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Research article

Advancing the foundations of data envelopment analysis through fuzzy multi-objective optimization

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Abstract: Evaluating the performance of decision-making units (DMUs) in environments with multiple inputs and outputs remains a critical challenge, particularly when efficiency assessments must account for trade-offs and decision-maker preferences. Traditional Data Envelopment Analysis (DEA) models often reduce multidimensional performance into a single scalar efficiency score, limiting their ability to reflect complex decision contexts. To address this limitation, we proposed a novel framework that integrated DEA with Fuzzy Multi-Objective Linear Programming (FMOLP) and the Ordinal Priority Approach (OPA). The proposed model reformulated DEA within a multi-objective optimization framework, enabling simultaneous consideration of multiple performance criteria and incorporating preference information through ordinal rankings. The model's applicability was demonstrated using a dataset of renewable energy units, evaluated against economic and environmental indicators. The results showed that, compared to classical DEA and common-weight models, the proposed FMOLP–OPA framework provides a more comprehensive and discriminative assessment by capturing trade-offs among objectives and reflecting strategic priorities. The findings highlight that efficiency evaluation should move beyond purely technical measures toward preference-driven, multi-objective frameworks. The proposed approach offers enhanced decision support for complex real-world problems with multiple stakeholders and conflicting objectives.

Keywords: data envelopment analysis (DEA); fuzzy multi-objective optimization; ordinal priority approach (OPA)
Mathematics Subject Classification: 90B50, 90C08, 90C29, 90C70

1. Introduction

Evaluating the relative efficiency of decision-making units (DMUs) is a central problem in operations research and performance management. Data envelopment analysis (DEA) has become a standard tool for this purpose because it does not require a priori specification of a production function. However, classical DEA models reduce multidimensional input-output performance to a single efficiency score, obscuring the trade-offs that decision-makers face in practice [1].

Classical DEA models, such as the CCR and BCC formulations, provide scalar efficiency rankings via linear programming. However, these models often combine multiple, sometimes conflicting objectives into a single score, thereby obscuring the trade-offs that are crucial in challenging decision-making situations. Especially when uniformity, fairness, and the expression of preferences across heterogeneous DMUs are crucial [2,3], this limitation has prompted interest in incorporating DEA into the broader framework of multi-objective linear programming (MOLP). One such development is the common set weights (CSW) model, which aims to ensure comparability across DMUs; however, it remains sensitive to preferences and may conceal unit-specific characteristics. Research has developed methods to address these issues by combining preference models, fuzzy logic, and mixed analytical approaches. Notable examples include [4], which studies Pareto-Koopmans efficiency under adverse outcomes, and [5], which presents a hybrid AHP-DEA multi-objective model for sustainability assessment. Advances highlight the desire to combine DEA with machine learning and robust optimization. Hybrid models that integrate DEA with ensemble learning algorithms can effectively handle nonlinear and high-dimensional contexts, as shown in [6,7]. Similarly, the researchers in [8] present a DEA goal-programming framework with uncertainty sets for robust financial decision-making.

Furthermore, the theoretical alignment between DEA efficiency frontiers and Pareto optimality in MOLP has been examined. The researchers in [9] proposed a unified framework for efficiency decomposition in two-stage DEA systems that aligns conceptually with Pareto efficiency. These advances underscore the need to investigate the structural features of DEA models more comprehensively within multi-objective settings. In this paper, we aim to provide theoretical foundations by situating DEA within the MOLP framework. This goal requires a thorough analysis of the relationships between DEA frontiers and Pareto-efficient sets, the implications of dichotomous variables, the interpretation of weights, and the robustness of efficiency scores across formulations. From this perspective, classical and contemporary DEA techniques can be examined coherently, providing a more solid foundation for future research and applications. We propose a novel framework that integrates fuzzy multi-objective linear programming (FMOLP) with the OPA within a common-weights DEA structure. The FMOLP component captures imprecise objectives and constraints via membership functions, while OPA derives strategic weights from ordinal expert rankings without requiring numerical inputs. Applied to 15 renewable energy DMUs, our model reveals that technical efficiency (CCR score) does not equate to strategic importance. For example, DMU 14 has a low CCR score of 0.30 but achieves high FMOLP satisfaction (0.98) and OPA weight (0.0799). In contrast, DMU 4 has moderate CCR (0.57) but zero FMOLP satisfaction, indicating failure under multi-objective conditions.

The major contributions are: (1) A theoretical proof (Proposition 4) establishing the equivalence between Pareto efficiency under common weights and the existence of a separating weight vector; (2) an integrated FMOLP-OPA model that combines fairness, fuzziness, and ordinal preferences; (3) a replicable implementation protocol; and (4) an empirical demonstration on renewable energy data. The remainder of this paper is organized as follows: In Section 2, we review related literature. In Section 3, we present background concepts of PPS, notation, and classical DEA. Section 4 develops the theoretical framework. In Section 5, we introduce the integrated FMOLP-OPA Model. Section 6 presents an empirical application. Section 7 discusses the results, and Section 8 concludes the research.

2. Literature review

Advances in integrating data DEA into multi-objective decision-making (MODM) systems have yielded hybrid models that enhance decision-making procedures across industries. These integrations aim to overcome the limitations of conventional DEA models by incorporating multiple factors and stakeholder demands into the analysis. Many MODM techniques also exist [10,11]. The researchers in [12] examined how DEA can be applied to multi-criteria decision analysis (MCDA) and emphasized the advantages of hierarchical arrangements and slack-based techniques for distinguishing management options. This research showed that slack-based models are more effective at distinguishing options when multiple objectives and indicators are present. The researchers in [13] presented an integrated approach that combines DEA and MOLP to evaluate hospitals providing stroke care. This approach provided a more comprehensive evaluation framework by integrating historical performance statistics with decision-makers' preferences. The researchers in [14] proposed a DEA-based nonlinear multi-objective model: an optimization model for software component selection that examines build and purchase options under optimal redundancy. Given constraints such as compatibility, reliability, and delivery time, the model aims to minimize overall cost while maximizing purchase value. The researchers in [15] developed a hybrid fuzzy multi-objective DEA model to assess the performance of decision-making units (DMUs). Their approach included optimistic and pessimistic efficiency estimates, providing a balanced assessment of DMUs, especially in the education sector. For multi-attribute decision-making problems, the researchers in [16] proposed an integrated approach that combines grey relational analysis (GRA), the analytic hierarchy process (AHP), and DEA. This strategy improved the robustness of the decision-making process by addressing cases involving uncertain attribute weights and fuzzy data. Integrating DEA with various multi-criteria decision-making techniques promises to broaden and enhance the effectiveness of decision-making processes across fields. Given the need to assess efficiency in complex decision-making scenarios, DEA, a nonparametric technique for assessing the efficiency of decision-making units (DMUs), is increasingly used in hybrid models to incorporate multiple quantitative and qualitative criteria. This review of the DEA-MODM interface highlights key advances in methodology, real-world applications, and emerging trends. The researchers in [17] demonstrated a powerful hybrid approach that combines DEA with multiple regression and MCDM-GIS methods for strategic infrastructure planning, particularly in airport site selection. This model highlighted DEA's utility in spatial and environmental decision-making contexts. The researchers in [18] developed a new DEA-MCDM model with interval-valued fuzzy-efficiency measures for road infrastructure assessment. The integration of DEA with fuzzy-efficiency techniques and the WASPAS approach highlighted the potential to combine robustness and flexibility in performance assessment, especially under uncertainty. In the energy sector, the researchers in [19] developed a DEA-GEMS model to evaluate new energy industrial clusters, demonstrating DEA's compatibility with sustainable development goals and sector innovation analysis. The researchers in [20] presented an example of integrating DEA with the (Adaptive Immune Differential Evolution) AIDE algorithm for multi-reservoir operations planning. They showed how MODM frameworks can support complex environmental and operational decisions. The researchers in [21] extended DEA to reverse logistics in the textile industry using ANFIS hybrid models and demonstrated how DEA supports predictive analytics and sustainability goals in supply chain contexts. The researchers in [22] discussed DEA hybrid models under different economies of scale and addressed DEA's adaptability to dynamic operating environments and strategic planning challenges. The researchers in [23] evaluated performance in the printing sector using a hybrid DEA-AHP model and demonstrated how DEA can be integrated with hierarchical methods to support complex prioritization and resource allocation. The researchers in [24] applied a hybrid DEA-machine-learning approach to analyze the efficiency of female poultry farmers and demonstrated how modern computational methods can enhance traditional DEA frameworks to capture the socioeconomic dimensions of efficiency. Overall, these surveys indicate that DEA-MODM integration has become a flexible framework capable of managing complex decision environments across industries. The integration of fuzzy logic, machine learning, geographic information systems, and multi-criteria decision analysis into DEA models marks a significant methodological shift. This subject ensures that DEA remains a leading tool for multi-objective efficiency assessment. Many MADM techniques assign weights to criteria [25,26]. In this research, we use the new OPA approach.

Our framework builds directly on the seminal CCR and BCC models [27,28], extends the common-weights approach of [29,30] to a fuzzy multi-objective setting, integrates the OPA approach [31,32] for preference derivation, and adopts the fuzzy DEA formulation of [33] to handle imprecision.

Two major gaps persist. First, the common set of weights (CSW) model improves fairness across DMUs but still collapses multiple objectives into a scalar. Second, neither classical nor CSW DEA incorporates stakeholder preferences or handles the inherent ambiguity in real-world performance data. Fuzzy DEA methods address uncertainty but rarely integrate preference-based prioritization.

Table 1 summarizes key DEA-MODM hybrid studies, highlighting that no one simultaneously incorporates common weights, fuzzy multi-objective satisfaction, and ordinal preference aggregation, the gap filled by our FMOLP-OPA framework.

Table 1. Summary of a Few DEA-MODM Hybrid Models and Limitations.

Reference No.	Method	Application Area	Key Contribution	Limitation Addressed by Our Model
[12]	DEA + MCDA (slack-based)	Fisheries management	Distinguishes management options	No common weights; no fuzzy preferences
[13]	DEA + MOLP	Hospital stroke care	Integrates historical data with DM preferences	No fuzzy handling; no ordinal preference aggregation
[14]	DEA + nonlinear multi-objective	Software component selection	Minimizes cost while maximizing value	No common weights; no fuzzy membership

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Reference No.	Method	Application Area	Key Contribution	Limitation Addressed by Our Model
[15]	Fuzzy multi-objective DEA	Education sector	Optimistic/pessimistic efficiency	No common weights; no OPA integration
[16]	DEA + GRA + AHP	General MADM	Handles uncertain weights	No fuzzy multi-objective framework
[17]	DEA + regression + MCDM-GIS	Airport site selection	Spatial decision support	No fuzzy objectives; no preference weights
[18]	DEA + interval-valued fuzzy + WASPAS	Road infrastructure	Handles uncertainty	No common weights; no OPA
[19]	DEA + GEMS	Energy industrial clusters	Sustainability assessment	No fuzzy multi-objective; no ordinal preferences
[24]	DEA + machine learning	Female poultry farmers	Socioeconomic efficiency analysis	No fuzzy logic; no preference aggregation
This study	FMOLP + OPA + Common-weights DEA	Renewable energy	Common weights + fuzzy satisfaction + ordinal preferences	–

3. Background (PPS, notation, classical DEA)

In this section, we describe the main mathematical structures of DEA models, focusing on the production possibility set, the efficiency frontier, and the basic elements of linear programming and duality. These provide the analytical foundation for the multi-objective DEA framework developed later in the paper.

3.1. Notation and assumptions

We consider a set of n decision-making units (DMUs), each using m inputs to produce s outputs. Let:
$x_{ij} \in \mathbb{R}_+$: the amount of input i used by DMU_j, $i = 1, \dots, m$
$y_{rj} \in \mathbb{R}_+$: the amount of output r used by DMU_j, $r = 1, \dots, s$

$$x_j = (x_{1j}, \dots, x_{mj})^T \in \mathbb{R}_+^m: input\ vector\ for\ DMU_j$$

$$y_j = (y_{1j}, \dots, y_{sj})^T \in \mathbb{R}_+^s: output\ vector\ for\ DMU_j$$

Let $X = [x_1, \dots, x_n] \in \mathbb{R}_+^{m\times n}$ and $Y = [y_1, \dots, y_n] \in \mathbb{R}_+^{s\times n}$ denote the input and output data matrices, respectively.
Assumptions typically include:

- Inputs and outputs are non-negative and bounded.
- All DMUs operate under comparable technological conditions.
- Convexity and free disposability are assumed (unless otherwise stated).

3.2. Production possibility set (PPS)

The production possibility set represents all feasible input-output combinations based on observed data. Under variable returns to scale and convexity assumptions, the PPS is defined as:

$$T = \{(x, y) \in \mathbb{R}_+^m \times \mathbb{R}_+^s | \boldsymbol{x} \geq \boldsymbol{X\lambda}, \boldsymbol{y} \leq \boldsymbol{Y\lambda},\ \boldsymbol{\lambda} \geq 0, \mathbf{1}^T\boldsymbol{\lambda} = \mathbf{1}\}$$

here, $\boldsymbol{\lambda} \in \mathbb{R}_+^n$ is the intensity vector representing the convex combination of observed DMUs. This formulation yields the variable returns to scale (VRS) technology. For constant returns to scale (CRS), the constraint $\mathbf{1}^T\boldsymbol{\lambda} = \mathbf{1}$ is dropped.

Under variable returns to scale (VRS/BCC model), the production possibility set T is convex. However, the efficient frontier (the set of Pareto-efficient DMUs) is not necessarily convex. As illustrated in [34], the VRS frontier is piecewise linear and can be non-convex because multiple extreme efficient DMUs may exist. This classic observation motivates our careful distinction, in Proposition 1 and Remark 1, between the convexity of the weight space and that of the efficiency frontier.

3.3. Efficiency and the DEA frontier

A DMU_o with an input-output pair (x_o, y_o) is considered efficient if no other feasible DMU dominates it. Mathematically, $(x_o, y_o) \in T$ is Pareto-efficient if:

$$\nexists (x, y) \in T \text{ such that } x \leq x_o,\ y \geq y_o, \quad \text{with at least one strict inequality.}$$

The set of all such non-dominated points defines the efficiency frontier, also known as the Pareto frontier, of the DEA model. This frontier serves as the benchmark for evaluating other DMUs.

3.4. Linear Programming and Duality in DEA

DEA models are formulated as linear programs that seek to assess the efficiency of a given DMU relative to the frontier. For instance, the input-oriented envelopment model for $\boldsymbol{DMU_o}$ as in model (1):

$$\begin{aligned} \min_{\theta,\lambda} \quad & \theta \\ \text{s.t.} \quad & \boldsymbol{X\lambda} \leq \boldsymbol{\theta x_o} \\ & \boldsymbol{Y\lambda} \geq \boldsymbol{y_o} \\ & \mathbf{1}^T\boldsymbol{\lambda} = \mathbf{1} \quad \text{(for VRS)} \\ & \boldsymbol{\lambda} \geq \mathbf{0} \end{aligned} \tag{1}$$

here, θ is a scalar efficiency score; $\theta^* < 1$ indicates inefficiency. The dual of this linear program corresponds to the multiplier model, where input/output weights (v_i, u_r) are used to construct the efficiency score.

Duality theory plays a central role in interpreting DEA results:

- Primal variables λ show peer comparisons.
- Dual variables represent shadow prices or trade-offs across inputs and outputs.
- Strong duality ensures both models yield the same efficiency score.

- Corresponding to the primal envelopment model (1), the dual model, commonly referred to as the multiplier DEA model, can be expressed as follows for DMUo in model (2):

$$\begin{aligned} &\max_{u,v} \sum_{r=1}^{s} u_r y_{ro} \\ &s.t. \quad \sum_{i=1}^{m} v_i x_{io} = 1, \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \\ &u_r \geq 0, \quad i = 1, \dots, s, \\ &v_i \geq 0, \quad i = 1, \dots, m. \end{aligned} \tag{2}$$

In this model, u_r and v_i represent the weights (shadow prices) assigned to outputs and inputs, respectively. The objective function maximizes the weighted sum of outputs for the evaluated DMU, normalized by the weighted sum of inputs. The inequality constraints ensure that, under the same set of weights, no DMU can obtain a weighted output-input ratio greater than one.

By the strong duality property of linear programming, the optimal objective value of the dual model equals the optimal efficiency score obtained from the primal envelopment model. While the primal model emphasizes peer benchmarking and reference technologies through the intensity variables, the dual model provides an economic interpretation of efficiency via implicit trade-offs, marginal rates of substitution, and relative importance of inputs and outputs.

4. Theoretical framework

In this section, we present a novel DEA framework formulated from a multi-objective optimization perspective. Building on the classical common-set-of-weights model, we reinterpret the efficiency problem as an MOLP formulation, preserving the vector structure of the performance measures and enabling deeper theoretical insights.

4.1. A multi-objective DEA model

Classical DEA models, including CCR and common-weight approaches, reduce multiple inputs and outputs to a single scalar efficiency score for each DMU. While this simplification is useful for ranking, it completely masks the trade-offs among competing performance dimensions. In many real-world applications, such as renewable energy policy, healthcare, or banking, a DMU may be technically efficient under one weighting scheme but strategically inefficient under another. Therefore, a multi-objective formulation is required to preserve the vector structure of performance. Model (3) below does not directly produce a final efficiency score. Instead, it is designed to generate the Pareto frontier and to compute the individual upper and lower bounds Z_j^+ and Z_j^- for each DMU's net performance. These bounds are later used in the fuzzy membership functions (Section 4.5) to obtain satisfaction degrees λ_j in the integrated FMOLP-OPA model (Section 5). Before presenting Model (3), we define all variables used in this multi-objective formulation. The decision variables in this model are:

- u_r: The weight assigned to the output r (for $r = 1, \dots, s$),
- v_i: The weight assigned to the input i (for $i = 1, \dots, m$).

These weights are common across all DMU_s, meaning the same u_r and v_i are applied to every DMU. This subject ensures fairness and comparability. The auxiliary variable $Z_j(u, v) = \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij}$ represents the net performance of DMU_j. Each Z_j is treated as a separate objective function. This model requires no additional decision variables. The constraints include:

- $u^T y_j - v^T x_j \leq 0$ for all j (ensuring no DMU exceeds the frontier
- $\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$ (normalization),
- $u_r \geq \epsilon$ and $v_i \geq \epsilon$ (non-negativity with a small positive lower bound ϵ to avoid zero weights).

In Model (3), the number of objective functions equals the number of $DMU_s(n)$. Specifically, for each DMU_j, we define an objective function $Z_j(u, v) = u^T y_j - v^T x_j$. The meaning of each objective is as follows: It measures the net performance of a particular DMU under a common set of weights (u, v). If Z_j is positive, DMU_j performs above the common frontier; if zero, it lies exactly on the frontier; if negative, it lies below. However, Model (3) is not solved directly as a multi-objective optimization problem with n objectives. Instead, it serves as a theoretical construct to obtain individual upper and lower bounds Z_j^+ and Z_j^-, respectively. These bounds are obtained by solving, for each DMU_j, a single-objective LP that maximizes (or minimizes) $Z_j(u, v)$ subject to the same constraints. This issue is standard practice in fuzzy multi-objective programming [35]. Therefore, we do not require a general MOLP solver; we solve only $2n$ independent single-objective LPs.

Here is an MOLP formulation for evaluating the efficiency of n DMUs with an input matrix $X \in \mathbb{R}_+^{m \times n}$ and output matrix $Y \in \mathbb{R}_+^{s \times n}$. Instead of collapsing the multiple objectives into a scalar, we treat input minimization and output maximization explicitly in the model (3):

Find $Z_j(u, v)$ such that $\max\left\{u^T y_j - v^T x_j\right\}_{j=1}^{n}$

subject to:

$$
\begin{aligned}
& u^T y_j - v^T x_j \leq 0, \qquad \forall j = 1, \dots, n \\
& \textstyle\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1 \\
& u_r \geq \epsilon \\
& v_i \geq \epsilon
\end{aligned}
\tag{3}
$$

Note that Model (3) is not a scoring model and does not produce a single efficiency value. Instead, for each DMU_j, the optimal objective value Z_j^* obtained from solving Model (3) as a single-objective LP (maximizing only the j^{th} objective subject to the same constraints) gives the upper bound Z_j^+. The lower bound Z_j^- is obtained similarly by minimizing the same objective. These bounds are denoted Z_j^+ and Z_j^- and are essential for constructing the fuzzy membership functions $\mu_{zj}(x)$ in Section 4.5.

In this model:

- Decision variables: u_r (output weights, $r = 1, \dots, s$) and v_i (input weights, $i = 1, \dots, m$).
- Objective functions: $Z_j(u, v) = u^T y_j - v^T x_j$ for each $DMU_j, j = 1, \dots, n$. Each Z_j is maximized simultaneously in the MOLP sense.

Constraints: The first set of constraints ($u^T y_j - v^T x_j \leq 0$) defines the common-weight production possibility set. The second constraint ($\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$) normalizes the weights. This normalization constraint is introduced solely to avoid an unbounded weight space. It does not affect the relative performance ordering of DMUs, because scaling (u, v) by any positive constant scales all Z_j equally and leaves the inequality constraints invariant.

The third set ($u_r, v_i \geq \epsilon$) ensures strictly positive weights.
Each objective function

$$Z_j(u,v) = \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij}$$

represents the net performance of DMU j evaluated under a *common* weighting scheme. The normalization constraint fixes the scale of the dual variables but preserves the linearity and comparability of all objectives. Consequently, maximizing any individual Z_j still corresponds to improving the relative performance of DMU j within the same production possibility set.

Importantly, the same type of normalization is standard in classical DEA multiplier models, where the weighted sum of inputs (or outputs) is held constant without affecting the interpretation of efficiency or dominance relations. In this multi-objective setting, the normalization constraint ensures that all objectives are evaluated on a common, invariant scale while preserving their relative trade-offs.

Therefore, the one-to-one correspondence between objectives and DMUs remains valid: Each objective measures the performance margin of a particular DMU relative to the common frontier, and the Pareto-efficient solutions of model (3) jointly characterize the DEA efficiency frontier. The normalization constraint serves a technical role only and does not undermine the conceptual link between objectives and DMU-specific performance.

This model seeks a common weighting scheme $(v_i\,, u_r)$ that supports the Pareto dominance of efficient DMUs while preserving the multi-criteria nature of the decision environment. Each objective in the MOLP corresponds to a DMU's net performance, maintaining a vector-valued representation. Propositions 1–3 are presented to establish that the proposed MOLP-based DEA model preserves desirable properties from classical DEA (convex frontier, monotonicity, scale invariance). These are not claimed as novel contributions but as consistency checks. The main theoretical advance is Proposition 4, which provides a formal equivalence between Pareto efficiency under common weights and the existence of a separating weight vector; a result not available in standard DEA.

Solving model (3) as a true multi-objective problem with n objectives would be computationally demanding and conceptually unnecessary for our purpose. Instead, we adopt the following standard approach from fuzzy linear programming [35]: For each DMU_j, we solve two single-objective linear programs, one maximizing $Z_j(u,v)$ and one minimizing $Z_j(u,v)$, subject to the same constraints as Model (3). The optimal values obtained are Z_j^+ (upper bound) and Z_j^- (lower bound), respectively. No additional MOLP technique (such as weighted sum, $\epsilon - constraint$, or goal programming) is applied to Model (3). The outputs of Model (3), Z_j^+ and Z_j^- for each DMU_j, are not final efficiency scores. They are intermediate values that feed into the fuzzy membership functions defined in Section 4.5 (Equations 14–15). Only after solving the integrated FMOLP-OPA model (Section 5, Model 48) do we obtain the final satisfaction degrees $\lambda_j \in [0,1]$, which represent a preference-aware, multi-objective efficiency measure.

A legitimate concern arises: Does the normalization constraint $\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$ break the one-to-one correspondence between each objective function $Z_j(u,v) = u^T y_j - v^T x_j$ and the performance of DMU_j? We now provide a formal justification that this is not the case.

The normalization constraint serves only to fix the scale of the weight vector (u,v). Without it, any positive multiple of a feasible weight vector would also be feasible and would produce proportionally scaled values of all Z_j. However, the relative orderings of DMUs by their z_j values remain invariant under positive scaling. More formally, for any feasible weight vector (u,v) satisfying $u^T y_j - v^T x_j \leq 0$, and any scalar $\alpha > 0$, the scaled vector $(\alpha u, \alpha v)$ satisfies the same inequality constraints and yields $Z_j(\alpha u, \alpha v) = \alpha Z_j(u,v)$. Therefore, the ratio Z_j/Z_k for any two DMU j and k is unchanged.

The normalization constraint $\sum u_{r+} \sum v_i = 1$ selects a unique representative from each ray of equivalent weight vectors. It does not alter which DMUs are Pareto-efficient, nor does it affect the sign or relative magnitude of any Z_j. Consequently, each Z_j still measures the net performance of DMU_j under a common weight set, and the one-to-one correspondence between objectives and DMUs remains valid. The same normalization is standard in classical DEA multiplier models (e.g., fixing $\sum v_i x_{io} = 1$ for DMU_o), and there it never raises doubts about the objective's meaning. Our formulation extends this normalization to the sum of all weights for technical convenience.

4.2. Theoretical properties (Propositions 1–4)

We now establish key theoretical properties of the proposed MOLP-based DEA model.
Proposition 1 (Convexity of the Feasible Weight Set).
The feasible set of weight vectors (v,u) defined by the constraints of Model (3), specifically:

- $u^T y_j - v^T x_j \leq 0,\ for\ all\ j = 1, \dots, n,$
- $\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$,
- $u_r \geq \epsilon, \quad v_i \geq \epsilon$

is a convex set.
Proof. All constraints are linear equalities or inequalities. The set defined by linear constraints is a convex polyhedron (or a convex polytope when bounded). Hence, the feasible region of (v,u) is convex. □
Remark 1 (Non-convexity of the Pareto-efficient frontier).

While the feasible set of weights (v,u) in Model (3) is convex, this does not imply convexity of the Pareto-efficient frontier of the DMUs. Indeed, as is well known from classical DEA theory [34], the production possibility set under variable returns to scale (VRS/BCC model) is convex. Yet, its efficient frontier is piecewise linear and non-convex. The same phenomenon can occur in our multi-objective formulation: The set of Pareto-efficient DMUs (in terms of input-output vectors) may be non-convex even though the weight space is convex. Therefore, Proposition 1 claims only convexity of the weight space, not convexity of the efficiency frontier.
Proposition 2 (Monotonicity).

If a DMU increases its output or reduces its inputs, its efficiency score (vector) under the proposed model will not worsen.
Proof.

Suppose $y'_j \geq y_j$ and $x'_j \leq x_j$ component-wise. Then, for any feasible weighting scheme (v,u), we have:

$$u^T y'_j - v^T x'_j \geq u^T y_j - v^T x_j$$

Hence, improving performance through input reduction or output increase cannot reduce the vector efficiency. □

Proposition 3 (Scale Invariance). Efficiency is invariant under proportional scaling of all inputs and outputs of a DMU.

Proof.

Let $x'_j = \alpha x_j$, $y'_j = \alpha y_j$ for $\alpha > 0$.

Then:

$$u^T y'_j - v^T x'_j = \alpha(u^T y_j - v^T x_j)$$

This Equation preserves the ordering of DMUs by efficiency while uniformly scaling the net score. □

The scale invariance property established in Proposition 3 is closely related to the role of the normalization constraint $\sum u_{r+} \sum v_i = 1$ in Model (3). Because scaling all inputs and outputs of a DMU by a common factor αα results in scaling its net performance z_j by α, the relative efficiency ordering among DMUs remains unchanged. The normalization constraint does not interfere with this property; it merely selects one particular scaling of the weight vector. Therefore, the correspondence between each objective z_j and DMU_j is preserved despite the normalization. This issue justifies treating z_j as a valid representation of $DMU_j{}'$s performance under common weights. □

Proposition 4 (Supported Pareto efficiency under common weights).

A DMU (x_o, y_o) is a *supported Pareto-efficient* solution of the common-weight MOLP model (3) if and only if there exists a feasible common weight vector (u, v), normalized such that $\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$, for which:

$$u^T y_j - v^T x_j \leq 0 \ and \ u^T y_o - v^T x_o = 0, \ \forall j.$$

Proof. The proof follows directly from the standard weighted-sum scalarization theorem in multi-objective linear programming [35,36]. In MOLP, a feasible solution is Pareto-efficient if and only if it optimizes a weighted sum of the objective functions for some non-negative weight vector. Applying this result to Model (3), where each objective $Z_j(u, v) = u^T y_j - v^T x_j$ corresponds to the net performance of DMU j under common weights, we obtain the stated equivalence. The weight vector (u, v) in Proposition 4 plays the role of the scalarization weights in the standard MOLP theorem.

Sufficiency: If such (u, v) exists, then (x_o, y_o) maximizes the weighted sum of objectives (with weight 1 on objective oo and weight 0 on all others) among all DMUs; hence, it is supported Pareto-efficient.

Necessity: If (x_o, y_o) is supported Pareto-efficient, the scalarization theorem guarantees the existence of a weight vector that separates it from the feasible set, yielding the required conditions. □

A remark on the distinction between supported and unsupported efficient solutions. The proposition above applies specifically to *supported* Pareto-efficient solutions; those that can be obtained by solving a weighted-sum scalarization of the original multi-objective problem. In general, multi-objective optimization may yield inefficient solutions that no weighted-sum approach can recover [36]. These unsupported solutions are Pareto-efficient but do not satisfy the separating weight vector condition. In the context of DEA, this distinction mirrors the classical observation [34] that the VRS efficiency frontier may contain unsupported extreme points. Proposition 4, therefore, characterizes only the subset of DMUs that are *supported* Pareto-efficient under common weights; this result does not exclude the existence of unsupported efficient DMUs.

Comparison with Classical CCR DEA. In the classical CCR model, a DMU can achieve an efficiency score of 1 using weights that are optimized specifically for that unit. These weights may not be feasible for other DMUs, meaning that CCR efficiency does not imply the existence of a common weight vector that separates the DMU from all others. In contrast, in the proposed common-weight

MOLP framework, supported Pareto efficiency is *equivalent* to the existence of such a separating vector. This subject follows directly from embedding DEA within the standard MOLP framework and applying the well-known scalarization theorem. The result reinforces the fairness and comparability advantages of the common-weight approach and clarifies that the theorem is a standard MOLP result rather than a novel DEA-specific proposition.

Comparison with Existing Models

The proposed model generalizes the common set of weights in the DEA model by interpreting them not as a means of scalarizing efficiency, but as a means of constructing the Pareto front. This subject contrasts with:

- CCR/BCC models, which assign weights per DMU and thus are not directly comparable across units.
- Classical common-weight models impose a single linear objective and lose Pareto detail.
- Our model maintains a multi-objective viewpoint, offering:
  - A family of efficient DMUs along the frontier.
  - Insights into the trade-offs between performance measures.
  - Adaptability to incorporate decision-makers' preferences (via weighted sums or the ε-constraint method).

Furthermore, this approach facilitates natural extensions:

- Preference-based efficiency via scalarization post hoc
- Robust efficiency under data uncertainty
- Hybrid models combining DEA with goal programming or utility theory.

Unlike classical DEA, where efficiency is defined per DMU with individualized weights, the proposed MOLP formulation ensures that efficient DMUs remain non-dominated even when all DMUs use the same weight set. Proposition 4 formalizes this distinction.

This result follows from the standard equivalence in multi-objective linear programming between supported Pareto-efficient solutions and weighted-sum scalarization [35,36]. The separating vector (u, v) corresponds to a supporting hyperplane of the common-weight frontier.

Corollary 1. In CCR DEA, the efficiency of a DMU does not imply the existence of a common separating vector, since weights are unit-specific. In the proposed common-weight MOLP formulation, efficiency is equivalent to the existence of such a vector.

4.3. Common set of weights in the DEA model

To illustrate the tension between scalar efficiency measurement and multi-objective structure, consider the commonly used common set of weights in the DEA model, which seeks a universal set of input and output weights applicable across all DMUs. Then, model (4) is presented based on the above-mentioned subjects:

$$Max \left\{ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}; \quad j = 1, \dots n \right\}$$

s.t.

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad i = 1, \dots n \tag{4}$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$$
$$u_r \geq \epsilon$$
$$v_i \geq \epsilon$$

The mathematical model (4) identifies a single set of weights $\{u_r, v_i\}$ that generally maximizes each DMU's net efficiency under a uniform weighting scheme. This model ensures fairness and comparability by applying the same evaluation criteria to all units. However, the implicit scaling of multiple objectives, here, weighted outputs versus weighted inputs, still obscures the necessary trade-offs between them. Furthermore, the constraint

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$$

acts as a normalization device but creates tension between interpretability and mathematical flexibility. From a multi-objective perspective, this formulation approximates the Pareto frontier in a single dimension. As such, it considers a particular efficiency profile and excludes other profiles that may be equally Pareto-optimal under alternative weighting schemes. This issue reinforces the need for a robust vector or preference approach to efficiency assessment, which we seek to address by developing a theoretical framework that explicitly incorporates the principles of multi-objective optimization into DEA. The model attempts to maximize the net efficiency of a given DMU under a set of globally consistent weights.

Although it promotes comparability, the scalar objective still simplifies a fundamentally multi-objective decision structure: improving multiple outputs and reducing various inputs. From an MOLP perspective, this model projects the Pareto frontier onto a single weighted efficiency score, potentially overlooking alternative Pareto-efficient solutions with different preferences or decision priorities.

4.4. General model of multi-objective linear programming

The general multi-objective model can be expressed as follows:

$$min\, Z_1, Z_2, \ldots, Z_k.$$
$$max\, Z_{k+1}, Z_{k+2}, \ldots, Z_{P,}$$
$$s.t.:$$
$$x \in X_{d,} X_d = \{x | g(x) \leq b_r, r = 1,2, \ldots, m\}$$

where $Z_1, Z_2, \ldots, Z_{k,}$ are the objectives with negative aspects (such as cost, shortage of goods, delivering goods to the customer after the due date) and $Z_{k+1}, Z_{k+2}, \ldots, Z_{P,}$ are the positive aspects, such as product quality, on-time delivery to customers, and after-sales service. X_d, is the set of justified solutions that are limited by the capacity of the machine centers, the prerequisite constraints on the sequence of tasks, the range in which the execution time of an activity can be compressed at a cost, etc. This study presents the mathematical model for the CSW problem, with details below, along with the relevant objective functions. Here, the aim is to optimize all objectives simultaneously and reach the best point within the feasible region.

4.5. Fuzzy decisions

Classical decision theory relies on clear outcomes within defined decision and state spaces and becomes complex under uncertainty. In contrast, fuzzy set theory handles imprecision by modeling decisions with fuzzy constraints and objective functions based on set commonality [35]. Fuzzy decision-making is categorized as symmetric, where objectives and constraints have equal importance, and asymmetric, where they are assigned different, proportional weights based on their significance [37]. Creating a symmetric and asymmetric model depends on the choice of operators. In fuzzy decision-making, selecting the appropriate operators is crucial for effective decision-making. The researchers in [38] identified eight key criteria for choosing appropriate operators in fuzzy decision-making, which are further examined concerning the CSW problem, followed by a discussion of suitable operators within a general fuzzy multi-objective programming model:

The vector x in the modified form $x^T = [x_1, x_2, \dots, x_n]$ that minimizes the objective function Z_k and maximizes the objective function Z_l is obtained according to the following assumptions.

$$Z_k = \sum_{i=1}^{n} c_{ki} x_i, k = 1,2, \dots p, \tag{5}$$

$$Z_l = \sum_{i=1}^{n} c_{li} x_i, l = p + 1. p + 2, \dots q \tag{6}$$

The constraints are as follows:

$$x \in X_d = \{x | g(x) = \sum_{i=1}^{n} a_{ri} x_i \leq b_r, r = 1,2, \dots, m, x \geq 0\} \tag{7}$$

where the values of C_{ki}, C_{li}, a_{ri}, and b_r are fuzzy or crisp, and the researcher in [35] solved the problem formed by equations (5) to (7) by fuzzy linear programming. He modeled fuzzy linear programming by separating each objective function Z_j into its maximum values Z_j^+ and its minimum values Z_j^-. It is explicitly stated here that Z_j^+ and Z_j^- for each DMU are obtained by solving model (4) as a single-objective LP, maximizing and minimizing $u^T y_j - v^T x_j$, respectively, without the common-weight cross-constraints.
This issue means that:

To obtain Z_k^+ and Z_k^- for each minimization objective (and similarly Z_l^+ and Z_l^- for maximization objectives using equations (8) and (9)), we solve Model (3) as a series of single-objective linear programs. Specifically, for each DMU_j:

- $Z_j^+ = max\{u^T y_j - v^T x_j\}$ subject to the constraints of Model (3),
- $Z_j^- = min\{u^T y_j - v^T x_j\}$ subject to the same constraints.

These $2n$ independent LP problems are solved using standard simplex-based solvers (e.g., Gurobi, CPLEX, or the linprog function in MATLAB). No multi-objective optimization algorithm is applied directly to Model (3).

The values Z_k^+ and Z_k^- (for minimization objectives) and Z_l^+ and Z_l^- (for maximization objectives) are obtained by solving Model (3) as a series of single-objective linear programs. Specifically, for each DMU_j, Z_j^+ is the optimal value when maximizing $u^T y_j - v^T x_j$ subject to the constraints of Model (3), without any cross-DMU objective aggregation. Similarly, Z_j^- is obtained by minimizing the same expression.

$$Z_k^+ = max\, Z_k\,, x \in X_{a,} Z_k^- = min\, Z_k\,, x \in X_d \tag{8}$$

$$Z_l^+ = max\, Z_l\,, x \in X_d, Z_l^- = min\, Z_l\,, x \in X_d \tag{9}$$

where Z_k^- and Z_l^+ are obtained by solving the multi-objective mathematical model as a single objective function; only one objective is calculated at a time, $x \in X_d$ means that the solutions must satisfy the constraints, and X_d is the set of optimal solutions obtained by solving the single objective function. Since it is assumed that each function Z_j's value varies linearly between Z_k^- and Z_l^+, it can be considered a fuzzy number with a fuzzy membership function $\mu_{zj}(x)$, as seen in Figures 1 and 2. A linear programming problem, such as (5) to (7), with the objective function and fuzzy constraints, can be presented as follows.

$$\tilde{Z}_k = \sum_{i=1}^{n} c_{ki}x_i \leq \sim Z_k^0, k = 1,2,\ldots,p \tag{10}$$

$$\tilde{Z}_l = \sum_{i=1}^{n} c_{li}x_i \geq \sim Z_l^0, l = p+1, p+2,\ldots,q \quad \text{(For Fuzzy Constraints)} \tag{11}$$

$$\tilde{g}_i(x) = \sum_{i=1}^{n} a_{ri}x_i \leq \sim b_r, r = 1,2,\ldots,h \tag{12}$$

$$(g_p(x) = \sum_{i=1}^{n} a_{pi}x_i \leq b_p, p = h+1,\ldots,m \qquad \text{For Crisp Constraints}$$
$$x_i \geq 0, \qquad i = 1,2,\ldots,n \tag{13}$$

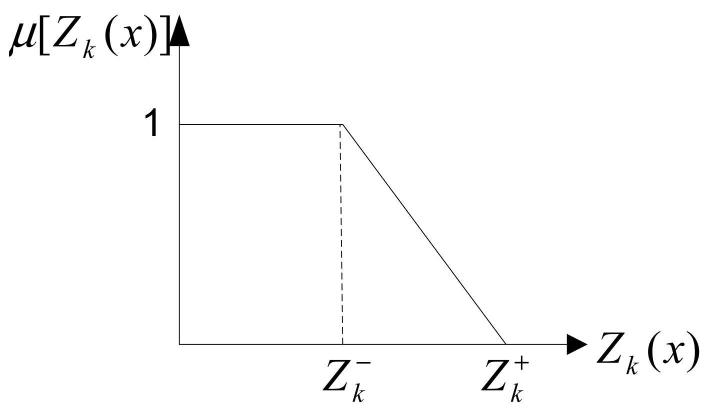

Figure 1. Objective function with a negative aspect ($\tilde{Z}_k$).

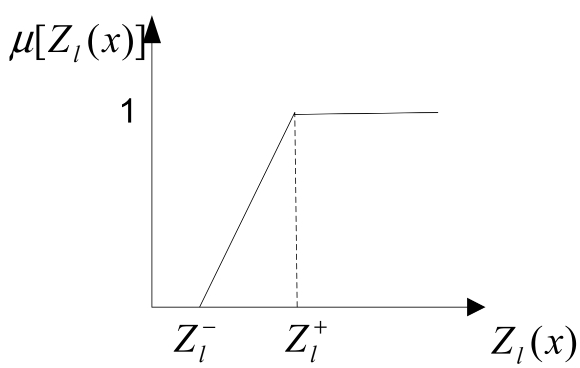

Figure 2. Objective function with a positive aspect ($\tilde{Z}_l$).

In the model consisting of equations (10)–(13), the symbol ~ expresses the fuzzy environment. The symbol $\leq \sim$ in the set of constraints represents the fuzzy state $\leq$ and has a linguistic interpretation as "necessarily less than or equal to," and the symbol $\geq \sim$ has a linguistic interpretation as "necessarily greater than or equal to". Additionally, in the above relationships, parameters Z_k^0 and Z_l^0 are the desired levels that the decision-maker wishes to reach. Assuming the membership functions are based on priority or linear compensation, the linear membership functions for the minimization (Z_k) and maximization (Z_l) objectives are given as Equations (14) and (15):

$$\mu_{zk}(x) = \begin{cases} 1 & for \quad Z_k \leq Z_k^-, \\ (Z_k(x) - Z_k^-)/(Z_k^+ - Z_k^-) for Z_k^- \leq Z_k(x) \leq Z_k^+, k = 1,2,\ldots p \\ 0 & for \quad z_k \geq Z_k^+. \end{cases} \tag{14}$$

$$\mu_{zl}(x) = \begin{cases} 1 & for \quad Z_l \geq Z_l^-, \\ (Z_l(x) - Z_l^-)/(Z_l^+ - Z_l^-) for Z_l^- \leq Z_l(x) \leq Z_l^+, k = 1,2,\ldots q \\ 0 & for \quad z_l \leq Z_l^+. \end{cases} \tag{15}$$

The linear membership function for fuzzy constraints is as follows:

$$\mu_{gr}(x) = \begin{cases} 1 & for \quad g_r(x) \leq b_r, \\ 1 - (g_r(x) - b_r)/d_r for b_r \leq g_r(x) \leq b_r + d_r, r = 1,2,\ldots h \\ 0 & for \quad g_r(x) \geq b_r + d_r. \end{cases} \tag{16}$$

"d_r" is a subjective constant value chosen to express the acceptable limit of violating the r^{th} inequality constraint (or the tolerance range of that constraint).

In the next section, we describe some essential operators in fuzzy decision-making.

Decision-making operators

First, we discuss the $min - max$ operator, which [37,38] have used for fuzzy multi-objective problems. The convex operator used in its expression enables the decision maker to assign different weights to different criteria. In fuzzy programming modeling, using the approach in [38], a fuzzy solution is obtained by combining all the presented fuzzy objectives or fuzzy constraints. The fuzzy solution for all fuzzy objectives and fuzzy constraints can be given as Equations (17) and (18):

$$\mu_D(x) = \left\{ \left\{ \bigcap_{q}^{j=1} \mu_{zj}(x) \right\} \cap \left\{ \bigcap_{h}^{r=1} \mu_{gr}(x) \right\} \right\} \tag{17}$$

The following Equation gives the optimal solution (x^*).

$$\mu_D(x^*) = \max_{x \in X_d} \mu_D(x) = \max_{x \in X_d} min \left[\min_{j=1,\ldots,q} \mu_{zj}(x), \min_{r=1,\ldots,h} \mu_{gr}(x) \right] \tag{18}$$

To find the optimal solution (x^*) in the above fuzzy model, the following crisp model can be solved equivalently [37].

$$Maximize \ \lambda \tag{19}$$
$$s.t.$$

$$\text{(For All the Objective Functions) } \lambda \leq \mu_{zj}(x), j = 1,2,\dots,q \tag{20}$$

$$\text{(For Fuzzy Constraints) } \lambda \leq \mu_{gr}(x), r = 1,2,\dots,h \tag{21}$$

$$\text{(For Crisp Constraints) } \quad g_p(x) \leq b_{ps}, p = 1,2,\dots,m \tag{22}$$

$$x_i \geq 0, i = 1,2,\dots,n, \quad and \quad \lambda \in [0,1] \tag{23}$$

where $\underset{D}{\mu}(x)$, $\mu_{zj}(x)$, and $\mu_{gr}(x)$ are the solution membership functions, objective functions, and constraints, respectively.

This solution describes the relationship between constraints and objective functions in a completely symmetric fuzzy environment [35]. In fuzzy decision-making, although objectives and constraints may vary in importance across problems, their interaction must be considered. The weighted ensemble model addresses this using a single utility function to reflect the decision-maker's preferences and to prioritize criteria accordingly [39]. In this case, a linear weighted utility function is created by multiplying each fuzzy objective's membership function by its respective weight and summing the results. The convex fuzzy model proposed by [40,41] and the weighted incremental model by [42] are:

$$\mu_D(x) = \sum_{j=1}^{q} w_j \mu_{zj}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x) \tag{24}$$

$$\sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, \quad w_j \; and \; \beta_r \geq 0 \tag{25}$$

where w_j and β_r are weighting coefficients that represent the relative importance between fuzzy objectives and fuzzy constraints, respectively. The following crisp single-objective programming is equivalent to the presented multi-objective fuzzy model:

$$max \sum_{j=1}^{q} w_j \lambda_j + \sum_{r=1}^{h} \beta_r \gamma_r \tag{26}$$

$$s.t.:$$

$$\lambda_j \leq \mu_{zj}(x), j = 1,2,\dots,q \tag{27}$$

$$\gamma_r \leq \mu_{gr}(x), r = 1,2,\dots,h. \tag{28}$$

$$g_p(x) \leq b_p, p = h+1,\dots,m. \tag{29}$$

$$\lambda_j, \;\; \gamma_r \in [0,1], \quad j = 1,2,\dots,q, \quad and \quad r = 1,2,\dots,h, \tag{30}$$

$$\sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, \quad w_j, \quad \beta_r \geq 0 \tag{31}$$

$$x_i \geq 0. i = 1,2,\dots,n. \tag{32}$$

Symmetric fuzzy multi-objective model

According to the previous explanations and equations (19)–(23), the proposed model for the symmetric fuzzy multi-objective case is given by the following expression. First, we will explain the new symbols in this relation.

Added symbols in symmetric fuzzy multi-objective programming

Z_l^+: Upper limit of the objective function changes l

b_r^+: Upper limit of the constraint changes r

b_r^-: Lower limit of the constraint changes r

λ: Satisfaction degree according to the relevant objective or constraint

In the proposed symmetric fuzzy multi-objective model, all objectives and constraints are treated as fuzzy and given equal importance, utilizing a single satisfaction degree λ that applies uniformly across the objective functions and related constraints, such as the machine center constraint.

$$Maximize \quad \lambda \tag{33}$$

$$\lambda \in [0,1] \tag{34}$$

To demonstrate the applicability of the proposed asymmetric fuzzy multi-objective model, it is applied to the CSW problem, and the results are analyzed after solving the model.

Asymmetric Fuzzy Multi-Objective Model

In this model, all fifteen fuzzy objective functions are included. Furthermore, there are varying degrees of satisfaction for the objective functions and constraints within the asymmetric model (i.e., $\lambda_1, \lambda_2, \dots, \lambda_{15}$). Consequently, by assigning values to the coefficients of the objective functions (which must sum to one according to Equation (31), the proposed Asymmetric Fuzzy Multi-Objective Programming Model is formulated as follows. First, we clarify the new symbols in the discussion of Asymmetric Fuzzy Multi-Objective Programming.

Symbols in Asymmetric Fuzzy Multi-Objective Programming

The Symbols in the asymmetric fuzzy multi-objective are as follows:

δ_j: weight coefficient representing the importance of the j^{th} objective;

γ_r: weight coefficient representing the importance of the r^{th} constraint;

λ_j: satisfaction degree of the jth objective function;

β_r: satisfaction degree of the r^{th} constraint;

$$\sum_{j=1}^{q} \delta_j + \sum_{r=1}^{h} \gamma_r = 1,$$

The above Equation indicates that the sum of the weights of the objective functions and fuzzy constraints must equal one.

$$Maximize \sum_{j=1}^{q} \delta_j \lambda_j + \sum_{r=1}^{h} \gamma_r \beta_r \tag{35}$$

$$\lambda_i, \beta_r \in [0,1] \tag{36}$$

4.6. Ordinal Priority Approach (OPA)

In this section, we describe the proposed model for determining the numerical weight of preference ranks for decision-making criteria based on experts' comments.

Symbols and signs

Tables 2 and 3 review the symbols and signs used in the presented mathematical model and show their definitions for indices, parameters, and variables.

Definition of sets

Table 2 reviews the sets used in the presented mathematical model for ranking criteria and shows their definitions.

Table 2. Sets Definitions.

Definition	Set
Experts Set, $\forall\, i \in I$	I
Criteria Set, $\forall\, j \in J$	J
Alternatives Set, $\forall\, k \in K$	K
Ranks Set, $\forall\, m \in M$	M

Parameters Definition

Table 3 presents the parameters used in the mathematical model and their definitions.

Table 3. Parameters Definitions.

Definition	Set
Experts Preference Rank	i
Criteria Preference Rank	j
Alternatives Preference Rank	r

Variables Definition

Table 4 presents the variables used in the mathematical model and their definitions.

Table 4. Variables Definition.

Definition	Set
Objective Function (Unrestricted in sign)	Z
The weight of the k^{th} alternative according to the j^{th} Criterion as assessed by the i^{th} expert at the r^{th}rank.	${W_{ijk}}^r$

Model Design

Most group decision-making methods rank alternatives based on expert-weighted indicators but often overlook the interrelated importance of alternatives, indicators, and experts. The proposed model addresses this by incorporating all three elements simultaneously, enabling experts to evaluate alternatives using indicators to determine the importance of each component.

Suppose there is a ranking of k alternatives based on the following steps:

• Ranking of experts (considering the organizational chart, academic degree, experience, etc.).

• Ranking of indicators by each expert.
• Ranking of alternatives according to each indicator by each expert.

Table 5. Experts' opinions.

Experts Ranking	Ranking of criteria by each expert	Ranking of alternatives according to each criterion by each expert
i^{th} Expert	The j^{th} criterion is based on the i^{th} expert opinion.	${A_{ijk}}^{1}$, ${A_{ijk}}^{2}, \dots, {A_{ijk}}^{\mathrm{r}}, \dots, {A_{ijk}}^{m}$

As in Table 5, the k^{th} alternative based on the j^{th} criterion by the i^{th} expert at the r^{th} rank is referred to as ${A_{ijk}}^{(r)}$, indicating that only independent alternatives have been provided by the experts according to the priority of the index and the alternatives, such that

$${A_{ijk}}^{1} > {A_{ijk}}^{2} > \cdots > {A_{ijk}}^{r} > {A_{ijk}}^{r+1} > \cdots > {A_{ijk}}^{m} \tag{37}$$

The only valid inference from the inequality ${A_{ijt}}^{r} > {A_{ijl}}^{r+1}$ (the superiority of the t^{th} alternative over the l^{th} alternative) is that ${W_{ijt}}^{n} > {W_{ijl}}^{n+1}$.

$${W_{ijk}}^{1} > {W_{ijk}}^{2} > \cdots > {W_{ijk}}^{r} > {W_{ijk}}^{r+1} \dots > {W_{ijk}}^{m-1} > {W_{ijk}}^{m} \tag{38}$$

In other words, the difference in weights associated with consecutive ranks in Equation (38) is positive:

$$\begin{gathered} {W_{ijk}}^{1} - {W_{ijk}}^{2} > 0, \\ {W_{ijk}}^{2} - {W_{ijk}}^{3} > 0, \\ {W_{ijk}}^{r} - {W_{ijk}}^{r+1} > 0, \dots \\ {W_{ijk}}^{m-1} - {W_{ijk}}^{m} > 0 \end{gathered} \tag{39}$$

In Equation (38), based on expert opinion, we conclude that the k^{th} alternative is preferable to the $(k+1)^{th}$ alternative. The degree of preference of the k^{th} alternative over the $(k+1)^{th}$ alternative is explained as follows: To observe the intensity of the preference, the ranks i, j, and r are set equal to the highest rank of each parameter (expert, index, and alternative). In other words, Equation (39) is modified as shown in Equation (40). Multiplying a non-zero positive number on both sides of an inequality does not affect its direction.

$$i\left(j\left(r\left({W_{ijk}}^{r} - {W_{ijk}}^{r+1}\right)\right)\right) > 0 \qquad \forall\, i, j, r \tag{40}$$

Therefore, to obtain appropriate values for the available (w), the mathematical model must be solved. In this model, we aim to maximize the preference for each alternative across all indices and experts. The mathematical model (41) is designed to maximize the advantage of the k^{th} alternative over the $(k+1)^{th}$ alternative based on its ranking as the r^{th} over the $(r+1)^{th}$rank,

$$\text{Max}\left\{ i\left(j\left(r\left({W_{ijk}}^{r} - {W_{ijk}}^{r+1}\right)\right)\right), ijm{W_{ijk}}^{m}\right\} \ \forall\, i, j, r \tag{41}$$

Given the number of alternatives and the ranks involved in decision-making, we develop a multi-objective mathematical model and optimize model (41) (MODM) to maximize the minimum objective, which yields model (6).

s.t:

$$\sum_{i}\sum_{j}\sum_{k} {W_{ijk}}^{r} = 1 \qquad \forall\, r$$

$${W_{ijk}}^{r} \geq 0$$

$$\begin{gathered} \text{Max } \ Min\left\{ i\left(j\left(r\left({W_{ijk}}^{r} - {W_{ijk}}^{r+1}\right)\right)\right), ijm{W_{ijk}}^{m}\right\} \ \forall\, i, j, r \\ \text{s.t:} \\ \sum_{i}\sum_{j}\sum_{k} {W_{ijk}}^{r} = 1 \qquad \forall\, r \\ {W_{ijk}}^{r} \geq 0 \end{gathered} \tag{42}$$

$$Z = Min\left\{ i\left(j\left(r\left({W_{ijk}}^{r} - {W_{ijk}}^{r+1}\right)\right)\right), ijm{W_{ijk}}^{m}\right\} \tag{43}$$

Using Equation (43), which represents a mathematical variable transformation, the nonlinear mathematical model (42) can be transformed into a linear mathematical model (44).

By solving model (44), the weights and ranks for each alternative, criterion, and expert are determined, taking into account the preferences associated with these two rankings. By solving the linear mathematical model (44), the weights of the other alternatives, criteria, and experts are as follows:

$$\begin{gathered} \text{Max: Z} \\ \text{s.t:} \\ Z \leq i\left(j\left(r\left({W_{ijk}}^{r} - {W_{ijk}}^{r+1}\right)\right)\right) \qquad \forall\, i, j, r \\ Z \leq \ ijm{W_{ijk}}^{m} \qquad \forall\, i, j \quad , \ r = m \\ \sum_{i}\sum_{j}\sum_{k} {W_{ijk}}^{r} = 1 \qquad \forall\, r \\ {W_{ijk}}^{r} \geq 0 \end{gathered} \tag{44}$$

$$Alternatives\ Weight = \sum_{k} W_{ijk}{}^{r} \qquad \forall\ i, j, r \tag{45}$$

$$Criteria\ Weight = \sum_{j} W_{ijk}{}^{r} \qquad \forall\ i, k, r \tag{46}$$

In this section, we outline the steps of the proposed group decision-making model.
Step 1. Determining experts and ranking them.
Step 2. Determining the indicators.
Step 3. Ranking the indicators by each expert.
Step 4. Ranking the alternatives based on each of the indicators by each expert ($\mathrm{W_{ijk}}^{(1)}, \mathrm{W_{ijk}}^{(2)}, \dots, \mathrm{W_{ijk}}^{(m)}$).
Step 5. Determining the alternatives, criteria, and experts' weights ($\mathrm{W_{ijk}}^{(1)}, \mathrm{W_{ijk}}^{(2)}, \dots, \mathrm{W_{ijk}}^{(m)}$).

5. Integrated FMOLP-OPA model

The proposed multi-objective DEA model provides a theoretically grounded and practically flexible alternative to traditional scalar-focused efficiency analysis. It redefines DEA within the established field of multi-objective optimization by treating inputs and outputs as explicit, competing objectives. This perspective enables a deeper understanding of performance trade-offs and provides a framework for integrating modern decision-making techniques, such as OPA.

5.1. Implications for modeling

In this study, we present several essential implications for advanced DEA models: Full Pareto Description: Unlike classical DEA models that reduce efficiency to a single score, the proposed model generates the entire Pareto frontier and provides a range of optimal equilibrium points rather than a single optimal point. This issue empowers decision-makers to choose the solution that best matches their strategic priorities. Common Weighting for Benchmarking: By introducing a common set of weights in a multi-objective framework, this model ensures comparability across DMUs and avoids bias from self-optimizing weight sets in traditional models. Analytical Bridge to Vector Optimization: This model provides a precise link between DEA and MOLP, enabling the use of established MOLP tools to analyze efficiency structures and provide dual interpretations. A foundation for preference-based evaluation: Most importantly, the MOLP formulation provides an ideal basis for integrating preference-based MCDA techniques (including OPA) to guide the selection of a preferred solution among Pareto-optimal alternatives.

All linear programs are solved using a standard educational package with default simplex settings and a feasibility tolerance of 10^{-6}. The lower bound ε is set to 10^{-6} to avoid zero weights, and the fuzziness parameter d_r is fixed at 0.05 to balance sensitivity and robustness.

Integration of DEA with OPA

The OPA, introduced in [31], is an MCDA method that enables decision-making based on ordinal judgments, thereby avoiding the need for precise numerical inputs or subjective weight estimation.

OPA is especially well-suited for strategic and policy-oriented assessments where stakeholders can rank the relative importance of objectives or DMUs but cannot provide primary input. Integrating OPA into the DEA context enables the following:

Prioritized selection from a Pareto set: Among the Pareto-optimal DMUs identified by the MOLP-based DEA model, OPA can prioritize options according to decision-makers' sequential preferences. Sequential weighting of inputs/outputs: OPA can derive implicit priority weights for inputs and outputs, preserving the decision-maker's objectives while remaining consistent with the DEA frontier. Decision support without quantification bias: Combining DEA and OPA reduces the reliance on subjective primary weights, addressing a well-known limitation of traditional DEA and MCDA methods. This synergy enables preference-enriched DEA models that can handle ambiguity, partial information, and multi-stakeholder input in a robust yet interpretable manner. Figure 3 shows the flow diagram and describes each stage of this research.

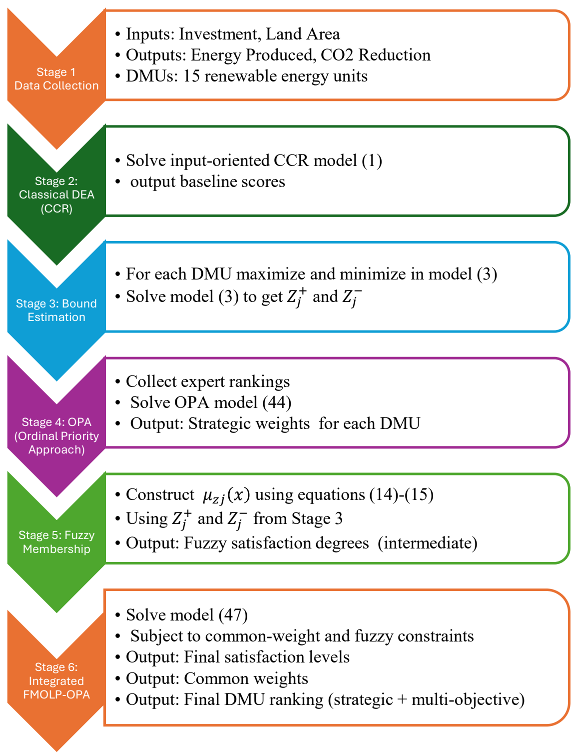

Figure 3. Flowchart of the Proposed FMOLP-OPA-DEA Framework.

Potential Extensions and Future Integration

Following the OPA integration, several research directions show promise:

Robust OPA-DEA hybrids: Extend the model to account for uncertainty in ordinal rankings (e.g., fuzzy OPA) and for data imprecision using robust DEA.

Dynamic Preference Modeling: Use OPA to incorporate evolving or scenario-dependent preferences in dynamic or intertemporal DEA frameworks.

Hierarchical MCDA–DEA Systems: Leverage OPA for priority setting at higher decision levels (e.g., regional policymakers), feeding into DEA-based evaluation at the operational level.

MCDA–DEA for Multi-Stakeholder Contexts: In settings such as renewable energy or public service delivery, multiple decision-makers often express differing objectives. OPA enables the aggregation of these ordinal opinions into a coherent evaluation process alongside DEA.

In this part, we use the benefits of input-oriented CCR and multi-objective DEA models with a common set of weights (Model 3), as well as the OPA approach. The OPA approach helps to calculate the weights of λ_j for 15 DMUs.

$$Max\ Z = \sum_{j=1}^{q} W_j \lambda_j$$

$$s.t.:$$

$$\Big(\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - Lower\ Bound\Big)/(Upper\ Bound - Lower\ Bound) \geq \lambda_j; \quad j = 1, \ldots, n$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n \tag{47}$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$$

$$\sum_{j=1}^{q} W_j = 1,$$

$$\begin{array}{ll} u_r \geq \epsilon, & r = 1, \ldots, s \\ v_i \geq \epsilon, & i = 1, \ldots, m \\ W_j \geq 0, & j = 1, \ldots, n \\ \lambda_j \in [0,1], & j = 1, \ldots, n \end{array}$$

In model (47), the weights of W_j are obtained using the OPA approach. Model (47) is the integrated formulation. The OPA weights W_j appear only in the objective function. The fuzzy satisfaction λ_j appears in the objective and constraints. The common weights (u, v) appear in all constraints. This decomposition clarifies each component's distinct role.

5.2. Integrated mathematical formulation

The complete model combines three components into one system:

Component 1 (Common weights): A single set of weights $(u_r - v_i)$ applied to all DMUs via the constraints $u^T y_j - v^T x_j \leq 0$ for all j.

Component 2 (Fuzzy objectives): For each DMU j, the net efficiency $u^T y_j - v^T x_j$ is transformed into a satisfaction degree λ_j using the membership function:
$\lambda_j = (u^T y_j - v^T x_j - L_j)/(U_j - L_j)$, where L_j and U_j are the lower and upper bounds obtained from single-objective solutions of model (3).

Component 3 (OPA preferences): The weights W_j from OPA serve as coefficients in the overall objective function.

The fully integrated model (48) is:

$$Max\ Z = \sum_{j=1}^{n} W_j\, \lambda_j$$
$$Subject\ to:$$
$$\lambda_j \le (u^T y_j - v^T x_j - L_j)/(U_j - L_j),\ \forall\, j \tag{48}$$
$$u^T y_j - v^T x_j \le 0, \quad \forall\, j$$
$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i\ = 1$$
$$u_r \ge \epsilon$$
$$v_i \ge \epsilon$$
$$\lambda_j \in [0, 1]$$

Here, W_j (from OPA) prioritize DMUs, λ_j (fuzzy satisfaction) measures multi-objective achievement, and common weights (u, v) ensure fairness.

The survey comprises the following parts: CCR baseline, bound estimation per DMU, OPA weight derivation, fuzzy membership construction, FMOLP solution using OPA weights, and solver settings.

The OPA procedure is based on ordinal rankings provided by four experts: two energy economists and two environmental engineers. Each expert ranks the objectives (economic efficiency, environmental sustainability, and reliability) from most to least important.

6. Empirical application

Table 6. DEA inputs and outputs (Input-Oriented CCR Model).

DMU	Inputs		Outputs	
	Investment (M USD)	Land Area (Ha)	Energy Produced (GWh)	CO_2 Reduction (Tons)
1	49.96	28.77	201.89	25.35
2	78.56	19.97	66.26	95.4
3	32.48	8.9	291.41	82.76
4	24.65	14.26	126.15	18.79
5	68.09	22.7	221.06	49.61
6	36.65	6.25	80.51	54.57
7	86.6	10.31	58.6	91.84
8	34.55	9.59	114.69	69.63
9	44.34	18.12	127.93	56.81
10	35.56	11.78	186.68	26.64
11	51.16	8.49	292.4	79.76
12	43.37	14.16	284.87	90.53
13	56.49	24.63	199.47	92.97
14	35.97	17.86	72.12	27.64
15	67.39	6.16	61.31	39.28

Table 6 shows a numerical example clarifying the theoretical model of new renewable energy with two inputs and two outputs, along with 15 DMUs.

Below is a simulated dataset of 15 DMUs in the renewable energy sector. Each DMU has been evaluated using the inputs of investment (in million USD) and Land area (in hectares), as well as the outputs of Energy Produced (in Gigawatt-Hours or GWh) and CO_2 Reduction (in tons).

We present the results of the efficiency input-oriented CCR and multi-objective DEA models, utilizing the common set of weights in Model (3), as shown in Table 7. Table 7 also presents the results of applying a simplified version of the multi-objective DEA model to the 15 DMUs in the renewable energy dataset.

Table 7. DEA Efficiency Scores (Input-Oriented CCR Model).

DMU	Input-Oriented CCR Model	Efficiency Score of Common Set of Weights Model (3)
1	0.45	25.3
2	0.51	95.3
3	1.00	82.7
4	0.57	18.7
5	0.36	49.6
6	0.93	54.5
7	0.95	91.8
8	0.79	69.6
9	0.50	56.0
10	0.59	26.6
11	1.00	79.7
12	0.82	90.5
13	0.65	92.9
14	0.30	27.6
15	0.68	39.27

Figure 4 also depicts the efficiency scores of the input-oriented CCR model.

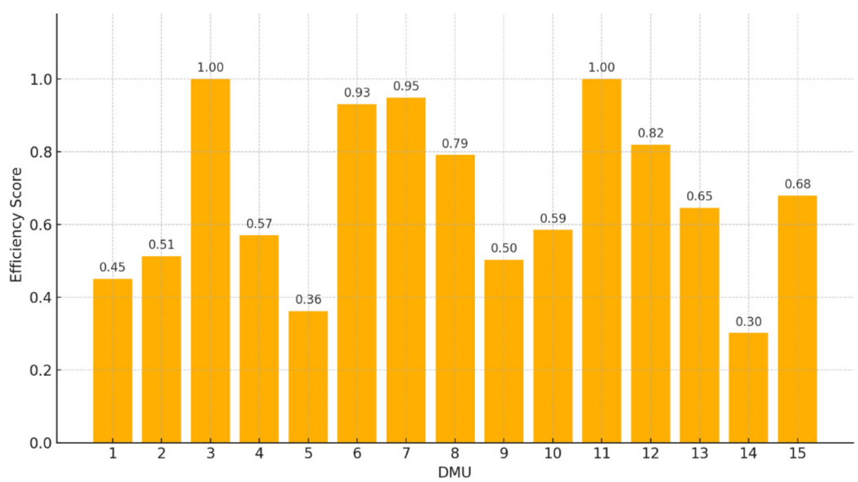

Figure 4. Efficiency Scores of the Input-Oriented CCR Model.

7. Discussion

Table 8 presents the weights of each DMU obtained from the OPA approach. These weights are extracted using equations (44)–(46). Because the OPA mathematical model approaches this group decision-making problem, there is no need to consider experts' agreement or disagreement, nor to perform any aggregation [31]. Then, by inputting the ranks suggested by experts into the OPA model, i.e., $i, j, and\ r,$ and solving the mathematical programming problem, the output of this model, i.e., W_j (weights of each DMU), is obtained.

Table 8. The weights of each DMU obtained from the OPA approach.

W1	W2	W3	W4	W5	W6	W7	W8
0.02542	0.097052	0.115487	0.094332	0.107882	0.072969	0.050956	0.029146
W9	W10	W11	W12	W13	W14	W15	
0.033592	0.077724	0.010139	0.045871	0.016432	0.079902	0.143096	

Table 9 shows the results of the numerical example of CSW using FMOLP.

Table 9. The results of the numerical example of CSW using FMOLP.

Variable	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	U1	U2	V1	V2
Value	0.59	0.90	1	0.00	0.93	0.05	0.68	0.83	0.97	0.20	1	0.96	0.96	0.98	0.88	0.01	0.07	0.01	0.91

Table 10. Numerical Comparison.

DMU	CCR DEA Efficiency (Table 7)	Common Weights Score (Table 7)	OPA Weights (Table 8)	FMOLP Satisfaction (Table 9)
1	0.45	25.35	0.02542	0.59
2	0.51	95.37	0.097052	0.90
3	1	82.76	0.115487	1
4	0.57	18.79	0.094332	0.00
5	0.36	49.61	0.107882	0.93
6	0.93	54.56	0.072969	0.05
7	0.95	91.81	0.050956	0.68
8	0.79	69.62	0.029146	0.83
9	0.5	56.8	0.033592	0.97
10	0.59	26.65	0.077724	0.20
11	1	79.76	0.010139	1
12	0.82	90.53	0.045871	0.96
13	0.65	92.95	0.016432	0.96
14	0.30	27.63	0.079902	0.98
15	0.68	39.27	0.143096	0.88

Table 10 reveals significant insights and numerical comparisons as follows:

CCR DEA vs. Common Weights

- Classical CCR DEA results in several DMUs (DMUs 3 and 11) achieving full efficiency (score = 1).
- However, when applying the Common Set of Weights, even efficient DMUs (such as DMU 3) show different performance levels (82.76 for DMU 3).
- This issue suggests that unified weight systems penalize certain DMUs that benefit from self-optimized weights in the CCR method.

OPA Weights Interpretation

- The weights obtained from OPA prioritize DMUs differently, as shown in Table 8. For instance, DMU 15 has the most OPA weight (0.143096), indicating strong strategic importance despite its CCR efficiency (0.68).
- DMU 11 has a CCR efficiency of 1, but it has the lowest OPA weight (0.010139), showing that technical efficiency alone is not sufficient in this case.

FMOLP Satisfaction Degrees

- FMOLP Satisfaction Values (Table 9) add nuance:
    - DMUs 3 and 11 still maintain complete satisfaction (e.g., 1).
    - DMU 4 reduces to 0.00, although it has moderate CCR and common weights scores, indicating that when considering multiple fuzzy objectives, DMU 4 reaches the minimum satisfaction level.
    - DMU 14, which initially has a low CCR (0.30), achieves a very high satisfaction (0.98), indicating that some decision units excel when multiple criteria and fuzzy preferences are considered.

Results in Tables 7–10 are based on FMOLP–OPA weights derived from the expert elicitation procedure described in Section 5.2.

Strategic Implication

- CCR DEA is purely technical and ignores trade-offs.
- Common Weights DEA improves comparability, but still compresses multiple objectives into a single view.
- FMOLP-OPA provides the richest, most preference-aware, and most strategic evaluation, especially when stakeholders have different objectives and trade-offs.
- CCR DEA is suitable for quick benchmarking, but it oversimplifies decision complexity.
- A common set of Weights improves fairness but still loses multi-objective nuance.
- FMOLP and OPA are superior for complex, real-world decision support because they align technical efficiency with stakeholder-driven preferences and multiple objectives.
- Some DMUs that are inefficient under CCR models become efficient under FMOLP, and vice versa, reflecting the multidimensional nature of performance.

Additionally, we can consider that:

- CCR DEA highlights technical best performers.
- Common Weights DEA highlights fairly benchmarked DMUs.
- FMOLP Satisfaction highlights strategically important and realistic DMUs when multiple objectives matter.

Thus, FMOLP-OPA models offer a significantly richer, more real-world-aligned understanding of DMU performance than traditional DEA models.

The comparison of DMUs 3, 4, and 14 is presented in Figure 5 as follows:

The following notes are about Figure 5:

X-axis (horizontal): The three Decision-Making Units (DMUs): DMU 3, DMU 4, and DMU 14. Y-axis (vertical): The scores of each DMU in three different evaluation models, ranging from 0.0 to 1.2 for visibility.

Each DMU has three colored bars: blue Bar: Score from Input-Oriented CCR DEA, red Bar: Score from Common Set of Weights DEA (normalized), green Bar: Score from FMOLP Satisfaction Degree. Additionally, Table 11 presents the information for Figure 5.

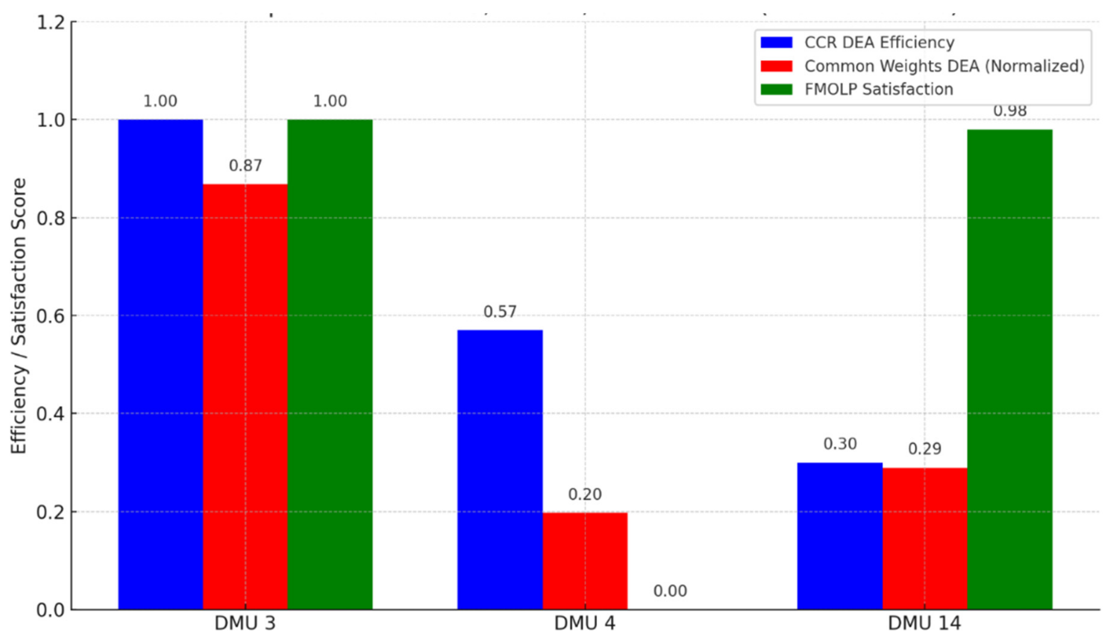

Figure 5. Comparison of DMUs 3, 4, and 14.

Table 11. Information regarding Figure 5.

Color	Meaning	Purpose
Blue	CCR DEA Efficiency	Measures pure technical efficiency (input minimization while keeping outputs constant).
Red	Common Weights DEA Score	Applies a global set of weights for all DMUs for fair comparison.
Green	FMOLP Satisfaction Degree	Measures how well DMUs satisfy multiple conflicting objectives with fuzzy preferences.

The efficiencies of some DMUs are as follows:

DMU 3:

- CCR DEA (Blue): Perfect 1.00; DMU 3 is fully efficient when evaluated using the traditional CCR DEA method.
- Common Weights DEA (Red): Slight drop, because when global fairness is enforced (everyone uses the same weights), DMU 3 performs less perfectly.

- FMOLP Satisfaction (Green): Still at 1.00; DMU 3 perfectly satisfies multiple fuzzy goals. DMU 3 remains a top performer across all perspectives.

DMU 4:

- CCR DEA (Blue): Moderate at 0.57, decent technical efficiency.
- Common Weights DEA (Red): Drops very low, meaning DMU 4's specific input/output mix is not favorable under a global standard.
- FMOLP Satisfaction (Green): Almost 0.00, critically bad in multi-objective fuzzy optimization. DMU 4 is sensitive to model choice; although it appears technically acceptable, it performs significantly worse when broader real-world goals are considered.

DMU 14:

- CCR DEA (Blue): Low at 0.30, traditionally very inefficient.
- Common Weights DEA (Red): Slightly better but still low.
- FMOLP satisfaction (green): Very high at 0.98.
- When multiple objectives are considered, DMU 14 performs well; despite technical inefficiency, it may be strategically excellent.

Thus, the key observations in the graph are as follows:

• The CCR DEA model's judgments are based solely on technical input/output ratios, ignoring strategic trade-offs.

• The common set of weights model enforces global fairness but still flattens the true strategic strengths of some DMUs.

• FMOLP better captures multi-objective and real-world satisfaction:

• Some DMUs, such as 14, that appear weak under technical DEA can be real stars under realistic objectives.

• Moreover, some other DMUs, such as DMU 4, that are mediocre under technical DEA, fail strategically.

Furthermore, the bar chart shows that CCR DEA is suitable for pure technical benchmarking.

8. Conclusion

This study presents a significant methodological advancement by integrating FMOLP and DEA preference-based modeling. As a next step, researchers can integrate concepts of varying returns to scale, fuzzy theory, multi-stakeholder input, and robust validation frameworks. Our research findings are as follows:

- CCR DEA is input-driven, fast, and suitable for initial evaluations; it also simplifies the real-world multi-objective framework.
- The CSW DEA improves comparability but still imposes a vision on complex data.
- FMOLP-OPA-enhanced DEA is a complex approach that enables realistic, flexible, and decision-maker-aligned assessments that establish actual multidimensional performance.

This transition from CCR to CSW and then to FMOLP-OPA is seen as an evolution from pure technical efficiency to uniform fairness, finally leading to multi-objective, preference-based strategic decision-making.

Additionally, the technical suggestions for further research are as follows:

Expand the Use of Alternative Returns-to-Scale Models. While we focus on Constant Returns to Scale (CRS) models (CCR), we strongly recommend that future research explore Variable Returns to

Scale (VRS) models, such as the BCC model. This is because many real-world systems (e.g., energy sectors and public services) exhibit increasing or decreasing returns to scale, and assuming CRS might oversimplify reality. In addition, the Super-Efficiency Models for Fine-Grained Ranking are as follows:

Extend the DEA analysis by applying Super-Efficiency Models to differentiate fully efficient DMUs (those with CCR scores of 1). This is because, in traditional DEA, multiple DMUs can be assigned a score of 1.0. Super-efficiency enables ranking beyond the frontier and identifying true outperformers. The next suggestion is to advance fuzzy modeling using interval-valued or type-2 fuzzy sets. Instead of crisp fuzzy membership functions, apply Interval-Valued Fuzzy Sets (IVFS) or Type-2 Fuzzy Sets. These methods handle greater uncertainty, enabling the model to better capture ambiguity in expert preferences or data collection. The next suggestion is to perform Robustness and Stability Testing Under Different Scenarios.

Additionally, the researcher can conduct Monte Carlo simulations or bootstrap analysis to evaluate the robustness of DEA–FMOLP–OPA rankings under different OPA weighting schemes and random noise in input-output data. This issue is crucial because it ensures that the models' conclusions are robust and valid, which is vital for making informed investment decisions. The next suggestion is to relate the DEA Results to Policy Implications. The researchers can move beyond purely efficiency rankings by linking DEA outcomes to strategic recommendations, such as investment priorities and environmental policy design (for CO_2 emissions). This suggestion is offered because practical insights increase the study's significance. In addition, researchers could integrate multi-stakeholder decision-making approaches, such as future work that incorporates multi-stakeholder OPA, in which different groups offer distinct preference structures. This proposal is motivated by the fact that real-world decisions often involve conflicting stakeholder preferences, and modeling these conflicts increases the model's realism. We also propose comparing the DEA models with improved machine learning methods. This issue can include modern approaches such as DEA with neural networks, random forests, or support vector machines (SVMs). This proposal is based on the premise that these hybrid approaches improve forecasting performance and enable the discovery of nonlinear patterns. The last proposal discusses transparency and interpretability and outlines how fuzzy theory, OPA, and efficiency bounds are derived in a logical, step-by-step process. This proposal guarantees that the model is clear and reproducible, a key requirement for high-impact scientific research.

A thorough sensitivity and robustness analysis of the FMOLP-OPA framework remains an important avenue for future investigation. This would entail systematically perturbing key parameters, such as the fuzzy tolerance levels and OPA preference weights, and reporting the resulting ranking stability through transparent summary statistics and experimental settings, thereby enabling rigorous scrutiny in future work.

Author contributions

Weiming Wang: Conceptualization, Methodology, Formal analysis, Investigation, Data curation, and Writing – original draft. Mohammadreza Feylizadeh: Conceptualization, Methodology, Formal analysis, Investigation, Validation, Writing – original draft, Writing – review & editing, and Supervision. Morteza Bagherpour: Methodology, Formal analysis, Investigation, Validation, Writing – review & editing, and Supervision. All authors have read and approved the final version of the manuscript and agree to be accountable for all aspects of the work.

Use of Generative-AI tools declaration

During the preparation of this manuscript, the authors partially used AI for language polishing, editing, and expression improvement. The authors reviewed and revised the AI-assisted content and take responsibility for the final published version.

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Conflict of interest

The authors declare no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

References

1. A. Mergoni, A. Emrouznejad, K. De Witte, Fifty years of data envelopment analysis, *Eur. J. Oper. Res.,* **326** (2025), 389–412. https://doi.org/10.1016/j.ejor.2024.12.049
2. J. Chu, W. Su, F. Li, Z. Yuan, Individual rationality and overall fairness in fixed cost allocation: An approach under DEA cross-efficiency evaluation mechanism, *J. Oper. Res. Soc.*, **74** (2023), 992–1007. https://doi.org/10.1080/01605682.2022.2079434
3. A. P. Singh, S. P. Yadav, S. K. Singh, A multi-objective optimization approach for DEA models in a fuzzy environment, *Soft Comput.*, **26** (2022), 2901–2912. https://doi.org/10.1007/s00500-021-06627-y
4. S. Maleki, A. Ebrahimnejad, R. Kazemi Matin, Pareto–Koopmans efficiency in two-stage network data envelopment analysis in the presence of undesirable intermediate products and nondiscretionary factors, *Expert Syst.*, **36** (2019), e12393. https://doi.org/10.1111/exsy.12393
5. P. Gupta, M. K. Mehlawat, U. Aggarwal, V. J. R. P. Charles, An integrated AHP-DEA multi-objective optimization model for sustainable transportation in mining industry, *Resour. Policy*, **74** (2021), 101180. https://doi.org/10.1016/j.resourpol.2018.04.007
6. Z. Pan, S. Fang, H. Wang, LightGBM technique and differential evolution algorithm-based multi-objective optimization design of DS-APMM, *IEEE Trans. Energy Convers.*, **36** (2020), 441–455. https://doi.org/10.1109/TEC.2020.3009480
7. J. Wu, J. Sun, L. Liang, Methods and applications of DEA cross-efficiency: Review and future perspectives, *Front. Eng. Manag.*, **8** (2021), 199–211. https://doi.org/10.1007/s42524-020-0133-1
8. N. Mohseny-Tonekabony, S. J. Sadjadi, E. Mohammadi, M. Tamiz, D. F. Jones, Robust, extended goal programming with uncertainty sets: an application to a multi-objective portfolio selection problem leveraging DEA, *Ann. Oper. Res.* **346** (2025), 1497–1552. https://doi.org/10.1007/s10479-023-05811-7

9. H. Li, J. Xiong, J. Xie, Z. Zhou, J. Zhang, A unified approach to efficiency decomposition for a two-stage network DEA model with application of performance evaluation in banks and sustainable product design, *Sustainability*, **11** (2019), 4401. https://doi.org/10.3390/su11164401
10. A. Mahmoudi, M. R. Feylizadeh, D. Darvishi. A note on "a multi-objective programming approach to solve grey linear programming, *Grey Syst. Theory Appl.,* **8** (2018), 35–45. https://doi.org/10.1108/GS-08-2017-0027
11. S. Noori, M. R. Feylizadeh, M. Bagherpour, F. Zorriassatine, R. M. Parkin. Optimization of material requirement planning by fuzzy multi-objective linear programming, *Proc. Inst. Mech. Eng. Part B J. Eng. Manuf.,* **222** (2008), 887–900. https://doi.org/10.1243/09544054JEM1014
12. S. Pascoe, On the Use of Data Envelopment Analysis for Multi-Criteria Decision Analysis, *Algorithms*, **17** (2024), 1–15. https://doi.org/10.3390/a17030089
13. M. Mirmozaffari, E. Shadkam, S. M. Khalili, M. Yazdani, Developing a novel integrated generalised data envelopment analysis (DEA) to evaluate hospitals providing stroke care services, *Bioengineering*, **8** (2021), 1–18. https://doi.org/10.3390/bioengineering8120207
14. P. Gupta, M. K. Mehlawat, D. Mahajan, Data envelopment analysis based multi-objective optimization model for evaluation and selection of software components under optimal redundancy, *Ann. Oper. Res.,* **312** (2022), 193–216. https://doi.org/10.1007/s10479-018-2842-y
15. A. P., Singh, S. P. Yadav, Performance evaluation of DMUs using hybrid fuzzy multi-objective data envelopment analysis, *Appl. Anal. Optim. Soft Comput. (Springer Proceedings in Mathematics & Statistics)*, **419** (2023), 329–343, https://doi.org/10.1007/978-981-99-0597-3_23
16. M. S. Pakkar, An integrated approach to grey relational analysis, analytic hierarchy process and data envelopment analysis, *J. Centrum Cathedra*, **9** (2016), 71–86. https://doi.org/10.1108/JCC-08-2016-0005
17. N. G. Raad, S. Rajendran, A hybrid robust SBM-DEA, multiple regression, and MCDM-GIS model for airport site selection: Case study of Sistan and Baluchestan Province, Iran, *Transp. Eng.*, **16** (2024), 100235. https://doi.org/10.1016/j.treng.2024.100235
18. D. Andjelković, G. Stojić, N. Nikolić, D. K. Das, M. Subotić, Ž. Stević, A novel data-envelopment analysis interval-valued fuzzy-rough-number multi-criteria decision-making (DEA-IFRN MCDM) model for determining the efficiency of road sections based on headway analysis, *Mathematics*, **12** (2024), 976. https://doi.org/10.3390/math12070976
19. R. Yu, J. Wang, T. C. E. Cheng, P. Yu, Assessment of new energy industrial clusters: An MCDM approach using DEA and GEMS, *Expert Syst. Appl.*, **252** (2024), 124231. https://doi.org/10.1016/j.eswa.2024.124231
20. L. Sinha, S. M. Narulkar, Optimal Operation of Multi-reservoir System Utilizing DEA, AIDE Algorithm and Flood Control Assessment by MCDM Approach, *Water Resour. Manag.*, **39** (2025), 1783–1802. https://doi.org/10.1007/s11269-024-04046-w
21. P. D. Sumo, X. Ji, L. Cai, Performance prediction of a textile reverse logistics system using DEA and ANFIS hybrid models, *J. Intell. Fuzzy Syst.*, **44** (2023), 5495–5505. https://doi.org/10.3233/JIFS-223418
22. A. Rozhnov, Investigation of hybrid DEA models in conditions of changing economies of scale, *The 16th Int. Conf. Manag. Large-Scale Syst. Dev. (MLSD)*, IEEE, 2023, 1–4. https://doi.org/10.1109/MLSD58227.2023.10303967

23. H. Singer, Application of a Performance Evaluation Model to the Paper and Paper Products Printing Sector: The DEA-AHP Hybrid Algorithm, *Optim. Ekon. Yönetim Bilim. Derg.*, **11** (2024), 215–238. https://doi.org/10.17541/optimum.1417219
24. A. Nandy, P. C. Nandi, M. Chatterjee, Efficiency Management of Women Poultry Farmers Using Hybrid DEA and Machine Learning Approach: A Case of SHG-based Production in Sub-Himalayan North Bengal, *Vision*, (2023), 09722629231159708. https://doi.org/10.1177/09722629231159708
25. N. Jamali, M. R. Feylizadeh, P. Liu. Prioritization of aircraft maintenance unit strategies using fuzzy Analytic Network Process: A case study, *J. Air Transp. Manag.* **93** (2021), 102057. https://doi.org/10.1016/j.jairtraman.2021.102057
26. S. Parsaei, M. A. Keramati, F. Zorriassatine, M. R. Feylizadeh. An order acceptance using FAHP and TOPSIS methods: A case study of Iranian vehicle belt production industry, *Int. J. Ind. Eng. Comput.*, **3** (2012), 211–224. https://doi.org/10.5267/j.ijiec.2011.08.016
27. R. D. Banker, A. Charnes, W. W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Manage. Sci.*, **30** (1984), 1078–1092. https://doi.org/10.1287/mnsc.30.9.1078
28. A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficiency of decision making units, *Eur. J. Oper. Res.*, **2** (1978), 429–444. https://doi.org/10.1016/0377-2217(78)90138-8
29. C. K. Hu, F. B. Liu, C. F. Hu, Efficiency measures in fuzzy data envelopment analysis with common weights, *J. Ind. Manag. Optim.*, **13** (2017), 237–249. https://doi.org/10.3934/jimo.2016014
30. C. Kao, H. Hung, Data envelopment analysis with common weights: the compromise solution approach, *J. Oper. Res. Soc.*, **56** (2005), 1196–1203. https://doi.org/10.1057/palgrave.jors.2601924
31. Y. Ataei, A. Mahmoudi, M. R. Feylizadeh, D. F. Li, Ordinal priority approach (OPA) in multiple attribute decision-making, *Appl. Soft Comput.*, **86** (2020), 105893. https://doi.org/10.1016/j.asoc.2019.105893
32. S. Cui, R. Wang, A Novel δ-SBM-OPA Approach for Policy-Driven Analysis of Carbon Emission Efficiency under Uncertainty in the Chinese Industrial Sector, 2024. https://doi.org/10.48550/arXiv.2408.11600
33. S. Lertworasirikul, S. C. Fang, J. A. Joines, H. L. Nuttle, Fuzzy data envelopment analysis (DEA): a possibility approach, *Fuzzy Sets Syst.*, **139** (2003), 379–394. https://doi.org/10.1016/S0165-0114(02)00484-0
34. W. W. Cooper, L. M. Seiford, K. Tone, *Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software*, Vol. 2, New York: Springer, 2007.
35. H. J. Zimmermann, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets Syst.*, **1** (1978), 45–55. https://doi.org/10.1016/0165-0114(78)90031-3
36. M. Ehrgott, *Multi-criteria optimization*. Berlin, Heidelberg: Springer, 2005.
37. H. J. Zimmermann, *Fuzzy sets, decision making, and expert systems*, Springer Science & Business Media, **10** (1987). https://doi.org/10.1007/978-94-009-3249-4
38. H. J. Zimmermann, *Fuzzy set theory—and its applications*, Springer Science & Business Media, 2011. https://doi.org/10.1007/978-94-010-0646-0

39. Y. J. Lai, C. L. Hwang, *Multiple objective decision making in Fuzzy Multiple Objective Decision Making: Methods and Applications,* Berlin, Springer Berlin Heidelberg, 1994. https://doi.org/10.1007/978-3-642-57949-3
40. R. G. Bellman, L. A. Zadeh, Decision making in a fuzzy environment, *Manage. Sci.*, **17** (1970), B141– B164. http://dx.doi.org/10.1287/mnsc.17.4.B141
41. M. Sakawa, Fuzzy sets and interactive multi-objective optimization. *Springer Science & Business Media,* 2013. https://doi.org/10.1007/978-1-4899-1633-4
42. R. N. Tiwari, S. Dharmar, J. Rao, Fuzzy goal programming—an additive model, *Fuzzy Sets Syst.*, **24** (1987), 27–34. https://doi.org/10.1016/0165-0114(87)90111-4

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