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Research article

Equilibrium strategy in a non-performing loan securitization game with regime switching

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Abstract: This paper investigates the equilibrium of a non-performing loan (NPL) securitization market characterized by strategic interactions between multiple banks and insurers. We develop a continuous-time framework in which the NPL recovery process is modeled by a Markov chain capturing stochastic macroeconomic regime shifts. Within this market, risk-averse banks and insurers—both represented by exponential utility preferences—seek to maximize their certainty-equivalent terminal wealth through optimal choices of securitization and guarantee ratios. Employing optimization techniques, we establish the equilibrium model and derive analytical, closed-form solutions for the equilibrium strategies as well as the endogenous insurance premium. Numerical experiments illustrate how equilibrium outcomes respond to key factors such as agents' risk aversion and macroeconomic transition probabilities, thereby validating the theoretical framework and offering a rigorous tool for practical decision-making and regulatory evaluation.

Keywords: non-performing loans; securitization; equilibrium game model; diffusion approximation; regime-switching
Mathematics Subject Classification: 91G30, 91A15

1. Introduction

Major economic shocks—such as the global financial crisis (GFC) of 2007–2009 and the more recent COVID-19 pandemic—have repeatedly triggered sharp increases in the stock of non-performing loans (NPLs) across banking systems worldwide [1, 2]. In the aftermath of severe recessions, banks in many jurisdictions have been left with heavily impaired loan portfolios, as prominently exemplified by the situation in Italy [3, 4]. The large-scale accumulation of NPLs poses a serious threat to financial stability: It erodes bank profitability, weakens capital positions, raises funding costs, and ultimately constrains the supply of new credit to the real economy [1, 5, 6]. These persistent challenges have elicited strong responses from policymakers and regulators worldwide, underscoring the urgency of timely and effective NPL resolution.

Against this backdrop, securitization has emerged as a key financial instrument for managing legacy NPLs, offering banks a strategic alternative to outright loan sales [7]. This mechanism involves pooling illiquid NPLs and transferring them to a special purpose vehicle (SPV), which in turn issues tranches of asset-backed securities to capital market investors [8]. By enabling risk transfer and balance-sheet deleveraging, NPL securitization allows banks to improve the quality of their credit portfolios, reduce portfolio risk, and release capital for new, more profitable lending activities [9]. Compared to direct loan sales, securitization can mitigate the "cost of deleveraging", as it alleviates information asymmetries and narrows the wide bid–ask spreads that often plague illiquid NPL markets [10]. From the investors' perspective, NPL-backed securities open up new investment opportunities, offering potentially attractive instruments for sophisticated investors willing to assume higher levels of risk [7, 11].

In practice, due to the high risk and informational asymmetries inherent in NPL assets, originating banks often employ credit enhancement mechanisms to make the securitized notes more attractive to investors [12, 13]. This frequently involves third-party guarantors, such as insurance companies, who agree to cover potential losses in exchange for a premium [14]. Such a structure creates a complex strategic environment. On one side, banks must decide the optimal portion of NPLs to securitize, weighing the benefits of risk transfer against the cost of insurance premiums. On the other side, insurers must determine the optimal amount of risk to underwrite by setting a premium that compensates for the potential losses from the guaranteed assets. The decisions of these agents are interdependent and are settled in a market framework.

However, the existing theoretical literature on securitization predominantly analyzes the market from a single-agent perspective, typically focusing on the originator's strategic decisions. This body of work often uses principal-agent models to design optimal contracts that address information asymmetries, such as setting risk retention levels to mitigate moral hazard [15, 16] or to signal asset quality under adverse selection [17]. Other studies examine how the possibility of securitization endogenously shapes the originator's operational strategies, like foreclosure policies [18]. While these models offer crucial insights into the originator's behavior, they share a common limitation: Other key market participants, particularly credit enhancers like insurance companies, are not modeled as strategic players. Instead, they are often treated as passive price-takers or as a part of the contractual constraint within the originator's optimization problem.

Furthermore, these micro-level models often neglect the macroeconomic determinants of credit risk. The valuation of credit-sensitive assets is inherently regime-dependent, with distinct patterns across stable and crisis periods. Recent studies employ regime-switching frameworks to capture this feature. For instance, corporate default clustering is better explained by models allowing for regime transitions [19]. Non-performing loans also switch between stationary and non-stationary regimes, with policy interventions exerting regime-specific effects on financial stability [20]. The relationship between non-performing loans and bank stability is increasingly understood as state-dependent, with non-linear models like the panel smooth transition regression model showing that the system switches between different risk regimes, often defined by the level of NPLs themselves. Consequently, policy interventions like a bank's income diversification strategy exert regime-specific effects on financial stability [21]. Market evidence is consistent: Sovereign credit default swap (CDS) spreads are driven by different factors in tranquil versus turbulent states [22], and pricing of CDS index tranches requires regime-switching to capture default dependence [23]. Ignoring such dynamics leads to an incomplete assessment of securitization risk and misrepresentation of banks' and insurers' strategic choices.

This paper addresses these shortcomings by developing a novel equilibrium framework that simultaneously captures the strategic interactions between banks and insurers under stochastic macroeconomic conditions. To achieve this, we construct a model in two interconnected parts. First, we establish a dynamic NPL repayment model where both the intensity and the amount of cash flow recovery are governed by a continuous-time Markov chain that represents transitions between macroeconomic states (e.g., expansion and contraction). Second, we build upon this foundation to establish a market equilibrium game model. This model features multiple banks (as originators) and insurance companies (as guarantors), where each participant, characterized by an exponential utility function, makes optimal decisions to maximize its terminal wealth.

Obtaining a tractable analytical solution for the equilibrium in NPL securitization markets presents several significant challenges, which motivate our methodological choices. First, the underlying aggregate recovery process is inherently complex. A direct diffusion approximation without simplifying assumptions, such as conditioning on regime occupation times, risks leading to an intractable analysis. Second, the strategic equilibrium itself is a high-dimensional problem. As the number of interacting banks and insurers increases, characterizing the market-clearing outcome requires solving a complex system of interdependent equations, making a closed-form characterization of the equilibrium elusive beyond highly stylized cases. Finally, modeling the macroeconomic environment involves a crucial trade-off between realism and tractability. While real-world credit cycles may encompass multiple states, extending the analysis beyond a two-state Markov chain introduces substantial technical hurdles that can obscure the core economic regimes.

A primary contribution of this work is the derivation of a closed-form solution for the market equilibrium. By solving the optimization equation system, we obtain analytical expressions for the optimal securitization ratio for banks, the optimal guarantee ratio for insurers, and the endogenously determined equilibrium insurance premium. The provision of an analytical solution is a significant advancement, as it allows for a transparent analysis of how equilibrium strategies respond to fundamental parameters, such as agents' risk aversion and macroeconomic volatility, which is often obscured in models that rely solely on numerical solutions. We further analyze the equilibrium under feasibility constraints, offering a detailed characterization for a two-bank, two-insurer market to illustrate the model's practical application.

Finally, the theoretical findings are complemented by a detailed numerical analysis. Through simulations, we illustrate the impact of key parameters on the equilibrium outcomes, providing tangible insights into the market dynamics. These results validate the theoretical model's conclusions and yield significant practical implications. For banks, our framework offers a robust tool for formulating NPL disposal and risk management strategies. For insurance companies and other guarantors, it provides a theoretical foundation for risk pricing and capital allocation. For regulators, the model serves as a new analytical instrument to understand and assess the systemic risks inherent in the NPL securitization market, particularly in relation to macroeconomic fluctuations.

The remainder of this paper is organized as follows. Section 2 details the dynamic process of NPL repayment and sets up the market equilibrium model. Section 3 derives and analyzes the equilibrium solutions under both unconstrained and constrained conditions. Section 4 uses numerical simulations to illustrate the impact of key parameters on equilibrium strategies and the premium. Section 5 concludes this work.

2. Model setup

2.1. NPL repayment dynamics

In contrast to conventional performing loans, non-performing loans (NPLs) exhibit substantial uncertainty both in repayment timing and in the magnitude of recovered cash flows. Such randomness stems from the credit deterioration of borrowers and the influence of macroeconomic conditions, making NPLs markedly different from standard debt instruments.

Let $\tau_m^{(k)}$ denote the time of the m-th repayment made by borrower k for $k = 1, 2, \ldots, n$, and let $Y^{(k)}$ represent the corresponding repayment amount. Formally, we model $\tau_m^{(k)}$ as the m-th jump time of a Cox process $\{N^{(k)}(t)\}_{0\leqslant t\leqslant T}$ with stochastic intensity $\{\lambda^{(k)}(t, X(t))\}_{0\leqslant t\leqslant T}$. Equivalently, starting from a standard Poisson process $\{\bar{N}^{(k)}(t)\}_{0\leqslant t\leqslant T}$ that is independent of the intensity, we define

$$N^{(k)}(t) = \bar{N}^{(k)}\Big(\Lambda^{(k)}(t)\Big), \quad \Lambda^{(k)}(t) = \int_0^t \lambda^{(k)}(s, X(s))\,\mathrm{d}s.$$

Hence, the m-th repayment time $\tau_m^{(k)}$ of borrower k can be characterized as

$$\tau_m^{(k)} = \inf\left\{t > 0 : N^{(k)}(t) \geqslant m\right\}.$$

This formulation naturally accommodates the clustering of defaults and repayments, which is a well-documented phenomenon in distressed debt portfolios.

The dynamics of repayment are further shaped by macroeconomic conditions. To capture regime shifts between economic expansion and contraction, we introduce a continuous-time, finite-state Markov chain $X = \{X(t), t \geqslant 0\}$ with state space $D = \{e_1, e_2, \ldots, e_N\}$, where $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)^\top$ is the i-th unit vector in $\mathbb{R}^N$. The transition intensity matrix of $X(t)$ is specified as

$$\boldsymbol{Q} = \begin{pmatrix} -q_{11} & q_{12} & \cdots & q_{1N} \\ \vdots & \ddots & \ddots & \vdots \\ q_{N1} & q_{N2} & \cdots & -q_{NN} \end{pmatrix}, \tag{2.1}$$

where $q_{ij} \geqslant 0$ for $i \neq j$, and $q_{ii} = \sum_{j=1, j\neq i}^N q_{ij}$ for $i = 1, 2, \ldots, N$. Let $\mathcal{G}_t^X$ denote the natural filtration generated by $X(t)$. This Markovian structure enables the repayment intensity to be explicitly linked to macroeconomic regimes, thereby embedding systemic risk factors directly into the cash-flow dynamics of securitized NPL portfolios.

Considering that both repayment times and repayment amounts are affected by macroeconomic conditions, we introduce for each borrower k a vector of regime-dependent repayment intensities

$$\boldsymbol{\lambda}^{(k)} = (\lambda_1^{(k)}, \lambda_2^{(k)}, \cdots, \lambda_N^{(k)})^\top, \tag{2.2}$$

where $\lambda_i^{(k)} \geq 0$ represents the repayment intensity under macroeconomic state e_i, and $\langle \cdot, \cdot \rangle$ denotes the inner product of two vectors. Then the repayment intensity process can be represented as a functional of the Markov chain $X(t)$, i.e.,

$$\lambda^{(k)}(t, X(t)) = \langle \boldsymbol{\lambda}^{(k)}, X(t) \rangle. \tag{2.3}$$

This specification ensures that repayment activity adjusts endogenously to the prevailing macroeconomic regime.

In a similar manner, the distribution of repayment amounts is allowed to vary across states. Define the vector of regime-dependent density functions as

$$\boldsymbol{f}^{(k)}(y) = (f_1^{(k)}(y), f_2^{(k)}(y), \cdots, f_N^{(k)}(y))^\top, \tag{2.4}$$

where $f_i^{(k)}(y)$ denotes the density of repayment amounts on regime i. Then, the probability density of the (non negative) repayment amount $Y^{(k)}$ at time t is given by

$$f^{(k)}(y, t, X(t)) = \langle \boldsymbol{f}^{(k)}(y), X(t) \rangle. \tag{2.5}$$

The cumulative cash flow collected from borrower k by time t is then

$$L^{(k)}(t) = \sum_{m=1}^{N^{(k)}(t)} Y_m^{(k)}. \tag{2.6}$$

Conditioning on the macroeconomic filtration $\mathcal{G}_T^X$, the jump sizes $Y_m^{(k)}$ with density $f^{(k)}(y, t, X(t))$ are independent of both past increments $L^{(k)}(t-)$ and future increments $L^{(k)}(T) - L^{(k)}(t)$, and they are also independent of the counting process $N^{(k)}(t)$. Aggregating across n borrowers, the total cumulative cash flow is

$$L_n(t) = \sum_{k=1}^{n} L^{(k)}(t). \tag{2.7}$$

To describe the time spent in each regime, define the occupation time of $X(t)$ in state i by

$$O_i(t) = \int_0^t \mathrm{I}_{\{X(s)=i\}}\, \mathrm{d}s, \quad i = 1, 2, \cdots, N,$$

where $\mathrm{I}_{\{\cdot\}}$ denotes the indicator function.

For each regime i, we can define the associated compound Poisson component

$$L_{n,i}(t) = \sum_{k=1}^{n} \sum_{m=1}^{N_i^{(k)}(t)} Y_{m,i}^{(k)}, \tag{2.8}$$

where $N_i^{(k)}(t)$ is a Poisson process with parameter $\lambda_i^{(k)}$, and $Y_{m,i}^{(k)}$ follows density $f_i^{(k)}(y)$.

Proposition 2.1. *Given $O_i(t) = t_i$ for $i = 1, 2, \cdots, N$, the aggregate cash flow can be decomposed as*

$$L_n(t) = \sum_{i=1}^{N} L_{n,i}(t_i), \tag{2.9}$$

where $\sum_{i=1}^{N} t_i = t$.

Proof. Conditional on $O_i(t) = t_i$ for all i, the additivity, stationary increments, and independence properties of Poisson processes imply that $L_n(t)$ can be expressed as Eq (2.9) by summing all jumps generated while $X(\cdot)$ occupies each regime. □

Define the standardized process as follows:

$$\bar{L}_n(t) = \frac{L_n(t) - \mu(t_1, \cdots, t_N)}{\sqrt{n}}, \tag{2.10}$$

where $\mu(t_1, \cdots, t_N)$ denotes the conditional expectation of $L_n(t)$.

Proposition 2.2. *Assume homogeneous borrowers, i.e., each borrower has the same repayment intensity λ_i and density function $f_i(y)$ in regime i. Given $O_i(t) = t_i$ for $i = 1, 2, \cdots, N$, as $n \to \infty$ the standardized process $\bar{L}_n(t)$ with fixed t converges to a normal distribution. Consequently, the cumulative cash flow admits the approximation*

$$L_n(t) \approx \mu(t_1, \cdots, t_N) + \sqrt{n \sum_{i=1}^{N} \lambda_i t_i \int_0^\infty y^2 f_i(y)\, \mathrm{d}y}\, W_t, \tag{2.11}$$

where

$$\mu(t_1, \cdots, t_N) = \sum_{i=1}^{N} n\lambda_i t_i \int_0^\infty y f_i(y)\, \mathrm{d}y, \tag{2.12}$$

and W_t is a standard Brownian motion independent of $X(t)$.

Proof. From Proposition 2.1, conditioning on $O_i(t) = t_i$, for $i = 1, 2, \cdots, N$, we have

$$\begin{aligned}\mathbb{E}_X\Big[e^{uL_n(t)}\Big] &= \prod_{i=1}^{N} \mathbb{E}\Big[e^{uL_{n,i}(t_i)}\Big] \\ &= \prod_{i=1}^{N} \prod_{k=1}^{n} \mathbb{E}\left[\exp\left\{u \sum_{m=1}^{N_i^{(k)}(t)} Y_{m,i}^{(k)}\right\}\right],\end{aligned} \tag{2.13}$$

where $\mathbb{E}_X[\cdot] = \mathbb{E}[\cdot \mid O_i(t) = t_i]$. Since $L_{n,i}(t_i)$ is a compound Poisson process,

$$\mathbb{E}\left[\exp\left\{u \sum_{m=1}^{N_i^{(k)}(t)} Y_{m,i}^{(k)}\right\}\right] = \exp\left\{\lambda_i^{(k)} t_i \Big(\int_0^\infty e^{uy} f_i^{(k)}(y)\, \mathrm{d}y - 1\Big)\right\}. \tag{2.14}$$

Under homogeneity, (2.13) gives

$$\mathbb{E}_X\Big[e^{uL_n(t)}\Big] = \exp\left\{\sum_{i=1}^{N} n\lambda_i t_i (\phi_{Y_i}(u) - 1)\right\}, \tag{2.15}$$

where $\phi_{Y_i}(u) = \int_0^\infty e^{uy} f_i(y)\, \mathrm{d}y$. Hence,

$$\mu(t_1, \cdots, t_N) = \mathbb{E}_X[L_n(t)] = \sum_{i=1}^{N} n\lambda_i t_i \int_0^\infty y f_i(y)\, \mathrm{d}y.$$

From Eq (2.15), the conditional moment generating function of $\bar{L}_n(t)$ can be written as

$$\mathbb{E}_{\mathbb{X}}\left[e^{u\bar{L}_n(t)}\right] = e^{-u\frac{\mu(t_1,\cdots,t_N)}{\sqrt{n}}} \exp\left\{\sum_{i=1}^{N} n\lambda_i t_i \left(\phi_{Y_i}\left(\frac{u}{\sqrt{n}}\right) - 1\right)\right\}. \tag{2.16}$$

By Taylor expansion, we have

$$\phi_{Y_i}\left(\frac{u}{\sqrt{n}}\right) = 1 + \frac{u}{\sqrt{n}}\mathbb{E}\left[Y_i\right] - \frac{u^2}{2n}\mathbb{E}\left[(Y_i)^2\right] + o\left(\frac{1}{n}\right). \tag{2.17}$$

As $n \to \infty$,

$$\mathbb{E}_X\left[e^{u\bar{L}_n(t)}\right] \approx \exp\left\{-\tfrac{u^2}{2}\sum_{i=1}^{N} \lambda_i t_i \int_0^{\infty} y^2 f_i(y)\,\mathrm{d}y\right\}.$$

Thus, given $O_i(t) = t_i$ for $i = 1, 2, \cdots, N$, the process $L_n(t)$ can be approximated by

$$L_n(t) \approx \mu(t_1, \cdots, t_N) + \sqrt{n\sum_{i=1}^{N} \lambda_i t_i \int_0^{\infty} y^2 f_i(y)\mathrm{d}y}W_t. \tag{2.18}$$

□

Remark 2.1. *The homogeneous portfolio assumption, albeit stylized, has been extensively employed in the credit risk literature as a tractable benchmark. By imposing identical default intensities and contagion parameters within a subportfolio, it permits closed-form characterizations of ordered default times, thereby facilitating calibration and transparent risk measurement. In contrast, fully heterogeneous specifications, though more realistic, typically entail substantial computational complexity and limited analytical insight. For example, see [24, 25].*

From Proposition 2.2, given $O_i(t) = t_i$ for $i = 1, 2, \cdots, N$, the process $L_n(t)$ can be rewritten as

$$L_n(t) = \mu(t_1, \cdots, t_N) + \sigma(t_1, \cdots, t_N)W_t, \tag{2.19}$$

where

$$\sigma(t_1, \cdots, t_N) = \sqrt{n\sum_{i=1}^{N} \lambda_i t_i \int_0^{\infty} y^2 f_i(y)\mathrm{d}y}. \tag{2.20}$$

For convenience, we denote

$$\bar{\mu}_j = \mathbb{E}_j\left[\mu(T_1, \cdots, T_N)\right], \quad \bar{\sigma}_j = \mathbb{E}_j\left[\sigma^2(T_1, \cdots, T_N)\right], \tag{2.21}$$

where $\mathbb{E}_j[\cdot] = \mathbb{E}[\cdot \mid X(0) = e_j]$ and $T_i = O_i(T)$ for $i = 1, 2, \cdots, N$.

2.2. Equilibrium model for NPL securitization

We now turn to the equilibrium formulation of non-performing loan (NPL) securitization. In practice, the securitization of distressed debt involves several key steps. First, banks or specialized asset management companies transfer part of their NPL exposures into structured products. Through this process, a fraction of the portfolio is securitized and sold to outside investors, while the remaining portion is retained on the banks' balance sheets. To mitigate the inherent default and recovery risks, insurance companies (or guarantors) provide credit enhancement by guaranteeing a share of the securitized assets in exchange for premium payments.

Thus, banks play the role of originators deciding how much of their NPL holdings to securitize, whereas insurers act as risk-sharing counterparties absorbing part of the repayment uncertainty. The equilibrium model presented below captures the interaction between these two sides of the market, together with the endogenous determination of the insurance premium.

Suppose there are $\bar{M}$ banks (or specialized asset management companies). For each $q \in \{1, \ldots, \bar{M}\}$, let ξ_q denote its relative scale, normalized such that $\sum_{q=1}^{\bar{M}} \xi_q = 1$. The government allocates one unit* of NPL exposure among the banks proportionally to their scale, so that bank q handles a fraction ξ_q.

$$u_q + v_q = \xi_q.$$

Banks. The terminal wealth of bank q is then given by

$$W_q^B(T) = u_q(1-p)e^{rT} + (\xi_q - u_q)L_n(T), \tag{2.22}$$

where p is the insurance premium per unit securitized asset. The first term captures the terminal wealth derived from the securitized portion of the portfolio: After paying the insurance premium p, the proceeds are reinvested at the risk-free rate until maturity. The second term reflects the stochastic cash flows generated by the retained share of NPLs, with repayments governed by the aggregate recovery process $L_n(T)$.

Each bank has exponential utility

$$U_q^B(w) = -\exp(-\gamma_q^B w),$$

where $\gamma_q^B > 0$ is its coefficient of absolute risk aversion. Bank q determines the optimal securitization share u_q by solving the following:

$$\max_{u_q} \mathbb{E}\Big[U_q^B\Big(W_q^B(T)\Big)\Big] = \max_{u_q} \mathbb{E}\Big[U_q^B\Big(u_q(1-p)e^{rT} + (\xi_q - u_q)L_n(T)\Big)\Big]. \tag{2.23}$$

Insurance companies. On the other side of the market, consider M insurance companies. Each $m \in \{1, \ldots, M\}$ guarantees a fraction y_m of the securitized portfolio. Its terminal wealth is

$$W_m^I(T) = y_m p e^{rT} - y_m p e^{CT} + y_m L_n(T), \tag{2.24}$$

*Note that the "one unit of NPL exposure" is a normalization for analytical convenience. The weights ξ_q are dimensionless. For example, to apply our model to a real-world NPL portfolio with a total notional value of S, all resulting equilibrium quantities expressed in monetary units (such as prices, payments, and wealth levels) should be scaled by S. The absolute NPL exposure for bank q would thus be $S \cdot \xi_q$. Consequently, due to the model's structure, equilibrium prices and rates of return are invariant to the total portfolio size.

where r is the risk-free rate, and C is the contractual transfer rate of return promised to investors on the securitized assets. On the right hand of (2.24), the first term denotes the premium income received by the insurer, reinvested at the risk-free rate until maturity. The second term corresponds to the contractual payment that must be transferred to investors in the securitized assets, representing the cost of providing credit enhancement. The third term captures the stochastic repayment flows arising from the guaranteed share of the NPL pool, determined by the aggregate recovery process.

The insurer's preferences are also exponential:

$$U_m^I(w) = -\exp(-\gamma_m^I w),$$

with $\gamma_m^I > 0$ as the risk-aversion coefficient. Insurance company m chooses y_m to maximize

$$\max_{y_m} \mathbb{E}\Big[U_m^I(W_m^I(T))\Big] = \max_{y_m} \mathbb{E}\Big[U_m^I\Big(y_m p e^{rT} - y_m p e^{CT} + y_m L_n(T)\Big)\Big], \tag{2.25}$$

subject to the market-clearing condition

$$\sum_{m=1}^{M} y_m = \sum_{q=1}^{\bar{M}} u_q. \tag{2.26}$$

To summarize, the above framework establishes the equilibrium environment for NPL securitization: Banks determine how much of their distressed assets to securitize, while insurers decide how much risk to guarantee, subject to a market-clearing condition. This formulation captures the strategic interaction between originators and insurers under risk aversion, and provides the foundation for the analytical derivation of equilibrium strategies.

3. Solution of the equilibrium

3.1. Solution of the unconstrained equilibrium

In this subsection, we solve for the unconstrained equilibrium of the NPL securitization model. Here, the term "unconstrained" means that we temporarily ignore the natural feasibility bounds on the decision variables: For each bank q, the securitization ratio u_q should lie in the interval $[0, \xi_q]$, and for each insurer m, the guaranteed proportion y_m should be nonnegative. By relaxing these bounds, the equilibrium strategies can be characterized in closed form through the optimal conditions and the market-clearing constraint. This unconstrained solution provides a tractable benchmark, from which we can later examine how the imposition of natural bounds modifies the equilibrium allocation of risks.

The following proposition summarizes the unconstrained equilibrium strategies of banks, insurers, and the endogenous premium p.

Proposition 3.1. *Without loss of generality, if the initial regime is $X(0) = e_j$, then the unconstrained* *equilibrium strategies are*

$$\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \vdots \\ \tilde{u}_{\bar{M}} \\ \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_M \\ \tilde{p} \end{pmatrix} = S^{-1}\boldsymbol{G} \begin{pmatrix} \bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j \\ \bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j \\ \vdots \\ \bar{\mu}_j - e^{rT} - \gamma_{\bar{M}}^B \xi_{\bar{M}} \bar{\sigma}_j \\ \bar{\mu}_j \\ \bar{\mu}_j \\ \vdots \\ \bar{\mu}_j \\ 0 \end{pmatrix}, \tag{3.1}$$

where

$$S^{-1} = \frac{1}{\sum_{q=1}^{\bar{M}} \frac{e^{rT}}{\gamma_q^B \bar{\sigma}_j} - \sum_{m=1}^{M} \frac{e^{rT} - e^{CT}}{\gamma_m^I \bar{\sigma}_j}}, \tag{3.2}$$

and

$$\boldsymbol{G} = \begin{pmatrix} -\frac{1}{\gamma_1^B \bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_1^B \gamma_1^B \bar{\sigma}_j^2} & \cdots & \frac{e^{rT}}{\gamma_1^B \gamma_{\bar{M}}^B \bar{\sigma}_j^2} & \frac{e^{rT}}{\gamma_1^B \gamma_1^I \bar{\sigma}_j^2} & \cdots & \frac{e^{rT}}{\gamma_1^B \gamma_M^I \bar{\sigma}_j^2} & -\frac{e^{rT}}{\gamma_1^B \bar{\sigma}_j} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{e^{rT}}{\gamma_1^B \gamma_{\bar{M}}^B \bar{\sigma}_j^2} & \cdots & -\frac{1}{\gamma_{\bar{M}}^B \bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_{\bar{M}}^B \gamma_{\bar{M}}^B \bar{\sigma}_j^2} & \frac{e^{rT}}{\gamma_{\bar{M}}^B \gamma_1^I \bar{\sigma}_j^2} & \cdots & \frac{e^{rT}}{\gamma_{\bar{M}}^B \gamma_M^I \bar{\sigma}_j^2} & -\frac{e^{rT}}{\gamma_{\bar{M}}^B \bar{\sigma}_j} \\ \frac{e^{CT} - e^{rT}}{\gamma_1^B \gamma_1^I \bar{\sigma}_j^2} & \cdots & \frac{e^{CT} - e^{rT}}{\gamma_{\bar{M}}^B \gamma_1^I \bar{\sigma}_j^2} & \frac{1}{\gamma_1^I \bar{\sigma}_j S^{-1}} + \frac{e^{CT} - e^{rT}}{\gamma_1^I \gamma_1^I \bar{\sigma}_j^2} & \cdots & \frac{e^{CT} - e^{rT}}{\gamma_1^I \gamma_M^I \bar{\sigma}_j^2} & -\frac{e^{CT} - e^{rT}}{\gamma_1^I \bar{\sigma}_j} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{e^{CT} - e^{rT}}{\gamma_1^B \gamma_M^I \bar{\sigma}_j^2} & \cdots & \frac{e^{CT} - e^{rT}}{\gamma_{\bar{M}}^B \gamma_M^I \bar{\sigma}_j^2} & \frac{e^{CT} - e^{rT}}{\gamma_1^I \gamma_M^I \bar{\sigma}_j^2} & \cdots & \frac{1}{\gamma_M^I \bar{\sigma}_j S^{-1}} + \frac{e^{CT} - e^{rT}}{\gamma_M^I \gamma_M^I \bar{\sigma}_j^2} & -\frac{e^{CT} - e^{rT}}{\gamma_M^I \bar{\sigma}_j} \\ -\frac{1}{\gamma_1^B \bar{\sigma}_j} & \cdots & -\frac{1}{\gamma_{\bar{M}}^B \bar{\sigma}_j} & -\frac{1}{\gamma_1^I \bar{\sigma}_j} & \cdots & -\frac{1}{\gamma_M^I \bar{\sigma}_j} & 1 \end{pmatrix}. \tag{3.3}$$

Proof. From Eq (2.19), since the aggregate cash-flow process $L_n(t)$ is normally distributed, the terminal wealth $W_q^B(T)$ of each bank also follows a normal distribution. Exploiting the certainty-equivalent form of exponential utility under normality, we obtain

$$\mathbb{E}\Big[U_q^B(W_q^B(T))\Big] = U_q^B\Big(\mathbb{E}[W_q^B(T)] - \tfrac{1}{2}\gamma_q^B \operatorname{Var}[W_q^B(T)]\Big). \tag{3.4}$$

Substituting into the optimization problem (2.23), the objective can be rewritten as

$$\max_{u_q} \Big(u_q(1-p)e^{rT} + (\xi_q - u_q)\,\mathbb{E}_j[\mu(t_1,\ldots,t_N)] - \tfrac{1}{2}\gamma_q^B(\xi_q - u_q)^2\,\mathbb{E}_j[\sigma^2(t_1,\ldots,t_N)]\Big), \tag{3.5}$$

where $\mathbb{E}_j[\cdot] = \mathbb{E}[\cdot \mid X(0) = e_j]$. Similarly, since $W_m^I(T)$ is also normally distributed, problem (2.25) can be expressed as

$$\max_{y_m} \Big(y_m(pe^{rT} - pe^{CT}) + y_m\mathbb{E}_j[\mu(t_1,\ldots,t_N)] - \tfrac{1}{2}\gamma_m^I y_m^2\,\mathbb{E}_j[\sigma^2(t_1,\ldots,t_N)]\Big). \tag{3.6}$$

Differentiating Eq (3.5) with respect to u_q yields the first-order condition

$$(1-p)e^{rT} - \mathbb{E}_j[\mu(t_1,\ldots,t_N)] + \gamma_q^B(\xi_q - u_q)\,\mathbb{E}_j[\sigma^2(t_1,\ldots,t_N)] = 0. \tag{3.7}$$

Likewise, differentiating Eq (3.6) with respect to y_m gives

$$(pe^{rT} - pe^{CT}) + \mathbb{E}_j[\mu(t_1, \ldots, t_N)] - \gamma_m^I y_m \, \mathbb{E}_j[\sigma^2(t_1, \ldots, t_N)] = 0. \tag{3.8}$$

Collecting the first-order conditions together with the market-clearing constraint Eq (2.26), the system can be represented compactly in matrix form as

$$\boldsymbol{A}\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \vdots \\ \tilde{u}_{\bar{M}} \\ \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_M \\ \tilde{p} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j \\ \bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j \\ \vdots \\ \bar{\mu}_j - e^{rT} - \gamma_{\bar{M}}^B \xi_{\bar{M}} \bar{\sigma}_j \\ \bar{\mu}_j \\ \bar{\mu}_j \\ \vdots \\ \bar{\mu}_j \\ 0 \end{pmatrix}, \tag{3.9}$$

where the coefficient matrix $\boldsymbol{A}$ is explicitly given by

$$\boldsymbol{A} = \begin{pmatrix} -\gamma_1^B\bar{\sigma}_j & 0 & \cdots & 0 & 0 & \cdots & 0 & -e^{rT} \\ 0 & -\gamma_2^B\bar{\sigma}_j & \cdots & 0 & 0 & \cdots & 0 & -e^{rT} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\gamma_{\bar{M}}^B\bar{\sigma}_j & 0 & \cdots & 0 & -e^{rT} \\ 0 & 0 & \cdots & 0 & \gamma_1^I\bar{\sigma}_j & \cdots & 0 & e^{CT} - e^{rT} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & \gamma_M^I\bar{\sigma}_j & e^{CT} - e^{rT} \\ -1 & -1 & \cdots & -1 & 1 & \cdots & 1 & 0 \end{pmatrix}. \tag{3.10}$$

To establish the existence of a unique solution, it suffices to show that $\det(\boldsymbol{A}) \neq 0$. By appropriate row operations, $\boldsymbol{A}$ can be transformed into an upper triangular matrix, i.e.,

$$\begin{pmatrix} -\gamma_1^B\bar{\sigma}_j & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & -e^{rT} \\ 0 & -\gamma_2^B\bar{\sigma}_j & \cdots & 0 & 0 & 0 & \cdots & 0 & -e^{rT} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\gamma_{\bar{M}}^B\bar{\sigma}_j & 0 & 0 & \cdots & 0 & -e^{rT} \\ 0 & 0 & \cdots & 0 & \gamma_1^I\bar{\sigma}_j & 0 & \cdots & 0 & e^{CT} - e^{rT} \\ 0 & 0 & \cdots & 0 & 0 & \gamma_2^I\bar{\sigma}_j & \cdots & 0 & e^{CT} - e^{rT} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \gamma_M^I\bar{\sigma}_j & e^{CT} - e^{rT} \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \sum_{q=1}^{\bar{M}} \frac{e^{rT}}{\gamma_q^B\bar{\sigma}_j} - \sum_{m=1}^{M} \frac{e^{CT}-e^{rT}}{\gamma_m^I\bar{\sigma}_j} \end{pmatrix}, \tag{3.11}$$

from which the determinant can be directly computed as follows:

$$\det(\boldsymbol{A}) = \left(\sum_{q=1}^{\bar{M}} \frac{e^{rT}}{\gamma_q^B\bar{\sigma}_j} - \sum_{m=1}^{M} \frac{e^{CT} - e^{rT}}{\gamma_m^I\bar{\sigma}_j}\right) \prod_{q=1}^{\bar{M}} (-\gamma_q^B\bar{\sigma}_j) \prod_{m=1}^{M} \gamma_m^I\bar{\sigma}_j \neq 0. \tag{3.12}$$

Therefore, the system (3.9) has a unique solution. The equilibrium strategies can be expressed as

$$\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \vdots \\ \tilde{u}_{\bar{M}} \\ \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_M \\ \tilde{p} \end{pmatrix} = \boldsymbol{A}^{-1} \begin{pmatrix} \bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j \\ \bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j \\ \vdots \\ \bar{\mu}_j - e^{rT} - \gamma_{\bar{M}}^B \xi_{\bar{M}} \bar{\sigma}_j \\ \bar{\mu}_j \\ \bar{\mu}_j \\ \vdots \\ \bar{\mu}_j \\ 0 \end{pmatrix}. \tag{3.13}$$

For computational convenience, the inverse $\boldsymbol{A}^{-1}$ can be derived by block-matrix techniques. Writing $\boldsymbol{A}$ in block form as

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{D} & E \end{pmatrix}, \tag{3.14}$$

where

$$\boldsymbol{B} = \begin{pmatrix} -\gamma_1^B \bar{\sigma}_j & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & -\gamma_2^B \bar{\sigma}_j & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\gamma_{\bar{M}}^B \bar{\sigma}_j & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \gamma_1^I \bar{\sigma}_j & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \gamma_2^I \bar{\sigma}_j & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \gamma_M^I \bar{\sigma}_j \end{pmatrix}, \tag{3.15}$$

$$\boldsymbol{C} = \left(-e^{rT}, -e^{rT}, \cdots, -e^{rT}, e^{CT} - e^{rT}, e^{CT} - e^{rT}, \cdots, e^{CT} - e^{rT}\right)^\top, \tag{3.16}$$

$$\boldsymbol{D} = (-1, -1, \cdots, -1, 1, 1, \cdots, 1), \tag{3.17}$$

and

$$E = 0. \tag{3.18}$$

Then the standard inversion formula yields

$$\boldsymbol{A}^{-1} = \begin{pmatrix} \boldsymbol{B}^{-1} + \boldsymbol{B}^{-1}\boldsymbol{C}S^{-1}\boldsymbol{D}\boldsymbol{B}^{-1} & -\boldsymbol{B}^{-1}\boldsymbol{C}S^{-1} \\ -S^{-1}\boldsymbol{D}\boldsymbol{B}^{-1} & S^{-1} \end{pmatrix} = S^{-1}\boldsymbol{G}, \tag{3.19}$$

where

$$S = E - \boldsymbol{D}\boldsymbol{B}^{-1}\boldsymbol{C} = \sum_{q=1}^{\bar{M}} \frac{e^{rT}}{\gamma_q^B \bar{\sigma}_j} - \sum_{m=1}^{M} \frac{e^{CT} - e^{rT}}{\gamma_m^I \bar{\sigma}_j}, \tag{3.20}$$

and

$$
\boldsymbol{G} = \begin{pmatrix} \frac{\boldsymbol{B}^{-1}}{S^{-1}} + \boldsymbol{B}^{-1}\boldsymbol{C}\boldsymbol{D}\boldsymbol{B}^{-1} & -\boldsymbol{B}^{-1}\boldsymbol{C} \\ -\boldsymbol{D}\boldsymbol{B}^{-1} & 1 \end{pmatrix}, \tag{3.21}
$$

with the clear form given in Eq (3.3). This completes the proof. □

Remark 3.1. *In Proposition 3.1, the general equilibrium of NPL securitization with multiple banks and insurers does not guarantee that the optimal securitization ratio u_q of bank q lies within the natural range $[0, \xi_q]$. This arises from the systemic and interactive complexity of the model, which depends on the banks' and insurers' risk aversion parameters, the distributional characteristics of NPL repayments, and the prevailing macroeconomic regime. Economically, when $u_q > \xi_q$, it can be interpreted as bank q effectively borrowing additional distressed assets from the "outside market" for securitization, thus acting as a short-seller of NPLs. Conversely, when $u_q < 0$, the bank is effectively holding more than its allocated share ξ_q, representing a long position in distressed assets. These cases reflect how heterogeneous risk preferences among insurers can reshape the allocation of NPL-related risks in equilibrium.*

Remark 3.2. *It is worth noting that similar "out-of-bound" unconstrained equilibrium solutions have also been documented in the reinsurance literature, where optimal ceded proportions may fall outside the interval* $[0, 1]$ *under certain utility or equilibrium settings; see, e.g., [26–28].*

3.2. Solution of the constrained equilibrium

In this subsection, we investigate the constrained equilibrium, where banks' securitization shares and insurers' guarantee proportions must respect their natural feasibility bounds, i.e., $u_q \in [0, \xi_q]$ and $y_m \geq 0$. Because the general multi-agent case involves intricate case distinctions and rarely admits closed-form solutions, we focus on a tractable benchmark with two banks and two insurers, i.e., $M = \bar{M} = 2$. This reduced setting, frequently adopted in the equilibrium literature (see, e.g., [29–31]), retains the essential interaction between heterogeneous institutions while allowing explicit characterization of constrained equilibria.

Under this framework, Proposition 3.2 characterizes the constrained equilibrium strategies by systematically examining all possible boundary situations.

Proposition 3.2. *Using the notation of Proposition 3.1, when the initial regime is $X(0) = e_j$, the constrained equilibrium strategies in the two-bank, two-insurer case are given as follows:*

- *If $0 \leqslant \tilde{u}_q \leqslant \xi_q$ for $q = 1, 2$, then $u_q^* = \tilde{u}_q$.*
- *If $0 \leqslant \tilde{u}_1 \leqslant \xi_1$ and $\tilde{u}_2 < 0$, then $u_1^* = \xi_1 - \frac{\gamma_2^B \xi_2}{\gamma_1^B}$ and $u_2^* = 0$.*
- *If $\tilde{u}_1 < 0$ and $0 \leqslant \tilde{u}_2 \leqslant \xi_2$, then $u_1^* = 0$ and $u_2^* = \xi_2 - \frac{\gamma_1^B \xi_1}{\gamma_2^B}$.*
- *If $\tilde{u}_1 < 0$ and $\tilde{u}_2 < 0$, then $u_1^* = u_2^* = 0$.*
- *In all remaining cases, the solution lies at the upper boundary, i.e., $u_1^* = \xi_1$ and $u_2^* = \xi_2$.*

The corresponding constrained equilibrium guarantee proportions are

$$
y_1^* = \frac{\gamma_2^I (u_1^* + u_2^*)}{\gamma_1^I + \gamma_2^I}, \qquad y_2^* = \frac{\gamma_1^I (u_1^* + u_2^*)}{\gamma_1^I + \gamma_2^I}.
$$

Proof. From Eq (3.1) in Proposition 3.1, when $M = \bar{M} = 2$, the equilibrium strategy takes the form

$$\begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{p} \end{pmatrix} = S^{-1}\boldsymbol{G}\boldsymbol{K}, \tag{3.22}$$

where S^{-1} represents the normalizing factor that balances banks' and insurers' risk exposures,

$$S^{-1} = \frac{1}{\sum_{q=1}^{2} \frac{e^{rT}}{\gamma_q^B \bar{\sigma}_j} - \sum_{m=1}^{2} \frac{e^{CT}-e^{rT}}{\gamma_m^I \bar{\sigma}_j}}, \tag{3.23}$$

and the interaction matrix $\boldsymbol{G}$ is given explicitly by

$$\boldsymbol{G} = \begin{pmatrix} -\frac{1}{\gamma_1^B \bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_1^B \gamma_1^B \bar{\sigma}_j^2} & \frac{e^{rT}}{\gamma_1^B \gamma_2^B \bar{\sigma}_j^2} & \frac{e^{rT}}{\gamma_1^B \gamma_1^I \bar{\sigma}_j^2} & \frac{e^{rT}}{\gamma_1^B \gamma_2^I \bar{\sigma}_j^2} & -\frac{e^{rT}}{\gamma_1^B \bar{\sigma}_j} \\ \frac{e^{rT}}{\gamma_1^B \gamma_2^B \bar{\sigma}_j^2} & -\frac{1}{\gamma_2^B \bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_2^B \gamma_2^B \bar{\sigma}_j^2} & \frac{e^{rT}}{\gamma_2^B \gamma_1^I \bar{\sigma}_j^2} & \frac{e^{rT}}{\gamma_2^B \gamma_2^I \bar{\sigma}_j^2} & -\frac{e^{rT}}{\gamma_2^B \bar{\sigma}_j} \\ \frac{e^{CT}-e^{rT}}{\gamma_1^B \gamma_1^I \bar{\sigma}_j^2} & \frac{e^{CT}-e^{rT}}{\gamma_2^B \gamma_1^I \bar{\sigma}_j^2} & \frac{1}{\gamma_1^I \bar{\sigma}_j S^{-1}} + \frac{e^{CT}-e^{rT}}{\gamma_1^I \gamma_1^I \bar{\sigma}_j^2} & \frac{e^{CT}-e^{rT}}{\gamma_1^I \gamma_2^I \bar{\sigma}_j^2} & -\frac{e^{CT}-e^{rT}}{\gamma_1^I \bar{\sigma}_j} \\ \frac{e^{CT}-e^{rT}}{\gamma_1^B \gamma_2^I \bar{\sigma}_j^2} & \frac{e^{CT}-e^{rT}}{\gamma_2^B \gamma_2^I \bar{\sigma}_j^2} & \frac{e^{CT}-e^{rT}}{\gamma_1^I \gamma_2^I \bar{\sigma}_j^2} & \frac{1}{\gamma_2^I \bar{\sigma}_j S^{-1}} + \frac{e^{CT}-e^{rT}}{\gamma_2^I \gamma_2^I \bar{\sigma}_j^2} & -\frac{e^{CT}-e^{rT}}{\gamma_2^I \bar{\sigma}_j} \\ -\frac{1}{\gamma_1^B \bar{\sigma}_j} & -\frac{1}{\gamma_2^B \bar{\sigma}_j} & -\frac{1}{\gamma_1^I \bar{\sigma}_j} & -\frac{1}{\gamma_2^I \bar{\sigma}_j} & 1 \end{pmatrix},$$

while the vector $\boldsymbol{K}$ collects the deterministic components driven by expectations and risk aversion:

$$\boldsymbol{K} = \begin{pmatrix} \bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j \\ \bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j \\ \bar{\mu}_j \\ \bar{\mu}_j \\ 0 \end{pmatrix}.$$

Expanding this expression yields the unconstrained solutions for banks' securitization shares:

$$\begin{aligned} \tilde{u}_1 =& \frac{S^{-1}}{\gamma_1^B} \Bigg(\left(\bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j\right) \left(-\frac{1}{\bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_1^B \bar{\sigma}_j^2} \right) \\ &+ \left(\bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j\right) \frac{e^{rT}}{\gamma_2^B \bar{\sigma}_j^2} + \bar{\mu}_j \left(\frac{e^{rT}}{\gamma_1^I \bar{\sigma}_j^2} + \frac{e^{rT}}{\gamma_2^I \bar{\sigma}_j^2} \right) \Bigg), \end{aligned} \tag{3.24}$$

and

$$\begin{aligned} \tilde{u}_2 =& \frac{S^{-1}}{\gamma_2^B} \Bigg(\left(\bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j\right) \left(-\frac{1}{\bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_2^B \bar{\sigma}_j^2} \right) \\ &+ \left(\bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j\right) \frac{e^{rT}}{\gamma_1^B \bar{\sigma}_j^2} + \bar{\mu}_j \left(\frac{e^{rT}}{\gamma_1^I \bar{\sigma}_j^2} + \frac{e^{rT}}{\gamma_2^I \bar{\sigma}_j^2} \right) \Bigg). \end{aligned} \tag{3.25}$$

To link the two expressions, we rearrange Eq (3.24) and obtain

$$\bar{\mu}_j \left(\frac{e^{rT}}{\gamma_1^I \bar{\sigma}_j^2} + \frac{e^{rT}}{\gamma_2^I \bar{\sigma}_j^2} \right) = -\left(\bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j\right) \left(-\frac{1}{\bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_1^B \bar{\sigma}_j^2} \right)$$

$$-\left(\bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j\right) \frac{e^{rT}}{\gamma_2^B \bar{\sigma}_j^2} + \tilde{u}_1 S \gamma_1^B. \tag{3.26}$$

Similarly, from Eq (3.25), we obtain

$$\bar{\mu}_j \left(\frac{e^{rT}}{\gamma_1^I \bar{\sigma}_j^2} + \frac{e^{rT}}{\gamma_2^I \bar{\sigma}_j^2} \right) = -\left(\bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j\right) \left(-\frac{1}{\bar{\sigma}_j S^{-1}} + \frac{e^{rT}}{\gamma_2^B \bar{\sigma}_j^2} \right)$$
$$-\left(\bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j\right) \frac{e^{rT}}{\gamma_1^B \bar{\sigma}_j^2} + \tilde{u}_2 S \gamma_2^B. \tag{3.27}$$

Substituting Eq (3.27) into Eq (3.24), we obtain another expression for the feasible securitization ratio of bank 1:

$$\tilde{u}_1 = \frac{\gamma_2^B (\tilde{u}_2 - \xi_2) + \gamma_1^B \xi_1}{\gamma_1^B}. \tag{3.28}$$

Similarly, substituting Eq (3.26) into Eq (3.25), we obtain

$$\tilde{u}_2 = \frac{\gamma_1^B (\tilde{u}_1 - \xi_1) + \gamma_2^B \xi_2}{\gamma_2^B}. \tag{3.29}$$

At this point, different boundary cases can be analyzed. If $0 \leqslant \tilde{u}_q \leqslant \xi_q$ for $q = 1, 2$, the unconstrained solutions are already feasible, so $u_q^* = \tilde{u}_q$. If $0 \leqslant \tilde{u}_1 \leqslant \xi_1$ but $\tilde{u}_2 < 0$, then $u_2^* = 0$, and from Eq (3.28),

$$u_1^* = \xi_1 - \frac{\gamma_2^B \xi_2}{\gamma_1^B}. \tag{3.30}$$

Similarly, if $\tilde{u}_1 < 0$ and $0 \leqslant \tilde{u}_2 \leqslant \xi_2$, then $u_1^* = 0$, and from Eq (3.29),

$$u_2^* = \xi_2 - \frac{\gamma_1^B \xi_1}{\gamma_2^B}. \tag{3.31}$$

If both $\tilde{u}_1$ and $\tilde{u}_2$ are negative, then clearly $u_1^* = u_2^* = 0$. Finally, if $\tilde{u}_1 > \xi_1$ and $\tilde{u}_2 > \xi_2$, both exceed their feasible bounds, so we take $u_1^* = \xi_1$ and $u_2^* = \xi_2$.

Turning to insurers, from Eq (3.22) we can compute $\tilde{y}_1, \tilde{y}_2$ as

$$\tilde{y}_1 = \frac{S^{-1}}{\gamma_1^I} \Bigg(\left(\bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j\right) \frac{e^{CT} - e^{rT}}{\gamma_1^B \bar{\sigma}_j^2}$$
$$+ \left(\bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j\right) \frac{e^{CT} - e^{rT}}{\gamma_2^B \bar{\sigma}_j^2} + \bar{\mu}_j \left(\frac{1}{\bar{\sigma}_j S^{-1}} + \frac{e^{CT} - e^{rT}}{\gamma_1^I \bar{\sigma}_j^2} + \frac{e^{CT} - e^{rT}}{\gamma_2^I \bar{\sigma}_j^2} \right) \Bigg), \tag{3.32}$$

and

$$\tilde{y}_2 = \frac{S^{-1}}{\gamma_2^I} \Bigg((\bar{\mu}_j - e^{rT} - \gamma_1^B \xi_1 \bar{\sigma}_j) \frac{e^{CT} - e^{rT}}{\gamma_1^B \bar{\sigma}_j^2}$$

$$+(\bar{\mu}_j - e^{rT} - \gamma_2^B \xi_2 \bar{\sigma}_j)\frac{e^{CT} - e^{rT}}{\gamma_2^B \bar{\sigma}_j^2} + \bar{\mu}_j\left(\frac{1}{\bar{\sigma}_j S^{-1}} + \frac{e^{CT} - e^{rT}}{\gamma_2^I \bar{\sigma}_j^2} + \frac{e^{CT} - e^{rT}}{\gamma_1^I \bar{\sigma}_j^2}\right)\Bigg). \tag{3.33}$$

These satisfy the proportionality relation

$$\tilde{y}_1 \gamma_1^I = \tilde{y}_2 \gamma_2^I, \tag{3.34}$$

which reflects risk sharing among insurers. Together with the market-clearing condition, we obtain the system

$$\begin{cases} y_1^* \gamma_1^I - y_2^* \gamma_2^I = 0, \\ y_1^* + y_2^* = u_1^* + u_2^*, \end{cases} \tag{3.35}$$

whose solution is

$$y_1^* = \frac{\gamma_2^I (u_1^* + u_2^*)}{\gamma_1^I + \gamma_2^I}, \quad y_2^* = \frac{\gamma_1^I (u_1^* + u_2^*)}{\gamma_1^I + \gamma_2^I}. \tag{3.36}$$

This completes the characterization of constrained equilibrium strategies. □

Proposition 3.2 shows that the constrained equilibrium coincides with the unconstrained solution whenever the securitization shares lie within their natural bounds. When the unconstrained solution falls outside, corner solutions arise: Banks may optimally choose zero or full securitization depending on their relative risk aversion and scale, while insurers allocate guarantees proportionally to their own risk tolerances. These results highlight how feasibility constraints shape the equilibrium structure in the two-bank, two-insurer setting.

In Proposition 3.2, we focus on the two-bank and two-insurer case to obtain an explicit analytical characterization of the constrained equilibrium. This benchmark is widely used in the equilibrium literature as it captures the essential strategic interactions while remaining tractable. Nonetheless, the proposed framework is not limited to this configuration. The same system of first-order conditions and the market-clearing constraint can, in principle, be formulated for any number of banks and insurers. The corresponding equilibrium can be derived using the block-matrix inversion technique introduced in Proposition 3.1, together with the case-by-case reasoning in Proposition 3.2. Although the resulting expressions become algebraically cumbersome, the two-by-two case remains an analytically convenient benchmark that preserves the key comparative-statics insights of the general multi-participant setting.

3.3. Two-state regime-switching model

In the previous subsections, we derived equilibrium strategies whose closed-form expressions depend on the regime-specific repayment parameters $\bar{\mu}$ and $\bar{\sigma}$. To obtain explicit formulas for these parameters, we now consider a two-state Markov chain, which in economic terms, corresponds to expansion (bull) and contraction (bear) regimes (see, e.g., [32]). Then, the transition intensity matrix of the Markov chain $X(t)$ is given by

$$\boldsymbol{Q} = \begin{pmatrix} -a_1 & a_1 \\ a_2 & -a_2 \end{pmatrix}. \tag{3.37}$$

Proposition 3.3. *Assume that the Markov chain has two states, and the initial state $X(0) = e_j$. Then the regime-dependent parameters $\bar{\mu}_j$ and $\bar{\sigma}_j$ are given by*

$$\bar{\mu}_j = \int_0^t n\left(\lambda_1 t_1 \int_0^\infty y f_1(y)\mathrm{d}y + \lambda_2(t - t_1) \int_0^\infty y f_2(y)\mathrm{d}y\right)\zeta_j(t, t_1)\mathrm{d}t_1, \tag{3.38}$$

and

$$\bar{\sigma}_j = \int_0^t n\left(\lambda_1 t_1 \int_0^\infty y^2 f_1(y)\mathrm{d}y + \lambda_2(t - t_1) \int_0^\infty y^2 f_2(y)\mathrm{d}y\right)\zeta_j(t, t_1)\mathrm{d}t_1, \tag{3.39}$$

respectively.

Proof. Following [33], when $X(0) = e_j$, the density function of the occupation time $O_i(t)$ is

$$\begin{aligned}\zeta_j(t, x) =& e^{-a_j t}\delta_t(x) + e^{-a_j x}e^{-(a-a_j)(t-x)}\left[a_j I_0\left(2\sqrt{a_i a_j x(t-x)}\right)\right.\\ &\left.+\sqrt{\frac{a_i a_j x}{t-x}} I_1\left(2\sqrt{a_i a_j x(t-x)}\right)\right], \quad i, j = 1, 2,\ i \neq j,\end{aligned} \tag{3.40}$$

where $0 \leq x < t$, $a = a_i + a_j$, $\delta_t(\cdot)$ represents the Dirac delta function, and $I_\rho(z), z > 0$ is the modified Bessel function of the first kind:

$$I_\rho(z) = \sum_{k=0}^\infty \frac{1}{k!\Gamma(k+\rho+1)}\left(\frac{z}{2}\right)^{2k+\rho}, \quad \rho = 0, 1.$$

Since the Markov chain has only two states, the expected repayment parameters under regime j can be written as

$$\bar{\mu}_j = \mathbb{E}_j\left[\mu(t_1, t - t_1)\right] = \int_0^t n\left(\lambda_1 t_1 \int_0^\infty y f_1(y)\mathrm{d}y + \lambda_2(t - t_1) \int_0^\infty y f_2(y)\mathrm{d}y\right)\zeta_j(t, t_1)\mathrm{d}t_1, \tag{3.41}$$

and

$$\begin{aligned}\bar{\sigma}_j =& \mathbb{E}_j\left[\sigma^2(t_1, t - t_1)\right]\\ =& \int_0^t n\left(\lambda_1 t_1 \int_0^\infty y^2 f_1(y)\mathrm{d}y + \lambda_2(t - t_1) \int_0^\infty y^2 f_2(y)\mathrm{d}y\right)\zeta_j(t, t_1)\mathrm{d}t_1.\end{aligned} \tag{3.42}$$

□

Substituting the explicit expressions of $\bar{\mu}_j$ and $\bar{\sigma}_j$ into the equilibrium formulas derived in the previous subsections, we obtain the analytical characterization of equilibrium strategies under the two-state regime-switching model.

Remark 3.3. *Note that before Proposition 3.3, without loss of generality, we assume that the Markov chain only has two states. When the Markov chain has more than two states, we cannot obtain the closed-form expressions of the density function of occupation time in the literature, which makes it very difficult to derive the analytic expressions of $\bar{\mu}_j$ and $\bar{\sigma}_j$.*

4. Numerical illustration

To illustrate the theoretical results, we conduct numerical experiments under a representative set of parameters, summarized in Table 1. Here, λ_i and β_i govern repayment dynamics, (a_1, a_2) describe the relative regime weights, γ_q^B and γ_m^I capture the risk aversion of banks and insurers, respectively, while r and T denote the risk-free rate and maturity horizon. The contractual transfer rate is represented by C, and (ξ_1, ξ_2) specify the relative sizes of the two banks. These parameter choices are designed to highlight the qualitative behavior of the equilibrium rather than to match a particular dataset†.

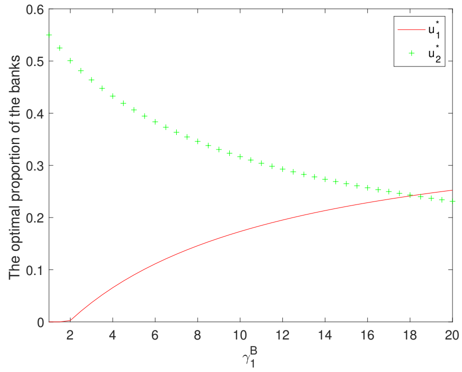
Table 1. Specification of model parameters for numerical illustration

Parameter	Value	Parameter	Value
λ_1	0.9	λ_2	0.8
β_1	2	β_2	3
a_1	0.2	a_2	0.7
γ_1^B	6	γ_2^B	8
γ_1^I	4	γ_2^I	5
r	0.01	T	2
ξ_1	0.4	ξ_2	0.6
C	0.02		

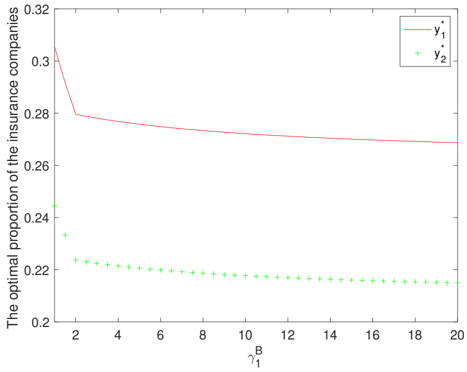
Figures 1 and 2 illustrate the impact of banks' risk-aversion parameters on equilibrium strategies and the resulting equilibrium premium. Figure 1(a) shows that as γ_1^B increases, Bank 1 becomes more risk-averse and thus securitizes a larger share of its non-performing loans to transfer risk, leading to a higher u_1^*. At the same time, Figure 2(a) indicates that the equilibrium insurance premium p^* also rises with γ_1^B, since greater securitization demand from Bank 1 increases insurers' required compensation. This higher premium, in turn, discourages Bank 2 from securitizing, thereby reducing u_2^*, as seen in Figure 1(c).

Taken together, the market-clearing condition ($u_1^* + u_2^* = y_1^* + y_2^*$) implies that the aggregate securitization share declines, consistent with the adjustment in insurers' guarantee proportions shown in Figure 1(b). A similar line of reasoning applies to Figure 1(c)–(d) and Figure 2(b), which illustrate the effects of variations in γ_2^B.

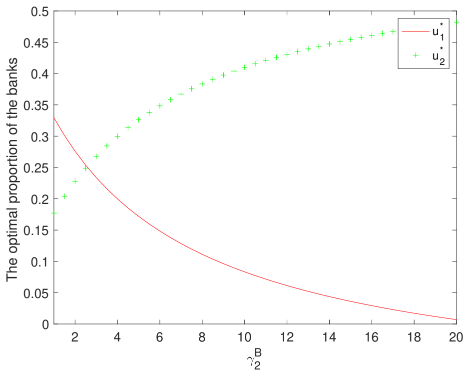
†If sufficient data on non-performing loan recoveries are available, specifically the amounts recovered at the end of each year, the parameter estimates can be obtained using maximum likelihood estimation.



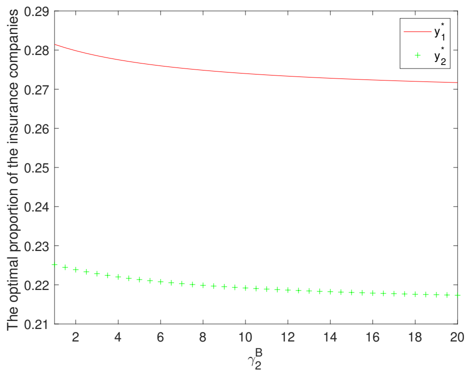
(a) Bank securitization proportions u^* with varying γ_1^B



(b) Insurer guarantee proportions y^* with varying γ_1^B

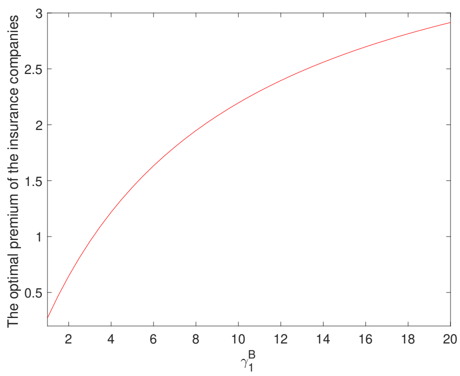


(c) Bank securitization proportions u^* with varying γ_2^B

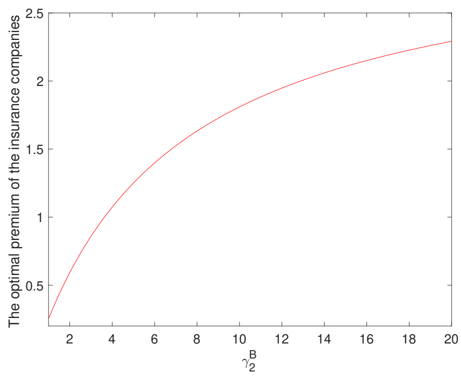


(d) Insurer guarantee proportions y^* with varying γ_2^B

Figure 1. Equilibrium strategies for different values of banks' risk-aversion parameters γ_1^B and γ_2^B.

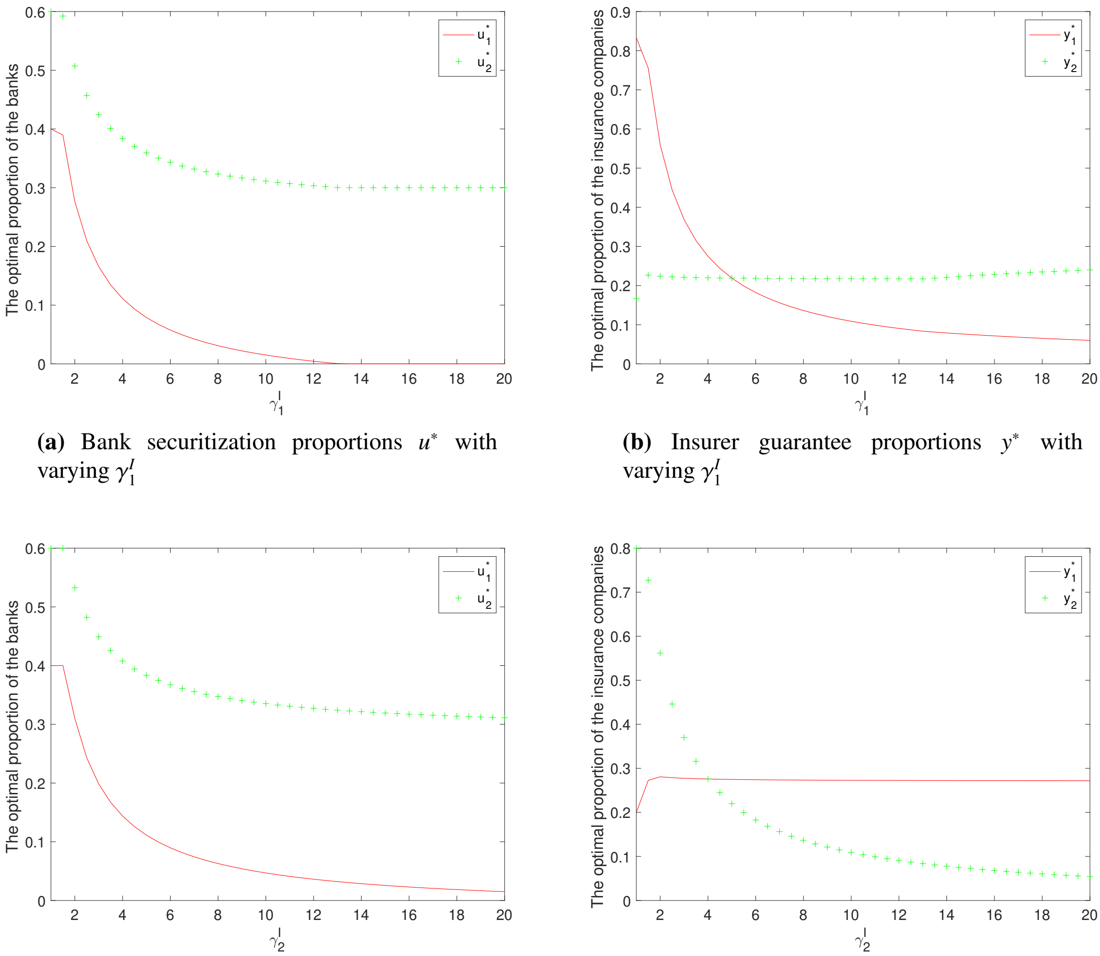


(a) Premium p^* with varying γ_1^B



(b) Premium p^* with varying γ_2^B

Figure 2. Equilibrium premium p^* for different values of banks' risk-aversion parameters γ_1^B and γ_2^B.



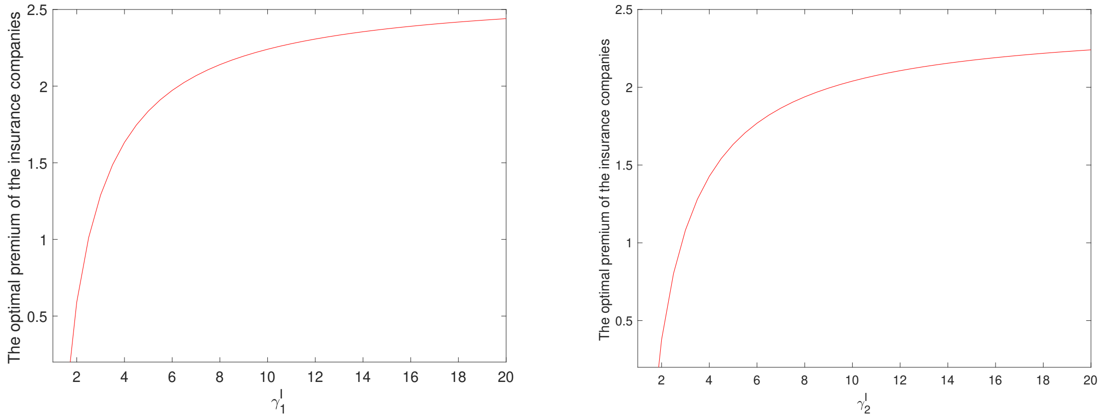
(a) Bank securitization proportions u^* with varying γ_1^I

(b) Insurer guarantee proportions y^* with varying γ_1^I

(c) Bank securitization proportions u^* with varying γ_2^I

(d) Insurer guarantee proportions y^* with varying γ_2^I

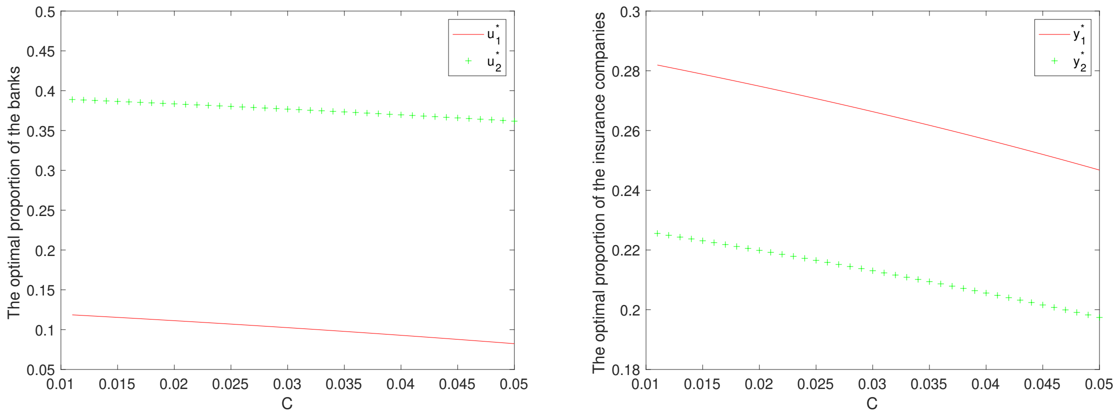
Figure 3. Equilibrium strategies for different values of insurers' risk-aversion parameters γ_1^I and γ_2^I.



(a) Premium p^* with varying γ_1^I **(b)** Premium p^* with varying γ_2^I

Figure 4. Equilibrium premium p^* for different values of insurers' risk-aversion parameters γ_1^I and γ_2^I.

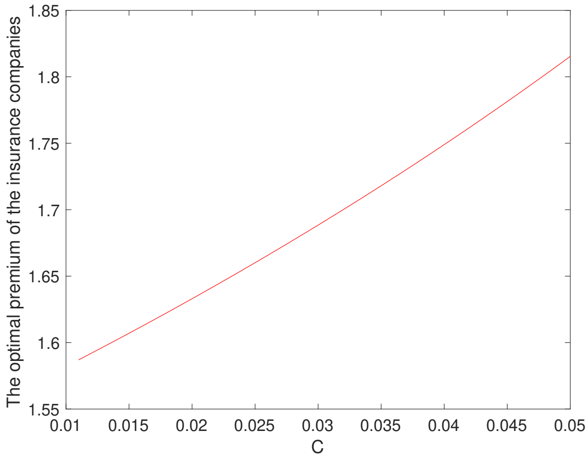
Figures 3 and 4 present the comparative statics with respect to insurers' risk-aversion parameters. Figure 3(a) shows that as γ_1^I increases, Insurer 1 becomes more conservative in absorbing NPL risk. This reduces its own guarantee share y_1^* (see Figure 3(b)) and simultaneously shrinks the securitization capacity available to the banks, as reflected in the declines of u_1^* and u_2^*. The market-clearing condition ($u_1^* + u_2^* = y_1^* + y_2^*$) implies that greater insurer risk aversion translates into a lower equilibrium securitization volume at the aggregate level. Interestingly, Figure 3(b) shows that y_2^* does not decrease monotonically with γ_1^I. This arises because the more conservative stance of Insurer 1 effectively cedes part of the guarantee market to Insurer 2. A symmetric reasoning applies to Figure 3(c)–(d), which report the effects of increasing γ_2^I.



(a) Bank securitization proportions u^* with varying C **(b)** Insurer guarantee proportions y^* with varying C

Figure 5. Equilibrium strategies for different values of contractual transfer rate C.

Figure 4 complements these findings by examining the equilibrium premium p^*. As either γ_1^I or γ_2^I increases, insurers require greater compensation for bearing repayment uncertainty. Consequently, the premium p^* rises monotonically with insurers' risk aversion, although the marginal rate of increase diminishes at higher values, reflecting the concavity of the adjustment.



(**a**) Premium p^* with varying C

Figure 6. Equilibrium premium p^* for different values of the contractual transfer rate C.

Figures 5 and 6 analyze the comparative statics of the contractual transfer rate C. Figure 5 shows that as C increases, both banks' securitization proportions u_1^* and u_2^* (Panel a) and insurers' guarantee proportions y_1^* and y_2^* (Panel b) decline steadily. The intuition is that a higher contractual transfer rate implies that insurers must remit a larger share of repayments to investors. This reduces the net benefit of providing guarantees, leading insurers to scale back their exposure. By the market-clearing condition, the reduced supply of guarantees feeds back to banks, lowering their equilibrium securitization volumes.

Figure 6 complements this by showing the effect on the equilibrium premium p^*. As C rises, insurers demand higher compensation to cover the increased contractual obligations, so p^* increases monotonically. However, the adjustment is gradual, reflecting the balance between rising contractual costs and the decreasing willingness of banks to securitize.

The numerical results above offer clear economic insights into the strategic behavior of banks and insurers. First, as the degree of banks' risk aversion (γ^B) increases, they securitize a larger fraction of their distressed assets to reduce exposure to repayment uncertainty. This behavior is consistent with observed post-crisis patterns in which banks seek to transfer credit risk through asset-backed structures. Second, insurers with higher risk aversion (γ^I) optimally reduce their guarantee ratios, reflecting a more conservative underwriting stance toward NPL-related risks. Third, from a regulatory perspective, the interaction between these two forces has important systemic implications: In stressed macroeconomic regimes, strongly risk-averse banks increase the demand for securitization, while cautious insurers limit guarantee supply, potentially exacerbating market illiquidity. Understanding this equilibrium feedback can thus help policymakers design capital-relief programs and guarantee mechanisms that stabilize the NPL securitization market under regime shifts.

5. Conclusions

This paper develops a multi-agent equilibrium model for the non-performing loan (NPL) securitization market, capturing the strategic interactions between originating banks and credit-enhancing insurers under stochastic macroeconomic conditions. By linking the NPL recovery process to a continuous-time Markov chain, we endogenously determine optimal risk exposures and the market price for credit guarantees. Our primary contribution is the derivation of an analytical, closed-form solution for the market equilibrium, which explicitly characterizes the optimal securitization and guarantee ratios and the equilibrium premium. This solution demonstrates how equilibrium strategies are fundamentally shaped by agents' risk aversion and expectations regarding macroeconomic state transitions, which are priced into the insurance premium and influence the strategic allocation of risk between market participants. The findings of this study offer a quantitative framework for strategic decision-making by banks and insurers, and provide a new tool for regulators to assess systemic risk related to macroeconomic shocks.

Author contributions

All the authors contributed equally to the paper.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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