5. Finally, we summarize our main conclusions in Section 6. The proofs of all propositions and lemmas are in Appendix A.

## 2. Literature review

This paper is primarily related to three aspects of literature: operational management of telemedicine systems, competitive contract coordination mechanisms in supply chains, and competition within healthcare systems.

Telemedicine has been a major trend in healthcare management in recent years. Some studies have focused on topics related to optimal decision-making in telemedicine, such as pricing strategies and resource allocation. Wang et al. [6] employed a mixed duopoly game approach to investigate capacity decisions for GH and optimal pricing and capacity decisions for telemedicine institutions (TF). Sun et al. [7] utilized a two-stage game model to examine profit-sharing schemes in healthcare alliances consisting of GH and TF. They derived the optimal pricing for TF and capacity allocation for GH. Rajan et al. [8] investigated the operational decisions of experts providing telemedicine services and pricing as face-to-face services. Tarakci et al. [9] proposed optimal investment in telemedicine technology and staffing using heuristic methods. Saghafian et al. [10] developed a novel agent-knowledge optimization model to explore the optimal strategy for determining which cases (patients) should be assigned to telemedicine doctors for further evaluation. Wang et al. [11] investigated the optimal introduction strategy and reimbursement policy for telemedicine by constructing a duopoly game model and comparing three types of scenarios. Qiao et al. [12] studied the optimization problem of resource allocation for remote consultation through a simulation model. Yu et al. [13] constructed a three-stage game model under the dual-channel service strategy, analyzed the decision relationship between offline and online services of GHs, and provided equilibrium decisions for patient arrival rates and referral thresholds. Finally, Wang et al. [14] developed a game model for a hospital with both Internet-based diagnosis and face-to-face medical channels. The necessary conditions were proposed for hospitals to operate Internet-based diagnosis channels and face-to-face channels simultaneously, and the optimal decision was obtained for the service level of Internet-based diagnosis platforms. Existing studies mostly focus on pricing and resource allocation issues in telemedicine, whereas our study concentrates on the downward referral problem between the general hospital's Internet diagnosis platform and CHs.

Service competition among medical institutions has a significant impact on system coordination [15]. The study of competitive contractual coordination mechanisms is widely used in the field of supply

chain management. Boyaci and Gallego [16] explored the competition and coordination relationship between two competing supply chains in the market and analyzed its impact. David and Adida [17] proposed a linear quantity discount contract for the problem of competition and coordination in a dual-channel supply chain and demonstrated its effectiveness in perfectly coordinating a dual-channel supply chain. Wang and Shin [18] investigated the impact of supply chain contracts on innovation investment, involving three different types of contracts: wholesale price contracts, wholesale price contracts related to product quality, and revenue-sharing contracts. They concluded that revenue-sharing contracts can coordinate supply chain decisions, whereas the other two contracts may lead to insufficient innovation investment. Peng et al. [19] analyzed a two-tier supply chain network system by a Stackelberg game. Through spanning revenue sharing (SRS) contracts, they achieved coordination in the supply chain, ensuring a win-win outcome for all members. Hosseini-Motlagh et al. [20] addressed the coordination problem of a sustainable supply chain (SSC) under competition by proposing three-way compensation contracts. Sun et al. [21] constructed three competition behavior models for a dual-channel supply chain, demonstrating that the Stackelberg model enables suppliers to achieve the highest profits while allowing retailers to gain first-mover advantages.

In recent years, many competition studies have been focused on healthcare systems. Andritsos and Aflaki [22] analyzed the impact of nonprofit and for-profit hospitals on elective care within the hospital market. Hua et al. [23] delved into the competition dynamics between private and public hospitals under government subsidy policies. Furthermore, Chen et al. [24] studied the issue of price competition and cooperation games between two private service providers, using a two-tier healthcare system as an example. Ghandour et al. [25] explored the competitive relationship between hospital investment and service quality. Jiang et al. [26] conducted a novel study on the performance-based contracting problem in the healthcare market, focusing on the asymmetric competition between cost information and quality of care access. They discovered that intensified hospital competition and bonus incentives could enhance patient welfare. Guan et al. [27] investigated the impact of privatization reforms on waiting times within the public healthcare system, particularly comparing two forms of privatization: the competitive model and the cooperative model. They found that private hospitals showed a preference for the cooperative model, especially in systems with high reimbursement rates and privatization levels. Subsequent literature further explored competition within two-tiered healthcare systems [24, 28, 29]. In our study, we tackle the issue of referral coordination between CHs and GHs within a competitive environment.

Table 1 compares our study with prior literature. Previous research has largely overlooked referral mechanisms in hierarchical healthcare systems with Internet diagnosis platforms, particularly the role of competition in referral coordination. This study addresses this gap by examining referral coordination in such systems through the lens of competitive supply chain contracts. From a mechanism design perspective, we construct the referral coordination mechanism from the view of competitive supply chain contracts, treating the healthcare system as a supply chain. By designing contracts, we balance the interests of two-level hospitals and the platform, optimizing referral coordination. In terms of practical applications, we propose a diverse network of coordinating agents, differentiated benefit-sharing strategies, and competition-based efficiency enhancement paths. These offer actionable guidance for the practice of reverse referral within Internet platforms, effectively promoting the efficient allocation of healthcare resources and the improvement of service quality, contributing both theoretically and practically to the implementation of tiered diagnosis and optimization of the healthcare system.

**Table 1.** Comparison between the literature and this study.

| Field   | Literature   | Literature contributions  | This study   |
|---|--|---|--|
| Operational management of telemedicine systems                | Wang et al. [6], Sun et al. [7], Rajan et al. [8], Tarakci et al. [9], Saghafian et al. [10], Wang et al. [11], Qiao et al. [12], Yu et al. [13], Wang et al. [14] | Prior research has examined pricing and resource allocation in telemedicine platforms, providing operational guidelines for effective implementation.                             | This research focuses on the downward referral issue between general hospitals' online platforms and community hospitals.  |
| Competitive contract coordination mechanisms in supply chains | Boyaci and Gallego [16], David and Adida [17], Wang and Shin [18], Peng et al. [19], Hosseini-Motlagh et al. [20], Sun et al. [21]                                 | The literature covers various competitive contract coordination mechanisms in supply chain management, including quantity discounts, revenue-sharing, and compensation contracts. | This research extends competitive coordination mechanisms to referral coordination in healthcare systems.  |
| Competition within healthcare systems                         | Andritsos and Aflaki [22], Hua et al. [23], Chen et al. [24], Ghandour et al. [25], Jiang et al. [26], Guan et al. [27], Guo et al. [28], Chen et al. [29]         | Studies have investigated competition in two-tiered healthcare systems and its impact on overall system performance.  | This research addresses referral coordination in a competitive environment involving one general hospital and multiple community hospitals under the impact of telemedicine platforms. |

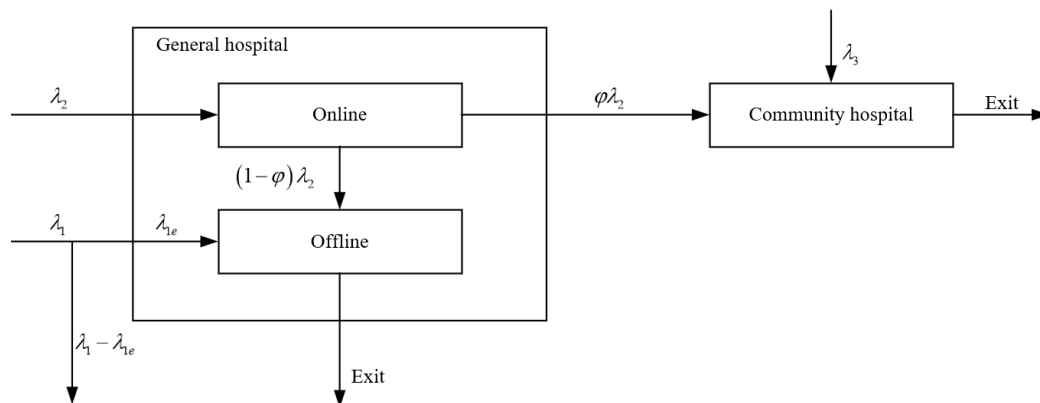
### 3. Model preliminaries

#### 3.1. Model description

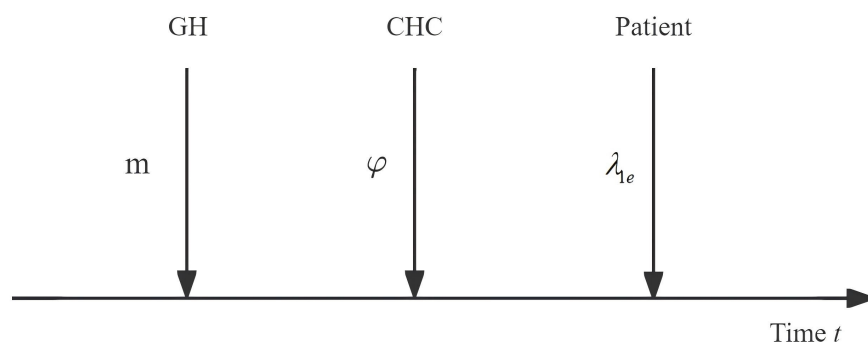
Consider a hierarchical healthcare system with an Internet diagnosis platform, which comprises a queuing network with two types of healthcare providers, CHs and GHs. Within the system, the CHs and GHs provide heterogeneous healthcare services to patients. CHs are limited to treating patients with relatively simple symptoms due to resource constraints, whereas GHs offer care to all patient types. Patients fall into two categories: the offline first-visit ones (including those from either a GH or CH) and online patients requiring subsequent referral to an offline CH or GH through the GH's online diagnosis platforms [7] (hereafter referred to simply as "online patients"). Two service system models are developed: Model 1 investigates the referral coordination problem between a single GH and CH, designated as a noncompetitive model, and Model 2 explores the referral coordination challenge

between a GH and multiple CHs (denoted as  $n$  CHs), where these CHs collectively handle their own patients and patients referred by the GH based on their capacities, introducing competitive dynamics (refer to [30]).

As shown in Figure 1, in the noncompetitive model, the arrival of potential offline first-visit patients at the GH, online patients from the GH, and offline first-visit patients from the CH follows a Poisson distribution with parameters  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$ . Online patients are initially unaware of their medical condition complexity, which is determined through preliminary online diagnosis. Subsequently, patients with conditions less complex than CH's treatment threshold  $\varphi$  ( $\varphi \in [0, 1]$ ) is determined based on referral payment offered by the GH and CH service capacity) are referred to the CH, and those with higher complexity are admitted offline at the GH for further treatment. The GH compensates the CH with a referral service fee to incentivize the acceptance of referred patients from the GH. Regarding potential offline first-visit patients at the GH, some proceed with treatment, and others defer, with the effective arrival rate denoted as  $\lambda_{1e}$  [31]. Therefore, the total patient arrival rate for the GH is  $\lambda_{1e} + (1 - \varphi)\lambda_2$ , and the patient arrival rate for the CH is  $\lambda_3 + \varphi\lambda_2$ .



**Figure 1.** Noncompetitive referral coordination model.



**Figure 2.** Sequence of decisions.

Under the outsourcing coordination mechanism, the outsourcing price  $m$  paid by the GH to the CH is set based on the treatment threshold  $\varphi$  of the CH. Therefore, considering the decision-making process of the participants in the hierarchical medical system, where the GH is the leader, the CH is the follower, and patients make medical choices based on the decisions of the two hospitals, a three-stage sequential game model is constructed.

As illustrated in Figure 2, the sequence of decisions is as follows:

Stage 1. The GH sets the outsourcing price  $m$  to maximize its expected profit.

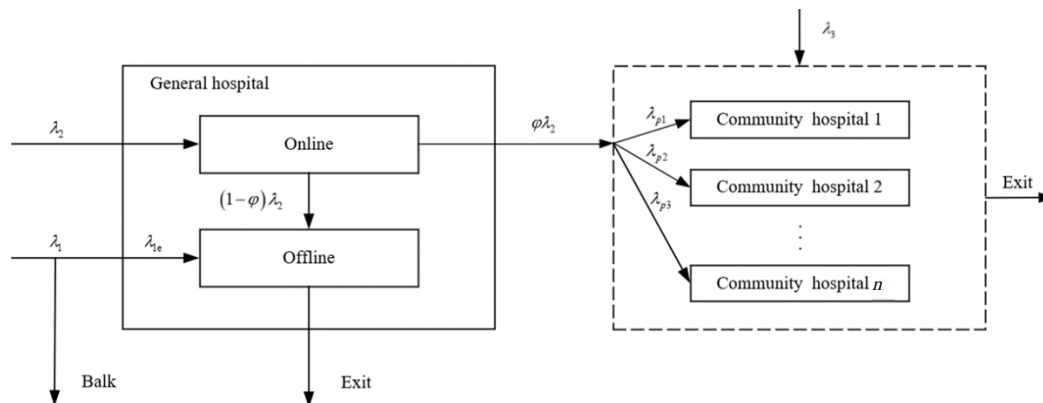
Stage 2. The CH determines the optimal treatment threshold  $\varphi$  to maximize its expected profit.

Stage 3. First-visit patients at the GH select that their effective arrival rate  $\lambda_{1e}$ , ensuring that their utility remains nonnegative.

In the competitive model depicted in Figure 3, the referral process for online patients mirrors that of Model 1. Following triage via the Internet diagnosis platform, patients remaining at the  $i$ th CH are denoted as  $\lambda_{3i} + (1/n)\varphi\lambda_2$ , where  $\lambda_{3i}$  denotes the patient arrival rate at the  $i$ th CH under the assumption of homogeneous characteristics. The outsourcing price set by the GH is  $m_i$ . Each CH sets an appropriate treatment threshold based on the given outsourcing price to accept mild patients from the online GH platform, thereby fostering competition. Increased participation by CHs intensifies this competitive dynamic. Definitions and variables used in this paper are detailed in Table 2.

**Table 2.** Notations and definitions.

| Notations             | Definitions   |
|-----------------------|---|
| $v$                   | Patient perceived value   |
| $x$                   | Complexity of the patient's disease   |
| $\lambda_1/\lambda_2$ | Potential arrival rate of offline first-visit patients in the GH/Arrival rate of online patients to the GH  |
| $\lambda_{1e}$        | Effective arrival rate of first-visit patients for the GH   |
| $\lambda_3$           | First-visit rate to a CH  |
| $d$                   | Waiting cost per unit time in the GH  |
| $p_1/p_2$             | Treatment price for offline/online patients in the GH   |
| $p_3$                 | CH treatment price  |
| $c_1/c_2$             | Unit cost of treatment for offline/online patients in the GH  |
| $c_3$                 | CH unit cost of treatment   |
| $c_3^w$               | Waiting cost per unit time in a CH  |
| $U$                   | Utility of the first-visit patient in the GH  |
| $\mu_1$               | Service capacity of the GH  |
| $\mu_2$               | Service capacity of a CH  |
| $\varphi$             | CH treatment threshold  |
| $\varphi_D^C$         | Referral thresholds for a CH under centralized decision-making in a competitive referral coordination model |
| $m$                   | Outsourcing price   |



**Figure 3.** Competitive referral coordination model.

### 3.2. System assumptions

Several assumptions reflect real-world conditions: (1) The GH and CHs adhere to a first-come-first-served queuing rule, with patient treatment times following independent exponential distributions with parameters  $\mu_1$  and  $\mu_2$ , respectively. The GH's service capacity is fixed due to practical constraints on expansion [32]. (2) The patient condition complexity  $x$  obeys a uniform distribution on  $[0, 1]$ . An  $x$  value closer to 1 indicates a more complex disease condition, whereas an  $x$  value closer to 0 indicates a less complex disease condition. (3) Diagnostic times are shorter than treatment times and are omitted [33], from the model for simplicity [34, 35]. Because the study population consists of online follow-up patients who had previously visited the GH, their online diagnostic process is streamlined and takes little time, exerting only a minimal impact on referral efficiency and system benefits. Therefore, excluding diagnostic time from the model does not compromise the practical interpretability of the conclusions. (4) The parameter  $p_1 - c_1 > p_2 - c_2 \geq p_3 - c_3$ , where  $p_1 - c_1 > p_2 - c_2$  ensures that the GH can improve its revenue by referring online patients, and the further parameter  $p_2 - c_2 \geq p_3 - c_3$  ensures that the revenue of the GH is not lower than that of a CH when treating equivalent patient volumes.

### 3.3. Patient utility and patient choice

Expected patient utility comprises perceived value, expected waiting cost, and medical service price. Here,  $v$  represents the perceived value of the patient, and  $d$  represents the waiting cost per unit time for the patient. Healthcare providers typically generate higher revenue by treating more severely ill patients [36]. Therefore, we assume a linear relationship between treatment price and patient condition complexity [37], that is,  $p = xp_i$  ( $i = 1, 2$ ). Here,  $p_1$  and  $p_2$  represent the price coefficients for initial patients from the offline GH and revisits from the online GH, respectively. Thus, the expected treatment price for initial patients from the offline GH is  $p_1/2$ , and for patients transferred to the GH through the online platform, the expected treatment price is  $\varphi p_2/2$  for mild cases and  $(1 + \varphi)p_2/2$  for severe cases. The treatment price at a CH is  $p_3$ , that is, the treatment price for initial offline and transferred revisit patients at a CH is  $p_3$ . Additionally, we assume that the unit treatment cost for offline patients at the GH and online patients entering the GH offline are  $c_1$  and  $c_2$ , respectively, and the unit treatment cost for a CH is  $c_3$ . Therefore, the net utility function for initial offline patients at the GH is given by

$$U = v - \frac{p_1}{2} - dW_1(\lambda_1 + \lambda_2 - \varphi\lambda_2, \mu_1), \quad (3.1)$$

where  $W_1$  denotes the expected waiting time of patients who are treated at the GH. This gives us  $W_1(\lambda_1 + \lambda_2 - \varphi\lambda_2, \mu_1) = (\lambda_1 + \lambda_2 - \varphi\lambda_2)/\mu_1$ .

Because patients cannot seek treatment that results in negative net utility, patients will choose to visit the GH for treatment only if  $U \geq 0$  is satisfied. When  $U = 0$  is satisfied, we can obtain the arrival rate of initial patients at the GH in equilibrium, as shown in Proposition 1. See the Appendix for proofs of all propositions and lemmas.

**Proposition 1.** *Given the treatment threshold  $\varphi$  of the CH, the effective arrival rate of initial patients at the GH's offline channel is given by*

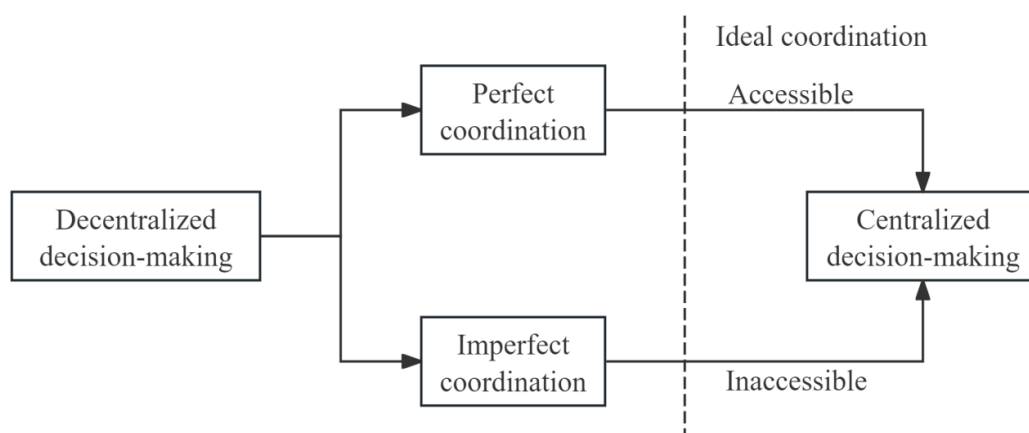
$$\lambda_{1e} = \frac{\mu_1(v - \frac{p_1}{2})}{d} - \lambda_2 + \varphi\lambda_2. \quad (3.2)$$

*Proposition 1 provides the equilibrium arrival rate for the GH's offline first-visit patients, where it is observed that the arrival rate for these patients is influenced by the GH's service capacity, patient perceived value, and waiting costs. As the GH's service capacity, patient perceived value, and waiting costs improve, the arrival rate for patients will increase.*

#### 4. Equilibrium analysis in competitive and noncompetitive systems

This section provides the equilibrium analysis for both competitive and noncompetitive systems. The analysis is presented in two parts, as summarized in Figure 4. The first part represents the centralized decision-making scenario under ideal coordination, with the optimal referral result from this scenario serving as the benchmark for the second part, which explores the decentralized decision-making framework. The second part further develops the analysis of two scenarios, perfect coordination and imperfect coordination, by comparing system performance under different decision modes.

##### 4.1. Noncompetitive referral coordination decision analysis



**Figure 4.** Decision classification.

Consider a referral coordination model between a GH and a CH, where online patients from the Internet diagnosis platform choose either the CH or the GH for further treatment based on their conditions. The GH prioritizes higher-revenue first-visit offline patients and refers mild cases to the CH, increasing the waiting cost for the CH. Therefore, the GH pays a referral service fee, denoted as  $m$ , to the CH. Here, we use  $\pi_1$  and  $\pi_2$  to represent the profits of the GH and CH, respectively. The objective functions for both hospitals are derived as follows:

$$\pi_1 = \left(\frac{p_1}{2} - \frac{c_1}{2}\right)\lambda_{1e} + \left(\frac{1+\varphi}{2}p_2 - \frac{1+\varphi}{2}c_2\right)(1-\varphi)\lambda_2 - m\varphi\lambda_2, \quad (4.1)$$

$$\pi_2 = (p_3 - c_3)\lambda_3 + (p_3 - c_3)\varphi\lambda_2 + m\varphi\lambda_2 - c_3^w(\lambda_3 + \varphi\lambda_2)W(\varphi), \quad (4.2)$$

where  $W(\varphi)$  denotes the expected waiting time of patients who are treated in CH. We then have  $W(\varphi) = (\lambda_3 + \varphi\lambda_2)/\mu_2$ .

For Equation 4.1, the first and second terms represent the GH's profits from offline first-visit patients and online patients, respectively, and the third term is the GH's referral cost for patients referred to the CH. For Equation 4.2, the first term is the CH's profit from treating its first-visit patients, the second term is the profit from treating online patients referred from the GH, and the third and fourth terms are, respectively, the referral benefits from the GH's and CH's total negative utility values based on patient waiting time.

#### 4.1.1. Systematic centralized decision analysis

In centralized decision-making, decision-makers prioritize optimizing the overall benefit of the healthcare system. The total profit of a noncompetitive referral system under the centralized decision-making model is derived as follows:

$$\pi = \left(\frac{p_1}{2} - \frac{c_1}{2}\right)\lambda_{1e} + \left(\frac{1+\varphi}{2}p_2 - \frac{1+\varphi}{2}c_2\right)(1-\varphi)\lambda_2 + (p_3 - c_3)\lambda_3 + (p_3 - c_3)\varphi\lambda_2 - c_3^w(\lambda_3 + \varphi\lambda_2)W(\varphi). \quad (4.3)$$

**Proposition 2.** *Under centralized decision-making, there exists an optimal treatment threshold that maximizes the total system profit, that is*

$$\varphi^c = \frac{\frac{1}{2}(p_1 - c_1) + (p_3 - c_3) - \frac{2c_3^w\lambda_3}{\mu_2}}{p_2 - c_2 + \frac{2c_3^w\lambda_2}{\mu_2}}. \quad (4.4)$$

Proposition 2 introduces the optimal treatment threshold  $\varphi^c$  within the framework of centralized decision-making, providing a theoretical basis for hierarchical medical care. Specifically, for patients with lower levels of medical complexity (below  $\varphi^c$ ), referral to the CH can save resources while reducing hospital operational costs. This not only alleviates the burden on the GH but also improves the efficiency of the CH. In contrast, patients with higher levels of medical complexity (above  $\varphi^c$ ) who require more advanced treatment should remain at the GH.

#### 4.1.2. Perfect coordination

Perfect coordination refers to achieving the optimal value of the objective function through service outsourcing mechanisms under the centralized decision-making model. The goal is to coordinate

referrals through payments, that is, the optimal treatment threshold is reached under the centralized decision-making model through the incentive of an outsourcing price, where  $\varphi^F = \varphi^c$  ( $\varphi^F$  represents the optimal treatment threshold under perfect coordination for the CH). The CH and GH negotiate to reach a consensus, adjusting outsourcing prices to maximize total system utility.

**Lemma 1.** *The utility functions of the CH and GH are strictly concave.*

*Based on Lemma 1, the optimal outsourcing price is given by maximizing the GH's profit.*

**Proposition 3.** *Given the system optimal treatment threshold  $\varphi^c$ , the optimal outsourcing price  $m^p$  of the GH is*

$$m^p = \frac{1}{2}(p_1 - c_1) - \varphi^c(p_2 - c_2). \quad (4.5)$$

Proposition 3 demonstrates that the GH can coordinate the healthcare system through optimal outsourcing payments, maximizing overall profit. The outsourcing price  $m^p$  serves as the key coordination mechanism between parties. By setting appropriate payment prices, the GH can optimize its profit while preventing inefficient resource utilization and revenue leakage from suboptimal referrals. This pricing mechanism incentivizes the CH to implement the optimal treatment threshold, aligning its interests with system-wide efficiency gains. The findings provide a theoretical framework for optimizing the allocation and coordination of healthcare resources.

#### 4.1.3. Imperfect coordination

Imperfect coordination occurs when the GH and CH make independent decisions to maximize their own profits without aiming for system optimization. In this scenario, local interest trade-offs and failures to fully consider overall system optimization may occur. The solution is based on backward derivation.

**Proposition 4.** *Given the outsourcing price  $m$ , the optimal treatment threshold for the CH under imperfect coordination is*

$$\varphi^{NP} = \frac{\mu_2(p_3 - c_3 + m)}{2c_3^w \lambda_2} - \frac{\lambda_3}{\lambda_2}. \quad (4.6)$$

Proposition 4 demonstrates how outsourcing prices influence the CH's treatment threshold. As the outsourcing price increases, the CH elevates its treatment threshold to accommodate more referrals, exemplifying a classic price-demand relationship. This in turn increases the arrival rate of patients who first visit the GH offline, achieving a win-win situation. Under imperfect coordination, the optimal treatment threshold increases with rising outsourcing prices. Based on Proposition 4, Proposition 5 provides the optimal outsourcing price of the GH under imperfect coordination.

**Proposition 5.** *Given the optimal response  $\varphi^{NP}$  of the CH and assuming that the waiting cost satisfies  $c_3^{w'} \leq c_3^w \leq c_3^{w''}$ , then the optimal outsourcing price of the GH is*

$$m^{NP} = \frac{A}{4c_3^w \lambda_2 + (p_2 - c_2)\mu_2}, \quad (4.7)$$

where

$$A = (p_1 - c_1)c_3^w \lambda_2 - 2(p_3 - c_3)c_3^w \lambda_2 + 2(p_2 - c_2)c_3^w \lambda_3 - (p_3 - c_3)(p_2 - c_2)\mu_2$$

$$\begin{aligned}
& + \frac{4c_3^w \lambda_2 \lambda_3}{\mu_2}, \\
c_3^{w'} &= \frac{[(p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 - 2(p_2 - c_2)\lambda_2]\mu_2}{4\lambda_2(\lambda_2 + \lambda_3)}, \\
c_3^{w''} &= \frac{[(p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 + 2(p_2 - c_2)(\lambda_3 - \lambda_2)]\mu_2}{4\lambda_2(2\lambda_2 - \lambda_3)}.
\end{aligned}$$

By Proposition 5, we can get  $\partial m^{NP} / \partial p_1 > 0$ , that is, the price of GH outsourcing increases with the unit price of treatment for its first-visit offline patients. This demonstrates that when the GH's offline first-visit fees increase, the GH tends to prioritize serving its own offline patients and therefore increases the outsourcing price to reduce the referral volume of online patients, thus reducing the pressure on its own resources.

#### 4.2. Competitive referral coordination model

Based on Section 3.1, the case is considered that  $n$  homogeneous CHs jointly receive referred patients from one online GH. In this case, if the two-level hospitals still aim at profit maximization, and  $\pi_1'$ ,  $\pi_{2i}$  denotes the profits of the GH and CH, respectively, then the objective function of the two-level hospitals is

$$\pi_1' = \left(\frac{p_1}{2} - \frac{c_1}{2}\right)\lambda_{1e} + \left(\frac{1+\varphi}{2}p_2 - \frac{1+\varphi}{2}c_2\right)(1-\varphi)\lambda_2 - \frac{1}{n} \sum_{i=1}^n m_i \varphi \lambda_2. \quad (4.8)$$

$$\pi_{2i} = (p_3 - c_3)\lambda_{3i} + (p_3 - c_3)\frac{\varphi\lambda_2}{n} + m_i\varphi\lambda_2 - c_3^w(\lambda_{3i} + \frac{1}{n}\varphi\lambda_2)W(\varphi). \quad (4.9)$$

For the GH, Equation 4.8 represents the total cost for referring patients to CHs; for CHs, Equation 4.9 describes the profit function for the  $i$ th CH. The first and second terms denote the revenue from treating its initial patients and referred patients from the online GH, respectively. The third and fourth terms represent the revenue from referral-outsourced payments from the GH and the total disutility based on the wait time of its patients, respectively.

##### 4.2.1. System centralized decision analysis

Taking the two-level hospitals as a whole, the total system utility objective function is given as

$$\begin{aligned}
\pi &= \left(\frac{p_1}{2} - \frac{c_1}{2}\right)\lambda_{1e} + \left(\frac{1+\varphi}{2}p_2 - \frac{1+\varphi}{2}c_2\right)(1-\varphi)\lambda_2 \\
&\quad - c_3^w(\lambda_3 + \varphi\lambda_2)W(\varphi) + (p_3 - c_3)\lambda_3 + (p_3 - c_3)\varphi\lambda_2.
\end{aligned} \quad (4.10)$$

**Proposition 6.** *The optimal treatment threshold for a competitive system under centralized decision-making is*

$$\varphi_D^C = \frac{\frac{1}{2}(p_1 - c_1) + (p_3 - c_3) - \frac{2c_3^w\lambda_3}{\mu_2}}{p_2 - c_2 + \frac{2c_3^w\lambda_2}{\mu_2}}. \quad (4.11)$$

Proposition 6 indicates that in a competitive situation, the system's optimal referral strategy is as follows: to maximize overall system utility, patients with a complexity of illness higher than  $\varphi_D^C$  should

be treated by the GH, whereas patients with a complexity of illness lower than  $\varphi_D^C$  should be referred to one of the lower-tier hospitals for treatment.

#### 4.2.2. Perfect coordination

Perfect coordination in a competitive model aims to maximize total utility in the system's centralized decision-making. This involves setting the CH's treatment threshold to the system's optimal threshold,  $\varphi_D^P = \varphi_D^C$ , and determining the corresponding parameters of the perfect coordination contract.

**Proposition 7.** *Given a system-optimal treatment threshold, the perfectly coordinated optimal outsourcing price for the GH needs to be satisfied for*

$$m^P = \frac{1}{2}(p_1 - c_1) - (p_2 - c_2)\varphi_D^C. \quad (4.12)$$

Proposition 7 provides the perfectly coordinated contract price, indicating that to achieve the system's optimal utility, the outsourcing mechanism needs to align with the GH outsourcing price specified in Equation 4.12.

#### 4.2.3. Imperfect coordination

Imperfect coordination does not aim for system optimality. Instead, each of the two-level hospitals makes decisions to maximize their respective utility. Equilibrium strategies for the GH and CH are derived using backward derivation.

**Proposition 8.** *Given the outsourcing price, the optimal treatment threshold for a CH under imperfect coordination is*

$$\varphi_D^{NP} = \frac{\mu_2(p_3 - c_3 + m)n}{2c_3^w\lambda_2} - \frac{n\lambda_3}{\lambda_2}. \quad (4.13)$$

**Proposition 9.** *Given that a CH makes the optimal response  $\varphi_D^{NP}$ , and assuming the waiting cost satisfies  $\underline{c}_3^w \leq c_3^w \leq \overline{c}_3^w$ , the optimal outsourcing price for the GH is*

$$m_i = \frac{\frac{1}{2}(p_1 - c_1) - \frac{1}{n}(p_3 - c_3) + \frac{(p_2 - c_2)n\lambda_3}{\lambda_2} - \frac{(p_3 - c_3)(p_2 - c_2)\mu_2 n}{2c_3^w\lambda_2} + \frac{2c_3^w\lambda_3}{n\mu_2}}{\frac{n+1}{n} + \frac{n(p_2 - c_2)\mu_2}{2c_3^w\lambda_2}}, \quad (4.14)$$

where  $\underline{c}_3^w = \left( \left( \frac{1}{2}(p_1 - c_1) + (p_3 - c_3) - (p_2 - c_2) \right) n^2 \mu_2 \right) / \left( 2 \left( n^2 \lambda_3 + (n + 1) \lambda_2 \right) \right)$ ,  
 $\overline{c}_3^w = \left( ((p_1 - c_1) + 2(p_3 - c_3)) \mu_2 \right) / (4\lambda_3)$ .

Propositions 8 and 9 provide equilibrium solutions for the two-tier hospital game under dynamic decision-making. Each hospital tier focuses solely on maximizing its utility. Given the GH's outsourcing price, each CH will optimally respond by determining its treatment threshold.

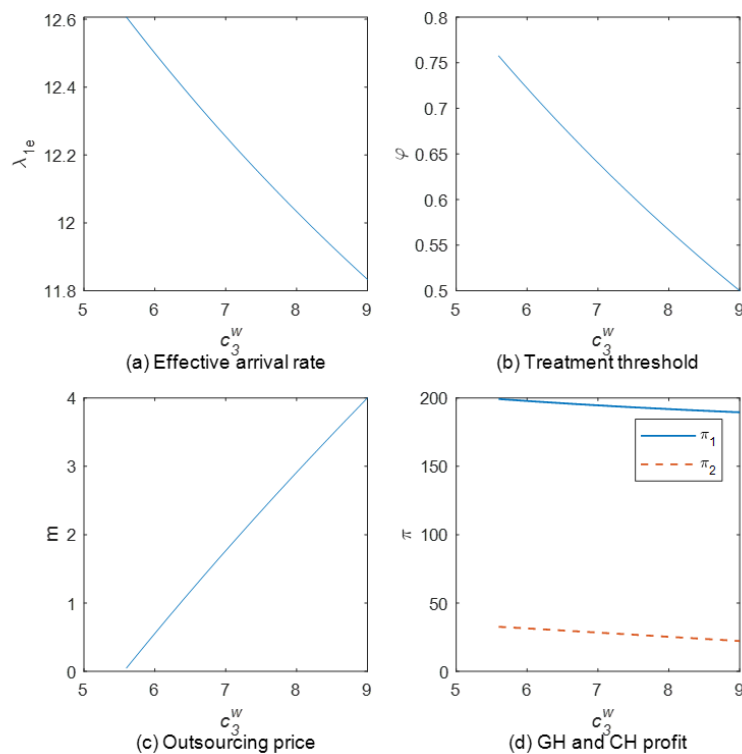
## 5. Numerical analysis

The numerical experiments comprise two parts: First, the influence of some parameters on the system's equilibrium decisions (GH offline first-visit patient arrival rate  $\lambda_{1e}$ , CH treatment threshold  $\varphi^*$ ,

outsourcing price  $m^*$ ) and equilibrium performances (GH profit  $\pi_1$ , CH profit  $\pi_2$ ) are analyzed. Second, the coordination of the outsourcing mechanism is validated, and the impact of competition on the referral coordination efficiency of the system is analyzed. In the equilibrium analysis section, we discuss the impacts of CH unit waiting cost  $c_3^w$ , service capability  $\mu_2$ , and GH online patient treatment price  $p_2$  on equilibrium decisions and performances. Without loss of generality, we set parameters as follows:  $p_1 = 40, p_2 = 24, p_3 = 15, v = 30, \mu_1 = 20, d = 15, c_1 = 10, c_2 = 8, c_3 = 6, \lambda_2 = 3, \lambda_3 = 5, \mu_2 = 9$ .

For the coordination analysis, we first explore how the optimal treatment threshold and total profit of the system vary under NS and CS, each with different unit waiting costs. Then, the coordination of the service outsourcing mechanism in CS and NS and the existence of perfectly coordinated equilibrium solutions in CS and NS are verified. Finally, based on the analysis of imperfect coordination under NS and CS, we investigate how to enhance coordination efficiency and provide some managerial insights.

### 5.1. Equilibrium analysis

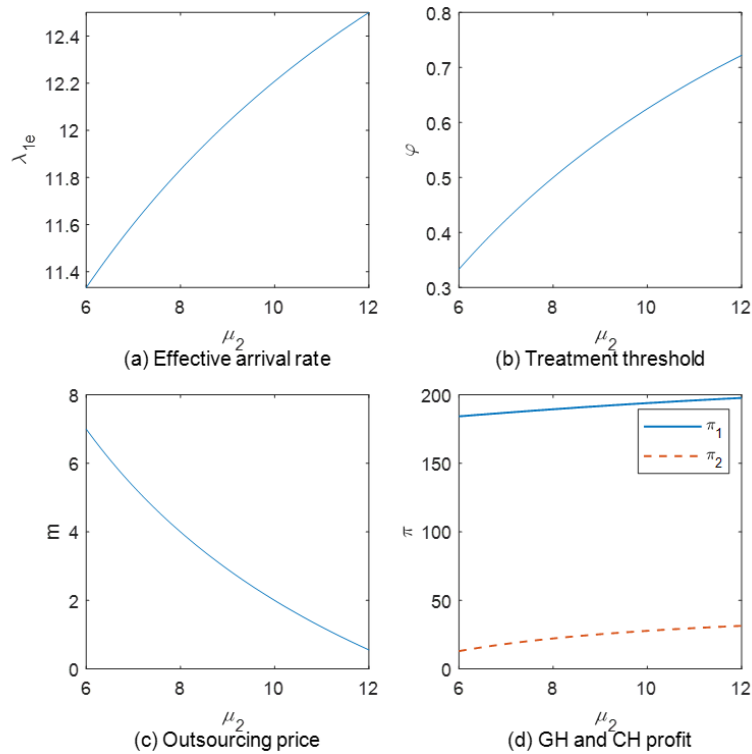


**Figure 5.** Effect of  $c_3^w$  on the equilibrium outcome.

*Note.* The parameters are  $p_1 = 40, p_2 = 24, p_3 = 15, v = 30, \mu_1 = 20, d = 15, c_1 = 10, c_2 = 8, c_3 = 6, \lambda_2 = 3, \lambda_3 = 5, \mu_2 = 9$ .

Figure 5 illustrates the effect of a CH's unit time waiting cost on the equilibrium outcome. Even though the GH incentivizes a CH to accept more online patients by increasing the outsourcing price, as  $c_3^w$  increases, the CH receives fewer online patients, and thus fewer first-visit offline patients enter the GH. This is because the effect of an increase in the unit waiting cost on lowering the treatment threshold

is greater than the effect of an increase in the outsourcing price on increasing the treatment threshold. Thus, the treatment threshold at the CH decreases as  $c_3^w$  increases, and consequently profits for two-level hospitals decrease as  $c_3^w$  increases.

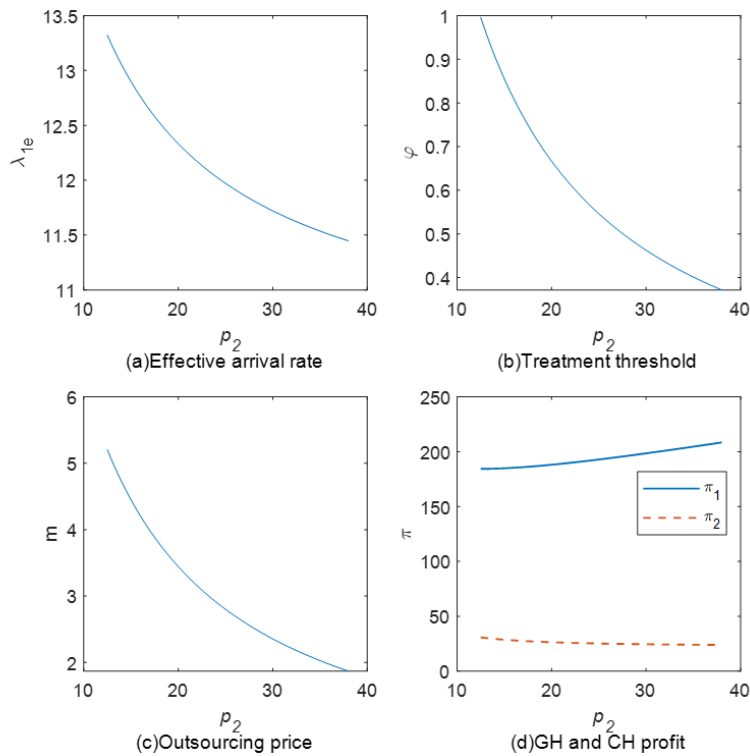


**Figure 6.** Effect of  $\mu_2$  on the equilibrium outcome.

*Note.* The parameters are  $p_1 = 40, p_2 = 24, p_3 = 15, v = 30, \mu_1 = 20, d = 15, c_1 = 10, c_2 = 8, c_3 = 6, \lambda_2 = 3, \lambda_3 = 5, c_3^w = 8$ .

Figure 6 illustrates the effect of CH service capacity on the equilibrium outcome. As  $\mu_2$  increases, the outsourcing price of the GH decreases, while the treatment threshold of the CH increases, and thus the number of initial offline patients of the GH increases. Then, the profits of the CH and GH both increase. This is because the thresholds for CH treatment are influenced by their service capacity and the price of outsourcing. Specifically, the increase in the treatment threshold due to the increased service capacity is more than the decrease in the treatment threshold due to the decrease of outsourcing price.

Figure 7 illustrates the impact of the treatment price for online patients at the GH on the equilibrium outcomes. With the increase in the treatment price for online patients at the GH, that is, as the price gap between first-visit and online patients' treatment decreases, the GH will decrease the outsourcing price to encourage the CH to reduce the treatment threshold. Consequently, the number of first-visit patients at the GH will decrease, thereby leading to a decrease in the CH's profit and an increase in the GH's profit.



**Figure 7.** Effect of  $p_2$  on the equilibrium outcome.

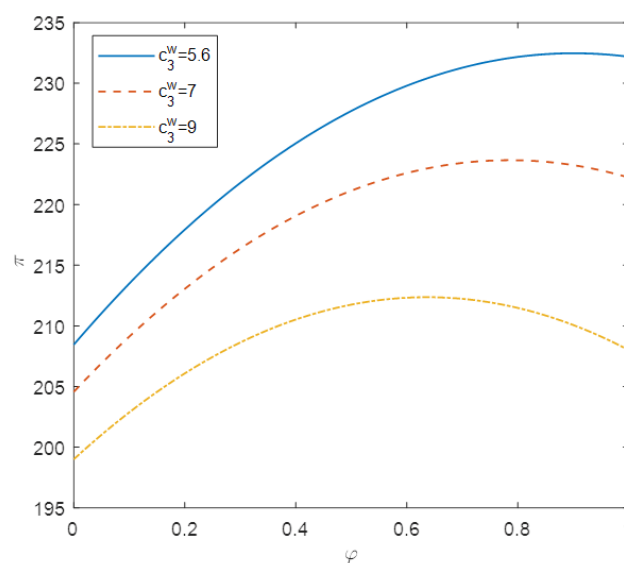
*Note.* The parameters are  $p_1 = 40$ ,  $p_3 = 15$ ,  $v = 30$ ,  $\mu_1 = 20$ ,  $d = 15$ ,  $c_1 = 10$ ,  $c_2 = 8$ ,  $c_3 = 6$ ,  $c_3^w = 8$ ,  $\lambda_2 = 3$ ,  $\lambda_3 = 5$ ,  $\mu_2 = 9$ .

Based on the above analysis, the following managerial insights can be provided: First, referral collaboration willingness is jointly constrained by the CH costs and service capacity, and there is an “offsetting effect” between service capacity and waiting costs. Specifically, a CH’s willingness to accept referrals of online patients is strongly associated with the per-unit-time waiting cost ( $c_3^w$ ) and service capacity ( $\mu_2$ ). When  $c_3^w$  decreases or  $\mu_2$  increases, the time pressure and capacity constraints faced by the CH in receiving referrals are alleviated, thereby strengthening its willingness to collaborate. In such cases, the GH can incentivize the CH to accept more online patients by offering lower outsourcing prices, thereby enhancing its efficiency. Conversely, when the treatment price for online patients at the GH is high, the GH should decrease the outsourcing price to reduce the number of online patients referred to CH, thus maximizing GH profits. Second, a higher unit waiting time cost for the CH is detrimental to two-level hospitals, resulting in lower profits for both institutions. Meanwhile, although an increase in CH service capacity ( $\mu_2$ ) may raise the outsourcing price charged by the GH and increase the treatment threshold for the CH, the additional profit generated by capacity expansion can offset the loss caused by higher waiting costs. As a result, the total system profit increases with  $\mu_2$ , though the growth rate slows once  $C_3^w > 5.6$ . These findings imply that GHs should prioritize selecting CHs characterized by low waiting costs and high service capacity, and provide targeted incentives and foster collaboration. CHs should enhance their bargaining position through process optimization or resource expansion, while remaining vigilant about the marginal deterioration risk of waiting costs. When  $c_3^w$

exceeds a critical threshold, a CH should control waiting time through process redesign (implementing flexible scheduling systems).

## 5.2. Coordination analysis

### 5.2.1. Centralized decision analysis



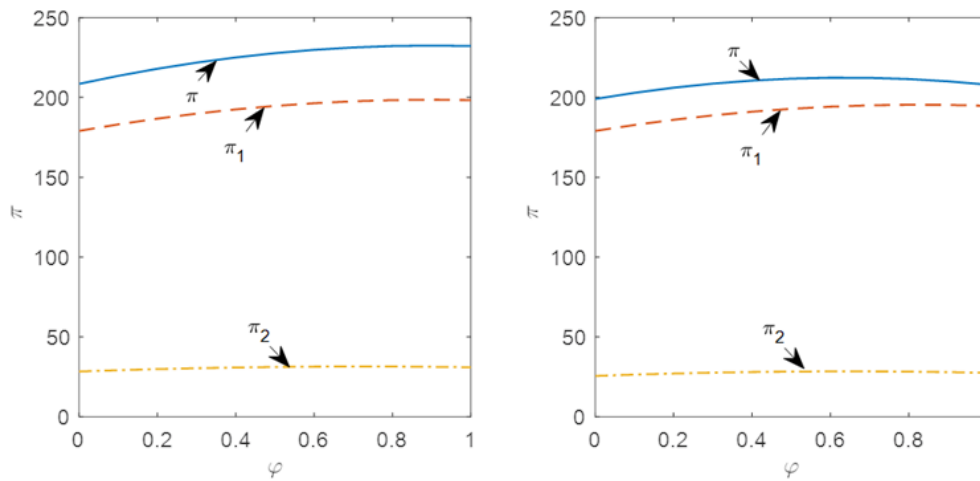
**Figure 8.** Total benefit function with different waiting cost.

*Note.* The parameters are  $p_1 = 40, p_2 = 24, p_3 = 15, v = 30, \mu_1 = 20, d = 15, c_1 = 10, c_2 = 8, c_3 = 6, \lambda_2 = 3, \lambda_3 = 5, \mu_2 = 9$ .

Figure 8 illustrates the change of the total profit function  $\pi$  concerning  $\varphi$  under different waiting costs. To ensure  $\varphi$  for  $[0, 1]$ ,  $c_3^w$  must be over 5.6, so  $c_3^w$  is set to start from 5.6. The system's total profit is maximized when  $c_3^w = 5.6$ . An increase in unit waiting time cost reduces the CH's optimal treatment threshold so that more GH online patients would enter its offline treatment. This is disadvantageous for receiving more offline initial patients. Additionally, higher waiting costs reduce the CH's efficiency and result in a decrease in the system's total profit.

Based on this analysis, the following managerial insights can be obtained: From the perspective of enhancing revenue, in NS and CS, increasing the service capacity of a CH can reduce the overall waiting cost, thereby accommodating more referred online patients and enhancing the profitability of the entire healthcare system.

### 5.2.2. Noncompetitive model coordination analysis



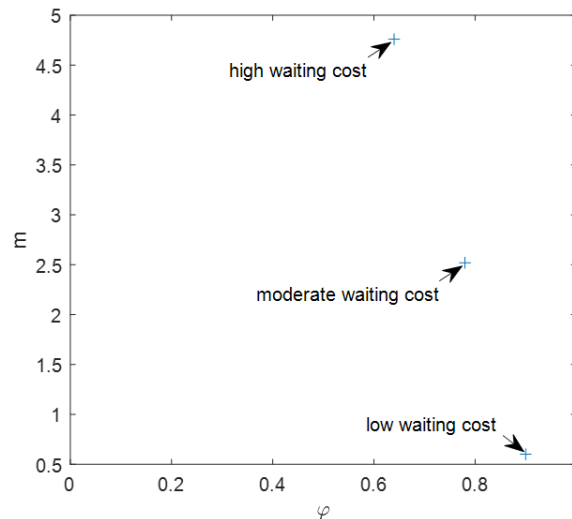
**Figure 9.** Coordination analysis of the noncompetitive outsourcing mechanism.

*Note.* The parameters are  $p_1 = 40, p_2 = 24, p_3 = 15, v = 30, \mu_1 = 20, d = 15, c_1 = 10, c_2 = 8, c_3 = 6, c_3^w = 8, \lambda_2 = 3, \lambda_3 = 5, \mu_2 = 9$ .

As Proposition 3 indicates, when the GH offers an outsourcing price  $m$ , the outsourcing contract can achieve perfect coordination. Figure 9 illustrates three different profits under scenarios  $c_3^w = 5.6$  and  $c_3^w = 9$  concerning the treatment threshold, including total profit, profit for the GH under the outsourcing contract, and profit for the CH. We can see that the profit functions of the two hospitals are both concave functions concerning the treatment threshold, and coordination is achieved when the optimal referral threshold maximizes the profits of the two hospitals.

Figure 10 demonstrates the options available for  $(\varphi, m)$  under the coordination scenario. In Figure 10, when coordination occurs, the available choices for the outsourcing contract are under low waiting cost ( $c_3^w = 5.6$ ), moderate waiting cost ( $c_3^w = 7$ ), and high waiting cost ( $c_3^w = 9$ ). From Figure 10, we observe that as patient waiting costs at the CH increase, the CH's referral threshold gradually decreases, whereas the GH's outsourcing price increases. This happens because as the CH's waiting costs increase, the operational cost of receiving referred patients rises. If the CH maintains its original referral threshold, the increase in waiting costs will reduce the CH's profits. Therefore, the CH's rational decision is to lower the referral threshold in order to maximize its profits. The reduction in the CH's referral threshold leads to a decrease in the number of patients referred to the CH by the GH, which will crowd out the GH's own service resources. This also implies that the CH's willingness to accept referrals decreases. To address this issue, the GH needs to incentivize the CH by raising the outsourcing price. Only when the outsourcing price reaches an adequate level will the CH be willing to continue accepting a certain proportion of referred patients despite the increase in waiting costs, ultimately balancing its operational costs with profit goals. Additionally, we have summarized some managerial insights based on these results. Internal service pricing by medical institutions directly affects their external cooperation strategies. When the profit from internal services is higher, institutions will rationally shift resources toward internal services by increasing outsourcing prices to maximize profit. A CH, as the party receiving referrals, needs to pay attention to the GH's internal price changes. If the GH increases

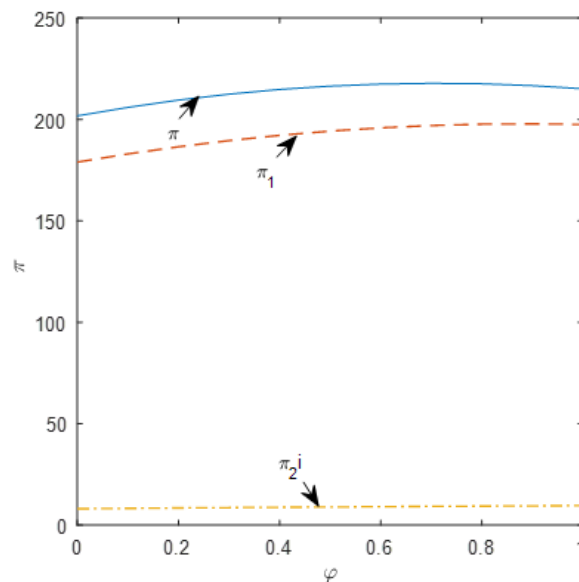
its offline first-visit prices, the CH should anticipate an increase in its outsourcing price and adjust its referral acceptance strategy in advance to ensure the maximization of cooperative benefits.



**Figure 10.** Perfectly coordinated solutions with different misdiagnosis rates for NS.

*Note.* The parameters are  $p_1 = 40, p_2 = 24, p_3 = 15, v = 30, \mu_1 = 20, d = 15, c_1 = 10, c_2 = 8, c_3 = 6, \lambda_2 = 3, \lambda_3 = 5, \mu_2 = 9$ .

### 5.2.3. Competition model coordination analysis



**Figure 11.** Coordination analysis of competitive outsourcing mechanism.

*Note.* The parameters are  $p_1 = 40, p_2 = 24, p_3 = 15, v = 30, \mu_1 = 20, d = 15, c_1 = 10, c_2 = 8, c_3 = 6, c_3^w = 8, \lambda_2 = 3, \lambda_3 = 5, \mu_2 = 9$ .

Similar to Figure 9, Figure 11 shows the coordination analysis of competitive outsourcing mechanism when ( $c_3^w = 8$ ). The profit functions of two-level hospitals are concave functions concerning the treatment threshold  $\varphi$ . Therefore, in the competitive scenario, coordination can still be achieved through outsourcing mechanisms, maximizing the profits of two-level hospitals. Additionally, the total profit functions in both systems exhibit similar changes concerning the treatment threshold. According to Propositions 2 and 7, the perfect coordination solutions are the same in both systems.

**Table 3.** The theoretical equilibrium results of centralized and decentralized decision-making.

| Decision-making mechanism     | Referral system | Coordination scenarios | Equilibrium results   |
|-------------------------------|-----------------|------------------------|---|
| Centralized decision-making   |                 |                        | $\varphi^c = \varphi^P = \varphi_D^C = \frac{\frac{1}{2}(p_1 - c_1) + (p_3 - c_3) \frac{2c_3^w \lambda_3}{\mu_2}}{p_2 - c_2 + \frac{2c_3^w \lambda_3}{\mu_2}} \sqrt{a^2 + b^2}$ |
| Decentralized decision-making | Noncompetitive  | Perfect coordination   | $\varphi^c = \varphi^P$ $m^P = \frac{1}{2}(p_1 - c_1) - \varphi^c(p_2 - c_2)$   |
|                               |                 | Imperfect coordination | $\varphi^{NP} = \frac{\mu_2(p_3 - c_3 + m)}{2c_3^w \lambda_2} - \frac{\lambda_3}{\lambda_2}$ $m^{NP} = \frac{A}{4c_3^w \lambda_2 + (p_2 - c_2)\mu_2}$                           |
|                               | Competitive     | Perfect coordination   | $\varphi_D^P = \varphi_D^C$ $m^P = \frac{1}{2}(p_1 - c_1) - (p_2 - c_2)\varphi_D^C$   |
|                               |                 | Imperfect coordination | $\varphi_D^{NP} = \frac{\mu_2(p_3 - c_3 + m)n}{2c_3^w \lambda_2} - \frac{n\lambda_3}{\lambda_2}$ $m_i = \frac{B}{\frac{n+1}{n} + \frac{n(p_2 - c_2)\mu_2}{2c_3^w \lambda_2}}$   |

**Table 4.** System coordination analysis.

|   | $\varphi$ | $m$    | $\pi_1$ | $\pi_2$ | $\pi$   | $e$   |
|---|-----------|--------|---------|---------|---------|-------|
| Centralized decision-making             | 0.708     | \      | \       | \       | 217.833 | 1     |
| Perfect coordination                    | 0.708     | 3.672  | 191.03  | 26.803  | 217.833 | 1     |
| Imperfect coordination (Noncompetitive) | 0.567     | 2.911  | 191.848 | 25.346  | 217.194 | 0.997 |
| Imperfect coordination (competitive)    | 0.8745    | 0.8217 | 197.842 | 19.102  | 216.944 | 0.996 |

Note. \ indicates that there is no relevant data to be filled in that cell.

The following analysis examines coordination under noncooperative games. The theoretical equilibrium results of centralized and decentralized decision-making are summarized in Table 3 for a better understanding of the numerical analysis results. System coordination efficiency is defined as  $e = (\pi_1 + \pi_2)/\pi$ , as seen in [38]. The results of system coordination analysis is shown in Table 4.

From Table 4, it can be seen that both the outsourcing price and coordination efficiency under imperfect coordination are lower than perfect coordination; by comparing the imperfect coordination of the outsourcing contract under NS and CS, it can be found that the coordination efficiency of the CS is slightly lower than that of the NS, the outsourcing price is significantly lower than that of the NS, and the treatment threshold and profit of GHs are significantly higher than those of the NS. Thus, for GHs, the introduction of competition not only helps to suppress the outsourcing price but also helps to increase the treatment incentives of CHs so that more online patients can be transferred to CHs and thereby improve the profit of the GH.

Competition reshapes system efficiency. Compared with the noncompetition mode, centralized decision-making and competitive outsourcing mechanisms (CS) can improve system performance through price suppression and stronger incentives. Under the CS mode, the system’s total profit shows greater tolerance to  $c_3^w$ , and both the outsourcing price and GH profit outperform those under the NS mode. Although the competitive mechanism slightly reduces overall coordination efficiency, it

can significantly lower the outsourcing price, increase the GH's treatment threshold and profit, and simultaneously incentivize CHs to accept more online-referred patients. These results indicate that a competition mechanism can effectively optimize the allocation of referral resources and energize the operation of a two-tier healthcare system. These insights suggest that GHs could introduce a "multi-CH bidding" mechanism to select high-efficiency partners.

## 6. Conclusion

This study investigates how competition affects system coordination in hierarchical healthcare with Internet diagnosis platforms under centralized and outsourcing mechanisms. Under centralized decision-making, we determine optimal treatment thresholds and referral strategies that maximize system-wide utility. For the outsourcing mechanism, we analyze optimal contract design, enabling the GH to achieve system-optimal utility while incentivizing CHs to adopt optimal treatment thresholds. Numerical results demonstrate that outsourcing mechanisms achieve perfect coordination in both CS and NS. Although the NS shows marginally higher coordination efficiency, the CS yields significantly lower outsourcing prices alongside substantially higher treatment thresholds and GH profits. From the GH's perspective, competition suppresses outsourcing costs while encouraging CHs to treat more online patients, thereby increasing GH profitability. However, it is also important to remain vigilant: Although competition can enhance coordination efficiency within the healthcare system, it may also induce the potential risk of medical resource concentration, which constitutes a core practical challenge in the real-world operation of healthcare systems. Driven by the marginal benefit advantage (i.e., higher profitability for GHs in treating complex diseases), high-quality medical resources may further agglomerate toward GHs. This makes it difficult for CHs to get high-quality medical resource support, directly constraining the improvement of CHs' service capacity.

This study does not model patient choice behavior in the referral process. Future research could incorporate online patient choice behavior and heterogeneous time-sensitivity preferences. Additionally, alternative coordination mechanisms, such as cost-sharing, outcome-based penalties, and contract design under information asymmetry, can be analyzed in the future.

## Author contributions

Jingyang Wang: Methodology, Supervision, Conceptualization; Xiaoyu Wang: Writing—original draft, Software, Formal analysis, Validation; Miao Yu: Writing—review and editing, Validation, Funding acquisition; Dandan Yu: Investigation, Data curation.

## Use of Generative-AI tools declaration

The authors declare that they did not utilize any artificial intelligence (AI) tools in the creation of this article.

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### Conflict of interest

The authors declare that they have no conflicts of interest.

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