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*Research article*

# Link exponential synchronization of discrete-time complex dynamic networks with infinite distributed delay

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**Abstract:** This paper investigates the link exponential synchronization (LES) problem of a class of discrete-time complex dynamic networks (DTCDNs) which consist of infinite distributed delay, node complex dynamic subsystems (NCDS), and link complex dynamic subsystems (LCDS) with mutual coupling relationships. A mathematical induction method is proposed to derive the global exponential stability criterion of error systems. The obtained LES is described using linear scalar inequalities, which are very simple and easy to verify based on standard software tools such as YALMIP. Finally, it was validated through an illustrative example.

**Keywords:** discrete-time complex dynamic networks (DTCDNs); link exponential synchronization (LES); infinite distributed delays

## 1. Introduction

In recent decades, complex dynamical networks (CDNs) have been extensively studied to model interconnected systems in nature and engineering. A CDN consists of dynamic nodes connected through specific interactions, effectively representing real-world systems such as smart grids [1, 2], social networks [3], smart transportation networks [4], and biological networks [5, 6]. From the perspective of graph theory [7], a CDN consists of many nodes interconnected by links. The network's dynamic behavior is shaped not only by the node dynamics, but also significantly regulated by the dynamics of the links themselves. For instance, drone networks [8], multiple unmanned surface vehicles networks [9], biological neural networks [10, 11], web-winding systems [12, 13], biological community [14, 15], and so on. Therefore, treating the link connections between nodes as a relatively independent dynamic subsystem and studying its dynamic characteristics holds significant practical importance.

In many practical applications, time delays are unavoidable due to finite signal propagation speeds, communication latencies, or memory effects in physical and biological systems. The presence of delays can significantly influence network dynamics, potentially destabilizing synchronization or inducing complex behaviors such as oscillations and bifurcations. Consequently, extensive research has focused on synchronization in CDNs with various delay types, including constant delays [16–18], time-varying delays [19], and distributed delays [20, 21]. For instance, in [17], the authors investigated synchronization in CDNs with discrete-time delays on time scales, providing unified stability criteria. Similarly, exponentially asymptotic synchronization in uncertain time-delay networks with multi-links was addressed in [18], while finite-time synchronization with a time-varying delay and hybrid coupling has been explored in [19]. These works highlight the critical role of delay effects in synchronization analysis and control.

However, many existing studies tend to focus solely on the dynamic characteristics of nodes, with controllers primarily applied to nodes to achieve dynamic behaviors such as stabilization or synchronization, while the connections between nodes are regarded merely as auxiliary and subordinate factors. For instance, [22] discussed node synchronization under constant connection relationships, [23] investigated related issues under time-varying and continuous connection relationships, and [24, 25] explored scenarios with switching connection relationships. In many real-world scenarios, such as social networks, transportation systems, and network operation processes, the strength or existence of links changes over time and is often governed by intrinsic dynamical laws. Traditional approaches typically treat links as static or purely time-varying without considering their own dynamic evolution. However, as demonstrated in the literature, neglecting the dynamic characteristics of links can lead to an incomplete understanding, and, in some cases, synchronization may fail due to the influence of dynamic links [26, 27]. For example, [26] shows that dynamic coupling connections can cause synchronization loss in CDNs, which has motivated the study of links as subsystems with their own independent dynamic characteristics, and the development of control strategies that explicitly account for link behavior.

From a perspective of large-scale systems, a CDN consists of two interconnected subsystems: the NCDS and the LCDS. By treating the weighted values of links as state variables of the LCDS and representing the LCDS using differential equations, synchronization is studied not only for node states but also for the coupling links themselves [28–33]. For instance, the wholly asymptotical stability of CDNs was investigated by designing both the node controller and the coupling mechanism of link dynamics in the Lyapunov sense [28]. The asymptotical structural balance of LCDS was developed by synthesising both the node controller and the whole dynamical behavior of links in [29]. Gao et al. [30] achieved asymptotic state synchronization of NCDS for the DTCDNs by synthesizing the NCDS controller and LCDS coupling term, using outgoing and incoming link vectors to provide simpler conditions, better geometric intuition, and easier generalization to nonlinear cases than matrix-based methods. Because link states are not precisely measurable in practical applications, the stabilization and synchronization of the NCDS were studied in [31] using the auxiliary role of the LCDS. Gao et al. [32] formally defined link synchronization and synthesized adaptive control schemes to achieve it, while Wang et al. [33] investigated state synchronization of controlled nodes via link dynamics, modeling the LCDS using a Riccati matrix differential equation. Link dynamics have also been incorporated into cluster synchronization [34] and outer synchronization [35] problems, and Zhao et al. [36] extended these ideas to stochastic link dynamics. Moreover, practical networks often face additional challenges such as uncertain couplings, external disturbances, and even malicious attacks on communication links. Robust synchronization under such conditions has been investigated using adaptive and impulsive control strategies [37–39]. For example, reference [38] studied robust synchronization of Markovian jump CDNs under link attacks using a dynamic event-triggered control approach, while Liu et al. [39] addressed observer-based secure synchronization in complex-valued networks under link attacks. These works underscore the importance of developing synchronization strategies that account for both link dynamics and external uncertainties.

DTCDNs have gained increasing attention due to their natural fit with digital control systems and sampled-data communications. Early studies focused on node dynamics and established basic synchronization criteria using Lyapunov–Krasovskii functionals and LMI techniques [29, 40, 41], treating the network as an NCDS coupled with an LCDS, often modeled as a Riccati matrix difference equation. Subsequent research expanded to tracking control of dynamic links [29] and structural balance in the LCDS [42]. More recently, Liang et al. [43] proposes a dual-event-driven control scheme for DTCDNs under discontinuous communication, Ma et al. [44] addresses exponential synchronization of nonlinear DTCDNs with communication delays via a delayed impulsive control framework, and Li et al. [45] achieve tracking control of DTCDNs with unavailable link states using an outgoing link vector for greater mathematical convenience. However, most existing approaches assume link states are measurable or estimable, which is often impractical. Despite these advances, the study of link synchronization in DTCDNs with time delays remains largely underexplored, and a systematic method to derive sufficient conditions for LES in such networks is still lacking.

Moreover, in the context of computer simulation, discrete-time networks offer distinct benefits over their continuous-time counterparts when handling digital information transmission. Consequently, exploring the dynamical properties of DTCDNs holds considerable importance. Based on [45], this paper considers DTCDNs with discrete time-varying delays and infinite distributed delays, and addresses the LES problem for a class of delayed DTCDNs that incorporate both node and link dynamics. A state feedback control law is proposed to guarantee global exponential stability of the resulting synchronization error system.

The main contributions of this work are summarized as follows:

(1) The proposed synchronization conditions are reduced to solving only a small number of simple linear scalar inequalities, which greatly lowers the computational burden.
(2) The controller gains are directly determined from the intrinsic parameters of the DTCDN, making the design process straightforward and intuitive.
(3) The proposed approach is directly founded on the LES definition, so it obviates the construction of any Lyapunov–Krasovskii functional.
(4) The proposed approach accommodates more complex and practically relevant delay scenarios, including discrete time-varying delays and infinite distributed delays.

The remainder of this paper is organized as follows. Section 2 presents the system model and provides the necessary preliminary information for the problem under study. The main theoretical findings, including a novel criterion for LES, are detailed in Section 3. An illustrative numerical example is given in Section 4 to demonstrate the effectiveness of the proposed results. Finally, Section 5 concludes the paper with a summary of the work.

*Notations*: The real number and integer set are denoted by $\mathbb{R}$ and $\mathbb{Z}$, respectively. Let $|S| = [|s_{ij}|]$, $[p, q]_{\mathbb{Z}}$ be the set that contains all the positive integers ranging from $p$ to $q$. The column-vectorizing operator is denoted by col(·). Let $\mathbb{R}^p$ be the linear space of all $p$-dimensional column vectors over $\mathbb{R}$. The symbol $\mathbb{R}^s_{\geq}$ denotes the set of all nonnegative vectors. The symbol $C(P_1, P_2)$ denotes the set that consists of all functions $\varphi : P_1 \to P_2$. A norm $\|(\cdot,\cdot)\|_{\sup}$ can be defined by

$$\|(\zeta, \tilde{\zeta})\|_{\sup} = \sup_{s\in(-\infty,0]_{\mathbb{Z}}} \max\left\{\|\zeta(s)\|_\infty, \|\tilde{\zeta}(s)\|_\infty\right\},$$
$$\forall \zeta \in C((-\infty, 0]_{\mathbb{Z}}, \mathbb{R}^n),\ \forall\ \tilde{\zeta} \in C((-\infty, 0]_{\mathbb{Z}}, \mathbb{R}^m).$$

## 2. Model description and preliminaries

Consider the DTCDNs with NCDS and LCDS. The DTCDN is modeled as follows:

$$\begin{aligned} x_i(t+1) = &\ Ax_i(t) + Cf(x_i(t)) + Dg(x_i(t-\tau_i(t))) + \sum_{m=1}^{+\infty} \mu_m w(x_i(t-m)) \\ &+ \sigma \sum_{l=1}^{N} \zeta_{il}(t)\varphi_l(x_l(t)) + u(x_i(t)), \end{aligned} \tag{2.1a}$$

$$\zeta_i(t+1) = B\zeta_i(t) + H(x(t))x_i(t), i \in [1, N]_{\mathbb{Z}}, t \geq 0, \tag{2.1b}$$

where $N$ is the number of nodes, $x_i(t) = [x_{i1}\ x_{i2}\ \cdots\ x_{in}]^T \in R^n$ is the state vector of $i$th node, $x(t) = [x_1\ x_2\ \cdots\ x_N]^T \in R^{nN}$, $\zeta_i(t) = [\zeta_{i1}\ \zeta_{i2}\ \cdots\ \zeta_{iN}]^T \in R^N$ is the outgoing links vector of $i$th node, $\zeta(t) = [\zeta_1\ \zeta_2\ \cdots\ \zeta_N]^T \in R^{N^2}$, $\sigma \in R$ is the common coupling strength of DTCDN, which is a given positive constant, $f(\cdot) \in R^n$, $g(\cdot) \in R^n$, and $w(\cdot) \in R^n$ represent the nonlinear vector functions of node state, $\varphi_l(x_l(t)) \in R^n$ is the inner coupling vector function about node state, $A = (a_{pq})_{n\times n}$, $C = (c_{pq})_{n\times n}$, $D = (d_{pq})_{n\times n}$, and $B = (b_{pq})_{N\times N}$ are constant matrices, $u(x_i(t)) \in R^n$ and $H(x(t)) = (h_{pq}(x(t)))_{N\times n}$ are the control input for NCDS and coupling term for LCDS, respectively, the positive integer $\tau_i(t)$ denotes the discrete time-varying delay satisfying $\check{\tau}_i \leq \tau_i(t) \leq \hat{\tau}_i$, and the constant $\mu_m \geq 0$ $(m = 1, 2, \cdots)$ satisfy the following convergent conditions:

$$\sum_{m=1}^{+\infty} \mu_m < +\infty, \quad \sum_{m=1}^{+\infty} m\mu_m < +\infty.$$

The initial conditions associated with system (2.1) are given by

$$x_i(s) = \varphi_i(s) \in C((-\infty, 0]_{\mathbb{Z}}, \mathbb{R}^n), \quad i \in [1, N]_{\mathbb{Z}}, \tag{2.2a}$$

$$\zeta_i(s) = \phi_i(s) \in C((-\infty, 0]_{\mathbb{Z}}, \mathbb{R}^N), \quad i \in [1, N]_{\mathbb{Z}}. \tag{2.2b}$$

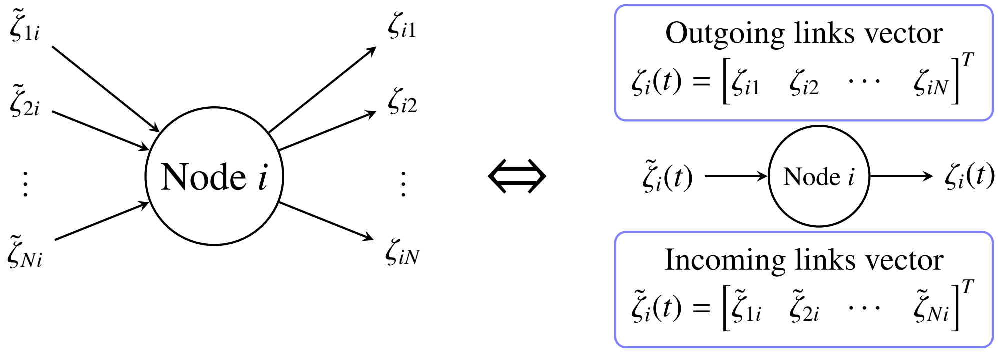



**Figure 1.** The schematic diagram of outgoing and incoming links vectors at the $i$th node.

**Remark 1.** *In Figure 1, consider a DTCDN consisting of N nodes. Let $\zeta_{ij}(t) \in \mathbb{R}$ ($i \in [1,N]_{\mathbb{Z}}$, $j \in [1,N]_{\mathbb{Z}}$.) denote the weight of the directed link from node i to node j at time t. In general, $\zeta_{ij}(t) \neq \zeta_{ji}(t)$; however, if $\zeta_{ij}(t) = \zeta_{ji}(t)$ holds for all $1 \leq i, j \leq N$, the network is said to be undirected. The diagonal entry $\zeta_{ii}(t)$ represents the self-connection weight of the ith node. For each node i, we define the outgoing links vector as $\zeta_i(t) = [\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{iN}]^T \in \mathbb{R}^N$, and the incoming links vector as $\tilde{\zeta}_i(t) = [\zeta_{1i}, \zeta_{2i}, \ldots, \zeta_{Ni}]^T \in \mathbb{R}^N$, where $i \in [1,N]_{\mathbb{Z}}$.*

We impose the following assumptions.

**Assumption 1.** *There are $\beta_\ell^f, \beta_\ell^g, \beta_\ell^w \in \mathbb{R}_>$ such that $f_\ell(0) = g_\ell(0) = \omega_\ell(0) = 0$, $|f_\ell(\alpha_1) - f_\ell(\alpha_2)| \leq \beta_\ell^f |\alpha_1 - \alpha_2|$, $|g_\ell(\alpha_1) - g_\ell(\alpha_2)| \leq \beta_\ell^g |\alpha_1 - \alpha_2|$, $|w_\ell(\alpha_1) - w_\ell(\alpha_2)| \leq \beta_\ell^w |\alpha_1 - \alpha_2|$ for any $\ell \in [1,n]_{\mathbb{Z}}$, $\alpha_1, \alpha_2 \in \mathbb{R}$, $\alpha_1 \neq \alpha_2$, where $f_\ell(\cdot)$, $g_\ell(\cdot)$, and $w_\ell(\cdot)$ are the coordinate components of $f(\cdot)$, $g(\cdot)$, and $w(\cdot)$, respectively.*

By introducing the node synchronization error $e_i^{(ij)}(t) = x_i(t) - x_j(t)$ and the link synchronization error $E_i^{(ij)}(t) = \zeta_i(t) - \zeta_j(t)$, the error dynamics system can be expressed as follows:

$$\begin{aligned} e_i^{(ij)}(t+1) = {} & Ae_i^{(ij)}(t) + Cf^*(e_i^{(ij)}(t)) + Dg^*(e_i^{(ij)}(t - \tau_i(t))) \\ & + \sum_{m=1}^{+\infty} \mu_m w^*(e_i^{(ij)}(t-m)) + \sigma \sum_{l=1}^{N} E_{il}^{(ij)} \varphi_l(x_l(t)) \\ & + u^*(e_i^{(ij)}(t)), \end{aligned} \tag{2.3a}$$

$$E_i^{(ij)}(t+1) = BE_i^{(ij)}(t) + H(x(t))e_i^{(ij)}(t), \tag{2.3b}$$

where

$$\begin{aligned} f^*(e_i^{(ij)}(\cdot)) &= f(e_i^{(ij)}(\cdot) + x_j(\cdot)) - f(x_j(\cdot)), \\ g^*(e_i^{(ij)}(\cdot)) &= g(e_i^{(ij)}(\cdot) + x_j(\cdot)) - g(x_j(\cdot)), \\ \omega^*(e_i^{(ij)}(\cdot)) &= \omega(e_i^{ij}(\cdot) + x_j(\cdot)) - \omega(x_j(\cdot)), \\ u^*(e_i^{(ij)}(\cdot)) &= u(e_i^{ij}(\cdot) + x_j(\cdot)) - u(x_j(\cdot)). \end{aligned}$$

Let

$$e_i^{(ij)}(t) = \mathrm{col}(e_{i1}^{(ij)},\ e_{i2}^{(ij)},\ \cdots,\ e_{in}^{(ij)}),$$

$$
\begin{aligned}
&E_i^{(ij)}(t) = \mathrm{col}(E_{i1}^{(ij)},\ E_{i2}^{(ij)},\ \cdots,\ E_{iN}^{(ij)}),\\
&f^*(e_i^{(ij)}(\cdot)) = \mathrm{col}(f_1^*(e_{i1}^{(ij)}(\cdot)), f_2^*(e_{i2}^{(ij)}(\cdot)),\ \cdots, f_n^*(e_{in}^{(ij)}(\cdot))),\\
&g^*(e_i^{(ij)}(\cdot)) = \mathrm{col}(g_1^*(e_{i1}^{(ij)}(\cdot)), g_2^*(e_{i2}^{(ij)}(\cdot)),\ \cdots, g_n^*(e_{in}^{(ij)}(\cdot))),\\
&w^*(e_i^{(ij)}(\cdot)) = \mathrm{col}(w_1^*(e_{i1}^{(ij)}(\cdot)), w_2^*(e_{i2}^{(ij)}(\cdot)),\ \cdots, w_n^*(e_{in}^{(ij)}(\cdot))).
\end{aligned}
$$

Thus, we can obtain the component system form of (2.3), expressed as follows:

$$
\begin{aligned}
e_{i\ell}^{(ij)}(t+1) = &\sum_{k=1}^{n} a_{\ell k} e_{ik}^{(ij)}(t) + \sum_{k=1}^{n} c_{\ell k} f_k^*(e_{ik}^{(ij)}(t)) + \sum_{k=1}^{n} d_{\ell k} g_k^*(e_{ik}^{(ij)}(t-\tau_i(t)))\\
&+ \sum_{m=1}^{+\infty} \mu_m w_\ell^*(e_{i\ell}^{(ij)}(t-m)) + \sigma \sum_{l=1}^{N} E_{il}^{(ij)}(t)\varphi_{l\ell}(x_{l\ell}(t))\\
&+ u_\ell^*(e_{i\ell}^{(ij)}(t)), i \in [1,N]_{\mathbb{Z}}, \ell \in [1,n]_{\mathbb{Z}}, t \geq 0,
\end{aligned} \tag{2.4a}
$$

$$
E_{ij}^{(ij)}(t+1) = \sum_{l=1}^{N} b_{jl} E_{il}^{(ij)}(t) + \sum_{k=1}^{n} h_{jk}(x(t)) e_{ik}^{(ij)}(t), j, i \in [1,N]_{\mathbb{Z}}, t \geq 0. \tag{2.4b}
$$

Under Assumption 1, we obtain that

$$
\begin{aligned}
&|f_\ell^*(\hbar)| \leq \beta_\ell^f |\hbar|,\ |g_\ell^*(\hbar)| \leq \beta_\ell^g |\hbar|,\ |w_\ell^*(\hbar)| \leq \beta_\ell^w |\hbar|,\\
&\hbar \in \mathbb{R},\ \ell \in [1,n]_{\mathbb{Z}}.
\end{aligned} \tag{2.5}
$$

**Assumption 2.** *The inner coupling vector function $\varphi_l(x_l(t))$ and coupling term $H(x(t))$ are bounded. Furthermore, there exits known constants $\hat{\varphi}_{l\ell}$ and $\hat{h}_{jk}$ such that the following inequality is satisfied:*

$$
|\varphi_{l\ell}(x_{l\ell}(t))| \leq \hat{\varphi}_{l\ell}, |h_{jk}(x(t))| \leq \hat{h}_{jk}, l, j \in [1,N]_{\mathbb{Z}}, \ell, k \in [1,n]_{\mathbb{Z}}, \tag{2.6}
$$

*where $\varphi_l(x_l(\cdot)) = \mathrm{col}(\varphi_{l1}(x_l(\cdot)), \varphi_{l2}(x_l(\cdot)),\ \cdots, \varphi_{ln}(x_l(\cdot)))$.*

**Definition 1.** *If there exist $\gamma, \mu \in \mathbb{R}_>$ and controller $u(x_i(t))$ such that the arbitrary solution $(e_i^{(ij)}(t), E_i^{(ij)}(t))$ of the error dynamical system* (2.4)*, satisfying*

$$
\left\|(e_i^{(ij)}(t), E_i^{(ij)}(t))\right\| \leq \mu e^{-\gamma t} \left\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\right\|_{\sup}, \forall t \in [0,\infty)_{\mathbb{Z}}, i, j \in [1,N]_{\mathbb{Z}},
$$

*where $(e_i^{(ij)^*},\ E_i^{(ij)^*}) \in C((-\infty,0]_{\mathbb{Z}}, \mathbb{R}^N) \times C((-\infty,0]_{\mathbb{Z}}, \mathbb{R}^N)$ is the initial functions corresponding to the solution $(e_i^{(ij)}(t), E_i^{(ij)}(t))$, then the DTCDN with NCDS* (2.1a) *and LCDS* (2.1b) *is LES.*

The goal of this paper is to construct a state feedback controller taking the form

$$
u(x_i(t)) = (R-A)x_i(t), R = (r_{pq})_{n\times n},\ i \in [1,N]_{\mathbb{Z}}, t \in [0,\infty)_{\mathbb{Z}} \tag{2.7}
$$

to make the DTCDN (2.1) achieve LES, where $r_{pq}$ represents the unknown controller gains to be solved.

## 3. Main results

Set

$$\Xi_\beta = |C|\Pi_1 + e^{\beta\hat{\tau}_i}|D|\Pi_2 + \alpha\Pi_3 + |R| + \Theta_{\beta,n},\ \Phi = \sigma[\hat{\varphi}_{\ell\jmath}],$$

$$\tilde{\Xi}_\beta = |B| + \Theta_{\beta,N},\ B = [b_{\jmath\ell}],\ \tilde{H} = [\hat{h}_{\jmath\ell}],$$

$$\Theta_{\beta,k} = -e^{-\beta}I_k,\ \alpha = \sum_{m=1}^{+\infty}\mu_m e^{\beta m},$$

$$\Pi_1 = \operatorname{diag}(\beta_1^f, \ldots, \beta_n^f),\ \Pi_2 = \operatorname{diag}(\beta_1^g, \ldots, \beta_n^g),$$

$$\Pi_3 = \operatorname{diag}(\beta_1^w, \ldots, \beta_n^w).$$

**Theorem 1.** *If there exists $\tilde{e}_i \in \mathbb{R}^n_>$, $\tilde{E}_i \in \mathbb{R}^N_>$ and $\beta \in \mathbb{R}_>$ such that*

$$\Xi_\beta\tilde{e}_i + \Phi\tilde{E}_i \preceq 0, \tag{3.1}$$

$$\tilde{\Xi}_\beta\tilde{E}_i + \tilde{H}\tilde{e}_i \preceq 0, \tag{3.2}$$

*then the error system* (2.4) *is globally exponentially stable, that is, the DTCDN* (2.1) *achieves LES via the controllers in* (2.7).

**Remark 2.** *The exponential decay rate $\beta$ in Theorem 1 quantifies the synchronization speed. A larger $\beta$ yields faster convergence. The feasible $\beta$ depends on the delay bounds $\check{\tau}_i, \hat{\tau}_i$ and the infinite distributed delay coefficients $\mu_m$. Specifically, larger upper discrete delays $\hat{\tau}_i$ introduce additional exponential weighting $e^{\beta\hat{\tau}_i}$ in $\Xi_\beta$, which tightens Inequality* (3.1) *and reduces the maximal allowable $\beta$. Similarly, slower-decaying $\mu_m$ (i.e., larger $\sum_{m=1}^{\infty} m\mu_m$) increases $\alpha = \sum_{m=1}^{\infty}\mu_m e^{\beta m}$, also limiting $\beta$. Thus, networks with shorter discrete delays and faster-decaying distributed delays achieve faster synchronization.*

**Remark 3.** *Unlike LMI-based approaches [29, 40] that require solving high-dimensional matrix inequalities, our method reduces synchronization conditions to simple linear scalar Inequalities* (3.1) *and* (3.2), *significantly lowering computational burden. Compared with adaptive control methods [45] that involve dynamic gain update laws, our controller gains are directly determined from system parameters. Moreover, unlike most existing works that treat link dynamics as measurable or ignore infinite distributed delays [30, 32], our method explicitly handles both node and link synchronization under infinite distributed delays without constructing any Lyapunov–Krasovskii functional.*

*Proof.* Choose a scalar $\Upsilon > 0$ such that

$$\Upsilon\tilde{e}_i \succ [1 \ \cdots \ 1]^T,\ \Upsilon\tilde{E}_i \succ [1 \ \cdots \ 1]^T.$$

Suppose $(e_i^{(ij)}(t),\ E_i^{(ij)}(t))$ is the solution of (2.4). For any fixed $e_i^{(ij)^*} \in C((-\infty,0]_{\mathbb{Z}},\mathbb{R}^n)$ and $E_i^{(ij)^*} \in C((-\infty,0]_{\mathbb{Z}},\mathbb{R}^N)$, define

$$\hat{e}_i(t) = \Upsilon\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup}\mathrm{e}^{-\beta t}\tilde{e}_i,\ t \in (-\infty,\infty)_{\mathbb{Z}}, \tag{3.3}$$

$$\hat{E}_i(t) = \Upsilon\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup}\mathrm{e}^{-\beta t}\tilde{E}_i,\ t \in (-\infty,\infty)_{\mathbb{Z}}, \tag{3.4}$$

where $(e^{(ij)^*},\ E^{(ij)^*})$ serve as the initial functions.

Next, mathematical induction is employed to prove the following expression:

$$|e_i^{(ij)}(t)| \le \hat{e}_i(t),\ |E_i^{(ij)}(t)| \le \hat{E}_i(t),\ t \in (-\infty,\infty)_{\mathbb{Z}}. \tag{3.5}$$

Evidently, based on the definition of $\|\cdot\|_{\sup}$ and the selection of $\Upsilon$, it follows that:

$$|e_i^{(ij)}(q)| \le \hat{e}_i(q),\ |E_i^{(ij)}(q)| \le \hat{E}_i(q),\ q \in (-\infty,0]_{\mathbb{Z}}.$$

Suppose that when $t \le q$, $q \ge 0$, Inequality (3.5) holds. When $t = q+1$, using (2.4a), (2.5), (2.7), and Assumption 2, we get

$$\begin{aligned}
|e_{i\ell}^{(ij)}(q+1)| \le\ & \sum_{k=1}^{n}|r_{\ell k}||e_{ik}^{(ij)}(q)| + \sum_{k=1}^{n}|c_{\ell k}||f_k^*(e_{ik}^{(ij)}(q))| + \sum_{k=1}^{n}|d_{\ell k}||g_k^*(e_{ik}^{(ij)}(q-\tau_i(q))|\\
& + \sum_{m=1}^{+\infty}\mu_m|w_\ell^*(e_{i\ell}^{(ij)}(q-m))| + \sigma\sum_{\jmath=1}^{N}|\varphi_{\ell\jmath}(x_{\ell\jmath}(q))||E_{i\jmath}^{(ij)}(q)|\\
\le\ & \sum_{k=1}^{n}|r_{\ell k}||e_{ik}^{(ij)}(q)| + \sum_{k=1}^{n}|c_{\ell k}|\beta_k^f|e_{ik}^{(ij)}(q))| + \sum_{k=1}^{n}|d_{\ell k}|\beta_k^g|e_{ik}^{(ij)}(q-\tau_i(q))|\\
& + \sum_{m=1}^{+\infty}\mu_m\beta_\ell^w|e_{i\ell}^{(ij)}(q-m)| + \sigma\sum_{\jmath=1}^{N}\hat{\varphi}_{\ell\jmath}|E_{i\jmath}^{(ij)}(q)|,\\
& i \in [1,N]_{\mathbb{Z}}, \ell \in [1,n]_{\mathbb{Z}}.
\end{aligned}$$

By applying the inductive hypothesis, we can derive

$$\begin{aligned}
|e_{i\ell}^{(ij)}(q+1)| \le\ & \sum_{k=1}^{n}|r_{\ell k}||\hat{e}_{ik}(q)| + \sum_{k=1}^{n}|c_{\ell k}|\beta_k^f|\hat{e}_{ik}(q))| + \sum_{k=1}^{n}|d_{\ell k}|\beta_k^g|\hat{e}_{ik}(q-\tau_i(q))|,\\
& + \sum_{m=1}^{+\infty}\mu_m\beta_\ell^w|\hat{e}_{i\ell}(q-m)| + \sigma\sum_{\jmath=1}^{N}\hat{\varphi}_{\ell\jmath}|\hat{E}_{i\jmath}(q)|,\\
& i \in [1,N]_{\mathbb{Z}}, \ell \in [1,n]_{\mathbb{Z}}.
\end{aligned} \tag{3.6}$$

By substituting Eqs (3.3) and (3.4) into (3.6), we obtain

$$
\begin{aligned}
|e_{i\ell}^{(ij)}(q+1)| \leq\ & \sum_{k=1}^{n} |r_{\ell k}| \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta q} \tilde{e}_{ik} \\
& + \sum_{k=1}^{n} |c_{\ell k}| \beta_k^f \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta q} \tilde{e}_{ik} \\
& + \sum_{k=1}^{n} |d_{\ell k}| \beta_k^g \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta (q-\hat{\tau}_i)} \tilde{e}_{ik} \\
& + \sum_{m=1}^{+\infty} \mu_m \beta_\ell^w \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta (q-m)} \tilde{e}_{i\ell} \\
& + \sigma \sum_{\jmath=1}^{N} \hat{\varphi}_{\ell \jmath} \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta q} \tilde{E}_{i\jmath} \\
=\ & \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta q} \\
& \times \left\{ \sum_{k=1}^{n} |r_{\ell k}| \tilde{e}_{ik} + \sum_{k=1}^{n} |c_{\ell k}| \beta_\ell^f \tilde{e}_{i\ell} + \sum_{k=1}^{n} |d_{\ell k}| \beta_k^g \mathrm{e}^{\beta \hat{\tau}_i} \tilde{e}_{ik} \right. \\
& \left. + \sum_{m=1}^{+\infty} \mu_m \beta_\ell^w \mathrm{e}^{\beta m} \tilde{e}_{i\ell} + \sigma \sum_{\jmath=1}^{N} \hat{\varphi}_{\ell \jmath} \tilde{E}_{i\jmath} \right\} \\
=\ & \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta q} \\
& \times \left\{ \sum_{k=1}^{n} \left[ |r_{\ell k}| + |c_{\ell k}| \beta_k^f + |d_{\ell k}| \beta_k^g \mathrm{e}^{\beta \hat{\tau}_i} \right] \tilde{e}_{ik} \right. \\
& \left. + \sum_{m=1}^{+\infty} \mu_m \beta_\ell^{(3)} \mathrm{e}^{\beta m} \tilde{e}_{i\ell} + \sigma \sum_{\jmath=1}^{N} \hat{\varphi}_{\ell \jmath} \tilde{E}_{i\jmath} \right\}, \\
& i \in [1, N]_{\mathbb{Z}}, \ell \in [1, n]_{\mathbb{Z}}.
\end{aligned} \tag{3.7}
$$

Since $i \in [1, N]_{\mathbb{Z}}$ and $\ell \in [1, n]_{\mathbb{Z}}$ are arbitrary, it follows that (3.7) is equivalent to

$$
\begin{aligned}
|e_i^{(ij)}(q+1)| \leq\ & \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta q} \\
& \times \left[ \left( |C| \Pi_1 + \mathrm{e}^{\beta \hat{\tau}_i} |D| \Pi_2 + \alpha \Pi_3 + |R| \right) \tilde{e}_i + \Phi \tilde{E}_i \right].
\end{aligned}
$$

By making use of (3.1) and (3.3), we obtain

$$
|e_i^{(ij)}(q+1)| \leq \Upsilon \|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup} \mathrm{e}^{-\beta (q+1)} \tilde{e}_i = \hat{e}_i(q+1).
$$

Similarly, using (2.4b) and Assumption 2, we get

$$\begin{aligned}|E_{i_J}^{(ij)}(q+1)| &\le \sum_{l=1}^{N}|b_{Jl}||E_{il}^{(ij)}(q)| + \sum_{k=1}^{n}|h_{Jk}(x(q))||e_{ik}^{(ij)}(q)| \\ &\le \sum_{l=1}^{N}|b_{Jl}||\hat{E}_{il}^{(ij)}(q)| + \sum_{k=1}^{n}\hat{h}_{Jk}|\hat{e}_{ik}^{(ij)}(q)| \\ &= \sum_{l=1}^{N}|b_{Jl}|\Upsilon\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup}\mathrm{e}^{-\beta q}\tilde{E}_{i\ell} \\ &\quad + \sum_{k=1}^{n}\hat{h}_{Jk}\Upsilon\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup}\mathrm{e}^{-\beta q}\tilde{e}_{ik} \\ &= \Upsilon\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup}\mathrm{e}^{-\beta q} \\ &\quad \times\left\{\sum_{l=1}^{N}|b_{Jl}|\tilde{E}_{i\ell} + \sum_{k=1}^{n}\hat{h}_{Jk}\tilde{e}_{ik}\right\}.\end{aligned}$$

Using (3.2) and (3.4), it is easy to obtain

$$|E_i^{(ij)}(q+1)| \le \Upsilon\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup}\mathrm{e}^{-\beta(q+1)}\tilde{E}_i = \hat{E}_i(q+1).$$

Therefore, (3.5) is true.

Then, in combination with Eqs (3.3)–(3.5), we get

$$\begin{aligned}\|(e_i^{(ij)}(t), E_i^{(ij)}(t))\| &= (\|e_i^{(ij)}(t)\|_2^2 + \|E_i^{(ij)}(t)\|_2^2)^{\frac{1}{2}} \\ &\le (\|\hat{e}_i(t)\|_2^2 + \|\hat{E}_i(t)\|_2^2)^{\frac{1}{2}} \\ &= \Upsilon\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\|_{\sup}\mathrm{e}^{-\beta t}(\|\tilde{e}_i\|_2^2 + \|\tilde{E}_i\|_2^2)^{\frac{1}{2}},\ \forall t \in [0,\infty)_{\mathbb{Z}}.\end{aligned}$$

Let $\mu = \Upsilon \max_{i\in[1,N]_{\mathbb{Z}}}(\|\tilde{e}_i\|_2^2 + \|\tilde{E}_i\|_2^2)^{\frac{1}{2}})$. Then,

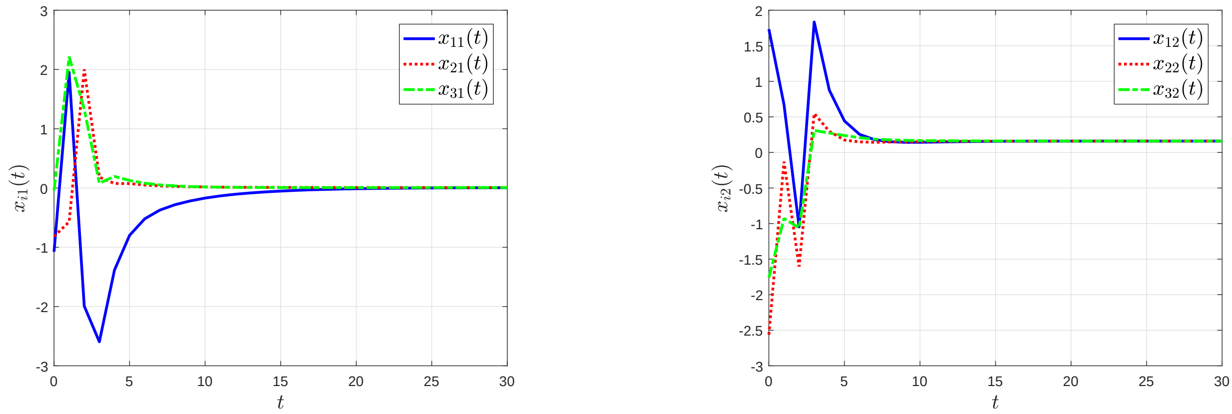
$$\left\|(e_i^{(ij)}(t), E_i^{(ij)}(t))\right\| \le \mu\mathrm{e}^{-\beta t}\left\|(e_i^{(ij)^*},\ E_i^{(ij)^*})\right\|_{\sup}, \forall t \in [0,\infty)_{\mathbb{Z}},$$

The arbitrariness of $e_i^{(ij)^*} \in C((-\infty,0]_{\mathbb{Z}}, \mathbb{R}^n)$ and $E_i^{(ij)^*} \in C((-\infty,0]_{\mathbb{Z}}, \mathbb{R}^N)$ guarantees LES of the error system (2.4), that is, the DTCDN (2.1) achieves LES via the controllers in (2.7).

## 4. Simulation example

Next, a specific numerical example is provided to demonstrate the effectiveness of the proposed results.

**Example 1.** *For $n = 2$ and $N = 3$, consider the DTCDNs with NCDS* (2.1a) *and LCDS* (2.1b) *with the controllers in* (2.7)*, and the parameters are as below:*




**(a)** Curves of node states $x_{i1}(t)$, $i = 1, 2, 3$.

**(b)** Curves of node states $x_{i2}(t)$, $i = 1, 2, 3$.

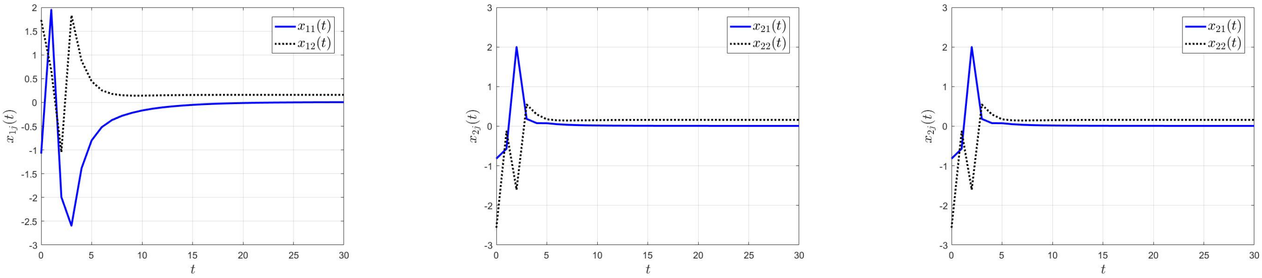
**Figure 2.** Curves of node states $x_{il}(t)$,$i = 1, 2, 3$, $l = 1, 2$.

$$
\begin{gathered}
a_{11} = 0.2, a_{12} = 0.1, a_{21} = 0.5, a_{22} = 0.5, \\
b_{11} = -1.6, b_{12} = 0.1, b_{13} = -4.1, \\
b_{21} = -1.6, b_{22} = 0.1, b_{23} = -4.1, \\
b_{31} = -1.6, b_{32} = 0.1, b_{33} = -4.1, \\
c_{11} = -1.4, c_{12} = 0.1, c_{21} = 0.2, c_{22} = -2.3, \\
d_{11} = -3, d_{12} = -2.5, d_{21} = 0.3, d_{22} = -1.2, \\
f_1(s) = 0.5\mathrm{sin(s)},\ f_2(s) = 0.4\mathrm{tanh(s)},
\end{gathered}
$$

$$
\begin{gathered}
g_1(s) = 0.3\mathrm{tanh(s)},\ g_2(s) = 0.25\mathrm{sin(s)}, \\
w_1(s) = 0.1\mathrm{sin(s)},\ w_2(s) = 0.08\mathrm{cos(s)}, \\
\varphi_1(s) = 0.12\mathrm{sin(s+1)},\ \varphi_2(s) = 0.1\mathrm{cos(s+1)}, s \in \mathbb{R}. \\
h_{11}(x_1) = 0.1\mathrm{sin(x_{11})} + 0.05\mathrm{cos(x_{12})},\ h_{12}(x_1) = 0.08\mathrm{tanh(x_{11})} + 0.03\mathrm{sin(x_{12})}, \\
h_{21}(x_2) = 0.09\mathrm{cos(x_{21})} + 0.04\mathrm{sin(x_{22})},\ h_{22}(x_2) = 0.07\mathrm{sin(x_{21})} + 0.06\mathrm{tanh(x_{22})}, \\
h_{31}(x_3) = 0.08\mathrm{tanh(x_{31})} + 0.05\mathrm{cos(x_{32})},\ h_{32}(x_3) = 0.1\mathrm{cos(x_{31})} + 0.04\mathrm{sin(x_{32})}.
\end{gathered}
$$

*Set* $\beta_1^f = 0.5, \beta_2^f = 0.4$, $\beta_1^g = 0.3, \beta_2^g = 0.25$, $\beta_1^w = 0.1, \beta_2^w = 0.08$, $\hat{\varphi}_{11} = \hat{\varphi}_{21} = \hat{\varphi}_{31} = 0.12$, $\hat{\varphi}_{12} = \hat{\varphi}_{22} = \hat{\varphi}_{32} = 0.1$, $\hat{h}_{11} = 0.15, \hat{h}_{12} = 0.11, \hat{h}_{21} = 0.13, \hat{h}_{22} = 0.13, \hat{h}_{31} = 0.13, \hat{h}_{32} = 0.14$. *Then, Assumptions 1 and 2 are satisfied.*

*Let* $\sigma = 0.2$, $r_{11} = 0.5$, $r_{12} = 0$, $r_{21} = 0$, *and* $r_{22} = 0.45$. *Clearly, the conditions required by Theorem 1 are satisfied, as can be readily checked.*

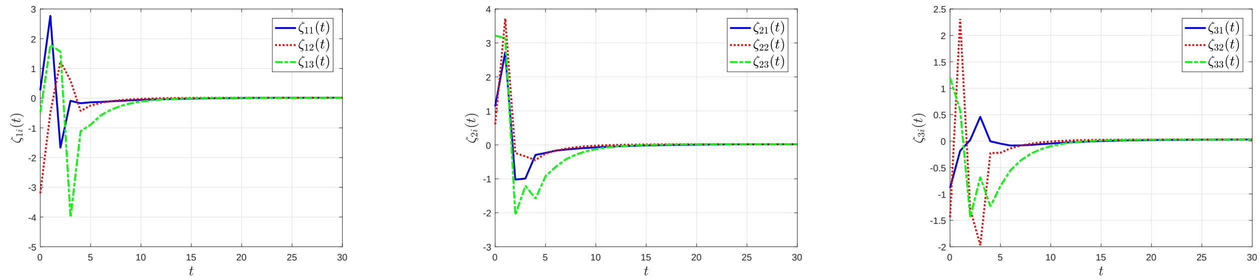



**(a)** Curves of node states $x_{1j}(t)$, $j = 1, 2$.

**(b)** Curves of node states $x_{2j}(t)$, $j = 1, 2$.

**(c)** Curves of node states $x_{3j}(t)$, $j = 1, 2$.

**Figure 3.** Curves of node states $x_{ij}(t)$, $i = 1, 2, 3,\ j = 1, 2$.

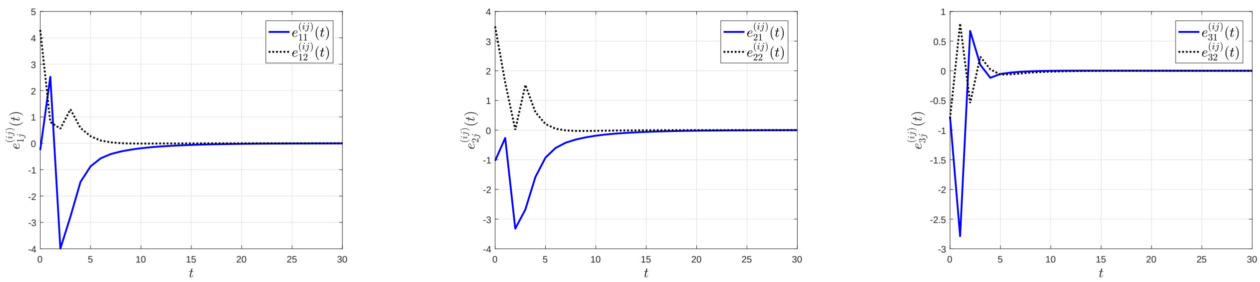



**(a)** Curves of link states $\zeta_{1i}(t)$, $i = 1, 2, 3$.

**(b)** Curves of link states $\zeta_{2i}(t)$, $i = 1, 2, 3$.

**(c)** Curves of link states $\zeta_{3i}(t)$, $i = 1, 2, 3$.

**Figure 4.** Curves of link states $\zeta_{ki}(t)$, $k = 1, 2, 3$, $i = 1, 2, 3$.

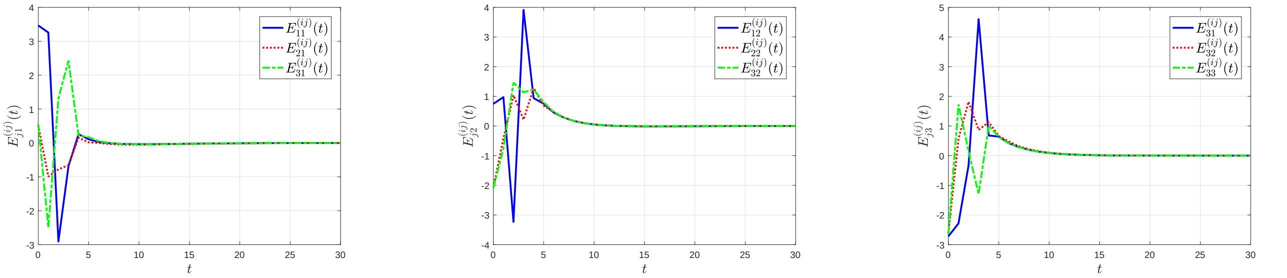



**(a)** The node error curves of $e_{1j}^{(ij)}(t)$, $j = 1, 2$.

**(b)** The node error curves of $e_{2j}^{(ij)}(t)$, $j = 1, 2$.

**(c)** The node error curves of $e_{3j}^{(ij)}(t)$, $j = 1, 2$.

**Figure 5.** The node error curves of $e_{kj}^{(ij)}(t)$, $k = 1, 2, 3,\ j = 1, 2$.




**(a)** The link error curves of $E_{j1}^{(ij)}(t)$, $j = 1, 2, 3$.

**(b)** The link error curves of $E_{j2}^{(ij)}(t)$, $j = 1, 2, 3$.

**(c)** The link error curves of $E_{j3}^{(ij)}(t)$, $j = 1, 2, 3$.

**Figure 6.** Combined figure: Link error curves $E_{jk}^{(ij)}(t)$, $j = 1, 2, 3$, $k = 1, 2, 3$.

*We set $\beta = 0.01$ and solve Inequalities (3.1) and (3.2) in Theorem 1 using MATLAB R2016b with the toolbox* YALMIP *software yields $\tilde{e}_i$ and $\tilde{E}_i$:*

$$\tilde{e}_i = [1.8466, 1.000]^T, \ \tilde{E}_i = [3.0523, 3.3531, 3.0442]^T.$$

*The simulation outcomes are displayed in Figures 2(a)–6(c).*

*(1) The initial states of the NCDS* (2.1a) *and LCDS* (2.1b) *are chosen as $x_1(s) = [-2.5949, 1.8348]^T$, $x_2(s) = [0.1836, 0.5476]^T$, $x_3(s) = [0.0818, 0.3101]^T$, $\zeta_1(s) = [-0.0893, -0.9915, 0.4589]^T$, $\zeta_2(s) = [0.5975, -0.3372, -1.9720]^T$, $\zeta_3(s) = [-4.0034, -1.2040, -0.6765]^T$, $s \in (-\infty, 0]_{\mathbb{Z}}$, Figures 2 and 3 show the state response curves of nodes, and Figure 4 presents the state response curves of the link subsystems.*

*(2) Figure 5 confirms that both node errors $e_{ik}^{(ij)}$ decay exponentially to zero.*

*(3) Figure 6 confirms that both link errors $E_{ik}^{(ij)}$ decay exponentially to zero.*

*Thus, it is apparent that the error states converge to zero rapidly. This implies, via Theorem 1, that LES is achieved for both the NCDS* (2.1a) *and LCDS* (2.1b) *under the action of controllers* (2.7).

## 5. Conclusions

This study investigates the LES problem for DTCDN that incorporate discrete delays, infinite distributed delays, and link structures. By directly applying the definition of LES, delay-dependent sufficient conditions are first established for the error dynamical system. Then, the corresponding controller gain is explicitly derived. Finally, a numerical example is provided to validate the theoretical findings.

In comparison with existing results in the literature, the proposed approach offers several distinct advantages:

(1) It relies directly on the LES definition, thereby eliminating the need to construct any Lyapunov–Krasovskii functional.

(2) The resulting sufficient conditions are expressed as linear scalar inequalities, which are straightforward to solve.

(3) With slight modifications, the proposed method can be extended to more general classes of DTCDNs. For example, general decay synchronization includes special cases such as polynomial synchronization, logarithmic synchronization, and other types of synchronization. We will consider this issue in our future research.

## Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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## Conflict of interest

The authors declare there is no conflict of interest.

## References

1. G. Chen, Z. Guo, Observer-based distributed control and synchronization analysis of inverter-based nonlinear power systems, *Nonlinear Dyn.*, **99** (2020), 2161–2183. https://doi.org/10.1007/s11071-019-05422-7
2. J. Schiffer, E. Fridman, R. Ortega, J. Raisch, Stability of a class of delayed port-Hamiltonian systems with application to microgrids with distributed rotational and electronic generation, *Automatica*, **74** (2016), 71–79. https://doi.org/10.1016/j.automatica.2016.07.029
3. P. Zhu, L. Cheng, C. Gao, Z. Wang, X. Li, Locating multi-sources in social networks with a low infection rate, *IEEE Trans. Network Sci. Eng.*, **9** (2022), 1853–1865. https://doi.org/10.1109/TNSE.2021.3137235
4. Z. Lv, S. Zhang, W. Xiu, Solving the security problem of intelligent transportation system with deep learning, *IEEE Trans. Intell. Transp. Syst.*, **22** (2021), 4281–4290. https://doi.org/10.1109/TITS.2020.3002185
5. D. J. Watts, S. H. Strogatz, Collective dynamics of ‘small-world’ networks, *Nature*, **393** (1998), 440–442. https://doi.org/10.1038/30918
6. S. H. Strogatz, Exploring complex networks, *Nature*, **410** (2001), 268–276. https://doi.org/10.1038/35065725
7. S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, D. U. Hwang, Complex networks: Structure and dynamics, *Phys. Rep.*, **424** (2006), 175–308. https://doi.org/10.1016/j.physrep.2005.10.009
8. J. Zhao, Y. Wang, P. Gao, Y. Peng, S. Li, Asymptotical tracking control for the complex network based on the dynamic topology, *ISA Trans.*, **148** (2024), 105–113. https://doi.org/10.1016/j.isatra.2024.02.025

9. L. Liu, D. Wang, Z. Peng, C. L. P. Chen, T. Li, Bounded neural network control for target tracking of underactuated autonomous surface vehicles in the presence of uncertain target dynamics, *IEEE Trans. Neural Networks Learn. Syst.*, **30** (2019), 1241–1249. https://doi.org/10.1109/TNNLS.2018.2868978

10. J. Veit, R. Hakim, M. P. Jadi, T. J. Sejnowski, H. Adesnik, Cortical gamma band synchronization through somatostatin interneurons, *Nat. Neurosci.*, **20** (2017), 951–959. https://doi.org/10.1038/nn.4562

11. M. Bartos, I. Vida, P. Jonas, Synaptic mechanisms of synchronized gamma oscillations in inhibitory interneuron networks, *Nat. Rev. Neurosci.*, **8** (2007), 45–56. https://doi.org/10.1038/nrn2044

12. P. R. Pagilla, N. B. Siraskar, R. V. Dwivedula, Decentralized control of web processing lines, *Trans. Control Syst. Technol.*, **15** (2006), 106–117. https://doi.org/10.1109/TCST.2006.883345

13. N. R. Abjadi, J. Soltani, J. Askari, G. R. A. Markadeh, Nonlinear sliding-mode control of a multi-motor web-winding system without tension sensor, *Control Theory Appl.*, **3** (2009), 419–427. https://doi.org/10.1049/iet-cta.2008.0118

14. H. Fort, M. Scheffer, E. Van Nes, The clumping transition in niche competition: A robust critical phenomenon, *J. Stat. Mech: Theory Exp.*, **2010** (2010), P05005. https://doi.org/10.1088/1742-5468/2010/05/P05005

15. E. A. Botts, B. F. N. Erasmus, G. J. Alexander, J. Lawlor, Small range size and narrow niche breadth predict range contractions in south african frogs, *Global Ecol. Biogeogr.*, **22** (2013), 567–576. https://doi.org/10.1111/geb.12027

16. Q. Li, B. Shen, Z. Wang, T. Huang, J. Luo, Synchronization control for a class of discrete time-delay complex dynamical networks: A dynamic event-triggered approach, *Trans. Cybern.*, **49** (2019), 1979–1986. https://doi.org/10.1109/TCYB.2018.2818941

17. Q. Cheng, J. Cao, Synchronization of complex dynamical networks with discrete time delays on time scales, *Neurocomputing*, **151** (2015), 729–736. https://doi.org/10.1016/j.neucom.2014.10.033

18. L. Li, J. Kurths, H. Peng, Y. Yang, Q. Luo, Exponentially asymptotic synchronization of uncertain complex time-delay dynamical networks, *Eur. Phys. J. B*, **86** (2013), 125. https://doi.org/10.1140/epjb/e2013-30517-6

19. B. Li, Finite-time synchronization for complex dynamical networks with hybrid coupling and time-varying delay, *Nonlinear Dyn.*, **76** (2014), 1603–1610. https://doi.org/10.1007/s11071-013-1232-x

20. Y. Liu, Z. Wang, J. Liang, X. Liu, Synchronization and state estimation for discrete-time complex networks with distributed delays, *IEEE Trans. Syst. Man Cybern. Part B Cybern.*, **38** (2008), 1314–1325. https://doi.org/10.1109/TSMCB.2008.925745

21. Z. Li, J. A. Fang, W. Zhang, X. Wang, Delayed impulsive synchronization of discrete-time complex networks with distributed delays, *Nonlinear Dyn.*, **82** (2015), 2081–2096. https://doi.org/10.1007/s11071-015-2309-5

22. A. Kazemy, J. Cao, Consecutive synchronization of a delayed complex dynamical network via distributed adaptive control approach, *Int. J. Control Autom. Syst.*, **16** (2018), 2656–2664. https://doi.org/10.1007/s12555-017-0718-6

23. L. Zhang, Y. Wang, Y. Huang, X. Chen, Delay-dependent synchronization for non-diffusively coupled time-varying complex dynamical networks, *Appl. Math. Comput.*, **259** (2015), 510–522. https://doi.org/10.1016/j.amc.2014.12.034

24. X. Ruan, J. Feng, C. Xu, J. Wang, Y. Zhao, Dynamic event-triggered pinning synchronization for switched impulsive complex networks with asynchronous switching, *IEEE Trans. Circuits Syst. II Express Briefs*, **69** (2022), 2211–2215. https://doi.org/10.1109/TCSII.2021.3123285

25. J. Wang, T. Ru, J. Xia, Y. Wei, Z. Wang, Finite-time synchronization for complex dynamic networks with semi-Markov switching topologies: An H event-triggered control scheme, *Appl. Math. Comput.*, **356** (2019), 235–251. https://doi.org/10.1016/j.amc.2019.03.037

26. A. Kazemy, K. Shojaei, Synchronization of complex dynamical networks with dynamical behavior links, *Asian J. Control*, **22** (2020), 474–485. https://doi.org/10.1002/asjc.1910

27. A. Kazemy, K. Shojaei, Adaptive synchronization of complex dynamical networks in presence of coupling connections with dynamical behavior, *J. Comput. Nonlinear Dyn.*, **14** (2019), 061003. https://doi.org/10.1115/1.4043146

28. P. Zhao, Y. Wang, Asymptotical stability for complex dynamical networks via link dynamics, *Math. Methods Appl. Sci.*, **43** (2020), 8706–8713. https://doi.org/10.1002/mma.6533

29. L. Liu, Y. Wang, Z. Gao, Tracking control for the dynamic links of discrete-time complex dynamical network via state observer, *Appl. Math. Comput.*, **369** (2020), 124857. https://doi.org/10.1016/j.amc.2019.124857

30. P. Gao, Y. Wang, L. Liu, L. Zhang, X. Tang, Asymptotical state synchronization for the controlled directed complex dynamic network via links dynamics, *Neurocomputing*, **448** (2021), 60–66. https://doi.org/10.1016/j.neucom.2021.03.095

31. Z. Gao, L. Liu, Y. Wang, P. Gao, Y. Li, Stabilization and synchronization control for complex dynamical networks with dynamic link subsystem, *Inf. Sci.*, **609** (2022), 1588–1600. https://doi.org/10.1016/j.ins.2022.07.153

32. P. Gao, Y. Wang, J. Zhao, L. Zhang, Y. Peng, Links synchronization control for the complex dynamical network, *Neurocomputing*, **515** (2023), 59–67. https://doi.org/10.1016/j.neucom.2022.10.024

33. Y. Wang, W. Wang, L. Zhang, State synchronization of controlled nodes via the dynamics of links for complex dynamical networks, *Neurocomputing*, **384** (2020), 225–230. https://doi.org/10.1016/j.neucom.2019.12.055

34. Q. Chen, Y. Peng, Y. Wang, H. Li, Cluster synchronization of delayed complex dynamical networks with the intracluster and intercluster link dynamics, *Math. Methods Appl. Sci.*, **48** (2025), 13542–13551. https://doi.org/10.1002/mma.11121

35. Q. Chen, Y. Wang, X. Tang, S. Li, Asymptotical outer synchronization control of the complex dynamical networks via unknown interaction and links dynamics, *Math. Methods Appl. Sci.*, **46** (2023), 4379–4390. https://doi.org/10.1002/mma.8765

36. J. Zhao, Y. Wang, P. Gao, S. Li, Y. Peng, Synchronization of complex dynamical networks with stochastic links dynamics, *Entropy*, **25** (2023), 1457. https://doi.org/10.3390/e25101457

37. L. Meng, D. Yang, Q. Zhao, Synchronization of uncertain complex dynamical networks, *Int. J. Software Eng. Knowl. Eng.*, **29** (2019), 923–939. https://doi.org/10.1142/S0218194019400060

38. L. Zhou, Y. Ma, J. Zou, Robust synchronization of Markovian jump complex dynamical networks under link attacks: Delay-dependent dynamic event-triggered control approach, *J. Franklin Inst.*, **361** (2024), 107251. https://doi.org/10.1016/j.jfranklin.2024.107251

39. M. Liu, J. Liu, J. Liang, Y. Sun, Y. Shu, Observer-based secure synchronization control of directed complex-valued dynamical networks under link attacks, *Nonlinear Dyn.*, **112** (2024), 12303–12318. https://doi.org/10.1007/s11071-024-09695-5

40. L. Liu, Y. Wang, H. Chen, Z. Gao, Synchronization control for discrete-time complex dynamical networks with dynamic links subsystem, *Mod. Phys. Lett. B*, **34** (2020), 2050352. https://doi.org/10.1142/S0217984920503522

41. L. Liu, Y. Wang, Z. Gao, Tracking control for the connection relationships of discrete-time complex dynamical network associated with the controlled nodes, *Int. J. Control Autom. Syst.*, **17** (2019), 2252–2260. https://doi.org/10.1007/s12555-018-0928-6

42. L. Liu, Y. Wang, X. Li, Z. Gao, Structural balance for discrete-time complex dynamical network associated with the controlled nodes, *Mod. Phys. Lett. B*, **34** (2020), 2050098. https://doi.org/10.1142/S0217984920500980

43. Z. Liang, S. Ding, Y. Jing, X. Xie, Dual-event-driven synchronization control for discrete-time complex dynamical networks, *IEEE Trans. Circuits Syst. I-regular papers*, **71** (2024), 851–861. https://doi.org/10.1109/TCSI.2023.3341150

44. Z. Ma, K. Teo, L. Wang, C. Wu, Q. Li, Synchronization of discrete-time complex dynamical networks with delays via impulsive distributed control, *Nonlinear Dyn.*, **114** (2026), 406. https://doi.org/10.1007/s11071-026-12278-1

45. B. Li, P. Li, Q. Chen, L. Zhong, Y. Wang, Tracking control for discrete-time complex dynamic network via link dynamics, *Int. J. Dyn. Control*, **13** (2025), 278. https://doi.org/10.1007/s40435-025-01771-x