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Research article

A hybrid intelligent method to dynamic real-time optimization

Wanlu Zhou[1], Huan Li[2,*], Jun Fu[1], Siheng Yao[1] and Ying Jin[1]

[1] State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang, Liaoning, 110000, China

[2] School of Mathematics and Statistics, Liaoning University, Shenyang, Liaoning, 110036, China

* **Correspondence:** Email: lihuan@lnu.edu.cn; Tel: +86-188-4102-9948.

Abstract: A novel hybrid intelligent dynamic real-time optimization (DRTO) algorithm is proposed for dynamic optimization problems under time-varying disturbances. First, due to the existence of time-varying disturbances in dynamic systems, a shrinking-horizon optimization framework that is progressively shortened at a fixed sampling period is employed to suppress them. Second, at each prediction horizon, a particle swarm optimization (PSO) algorithm explores the entire search space to identify the region containing the global optimum. A sequential quadratic programming (SQP) algorithm is then applied within this detected area to obtain a global optimal prediction control sequence. Subsequently, the first control component of the control sequence is implemented to update the system state initial estimate based on the latest disturbance measurement for the next prediction horizon. While the horizon shrinks, the optimization process is repeated until a near-global optimal control sequence is generated. Finally, simulation results of the cart-spring-damper (CSD) system demonstrate the effectiveness of the proposed hybrid intelligent algorithm, whose optimization performance outperforms both a pure deterministic algorithm and existing intelligent algorithms.

Keywords: hybrid intelligent; dynamic real-time optimization; time-varying disturbances; shrinking-horizon optimization; improved particle swarm optimization
Mathematics Subject Classification: 90C26, 90C39

1. Introduction

Real-time optimization (RTO) has become an indispensable tool in modern engineering, finding widespread applications in chemical engineering [1–3], refinery operations [4, 5], and areas such as vehicle driving systems [6, 7]. The fundamental objective of RTO is to compute optimal operating strategies or control inputs online, adapting to real-time measurements and changing environmental conditions. However, conventional RTO typically relies on steady-state detection and assumes that the process operates at a stationary condition before optimization is triggered. This paradigm becomes inadequate when processes are subject to frequent and significant disturbances, such as those arising from renewable energy integration [8,9] or load following in power plants [10–12]. Under such transient conditions, the system may never reach a true steady state, leading to outdated or suboptimal setpoints and potential violations of dynamic constraints.

In response to the limitations of conventional RTO, a dynamic real-time optimization (DRTO) approach that integrates optimization and control within a unified framework has emerged. DRTO extends the RTO paradigm by explicitly considering the temporal evolution of the system and the presence of uncertainties, typically solved in a moving or receding horizon fashion [13–15]. The existing approaches are predominantly deterministic algorithms [16–22], which are grounded in mathematical programming theory and exploit analytical properties such as gradients and Hessians to generate a sequence of iterates converging to a local optimum. The main advantage of deterministic algorithms lies in their ability to converge rapidly to a high-precision solution when the initial guess is sufficiently close to the optimum. However, these methods are prone to becoming trapped in local minima, particularly in nonconvex problems. In recent years, some improved gradient-based optimization methods [23,24] have attempted to escape poor local minima, but these techniques are typically limited to low-dimensional systems with simple structures.

Moreover, heuristic strategies, including particle swarm optimization (PSO) [25,26], genetic algorithms (GAs) [27–29], ant colony optimization (ACO) [30–32], and simulated annealing (SA) [33,34], have gained popularity due to their ability to perform global searches without requiring gradient information. These methods are generally easy to implement and can locate near-global optimal solutions for dynamic optimization problems [35–37]. n recent years, various hybrid and multi-swarm methods of PSO have been developed to enhance the exploration and exploitation capabilities of the algorithm. For instance, hybrid multi-swarm PSO (HMSPSO) further combines parallel multi-swarm PSO with real-coded genetic operators, including crossover and mutation, to enhance solution quality and accelerate convergence [38]. Another notable hybrid approach is real-coded genetic algorithm PSO (RCGA-PSO), which integrates evolutionary selection mechanisms with swarm optimization strategies, effectively balancing global exploration and local exploitation [39]. In addition to PSO-based hybrids, RCGAs have been widely adopted for optimizing dynamic systems. For example, the RCGA with fuzzy control (F-RCGA) employs fuzzy logic to adaptively control genetic parameters such as crossover and mutation probabilities during the optimization process, improving adaptability to varying problem landscapes [40]. However, these methods are not without limitations. For instance, PSO is known to converge rapidly during the initial phase but slows down considerably as it approaches the optimum, resulting in poor fine-tuning capability [41]. Moreover, heuristic methods cannot guarantee satisfaction of optimality conditions and typically rely on heuristic termination criteria such as maximum iterations or a fitness threshold. Consequently, achieving high-precision solutions often demands substantial computational effort.

Based on the above, this paper presents a novel hybrid intelligent optimization approach that strategically integrates both paradigms. The methodology is developed within a shrinking-horizon optimization framework to address nonlinear systems subject to time-varying disturbances, thereby enabling effective disturbance rejection throughout the operating horizon. Within this framework, an improved ring-topology PSO algorithm in [42] is first employed to conduct global search and identify regions potentially containing the global optimum. Subsequently, a sequential quadratic programming (SQP) algorithm in [22] is applied within these detected promising regions, leveraging gradient information to refine the solution with superior accuracy compared to standalone PSO approaches. Consequently, the proposed hybrid intelligent DRTO method synergistically combines the strengths of the heuristic and the deterministic approaches, which not only facilitates effective stochastic exploration of globally optimal regions but also ensures the optimality of the final solution.

The main contributions of this paper are twofold:

1. A novel hybrid intelligent DRTO algorithm is proposed for nonlinear systems with time-varying disturbances. By integrating an improved PSO with ring topology and a gradient-based optimizer, the algorithm significantly enhances global search capability while maintaining solution accuracy.
2. A shrinking-horizon optimization framework is adapted that effectively mitigates time-varying disturbances. Within this framework, the optimization problem dimension decreases progressively as the sampling horizon advances, leading to a linear reduction in computational burden.

The rest of this paper proceeds as follows: Section 2 formulates the dynamic optimization problem under time-varying disturbances and introduces the shrinking-horizon optimization framework. Section 3 provides a brief review of related work on PSO and SQP and then details the proposed hybrid intelligent DRTO algorithm that integrates an improved PSO with SQP. Section 4 presents numerical simulation results on a cart-spring-damper (CSD) system and comparative analyses that validate the effectiveness of the proposed method compared to standalone PSO and SQP. Finally, Section 5 concludes the paper with a summary of key findings and outlines directions for future research.

2. Problem formulation

The dynamic optimization with time-varying disturbances is considered using the control vector parameterization (CVP) technique [43–46], formulated as follows:

$$\begin{aligned} \min_{u \in U} \quad & S(x(t_f), u) \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t), u, \hat{d}(t)), \forall t \in T, \\ & x(t_0) = \hat{x}_0, \\ & u_{min} \leq u \leq u_{max}, \end{aligned} \tag{2.1}$$

where $u \in U \subset \mathbb{R}^{n_u}$ represents the time-invariant control input sequences, with U being nonempty and compact; $x(t) \in X \subset \mathbb{R}^{n_x}$ characterizes the system states subject to initial condition $\hat{x}_0$, where X is compact and nonempty. The time-varying disturbance $\hat{d}(t) : T \to D \subset \mathbb{R}^{n_{\hat{d}}}$ is assumed to be measurable and bounded, where D is also a compact and nonempty set. The objective function $S : X \times U \to \mathbb{R}$ and the dynamic process model $f : X \times U \times D \to \mathbb{R}^{n_x}$ are assumed to be continuously differentiable in their respective arguments. $T := [t_1, t_f]$ defines the operating time horizon of the system.

The time-varying disturbance $\hat{d}(t) \in \mathbb{R}^{n_{\hat{d}}}$ is modeled as a measurable signal with zero-order hold (ZOH), which is held constant over each sampling interval Δt and updated at each sampling instant t_j. Specifically, the disturbance can be formulated as follows:

$$\hat{d}_j(t) = \{\hat{d}_j^k\}, \quad j = 1, 2, \ldots, J, \quad k = 0, 1, \ldots, J - j, \quad t \in [t_j, t_f],$$

where J is the total number of sampling intervals within the optimization horizon, $\hat{d}_j^1$ denotes the online measurement of the disturbance at the current sampling instant t_j, and the subsequent values $\hat{d}_j^2, \hat{d}_j^3, \ldots, \hat{d}_j^{J-j}$ are predictions provided by the trend detection mechanism described below.

From historical data, the trend detection mechanism [47, 48] can estimate the slow change of future disturbance by detecting constant, linear, or higher-order polynomial trends in the multivariate disturbance signal. Consequently, this approach enables the integration of the identified trend as a predictive disturbance component within the optimization framework.

To address this time-varying disturbance in a closed-loop manner, we propose a shrinking-horizon optimization strategy. This approach decomposes the original problem (2.1) into a sequence of dynamic optimization subproblems (2.2), each defined over a shrinking horizon $[t_j, t_j^f]$ (where the terminal time t_j^f is fixed; i.e., $t_j^f = t_f$). At each sampling instant t_j ($j = 1, 2, \ldots, J$), the control inputs are reoptimized over the remaining horizon based on the current state $\hat{x}_j$ and the latest disturbance estimate $\hat{d}_j$. Hence, this closed-loop framework enables the controller to adapt to time-varying disturbances while maintaining feasibility of path constraints. For computational efficiency, the period of the shrinking-horizon update Δt is aligned with the temporal resolution of disturbance parameterization, that is, $n_{\hat{d}} = J$, where J denotes the total number of sample intervals. Consequently, Problem (2.1) is transformed into the following form:

$$
\begin{aligned}
\min_{u_j \in U} \quad & S(x_j(t_j^f), u_j) \\
\text{s.t.} \quad & \dot{x}_j(t) = f(x_j(t), u_j, \hat{d}_j), \forall t \in T_j, \\
& x_j(t_j) = \hat{x}_j, \\
& u_{min} \le u_j \le u_{max}, j = 1, 2, \ldots, J,
\end{aligned} \tag{2.2}
$$

where $u_j \in U \subset \mathbb{R}^{n_u}$ represents the time-invariant control input variables; $x_j(t) \in X \subset \mathbb{R}^{n_x}$ denotes the state variables with initial conditions $\hat{x}_j$. The sampling periods are indicated by $T_j := [t_j, t_f] \subset T$, $j = 1, 2, \ldots, J$ (J is the number of samples). In this case, the previous time horizon $[t_j, t_f]$ is reduced by a uniform sampling interval Δt. Consequently, while the aforementioned procedure operates in batch mode, a shrinking-horizon optimization mechanism [15] is introduced here for the fixed terminal time problem.

3. Algorithm design

The main task of this section is to design a shrinking-horizon DRTO framework for nonlinear systems subject to time-varying disturbances. First, an improved PSO method is adapted to explore the optimal global region of the decision variables under the current disturbance value. Then, a gradient-based optimization method is employed to refine the solution locally, ensuring that the optimality conditions are satisfied. This hybrid intelligent DRTO approach integrates the global search capability of heuristic algorithms with the fast convergence properties of deterministic gradient-based methods, thereby enabling near-global optimization in the presence of time-varying disturbances.

3.1. Improved PSO

PSO is a population-based heuristic that simulates the collective behavior of organisms in nature, such as bird flocking or fish schooling, to locate optimal regions within a search space [37, 49, 50].

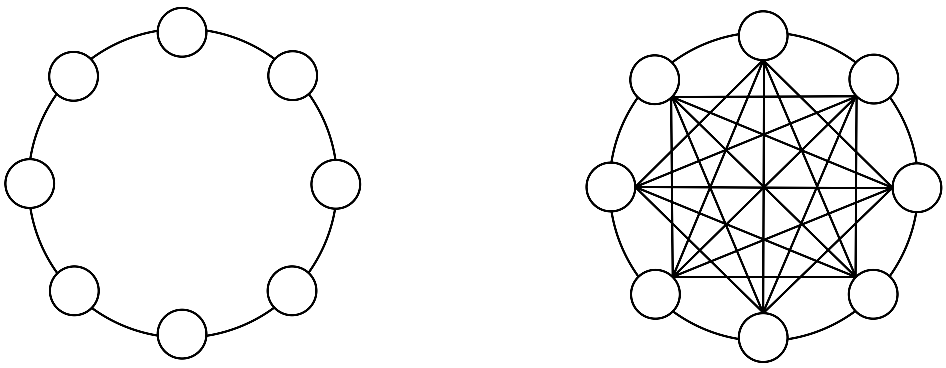
In the PSO paradigm, a swarm of particles traverses the multidimensional decision space, with each particle representing a candidate solution to the optimization problem. The movement of each particle is influenced by its own historical best position and the best position discovered by its neighboring particles, thereby balancing individual exploration with social cooperation.

The core of the PSO algorithm lies in iteratively updating the motion state of particles through mathematical rules. The update rules for their velocity and position are as follows:

$$\begin{aligned} v_{id}(k+1) &= wv_{id}(k) + c_1r_1(pbest_{id} - x_{id}(k)) + c_2r_2(gbest_d - x_{id}(k)), \\ x_{id}(k+1) &= x_{id}(k) + v_{id}(k+1), \end{aligned} \tag{3.1}$$

where $v_{id}(k)$ and $x_{id}(k)$ are the velocity and position of particle i in dimension d at iteration k, respectively; w is the inertia weight; c_1 and c_2 are acceleration coefficients; r_1 and r_2 are random numbers uniformly distributed in $[0, 1]$; $pbest_{id}$ denotes the personal best position of particle i; and $gbest_d$ represents the global best position found by the entire swarm.

The PSO algorithm has two topological structure models [50, 51]: Local neighborhood topology (l_{best}) and global neighborhood topology (g_{best}), as shown in Figure 1 (adopted from [51]). In the l_{best} model, particles are updated by obtaining information about adjacent particles. In the g_{best} model, particles are updated using the information of all particles in the entire population. Generally speaking, PSO using the g_{best} topology has a fast convergence speed and short running time but tends to fall into premature convergence easily, thus performing poorly when dealing with complex or multimodal optimization problems. In contrast, the PSO with l_{best} topology converges more slowly, but its particles can explore the entire search space more deeply, thereby enhancing the global search capability. This paper adopts the PSO with l_{best} topology as shown in Figure 1 (a) for algorithm design.



(a) l_{best} model. **(b)** g_{best} model.

Figure 1. PSO topology models.

In addition, the inertia weight is a key parameter in PSO used to balance global exploration and local exploitation capabilities. In the velocity update formula, the inertia weight ω controls the extent to which a particle inherits its previous velocity: A larger ω helps particles search for new regions in a vast space, enhancing global exploration ability; a smaller ω enables particles to conduct fine-grained searches near the current optimal solution, improving local exploitation efficiency. To overcome the limitation of fixed weights in adapting to complex search processes, dynamic adjustment strategies are widely adopted, among which the linearly decreasing weight is one of the most common approaches. This method gradually reduces the weight from a larger value to a smaller one as iterations progress, thereby emphasizing global exploration in the early stages of the algorithm and focusing on local convergence in the later stages. The formula for linearly decreasing the inertia weight is as follows:

$$\omega(k) = \omega_{\max} - \left(\frac{\omega_{\max} - \omega_{\min}}{K_{\text{iter}}^{\text{PSO}}}\right) k, \tag{3.2}$$

where $K_{\text{iter}}^{\text{PSO}}$ is the maximum number of iterations for the improved PSO algorithm.

3.2. SQP method

Although the improved PSO described in the preceding subsection excels at identifying promising regions within the search space, its stochastic nature and lack of curvature information lead to slow convergence when refining solutions to high precision. To overcome this limitation, an SQP method is employed once the PSO phase has successfully identified a neighborhood that potentially contains the global optimum. The SQP is one of the most powerful algorithms for solving nonlinear constrained optimization problems [22, 44, 52]. Its core idea is intuitive: At each iteration, a quadratic programming subproblem is used to approximate the original nonlinear problem around the current point $\hat{x}_k$. Solving this subproblem yields a search direction, followed by a line search. This process is repeated until convergence. By leveraging Hessian information, SQP achieves rapid convergence to a local optimum that satisfies the necessary optimality conditions while naturally handling the constraints on control inputs. This approach complements the global exploration capability of the heuristic search, as detailed in Algorithm 1 (adapted from [22]).

3.3. Hybrid intelligent DRTO algorithm

Based on the above analysis in this section, the PSO is an efficient heuristic optimization algorithm, but it converges very slowly around the global optimal solution, which implies that its solutions are less fine-tunable. On the other hand, the SQP possesses strong local search capability and can converge quickly to a highly accurate local optimum. Therefore, inspired by [51], this paper proposes a hybrid intelligent DRTO algorithm (as shown in Figure 2), which combines the advantages of PSO and SQP, namely avoiding entrapment in local optima and ensuring the theoretical optimality of the obtained solution. First, the infinite-dimensional optimal control problem is reduced to a finite-dimensional optimization problem using the CVP technique. Then, an improved ring-topology PSO algorithm is employed to search the entire space and identify the region containing the global optimum. Finally, the SQP algorithm is deployed within the detected global optimal region to converge rapidly to a high-precision solution.

Algorithm 1 SQP for dynamic optimization with time-varying disturbance.

Input: Current sampling index j; initial control guess $u^{(0)}$; initial state $\hat{x}_j$; current disturbance $\hat{d}_j$; time interval $[t_j, t_f]$; control bounds $u_{\min}, u_{\max}$; maximum iterations $K_{\max}^{\text{NLP}}$; optimality tolerance ϵ_{opt}

Initialize $k = 0$, $u^{(k)} = u^{(0)}$, $H^{(k)} = I$ (identity matrix)
while $k < K_{\max}^{\text{NLP}}$ **do**
  Forward integration: Integrate system $\dot{x} = f(x, u^{(k)}, \hat{d}_j)$ from t_j to t_f to obtain $x(t)$ and cost $S^{(k)}$
  Gradient computation: Compute $\nabla S^{(k)}$ via costate method
  if $\|\nabla S^{(k)}\| < \epsilon_{\text{opt}}$ **then**
    break
  else
    Hessian update: Update $H^{(k+1)}$ using Broyden–Fletcher–Goldfarb–Shanno (BFGS)
    QP subproblem: Solve $\min_{\Delta u} \frac{1}{2}\Delta u^T H^{(k)} \Delta u + \nabla S^{(k)T} \Delta u$ subject to $u_{\min} - u^{(k)} \leq \Delta u \leq u_{\max} - u^{(k)}$ to obtain direction $\Delta u^{(k)}$
    Line search: Find step size $\alpha^{(k)} \in (0, 1]$ satisfying Armijo condition, $u^{(k+1)} \leftarrow u^{(k)} + \alpha^{(k)} \Delta u^{(k)}$
    $k \leftarrow k + 1$
  end if
end while

Output: $u_j^* = u^{(k)}$

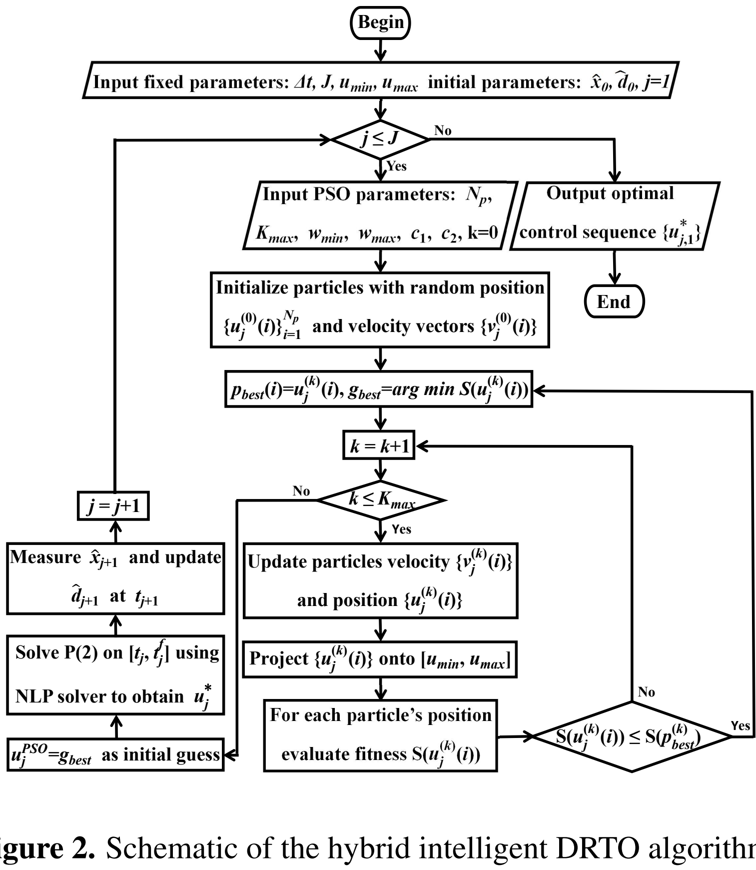


Figure 2. Schematic of the hybrid intelligent DRTO algorithm.

Algorithm 2 Hybrid intelligent DRTO for nonlinear system with time-varying disturbances.

Input: Total sampling number J; fixed sampling period Δt; control input constraint parameters $u_{\max}$ and $u_{\min}$; initial state $\hat{x}_1$; initial disturbance measurement value $\hat{d}_1$

for $j = 1$ to J **do**
  Input: PSO parameters N_p, $K_{\max}^{\text{PSO}}$, $w_{\max}$, $w_{\min}$, c_1, c_2 and $k = 0$
  Initialize particles: $\{u_j^{(0)}(i)\}_{i=1}^{N_p}$, velocities $\{v_j^{(0)}(i)\}$
  $p_{\text{best}}(i) \leftarrow u_j^{(0)}(i)$, $g_{\text{best}} \leftarrow \arg\min S(u_j^{(0)}(i))$
  while $|S_{\text{best}}^{(k)}(u) - S_{\text{best}}^{(k-5)}(u)| > \epsilon_{\text{PSO}}$ or $k \le K_{\max}^{\text{PSO}}$ **do**
    $w^{(k)} \leftarrow w_{\max} - \frac{k}{K_{\max}^{\text{PSO}}}(w_{\max} - w_{\min})$
    for each particle i **do**
      $v_j^{(k)}(i) \leftarrow w^{(k)} v_j^{(k-1)}(i) + c_1 r_1 (p_{\text{best}}(i) - u_j^{(k-1)}(i)) + c_2 r_2 (g_{\text{best}} - u_j^{(k-1)}(i))$
      $u_j^{(k)}(i) \leftarrow u_j^{(k-1)}(i) + v_j^{(k)}(i)$
      Project $u_j^{(k)}(i)$ onto $[u_{\min}, u_{\max}]$
      Evaluate $S(u_j^{(k)}(i))$ by integrating system dynamics
      if $S(u_j^{(k)}(i)) < S(p_{\text{best}}(i))$ **then**
        $p_{\text{best}}(i) \leftarrow u_j^{(k)}(i)$
      else
        $p_{\text{best}}(i) \leftarrow p_{\text{best}}(i)$
      end if
    end for
    Update g_{best} from $\{p_{\text{best}}(i)\}$
    $k \leftarrow k + 1$
  end while
  $u_j^{\text{PSO}} \leftarrow g_{\text{best}}$
  Use u_j^{PSO} as initial guess
  Solve Problem (2.2) on $t \in [t_j, t_f]$ to obtain locally optimal u_j^* by Algorithm 1
  At t_{j+1}, implement the control $u_j^*(t_j)$ to get the calculated value $\hat{x}_{j+1}$ and update the disturbance measurement value $\hat{d}_{j+1}$
  Reduce the time horizon for a batch operation by Δt, i.e., $t_{j+1} = t_j + \Delta t$, $t_{j+1}^f = t_j^f$
  Output: $u_j(t) \leftarrow \bar{u}_j^*(t_j)$ $(t \in [t_j, t_{j+1}))$
end for
Optimization completed, obtaining the optimal control sequence u_{seq} over $[t_0, t_f]$

A key issue in hybrid intelligent DRTO methods is the transition condition between the two algorithms, that is, the termination of PSO and the initiation of SQP. Inspired by [35, 36, 51], the value of the cost function and the number of iterations of the PSO algorithm are adopted as two trigger conditions: One is that the relative improvement of the global best value over the last i iterations is less than a threshold ϵ_{PSO}, indicating that the PSO has entered the neighborhood of the global optimum and necessary to trigger the SQP search; the other is that the number of iterations of PSO reaches a preset maximum. The transition from PSO to SQP is performed as soon as either of these two conditions is satisfied. The trigger conditions are expressed as follows:

$$
\begin{aligned}
|S^{(k)}_{\text{best}}(u) - S^{(k-i)}_{\text{best}}(u)| &< \epsilon_{\text{PSO}}, \\
k &> K^{\text{PSO}}_{\max},
\end{aligned}
\tag{3.3}
$$

where $S^{(k)}_{\text{best}}(u)$ is the fitness of best particle discovered in the kth iteration; $K^{\text{PSO}}_{\max}$ is the maximum number of iterations of the PSO; u is the decision variable.

The result obtained by the hybrid intelligent DRTO algorithm is summarized in Algorithm 2.

The existence of a unique solution to the Problem (2.1) is established under the following assumption [51].

Assumption 1. *The cost function $S(x(t_f), u)$ and the right-hand-side function $f(x(t), u, \hat{d})$ in Problem (2.1) are Lipschitz continuous and differentiable with respect to all their arguments.*

Under Assumption 1, given the initial state $\hat{x}_1$ and initial disturbance $\hat{d}_1$, Algorithm 2 solves Problem (2.2) within each sampling interval $j = 1, 2, \ldots, J$ under the shrinking-horizon framework by performing an improved PSO-based global search and an SQP local optimization. This generates an optimal control sequence $u^* = \{u_j(t) | t \in [t_j, t_{j+1}), j = 1, 2, \ldots, J\}$ for Problem (2.1), which satisfies the control constraints $[u_{\min}, u_{\max}]$ over the entire operating horizon $[t_0, t_f]$ and effectively mitigates the impact of time-varying disturbances.

4. Simulation study and analyses

In this section, the effectiveness of the proposed hybrid intelligent DRTO method is illustrated using a numerical example on a cart-spring-damper (CSD) system under time-varying disturbance (see Figure 3 [53–55]). The simulation results are further compared to a deterministic optimization algorithm and four intelligent optimization algorithms, namely SQP [22], PSO [25], RCGA [29], ACO [32], and SA [34]. The simulation implementation is performed in MATLAB Version (R2024a) on a 12th Gen Intel(R) Core(TM) i7-12700KF @3.61 GHz.

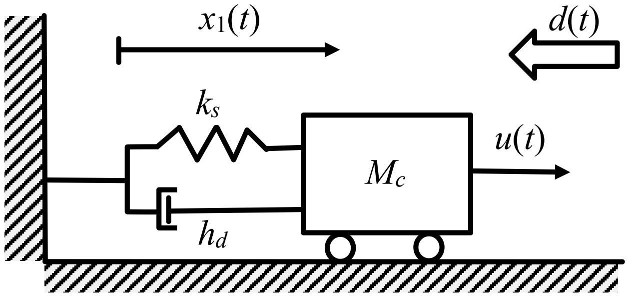


Figure 3. Graphical representation of the CSD system with the time-varying disturbance.

4.1. Dynamics of the CSD system

The system model follows a structure similar to that presented in [53–55], with the dynamics expressed as:

$$
M_c \ddot{x}(t) + h_d \dot{x}(t) + k_s x(t) = u(t) + d(t),
\tag{4.1}
$$

where $x(t)$ describes the displacement of the cart from the equilibrium position, M_c denotes the mass of the cart, h_d quantifies the damping coefficient, k_s denotes the spring constant, $u(t)$ represents the control input, and $d(t)$ represents the environmental forces (wind resistance) that act on the CSD system.

The system model is expressed in the state-space representation as follows:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= -\frac{k_0}{M_c} e^{-x_1(t)} x_1(t) - \frac{h_d}{M_c} x_2(t) + \frac{u(t)}{M_c} + \frac{\hat{d}(t)}{M_c}, \end{aligned} \tag{4.2}$$

where $M_c = 1kg$, $h_d = 0.3N \cdot s/m$, $k_s = k_0 e^{-x_1(t)}$ with $k_0 = 2.5N/m$.

The goal is to find the optimal control input that minimizes the following cost function:

$$S(x(t), u(t)) = \int_0^{t_f} (q_1 x_1^2(t) + q_2 x_2^2(t) + r u^2)\, dt, \tag{4.3}$$

where $q_1 = 10$, $q_2 = 1$, and $r = 0.1$.

The time-varying disturbance is modeled as a piecewise constant signal with random variations at each sampling interval:

$$\hat{d}(t) = 2 + 3\xi(t), \forall t \in [0, t_f], \tag{4.4}$$

where $\xi(t) \sim \mathcal{N}(\mu_d, \sigma_d^2)$ with $\mu_d = 0$ and $\sigma_d = 1$, truncated to the range $[d_{\min}, d_{\max}] = [-9, 9]$, ensuring realistic bounded disturbances.

4.2. Parameter settings

In this subsection, other parameter settings required for the simulation are presented, as shown in Table 1.

Table 1. Parameter settings for the simulation.

System configuration			**PSO parameters**		
Parameter	**Symbol**	**Value**	**Parameter**	**Symbol**	**Value**
Terminal time	t_f	10 s	Number of particles	N_p	20
Sampling period	Δt	1 s	Maximum iterations	$K_{\max}^{\text{PSO}}$	50
Control input lower bound	$u_{\min}$	-5 N	Maximum inertia weight	$w_{\max}$	0.9
Control input upper bound	$u_{\max}$	5 N	Minimum inertia weight	$w_{\min}$	0.4
Initial position	$x_1(0)$	0.5 m	Cognitive coefficient	c_1	2.0
Initial velocity	$x_2(0)$	0 m/s	Social coefficient	c_2	2.0

Additionally, the termination conditions for the improved PSO and the SQP solver in the proposed method are defined as follows:

(1) Improved PSO convergence:

$$|S_{\text{best}}^{(k)} - S_{\text{best}}^{(k-1)}| < \epsilon_{\text{PSO}}, \quad \epsilon_{\text{PSO}} = 10^{-6} \quad or \quad K_{\max}^{\text{PSO}} = 50 \quad iterations.$$

(2) Nonlinear programming (NLP) solver:

$$||\nabla S(u)|| < \epsilon_{\text{NLP}}, \quad \epsilon_{\text{NLP}} = 10^{-6}.$$

4.3. Simulation results and analysis

The MATLAB simulation results presented in Figure 4–Figure 6 illustrate the performance of the proposed hybrid intelligent DRTO framework under time-varying disturbances. The measurement disturbance applied to the system (4.2)–(4.4) is depicted in Figure 4, which was recorded upon the termination of Algorithm 2. The disturbance trajectory is generated by obtaining a random value within the range $[-9, 9]$ at each sampling instant, thereby constructing the overall disturbance sequence. It is observed that the trajectory exhibits aperiodic fluctuations and nonstationary characteristics, which are consistent with the defined disturbance model $\hat{d}(t)$. Under the time-varying disturbance shown in the previous figure, the system (4.2)–(4.4) is solved at each sampling period based on the current system initial state and disturbance observation. The solution yields an optimal control prediction sequence that satisfies the input constraints $[-5, 5]$, aiming to effectively suppress the time-varying disturbance acting on the system. Algorithm 2 employs a receding horizon strategy, applying only the first control input of the sequence to the plant. The final optimal control sequence trajectory at the end of Algorithm 2 is shown in Figure 5, and the corresponding optimal state trajectory is depicted in Figure 6. The state trajectory originates from the point $[0.5, 0]$ at time $t = 0$.

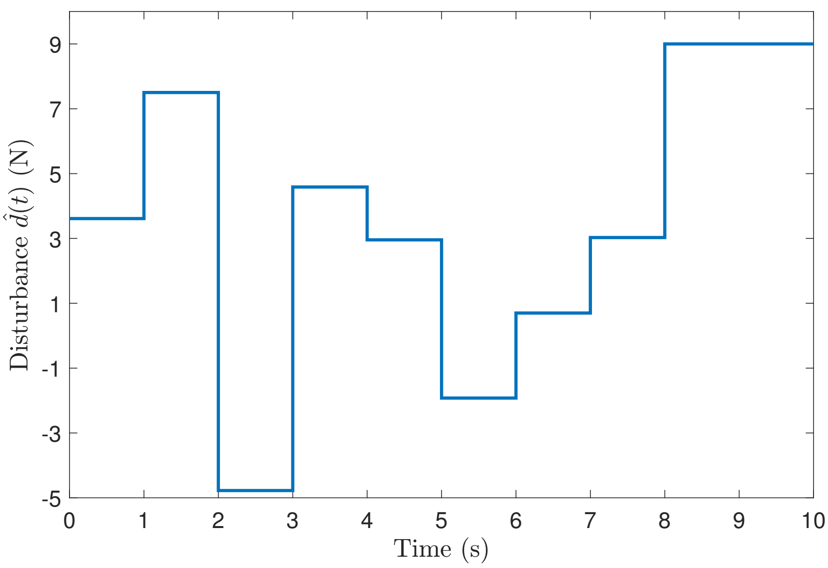


Figure 4. Disturbance trajectory acting on the CSD system (4.2)–(4.4) upon termination of Algorithm 2.

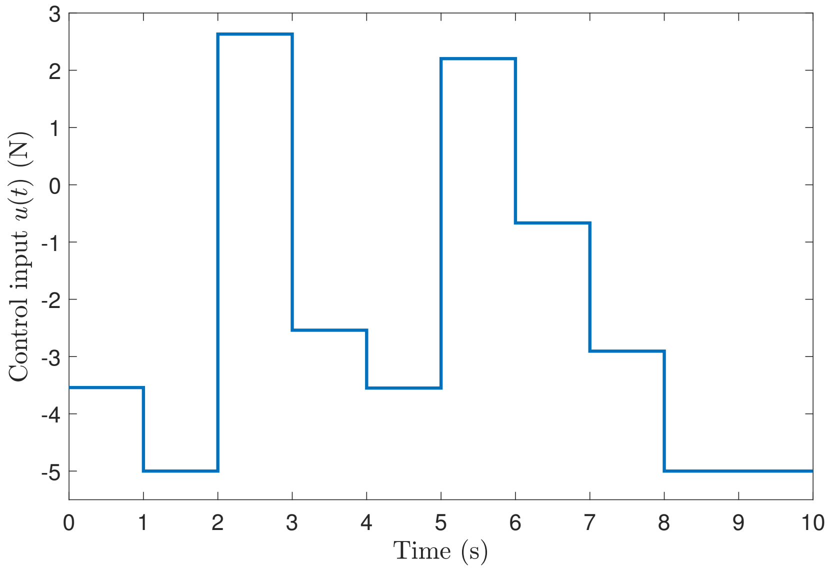


Figure 5. The control input profile for the CSD system (4.2)–(4.4) upon termination of Algorithm 2. The temporal domain $[0, 10]$ is discretized into 10 uniformly spaced intervals.

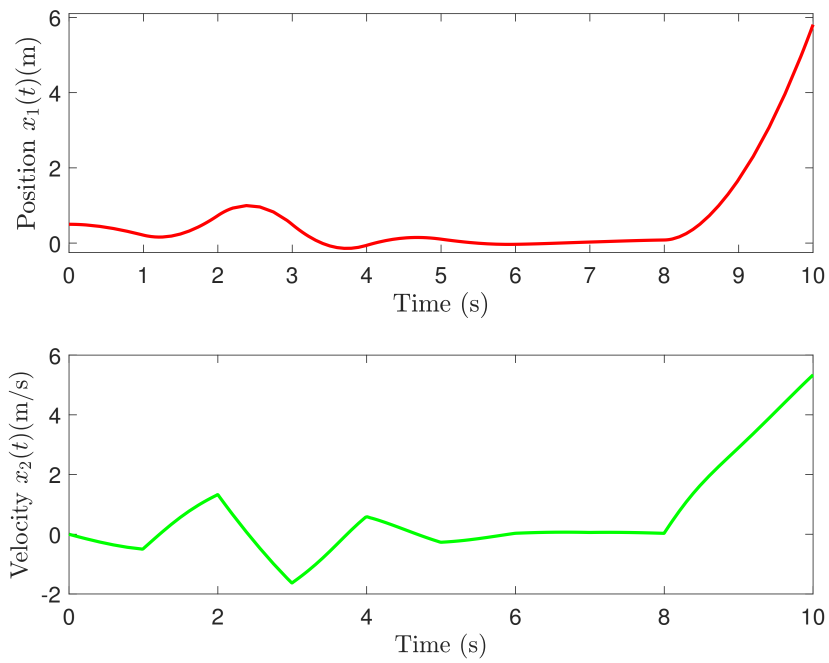


Figure 6. Optimal state trajectories for the CSD system (4.2)–(4.4) corresponding to the control input.

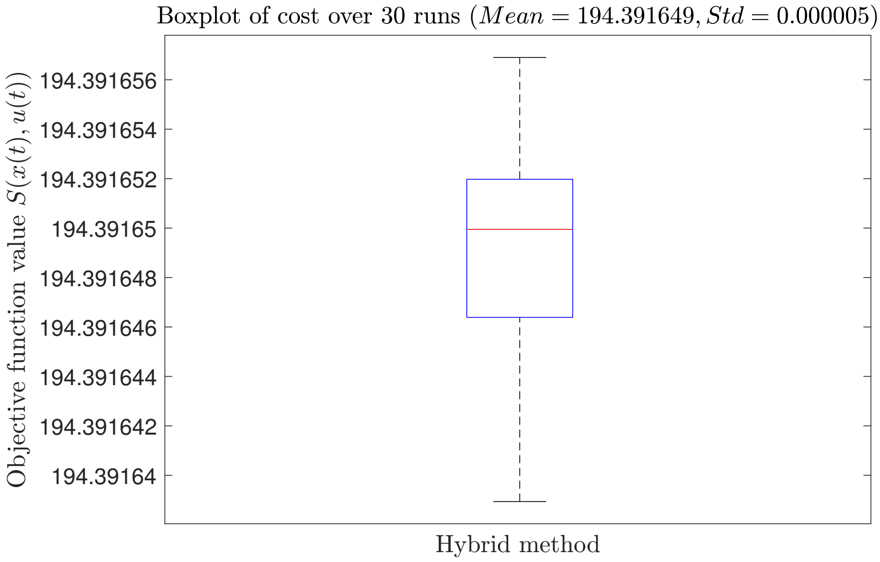


Figure 7. Box plots of objective function values for the CSD system (4.2)–(4.4) upon termination of Algorithm 2 over 30 independent runs.

Additionally, PSO-based algorithms are stochastic in nature. To further assess the stability of the proposed hybrid method, 30 independent runs were conducted under identical conditions with the same fixed disturbance sequence to ensure a fair comparison. The distribution of the objective function values is visualized using box plots in Figure 7. As shown in the figure, the proposed hybrid method exhibits a highly concentrated distribution of objective function values, with the difference between the maximum and minimum values being less than 10^{-4}, a small interquartile range, and no outliers. This indicates that nearly identical optimal performance is achieved in every run. Such a concentrated distribution, characterized by near-zero variance and the absence of outliers, demonstrates that the method consistently achieves near-optimal performance across different runs with random initial positions, confirming the excellent robustness of the hybrid approach to the stochastic nature of the PSO-based global search phase.

4.4. Comparison with the related methods

To further demonstrate the superior performance of the proposed hybrid intelligent DRTO method, this subsection presents a comparative study on the CSD system (4.2)–(4.4) against five benchmark algorithms, including a deterministic optimization method (SQP [22]) and four intelligent optimization algorithms (PSO [25], RCGA [29], ACO [32], and SA [34]). All compared methods are implemented within the shrinking-horizon framework to ensure a fair evaluation. For the PSO-based benchmark, the standalone PSO method employs the same ring-topology structure as the hybrid method. The simulation parameters for the PSO-based method are identical to those listed in Table 1. The parameter settings for the other four benchmark algorithms (RCGA, ACO, SA, and SQP) are summarized in Table 2.

Table 2. Parameter settings for the benchmark algorithms (RCGA, ACO, SA, and SQP).

RCGA parameters			**ACO parameters**		
Parameter	**Symbol**	**Value**	**Parameter**	**Symbol**	**Value**
Population size	N_p	20	Archive size	K	50
Max generations	$G_{\max}$	50	Max iterations	$I_{\max}$	50
Crossover probability	p_c	0.8	Exploration rate	q	0.5
Mutation probability	p_m	0.1	Deviation factor	ξ	0.5
SA parameters			**SQP parameters**		
Parameter	**Symbol**	**Value**	**Parameter**	**Symbol**	**Value**
Initial temperature	T_0	100	Initial position	$x_1(0)$	0.5 m
Cooling rate	α	0.95	Initial velocity	$x_2(0)$	0 m/s
Max iterations	$I_{\max}$	1000	Initial guess	u_0	0
Iterations per temp	L	20	Optimality tolerance	ϵ	10^{-6}

The performance comparison results are reported in Table 3. As shown in Table 3, the proposed hybrid method achieves the lowest objective value (194.3917) among all six methods, outperforming SA (195.8261), RCGA (200.3393), ACO (200.7753), SQP (202.2053), and PSO (246.7610). In terms of computational efficiency, the hybrid method (10.4321 s) is significantly faster than SA (63.3263 s), ACO (86.8133 s), and RCGA (24.2285 s) while being slightly slower than pure PSO (9.6296 s) but with substantially better solution quality (a 21.2% improvement). Compared with the pure SQP method, the hybrid approach achieves better optimality (194.3917 vs. 202.2053) with a shorter computation time (10.4321 s vs. 16.5801 s). These results indicate that the proposed hybrid strategy effectively balances solution quality and computational efficiency: It outperforms all five benchmark algorithms in terms of optimality and is significantly faster than SA, ACO, and RCGA. The ability to achieve superior optimization performance within a reasonable time frame renders the proposed hybrid DRTO framework highly attractive for online applications where both accuracy and speed are critical.

Table 3. Performance comparison of six different DRTO algorithms.

Method	**Iterations**	**Objective value**	**Computational time (s)**
Proposed hybrid method	225	194.3917	10.4321
SA [34]	10000	195.8261	63.3263
RCGA [29]	500	200.3393	24.2285
ACO [32]	500	200.7753	86.8133
SQP [22]	206	202.2053	16.5801
PSO [25]	66	246.7610	9.6296

5. Conclusion and future work

A hybrid intelligent DRTO method for optimal control of nonlinear systems under time-varying disturbances is proposed, which combines the advantages of the improved PSO and SQP methods to improve the solution accuracy of PSO and avoid falling into local optima obtained by the SQP method. The proposed framework is then applied to a CSD system with time-varying disturbances, and the simulation results confirm that the proposed hybrid intelligent algorithm outperforms both deterministic and existing intelligent algorithms in terms of optimization performance. Moreover, because the SQP phase refines the PSO solution by solving a sequence of quadratic programming subproblems, the final solution theoretically satisfies the necessary optimality conditions (i.e., the Karush–Kuhn–Tucker conditions), providing a rigorous theoretical foundation for the proposed approach.

The proposed algorithm provides a feasible framework for dynamic optimization under time-varying disturbances, and several meaningful extensions can be pursued: (1) Practical industrial processes often require adherence to safety and quality limits throughout operation, which are naturally expressed as path constraints (including point constraints). Extending the hybrid intelligent DRTO method to handle dynamic systems with path constraints under time-varying disturbances represents a valuable direction for further research. (2) Many industrial applications involve conflicting performance criteria. For instance, [56] developed a Tchebycheff-based multiobjective dynamic real-time optimization (MOO-DRTO) framework combined with a hybrid PSO-SQP approach for cycling energy systems, successfully generating Pareto-optimal solutions for a CO_2 capture process under time-varying conditions. Integrating similar multiobjective techniques into our algorithm would enable systematic handling of trade-offs between competing objectives under time-varying disturbances, broadening its applicability to the complex industrial process.

Author contributions

Wanlu Zhou: Writing original draft, methodology, algorithm design, formal analysis, writing-review & editing; Huan Li: Writing-Algorithm design, formal analysis, supervision; Jun Fu: Supervision, conceptualization; Siheng Yao: Validation, writing-review & editing. Ying Jin: Writing-review & editing.

Use of Generative-AI tools declaration

The authors declare that no generative AI tools were used in the writing, revision, or data processing of this manuscript.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. Jun Fu is an editorial board member for the *Journal of Industrial and Management Optimization* and was not involved in the editorial review or the decision to publish this article. All authors declare that there are no competing interests.

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