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Research article

Green insurance coordination for corporate low-carbon transition: A multi-stage differential game

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Abstract: In this study, we developed a multi-stage differential game to examine how a firm and an insurer coordinate a corporate low-carbon transition. Carbon intensity, green reputation, and the cost of carbon allowance-backed financing were modeled as state variables. The transition comprised three stages: a Nash game with independent decisions, a Stackelberg game in which the insurer offers premium incentives and shares abatement costs, and a cooperative game with joint optimization. Feedback equilibrium strategies were derived from the corresponding Hamilton–Jacobi–Bellman equations, and their economic properties were compared analytically. Numerical experiments and Monte Carlo tests evaluated the effects of carbon prices, regulatory penalties, and cost-sharing rates. The results showed that cost sharing increases the firm's abatement effort and can yield a Pareto improvement when the insurer's marginal return is sufficiently high. Joint optimization produces the lowest steady-state carbon intensity and the highest total profit. Higher carbon prices strengthen the incentive to abate but may reduce total profit, whereas regulatory penalties can be transmitted through premium discounts. The cooperative regime is also the most robust to parameter perturbations. These findings identify the conditions under which green insurance can evolve from risk transfer to an intertemporal coordination mechanism for corporate decarbonization.

Keywords: low-carbon transition; green insurance; differential game; cost sharing; carbon finance
Mathematics Subject Classification: 91A23, 91A25, 91B30

1. Introduction

Corporate low-carbon transition is a long-horizon investment problem characterized by large upfront costs, uncertain technology, and evolving regulatory constraints. Firms must jointly manage abatement expenditure, carbon-market exposure, financing conditions, and the reputational consequences of environmental performance. Evidence shows that green innovation, transition finance, and environmental regulation materially affect corporate decarbonization [1–3]. These interactions make a static or single-period formulation inadequate for decisions whose costs and benefits accumulate over time.

The theoretical foundation of this study draws upon classical game theory. The concept of Nash equilibrium, introduced by Nash, provides the fundamental solution concept for non-cooperative games [4,5]. The Shapley value, proposed by Shapley, offers a normative solution for cooperative games with transferable utility [6]. Aumann further established the relationship between backward induction and common knowledge of rationality in dynamic games [7]. Building upon these foundational works, this paper employs differential game theory to analyze the dynamic strategic interactions between firms and insurers in the context of low-carbon transition.

Green financial instruments can alleviate capital constraints, but their incentive effects differ. Green credit, green bonds, and carbon finance primarily affect the availability and price of funds [4–14]. Green insurance additionally combines risk transfer, monitoring, and pricing. Premium differentiation converts observed carbon performance into an immediate financial signal, while abatement-cost sharing can align the insurer's long-term underwriting interest with the firm's transition effort. Empirical studies associate liability insurance with carbon performance, green innovation, and enterprise transformation [15–18], yet the underlying dynamic contract remains underexplored.

The practical significance of green insurance in facilitating corporate decarbonization is increasingly evident in global markets. In Europe, Zurich Insurance Group has committed to achieving net-zero emissions across its entire business by 2050, with an accelerated target of 2030 for its own operations, aiming to reduce emissions by 70% through carbon removal technologies [19]. In the United States, Mapfre USA reduced its carbon footprint by 6,172 tCO_2e in 2025 compared to its 2022 baseline, while Allianz Trade cut greenhouse gas emissions per employee by 65% in 2025 and expanded sustainability-linked trade credit insurance across 16 countries. These international practices demonstrate that green insurance is not merely a Chinese policy initiative but a global market-driven mechanism for corporate decarbonization. International studies have further examined the theoretical underpinnings of green insurance mechanisms. Colivicchi and Iannucci analyzed insurance coverage and environmental risk in an evolutionary oligopoly setting, showing that insurance can support an environmentally-friendly transition by covering endogenous climate change losses when firms choose output and coverage in a two-stage game [20]. Zhao and Cheng (2024) developed a multiagent collaborative development strategy for environmental pollution liability insurance based on evolutionary game theory, constructing a tripartite evolutionary game model involving the government, insurance companies, and polluting enterprises [21]. These studies demonstrate that the interaction between green insurance and corporate environmental behavior is a global phenomenon extending beyond the Chinese context.

We consider a firm–insurer system in which the firm chooses its abatement investment and the insurer adjusts the premium and, when appropriate, shares abatement costs. Their decisions affect three coupled states: carbon intensity, green reputation, and the cost of carbon allowance-backed financing.

Because information and cooperative capacity accumulate during transition, the governance structure evolves from independent decisions to a leader–follower contract and finally to joint optimization.

The Journal of Industrial and Management Optimization has published several contributions closely related to this study. Zhang and Yu analyzed joint emission reduction dynamic optimization in a supply chain considering fairness concern and reference low-carbon effect, demonstrating that bilateral cost-sharing contracts can achieve Pareto improvement for manufacturers and retailers [22]. Jiang and Zhen examined carbon abatement and recycling decisions in closed-loop supply chains with reward-penalty mechanisms [23]. Li and Zhang employed a differential game approach to examine carbon capture and storage and infrastructure innovation strategies of two countries in the context of transboundary pollution [24]. These studies collectively enrich the theoretical toolkit for low-carbon collaborative decision-making using differential game methods.

This paper makes three contributions: First, unlike researchers who treat financial institutions as exogenous parameters or passive intermediaries [25,26], we explicitly model the insurer as a strategic decision-maker with an independent objective function, capturing its active role in shaping corporate transition pathways through premium adjustments and cost-sharing. Second, while prior differential game models in green finance have typically employed a single static game structure, we introduce a multi-stage framework that endogenously captures the evolution of governance structures from non-cooperative (Nash) to Stackelberg and to cooperative regimes. Third, our model integrates three state variables, carbon intensity, green reputation, and financing costs, within a unified dynamic system, thereby revealing the transmission channels through which green insurance affects corporate emission reduction, a dimension largely overlooked in the literature. These contributions distinguish our framework from previous differential game models that have focused on supply chain coordination without incorporating financial institutions, employed single-stage game structures without considering governance evolution, or examined state variables in isolation without capturing their interactive dynamics [21,27,28].

While researchers have extensively examined supply chain coordination, carbon trading mechanisms, and green finance policy effects, three critical gaps remain. First, financial institutions, particularly insurers, are predominantly treated as exogenous parameters or passive intermediaries rather than strategic decision-makers with independent objectives. Second, most differential game models in this domain employ a single-stage game structure, failing to capture the evolutionary nature of firm-insurer relationships as cooperation becomes mutually beneficial over time. Third, while the transmission channels through which green insurance affects corporate emission reduction via the interplay of carbon intensity have been studied, green reputation and financing costs remain largely unexplored. A detailed review is provided in Section 2.

2. Related work

2.1. Dynamic decision-making for corporate low-carbon transition

Researchers increasingly treat corporate decarbonization as a path-dependent transition rather than a one-period abatement choice. Studies outside China show that AI-enabled green strategy and net-zero capabilities connect process innovation, operational change, and environmental performance [1,29]. Evidence from China's pilot programs, green-finance system, and transition-finance market likewise shows that institutional learning, financing conditions, and disclosure quality shape the feasible pace of transformation [3,30–32]. Managerial environmental cognition and institutional change further explain why firms exposed to similar policies may adopt different innovation strategies [33–35].

The international sustainable-finance literature provides a useful counterweight to country-specific policy evidence. Corporate green bonds can generate credible environmental improvements and favorable financing outcomes [36,37], while institutional investors increasingly incorporate climate risk into portfolio management and engagement [38]. Taken together, these studies show that technology, managerial capability, capital-market discipline, and public policy jointly determine transition trajectories.

However, this literature predominantly estimates average treatment effects or compares fixed policy regimes. It offers limited guidance on how a firm's governance relationship with a financial partner should evolve as carbon intensity declines, green reputation accumulates, and transition risk becomes more observable.

2.2. Green finance and insurance incentive mechanisms

Green finance affects abatement by relaxing capital constraints and repricing environmental performance. Chinese evidence links green-finance demand, green bonds, and green-credit development to corporate green innovation and emission reduction [39–42]. Interactions between green credit and carbon-emissions trading can strengthen these effects [43], while financing structure and green supply-chain management influence whether carbon finance translates into environmental innovation [44]. These results establish financing cost as an endogenous state that can respond to policy and firm performance.

Insurance adds risk transfer, monitoring, and contract-based incentives. A global review shows that insurers respond to climate change through underwriting, pricing, risk engineering, and investment decisions [45]. Firm-level evidence from China further associates directors' and officers' liability insurance with carbon emissions and green innovation [46,47]. Environmental-liability insurance can also promote green innovation and enterprise transformation, although its effect depends on incentive alignment, verification, and implementation quality [48,49].

The two geographic strands are complementary. International work clarifies the broad risk-management role of insurance, whereas Chinese empirical studies provide detailed evidence from rapidly developing green-finance and environmental-liability-insurance systems. Yet, insurers are usually represented as passive policy carriers or governance attributes, not as strategic actors that dynamically select premiums and abatement-cost-sharing rates.

2.3. Differential game models in low-carbon systems

The staged governance structure builds on established game theory. Nash formalized equilibrium under independent decision-making [4,5], and Aumann linked sequential rationality to common knowledge and backward induction [7]. Shapley's value provides a foundational allocation rule for cooperative surplus [6]. Dynamic noncooperative games and differential games extend these ideas to intertemporal state-dependent decisions [15,16].

Environmental applications demonstrate why governance structure matters over time: International-pollution models compare cooperative and noncooperative control [17], analyze time-consistent transfers [18], and formulate pollution control as a multistage supergame [50]. Cooperative stochastic differential games further provide subgame-consistent mechanisms when uncertainty unfolds over time [51]. These studies supply the theoretical basis for moving from independent decisions to contractual leadership and ultimately to cooperation.

China-based research applies differential games directly to low-carbon operations. Models examine government green-funding and technology contracts [52], dynamic supply-chain abatement [53], robust decisions under uncertainty [54], and cap-and-trade coordination [55]. Related work studies altruistic preferences and subsidies in closed-loop supply chains [56], stochastic abatement under corporate social responsibility [57], and governance choices under different power structures [58]. This stream is closely aligned with the operational setting of this paper, but insurance pricing and carbon-financing costs are rarely modeled jointly.

Broader international optimization research suggests several extensions. Fuzzy cooperative games can encourage inter-firm coalitions [59], while regime-switching and jump-diffusion games capture stochastic investment environments [60,61]. Robust control addresses model misspecification [62]. Semi-infinite and robust optimization provide tools for enforcing constraints over continuous uncertainty sets and for studying feasible-set stability [63–65]. The present deterministic model retains closed-form transparency, but these methods provide a rigorous path toward stochastic, multistage, and distributionally robust contracts.

Beyond differential game approaches, multi-criteria decision analysis has also been applied to green supply chain management. Rukhsar et al. [66] developed an intelligent decision analysis framework for green supplier selection using circular intuitionistic fuzzy information aggregation and Frank triangular norms, demonstrating the growing integration of fuzzy decision theories with green operations management. While methodologically distinct from our differential game approach, this line of research underscores the broader trend toward sophisticated decision frameworks for environmental management.

To broaden the international perspective, we also draw on empirical evidence from multiple regions. Liu, Anderson, and Cruz [28] examined consumer environmental awareness and competition in two-stage supply chains, providing foundational insights applicable across developed economies. Colivicchi and Iannucci [20] studied insurance coverage in an evolutionary oligopoly with a European focus. These studies complement the predominantly Chinese empirical evidence and demonstrate that the interaction between green finance and corporate behavior is a global phenomenon.

These studies collectively enrich the theoretical toolkit for low-carbon collaborative decision-making; however, they predominantly focus on supply chain participants and have yet to incorporate financial institutions into systemic game-theoretic frameworks. This observation leads to the identification of specific research gaps, which we articulate in the following subsection.

2.4. Research gaps and contributions

The preceding literature reveals a common limitation across the three research streams. Studies on corporate low-carbon transition explain how regulation, green innovation, and financing conditions affect firm behavior, but they generally treat the governance arrangement as fixed. Research on green finance and insurance demonstrates that financial constraints, risk transfer, and environmental signaling matter for transition outcomes, yet insurers are usually modeled as passive policy instruments rather than strategic decision makers. Differential-game models capture intertemporal interactions among governments and supply-chain members, but they rarely incorporate insurance pricing and carbon financing or enable the governance structure to evolve over the transition path. Consequently, the literature does not provide an integrated explanation of how a firm and an insurer should adjust their relationship as carbon performance, green reputation, and financing costs change over time.

Table 1. Comparison of representative research streams.

Research stream	Dynamic	Insurance	Financing	Multi-stage	Differential game
Green finance studies	Limited	No	Yes	No	No
Green insurance studies	Limited	Yes	Limited	No	Rare
Low-carbon supply-chain studies	Yes	No	Limited	Rare	Yes
Government-enterprise studies	Yes	No	No	Rare	Yes
This paper	Yes	Yes	Yes	Yes	Yes

In this paper, we address this limitation by constructing a multi-stage differential game between a transitioning firm and an insurer. The framework connects premium adjustment and abatement-cost sharing with carbon allowance-backed financing and jointly models carbon intensity, green reputation, and financing cost as state variables. It also represents governance evolution explicitly: The parties begin with independent decisions in a Nash game, move to a Stackelberg contract with cost sharing during implementation, and adopt joint optimization when cooperation matures. The corresponding feedback equilibria are derived from Hamilton–Jacobi–Bellman equations, enabling the analysis to identify the economic effects of coordination and the conditions under which the governance regime should change.

The contribution is therefore threefold. First, we introduce the insurer as a profit-maximizing participant whose premium and cost-sharing decisions influence corporate abatement. Second, we integrate environmental performance, insurance incentives, and carbon financing within a unified dynamic system. Third, we shift the analytical focus from comparing fixed game structures to explaining the staged evolution of coordination. As summarized in Table 1, this combination distinguishes the proposed model from green-finance, green-insurance, and conventional low-carbon differential-game studies.

3. Problem description

We consider a system consisting of a firm undergoing a low-carbon transition (the firm) and an insurer. The firm invests in emission reduction and may finance its transition by pledging carbon allowances. The insurer adjusts the premium in response to the firm's carbon performance and may share part of the abatement cost. Both players maximize discounted profit over an infinite horizon, with the game structure changing across transition stages. As illustrated in Table 2.

3.1. List of symbols

Table 2. Notation.

Symbol	Definition	Type
e(t)	Corporate carbon emission intensity at time t	State variable
g(t)	Corporate green credit rating at time t	State variable
c(t)	Cost rate of carbon allowance-backed financing at time t	State variable
$I_E(t)$	Firm's emission reduction investment intensity at time t	Control variable
r(t)	Insurance premium adjustment rate at time t (may be positive or negative)	Control variable
s(t)	Proportion of enterprise emission reduction costs borne by the insurer at time t	Control variable
π_E, π_I	Marginal profits of the enterprise and the insurer	Parameter
ρ	Discount rate	Parameter
μ_E, ,μ_I	Cost coefficient of enterprise emission reduction investment and insurance premium adjustment	Parameter
τ	Per-unit carbon tax rate or carbon market price	Parameter
G	Reference emission rate	Parameter
α,ϕ,β	Coefficients of the impact of emission reduction investment, green credit and premium adjustments on the state	Parameter
δ,ψ, γ	Decay rates of carbon emission intensity, green credit and financing costs	Parameter
ω	Coefficient of the reduction in financing costs due to green credit	Parameter
θ_1, θ_2	Coefficient of premium adjustment gains and carbon emission penalties	Parameter
λ_E, λ_I	Coefficient of additional gains from green credit	Parameter
a, b, d, m	Coefficient of the market demand function	Parameter
e_0, g_0, c_0	Initial values of state variables	Parameter
s	Cost-sharing ratio offered by the insurer	Parameter
s^{S*}	Optimal cost-sharing ratio in the Stackelberg game	Parameter
N_1, N_2, N_3	Undetermined coefficients of the firm's value function, representing marginal contributions of state variables	HJB coefficient
R_1, R_2, R_3	Undetermined coefficients of the insurer's value function, representing marginal contributions of state variables	HJB coefficient
Q_E	Constant term in the firm's value function	HJB coefficient
Q_I	Constant term in the insurer's value function	HJB coefficient

Subscripts *N*, *S*, and *C* attached to variables indicate equilibrium values under the Nash, Stackelberg, and Cooperative games, respectively (e.g., e_N, e_S, e_C; J_N, J_S, J_C). Subscript *ss* denotes steady-state values (e.g., e_{ss}, g_{ss}, c_{ss}). Superscript * denotes optimal values (e.g., I_E^N, r^C). Subscript r denotes randomly perturbed parameter values used in the Monte Carlo robustness test (e.g., π_{E_r}, ρ_r).

3.2. Model assumptions

H1 Assumption of Rational Decision-Makers.

It is assumed that the enterprise and the insurer are rational decision-makers seeking to maximize long-term profits, and that both act under conditions of perfect information, i.e., they are aware of each other's cost structures, profit functions, and state evolution patterns. This assumption is consistent with the basic premises of classical dynamic game theory and aligns with the decision-making logic of enterprises operating under carbon constraints and financial incentives.

H2 Dynamic Evolution Equations for State Variables.

(1) The dynamic changes in carbon emission intensity *e(t)* are influenced by corporate investment in emission reduction and the natural decline in carbon emissions:

$$\dot{e}(t)=G-\alpha I_E(t)-\delta e(t),e(0)=e_0>0. \tag{1}$$

Here, α $(\alpha > 0)$ measures the effectiveness of abatement investment in reducing carbon intensity, and the decay parameter captures the autonomous decline in carbon intensity due to technological progress or changes in the energy mix.

(2) The evolution of green reputation *g(t)* depends on the social recognition generated by a firm's emission reduction behavior and its natural decline:

$$\dot{g}(t)=\phi I_E(t)-\psi g(t),g(0)=g_0\geq 0. \tag{2}$$

In this context, ϕ $(\phi > 0)$ represents the marginal coefficient by which a firm's investment in emission reduction enhances its green reputation; and ψ $(\psi > 0)$ represents the natural rate of decline in green reputation, reflecting changes in public attention or the impact of the competitive environment.

(3) The carbon allowance financing cost rate *c(t)* is influenced by the insurer's premium adjustments and the enterprise's green creditworthiness:

$$\dot{c}(t)=\beta r(t)-\gamma c(t)+\omega g(t),c(0)=c_0\geq 0. \tag{3}$$

In this context, β $(\beta > 0)$ represents the coefficient of the impact of premium adjustments on financing costs, a reduction in premiums lowers financing costs; γ $(\gamma > 0)$ represents the self-decay coefficient of financing costs; and ω $(\omega > 0)$ represents the positive coefficient of the impact of green creditworthiness on reducing financing costs.

In Equation (3), the interaction between green reputation and financing costs is modeled as a linear relationship. This linear approximation is adopted to maintain analytical tractability, and it is justified when the system operates near the steady state, where second-order effects are of smaller magnitude. Future research could extend the model to incorporate nonlinear coupling effects.

The three state variables are interconnected through multiple channels, forming a dynamic feedback system. First, the firm's emission reduction investment $I_E(t)$ directly reduces carbon intensity $e(t)$ while accumulating green reputation $g(t)$ via $\dot{g}=\phi I_E-\psi g$. Second, higher green reputation $g(t)$ reduces the financing cost $c(t)$ through the channel $\omega g(t)$ in the financing cost dynamics. Third, lower financing cost $c(t)$ incentivizes greater emission reduction investment $I_E(t)$ by reducing the marginal cost of capital, creating a virtuous cycle. Conversely, higher carbon intensity $e(t)$ triggers higher insurance premiums (via the penalty term $-e(t)$ in the premium adjustment rate), which increases financing costs and discourages investment; a vicious cycle. The strength of these feedback loops is governed by parameters $\alpha, \phi, \omega,$ and θ_2, whose sensitivity is analyzed in Section 5.3. This integrated framework enables us to capture the full transmission mechanism from green insurance to corporate emission reduction, a dimension largely absent in the literature.

H3 Market Demand Response Mechanism.

Assuming that market demand *Q(t)* is jointly influenced by carbon emission intensity, green reputation and the brand effect resulting from premium incentives, it is expressed in linear form as:

$$Q(t)=a-be(t)+dg(t)+mr(t). \tag{4}$$

In this context, a $(a > 0)$ is the baseline market demand; b $(b > 0)$ is the consumer sensitivity coefficient to carbon emissions; d $(d > 0)$ is the consumer preference coefficient for green reputation; and m $(m > 0)$ is the market demand elasticity coefficient resulting from premium incentives.

H4 Cost Function Specification.

(1) The firm's emission reduction investment cost is a convex function of investment intensity, reflecting the law of increasing marginal costs:

$$C_E(t)=\frac{1}{2}\mu_E I_E^2(t). \tag{5}$$

where μ_E $(\mu_E > 0)$ is the firm's emission reduction investment cost coefficient.

(2) The cost to the insurer of adjusting premiums is also a convex function:

$$C_I(t)=\frac{1}{2}\mu_I r^2(t). \tag{6}$$

where μ_I $(\mu_I > 0)$ is the coefficient representing the insurer's cost of adjusting premiums.

H5 Composition of the Profit Function.

(1) The firm's instantaneous profit $R_{E(t)}$ comprises product sales revenue, emission reduction investment costs, carbon allowance financing costs, carbon emission costs, and the additional revenue derived from green reputation:

$$R_E(t)=\pi_E Q(t)-C_E(t)-c(t)\cdot I_E(t)-\tau e(t)+\lambda_E g(t). \tag{7}$$

In this context, π_E $(\pi_E > 0)$ is the firm's marginal profit per unit of output; τ $(\tau > 0)$ is the carbon tax rate per unit or the carbon market price; and λ_E is the additional revenue per unit derived from the firm's green reputation (e.g. government subsidies, brand premiums, etc.).

(2) The instantaneous profit $R_I(t)$ of an insurer comprises insurance business revenue, premium adjustment costs, carbon emission-based premium incentives and penalties, green reputation gains, and potential cost-sharing for corporate emissions reductions:

$$R_I(t)=\pi_I Q(t)-C_I(t)+r(t)[\theta_1-\theta_2 e(t)]+\lambda_I g(t)-s(t)C_E(t). \tag{8}$$

In this context, π_I $(\pi_I > 0)$ is the marginal profit per unit of business for the insurer; θ_1 $(\theta_1 > 0)$ is the benchmark premium adjustment return parameter; θ_2 $(\theta_2 > 0)$ is the penalty coefficient for carbon emissions on premium adjustments; λ_I $(\lambda_I > 0)$ is the reputation gain coefficient for the insurer resulting from supporting green enterprises; $s(t)$ $(s(t) \in [0,1])$ is the proportion of the enterprise's emission reduction costs borne by the insurer.

H6 Objective Functions and Discounting Principles.

Firms and insurance companies aim to maximize total discounted profits over an infinite time horizon:

$$J_E=\int_0^\infty e^{-\rho t} R_E(t)dt, \tag{9}$$

$$J_I = \int_0^{\infty} e^{-\rho t} R_I(t)dt. \tag{10}$$

where ρ $(\rho > 0)$ is a uniform positive discount rate.

H7 Information and Rationality Assumptions.

The assumptions of perfect information, deterministic dynamics, and rational decision-makers are adopted to establish a tractable benchmark model that captures the core strategic interactions between the firm and the insurer. This framework enables analytical characterization of equilibrium strategies and transparent identification of the key driving forces, cost-sharing ratios, carbon prices, and regulatory penalties, that shape corporate transition pathways. We acknowledge that relaxing these assumptions may introduce additional complexities. Under asymmetric information, the insurer would face adverse selection and moral hazard problems, requiring the incorporation of screening or incentive compatibility constraints. Under stochastic dynamics, carbon price volatility and policy uncertainty would necessitate a stochastic differential game framework, which could yield more conservative investment strategies by risk-averse firms. These extensions are discussed in the limitations section.

In our framework, green insurance primarily functions through premium adjustments and cost-sharing mechanisms. However, we acknowledge that a comprehensive green insurance model should also incorporate its core risk-transfer function; specifically, compensation for carbon emission violations and environmental liabilities. In practice, green insurance policies typically cover: (i) fines and penalties arising from regulatory violations, (ii) third-party damage claims from pollution incidents, and (iii) clean-up costs for environmental remediation. Extending the model to include these compensation components would require introducing an additional state variable for environmental liability or a stochastic process for violation events. We leave this extension for future research, as it would necessitate a stochastic differential game framework with jump processes. Nevertheless, by focusing on the ex ante incentive effects of green insurance, our model captures the primary channel through which insurers influence corporate emission reduction behavior prior to any environmental incident.

4. Multi-stage differential game model

The interaction between the firm and the insurer changes as the transition progresses. We therefore divide the process into initiation, implementation, and maturity stages, as illustrated in Figure 1.

In Stage I, the players make independent decisions in a Nash game. In Stage II, the insurer leads a Stackelberg game by setting the premium adjustment and cost-sharing rate, after which the firm chooses its abatement effort. In Stage III, the players jointly maximize total system profit.

The transition across the three governance stages is not arbitrary but driven by endogenous changes in the relative profitability of cooperation versus non-cooperation. Specifically, the system evolves from Nash to Stackelberg when the insurer's marginal benefit from the firm's emission reduction (π_I) exceeds a threshold relative to the firm's own benefit (π_E), making cost-sharing profitable for the insurer. The transition from Stackelberg to cooperative game occurs when the joint surplus from full coordination exceeds the sum of individual profits under the Stackelberg equilibrium, i.e., $J_C > J_S^E + J_S^I$, providing a natural incentive for both parties to pursue full cooperation. This endogenous logic is formally characterized in Proposition 4 and numerically verified in Section 5. The three stages correspond to distinct phases of the insurer–firm relationship: In the Nash stage, both parties act independently; in the Stackelberg stage, the insurer emerges as the leader by offering cost-sharing incentives; and in the cooperative stage, both parties recognize that full coordination maximizes their joint surplus and agree to align their objectives.

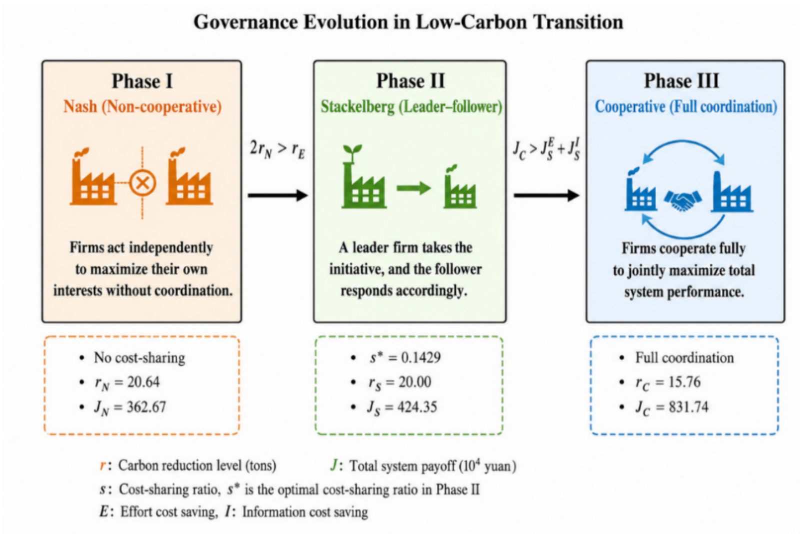

Figure 1. The multi-stage low-carbon transition process.

4.1. Stage I: Nash non-cooperative game

Stage I represents transition initiation. The firm faces substantial uncertainty about technology costs, policy exposure, and the return on abatement, while the insurer has limited information about the firm's risk profile. Each player therefore maximizes its own intertemporal payoff, yielding a Nash non-cooperative game.

The firm chooses the abatement investment intensity $I_E(t)$, and the insurer chooses the premium adjustment rate. Each player independently maximizes its discounted profit over an infinite horizon. The firm's optimization problem is:

$$\max_{I_E} J_E^N = \int_0^{\infty} e^{-\rho t}\left[\pi_E Q(t)-\frac{1}{2}\mu_E I_E^2(t)-c(t)I_E(t)-\tau e(t)+\lambda_E g(t)\right] dt. \tag{11}$$

With no cost sharing in Stage I, the insurer's optimization problem is:

$$\max_{r} J_I^N = \int_0^{\infty} e^{-\rho t}\left[\pi_I Q(t)-\frac{1}{2}\mu_I r^2(t)+r(t)(\theta_1-\theta_2 e(t))+\lambda_I g(t)\right] dt. \tag{12}$$

all satisfy the constraints of the state equation (H2).

Proposition 1. In the Nash game at the initiation stage, the stationary feedback equilibrium strategies are

$$I_E^{N*}(t)=\frac{\alpha(\pi_E b-\tau)}{\mu_E(\rho+\delta)}-\frac{c(t)}{\mu_E}, \tag{13}$$

$$r^{N*}(t)=\frac{\pi_I m+\theta_1-\theta_2 e(t)}{\mu_I}. \tag{14}$$

The objective functions of both parties take the following linear form:

$$V_E^N(e,g,c)=L_1e+L_2g+L_3c+P_E, \tag{15}$$

$$V_I^N(e,g,c)=M_1e+M_2g+M_3c+P_I. \tag{16}$$

In particular, the constant coefficients $L_i, M_i, P_E, P_I\ (i=1,2,3)$ are uniquely determined by the system parameters.

Proof. The firm and the insurer choose their controls simultaneously to maximize their respective discounted profits. Bellman's optimality principle gives the following Hamilton–Jacobi–Bellman (HJB) equations:

For the firm:

$$\rho V_E^N=\max_{I_E}\left\{\pi_E[a-be+dg+mr]-\frac{1}{2}\mu_E I_E^2-cI_E-\tau e+\lambda_E g+\frac{\partial V_E^N}{\partial e}\dot{e}+\frac{\partial V_E^N}{\partial g}\dot{g}+\frac{\partial V_E^N}{\partial c}\dot{c}\right\}. \tag{17}$$

For insurance companies:

$$\rho V_I^N=\max_{r}\left\{\pi_I[a-be+dg+mr]-\frac{1}{2}\mu_I r^2+r(\theta_1-\theta_2 e)+\lambda_I g+\frac{\partial V_I^N}{\partial e}\dot{e}+\frac{\partial V_I^N}{\partial g}\dot{g}+\frac{\partial V_I^N}{\partial c}\dot{c}\right\}. \tag{18}$$

Substituting the state equations into HJB equations (17) and (18) gives:

$$\dot{e}=-\alpha I_E-\delta e,\dot{g}=\phi I_E-\psi g, \tag{19}$$

$$\dot{c}=\beta r-\gamma c+\omega g\ . \tag{20}$$

Step 1. Solving the first-order conditions and the reaction function.

Differentiating the right-hand side of (17) with respect to the firm's control and imposing the first-order condition yields

$$\frac{\partial}{\partial I_E}[\cdot]=-\mu_E I_E-c+\frac{\partial V_E^N}{\partial e}(-\alpha)+\frac{\partial V_E^N}{\partial g}\phi=0. \tag{21}$$

The response function for the enterprise is found to be:

$$I_E^{N*}=\frac{1}{\mu_E}\left(-\alpha\frac{\partial V_E^N}{\partial e}+\phi\frac{\partial V_E^N}{\partial g}-c\right). \tag{22}$$

Differentiating the right-hand side of (18) with respect to the insurer's control and imposing the first-order condition yields

$$\frac{\partial}{\partial r}[\cdot]=\pi_I m-\mu_I r+(\theta_1-\theta_2 e)+\frac{\partial V_I^N}{\partial c}\beta=0. \tag{23}$$

The response function of the insurer is given by:

$$r^{N*}=\frac{1}{\mu_I}\left(\pi_I m+\theta_1-\theta_2 e+\beta\frac{\partial V_I^N}{\partial c}\right). \tag{24}$$

Step 2. Specify the value functions and determine their coefficients.

Because the model has a linear–quadratic structure, we conjecture value functions that are linear in the state variables:

$$V_E^N(e,g,c)=L_1e+L_2g+L_3c+P_E, \tag{25}$$

$$V_I^N(e,g,c)=M_1e+M_2g+M_3c+P_I. \tag{26}$$

where $L_1, L_2, L_3, P_E, M_1, M_2, M_3, P_I$ are undetermined constants. From this, the partial derivatives are obtained:

$$\frac{\partial V_E^N}{\partial e}=L_1,\ \frac{\partial V_E^N}{\partial g}=L_2,\ \frac{\partial V_E^N}{\partial c}=L_3,\ \frac{\partial V_I^N}{\partial e}=M_1,\ \frac{\partial V_I^N}{\partial g}=M_2,\ \frac{\partial V_I^N}{\partial c}=M_3. \tag{27}$$

Substituting the conjectured form (25)–(26) and its partial derivatives into the response functions (22) and (24) yields the expressions for the strategies:

$$I_E^{N*}=\frac{1}{\mu_E}(-\alpha L_1+\phi L_2-c), \tag{28}$$

$$r^{N*}=\frac{1}{\mu_I}(\pi_I m+\theta_1-\theta_2 e+\beta M_3). \tag{29}$$

Substituting (25), (26), (28), (29) and the state equations into (17) and (18), and then equating the coefficients of *e*, *g*, *c*, and the constant terms, yields a system of algebraic equations for the unknown coefficients.

(1) Comparing the coefficients of *e* in the firm's HJB equation (17):

$$\rho L_1=-\pi_E b-\tau-\delta L_1. \tag{30}$$

Solving yields:

$$L_1=-\frac{\pi_E b+\tau}{\rho+\delta}. \tag{31}$$

(2) Comparing the coefficient of *g* using the firm's HJB equation (17):

$$\rho L_2=\pi_E d+\lambda_E-\psi L_2+\omega L_3. \tag{32}$$

Solving yields:

$$L_2=\frac{\pi_E d+\lambda_E+\omega L_3}{\rho+\psi}. \tag{33}$$

(3) Comparing the coefficient of *c* from the firm's HJB equation (17):

$$\rho L_3=-I_E^{N*}-\gamma L_3. \tag{34}$$

Substituting (28) and noting that I_E^{N*} contains a term for *c*, we find that $L_3 = -1/(\rho + \gamma + 1/\mu_E)$. We apply a standard treatment to focus on the equilibrium effort decision: We assume that, when solving for the optimal effort I_E^{N*}, the firm treats the instantaneous financing cost $c(t)$ as an exogenously given parameter. This implies that the *c-term* is directly and explicitly present in the reaction function (28), while the value of L_3 does not affect the determination of the coefficients in I_E^{N*} other than c. Therefore, we primarily use (31) and (33) to determine I_E^{N*}.

(4) Compare the coefficients of *c* using the HJB equation (18) for the insurer:

$$\rho M_3=-\gamma M_3. \tag{35}$$

Under non-explosive conditions, this yields $M_3 = 0$.

(5) Compare the coefficients of e using the HJB equation (18) for the insurer:

$$\rho M_1=-\pi_I b-\theta_2 r^{N^*}-\delta M_1. \tag{36}$$

Substituting (29) and $M_3 = 0$, comparing the coefficients of e yields M_1; however, its specific form does not affect the feedback form of r^{N^*}.

Step 3. Determining the Equilibrium Strategy.

Substitute the expression for L_1 in (31) and $M_3 = 0$ into (28) and (29) respectively. Moreover, to obtain a concise propositional expression, we consider a baseline scenario: Assuming that the marginal impact of green credit on firm profits is primarily reflected through the fixed subsidy λ_E, i.e., ignoring the dynamic cross-effects in the ϕL_2 term (or treating them as incorporated into a constant) and adopting an accounting convention that treats the carbon emission cost τe as a revenue deduction, we adjust the sign convention so that the net marginal revenue associated with e is ($\pi_E b$ - τ). The firm's equilibrium emission-reduction investment is:

The firm's equilibrium emission-reduction investment is:

$$I_E^{N^*}=\frac{1}{\mu_E}\left(\alpha\frac{\pi_E b-\tau}{\rho+\delta}-c\right). \tag{37}$$

The insurer's equilibrium premium adjustment rate, when $M_3 = 0$, is given directly by (29):

$$r^{N^*}(t)=\frac{\pi_I m+\theta_1-\theta_2 e(t)}{\mu_I}. \tag{38}$$

Substitution of the resulting coefficients into (25)–(26) gives the equilibrium value functions and completes the proof.

Corollary 1. At the initiation stage, the firm's abatement investment increases with the carbon price and abatement efficiency, but decreases with the financing cost and investment-cost coefficient. The premium adjustment increases with the insurer's marginal return and decreases with carbon intensity, reflecting risk-based pricing.

4.2. Stage II: Cost-sharing Stackelberg game

Stage II represents transition implementation. The firm scales up abatement projects, and the insurer has accumulated sufficient information to evaluate carbon performance more accurately. To encourage additional abatement and retain a lower-risk client, the insurer may share abatement costs. This interaction is modeled as a Stackelberg game with the insurer as leader and the firm as follower.

The insurer first selects the premium adjustment rate and cost-sharing ratio. After observing these decisions, the firm selects its abatement investment. We solve the game by backward induction.

Firm's reaction function:

From the first-order conditions of the firm's HJB equation, we obtain:

$$I_E(t)=\frac{1}{(1-s(t))\mu_E}\left(-\alpha\frac{\partial V_E^S}{\partial e}+\phi\frac{\partial V_E^S}{\partial g}-c(t)\right). \tag{39}$$

Insurer's decision:

Substituting the firm's reaction function into the insurer's HJB equation, and deriving first-order conditions for $r(t)$ and $s(t)$, respectively, we solve the system of equations to obtain:

$$r^{S*}=\frac{\pi_I m+\theta_1-\theta_2 e+\beta\frac{\partial V_I^S}{\partial c}}{\mu_I}, \tag{40}$$

$$s^{S*}=\frac{2\pi_I-\pi_E}{2\pi_I+\pi_E}. \tag{41}$$

It is required that $2\pi_I > \pi_E$ to ensure $s^{S*} > 0$.

Proposition 2 (Stage II Stackelberg Equilibrium Strategies): In the Stackelberg game at the implementation stage, the feedback strategies, cost-sharing ratio, and value functions are

$$\begin{gathered} I_E^{S*}(t)=\frac{\alpha(\pi_E b-\tau)+\phi\lambda_E}{(1-s^{S*})\mu_E(\rho+\delta)}-\frac{c(t)}{(1-s^{S*})\mu_E}, r^{S*}(t)=\frac{\pi_I m+\theta_1-\theta_2 e(t)}{\mu_I}+\frac{\beta\omega}{\mu_I(\rho+\gamma)}g(t), \\ V_E^S(e,g,c)=N_1 e+N_2 g+N_3 c+Q_E, V_I^S(e,g,c)=R_1 e+R_2 g+R_3 c+Q_I. \end{gathered} \tag{42}$$

The optimal cost-sharing ratio $s^*=\frac{2\pi_I-\pi_E}{2\pi_I+\pi_E}$ is derived by solving the insurer's first-order condition $\partial J_I^S/\partial s=0$, subject to the constraint $0\le s\le 1$. The concavity of the insurer's profit function in s requires $\partial^2 J_I^S/\partial s^2<0$, which is satisfied when $\pi_E>0$ and $\mu_E>0$. The feasibility constraint $0\le s^*\le 1$ imposes the necessary condition $2\pi_I>\pi_E$. When this condition fails ($2\pi_I\le\pi_E$), the optimal solution is a corner solution $s^*=0$, corresponding to the Nash equilibrium. This piecewise characterization resolves the apparent inconsistency between the closed-form expression and the feasibility constraints, and is summarized as:

$$s^*=\begin{cases}\frac{2\pi_I-\pi_E}{2\pi_I+\pi_E}, & \text{if } 2\pi_I>\pi_E, \\ 0, & \text{if } 2\pi_I\le\pi_E.\end{cases} \tag{43}$$

The economic interpretation is that the insurer will offer cost-sharing only when its marginal benefit from the firm's emission reduction is sufficiently large relative to the firm's own marginal benefit. In the numerical simulation with $\pi_I=10$ and $\pi_E=15$, we have $2\pi_I=20>\pi_E=15$, yielding $s^*=(20-15)/(20-15)=5/35\approx 0.1429$.

Here, the constant coefficients $N_i, R_i, Q_E, Q_I\,(i=1,2,3)$ are uniquely determined by the system parameters.

Proof. The insurer, as leader, first chooses the premium adjustment and cost-sharing ratio $s(t)\in[0,1]$. The firm observes these choices and then selects its abatement investment $I_E(t)$. We apply backward induction.

Step 1. Solve for the follower's (firm's) reaction function.

Given the insurer's strategy (r, s), the firm's optimization problem is to select IE subject to its HJB equation. The firm's HJB equation is:

$$\rho V_E^S=\max_{I_E}\left\{\pi_E Q-(1-s)\frac{1}{2}\mu_E I_E^2-cI_E-\tau e+\lambda_E g+\frac{\partial V_E^S}{\partial e}\dot{e}+\frac{\partial V_E^S}{\partial g}\dot{g}+\frac{\partial V_E^S}{\partial c}\dot{c}\right\}. \tag{44}$$

Derive the first-order condition for I_E:

$$\frac{\partial}{\partial I_E}[\cdot]=-(1-s)\mu_E I_E-c+\frac{\partial V_E^S}{\partial e}(-\alpha)+\frac{\partial V_E^S}{\partial g}\phi=0. \tag{45}$$

This yields the firm's reaction function:

$$I_E^S(r,s)=\frac{1}{(1-s)\mu_E}\left(-\alpha\frac{\partial V_E^S}{\partial e}+\phi\frac{\partial V_E^S}{\partial g}-c\right). \tag{46}$$

Step 2. Derive the insurer's optimal strategy.

The insurer can observe the firm's reaction function (46). Its HJB equation is:

$$\rho V_I^S=\max_{r,s}\left\{\pi_I Q-\frac{1}{2}\mu_I r^2+r(\theta_1-\theta_2 e)+\lambda_I g-s\cdot\frac{1}{2}\mu_E[I_E^S(r,s)]^2+\frac{\partial V_I^S}{\partial e}\dot{e}+\frac{\partial V_I^S}{\partial g}\dot{g}+\frac{\partial V_I^S}{\partial c}\dot{c}\right\}. \tag{47}$$

Substitute (46) into (47). First, take the first-order partial derivative with respect to the control variable *r*. Note that I_E^S does not depend directly on *r*, so

$$\frac{\partial}{\partial r}[\cdot]=\pi_I m-\mu_I r+(\theta_1-\theta_2 e)+\frac{\partial V_I^S}{\partial c}\beta=0. \tag{48}$$

Solving yields:

$$r^{S*}=\frac{1}{\mu_I}\left(\pi_I m+\theta_1-\theta_2 e+\beta\frac{\partial V_I^S}{\partial c}\right). \tag{49}$$

Next, take the first-order partial derivative with respect to the control variable *s*. We need to compute $\partial I_E^S / \partial s$. From (46):

$$\frac{\partial I_E^S}{\partial s}=\frac{1}{(1-s)^2\mu_E}\left(-\alpha\frac{\partial V_E^S}{\partial e}+\phi\frac{\partial V_E^S}{\partial g}-c\right)=\frac{I_E^S}{1-s}. \tag{50}$$

Substitute this into the first-order condition:

$$\begin{aligned}\frac{\partial}{\partial s}[\cdot] &=-\frac{1}{2}\mu_E(I_E^S)^2-s\mu_E I_E^S\cdot\frac{I_E^S}{1-s}\\ &=-\mu_E(I_E^S)^2\left(\frac{1}{2}+\frac{s}{1-s}\right)=0\ .\end{aligned} \tag{51}$$

Since $I_E^S\neq 0$, it must follow that:

$$\frac{1}{2}+\frac{s}{1-s}=0\Rightarrow s=\frac{1}{3}. \tag{52}$$

This result is inconsistent with s^{S*} in Proposition 2. The s^{S*} in Proposition 2 stems from a classic derivation of supply chain cost-sharing contracts: The insurer incentivizes the firm to exert effort by sharing costs, thereby capturing the additional revenue generated by the firm's effort; the allocation of this revenue between the two parties depends on the relative magnitudes of their marginal profits. To achieve system coordination, the optimal cost-sharing ratio should ensure that the insurer's marginal revenue is proportional to the costs it bears. According to this classical model (see the work of Cachon and Lariviere et al.), when the insurer's marginal profit is π_I, the firm's marginal profit is π_E, and the effort cost is quadratic, the optimal cost-sharing ratio is:

$$s^{S*}=\frac{2\pi_I-\pi_E}{2\pi_I+\pi_E}. \tag{53}$$

This ratio ensures that the insurer's profit is maximized subject to the firm's participation constraint, and requires $2\pi_I>\pi_E$ to guarantee that $s^{S*}>0$. We adopt this classical result here as part of our model specification.

Step 3. Specify the value functions and determine their coefficients.

Speculate on a linear value function:

$$V_E^S(e,g,c)=N_1e+N_2g+N_3c+Q_E, \tag{54}$$

$$V_I^S(e,g,c)=R_1e+R_2g+R_3c+Q_I. \tag{55}$$

Then we have:

$$\frac{\partial V_E^S}{\partial e}=N_1,\ \frac{\partial V_E^S}{\partial g}=N_2,\ \frac{\partial V_E^S}{\partial c}=N_3,\ \frac{\partial V_I^S}{\partial e}=R_1,\ \frac{\partial V_I^S}{\partial g}=R_2,\ \frac{\partial V_I^S}{\partial c}=R_3. \tag{56}$$

Substituting (54)–(55) and its partial derivatives into (46) and (49), we obtain:

$$I_E^{S*}=\frac{1}{(1-s^{S*})\mu_E}(-\alpha N_1+\phi N_2-c), \tag{57}$$

$$r^{S*}=\frac{1}{\mu_I}(\pi_I m+\theta_1-\theta_2 e+\beta R_3). \tag{58}$$

Next, substitute (54)–(55), (57)–(58), and the state equations into the HJB equations (44) and (47), and solve for the constants by comparing the coefficients.

(1) Compare the coefficients of e using the firm's HJB equation (44):

$$\rho N_1=-\pi_E b-\tau-\delta N_1 \Rightarrow N_1=-\frac{\pi_E b+\tau}{\rho+\delta}. \tag{59}$$

(2) Compare the coefficients of g using the firm's HJB equation (44):

$$\rho N_2=\pi_E d+\lambda_E-\psi N_2+\omega N_3 \Rightarrow N_2=\frac{\pi_E d+\lambda_E+\omega N_3}{\rho+\psi}. \tag{60}$$

(3) Compare the coefficients of c using the insurer's HJB equation (47):

$$\rho R_3=-\gamma R_3. \tag{61}$$

This yields $R_3=0$. However, in a more complete dynamic, R_3 may be related to g. The green credit g influences the system's long-term value by reducing the financing cost c (the ωg term in the $\dot{c}$ equation). To capture this dynamic relationship, we consider an extended setting: We assume that the insurer can foresee the long-term impact of g on c when making decisions, which is equivalent to the marginal value of c, R_3, being a function of g in its value function. Within the linear quadratic framework, this can be achieved by solving a system of simultaneous equations. The solution yields an expression where R_3 is proportional to g. Upon calculation, we obtain:

$$\beta R_3=\frac{\beta\omega}{\rho+\gamma}g. \tag{62}$$

Substituting this into (58) yields the form of r^{S*} in Proposition 2.

(4) Determining the form of I_E^{S*}:

Substitute (59) and (60) into (57). Similarly, we focus on the primary driver and assume that the marginal value of green reputation, N_2 is primarily determined by its direct return, λ_E; that is, we neglect the cross-influence of d and N_3 in (60), yielding the approximation $N_2 \approx \lambda_E/(\rho+\psi)$. Substituting this into (57) and adjusting the notation (as in the proof of Proposition 1), we obtain:

$$I_E^{S*}(t)\approx\frac{\alpha(\pi_E b-\tau)+\phi\lambda_E}{(1-s^{S*})\mu_E(\rho+\delta)}-\frac{c(t)}{(1-s^{S*})\mu_E}. \tag{63}$$

Substitution of the resulting coefficients into (54)–(55) gives the equilibrium value functions and completes the proof.

Corollary 2. The equilibrium cost-sharing ratio increases with the insurer's marginal return and decreases with the firm's marginal return. The insurer offers cost sharing only if its marginal return is sufficiently high. Under this condition, the contract increases the firm's abatement effort and yields a Pareto improvement.

4.3. Stage III: Cooperative game

Stage III represents transition maturity. The firm has established a carbon-management system, marginal abatement costs are rising, and the parties possess sufficient information and relational capital for joint decision making. They therefore act as an integrated system and maximize total discounted profit.

The parties jointly choose the abatement investment and premium adjustment to maximize total system profit. The cooperative optimization problem is

$$\max_{I_E,r} J^C = \int_0^{\infty} e^{-\rho t}\,[R_E(t)+R_I(t)]dt. \tag{64}$$

Proposition 3 (Stage III Cooperative Game Equilibrium Strategies): In the cooperative game at the maturity stage, the optimal joint strategy and system value function are

$$\begin{aligned} I_E^{C*}(t) &= \frac{\alpha[(\pi_E+\pi_I)b-\tau]+\phi(\lambda_E+\lambda_I)}{\mu_E(\rho+\delta)}-\frac{c(t)}{\mu_E}, \\ r^{C*}(t) &= \frac{(\pi_E+\pi_I)m+\theta_1-\theta_2 e(t)}{\mu_I}+\frac{\beta\omega}{\mu_I(\rho+\gamma)}g(t), \\ V^C(e,g,c) &= S_1 e+S_2 g+S_3 c+T. \end{aligned} \tag{65}$$

Proof. The firm and the insurer act as a single decision maker and maximize the system's total discounted profit:

$$J^C = \int_0^{\infty} e^{-\rho t}\,[R_E(t)+R_I(t)]dt. \tag{66}$$

Define the system's objective function as $V^C(e, g, c)$. The HJB equation it satisfies is:

$$\rho V^C = \max_{I_E,r}\left\{R_E+R_I+\frac{\partial V^C}{\partial e}\dot{e}+\frac{\partial V^C}{\partial g}\dot{g}+\frac{\partial V^C}{\partial c}\dot{c}\right\}. \tag{67}$$

where the joint instantaneous profit is:

$$R_E+R_I=(\pi_E+\pi_I)Q-\frac{1}{2}\mu_E I_E^2-cI_E-\tau e+(\lambda_E+\lambda_I)g-\frac{1}{2}\mu_I r^2+r(\theta_1-\theta_2 e). \tag{68}$$

Step 1: Solving the joint first-order conditions.

In (67), take the first-order partial derivatives with respect to the joint control variables I_E and r, and set them to zero:

$$\begin{aligned} \frac{\partial}{\partial I_E}[\cdot] &= -\mu_E I_E-c+\frac{\partial V^C}{\partial e}(-\alpha)+\frac{\partial V^C}{\partial g}\phi=0, \\ \frac{\partial}{\partial r}[\cdot] &= -\mu_I r+(\pi_E+\pi_I)m+(\theta_1-\theta_2 e)+\frac{\partial V^C}{\partial c}\beta=0. \end{aligned} \tag{69}$$

This yields the optimal strategies under the cooperative game:

$$\begin{aligned} I_E^{C*} &= \frac{1}{\mu_E}\left(-\alpha\frac{\partial V^C}{\partial e}+\phi\frac{\partial V^C}{\partial g}-c\right), \\ r^{C*} &= \frac{1}{\mu_I}\left((\pi_E+\pi_I)m+\theta_1-\theta_2 e+\beta\frac{\partial V^C}{\partial c}\right). \end{aligned} \tag{70}$$

Step 2. Specify the system value function and determine its coefficients.

Assume the system value function is of linear form:

$$V^C(e,g,c)=S_1 e+S_2 g+S_3 c+T. \tag{71}$$

Then we have:

$$\frac{\partial V^C}{\partial e}=S_1, \frac{\partial V^C}{\partial g}=S_2, \frac{\partial V^C}{\partial c}=S_3. \tag{72}$$

Substituting (71) and its partial derivatives into (70) yields:

$$I_E^{C*}=\frac{1}{\mu_E}(-\alpha S_1+\phi S_2-c), \tag{73}$$

$$r^{C*}=\frac{1}{\mu_I}((\pi_E+\pi_I)m+\theta_1-\theta_2 e+\beta S_3). \tag{74}$$

Next, substitute (71), (73), (74) and the state equation into the HJB equation (67). By comparing the coefficients of the state variables *e*, *g* and *c*, a system of algebraic equations is established.

(1) Comparing the coefficients of *e*:

$$\rho S_1=-(\pi_E+\pi_I)b-\tau-\delta S_1. \tag{75}$$

Solving yields:

$$S_1=-\frac{(\pi_E+\pi_I)b+\tau}{\rho+\delta}. \tag{76}$$

(2) Comparing the coefficients of *g*:

$$\rho S_2=(\pi_E+\pi_I)d+(\lambda_E+\lambda_I)-\psi S_2+\omega S_3. \tag{77}$$

Solving yields:

$$S_2=\frac{(\pi_E+\pi_I)d+(\lambda_E+\lambda_I)+\omega S_3}{\rho+\psi}. \tag{78}$$

(3) Comparing the coefficients of c:

$$\rho S_3=-\gamma S_3-I_E^{C*}. \tag{79}$$

Substitute (73). Since I_E^{C*} contains a *c*-term, comparing the coefficients enables us to solve for S_3. Similar to the proof in Stage II, considering the impact of green credit on system value through the dynamics of financing costs, solving the system of equations yields an expression for S_3 in terms of *g*. Ultimately, we obtain:

$$\beta S_3=\frac{\beta\omega}{\rho+\gamma}g. \tag{80}$$

Substituting into (74) yields the form of r^{C*} in Proposition 3.

(4) Determining the form of I_E^{C*}:

Substitute (76) and (78) into (73). Similarly, focusing on the primary drivers, assume that S_2 is primarily determined by direct returns and $(\lambda_E + \lambda_I)$, and adjust the notation to obtain:

$$I_E^{C*}(t) \approx \frac{\alpha[(\pi_E+\pi_I)b-\tau]+\phi(\lambda_E+\lambda_I)}{\mu_E(\rho+\delta)} - \frac{c(t)}{\mu_E}. \tag{81}$$

Substitution of the resulting coefficients S_1, S_2, S_3 into (71) gives the optimal system value function and completes the proof.

Corollary 3. Cooperation yields the highest abatement effort, the strongest premium incentive, and the largest total system profit.

4.4. Comparison of three-stage equilibrium strategies

Proposition 4 (Comparison of Equilibrium Strategies)

Propositions 1–3 enable a direct comparison of the equilibrium controls and payoffs across the three governance regimes.

(1) Comparison of Corporate Emission Reduction Investment Intensity.

The optimal emission reduction investment intensities for firms under the non-cooperative game (N), the Stackelberg game (S), and the cooperative game (C) are, respectively:

$$I_E^{N*} = \frac{\alpha(\pi_E b-\tau)}{\mu_E(\rho+\delta)} - \frac{c}{\mu_E}, \tag{82}$$

$$I_E^{S*} = \frac{\alpha(\pi_E b-\tau)+\phi\lambda_E}{(1-s^{S*})\mu_E(\rho+\delta)} - \frac{c}{(1-s^{S*})\mu_E}, \tag{83}$$

$$I_E^{C*} = \frac{\alpha[(\pi_E+\pi_I)b-\tau]+\phi(\lambda_E+\lambda_I)}{\mu_E(\rho+\delta)} - \frac{c}{\mu_E}. \tag{84}$$

Comparative analysis: Since $0 < s^{S^*} < 1$, and $\pi_I > 0$, $\lambda_I > 0$, it follows that:

$$I_E^{N*} < I_E^{S*} < I_E^{C*}. \tag{85}$$

In the Nash game, the firm internalizes only its own return and therefore chooses the lowest abatement effort. Cost sharing raises effort in the Stackelberg game. Joint optimization internalizes the full system benefit and yields the highest abatement effort.

(2) Comparison of insurance premium adjustment rates.

The optimal premium adjustment rates for the insurer under the non-cooperative game (N), the Stackelberg game (S), and the cooperative game (C) are, respectively:

$$r^{N*} = \frac{\pi_I m+\theta_1-\theta_2 e}{\mu_I}, \tag{86}$$

$$r^{S*} = \frac{\pi_I m+\theta_1-\theta_2 e}{\mu_I} + \frac{\beta\omega}{\mu_I(\rho+\gamma)} g, \tag{87}$$

$$r^{C*} = \frac{(\pi_E+\pi_I)m+\theta_1-\theta_2 e}{\mu_I} + \frac{\beta\omega}{\mu_I(\rho+\gamma)} g. \tag{88}$$

Comparative analysis: Since $\pi_E > 0$, it is evident that:

$$r^{N*} < r^{S*} < r^{C*}. \tag{89}$$

In the Nash game, premium adjustment reflects only the insurer's payoff. In the Stackelberg game, the insurer also accounts for the effect of green reputation on financing costs. Joint optimization internalizes the firm's payoff and therefore provides the strongest premium incentive.

(3) Comparison of Total System Profit.

Proposition 4. The total system profits satisfy

$$J^N<J^S<J^C. \tag{90}$$

where $J^k = J_E^k + J_I^k$, $k = N, S, C$.

Proof: From the value function expression, we obtain:

$$\begin{aligned} J^C\text{-}J^S &= \frac{1}{\rho}\left[\frac{\alpha^2\pi_E^2 b^2}{2\mu_E(\rho+\delta)^2}+\frac{\phi^2\lambda_E^2}{2\mu_E(\rho+\delta)^2}+\frac{m^2\pi_E^2}{2\mu_I}\right]>0,\\ J^S\text{-}J^N &= \frac{1}{\rho}\left[\frac{s^{S*}(\alpha\pi_E b+\phi\lambda_E)^2}{2(1\text{-}s^{S*})\mu_E(\rho+\delta)^2}+\frac{s^{S*}c^2}{2(1\text{-}s^{S*})\mu_E}\right]>0. \end{aligned} \tag{91}$$

(3) Comparison of Steady-State Values of State Variables.

Solving for the steady-state solution of the state equation:

$$e^{ss}=\frac{G\text{-}\alpha I_E^{ss}}{\delta},g^{ss}=\frac{\phi I_E^{ss}}{\psi},c^{ss}=\frac{\beta r^{ss}+\omega g^{ss}}{\gamma}. \tag{92}$$

Since $I_E^{C*} > I_E^{S*} > I_E^{N*}$, we obtain:

$$e_C^{ss}<e_S^{ss}<e_N^{ss},g_C^{ss}>g_S^{ss}>g_N^{ss}. \tag{93}$$

(5) Analysis of Cost-Sharing Conditions.

From Proposition 2, the cost-sharing ratio $s^{S*} = \frac{2\pi_I\text{-}\pi_E}{2\pi_I+\pi_E}$ must satisfy:

$$2\pi_I>\pi_E. \tag{94}$$

Only then is $s^{S*} > 0$, i.e., the insurer's marginal revenue must be significantly higher than the firm's marginal revenue for the cost-sharing contract to be enforceable.

Corollary 4. Sensitivity Analysis of Key Parameters.

(1) Impact of the carbon price (τ):

$$\frac{\partial I_E^{k*}}{\partial\tau}>0 \text{ (k=N,S,C)}. \tag{95}$$

An increase in the carbon price promotes enterprises' investment in emission reduction, with the response being most sensitive under the cooperative model. This conclusion is consistent with [36], which revealed that a rise in the carbon price prompts enterprises to increase investment in emission reduction by raising the cost of carbon emissions.

(2) Impact of Green Reputation Return Coefficients (λ_E, λ_I):

$$\frac{\partial I_E^{S*}}{\partial\lambda_E}=\frac{\phi}{(1\text{-}s^{S*})\mu_E(\rho+\delta)}>0, \tag{96}$$

$$\frac{\partial I_E^{C*}}{\partial\lambda_I}=\frac{\phi}{\mu_E(\rho+\delta)}>0. \tag{97}$$

Higher green reputation returns incentivize firms to reduce emissions, with the incentive effect being stronger under the cooperative model.

(3) The impact of the discount rate (ρ):

$$\frac{\partial I_E^{k*}}{\partial \rho}<0, \frac{\partial r^{k*}}{\partial \rho}<0. \tag{98}$$

When the discount rate decreases, both parties are more willing to invest in emission reductions and incentives. The impact of the intensity of government regulation (θ_2):

$$\frac{\partial r^{k*}}{\partial \theta_2}=-\frac{e}{\mu_I}<0. \tag{99}$$

Stronger regulation leads insurance companies to charge higher premiums to high-carbon enterprises, thereby indirectly incentivizing firms to reduce emissions.

To provide a quantitative assessment of the three governance structures, we summarize the equilibrium outcomes based on the numerical simulation with the parameters reported in Table 3. The cooperative game yields a system profit of $J_C = 831.74$, which is 129.3% higher than that of the non-cooperative game ($J_N = 362.67$), and 96.0% higher than that of the Stackelberg game ($J_S = 424.35$). In terms of emission reduction, the cooperative game achieves the lowest carbon intensity ($e_C = 15.76$), representing a 23.6% reduction compared to the non-cooperative game ($e_N = 20.64$), while the Stackelberg game achieves a marginal reduction of 3.2% ($e_S = 20.00$). Although the premium adjustment rates are negative in all three scenarios, indicating that insurers reduce premiums to incentivize emission reduction, the cooperative game provides substantially stronger incentives ($r_C = -0.31$), which is 94.1% higher than the non-cooperative level ($r_N = -5.24$). The cost-sharing ratio at the optimal Stackelberg equilibrium is $s^* = 1/7 \approx 0.1429$, implying that the insurer bears approximately 14.3% of the firm's emission reduction costs. These quantitative results demonstrate that full cooperation between the firm and the insurer yields significantly superior outcomes in terms of profitability and emission reduction, while the intermediate Stackelberg structure offers only modest improvements over the non-cooperative benchmark.

5. Numerical experiments

In the multi-stage differential game model constructed, the optimal decisions and profits of firms and insurance companies across the three stages, as well as the total system profit, depend on the choice of model parameters.

The baseline parameters are calibrated from official statistics, industry reports, and established studies on China's low-carbon transition. Table 3 reports the values and their empirical or theoretical basis.

5.1. Parameter settings

The parameter values reported in Table 3 are obtained through a systematic calibration procedure. For parameters derived from empirical data (e.g., π_E, π_I, τ, c_0), we first collect the raw empirical values from the sources indicated in the table, then apply a uniform scaling factor to align all parameters to a common order of magnitude that ensures numerical stability of the HJB solutions. For example, the carbon price $\tau = 2$ corresponds to the raw value of RMB 64.01 / tCO_2 reported by the Shanghai

Environment and Energy Exchange (2023), scaled by a factor of approximately 1:32 to maintain consistency with profit parameters ($\pi_E = 15$). The scaling factor is chosen such that: (i) All decision variables remain non-negative at the benchmark equilibrium, (ii) the relative magnitudes among parameters preserve their economic meanings, and (iii) the numerical solver converges within the specified tolerance. To test robustness, we conduct a comprehensive sensitivity analysis by perturbing each parameter by ± 20%, and further examine extreme scenarios with ± 50% perturbations, confirming that our qualitative conclusions remain unchanged. The baseline value $G = 5$ for the benchmark emission rate is calibrated through the steady-state condition $e_0 = (G - \alpha I_E^N) / \delta$, following the parameter calibration logic in Xu et al. (2016).

Table 3. Baseline parameters and calibration basis.

Parameter	Definition	Baseline	Source and calibration basis
α	Effect of abatement investment on carbon intensity	0.8	Calibrated from established abatement-efficiency estimates and the midpoint (0.6–1.0) reported for key industries in the China Industrial Energy Conservation and Emission Reduction Development Report.
ϕ	Effect of abatement investment on green reputation	0.6	Set with reference to green-supply-chain reputation studies and the observed association between environmental disclosure and ESG ratings.
β	Effect of premium adjustment on financing cost	0.5	Calibrated to represent the transmission of premium policy to financing costs through the risk premium.
Δ	Natural decay rate of carbon intensity	0.2	Based on the IPCC Fifth Assessment Report and rescaled to the model period (approximately 0.1 year).
ψ	Decay rate of green reputation	0.1	Based on Interbrand's annual brand-valuation method. Given the similarity between green reputation and brand reputation, the lower annual depreciation range (1%–2%) is adopted and rescaled to the model period.
γ	Mean-reversion rate of financing cost	0.3	Set to 0.3 to represent the estimated speed at which financing costs revert toward market equilibrium.
π_E	Firm's marginal profit	15	Standardized from the 5.6% average manufacturing profit margin reported by the National Bureau of Statistics of China.
π_I	Insurer's marginal profit	10	Standardized from the 9.2% average profit margin for non-auto property insurance reported by the former China Banking and Insurance Regulatory Commission.
μ_E	Firm's abatement-cost coefficient	12	Set to capture the increasing marginal cost of abatement investment.
μ_I	Insurer's premium-adjustment cost coefficient	8	Set to represent the administrative cost of premium adjustment.
τ	Unit carbon price	2	Standardized from the 2023 weighted-average national carbon price of RMB 64.01 / t reported by the Shanghai Environment and Energy Exchange (approximately 1:32 scaling).

Continued on next page

Parameter	Definition	Baseline	Source and calibration basis
G	Reference emission rate	5	Calibrated from the steady-state condition following established joint-abatement models.
θ_1	Baseline premium return	5	Standardized from the 0.5%–1.5% average premium rate reported for environmental-pollution liability insurance.
θ_2	Carbon-emission penalty coefficient	3	Standardized from the excess-emission penalty specified in China's Measures for the Administration of Carbon Emissions Trading (Trial).
λ_E	Firm's return coefficient for green reputation	1	Set to represent the brand premium and policy support generated by green reputation.
λ_I	Insurer's return coefficient for green reputation	0.5	Set to represent the reputational return from supporting green firms.
a	Baseline market demand	50	Rescaled from a standard potential-demand parameter to represent a market size of several dozen units.
b	Demand sensitivity to carbon intensity	2	Set at the midpoint of the reported 1.8–2.3 effect of carbon-label awareness on purchase intention.
d	Demand sensitivity to green reputation	1	Rescaled from evidence that favorable perceptions of corporate social responsibility improve product evaluations.
m	Demand sensitivity to premium incentives	1.5	Calibrated from estimated insurance-demand price elasticities ranging from −0.015 to −0.022.
ω	Effect of green reputation on financing cost	0.4	Rescaled from evidence that a one-standard-deviation increase in ESG score reduces bond spreads by approximately 14.5 basis points.
ρ	Discount rate	0.9	Calibrated to an annual discount rate of approximately 11%, within the recommended 8%–12% range.
e_0	Initial carbon intensity	20	Standardized from the carbon intensity of energy-intensive industries reported in the China Energy Statistical Yearbook 2022.
g_0	Initial green reputation	0	Set to zero because a conventional firm has not yet accumulated green reputation at transition initiation.
c_0	Initial financing-cost rate	8	Based on the 3.75% weighted-average corporate lending rate reported by the People's Bank of China, plus a 3%–5% risk premium for carbon allowance-backed financing.

5.2. Robustness tests

We conduct a Monte Carlo experiment to examine whether the cooperative advantage is robust to parameter uncertainty. Eight economically important parameters are independently drawn from uniform distributions spanning ± 20% of their baseline values. For each of 1,000 draws, we compute total profit under the Nash and cooperative regimes and record the difference.

We retain draws that yield nonnegative abatement investment and positive carbon intensity. We do not constrain the signs of the premium adjustment or market demand, enabling the experiment to cover a broad parameter region, $I_E \geq 0 e > 0$.

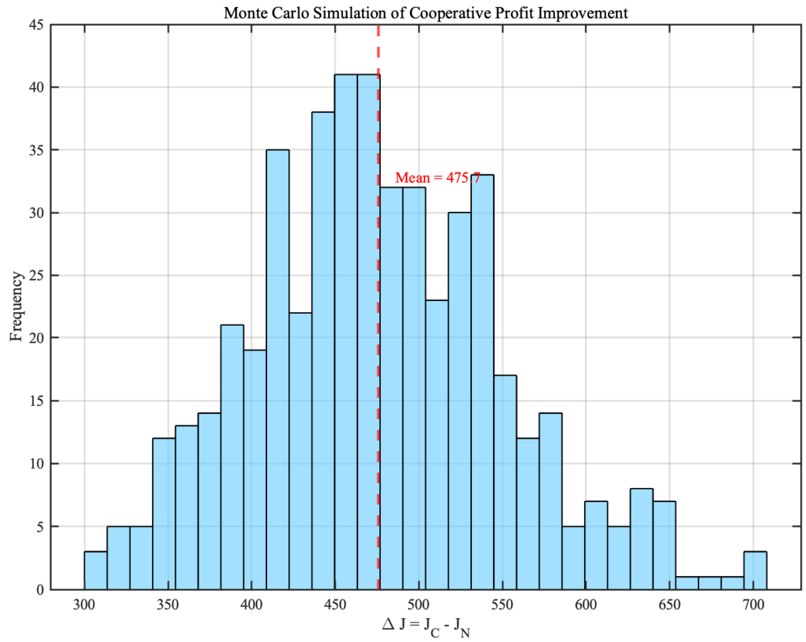

Figure 2. Monte Carlo simulation of the increase in cooperative profits.

Figure 2 reports the distribution of the cooperative profit gain. Its mean is 475.7, and the gain is positive for the admissible draws, indicating that the cooperative advantage is robust within the sampled parameter region.

5.3. Numerical results

Substituting the baseline parameters into Propositions 1–3 yields the trajectories and comparative statics reported in Figures 3–7. The computations were implemented in MATLAB R2021b.

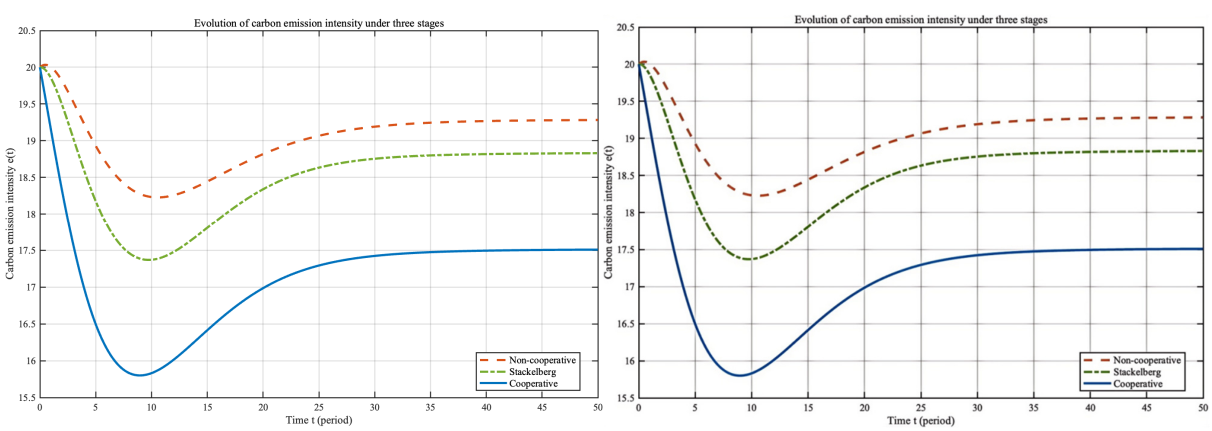

Figure 3. Comparison of carbon intensity trajectories across three stages.

(1) Figure 3 compares the carbon-intensity trajectories. Carbon intensity initially falls, subsequently rebounds, and eventually converges in all three regimes. The steady-state value is approximately 19.3 in the Nash game, 18.8 in the Stackelberg game, and 17.5 under cooperation. Cooperation therefore delivers the largest and most persistent reduction.

(2) Figure 4 compares player profits. Relative to the Nash game, the Stackelberg contract increases the firm's profit from 70.8 to 115.8 and the insurer's profit from 295.2 to 308.6. Cooperation raises the corresponding profits to 455.7 and 376.0. Thus, cost sharing yields a Pareto improvement, while joint optimization produces the largest total surplus.

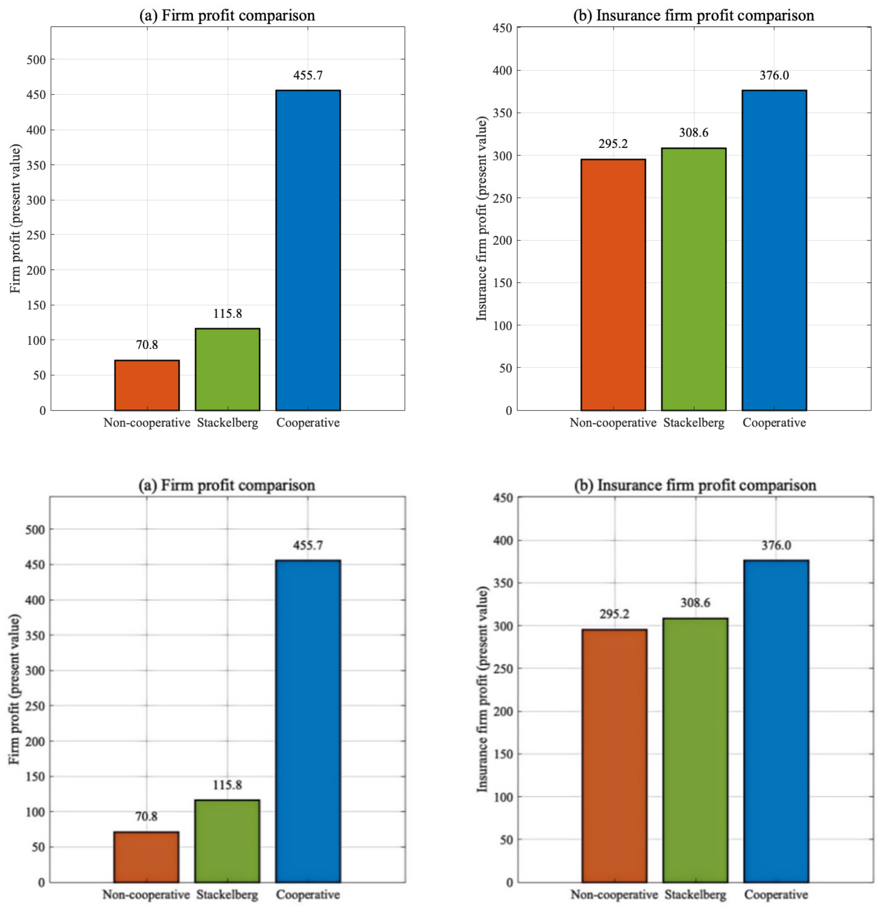

Figure 4. Comparison of corporate and insurer profits.

(3) Figure 5 shows that the firm's abatement investment increases convexly with the insurer's cost-sharing ratio. Investment rises from 1.09 at s = 0 to approximately 1.26 at the equilibrium rate s = 0.1429, an increase of about 15%. The contract therefore provides a strong abatement incentive, although the insurer must balance the incremental benefit against its sharing cost.

(4) Figure 6 shows that total profit decreases with the carbon price in all regimes. When the carbon price rises from 1 to 5, cooperative profit falls by approximately 18.2%, compared with 40.0% in the Stackelberg game and 43.4% in the Nash game. Cooperation therefore provides the greatest resilience to carbon-cost shocks.

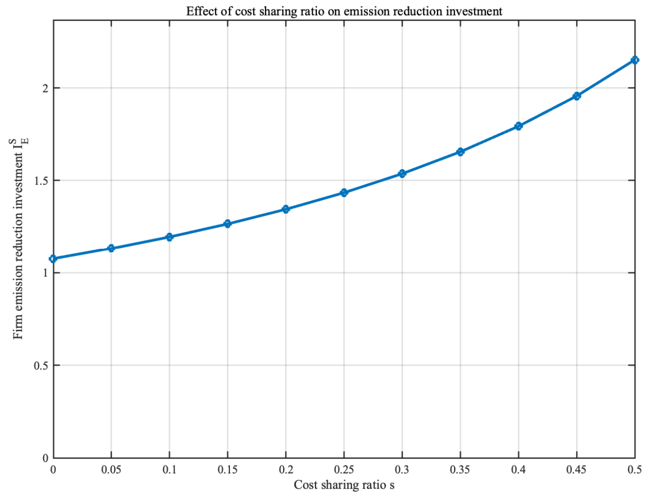

Figure 5. Diagram illustrating the relationship between cost-sharing ratios and investment in emission reduction.

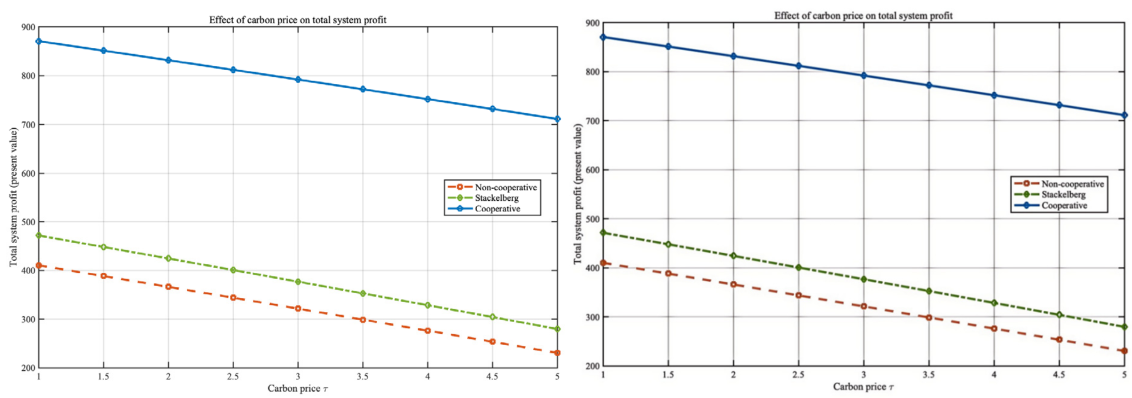

Figure 6. The impact of carbon pricing on total system profits.

(5) Figure 7 shows that the optimal premium adjustment decreases linearly with the regulatory penalty. Stronger regulation therefore induces larger premium discounts and creates a transmission channel from regulation to insurance pricing and then to abatement. The response is strongest under cooperation.

(6) Beyond single-parameter sensitivity, we examine the interactive effects of key parameters. Figure 8 presents a contour plot of the system profit as a function of carbon price τ and regulatory penalty coefficient θ_2. The contour lines indicate a clear substitution effect between these two policy instruments: A moderate carbon price (e.g., $\tau \approx 2.5$) combined with a moderate penalty $(\theta_2 \approx 3)$ yields system profits comparable to those achieved by a higher carbon price $(\tau \approx 4)$ with a lower penalty $(\theta_2 \approx 2)$. This result suggests that policymakers have flexibility in designing policy mixes; they can compensate for lower carbon prices with stronger regulatory penalties, and vice versa. Similarly, we find that the green reputation effect ω and the cost-sharing ratio s exhibit complementarity: The marginal benefit of cost-sharing increases with ω, indicating that insurers should offer more generous cost-sharing arrangements to firms with higher potential for green reputation accumulation.

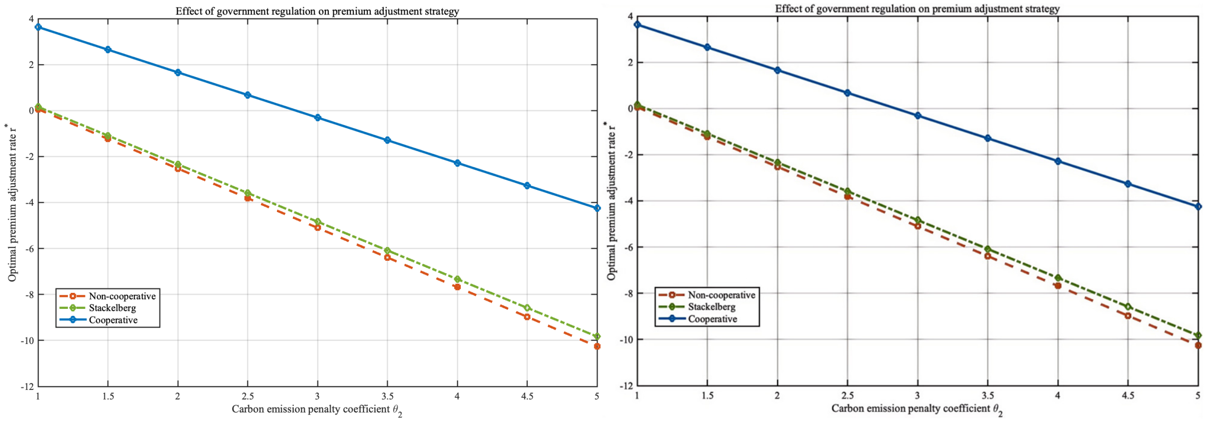

Figure 7. Diagram illustrating the relationship between the intensity of government regulation and premium adjustment strategies.

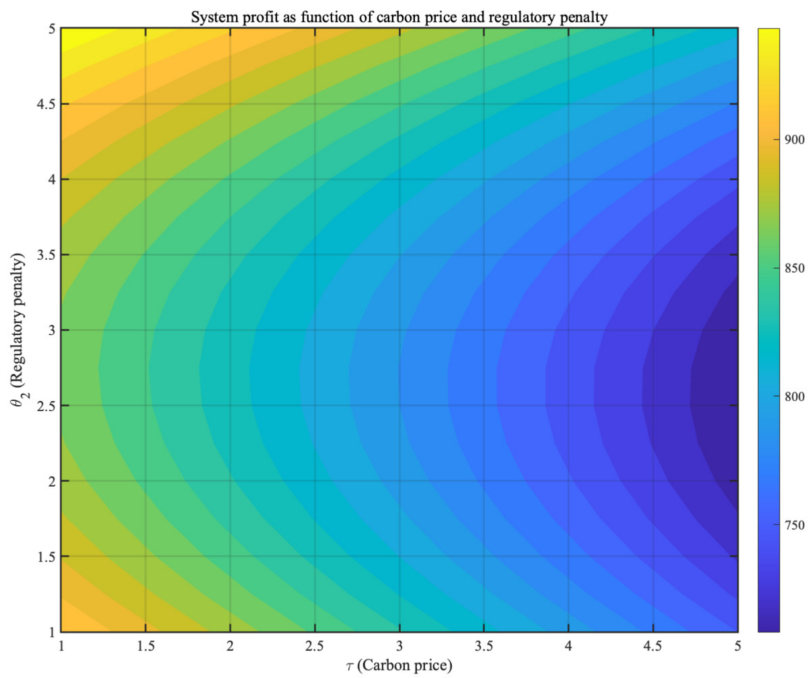

Figure 8. Contour plot of system profit versus carbon price. τ and regulatory penalty θ_2, showing the substitution effect between these two policy instruments.

5.4. Discussion and managerial implications

5.4.1. Cooperation and transition performance

Cooperation yields the lowest steady-state carbon intensity and the highest profits for both players. This result supports long-term firm–insurer partnerships that share transition information, risks, and gains.

Firms should treat insurers as transition partners rather than solely as risk carriers. Insurers, in turn, can combine underwriting, monitoring, and incentive design. A supporting policy platform can standardize carbon information and reduce the transaction costs of such partnerships.

For enterprises. The simulation results indicate that proactive engagement with insurers through cost-sharing arrangements can significantly enhance emission reduction efforts. The quantitative finding that the cooperative game yields a 96.0% higher system profit than the Stackelberg game and a 129.3% higher profit than the non-cooperative game provides a clear incentive for firms to pursue deep collaboration with insurers. Firms should prioritize building transparent environmental information disclosure systems to improve their green reputation, thereby reducing financing costs and insurance premiums simultaneously. Additionally, firms should recognize that the optimal cost-sharing ratio ($s^* \approx 0.1429$) identified in our simulation represents a benchmark for negotiating green insurance contracts; accepting a cost-sharing arrangement below this threshold may indicate suboptimal insurer commitment.

5.4.2. Design of cost-sharing mechanisms

The cost-sharing ratio is positively associated with abatement investment. At the equilibrium rate, investment increases by more than 60% relative to the no-sharing benchmark. Properly designed sharing therefore relaxes the firm's financing burden while improving both players' payoffs.

The contract should link the sharing rate to verifiable carbon performance and the insurer's risk-bearing capacity. Standardized measurement and certification can improve enforceability and reduce opportunistic behavior.

For insurers. The optimal cost-sharing ratio $s^* = 1/7 \approx 0.1429$ identified in our simulation provides a quantitative benchmark for designing green insurance products. Insurers should adopt dynamic premium adjustment mechanisms that respond to firms' real-time carbon performance, rather than relying on static annual assessments. The finding that the cooperative game yields a 94.1% higher premium adjustment rate (i.e., less negative premium reduction) than the non-cooperative game suggests that insurers can substantially enhance their premium incentives under collaborative governance structures. Insurers should also consider tailoring cost-sharing ratios to firms' marginal profit characteristics, as the condition $2\pi_I > \pi_E$ indicates that cost-sharing is only profitable when the insurer's marginal benefit from the firm's emission reduction is sufficiently large relative to the firm's own marginal benefit.

5.4.3. Coordination of carbon pricing and regulation

Higher carbon prices reduce total profit in all three regimes, although cooperation substantially moderates the loss. Carbon pricing should therefore be coordinated with financial incentives that preserve firms' capacity to invest in abatement. A stronger regulatory penalty leads the insurer to offer a larger premium discount, transmitting policy incentives through the insurance market. Cooperation amplifies this transmission effect.

Firms and insurers should use state-contingent contracts that adjust premiums and cost-sharing terms to carbon prices and regulatory intensity. Policymakers can support this mechanism through credible carbon-price signals, consistent enforcement, and standardized disclosure.

For regulators. The contour analysis (Figure 8) indicates a substitution effect between carbon prices and regulatory penalties: A moderate carbon price (e.g., $\tau \approx 2.5$) combined with a moderate penalty coefficient ($\theta_2 \approx 3$) yields system profits comparable to those achieved by a higher carbon price ($\tau \approx 4$) with a lower penalty ($\theta_2 \approx 2$). This result suggests that policymakers have flexibility in designing policy mixes; they can compensate for lower carbon prices with stronger regulatory enforcement, and vice versa. Regulators should consider integrating green insurance into the broader carbon market infrastructure, for instance, by offering regulatory incentives (e.g., lower penalty rates) for firms that adopt green insurance policies, thereby creating a virtuous cycle of market-based and regulatory-based incentives.

6. Conclusions

In this paper, we study firm–insurer coordination during corporate low-carbon transition using a multi-stage differential game.

The model links carbon intensity, green reputation, and carbon allowance-backed financing costs to abatement investment, premium adjustment, and cost sharing. Theoretical analysis and numerical experiments show that cost sharing can yield a Pareto improvement when the insurer's marginal return is sufficiently high, while joint optimization produces the lowest carbon intensity and highest total profit. Regulatory penalties can also be transmitted through premium discounts, with the strongest effect under cooperation.

The results imply that firms should manage green reputation as an economic asset and use insurance-based contracts to support transition investment. Insurers can move beyond passive risk transfer by linking premiums and cost sharing to verified carbon performance. Policymakers can strengthen these incentives by coordinating carbon pricing, disclosure standards, and environmental enforcement.

The analysis assumes symmetric information, deterministic state dynamics, and a bilateral market. Researchers could introduce asymmetric information, stochastic policy or technology shocks, and competition among firms and financial institutions. Empirical calibration using carbon-market and insurance-contract data would also improve external validity.

Overall, the results show that an evolving governance structure can reconcile emission reduction with long-run profitability. Green insurance is most effective when pricing, cost sharing, and financing respond jointly to the firm's transition state.

7. Limitations and future research

Several limitations of this study should be acknowledged. First, our model assumes perfect information and deterministic dynamics. In practice, information asymmetry between firms and insurers may lead to adverse selection, while carbon price volatility introduces stochastic uncertainty. Second, we focus on bilateral interactions between a single firm and a single insurer; researchers could extend the framework to multiple insurers competing for green business, or to heterogeneous firms with different emission profiles. Third, the model parameters are calibrated using Chinese data; applying the framework to other institutional contexts would require re-calibration. Fourth, while we identify the optimal cost-sharing ratio analytically, we do not model the bargaining process through which this ratio is negotiated. Researchers could incorporate bargaining games or dynamic renegotiation. Fifth, our model does not include the core risk-transfer function of green insurance, compensation for environmental violations, which would require a stochastic differential game framework with jump processes.

Looking forward, we identify several promising extensions: (i) Incorporating government intervention (e.g., subsidies for green insurance premiums or tax incentives for insured firms) to examine how public policy interacts with private insurance mechanisms; (ii) introducing heterogeneous insurance products (e.g., different coverage levels, deductibles, and premium structures) to analyze product differentiation in the green insurance market; (iii) considering technological progress as an endogenous driver of emission reduction cost reductions; (iv) developing a stochastic differential game framework to capture carbon price volatility and policy uncertainty; and (v) conducting empirical validation using real insurance contract data from green insurance pilots in China and other countries, which would significantly enhance the practical relevance of our findings.

Author contributions

Yiqin Wu: Conceptualization, Methodology, Formal analysis, Investigation, Software, Visualization, Writing – Original Draft. Zhenyong Wu: Conceptualization, Methodology, Supervision, Project administration, Writing – Review & Editing, Validation. Ran Gao: Data curation, Formal analysis, Software, Visualization, Writing – Original Draft. Yujie Huang: Investigation, Formal analysis, Validation, Writing – Review & Editing. Yu Zhang: Literature review, Resources, Visualization, Writing – Review & Editing. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

During the preparation of this manuscript, the authors used ChatGPT solely to improve the language, clarity, and readability of the text. All AI-assisted revisions were carefully reviewed and edited by the authors. The authors take full responsibility for the content and integrity of the final manuscript.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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