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Research article

Pricing and sales-cycle optimization for deteriorating fresh agricultural products with live-streaming investment and discounts

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Abstract: Live-streaming e-commerce creates a channel for selling fresh agricultural products, but e-tailers that operate with a fixed order quantity must balance demand conversion, inventory clearance, and deterioration losses. This paper developed a two-stage inventory model with a Weibull time-varying deterioration rate, live-streaming investment, deterioration-stage discounts, and live-streaming sales commission costs. The model jointly determines the regular selling price, sales-cycle length, and live-streaming investment intensity to maximize average profit per unit time. A second-order Maclaurin approximation and a structural solution method are used to identify candidates at the inventory-depletion boundary and exogenous sales-cycle bounds. Results showed that live-streaming investment and deterioration-stage discounts accelerate demand and clearance but do not necessarily increase profit, because their benefits may be offset by investment costs, sales commission costs, lower unit revenue, and deterioration-related costs. The optimal sales cycle is determined by boundary comparison rather than an interior first-order condition. The study provides an integrated basis for coordinated pricing, live-streaming investment, and inventory-clearance decisions in fresh-product e-commerce.

Keywords: fresh agricultural products; live-streaming investment; deterioration-stage discounts; weibull deterioration rate; sales cycle decisions; inventory management; average profit optimization
Mathematics Subject Classification: 90B05, 90B50, 90C30

1. Introduction

In recent years, the development of online retailing, rural e-commerce, and live-streaming e-commerce has been reshaping the circulation of agricultural products and creating operational scenarios for the online sales of fresh agricultural products. In 2025, China's national online retail sales reached RMB 15.9722 trillion, representing a year-on-year increase of 8.6% [1]. In the same year, rural online retail sales and online retail sales of agricultural products reached RMB 3.0 trillion and RMB 783.31 billion, respectively, with year-on-year growth rates of 6.7% and 9.9%; leading e-commerce platforms conducted more than four million live-streaming sessions in support of agricultural product sales, with orders of specialty agricultural products exceeding 10 billion, and average daily shipments surpassing 20 million orders [2]. In addition, the Report on the High-Quality Development of Live-Streaming E-commerce shows that China's live-streaming e-commerce market reached approximately RMB 5.8 trillion in 2024, and that the compound annual growth rate of China's live-streaming e-commerce market from 2024 to 2026 is expected to be 18.0% [3]. These data indicate that live-streaming e-commerce is no longer merely a promotional tool but has become an important operational mode linking product display, consumer interaction, and inventory sales. In practice, this operating mode is particularly relevant to self-operated fresh agricultural product e-tailers that sell short-shelf-life products, such as berries, leafy vegetables, seasonal fruits, and other fresh produce, through front-warehouse-based fulfillment systems [4]. Since these products are usually procured, sorted, and allocated to the fulfillment system before the selling window starts, the batch quantity available during the sales cycle is fixed and cannot be replenished or adjusted within that cycle. During the sales cycle, live-streaming sales can provide product display, interactive communication, and scenario-based presentation. Empirical evidence from fresh-product live broadcasts further indicates that live-broadcast features affect purchase intention through perceived value and trust [5]. As products approach and exceed the freshness threshold, deterioration-stage discounts are introduced to stimulate demand and accelerate inventory clearance. Therefore, self-operated fresh agricultural product e-tailers need to balance live-streaming investment, discount-induced demand stimulation, inventory holding, and deterioration-related losses within a limited sales cycle.

However, fresh agricultural products have limited shelf lives and time-varying deterioration characteristics, and the demand increase generated by live-streaming does not necessarily translate into profit growth. On the one hand, live-streaming sales can enhance consumer trust and improve demand conversion through product demonstration, real-time interaction, and real-time responses to consumers' questions. On the other hand, live-streaming investment, live-streaming sales commission costs, and traffic maintenance increase operating costs, while deterioration-stage discounts reduce unit sales revenue. If the sales cycle is too short, potential demand may not be sufficiently realized; if it is too long, inventory holding costs, deterioration losses, and end-of-cycle disposal risks may increase. Therefore, the core issue addressed in this paper is not simply how to increase live-streaming traffic or deepen discounts, but how to coordinate live-streaming investment, deterioration-stage discounts, inventory clearance, and profit efficiency under a fixed order quantity.

Deteriorating inventory studies provide a foundation for this problem. Because deteriorating items experience quality decay or quantity loss during the sales cycle, existing research has examined how deterioration-rate specifications, ordering and pricing policies, and inventory control shape the management of deterioration losses. Zhang and Mo [6] developed an ordering and pricing model incorporating a Weibull deterioration function and price discounts. Jiang et al. [7], Ai et al. [8], Li et al. [9], and Cui et al. [10] considered joint ordering or replenishment, pricing, and preservation-technology decisions in deteriorating-inventory settings. Kaushik [11–13] further examined inventory models with Weibull deterioration under ramp-type demand, linear demand, and changing consumer preferences. Collectively, these studies show that both the deterioration specification and the demand environment are central to inventory decisions.

Recent deteriorating-inventory studies have extended the decision scope to markdown policies, multiple shelf-life phases, waste reduction, and dynamic control. Moshtagh et al. [14,15] studied markdown policies for products with fixed shelf lives and dynamic inventory-pricing control for perishable products with multiple shelf-life phases, respectively. Zhou et al. [16] analyzed two-period pricing and inventory decisions for perishable products with partial lost sales. Hasiloglu-Ciftciler and Kaya [17] developed a bi-objective dynamic-programming model for a two-branch retail system, jointly determining ordering, inventory sharing, and the prices of new and old perishable foods to maximize profit and minimize waste. Salmasnia and Kohan [18] incorporated quality deterioration, quantity deterioration, pricing, inventory control, and preservation technology investment into a unified framework. Kavoosi et al. [19] and Mohamadi et al. [20] further used deep reinforcement learning methods to address dynamic pricing, inventory control, and vendor-managed inventory problems for perishable products. Hou et al. [21] analyzed pricing and inventory strategies for perishable products in competitive markets with strategic consumers. These studies show that deteriorating inventory management has evolved from traditional EOQ or replenishment models toward pricing, preservation, dynamic control, and waste reduction. However, most of these studies still focus on traditional sales, general e-commerce, or preservation investment scenarios, and insufficient attention has been paid to inventory clearance under the joint effects of demand conversion, live-streaming sales commission costs, and deterioration-stage discounts in live-streaming sales.

Compared with general deteriorating items, fresh agricultural products place particular emphasis on freshness perception, sales-cycle arrangements, and demand responses. Chen and Kang [22] studied joint pricing and inventory replenishment in dual-channel agri-food sales. Related studies examined dynamic pricing in competitive dual-channel fresh-food supply chains [23], joint freshness-keeping and promotion efforts [24], information-sharing decisions [25], and sales-mode choice conditional on product freshness [26]. Zhang et al. [27] studied joint pricing and inventory decisions for perishable agricultural products in an e-commerce setting. In a broader dual-channel online context, Cao et al. [28] examined how online-channel selection, pricing, and service levels affect demand and profit. Du and Lu [29] examined visualization-service investment strategies in a fresh agricultural product supply chain comprising a manufacturer and a self-operated e-tailer. Together, these studies show that fresh-product operations involve pricing, freshness, demand response, and channel-related decisions. Nevertheless, they primarily address dual-channel structures, preservation, promotion, information sharing, or sales-mode choice, rather than the joint interaction among live-streaming investment, deterioration-stage discounts, live-streaming sales commission costs, inventory deterioration, and the sales-cycle decision under fixed-batch procurement.

Live-streaming e-commerce studies provide an important basis for understanding demand conversion and operational decision-making in fresh agricultural product operations. Unlike ordinary online sales, live-streaming sales can influence consumer purchase decisions through real-time interaction, product demonstration, streamer explanation, and scenario-based presentation. Xiao et al. [30] discussed the operational and supply-chain characteristics of live-streaming e-commerce, emphasizing traffic conversion, streamer incentives, consumer interaction, and supply-chain responsiveness. From the consumer-behavior perspective, Jin and Zhang [31] showed that streamer characteristics, consumer interaction, and trust mechanisms influence purchase intention for green agricultural products. Sun et al. [32] further found that visibility affordance, metavoicing affordance, and guidance shopping affordance can influence purchase intention through live-streaming engagement. Guo et al. [33] showed that live-streaming features increase consumers' perceived value and reduce perceived uncertainty in cross-border online shopping, while monetary savings can further strengthen the value effect of live streaming. Based on survey evidence from China, Sima et al. [34] found that price concessions and interaction frequency on live-streaming platforms positively affect purchase intention, and that consumer trust in e-commerce strengthens the effect of price concessions. These studies suggest that live streaming affects demand not only by increasing product exposure but also by improving information transmission, reducing perceived uncertainty, strengthening trust, and enhancing consumers' responses to monetary incentives. Building on these demand-conversion mechanisms, operational studies have examined how promotional exposure, live-streaming decisions, channel structures, and related costs affect inventory, pricing, and channel choices. San-José et al. [35] incorporated advertising frequency into an inventory model with price- and time-dependent demand. Liang and Wang [36] examined dynamic pricing and production decisions across a live-streaming presale stage and a conventional-sales stage for fresh agricultural products. At the supply-chain level, Wang et al. [37] and Zhang et al. [38] studied traffic investment, channel selection, and sales-mode choice in agricultural-product live-streaming supply chains. Yu et al. [39] analyzed competitive manufacturers' live-streaming channel strategies and showed that product substitutability, commission rates, and market-expansion effects shape channel selection and pricing outcomes. Chen et al. [40] studied pricing and coordination in a dual-channel supply-chain model with streamer marketing effort. Zhang et al. [41], Zhang and Zhang [42], Yang et al. [43], and Zhou et al. [44] respectively examined product-quality choice, hybrid-channel pricing and channel selection, live-stream promotional pricing, and platform promotion strategies. Chen et al. [45] and Wang et al. [46] investigated live-streaming selling modes, return-freight insurance, and the introduction of live streaming in dual-channel retailing. Taken together, these studies examined either demand-conversion mechanisms or particular live-streaming operational choices. The joint interaction among live-streaming investment, deterioration-stage discounts, live-streaming sales commission costs, inventory deterioration, and the sales-cycle decision under fixed-batch procurement remains insufficiently examined.

Table 1 compares representative studies across the modeling features most closely related to the present study. Existing research has examined Weibull deterioration, two-stage structures, stage-based discounts, digital service or live-streaming investment, and revenue-linked commission costs in different combinations. The present study integrates these elements within a fixed-batch deteriorating-inventory framework and further characterizes the sales-cycle decision by comparing the inventory-depletion boundary with the feasible exogenous bounds.

Table 1. Comparison between this study and representative studies on deteriorating inventory and live-streaming e-commerce.

Study	Weibull deterioration rate	Two-stage structure	Stage-based discounts	Live-streaming promotion	Live-streaming sales commission cost
[9] Li et al. (2016)		√			
[6] Zhang and Mo (2020)	√		√		
[8] Ai et al. (2020)		√			
[11] Kaushik (2023)	√				
[16] Zhou et al. (2023)		√	√		
[37] Wang et al. (2023)				√	
[38] Zhang et al. (2023)				√	√
[12] Kaushik (2024)	√				
[36] Liang and Wang (2024)		√		√	√
[44] Zhou et al. (2024)				√	√
[13] Kaushik (2025)	√				
[14] Moshtagh et al. (2025a)			√		
[15] Moshtagh et al. (2025b)		√	√		
[40] Chen et al. (2025, streamer effort)				√	
[41] Zhang et al. (2025)					√
[42] Zhang and Zhang (2025)				√	
[45] Chen et al. (2025, selling mode)				√	
[46] Wang et al. (2025)				√	
[43] Yang et al. (2026)			√	√	√
Present study	√	√	√	√	√

The main contributions of this paper are as follows:

First, this study formulates a two-stage inventory model for a self-operated fresh agricultural product e-tailer operating under fixed-batch procurement. The model jointly incorporates Weibull time-varying deterioration, deterioration-stage discounting, live-streaming investment, and live-streaming sales commission costs into inventory evolution and profit evaluation.

Second, the demand specification captures the joint role of live-streaming investment and deterioration-stage discounts in demand conversion and inventory clearance, while the cost structure incorporates the corresponding revenue-linked commission cost. This formulation makes it possible to assess whether additional sales stimulation improves average profit per unit time.

Third, for a given regular selling price and live-streaming investment intensity, the study establishes that the interior stationary point of the sales-cycle subproblem does not maximize average profit per unit time. The optimal sales cycle is therefore identified by comparing the inventory-depletion boundary with the exogenous sales-cycle bounds.

2. Problem setting and assumptions

2.1. Problem setting

This paper considers the inventory and sales-cycle decision problem of a self-operated fresh agricultural product e-tailer in a live-streaming sales context. Before the start of each sales cycle, the firm procures a fixed batch of products and relies on the front warehouse or a corresponding local fulfillment system to complete product storage, live-streaming sales, and order fulfillment during the cycle. Thus, the sales cycle in this paper can be interpreted as the selling window after a batch of fresh agricultural products has been allocated to the fulfillment system, and Q denotes the saleable quantity available at the beginning of this window.

Fresh agricultural products have limited shelf lives, and their inventory status changes as the sales cycle progresses. In the early stage of the sales cycle, product quality is relatively stable, and the firm mainly relies on the regular selling price and live-streaming presentation to achieve normal sales. As products gradually approach and exceed the freshness threshold, inventory enters a period in which deterioration risk begins to accumulate. At this point, the firm usually needs to introduce a discount strategy while continuing live-streaming sales to stimulate demand and accelerate inventory turnover.

Accordingly, the firm jointly determines the regular selling price, sales-cycle length, and live-streaming investment intensity to maximize average profit per unit time, while balancing sales revenue, live-streaming-related costs, inventory holding costs, and deterioration-related losses.

2.2. Model assumptions

To characterize the joint decision-making process of self-operated fresh agricultural product e-tailers regarding pricing, sales cycle length, and live-streaming investment in a live-streaming sales context, while ensuring the analytical tractability and solvability of the model, the following assumptions are made.

Assumption 1. Consistent with non-instantaneous deterioration settings [8,9], the product remains in the non-deterioration stage during the interval $[0,l]$, and no deterioration occurs in this stage. When $t > l$, the product enters the deterioration risk accumulation stage, and its deterioration rate is described by a Weibull-type time-varying function with a freshness threshold [6,11], defined as $\theta(t)=\alpha\beta(t-l)^{\beta-1}$, where $\alpha > 0$ is the scale parameter of the deterioration rate, and $\beta > 1$ is the shape parameter, indicating that the product deterioration rate increases over time. It should be noted that the "deterioration stage" in this paper does not mean that all products have deteriorated. Rather, it denotes the stage after the freshness threshold in which the remaining products are exposed to time-varying deterioration risk. In this stage, products that have not deteriorated can still be sold at a discount, whereas deteriorated products immediately exit the inventory system.

Assumption 2. Demand in the non-deterioration stage is given by $D_1 = a - bp + \gamma\tau$, where $a > 0$ denotes the potential market demand, $b > 0$ is the price sensitivity coefficient, τ represents the live-streaming investment intensity, and $\gamma > 0$ is the sensitivity coefficient of demand to live-streaming investment. Demand in the deterioration stage is specified as $D_2 = (a - b\rho p + \gamma\tau)(1 + \omega(1-\rho))$, where $0 < \rho < 1$ is the discount coefficient, and $\omega > 0$ is the live-streaming-discount synergy coefficient. The term $1-\rho$ represents the discount depth. The factor $1+\omega(1-\rho)$ captures the additional demand amplification arising when the deterioration-stage discount is communicated and implemented through live-streaming sales. In live-streaming platforms, price concessions and interaction frequency have been shown to positively affect purchase intention, and consumer trust can further strengthen the effect of price concessions [34]. Hence, ω represents the extent to which live-streaming sales enhance the demand-conversion effect of the same deterioration-stage discount.

Assumption 3. Motivated by the front-warehouse setting in fresh food e-commerce [4], this paper assumes a fixed-batch procurement mode. Before the selling window begins, the firm procures and allocates a single batch of Q units to the fulfillment system, and no replenishment occurs during the sales cycle. Accordingly, this paper focuses on within-cycle operational decisions under a given batch quantity rather than cross-cycle order-quantity optimization. Stockouts are not allowed, and the inventory level during the sales cycle should satisfy the non-negativity constraint. If there is remaining inventory at the end of the cycle, it is regarded as leaving the normal sales state of the current cycle and is included in disposal losses. If the inventory-depletion boundary is triggered, the boundary sales cycle at which inventory is exactly depleted is determined by $I_2(L_B)=0$. The sales cycle length L satisfies $L \geq l$.

Assumption 4. Drawing on fresh agricultural product live-streaming studies [36,38], live-streaming investment cost describes the firm's expenditure on improving live-streaming presentation, content planning, interactive explanation, and demand-conversion efficiency, and is specified as $C_{effort}=\dfrac{k\tau^2}{2}$, where $k>0$ is the live-streaming investment cost coefficient; a larger k means that the same increase in τ requires a greater expenditure. In addition, live-streaming sales commission cost captures revenue-linked commissions in live-streaming sales and is denoted by $C_{com}=\eta R$, where $\eta \in (0,1)$ represents the live-streaming sales commission rate, and R denotes the total live-streaming sales revenue over the entire cycle. In addition, constrained by the firm's marketing budget, streamer time availability, and platform traffic capacity, the live-streaming investment intensity satisfies the exogenous upper-bound constraint $0 \leq \tau \leq \tau_{max}$, where $\tau_{max} \geq 0$ denotes the maximum live-streaming investment intensity available to the e-tailer during a sales cycle.

3. Model formulation

3.1. Deterioration rate and demand functions

Consider a self-operated fresh agricultural product e-tailer that sells a single type of perishable fresh agricultural product through live streaming within one sales cycle. At the beginning of the cycle, the firm procures a fixed quantity Q of products in a single batch and makes no replenishment during the cycle. The product remains in the non-deterioration stage in the first stage, during which no deterioration occurs. After exceeding the freshness threshold, the product enters the deterioration stage and begins to suffer time-varying losses.

This paper represents the time-varying deterioration rate of the product during the deterioration stage in the following Weibull form:

$$\theta(\mathrm{t})=\begin{cases}0, & 0\le t<l\\ \alpha\beta(t-l)^{\beta-1}, & l\le t<L\end{cases} \tag{1}$$

where $\alpha>0$ is the deterioration-rate scale coefficient, $\beta>1$ is the deterioration-rate shape parameter, l is the length of the non-deterioration stage, and L is the sales cycle length, with $L>l$.

Demand is divided into two stages. According to Assumption 2, demand in the non-deterioration stage and the deterioration stage is given by $D_1=a-bp+\gamma\tau$ and $D_2=(a-bp\rho+\gamma\tau)(1+\omega(1-\rho))$, respectively. The definitions of the related parameters are provided in Table 2 and Assumption 2.

Table 2. Notations.

Symbol	Description
p	Regular selling price
L	Sales cycle length
τ	Live-streaming investment intensity
ρ	Discount coefficient in the deterioration stage
Q	Fixed order quantity
a	Potential market demand
b	Price sensitivity coefficient
γ	Live-streaming demand sensitivity coefficient
k	Live-streaming investment cost coefficient
c	Unit purchasing cost
h	Unit inventory holding cost per unit time
s	Unit deterioration and disposal cost
K	Fixed ordering cost
α	Deterioration-rate scale coefficient
β	Deterioration-rate shape parameter
l	Length of the non-deterioration stage
ω	Live-streaming-discount synergy coefficient
η	Live-streaming sales commission rate

3.2. Two-stage inventory dynamics

To characterize the dynamic changes in inventory caused by demand consumption and product deterioration during the sales cycle, let $I_1(t)$ and $I_2(t)$ denote the inventory levels at time t in the non-deterioration stage and the deterioration stage, respectively. The inventory differential equations for the two stages are then formulated as follows.

3.2.1. Inventory model in the non-deterioration stage

During the non-deterioration stage $0\le t\le l$, fresh agricultural products do not deteriorate, and the inventory level is depleted only by continuous market demand. Therefore, the inventory differential equation is

$$\frac{dI_1(t)}{dt} = -D_1, 0 \le t \le l \tag{2}$$

With the boundary condition $I_1(0) = Q$, where Q is the fixed initial order quantity; solving this first-order linear differential equation gives the inventory level at any time during the non-deterioration stage as

$$I_1(t) = Q - D_1 t, 0 \le t \le l \tag{3}$$

When $t = l$, the non-deterioration stage ends, and the inventory level at this moment becomes the initial inventory for the deterioration stage, namely,

$$I_1(l) = Q - D_1 l \tag{4}$$

3.2.2. Inventory model in the deterioration stage

After entering the deterioration stage, the inventory level is affected by both continuous demand consumption and Weibull time-varying deterioration. Therefore, the inventory differential equation is given by

$$\frac{dI_2(t)}{dt} + \theta(t) I_2(t) = -D_2, l \le t \le L \tag{5}$$

The boundary condition is the initial inventory level at the beginning of the deterioration stage, namely $I_2(l) = I_1(l)$, where $\theta(\mathrm{t}) = \alpha\beta(t-l)^{\beta-1}$ is the Weibull time-varying deterioration rate.

This first-order linear nonhomogeneous differential equation is solved by using the integrating factor method. The integrating factor is $e^{\int_l^t \theta(x)dx} = e^{\alpha(t-l)^\beta}$. Substituting the boundary condition into the solution yields the exact analytical expression for the inventory level at any time during the deterioration stage:

$$I_2(t) = \mathrm{e}^{-\alpha(t-l)^\beta}[I_1(l) - D_2 \int_l^t \mathrm{e}^{\alpha(x-l)^\beta} dx], l \le t \le L \tag{6}$$

when $t = L$, the sales cycle ends, and the inventory level at this moment represents the disposal quantity remaining at the end of the cycle, namely,

$$I_2(L) = \mathrm{e}^{-\alpha(L-l)^\beta}[I_1(l) - D_2 \int_l^L \mathrm{e}^{\alpha(x-l)^\beta} dx] \tag{7}$$

3.3. Second-order Maclaurin approximation

The exact solution for the inventory level in the deterioration stage contains two types of transcendental exponential terms, $\mathrm{e}^{\alpha(x-l)^\beta}$ and $\mathrm{e}^{-\alpha(t-l)^\beta}$, which make direct symbolic integration and subsequent optimization difficult. Let the expansion variables be $z = \alpha(x-l)^\beta$ and $z = -\alpha(t-l)^\beta$, respectively. Since $l \le x \le t \le L$, both expansion variables satisfy $|z| \le \alpha(L-l)^{\beta}$. Therefore, $\alpha(L-l)^{\beta} < 0.3$ is used as the applicability condition for the second-order Maclaurin approximation. Under this condition, the exponential terms can be expanded around $z=0$ by using a second-order Maclaurin expansion.

First, applying the second-order Maclaurin expansion to the general exponential function e^{z} around $z=0$ gives

$$e^{z} = 1 + z + \frac{z^{2}}{2} + o(z^{3}) \tag{8}$$

where $o(z^{3})$ denotes the higher-order truncation term after the second-order Maclaurin expansion.

Substituting $z=\alpha(x-l)^{\beta}$ into Eq (8), we have $e^{\alpha(x-l)^{\beta}} \approx 1+\alpha(x-l)^{\beta}+\frac{1}{2}\alpha^{2}(x-l)^{2\beta}$.

Then, substituting this approximation into the inner integral term in Eq (6) and integrating term by term, we obtain

$$\begin{aligned}&\int_{l}^{t} e^{\alpha(x-l)^{\beta}} dx \approx \int_{l}^{t} [1+\alpha(x-l)^{\beta}+\frac{1}{2}\alpha^{2}(x-l)^{2\beta}]dx \\ &= (t-l)+\frac{\alpha}{\beta+1}(t-l)^{\beta+1}+\frac{\alpha^{2}}{2(2\beta+1)}(t-l)^{2\beta+1}\end{aligned} \tag{9}$$

Similarly, substituting $z=-\alpha(t-l)^{\beta}$ into Eq (8), we know that

$$\mathrm{e}^{-\alpha(t-l)^{\beta}} \approx 1-\alpha(t-l)^{\beta}+\frac{1}{2}\alpha^{2}(t-l)^{2\beta} \tag{10}$$

Substituting Eqs (9) and (10) into the exact inventory solution in the deterioration stage, and using the end-of-stage inventory in the non-deterioration stage, $I_{1}(l)=Q-D_{1}l$, yields the second-order approximation of the inventory level in the deterioration stage with the fixed order quantity Q:

$$I_{2}(t) \approx \left[1-\alpha(t-l)^{\beta}+\frac{1}{2}\alpha^{2}(t-l)^{2\beta}\right]\left\{Q-D_{1}l-D_{2}\left[(t-l)+\frac{\alpha}{\beta+1}(t-l)^{\beta+1}+\frac{\alpha^{2}}{2(2\beta+1)}(t-l)^{2\beta+1}\right]\right\} \tag{11}$$

To further improve the accuracy of the approximation, the expanded inventory function is further expanded with respect to α around 0, and only terms up to the second order in α are retained. Third- and higher-order terms are removed, thereby obtaining a second-order approximate (SOA) inventory function for subsequent inventory integration and cost calculation. After applying the second-order Maclaurin expansion to the exponential terms, the truncation error is dominated by third- and higher-order terms, with the leading order given by $o(\alpha^{3}(t-l)^{3\beta})$. Under the applicability condition $\alpha(L-l)^{\beta} < 0.3$, and given $l < t < L$, it follows that $\alpha(t-l)^{\beta} \le \alpha(L-l)^{\beta} < 0.3$. Therefore, the effect of higher-order truncation terms on the inventory integral and profit function is relatively small. To further verify the reliability of the approximation, the deviation between the second-order approximation and the exact solution will be examined in the numerical analysis by simulating the original exact ordinary differential equation.

3.4. Inventory integration and average profit function

3.4.1. Inventory integration

To facilitate the subsequent construction of the inventory holding cost and profit function, the inventory integrals of the two stages need to be calculated separately.

According to Eq (3), the inventory level in the non-deterioration stage $0 \le t \le l$ is $I_1(t) = Q - D_1 t$. Therefore, the inventory integral in the first stage is

$$\int_0^l I_1(t)dt = \int_0^l (Q - D_1 t)dt = Ql - \frac{1}{2}D_1 l^2 \tag{12}$$

According to Eq (4), the initial inventory level in the deterioration stage is $I_1(l) = Q - D_1 l$. Based on the second-order Maclaurin approximation in Eq (11) and the treatment that retains terms up to the second order in α at the inventory-function level, the inventory level in the second stage can be approximated as

$$\begin{aligned} I_2(t) &\approx (Q - D_1 l) - D_2(t-l) - \alpha(Q - D_1 l)(t-l)^{\beta} + \frac{\alpha\beta D_2}{\beta+1}(t-l)^{\beta+1} \\ &+ \frac{\alpha^2}{2}(Q - D_1 l)(t-l)^{2\beta} - \frac{\alpha^2\beta^2 D_2}{(\beta+1)(2\beta+1)}(t-l)^{2\beta+1}, l \le t \le L \end{aligned} \tag{13}$$

By integrating Eq (13) term by term over the interval $l \le t \le L$, the approximate inventory integral in the second stage is obtained as

$$\begin{aligned} \int_l^L I_2(t)dt &\approx \int_l^L [(Q - D_1 l) - D_2(t-l) - \alpha(Q - D_1 l)(t-l)^{\beta} + \frac{\alpha\beta D_2}{\beta+1}(t-l)^{\beta+1} \\ &+ \frac{\alpha^2}{2}(Q - D_1 l)(t-l)^{2\beta} - \frac{\alpha^2\beta^2 D_2}{(\beta+1)(2\beta+1)}(t-l)^{2\beta+1}] \\ &= (Q - D_1 l)(L-l) - \frac{D_2(L-l)^2}{2} - \frac{\alpha(Q - D_1 l)(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha\beta D_2(L-l)^{\beta+2}}{(\beta+1)(\beta+2)} \\ &+ \frac{\alpha^2(Q - D_1 l)(L-l)^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha^2\beta^2 D_2(L-l)^{2\beta+2}}{2(\beta+1)^2(2\beta+1)} \end{aligned} \tag{14}$$

Combining Eqs (12) and (14), the total inventory integral over the entire sales cycle is

$$\int_0^L I(t)dt = \int_0^l I_1(t)dt + \int_l^L I_2(t)dt = Ql - \frac{D_1 l^2}{2} + (Q - D_1 l)(L-l) - \frac{D_2(L-l)^2}{2}$$
$$- \frac{\alpha(Q-D_1 l)(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha\beta D_2(L-l)^{\beta+2}}{(\beta+1)(\beta+2)} \tag{15}$$
$$+ \frac{\alpha^2(Q-D_1 l)(L-l)^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha^2\beta^2 D_2(L-l)^{2\beta+2}}{2(\beta+1)^2(2\beta+1)}$$

3.4.2. Cost and average profit function

Under a fixed order quantity Q, the economic outcome within one sales cycle is determined by sales revenue and the cost components specified in the model. These components comprise fixed ordering cost, purchasing cost, inventory holding cost, deterioration and disposal cost, and the two live-streaming-related costs defined in Assumption 4. The following expressions specify total sales revenue and each corresponding cost item.

The total sales revenue within one cycle is

$$R = p\left(\int_0^l D_1 dt + \rho\int_l^L D_2 dt\right) = pD_1 l + p\rho D_2(L-l) \tag{16}$$

The live-streaming sales commission cost within one cycle is

$$C_{com} = \eta R = \eta(pD_1 l + p\rho D_2(L-l)) \tag{17}$$

The live-streaming investment cost within one cycle is

$$C_{effort} = \frac{k\tau^2}{2} \tag{18}$$

Accordingly, the live-streaming-related cost within one cycle is

$$C_{live} = C_{com} + C_{effort} = \eta(pD_1 l + p\rho D_2(L-l)) + \frac{k\tau^2}{2} \tag{19}$$

The purchasing cost within one cycle is

$$C_O = cQ \tag{20}$$

The inventory holding cost within one cycle is

$$C_h = h\int_0^L I(t)dt = h(Ql - \frac{D_1 l^2}{2} + (Q - D_1 l)(L-l) - \frac{D_2(L-l)^2}{2}$$
$$-\frac{\alpha(Q-D_1 l)(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha\beta D_2(L-l)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha^2(Q-D_1 l)(L-l)^{2\beta+1}}{2(2\beta+1)} \tag{21}$$
$$-\frac{\alpha^2\beta^2 D_2(L-l)^{2\beta+2}}{2(\beta+1)^2(2\beta+1)})$$

Based on the strict SOA inventory function for the second stage, the process deterioration quantity can be obtained as

$$B = \int_l^L \theta(t) I_2(t)dt \approx \alpha(Q-D_1 l)(L-l)^{\beta} - \frac{\alpha\beta D_2(L-l)^{\beta+1}}{\beta+1}$$
$$-\frac{\alpha^2(Q-D_1 l)(L-l)^{2\beta}}{2} + \frac{\alpha^2\beta^2 D_2(L-l)^{2\beta+1}}{(\beta+1)(2\beta+1)} \tag{22}$$

According to the inventory treatment rule stated above, if remaining inventory exists at the end of the cycle, it is regarded as a disposal quantity. Therefore, the end-of-cycle disposal quantity is $I_2(L)$.

By setting $t = L$ in Eq (13), we obtain

$$I_2(L) \approx (Q - D_1 l) - D_2(L-l) - \alpha(Q-D_1 l)(L-l)^{\beta} +$$
$$\frac{\alpha\beta D_2(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha^2(Q-D_1 l)(L-l)^{2\beta}}{2} - \frac{\alpha^2\beta^2 D_2(L-l)^{2\beta+1}}{(\beta+1)(2\beta+1)} \tag{23}$$

From the perspective of inventory evolution, the initial inventory $I_1(l)$ in the second stage has only three possible destinations over the interval $[l, L]$: first, it is sold at the demand rate D_2; second, it deteriorates naturally during the sales process under the time-varying deterioration rate $\theta(t)$; and third, it forms the remaining disposal quantity at the end of the cycle. Therefore, the inventory balance relationship in the second stage satisfies

$$I_1(l) = D_2(L-l) + B + I_2(L) \tag{24}$$

Combining Eq (4) and $I_1(l) = Q - D_1 l$ gives

$$Q - D_1 l = D_2(L-l) + B + I_2(L) \tag{25}$$

Thus, the total deterioration and disposal cost within one cycle is

$$C_s = s(B + I_2(L)) = s(Q - D_1 l - D_2(L-l)) \tag{26}$$

It should be noted that Eq (26) is only a compact expression obtained from the inventory balance relationship in the second stage. It does not replace the previous derivation based on the deterioration-rate integral. Rather, this expression is derived from the inventory balance relationship, which avoids repeatedly handling a complicated deterioration-rate integral while ensuring the accuracy of cost calculation.

Accordingly, the total cost function within one sales cycle can be written as

$$TC = K + C_O + C_s + C_{live} + C_h \tag{27}$$

The corresponding total profit function within one cycle is

$$\pi_{cycle} = R - TC \tag{28}$$

Since this paper aims to maximize average profit per unit time, the average profit function is

$$\pi_{avg} = \frac{\pi_{cycle}}{L} \tag{29}$$

Equation (29) is the objective function used for the subsequent optimization analysis.

4. Model analysis and solution method

Since the average profit function contains the regular selling price, live-streaming investment intensity, sales cycle length, and inventory integral terms under the Weibull time-varying deterioration rate, directly proving its joint concavity with respect to all decision variables is difficult. To exploit the structural properties of the model, this paper adopts a subproblem-based solution procedure. Specifically, the price-live-streaming investment subproblem is first analyzed for a given sales cycle length; the sales-cycle boundary property is then examined for a given price and live-streaming investment intensity; and feasible boundary candidates are finally constructed and compared to determine the optimal strategy.

4.1. Structural properties of the optimal solution

Theorem 1: For any given sales cycle length L, if the live-streaming investment cost coefficient satisfies $k > \dfrac{(1-\eta)\gamma^2\{l+\rho[1+\omega(1-\rho)](L-l)\}^2}{2b\{l+\rho^2[1+\omega(1-\rho)](L-l)\}}$, then the average profit function π_{avg} is jointly concave with respect to the regular selling price p and live-streaming investment intensity τ.

Proof. For a given L, substituting Eqs (16), (19)–(21), and (26) into Eq (29) and collecting terms in p and τ yields

$$\pi_{\mathrm{avg}} = A_0 + A_1 p + A_2\tau - A_3 p^2 + A_4 p\tau - A_5\tau^2 \tag{30}$$

where

$$A_0 = \frac{1}{L}\Big\{-K - Qc - sQ - h[QL - \frac{\alpha Q(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha^2 Q(L-l)^{2\beta+1}}{2(2\beta+1)}]$$
$$+a[sl + \frac{hl^2}{2} + hl(L-l) - \frac{\alpha hl(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha^2 hl(L-l)^{2\beta+1}}{2(2\beta+1)}] \qquad ,$$
$$+a[1+\omega(1-\rho)][s(L-l) + \frac{h(L-l)^2}{2} - \frac{\alpha\beta h(L-l)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha^2\beta^2 h(L-l)^{2\beta+2}}{2(\beta+1)^2(2\beta+1)}]\Big\}$$

$$A_1 = \frac{1}{L}\Big\{-b\left[sl + \frac{hl^2}{2} + hl(L-l) - \frac{\alpha hl(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha^2 hl(L-l)^{2\beta+1}}{2(2\beta+1)}\right]$$
$$-b\rho[1+\omega(1-\rho)]\left[s(L-l) + \frac{h(L-l)^2}{2} - \frac{\alpha\beta h(L-l)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha^2\beta^2 h(L-l)^{2\beta+2}}{2(\beta+1)^2(2\beta+1)}\right],$$
$$+(1-\eta)a\big[l + \rho[1+\omega(1-\rho)](L-l)\big]\Big\}$$

$$A_2 = \frac{1}{L}\Big\{\gamma[sl + \frac{hl^2}{2} + hl(L-l) - \frac{\alpha hl(L-l)^{\beta+1}}{\beta+1} + \frac{\alpha^2 hl(L-l)^{2\beta+1}}{2(2\beta+1)}]$$
$$+\gamma[1+\omega(1-\rho)][s(L-l) + \frac{h(L-l)^2}{2} - \frac{\alpha\beta h(L-l)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha^2\beta^2 h(L-l)^{2\beta+2}}{2(\beta+1)^2(2\beta+1)}]\Big\},$$

$$A_3 = \frac{(1-\eta)b[l + \rho^2[1+\omega(1-\rho)](L-l)]}{L},\ A_4 = \frac{(1-\eta)\gamma[l + \rho[1+\omega(1-\rho)](L-l)]}{L},\quad A_5 = \frac{k}{2L}.$$

The first-order partial derivatives and the relevant second-order partial derivatives of π_{avg} with respect to p and τ give

$$\frac{\partial \pi_{\text{avg}}}{\partial p} = A_1 - 2A_3 p + A_4\tau,\ \frac{\partial \pi_{\text{avg}}}{\partial \tau} = A_2 + A_4 p - 2A_5\tau,$$

$$\frac{\partial^2 \pi_{\text{avg}}}{\partial p^2} = -2A_3,\ \frac{\partial^2 \pi_{\text{avg}}}{\partial \tau^2} = -2A_5,\ \frac{\partial^2 \pi_{\text{avg}}}{\partial p \partial \tau} = A_4$$

Thus, the Hessian matrix of π_{avg} with respect to p and τ is $H_{p,\tau} = \begin{pmatrix} -2A_3 & A_4 \\ A_4 & -2A_5 \end{pmatrix}$.

The first leading principal minor is $-2A_3$. Since $0<\eta<1$, $b>0$, $L>l>0$, $0<\rho<1$ and $\omega>0$, we have

$$-2A_3 = -\frac{2(1-\eta)b[l + \rho^2[1+\omega(1-\rho)](L-l)]}{L} < 0.$$

The second leading principal minor of $H_{p,\tau}$ is $4A_3A_5 - A_4^2$. The following inequality

$$2bk\left[l+\rho^2[1+\omega(1-\rho)](L-l)\right]-(1-\eta)\gamma^2\left[l+\rho[1+\omega(1-\rho)](L-l)\right]^2>0$$

implies $4A_3A_5 - A_4^2 > 0$.

Equivalently, when $k > \dfrac{(1-\eta)\gamma^2\{l+\rho[1+\omega(1-\rho)](L-l)\}^2}{2b\{l+\rho^2[1+\omega(1-\rho)](L-l)\}}$, the Hessian matrix is negative definite. Therefore, π_{avg} is jointly concave with respect to p and τ.

Solving the first-order conditions $\dfrac{\partial \pi_{\text{avg}}(p,L,\tau)}{\partial p}=0$, $\dfrac{\partial \pi_{\text{avg}}(p,L,\tau)}{\partial \tau}=0$ yields the optimal responses

$$p^* = \frac{2A_1A_5 + A_2A_4}{4A_3A_5 - A_4^2} \tag{31}$$

$$\tau^* = \frac{A_1A_4 + 2A_2A_3}{4A_3A_5 - A_4^2} \tag{32}$$

Theorem 2: For any given regular selling price p and live-streaming investment intensity τ, suppose that the sales cycle length satisfies $L_{\min} \le L \le L_{\max}$. The optimal sales cycle length can be characterized as follows:

If $I_2(L_{\max}) \ge 0$, then the effective feasible interval is $[L_{\min}, L_{\max}]$, and the optimal sales cycle length is

$$L^* \begin{cases} L_{\max}, \pi_{\text{avg}}(p, L_{\max}, \tau) > \pi_{\text{avg}}(p, L_{\min}, \tau); \\ L_{\min}, \pi_{\text{avg}}(p, L_{\max}, \tau) < \pi_{\text{avg}}(p, L_{\min}, \tau). \end{cases}$$

If $I_2(L_{\max}) < 0$, and there exists an inventory-depletion boundary L_B such that $I_2(L_B) = 0$, then the effective feasible interval is $[L_{\min}, L_B]$, and the optimal sales cycle length is

$$L^* \begin{cases} L_B, \pi_{\text{avg}}(p, L_B, \tau) > \pi_{\text{avg}}(p, L_{\min}, \tau); \\ L_{\min}, \pi_{\text{avg}}(p, L_B, \tau) < \pi_{\text{avg}}(p, L_{\min}, \tau). \end{cases}$$

Proof. Rearranging π_{avg} according to the structure of the sales cycle length L, we obtain $\pi_{\text{avg}} = \dfrac{M_0 - C_h}{L} + M_1$, where M_1L denotes the revenue and deterioration-cost terms that vary linearly with the sales cycle length L for a given p and τ, M_0 denotes the profit terms independent of L for a given p and τ, and C_h denotes the inventory holding cost generated by the inventory integral.

Accordingly, $M_1 = [(1-\eta)\rho p + s]D_2$ and $M_0 = \{[(1-\eta)p + s]D_1 - [(1-\eta)\rho p + s]D_2\}l - K - cQ - sQ - \frac{k\tau^2}{2}$. Taking the first-order derivative of π_{avg} with respect to L, we have $\frac{\partial \pi_{\text{avg}}}{\partial L} = \frac{C_h - L\frac{\partial C_h}{\partial L} - M_0}{L^2}$. Let $G(L) = C_h - L\frac{\partial C_h}{\partial L} - M_0$, then $\frac{\partial \pi_{\text{avg}}}{\partial L} = \frac{G(L)}{L^2}$. Therefore, $\frac{\partial \pi_{\text{avg}}}{\partial L}$ has the same sign as $G(L)$. If L is an interior stationary point, then $G(L) = 0$. Taking the second-order derivative of π_{avg} with respect to L gives $\frac{\partial^2 \pi_{\text{avg}}}{\partial L^2} = \frac{L^2 G'(L) - 2LG(L)}{L^4}$. At an interior stationary point, since $G(L) = 0$, we have $\frac{\partial^2 \pi_{\text{avg}}}{\partial L^2} = \frac{G'(L)}{(L)^2}$. Moreover, $G'(L) = -L\frac{\partial^2 C_h}{\partial L^2}$. Since the inventory holding cost is $C_h = h(\int_0^l I_1(t)dt + \int_l^L I_2(t)dt)$, it follows that $\frac{\partial^2 C_h}{\partial L^2} = h\frac{\partial I_2(L)}{\partial L}$. The inventory level in the second stage satisfies the inventory differential equation $\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = -D_2$; when $t = L$, it follows that $\frac{\partial I_2(L)}{\partial L} = -D_2 - \theta(L)I_2(L)$, since $D_2 > 0$, $\theta(L) = \alpha\beta(L-l)^{\beta-1} > 0$, and $I_2(L) > 0$, we have $\frac{\partial I_2(L)}{\partial L} = -D_2 - \theta(L)I_2(L) < 0$. Therefore, $G'(L) = -L\frac{\partial^2 C_h}{\partial L^2} = -Lh\frac{\partial I_2(L)}{\partial L} > 0$. Consequently, $\frac{\partial^2 \pi_{\text{avg}}}{\partial L^2} = \frac{G'(L)}{(L)^2} > 0$. Thus, if an interior stationary point exists for the average profit function with respect to the sales cycle length L, it is a local minimum rather than a local maximum. Since $\pi_{\text{avg}}(L)$ is continuously differentiable over the effective feasible interval, any interior local maximum, if it exists, must satisfy the first-order necessary condition. However, the interior stationary point satisfying the first-order condition has been shown to be a local minimum. Therefore, the profit-maximizing sales cycle length can only be attained at the boundary of the effective feasible interval. This implies that the optimal sales cycle length cannot be determined solely by the interior first-order condition $G(L) = 0$; instead, the inventory-depletion boundary and the exogenous bounds must be further examined.

When $I_2(L_{max}) \geq 0$, the inventory constraint remains feasible at the exogenous upper bound. In this case, the effective feasible interval is $[L_{\min}, L_{\max}]$, and the optimal sales cycle length satisfies $L^* \in \{L_{\min}, L_{\max}\}$. If the average profits at the two endpoints are unequal, the optimal sales cycle length is unique and is determined by the endpoint with the larger average profit. If the two endpoint profits are equal, both $L_{\min}$ and $L_{\max}$ are optimal sales-cycle candidates. Figure 1 illustrates the inventory trajectory and the corresponding sales-cycle boundaries in this case.

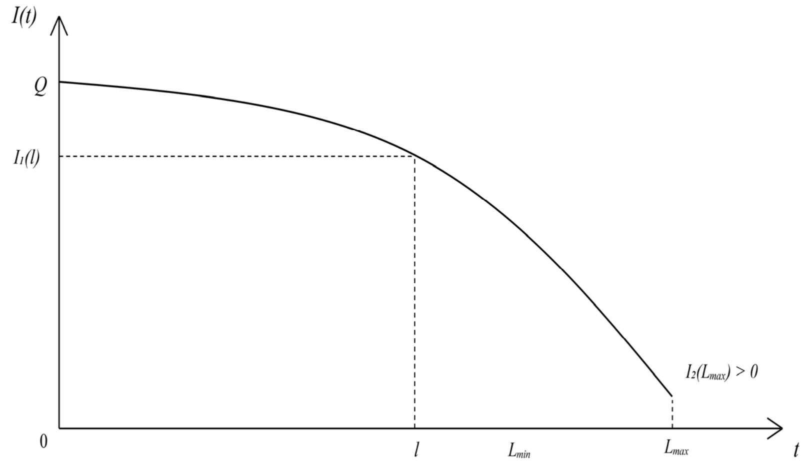

Figure 1. Inventory trajectory when the exogenous upper bound remains feasible.

When $I_2(L_{max}) < 0$, the inventory constraint is triggered before the exogenous upper bound. In this case, there exists an inventory-depletion boundary sales cycle $L_B \in (L_{\min}, L_{\max})$, satisfying $I_2(L_B) = 0$, and the effective feasible interval is $[L_{\min}, L_B]$. Since an interior point does not yield the maximum, the optimal sales cycle length satisfies $L^* \in \{L_{\min}, L_B\}$. If the average profits at the two endpoints are unequal, the optimal sales cycle length is unique and is determined by the one with the larger average profit between $L_{\min}$ and L_B. If the two endpoint profits are equal, both $L_{\min}$ and L_B are optimal sales-cycle candidates. Figure 2 illustrates the inventory-depletion boundary and the corresponding feasible interval in this case.

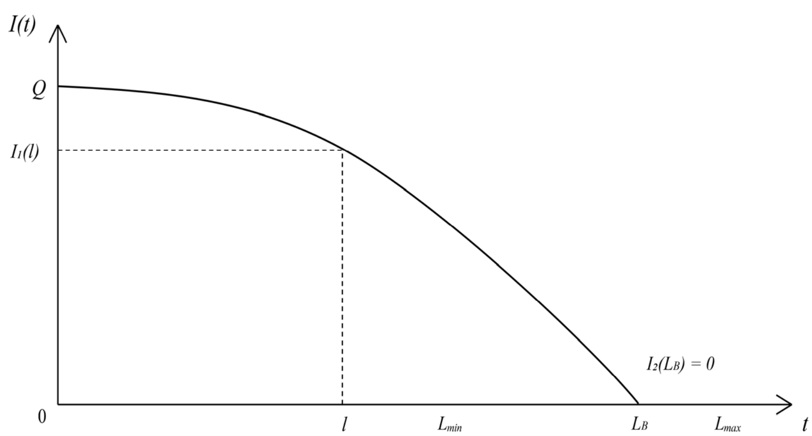

Figure 2. Inventory trajectory when the inventory-depletion boundary is triggered.

It should be noted that the inventory-depletion boundary L_B is implicitly determined by $I_2(L_B; p, \tau) = 0$. Its value changes with the regular selling price p and live-streaming investment intensity τ, and it is generally difficult to obtain an explicit analytical expression. Therefore, during the iterative procedure, the regular selling price, live-streaming investment intensity, and inventory-depletion boundary are updated alternately. The converged feasible candidate solutions are then substituted into the average profit function for comparison, so as to determine the final optimal strategy.

4.2. Algorithm design

Step 1. Input the model parameters and the feasible intervals of the decision variables, $p \in [p_{\min}, p_{\max}]$, $L \in [L_{min}, L_{max}]$, $\tau \in [\tau_{min}, \tau_{max}]$. Set the convergence tolerance as $\varepsilon = 10^{-6}$ and the maximum number of iterations as $N_{\max} = 120$. Generate multiple initial values $L^{(0)}$ within the feasible interval of the sales cycle length.

Step 2. Select an initial value $L^{(0)}$ and set $j = 0$. Substitute $L^{(j)}$ into Eqs (31) and (32) to obtain the optimal regular selling price and live-streaming investment intensity under the current sales cycle length, $p^{(j)}$, $\tau^{(j)}$. If the response solution does not satisfy the variable bounds or inventory feasibility constraints, then p and τ are re-optimized within the feasible region for the given $L^{(j)}$.

Step 3. Update the sales cycle length according to Theorem 2 based on $p^{(j)}$ and $\tau^{(j)}$. If $I_2(L_{max}; p^{(j)}, \tau^{(j)}) \geq 0$, then the value with the larger average profit between L_{min} and L_{max} is selected as $L^{(j+1)}$. If $I_2(L_{max}; p^{(j)}, \tau^{(j)}) < 0$, the inventory-depletion boundary L_B satisfying $I_2(L_B; p^{(j)}, \tau^{(j)}) = 0$ is first solved. The value with the larger average profit between L_{min} and L_B is then selected as $L^{(j+1)}$.

Step 4. Given $L^{(j+1)}$, substitute it into Eqs (31) and (32) to obtain $p^{(j+1)}$ and $\tau^{(j+1)}$. If $L^{(j+1)} = L_B$, the inventory-depletion boundary condition $I_2(L^{(j+1)}, p^{(j+1)}, \tau^{(j+1)}) = 0$ is imposed during the solution process. Otherwise, the inventory-feasibility constraint $I_2(L^{(j+1)}, p^{(j+1)}, \tau^{(j+1)}) \geq 0$ is maintained.

Step 5. If $\max\left\{\left|L^{(j+1)} - L^{(j)}\right|,\ \left|p^{(j+1)} - p^{(j)}\right|,\ \left|\tau^{(j+1)} - \tau^{(j)}\right|\right\} < \varepsilon$ and the change in average profit between two consecutive iterations is less than ε, then the iteration for the current initial value is terminated. Otherwise, set $j = j + 1$ and return to Step 2. If $j = N_{\max}$, the calculation for the current initial value is terminated.

Step 6. Repeat Steps 2–5 for all initial values of the sales cycle length. Substitute all converged feasible candidate solutions into the average profit function and select the feasible solution with the largest average profit as the alternating-iteration candidate solution. Then, compare the average profits of all candidate solutions and take the feasible solution with the largest average profit as the final optimal strategy, (p^*, L^*, τ^*).

5. Numerical analysis and discussion

5.1. Baseline parameters and optimal solution

With reference to the numerical examples of Zhang and Mo [6], Liang and Wang [36], and Zhang et al. [38], the baseline parameters are calibrated to reflect Weibull deterioration and price discounts, the two-stage fresh-product sales process, and live-streaming effort and commission mechanisms: $a = 520$, $b = 25$, $\gamma = 3$, $\rho = 0.85$, $k = 18$, $c = 4$, $h = 0.3$, $s = 0.5$, $K = 1000$, $\alpha = 0.05$, $\beta = 1.3$, $l = 5$, $\omega = 1$, $\eta = 0.05$, and $Q = 2000$. In the numerical solution, the search intervals for

p and L are set as $p \in [c+\varepsilon, 20]$ and $L \in [l+\varepsilon, 20]$, respectively, while the live-streaming investment intensity is constrained by $0 \le \tau \le \tau_{max}$, with $\tau_{max} = 20$ in the baseline case. Here, ε is a sufficiently small positive number used to prevent p and L from falling on singular boundary points. Solving the model with MATLAB yields $p^* = 14.29$, $L^* = 8.17$, and $\tau^* = 11.74$, and the corresponding optimal average profit is $\pi_{\text{avg}}^* = 1346.08$.

5.2. Error analysis of the Maclaurin approximation

To intuitively examine how well the second-order Maclaurin approximation captures the original inventory evolution process, Figure 3 compares the inventory trajectories obtained from the original exact ordinary differential equation (ODE) and the SOA inventory function under the optimal strategy.

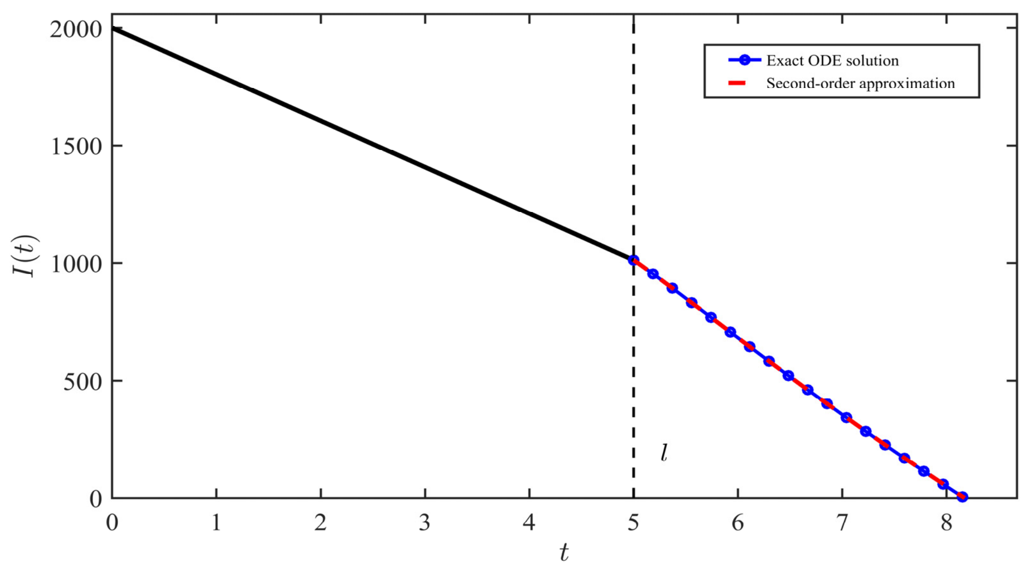

Figure 3. Comparison of inventory trajectories between the original ODE and the second-order Maclaurin approximation under the optimal strategy.

As shown in Figure 3, after $t = l$, the inventory trajectory generated by the original exact ODE in the second stage almost overlaps with that obtained from the second-order Maclaurin approximation, with only a minor deviation near the end of the cycle. This indicates that the second-order approximation can effectively characterize the dynamic evolution of inventory in the deterioration stage, thereby supporting the validity of the approximate model at the trajectory level. Based on this trajectory comparison, Table 3 further evaluates the accuracy of the SOA model from the perspectives of key state variables and economic indicators.

Table 3. Accuracy comparison between the SOA model and the exact ODE solution.

Key indicator	SOA model	Exact ODE solution	Relative error
$I_2(L)$	0.00	0.00	—
$\int_0^L I(t)dt$	9100.78	9099.89	0.01%
B^*	94.71	95.96	1.30%
C_h^*	2730.23	2729.96	0.01%
π_{avg}^*	1346.08	1346.04	0.00%

As shown in Table 3, the SOA model and the original Weibull ODE solution are highly consistent in terms of the full-cycle inventory integral and inventory holding cost, with relative errors of only 0.01%. More importantly, the relative error in average profit per unit time is only 0.0032%. This error is calculated from the unrounded computational results, whereas the corresponding values reported in Table 3 are displayed to two decimal places. Therefore, the displayed values do not affect the accuracy of the reported profit error, indicating that the approximation does not materially influence subsequent profit evaluation or optimal decision analysis. The relative error in the process deterioration quantity B^* is 1.3002%, which is higher than that of the other indicators. This is because B^* is calculated from the deterioration-rate-weighted inventory integral and is therefore more sensitive to local deviations in the inventory trajectory during the deterioration stage. Nevertheless, this deviation does not materially affect the economic evaluation of the model, as the error in average profit per unit time remains extremely small. In addition, no relative error is reported for terminal inventory $I_2(L)$, because both results are numerically close to zero, and a relative error measure would not be informative. Overall, Table 3 confirms that the second-order Maclaurin approximation has satisfactory numerical accuracy under the baseline setting.

5.3. Validation of the optimal solution

As shown in Figure 4, the average profit per unit time first increases and then decreases as the sales cycle length L increases. It reaches its maximum near $L^* = 8.17$, with π_{avg}^* approximately equal to 1346.08. This pattern indicates that, when the sales cycle is too short, the firm cannot sufficiently realize the additional demand potential generated by live-streaming traffic attraction and discount sales, and the cycle profit cannot be fully captured. However, when the sales cycle continues to lengthen, inventory holding costs, deterioration losses, and the decline in later-stage selling efficiency gradually offset the additional revenue, causing the average profit per unit time to decrease. Therefore, the sales cycle length should strike an optimal balance between demand conversion and inventory losses. Figure 5 further fixes the sales cycle length at $L = L^*$ and examines the joint effect of the regular selling price p and live-streaming investment intensity τ on average profit per unit time. The profit surface forms a clear high-value region around the optimal solution. Moreover, the computed optimum almost coincides with the highest feasible grid point on the profit surface, indicating that, under the optimal sales cycle length, the combination of price and live-streaming investment intensity exhibits a stable optimal response relationship. It should be noted that the computed optimum is obtained through continuous optimization, whereas the grid maximum is selected from a finite plotting grid. Therefore, it is normal for the grid-based maximum to be slightly lower than the continuous optimum. Taken together, Figures 4 and 5 validate the rationality of the optimal solution from the perspectives of the sales cycle length and the joint price–live streaming investment decision. This suggests that the obtained solution is not an isolated numerical result, but is consistent with both the variation pattern of the profit function and the economic mechanism of the model.

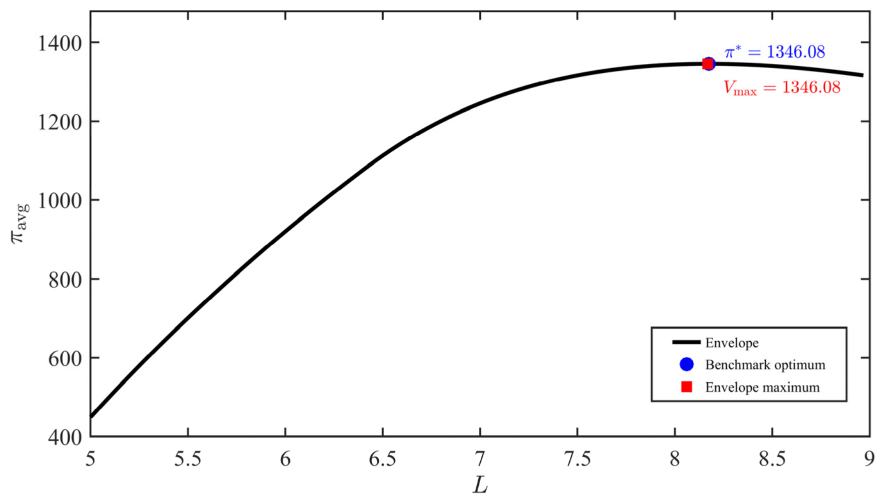

Figure 4. Effect of sales cycle length on average profit.

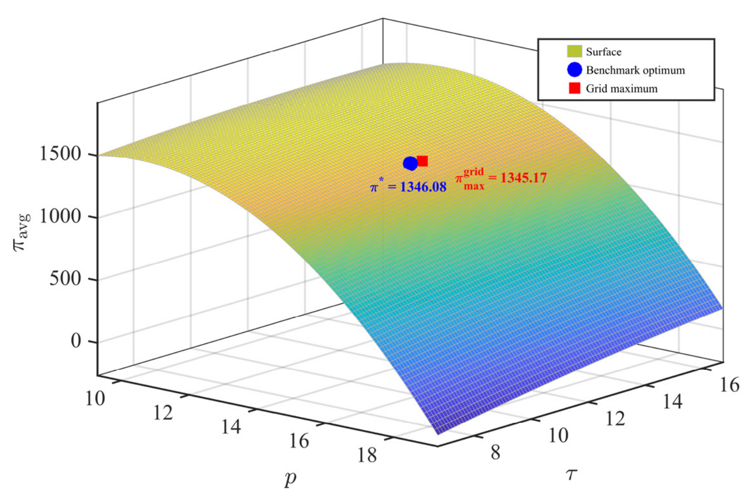

Figure 5. Effects of regular selling price and live-streaming investment intensity on average profit.

5.4. Sensitivity analysis

To examine how changes in key operating and market conditions affect the optimal policy, a one-factor sensitivity analysis is conducted from three perspectives: live-streaming promotion and market response, deterioration and inventory pressure, and cost structure. The results are reported in Tables 4–6.

Table 4. Live-streaming promotion and market response parameters.

Parameters	Change (%)	Value	p^*	L^*	τ^*	π^*_{avg}
ρ	15	0.98	13.53	8.53	12.03	1315.44
	10	0.94	13.81	8.42	11.97	1331.99
	−25	0.64	14.79	7.36	10.61	1245.09
	−50	0.42	13.64	6.46	9.50	974.73
γ	50	4.50	15.62	7.84	17.77	1541.62
	25	3.75	14.89	8.02	14.74	1432.60
	−25	2.25	13.85	8.30	8.77	1280.47
	−50	1.50	13.53	8.39	5.83	1234.51
ω	50	1.50	14.46	8.09	11.74	1388.46
	25	1.25	14.38	8.13	11.74	1367.52
	−25	0.75	14.22	8.22	11.74	1324.13
	−50	0.50	14.14	8.27	11.74	1301.63
b	50	37.50	9.92	8.87	7.57	399.14
	25	31.25	11.64	8.52	9.24	769.46
	−25	18.75	18.93	7.82	15.88	2352.87
	−50	12.50	20.00	6.08	13.85	3939.65

Table 5. Deterioration and inventory pressure parameters

Parameters	Change (%)	Value	p^*	L^*	τ^*	π^*_{avg}
α	50	0.08	14.02	7.90	11.60	1316.39
	25	0.06	14.19	8.05	11.65	1330.57
	−25	0.04	14.44	8.32	11.84	1363.30
	−50	0.03	14.60	8.50	11.96	1382.57
β	50	1.95	13.23	7.51	11.87	1303.50
	25	1.63	14.08	7.99	11.68	1329.67
	−10	1.17	14.39	8.25	11.76	1352.40
	−20	1.04	14.48	8.33	11.79	1358.46
Q	50	3000	12.04	8.97	17.41	1035.62
	25	2500	13.77	8.97	14.37	1265.93
	−25	1500	14.07	6.75	9.16	1382.79
	−50	1000	13.73	5.07	6.32	1354.68

Table 6. Cost structure parameters.

Parameters	Change (%)	Value	p^*	L^*	τ^*	π^*_{avg}
k	50	27	13.96	8.27	7.80	1295.96
	25	22.5	14.10	8.23	9.37	1315.90
	−25	13.50	14.65	8.08	15.69	1397.06
	−50	9	14.96	7.94	20.00	1494.01
η	50	0.08	14.33	8.21	11.41	1268.93
	25	0.06	14.31	8.19	11.57	1307.46
	−25	0.04	14.29	8.16	11.90	1384.79
	−50	0.03	14.28	8.14	12.06	1423.59
h	50	0.45	14.24	8.14	11.72	1179.16
	25	0.38	14.27	8.16	11.73	1262.60
	−25	0.22	14.34	8.19	11.75	1429.60
	−50	0.15	14.37	8.21	11.75	1513.16
s	50	0.75	14.28	8.16	11.74	1343.19
	25	0.63	14.29	8.17	11.74	1344.64
	−25	0.38	14.31	8.18	11.73	1347.53
	−50	0.25	14.32	8.19	11.73	1348.99

As shown in Table 4, live-streaming promotion and market response parameters strongly affect the optimal strategy. When ρ increases from the baseline value of 0.85 to 0.94 and 0.98, the optimal average profit decreases to 1331.99 and 1315.44, respectively, both of which lower than the baseline profit of 1346.08. Since a larger ρ indicates a weaker discount, this result suggests that an insufficient discount level leads to weak demand stimulation in the deterioration stage and reduces the promotional benefit of discounts in the live-streaming context. When ρ decreases by 25% and 50%, the optimal average profit drops to 1245.09 and 974.73, respectively, indicating that excessively deep discounts significantly reduce unit sales revenue. Therefore, neither a too high nor a too low value of the discount coefficient is desirable; rather, an appropriate trade-off between demand stimulation and unit revenue is required. When the live-streaming demand sensitivity coefficient γ increases by 50%, τ^* rises from 11.74 to 17.77, an increase of approximately 51.36%, while π^*_{avg} increases by about 14.53%. This result suggests that, when demand becomes more responsive to live-streaming investment, the firm tends to increase τ to capture the additional demand potential. By contrast, the live-streaming-discount synergy coefficient ω has a positive effect on profit, but its effect on live-streaming investment intensity is limited. The price sensitivity coefficient b has the greatest effect on π^*_{avg}. When b decreases by 50%, the optimal price reaches the upper bound of the search interval, indicating that the profit increase in this case is partly constrained by the imposed price range. A higher consumer price sensitivity compresses both the firm's pricing space and the return on live-streaming investment, making it a key market factor that constrains profit improvement.

As shown in Table 5, deterioration and inventory pressure parameters affect the optimal strategy by changing inventory exposure time and deterioration losses. As the deterioration-rate scale coefficient α increases, the optimal average profit declines, and the firm tends to shorten the sales cycle length. When α decreases, the sales cycle length extends from the baseline value to 8.50, indicating that a lower deterioration intensity gives the firm more room to extend the sales cycle and realize demand over a longer selling window. Since the Weibull shape parameter must satisfy $\beta > 1$, this paper adopts feasible perturbation ratios of 50%, 25%, -10%, and -20%. The results show that an increase in β strengthens the time-increasing characteristic of the deterioration rate, so the firm needs to shorten the sales cycle to reduce deterioration losses. When the fixed order quantity Q increases by 50%, the firm needs to raise live-streaming investment by approximately 48.30% and extend the sales cycle by about 9.79% to clear the additional inventory. However, π^*_{avg} decreases by approximately 23.06%. This result indicates that expanding the batch quantity beyond the firm's inventory-clearance capacity may generate a disproportionately large profit loss, because additional inventory requires stronger live-streaming investment and a longer sales cycle, which further increase operating costs, holding costs, and deterioration risk.

By contrast, reducing Q from 2000 to 1500 increases average profit per unit time from 1346.08 to 1382.79, whereas a further reduction to 1000 lowers it to 1354.68. Although the latter value remains 0.64% above the baseline profit, it is approximately 2.03% below the profit obtained at $Q=1500$. This non-monotonic pattern indicates that an initial reduction in batch size relieves inventory-clearance pressure, whereas an excessively small batch restricts sales scale and increases the fixed ordering cost allocated per unit time. Therefore, although Q is exogenous to the within-cycle model, it remains a pre-cycle planning variable. A practical screening procedure is to construct a discrete set of procurement-feasible batch sizes, re-optimize p, L, and τ for each candidate under the same decision bounds and inventory-feasibility conditions, and select the candidate that yields the highest optimized average profit per unit time. Among the tested values $Q\in\{1000, 1500, 2000, 2500, 3000\}$, $Q=1500$ yields the highest average profit of 1382.79 under the baseline calibration. This result illustrates the batch-size screening procedure rather than establishing a universally optimal order quantity.

As shown in Table 6, a lower live-streaming investment cost coefficient k makes live-streaming investment more cost-effective. Accordingly, the optimal policy increases the live-streaming investment intensity τ to stimulate demand and improve profit. Conversely, when the value k is high, additional live-streaming investment becomes less profitable and may reduce average profit. An increase in the live-streaming sales commission rate η mainly erodes the firm's net revenue, while its effects on sales cycle length and pricing decisions are limited. Under the current inventory-feasibility constraints, the optimal policy remains relatively stable, and the increase in inventory holding cost h is reflected primarily in a direct compression of profit margins. By contrast, the unit deterioration and disposal cost s has a relatively weak effect. Even when s increases by 50%, π^*_{avg} decreases by only about 0.21%. This indicates that, under the baseline optimal strategy, sales cycle planning and inventory control already limit deterioration losses effectively. As a result, the marginal effect of the unit deterioration and disposal cost is relatively small. Overall, the price sensitivity coefficient b, fixed order quantity Q, live-streaming demand sensitivity coefficient γ, live-streaming investment cost coefficient k, and inventory holding cost h are the key factors affecting optimal profit. Specifically, b determines the firm's pricing space, Q determines inventory clearance pressure, γ and k jointly determine the benefit–cost balance of live-streaming investment, and h directly affects the cost of carrying inventory.

5.5. *Robustness of the live-streaming investment capacity limit*

Table 6 shows that reducing the live-streaming investment cost coefficient from its baseline value of $k=18$ to $k=9$ makes the optimal investment intensity reach the baseline capacity limit

$\tau_{\max}=20$. To assess whether the selected capacity limit affects the optimal policy, the model is re-optimized for alternative values of $\tau_{\max}$. For each $k=18$ and $k=9$, $\tau_{\max}$ takes the values 15, 20, 25, and 30. Except for k and $\tau_{\max}$, all parameters and feasibility constraints remain at their baseline settings. The regular selling price, sales-cycle length, and live-streaming investment intensity are jointly re-optimized. The results are reported in Table 7.

Table 7. Optimal policies under alternative live-streaming investment capacity limits.

k	$\tau_{\max}$	p^*	L^*	τ^*	π^*_{avg}	Profit change relative to $\tau_{\max}=20$ within the same k (%)
18	15	14.30	8.17	11.74	1346.08	0.00
18	20	14.30	8.17	11.74	1346.08	0.00
18	25	14.30	8.17	11.74	1346.08	0.00
18	30	14.30	8.17	11.74	1346.08	0.00
9	15	14.46	8.02	15.00	1459.51	−2.31
9	20	14.96	7.94	20.00	1494.01	0.00
9	25	15.36	7.90	23.66	1501.57	0.51
9	30	15.36	7.90	23.66	1501.57	0.51

When $k=18$, the optimal live-streaming investment is 11.74, which is below the smallest tested capacity limit of 15. Increasing $\tau_{\max}$ from 15 to 30, therefore, leaves the optimal price, sales-cycle length, investment intensity, and average profit unchanged, indicating that the baseline capacity limit does not restrict the optimum. When $k=9$, the optimal investment intensity reaches the available capacity limit at $\tau_{\max}=15$ and $\tau_{\max}=20$. Increasing the capacity limit from 20 to 25 raises the optimal investment intensity from 20.00 to 23.66 and the average profit from 1494.01 to 1501.57, an improvement of 0.51%. A further increase in the limit to 30 does not change the optimal policy or average profit. Thus, $\tau_{\max}=20$ is non-binding when $k=18$, whereas under the tested low-cost case $k=9$, a capacity of approximately 23.66 is required to attain the capacity-unconstrained optimum.

5.6. Approximation accuracy across sensitivity scenarios

To examine the numerical accuracy of the second-order Maclaurin approximation over the parameter scenarios considered in the sensitivity analysis, 44 one-factor perturbation scenarios are evaluated. For each scenario, the optimal decision vector (p^*, L^*, τ^*) is first obtained using the SOA model. This decision vector is then fixed, and the inventory-related measures and average profit per unit time are calculated separately using the second-order Maclaurin approximation and the exact Weibull inventory ODE under the same parameter scenario and decision point. The two sets of results are subsequently compared.

For each scenario, the expansion argument $z=\alpha(L-l)^{\beta}$ was calculated from the full-precision SOA solution, where L denotes the scenario-specific optimal sales-cycle length. It remained below 0.3 in all 44 scenarios. The numerical values reported in Tables 4–6 are rounded solely for presentation.

Table 8. Numerical error validation of the second-order Maclaurin approximation across all sensitivity scenarios.

Parameter group	No. of scenarios	Max. rel. error in $\int_0^L I(t)dt$ (%)	Max. rel. error in B (%)	Max. rel. error in π^*_{avg} (%)
Live-streaming and market-response parameters	16	0.03	2.23	0.02
Deterioration and inventory-pressure parameters	12	0.03	2.48	0.01
Cost-structure parameters	16	0.01	1.43	0.00
All sensitivity scenarios	44	0.03	2.48	0.02

As shown in Table 8, the maximum relative error in the full-cycle inventory integral is 0.03% for the live-streaming promotion and market-response parameter group and the deterioration and inventory-pressure parameter group, and 0.01% for the cost-structure parameter group. The overall maximum is 0.03%, indicating that the approximation accurately captures the cumulative inventory level across the sensitivity-analysis range. The maximum relative errors in the process natural deterioration quantity B are 2.23%, 2.48%, and 1.43% across the three parameter groups. Although these values are relatively higher, the corresponding absolute deviation remains small. In the worst-case scenario, the difference between the approximated and unapproximated values of B is only 2.1658 units, accounting for 0.1083% of the fixed order quantity. Since B affects profit through the deterioration and disposal cost component, its deviation has only a limited effect on the profit calculation. The maximum relative errors in average profit per unit time are 0.02%, 0.01%, and 0.00%, respectively. For the cost-structure parameter group, the displayed value of 0.00% results from rounding the largest calculated error among its 16 scenarios, 0.003915%, to two decimal places. Across all 44 scenarios, the unrounded maximum relative error in average profit per unit time is 0.022558%, which is displayed as 0.02% in Table 8. Overall, the errors reported in Table 8 are very small and can be considered negligible, confirming that the second-order Maclaurin approximation provides reliable numerical support for the sensitivity analysis.

6. Conclusions

This paper develops a two-stage inventory model for a self-operated fresh agricultural product e-tailer operating under fixed-batch procurement. The model jointly considers Weibull time-varying deterioration, live-streaming investment, deterioration-stage discounts, and live-streaming sales commission costs, and determines the regular selling price, sales-cycle length, and live-streaming investment intensity to maximize average profit per unit time. Numerical validation against the exact Weibull inventory ODE shows that the second-order Maclaurin approximation remains accurate over all examined sensitivity scenarios.

The results show that live-streaming investment and deterioration-stage discounts can stimulate demand and accelerate inventory clearance, but neither automatically improves average profit per unit time. Additional demand must be evaluated together with the quadratic live-streaming investment cost, the revenue-linked live-streaming sales commission cost, the reduction in unit sales revenue caused by discounts, inventory holding costs, and deterioration-related losses. Therefore, alternative deterioration-stage discount policies should be evaluated jointly with pricing and live-streaming investment, rather than by making isolated increases in promotional spending or discount depth.

For a given regular selling price and live-streaming investment intensity, the interior stationary point of the sales-cycle subproblem does not maximize average profit per unit time. The sales-cycle decision should therefore be made by comparing the profit at the inventory-depletion boundary with the profits at the feasible exogenous sales-cycle bounds. This result explains why an excessively short sales cycle may leave demand unrealized, whereas an excessively long sales cycle intensifies inventory holding and deterioration losses.

Related perishable-product studies likewise connect markdown decisions with inventory conditions and waste-reduction objectives. Zhou et al. [16] examined two-period pricing and inventory decisions under partial lost sales, while Hasiloglu-Ciftciler and Kaya [17] showed that greater emphasis on waste reduction leads to deeper discounts. The present results confirm the inventory-clearance role of markdowns but further show that, across the examined scenarios, both overly weak and overly deep deterioration-stage discounts reduce average profit per unit time. The conditional profitability of live streaming is consistent with Yang et al. [43], who found that live-streaming promotion does not always outperform platform promotion and becomes advantageous only when streamer commissions are low and consumers' product valuations increase sufficiently. This study extends that result to fixed-batch deteriorating inventory by showing that profitability also depends on sales-cycle length, clearance pressure, and deterioration losses. Unlike the state-dependent dynamic production and pricing setting of Moshtagh et al. [15], the fixed-batch within-cycle setting considered here yields a boundary-based sales-cycle decision: the inventory-depletion boundary must be compared with the feasible exogenous bounds rather than relying on an interior stationary point.

Managerial implications

For self-operated fresh agricultural product e-tailers operating under fixed-batch procurement, the results yield three decision rules at the pre-cycle and within-cycle stages. The numerical values reported below are calibrated benchmarks rather than universal thresholds; firms should apply the same comparisons using their own demand, cost, inventory, and capacity data.

(1) Batch-size screening before procurement. Because the procurement batch is fixed during a sales cycle, it should be selected before procurement according to the firm's inventory-clearance capability rather than merely to enlarge sales volume. The firm should compare a discrete set of procurement-feasible batch sizes. For each candidate batch size, it should assess the regular selling price, sales-cycle length, and live-streaming investment plan under the same inventory-feasibility conditions and select the batch size with the highest expected average profit per unit time. In the calibrated results, $Q=1500$ yields an average profit of 1382.79, compared with 1354.68 at $Q=1000$, 1346.08 at $Q=2000$, and 1035.62 at $Q=3000$. Thus, neither a larger nor smaller batch is inherently preferable: an excessive batch intensifies clearance pressure and deterioration exposure, whereas an insufficient batch constrains sales scale and increases the fixed ordering cost allocated per unit time.

(2) Capacity-aware live-streaming planning. The upper bound on live-streaming investment should be treated as an operational-capacity constraint rather than merely as a numerical search range.

If the optimal investment intensity satisfies $\tau < \tau_{\max}$, the available capacity does not constrain the operating policy; if $\tau = \tau_{\max}$, the capacity ceiling may limit attainable operating profit. In the baseline case, $\tau^* = 11.74$, which is below the smallest tested capacity limit of 15. By contrast, in the low-cost case with $k = 9$, increasing $\tau_{\max}$ from 20 to 25 raises the optimized average profit from 1494.01 to 1501.57, whereas a further increase to 30 produces no additional gain in the tested scenario. When the ceiling binds, the firm should compare feasible capacity scenarios while reassessing the regular price, sales-cycle length, and live-streaming investment as one operating plan. The actual capacity-expansion decision should additionally consider the incremental cost of streamer time, traffic resources, and marketing capacity.

(3) Deterioration-stage clearance-policy design. The deterioration-stage discount should be evaluated as a planned clearance policy rather than mechanically deepened when clearance pressure increases. The firm should compare feasible discount policies together with the planned regular price, sales-cycle length, and live-streaming investment intensity. The calibrated results identify a 15% deterioration-stage discount $\rho = 0.85$ as the most profitable among the tested policies; however, this value is a reference rather than a universal rule. Both overly weak and overly deep discounts can reduce average profit because the faster inventory clearance may not compensate for the associated reduction in unit sales revenue, commission cost, holding cost, and deterioration-related loss.

Limitations and future research: Despite these implications, the analysis has several limitations. To isolate the joint effects of live-streaming investment, deterioration-stage discounting, and Weibull time-varying deterioration, the model assumes deterministic demand and a single-product sales setting. It therefore does not capture demand uncertainty caused by live-streaming traffic shocks, strategic consumers' intertemporal purchasing decisions, substitution among multiple fresh products, or within-cycle adjustments in live-streaming investment. Future research may incorporate these features in stochastic and multi-product settings. It may also examine how environmental objectives or constraints, such as food-waste reduction and carbon emissions, alter the joint pricing, sales-cycle, inventory, and live-streaming decisions.

Author contributions

Lijun Wu: Conceptualization, methodology, formal analysis, software, validation, visualization, data curation, and writing–original draft; Xuan Li: Investigation, data curation, and writing–review and editing; Haiping Ren: Conceptualization, methodology, supervision, validation, project administration, and writing–review and editing. All authors have read and approved the final version of the manuscript.

Use of Generative-AI tools declaration

The authors declare that DeepSeek was used only to polish sentences, correct grammatical errors, and improve linguistic clarity in this manuscript. All research content is original, and the authors take full responsibility for the final published article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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