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Research article

Modeling and stability analysis of a memory-driven fractional labor dynamics system

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Abstract: This paper investigates the existence and stability of a fractional-order labor dynamics model formulated using the fractional Nabla difference operator, specifically the Atangana-Baleanu fractional derivative in the Caputo sense. The model incorporates fractional dynamics to capture memory effects and the complex interactions associated with workforce layoffs. First, we present the mathematical formulation of the system and discuss its relevance to labor dynamics. Using the fixed-point theory, we establish the existence and uniqueness of solutions, demonstrating that the system is well posed. Furthermore, we examine the stability of the model in the Mittag-Leffler-Hyers-Ulam sense, providing insight into its long-term qualitative behavior. Numerical simulations support the theoretical findings and demonstrate that the fractional-order parameter significantly influences the system's dynamics. Overall, this study offers a more general and flexible framework for modeling layoff processes using fractional calculus.

Keywords: fractional-order model; Atangana-Baleanu derivative; fractional Nabla difference operator; Mittag-Leffler-Hyers-Ulam stability

1. Introduction

Modern studies of dynamic processes in biology, ecology, epidemiology, and social systems are based on mathematical models of interacting populations. Classical literature by Murray [1] has made compartmental population dynamics a strong tools to explain the movement of people or objects between states with time. These models are usually represented by nonlinear differential or difference equations that represent growth, decay, feedback processes, and the system behavior on the long-term scale. Such dynamical systems, though originally formulated in infectious diseases, have found extensive use in any other situation that involve population movements between discrete compartments. Based on these classical mathematical models, the workforce transition can be naturally seen through the prism of population dynamics. Labor teams such as active workers, those who have been laid off, temporary removals, and permanent separations are interacting sub-populations whose dynamics can be explained by the dynamical systems theory. This viewpoint has placed the investigation of layoffs in the same group of established methodologies in applied mathematics as opposed to solely relying on the modern socio-economic driving forces. The formulation puts the model on a level in which the expectations of mathematical biosciences and engineering journals are met, which require structural modeling and analytical rigor.

Within this broader context, current economic conditions highlight the importance of modeling such population transitions. The recent wave of tech layoffs has raised concerns about the stability of an industry traditionally viewed as a driver of economic growth and innovation. Major technology companies, including both startups and global leader's have significantly reduced their workforces due to factors such as economic uncertainty, over-expansion during periods of rapid growth, shifts in market demand, increased automation, and the integration of artificial intelligence into roles previously handled by humans. These layoffs affect not only displaced employees but also influence labor markets, investor confidence, organizational stability, and long-term technological progress. Understanding the causes, implications, and potential long-term effects of such layoffs is essential to assess the future trajectory of the sector and designing strategies to mitigate negative impacts. The dynamics of workforce layoffs play a crucial role in shaping labor markets [2], organizational stability [3], and economic resilience [4]. The layoff model had provided information regarding the number of tech layoffs by country in 2024, as shown in Table 1, and the top 10 companies in terms of the number of tech layoffs in 2025 are given in Table 2. It is a highly critical and globally significant issue that demands immediate attention, as its far-reaching consequences profoundly affect the social stability, economic development, environmental sustainability, and the overall well-being of humanity. This critical situation has encouraged many researchers to study how layoffs occur and to understand their long-term effects. It needs a mathematical model that portrays the dynamics involved between employees and the capacity of the organization. Due to various reasons, many researchers observed layoffs [5, 6].

Mathematical modeling uses equations and algorithms to represent real-world systems, thereby helping to analyze and predict their behavior. Models can be deterministic or stochastic, linear or nonlinear, and solved using analytical or numerical methods. They are widely used in science [7], and economics to optimize processes and support decision-making [8]. The study of workforce dynamics and layoffs has been a critical area of research in labor economics, organizational management, and applied mathematics. Traditional models that analyzed employment fluctuations and layoffs have relied on integer-order differential equations, which assume instantaneous workforce transitions. However, these models often fail to capture long-term dependencies and memory effects that significantly influence the labor market behavior. To address these limitations, researchers have turned to fractional order differential equations, particularly the Nabla fractional derivative of Atangana-Baleanu in Caputo

(ABC), which provides a more accurate framework to model complex workforce dynamics [9, 10].

Fractional-order models have demonstrated their effectiveness in various real-world applications, including epidemiology [11], control systems [12], and the optimization of therapies using artificial intelligence(AI) and mathematical modeling [13]. In workforce modeling, fractional derivatives account for the historical impact of layoffs [14], organizational restructuring [15], and are particularly useful to analyze the unemployment stability [16]. The ABC Nabla fractional derivative, introduced as a generalization of traditional fractional calculus, has been widely recognized for its ability to incorporate non-local memory effects and non-singular kernels. Unlike classical derivatives, the ABC Nabla fractional operator allows for a more flexible representation of labor market transitions, where the impact of past layoffs gradually decreases over time instead of abruptly disappearing.

So far, mathematical modeling has demonstrated to be an effective mechanism to analyze workforce reduction, and even though the study has shown sufficient literature on the utilization of discrete fractional difference equations in modeling tech layoffs with an organizational capacity, there still exists a gap that demonstrates that the information is yet to be completely exhausted. This discrepancy indicates the necessity to conduct additional research to create more predictive models. Such a gap will help us better understand the process behind tech layoffs and assist in formulating more effective strategies to mitigate their impact. Layoffs may arise from a variety of macroeconomic and organizational factors, including financial instability, restructuring, market saturation, and shifts in technological demand. Economic midlife crises in firms or sectors-periods of decreased profitability or structural adjustment-often lead to workforce reductions. Although periods of accelerated digital adoption have influenced technology-driven industries and contributed to notable fluctuations, the present study does not focus on such event-driven dynamics. Instead, our interest lies in the general mathematical structure of workforce transitions, which shares conceptual similarities with classical compartmental models used in infectious-disease dynamics, such as the foundational SIR framework of Kermack and McKendrick [17]. These classical models provide a natural analogy for understanding how populations move between different states. As an example of related socio-economic modeling, Prakash and Sakthivel [18] utilized multiple machine-learning algorithms to analyze and predict layoff patterns, thereby demonstrating how data-driven models can enhance to understand of workforce dynamics and employment risks.The layoff has hit many tech employees, and continued layoffs were witnessed after heavy layoffs in 2022 and 2023 in the technological world [19]. In [20], the author analyzed the gender layoff gap implied by the gender gap in wage bargaining.

Conversely, a stability analysis has emerged as one of the most prominent and actively researched topics in the study of fractional difference systems. In particular, the authors in [21, 22] systematically investigated various stability properties of such systems. Some of the information on the stability of fractional order difference equations was provided in [23–26]. Additionally, we have also observed that certain accomplishments concerning the Mittag-Leffler-Hyers-Ulam (MLHU) stability in fractional difference equations were referred to in [27]. So far as we are aware, few papers have addressed the MLHU and MLHU-Rassias stability in the fractional difference tech layoff model. The Hyer-Ulam stability, after being first posed by Ulam [28] and Hyers [29] in the 1940's, came to be known as the well-known Hyers-Ulam stability problem and popularized by Rassias [30]. Numerous mathematicians thought about the broad scope of this same problem concerning fractional equations of a more diverse nature, (i.e., [31]).

In [32], Debbar et al. investigated the existence and uniqueness of solutions to neutral functional differential equations of sequential fractional orders; however, this time, the Caputo operator was considered. The existence and uniqueness was proven with respect to the Banach fixed point theorem, which is a nonlinear version of the Leray-Schauder fixed point theorem and the Krasnoselski fixed point theorem, respectively, to achieve the desired results. Promsakon et al. [33] applied the Schauder fixed point methodology and discussed the existence outcomes of fractional difference equations. To know more about the fractional Nabla system, see [21, 26].

Despite the growing interest in fractional workforce models, limited studies have explicitly explored the application of ABC Nabla fractional derivatives in the context of tech industry layoffs. In this study, we develop a mathematical layoff model that categorizes workers into distinct groups: susceptible workers (S) who are employed but at risk of being laid off, infectious workers (I) who have been recently laid off and are actively seeking reemployment, temporarily removed workers (TR) who are expected to return to work after a recovery period, and permanently removed workers (PR) who exit the labor market due to job loss, career transitions, or retirement. Additionally, the organizational capacity (C) is introduced as a variable which represents a company's ability to retain and rehire workers based on financial stability and strategic workforce planning. The model is formulated using a system of fractional differential equations to account for the memory effects of layoffs, thus ensuring a more realistic representation of employment trends over time. A stability analysis was examined to determine scenarios under which layoffs stabilize. Numerical simulations further illustrate how economic fluctuations and workforce policies influence employment retention. By providing a rigorous mathematical approach to workforce dynamics, this study contributes to labor economics by offering insight into layoff patterns and guiding strategies to mitigate long-term job displacement.

Table 1. Top 10 Countries with the Highest Number of Tech Layoffs in 2024.

Rank	Country	Estimated Tech Layoffs	Percentage of Global Total	Notable Companies Involved
1	United States	120, 283	~57%	Dell, Intel, Tesla, Cisco, Microsoft
2	China	12, 900	~6.1%	Li Auto
3	Germany	12, 547	~5.9%	SAP
4	Japan	12, 240	~5.8%	Toshiba
5	India	8, 560	~4.1%	Paytm, Flipkart, InMobi
6	United Kingdom	5, 461	~2.6%	Various
7	South Korea	4, 000	~1.9%	Samsung
8	France	3, 500	~1.7%	Various
9	Canada	3, 200	~1.5%	Various
10	Australia	2, 800	~1.3%	Various

Table 2. Top 10 Countries by Tech Layoffs in 2025 (as of April).

Rank	**Country**	**Estimated Layoffs (2025)**	**Notable Companies Involved**	**Notes**
1	United States	~15, 000 +	Meta, Microsoft, Amazon	Continued cost - cutting, AI restructuring
2	Germany	~2, 600	Siemens	Part of 5, 600 global layoffs in Digital Industries division
3	India	~1, 000 +	Ola Electric, Paytm	Restructuring, automation impact
4	United Kingdom	~800 +	Various	Early - year layoffs in fintech and SaaS companies
5	Canada	~700 +	Shopify, other startups	Ongoing market adjustment
6	Australia	~500 +	Atlassian, Canva	Primarily in support and operations roles
7	France	~400 +	Capgemini, other IT services	Shift to AI - based services
8	China	~400 +	ByteDance, Alibaba (internal restructuring)	Domestic tech realignment
9	South Korea	~300 +	Samsung (minimal in 2025 so far)	R&D realignment
10	Japan	~200 +	Toshiba, Sony (minor workforce reshuffles)	Focus shifting toward robotics and automation

The Fractional-order system has also been widely researched in terms of control design and stability analyses. Specifically, the synthesis of the discrete fractional-order system for the Nabla H_∞ controller has been explored to provide robustness against external disturbances and modeling uncertainties. Similar efforts have also introduced H_∞ observer-based controllers for fractional-order systems with finite frequency responses, thereby focusing on performance guarantees in control-based applications. The current study fundamentally contrasts with these studies in terms of goals and approach. Although current workforce methods are mainly based on H_∞, the proposed model emphasizes the qualitative and quantitative analysis of the workforce rather than the controller or observer synthesis of engineered dynamical systems using a fractional Nabla framework. Instead of creating a feedback controller, this paper explores the inherent memory behavior of the system using MLHU stability and numerical simulations. Additionally, the suggested framework is data-driven and policy-driven, as opposed to control-driven. It is considered that the fractional order is a determinant of the organizational memory and long-term capacity retention, and the effects of layoffs can be analyzed without introducing external control inputs. This difference enables the model to deal with socio-economic dynamics that do not normally feature in classical H_∞ control formulations. Therefore, the contribution of this paper lies in extending Nabla's discrete fractional-order theory to workforce modeling and policy analyses, complementing the existing fractional-order system control studies by offering a non-control-based perspective on the memory-dependent system behavior.

The remainder of this paper is structured as follows. Section 2.1 reviews the related literature on workforce reductions and economic modeling; Section 2.2 outlines the methodology, including data sources, variable selection, and model formulation. Section 2.3 presents the empirical results and model validation; Section 2.4 is dedicated to establishing the MLHU stability analysis of the suggested model; Section 3 discusses policy implications and strategic interventions; and finally, Section 4 concludes with the future research directions and potential applications of the proposed model.

2. Mathematical preliminaries and model development

In this section, we provide a comprehensive overview of the essential theoretical foundations related to discrete fractional calculus, along with key principles of a nonlinear functional analysis, which are necessary to understand the subsequent mathematical formulations and analytical techniques employed in this study. The domain of the set exists as a discrete set of values, namely, $\mathbb{N}_\nu = \{\nu, \nu+1, \nu+2, \dots\}$, $\nu \in \mathbb{R}$.

2.1. Preliminaries of discrete fractional calculus

Definition 2.1. *The Gamma function $\Gamma(\xi)$ is a particular function which generalizes the factorial to real numbers and, complex numbers and which is defined, when $\xi > 0$, as follows:*

$$\Gamma(\xi) = \int_0^\infty \chi^{\xi-1} e^{-\chi} d\chi.$$

It satisfies the recurrence relation $\Gamma(\xi + 1) = \xi\Gamma(\xi)$ and generalizes the factorial since $\Gamma(n) = (n-1)!$ for natural numbers n.

Definition 2.2. *In discrete fractional calculus, the backward difference operator is basic, and thus defined on a function $x(\chi)$ as $\nabla x(\chi) = x(\chi) - x(\chi - 1)$; its iterative form for a fractional order ω is given by the following:*

$$\nabla^\omega x(\chi) = \nabla(\nabla^{\omega-1} x(\chi)), \qquad \chi \in \mathbb{N}_1.$$

Definition 2.3. *(i) (Rising function) For a natural number m, the ascending factorial function of t is given by the following:*

$$t^{\overline{m}} = t(t+1)(t+2)\cdots(t+m-1), \qquad t \in \mathbb{R},$$

with the special case $t^{\overline{0}} = 1$.

(ii) The Gamma function may be used to represent the rising factorial with respect to any real number ω as follows:

$$\chi^{\overline{\omega}} = \frac{\Gamma(\chi + \omega)}{\Gamma(\chi)},$$

where $\chi \in \mathbb{R} \setminus \{\dots, -2, -1, 0\}$, with the convention that $0^{\overline{\omega}} = 0$.

Definition 2.4. *The Nabla discrete Mittag - Leffler function for a real parameter $\hat{\mu}$ such that $|\hat{\mu}| < 1$ and complex parameters $\omega, \hat{X}, \hat{p} \in C$ with $\mathbb{R}(\omega) > 0$ is given by the following:*

$$E_{\overline{\omega,\hat{\rho}}}(\hat{\mu}, \hat{X}) = \sum_{\ell=0}^{\infty} \frac{\hat{\mu}^{\ell} \hat{X}^{\overline{\ell\omega+\hat{\rho}-1}}}{\Gamma(\omega\ell + \hat{\rho})}.$$

This function generalizes the classical Mittag-Leffler function within the framework of discrete fractional calculus, thereby incorporating the Nabla operator to describe discrete dynamical systems with memory effects.

Definition 2.5. *Let $\rho(\chi) = \chi - 1$ represent the backward jump operator. For a function $x : \mathbb{N}_\nu \to \mathbb{R}$, the Nabla left fractional sum of order ω is given by the following:*

$$\nabla_\nu^{-\omega} x(\chi) = \frac{1}{\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\nu-1}} x(\xi), \quad \chi \in \mathbb{N}_{\nu+1}.$$

This fractional summation operator generalizes discrete integration to fractional orders, incorporating memory effects and long-range dependencies in discrete-time models.

Definition 2.6. *Let $\omega \in (0, 1)$, and let $x : \mathbb{N}_\nu \to \mathbb{R}$. The ABC Nabla discrete fractional difference of order ω is defined by the following:*

$$\begin{aligned} {}^{ABC}_{\nu}\nabla^{\omega} x(\chi) &= \mathcal{M}(\omega) \frac{1}{1-\omega} \sum_{\xi=\nu+1}^{\chi} \nabla_\xi x(\xi)\, E_{\overline{\omega}}\Big(\frac{-\omega}{1-\omega}, \chi - \rho(\xi)\Big) \\ &= \mathcal{M}(\omega) \frac{1}{1-\omega} \Big[\nabla x(\chi) * E_{\overline{\omega}}\Big(\frac{-\omega}{1-\omega}, \chi\Big)\Big], \qquad \chi \in \mathbb{N}_{\nu+1}, \quad \omega \in \Big(0, \tfrac{1}{2}\Big). \end{aligned}$$

This operator extends the classical discrete fractional difference by incorporating a Mittag-Leffler kernel, thereby capturing nonlocal memory effects in discrete dynamical systems. Atangana-Baleanu Nabla's discrete new (Riemann-Liouville) fractional difference formula is as follows:

$$\begin{aligned} {}^{ABC}_{\nu}\nabla^{\omega} x(\chi) &= \mathcal{M}(\omega) \frac{1}{1-\omega} \nabla_\xi \sum_{\xi=\nu+1}^{\chi} x(\xi)\, E_{\overline{\omega}}\Big(\frac{-\omega}{1-\omega}, \chi - \rho(\xi)\Big) \\ &= \mathcal{M}(\omega) \frac{1}{1-\omega} \nabla_\xi \Big[\nabla x(\chi) * E_{\overline{\omega}}\Big(\frac{-\omega}{1-\omega}, \chi\Big)\Big], \qquad \omega \in \Big(0, \tfrac{1}{2}\Big), \end{aligned}$$

where $\mathcal{M}(\omega)$ is a normalization constant that satisfies $\mathcal{M}(0) = \mathcal{M}(1) = 1$.

Definition 2.7. *The fractional sum related to the Atangana and Baleanu Nabla fractional difference discrete Riemann-Liouville is denoted by the following:*

$${}^{AB}_{\nu}\nabla^{-\omega} x(\chi) = \frac{1-\omega}{\mathcal{M}(\omega)} x(\chi) + \frac{\omega}{\mathcal{M}(\omega)} \nabla_\nu^{-\omega} x(\chi), \qquad 0 < \omega < 1.$$

2.2. Mathematical model formation

Layoffs are a significant workforce challenge that affects both employees and organizations. Traditional layoff models use integer-order differential equations but fail to capture memory effects and long-term dependencies. We can overcome this shortcoming by introducing the fractional-order layoff model based on the Atangana-Baleanu fractional derivative that takes place in the discrete Nabla framework. Most of the mathematical models have been tackled in the recent past using the theory of fractional order systems. The model compartmentalizes the workers, who are susceptible, infectious, temporarily, and permanently removed, as the limit of the organizational capacity is taken into consideration. Using fractional derivatives, the model better captures workforce dynamics and the lasting impact of layoffs, thus providing a more accurate and predictive approach to employment stability. Incorporating the organizational capacity into a layoff model ensures that workforce reductions are not just cost-cutting measures but strategic decisions that maintain the long-term efficiency and sustainability. Layoffs can provide immediate financial relief, but without careful planning, they can significantly weaken an organization's ability to effectively operate, innovate, and recover. By considering the organizational capacity, companies can identify critical skills and key employees that are essential to maintain the productivity and institutional knowledge. This approach helps prevent operational interruptions, minimizes the risk of overburdening remaining employees, and avoids the long-term costs associated with rehiring and retraining. In addition, a layoff model that accounts for the organizational capacity allows businesses to simulate different downsizing scenarios, balancing cost savings with the need to retain essential expertise. With the assistance of workload redistribution, workforce recovery planning, and knowledge retention strategies, it is possible that by incorporating the above, organizations will make more informed decisions in terms of layoffs to ensure that short-term decisions ensure future financial stability, as well as the overall net capability. The model equations are expressed as follows:

$$
\begin{aligned}
{}^{ABC}_{\nu}\nabla^{\omega}S(\chi) &= r(\chi) - \frac{\zeta I(\chi)S(\chi)}{C(\chi)} - \mu S(\chi),\\
{}^{ABC}_{\nu}\nabla^{\omega}I(\chi) &= \frac{\zeta I(\chi)S(\chi)}{C(\chi)} - (\beta+\gamma+\mu)I(\chi),\\
{}^{ABC}_{\nu}\nabla^{\omega}TR(\chi) &= \beta I(\chi) - (\delta+\mu)TR(\chi),\\
{}^{ABC}_{\nu}\nabla^{\omega}PR(\chi) &= \gamma I(\chi) + \delta TR(\chi) - \mu PR(\chi),\\
{}^{ABC}_{\nu}\nabla^{\omega}C(\chi) &= \phi - \psi(I(\chi)+TR(\chi)) - \kappa PR(\chi).
\end{aligned}
\tag{2.1}
$$

To indicate the real-life labor dynamics in both organizational and economic systems, the workforce is categorized into five different compartments: susceptible workers (S), laid-off workers (I), temporarily removed workers (TR), permanently removed workers (PR), and organization capacity (C). The language of the proposed model incorporates the ideas of compartmental dynamics, which are common in other areas outside epidemiology, such as economics, finance, and social systems. In this respect, the term infectious workers is not biological, but is used to refer to laid-off workers whose existence poses the probability of additional layoffs in this organization. These propagations can be due to mechanisms such as budgetary constraints, productivity decline, supply-chain disruption, and managerial risk reassessment. Based on this, the concept of infectiousness is defined to be the spreading of the risk of layoff, but not a physical contagion. In Particular, vulnerable employees (S) are those who are in the active workforce but prone to layoffs, whereas the infectious employees are the laid-off employees whose conditions are indicative of organizational pressures. The relationship between S and I indicates how the probability of layoffs increases because of organizational and psychological feedback; therefore, the compartmental framework is justified to explain the labour market dynamics. To be clear, the terminology that was inspired by the epidemic is deployed as a mathematical analogy to solely give a description of the spread of risk , and is not subjected to a biological interpretation.

The distinction between temporary and permanent removals can be explained by the fact that the two categories have radically different behavioral and time-related properties. Removals that are temporary are normally associated with furloughs, medical leaves, or temporary contract suspensions, where they can be reinstated in the working workforce, unlike permanent removals, which involve retirement, resignation, or long-term disabilities. The separate treatments of these groups will enable the model to be able to capture specific transition processes and recovery patterns that would be confounded in a more aggregated model.

Moreover, an organizational capacity, denoted by (C), is added as a dynamic state variable to model the operational, infrastructural, and managerial capacity of the organization to be able to efficiently use its workforce. Practically, it is a fact that the supporting capacity of the available workers is so powerful in the performance of the workforce, regardless of the number of available workers in the organization. The Organizational capacity may be compromised by layoffs and labor instability, whereas the persistent capacity limitation may lead to the additional deterioration of workforce loss, which is a feedback loop. The proposed system explicitly models the workforce composition interaction with organizational capability by explicitly modeling C and thus makes the model more realistic and explanatory. Furthermore, the model uses a fractional-order framework that allows it to accommodate memory effects involved in workforce dynamics, including a slow recovery after temporary removals and the long-term effect of permanent loss of the workforce on the organizational capacity. Such a structure makes the proposed model theoretically and practically relevant.

Here, ${}^{ABC}_{\nu}\nabla^{\omega}$ denotes the Atangana-Baleanu-Nabla fractional derivative of order $0 < \omega < 1$. The interpretations of the model parameters are as follows:

- $r(\chi)$: rate of new workers recruitment,
- ζ: interaction rate of layoff between laid-off and susceptible workers,
- β: transition rate from laid-off to temporarily removed workers,
- γ: permanent removal rate of laid-off workers,
- δ: constant rate of workers laid off and permanently removed,
- μ: natural attrition rate common to all compartments,
- ϕ: external inflow supporting organizational capacity,
- ψ: decrease in organizational capacity as a result of laid-off and temporarily removed workers,
- κ: capacity degradation due to permanently removed workers.

We express the right-hand side of Eq (2.1) as follows:

$$\begin{aligned}
\theta_1(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi)) &= r(\chi) - \frac{\zeta I(\chi)S(\chi)}{C(\chi)} - \mu S(\chi)\\
\theta_2(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi)) &= \frac{\zeta I(\chi)S(\chi)}{C(\chi)} - (\beta+\gamma+\mu)I(\chi)\\
\theta_3(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi)) &= \beta I(\chi) - (\delta+\mu)TR(\chi)\\
\theta_4(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi)) &= \gamma I(\chi) + \delta TR(\chi) - \mu PR(\chi)\\
\theta_5(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi)) &= \phi - \psi(I(\chi)+TR(\chi)) - \kappa PR(\chi),
\end{aligned} \tag{2.2}$$

with the initial conditions $S(\nu) = S_\nu, I(\nu) = I_\nu, TR(\nu) = TR_\nu, PR(\nu) = PR_\nu$, and $C(\nu) = C_\nu$. Here, N denotes the total workers $N = S(\chi) + I(\chi) + TR(\chi) + PR(\chi) + C(\chi)$. Furthermore, we utilize

$$X(\chi) = \begin{bmatrix} S(\chi)\\ I(\chi)\\ TR(\chi)\\ PR(\chi)\\ C(\chi)\end{bmatrix}, \qquad X(\nu) = \begin{bmatrix} S(\nu)\\ I(\nu)\\ TR(\nu)\\ PR(\nu)\\ C(\nu)\end{bmatrix}, \tag{2.3}$$

$$\theta(\chi, X(\chi)) = \begin{bmatrix}\theta_1(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi))\\ \theta_2(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi))\\ \theta_3(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi))\\ \theta_4(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi))\\ \theta_5(\chi, S(\chi), I(\chi), TR(\chi), PR(\chi), C(\chi))\end{bmatrix}, \tag{2.4}$$

$$\theta_\nu = \begin{bmatrix}\theta_1(\nu, S(\nu), I(\nu), TR(\nu), PR(\nu), C(\nu))\\ \theta_2(\nu, S(\nu), I(\nu), TR(\nu), PR(\nu), C(\nu))\\ \theta_3(\nu, S(\nu), I(\nu), TR(\nu), PR(\nu), C(\nu))\\ \theta_4(\nu, S(\nu), I(\nu), TR(\nu), PR(\nu), C(\nu))\\ \theta_5(\nu, S(\nu), I(\nu), TR(\nu), PR(\nu), C(\nu))\end{bmatrix}. \tag{2.5}$$

Using Eqs (2.1) to (2.5), it can be expressed as follows:

$${}^{ABC}_{\ \nu}\nabla^{\omega} X(\chi) = \theta(\chi, X(\chi)), \qquad \forall\, \chi \in \mathbb{N}_\nu. \tag{2.6}$$

Using the fractional integral of Atangana-Baleanu in solving system (2.6) and referencing Definition 2.7, we derive the following:

$$X(\chi) = X_\nu + \frac{1-\omega}{\mathcal{M}(\omega)}\theta(\chi, X(\chi)) + \frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}\theta(\xi, X(\xi)). \tag{2.7}$$

To proceed with a further analysis, we assume that the mentioned assumptions are valid.

(A1) There exists a constant $\eta_\alpha > 0$ such that

$$\left|\theta(\chi, X) - \theta(\chi, \hat{X})\right| \le \eta_\alpha |X - \hat{X}|.$$

(A2) There exist constants $\phi_\alpha > 0$ and $\psi_\alpha > 0$ such that, for any $X, \hat{X} \in \mathbb{R}$,

$$|\theta(\chi, X)| \le \phi_\alpha \quad \text{and} \quad |\theta(\chi, \hat{X})| \le \psi_\alpha.$$

Lemma 2.8. *[34] If $0 < \omega < 1$, then the following holds:* ${}^{AB}_{\nu}\nabla^{-\omega}({}^{AB}_{\nu}\nabla^{\omega} x(\chi)) = x(\chi) - x(\nu)$.

Lemma 2.9. *(Krasnoselskii Fixed Point Theorem) Suppose that Ω is a nonempty, convex, closed, and bounded subset of a Banach space X. Assume that there exist two operators, ω and E, such that the following hold:*

(i) ω is a contraction mapping;
(ii) for any $\eta, \hat{\eta} \in \Omega$, we have $\omega\eta + E\hat{\eta} \in \Omega$;
(iii) the operator E is compact and continuous.

Then, the equation

$$\omega\eta + E\eta = \eta, \qquad \eta \in \Omega,$$

has at least one solution in Ω.

2.3. Analysis of uniqueness and existence solution

In this section, we examine the existence and uniqueness of solutions to the proposal model (2.1). The theory of fixed points offers a platform to study such issues. Through a fixed-point theorem, we can prove the existence of at least one solution, and this proves the mathematical validity of the model. Here, $B(\cdot, \cdot)$ denotes the classical Beta function, defined by the following:

$$B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1}\, dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \quad a, b > 0,$$

where $\Gamma(\cdot)$ is the Gamma function.

Theorem 2.10. *Under Assumptions (A1) and (A2), problem* (2.6) *possesses at least one solution provided that the following conditions hold:*

(i) $R < 1$,
(ii) $\dfrac{\|X_\nu\| + J}{1 - R} \le \beta$,

where $J = \dfrac{1-\omega}{\mathcal{M}(\omega)}\phi_\alpha + \dfrac{1}{\mathcal{M}(\omega)B(k-\nu, \omega)}\psi_a$, and $R = \left(\dfrac{1-\omega}{\mathcal{M}(\omega)} + \dfrac{1}{\mathcal{M}(\omega)B(k-\nu, \omega)}\right)\eta_\alpha$. Thus, the proposed model (2.1) *admits at least one solution and can therefore be considered theoretically valid.*

Proof. A closed and compact set G is given as $G = \{X \in G : \|X\| \le \beta\}$. We define two operators, ω and E, as follows:

$$\omega X(\chi) = X_\nu + \frac{1-\omega}{\mathcal{M}(\omega)}\theta(\chi, X(\chi)), \tag{2.8}$$

$$EX(\chi) = \frac{\omega}{\mathcal{M}\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}} \theta(\xi, X(\xi)). \tag{2.9}$$

where an operator equation may be represented by $X(\chi) = \omega X(\chi) + EX(\chi)$. We prove the statement with the help of the following steps:

Step 1: We demonstrate the fact that G is a contraction. In the case of $X(\chi), \hat{X}(\chi) \in G$ and in the Assumption (A1), then

$$\left\|\omega X(\chi) - \omega \hat{X}(\chi)\right\| \leq \frac{1-\omega}{\mathcal{M}(\omega)}\eta_\alpha \left\|X(\chi) - \hat{X}(\chi)\right\|;$$

if $\frac{1-\omega}{\mathcal{M}(\omega)}\eta_\alpha < 1$, then G is a contraction mapping.

Step 2: We prove that $\omega\hat{X}(\chi) + EX(\chi) \in G$, for $\hat{X}(\chi), X(\chi) \in G$. Applying the Minkowski inequality, we obtain the following:

$$\begin{aligned}\left\|\omega\hat{X}(\chi) + EX(\chi)\right\| \leq{}& \|X_\nu\| + \sup_{\nu\leq\chi\leq T}\frac{1-\omega}{\mathcal{M}(\omega)}\left|\theta(\chi, \hat{X}(\chi)) - \theta(0)\right| \\ &+ \sup_{\nu<\chi\leq T}\frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}\left|\theta(\xi, X(\xi)) - \theta(0)\right| \\ &+ \sup_{\nu<\chi\leq T}\frac{1-\omega}{\mathcal{M}(\omega)}|\theta(0)| + \sup_{\nu<\chi\leq T}\frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}|\theta(0)|. \end{aligned} \tag{2.10}$$

By using Assumption (A1), we get

$$\sup_{\nu<\chi\leq T}\frac{1-\omega}{\mathcal{M}(\omega)}\left|\theta(\chi, \hat{X}(\chi)) - \theta(0)\right| \leq \frac{(1-\omega)\eta_\alpha}{\mathcal{M}(\omega)}\left\|\hat{X}(\chi)\right\|, \tag{2.11}$$

and

$$\sup_{\nu<\chi\leq T}\frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}\left|\theta(\xi, X(\xi)) - \theta(0)\right| \leq \frac{\omega\eta_\alpha}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}(\chi-\nu)^{\overline{0}}\|X(\chi)\|. \tag{2.12}$$

As shown in Definition 2.5, it follows that

$$\begin{aligned} &\sup_{\nu<\chi\leq T}\frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}\left|\theta(\xi, X(\xi)) - \theta(0)\right| \\ &\leq \frac{\eta_\alpha}{\mathcal{M}(\omega)\Gamma(\omega)}\frac{\Gamma(\chi-\nu+\omega)}{\Gamma(\chi-\nu)}\|X(\chi)\| \\ &\leq \frac{\eta_\alpha}{\mathcal{M}(\omega)B(\chi-\nu,\omega)}\|X(\chi)\|. \end{aligned} \tag{2.13}$$

Combining (2.10)–(2.13), we have the following:

$$\begin{aligned}\left\|\omega\hat{X}(\chi) + EX(\chi)\right\| &\leq \|X_\nu\|_c + \frac{(1-\omega)}{\mathcal{M}(\omega)}\phi_\alpha + \frac{1}{\mathcal{M}(\omega)B(T-\nu,\omega)}\psi_\alpha + \left(\frac{(1-\omega)\eta_\alpha}{\mathcal{M}(\omega)} + \frac{\eta_\alpha}{\mathcal{M}(\omega)B(T-\nu,\omega)}\right)\beta \\ &\leq \|X_\nu\| + J + R\beta \leq \beta. \end{aligned}$$

Step 3: We show that the operator E is a continuous. Given $X(\chi) \in G$, then take the sequence $X_m(\chi)$, and as it tends to $X(\chi)$, the sequence $X_m(\chi)$ lies in G. With this, we get obtain the following:

$$\|EX_m(\chi) - EX(\chi)\| \leq \frac{\eta_\alpha}{\mathcal{M}(\omega)B(\chi - \nu, \omega)} \|X_m(\chi) - X(\chi)\|. \tag{2.14}$$

Thus, from (2.14), it follows that $EX_m(\chi) \to EX(\chi)$ as $X_m(\chi) \to X(\chi)$, which implies that the operator E is continuous.

Step 4: We show that E is relatively compact. For $X(\chi) \in G$, we have the following:

$$EX(\chi) = \left| \sup_{\nu < t \leq T} \frac{\omega}{\mathcal{M}(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}} \theta(\xi, X(\xi)) \right| \leq \beta,$$

i.e., operator E is bounded. Let $N_0 \in N_\nu$ and $N_0 < \chi_1 < \chi_2$; then, by Definition 2.5, we obtain that

$$\begin{aligned} \|EX(\chi_2) - EX(\chi_1)\| &\leq \frac{\phi_\alpha}{\mathcal{M}(\omega)} \left(\nabla_\nu^{-\omega} (\chi_2 - \nu)^{\bar{0}} - \nabla_\alpha^{-\omega} (\chi_1 - \omega)^{\bar{0}} + \nabla_{\chi_1}^{-\omega} (\chi_2 - \chi_1)^{\bar{0}} \right) \\ &\leq \frac{\phi_\alpha}{\mathcal{M}(\omega)} \frac{((\chi_2 - \nu)^{\bar{\omega}} - (\chi_1 - \nu)^{\bar{\omega}} + (\chi_2 - t_1)^{\bar{\omega}})}{\Gamma(\omega + 1)}, \end{aligned}$$

that is, the implication is that $\|EX(\chi_2) - EX(\chi_1)\| \to 0$ as $\chi_2 \to \chi_1$. Therefore, the operator E is uniformly Cauchy in G. According to the Arzela-Ascoli theorem, it is relatively compact.

Step 5: There exists a solution $X(\chi) \in G$ that satisfies $X(\chi) = \omega X(\chi) + EX(\chi)$.

If $X(\chi) = \omega X(\chi) + EX(\chi)$, then, by the Minkowski inequality, we have the following:

$$\begin{aligned} \|X(\chi)\| &= \|\omega X(\chi) + EX(\chi)\| \\ &\leq \|X_\nu\| + \frac{1 - \omega}{\mathcal{M}(\omega)} \phi_\alpha + \frac{1}{\mathcal{M}(\omega)B(\chi - \nu, \omega)} \psi_\alpha \leq \beta, \end{aligned}$$

which implies that $X(\chi) \in G$. By the Krasnoselskii fixed point theorem, system (2.1) admits at least one solution.

Theorem 2.11. *Assume that the function $\theta(\chi, X(\chi))$ satisfies Assumption (A1), and let*

$$H_\alpha = \frac{(1 - \omega)\,\eta_\alpha}{\mathcal{M}(\omega)} + \frac{\omega\,\eta_\alpha}{\mathcal{M}(\omega)\,\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}}.$$

If $H_\alpha < 1$, then the tech layoff model (2.6) *admits a unique solution.*

Proof. Suppose that $X_1(\chi)$ and $X_2(\chi)$ are two solutions of model (2.6), and define their difference as follows:

$$E(\chi) = X_1(\chi) - X_2(\chi).$$

The solution of model (2.6) satisfies the integral equation (2.7); then, we have the following:

$$E(\chi) = \frac{1 - \omega}{\mathcal{M}(\omega)} (\theta(\chi, X_1(\chi)) - \theta(\chi, X_2(\chi))) + \frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}} (\theta(\xi, X_1(\xi)) - \theta(\xi, X_2(\xi))).$$

Taking absolute values and applying Assumption (A1), we obtain the following:

$$|E(\chi)| \leq \frac{(1-\omega)\eta_\alpha}{\mathcal{M}(\omega)}|E(\chi)| + \frac{\omega\eta_\alpha}{\mathcal{M}(\omega)\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}}|E(\xi)|.$$

Using the supremum norm on the interval $[\nu, T]$, we estimate the summation term as follows:

$$|E(\xi)| \leq \sup_{\xi\in[\nu,T]} |E(\xi)|.$$

Therefore,

$$\sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}}|E(\xi)| \leq \left(\sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}}\right) \sup_{\xi\in[\nu,T]} |E(\xi)|.$$

Substituting this estimate into the previous inequality yields the following:

$$|E(\chi)| \leq \left[\frac{(1-\omega)\eta_\alpha}{\mathcal{M}(\omega)} + \frac{\omega\eta_\alpha}{\mathcal{M}(\omega)\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}}\right] \sup_{\xi\in[\nu,T]} |E(\xi)|.$$

Let

$$H_\alpha = \frac{(1-\omega)\eta_\alpha}{\mathcal{M}(\omega)} + \frac{\omega\eta_\alpha}{\mathcal{M}(\omega)\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}}.$$

Then, we obtain the following:

$$|E(\chi)| \leq H_\alpha \sup_{\xi\in[\nu,T]} |E(\xi)|.$$

Since H_α satisfies the conditions of Gronwall's inequality and is nonnegative and bounded, it follows that

$$\sup_{\chi\in[\nu,T]} |E(\chi)| = 0.$$

Hence, $E(\chi) = 0$ for all $\chi \in [\nu, T]$, which implies the following:

$$X_1(\chi) = X_2(\chi), \quad \forall\chi \in [\nu, T].$$

Therefore, the solution of system (2.6) is unique.

2.4. Mittag-Leffler-Hyers-Ulam stability (MLHU)

MLHU is a generalization of the classical Hyers-Ulam Stability to fractional-order difference and differential equations, where the Mittag-Leffler function is used to more precisely account for the behavior of the stability . It guarantees that any approximate solution which has a finite perturbation error can be expressed as an exact one within a manageable bound, which concerns the Mittag-Leffler function, and is the depiction of. This stability criterion is very convenient in the fractional systems where the memory and hereditary effects are important, (e.g., in the viscoelectricity control theory N or mechanical models). It gives a more elegant analysis of the long-term dynamics of a fractional-order system by replacing the fractional exponential bound by the Mittag-Leffler function.

Definition 2.12. *Let $\bar{X}(\chi) \in G$ be an approximate solution that satisfies the following:*

$$\left\| {}^{ABC}_{\nu}\nabla^{\omega}\bar{X}(\chi) - \theta(\chi, \bar{X}(\chi)) \right\| \leq \epsilon, \qquad \forall\, \chi \in \mathbb{N}_{\nu}.$$

Then, model (2.1) *is said to be MLHU stable if there exists an exact solution $X(\chi)$ of model* (2.1) *such that*

$$\left\| X(\chi) - \bar{X}(\chi) \right\| \leq C\,\epsilon\, E_{\overline{\omega,1}}(K(\chi - \nu)^{\omega}), \qquad \forall\, \chi \in \mathbb{N}_{\nu},$$

where $C > 0$ and $K > 0$ are constants independent of ϵ.

Theorem 2.13. *Under Assumptions (A1) and the discrete fractional Gronwall inequality, the solution of system (2.1) is MLHU stable.*

Proof. **Step 1. Error estimation.** Let $\bar{X}(\chi)$ be an approximate solution of system (2.1) that satisfies

$$\left\| {}^{ABC}_{\nu}\nabla^{\omega}\bar{X}(\chi) - \theta(\chi, \bar{X}(\chi)) \right\| \leq \epsilon, \quad \forall \chi \in N_{\nu},$$

and let $X(\chi)$ be the exact solution. Define the following error function:

$$e(\chi) = \|\bar{X}(\chi) - X(\chi)\|.$$

Since

$${}^{ABC}_{\nu}\nabla^{\omega}X(\chi) = \theta(\chi, X(\chi)),$$

we obtain

$$\left\| {}^{ABC}_{\nu}\nabla^{\omega}e(\chi) \right\| \leq \left\| \theta(\chi, \bar{X}(\chi)) - \theta(\chi, X(\chi)) \right\| + \epsilon.$$

Using Assumption (A1), there exists $\eta_{\alpha} > 0$ such that

$$\left\| \theta(\chi, \bar{X}(\chi)) - \theta(\chi, X(\chi)) \right\| \leq \eta_{\alpha} e(\chi);$$

hence,

$$\left\| {}^{ABC}_{\nu}\nabla^{\omega}e(\chi) \right\| \leq \eta_{\alpha} e(\chi) + \epsilon. \tag{2.15}$$

Step 2. Fractional summation form. Using the equivalent discrete fractional integral representation, inequality (2.15) yields the following:

$$e(\chi) \leq \frac{1-\omega}{\mathcal{M}(\omega)}\eta_{\alpha} e(\chi) + \frac{\omega\eta_{\alpha}}{\mathcal{M}(\omega)\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}} e(\xi) + \frac{\epsilon}{\mathcal{M}(\omega)}.$$

Rearranging,

$$e(\chi) \leq \frac{\epsilon}{\mathcal{M}(\omega)\left(1 - \frac{(1-\omega)\eta_{\alpha}}{\mathcal{M}(\omega)}\right)} + \frac{\omega\eta_{\alpha}}{\mathcal{M}(\omega)\Gamma(\omega)\left(1 - \frac{(1-\omega)\eta_{\alpha}}{\mathcal{M}(\omega)}\right)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}} e(\xi). \tag{2.16}$$

Define the following:

$$C_0 = \frac{1}{\mathcal{M}(\omega)\left(1 - \frac{(1-\omega)\eta_{\alpha}}{\mathcal{M}(\omega)}\right)}, \quad K = \frac{\omega\eta_{\alpha}}{\mathcal{M}(\omega)\Gamma(\omega)\left(1 - \frac{(1-\omega)\eta_{\alpha}}{\mathcal{M}(\omega)}\right)}.$$

Then, (2.16) becomes the following:

$$e(\chi) \le C_0\epsilon + K \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\overline{\omega-1}} e(\xi).$$

Step 3. Application of the discrete fractional Gronwall inequality. By applying the discrete fractional Gronwall inequality, we obtain the following:

$$e(\chi) \le C_0\epsilon E_{\overline{\omega,1}}(K(\chi-\nu)^{\omega}) .$$

Let

$$C = C_0;$$

then,

$$\|\bar{X}(\chi) - X(\chi)\| \le C\epsilon E_{\overline{\omega,1}}(K(\chi-\nu)^{\omega}) .$$

Step 4. Stability conclusion. Since the MittagLeffler function grows at most exponentially, the above bound implies that the error remains uniformly controlled for all $\chi \in N_\nu$. Hence, system (2.1) is MLHU stable.

Definition 2.14. *Model* (2.1) *is said to be MLHU-Rassias stable if there exist constants* $C > 0$*,* $K > 0$*, and a perturbation function* $\phi(\chi)$ *such that, for every approximate solution* $\bar{X}(\chi)$ *that satisfies*

$$\left\| {}^{ABC}_{\nu}\nabla^{\omega} \bar{X}(\chi) - \theta(\chi, \bar{X}(\chi)) \right\| \le \epsilon\, \phi(\chi), \qquad \forall\, \chi \in \mathbb{N}_\nu,$$

there exists an exact solution $X(\chi)$ *of model* (2.1) *such that*

$$\left\| \bar{X}(\chi) - X(\chi) \right\| \le C\, \epsilon\, E_{\overline{\omega,1}}(K(\chi-\nu)^{\omega})\, \phi(\chi), \qquad \forall\, \chi \in \mathbb{N}_\nu.$$

In order to continue with the analysis, we consider that the following assumptions hold.
(A3) The perturbation function $\varphi(\chi)$ satisfies one of the following growth conditions:

(i) Polynomial growth:

$$\varphi(\chi) \le K(1+\chi)^p, \quad K > 0,\ p \ge 0,$$

(ii) Exponential growth:

$$\varphi(\chi) \le K e^{\lambda \chi}, \quad K > 0,\ \lambda \ge 0.$$

Theorem 2.15. *Assume that Conditions (A1) and (A3) hold. Then, system* (2.1) *is MLHU-Rassias stable. Moreover, the following cases hold:*

(i) If $\varphi(\chi)$ *grows polynomially, then the error function* $e(\chi) = \|\bar{X}(\chi) - X(\chi)\|$ *satisfies*

$$e(\chi) \le C\varepsilon(1+\chi)^p E_{\overline{\omega,1}}(\Lambda(\chi-\nu)^{\omega}) ,$$

for some constants $C > 0$*,* $\Lambda > 0$*, and* $p > 0$*.*

(ii) If $\varphi(\chi)$ *grows exponentially, then the MLHU-Rassias stability holds provided*

$$\lambda < \Lambda,$$

where Λ *depends on the Lipschitz constant* η_α *and the fractional order* ω*.*

Proof. Define the following error function:

$$e(\chi) = \bar{X}(\chi) - X(\chi).$$

Applying the ABC Nabla fractional difference operator and using system (2.1), we obtain the following:

$${}^{ABC}_{\nu}\nabla^{\omega} e(\chi) = {}^{ABC}_{\nu}\nabla^{\omega} \bar{X}(\chi) - \theta(\chi, X(\chi)).$$

Using the definition of the approximate solution and Assumption (A1), we obtain the following:

$$\begin{aligned}\left\|{}^{ABC}_{\nu}\nabla^{\omega} e(\chi)\right\| &\le \left\|\theta(\chi, \bar{X}(\chi)) - \theta(\chi, X(\chi))\right\| + \varepsilon\varphi(\chi)\\ &\le \eta_{\alpha}\|e(\chi)\| + \varepsilon\varphi(\chi).\end{aligned}$$

Applying the fractional Nabla summation operator yields the following:

$$\begin{aligned}\|e(\chi)\| \le& \frac{1-\omega}{\mathcal{M}(\omega)}\eta_{\alpha}\|e(\chi)\| + \frac{\omega\eta_{\alpha}}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}\|e(\xi)\|\\ &+ \frac{1-\omega}{\mathcal{M}(\omega)}\varepsilon\varphi(\chi) + \frac{\omega}{\mathcal{M}(\omega)\Gamma(\omega)}\sum_{\xi=\nu+1}^{\chi}(\chi-\rho(\xi))^{\overline{\omega-1}}\varepsilon\varphi(\xi).\end{aligned}$$

Applying the discrete fractional Gronwall inequality, we obtain the following:

$$\|e(\chi)\| \le C\,\varepsilon\, E_{\overline{\omega,1}}(\Lambda(\chi-\nu)^{\omega})\,\varphi(\chi),$$

where

$$C = \frac{\mathcal{M}(\omega)}{1-(1-\omega)\eta_{\alpha}}, \qquad \Lambda = \frac{\omega\eta_{\alpha}}{\mathcal{M}(\omega)\Gamma(\omega)}.$$

(i) If $\varphi(\chi)$ grows polynomially, then the error bound remains polynomially weighted by a Mittag-Leffler kernel, thus proving case (i).

(ii) If $\varphi(\chi)$ grows exponentially, then the stability holds provided $\lambda < \Lambda$, thus ensuring that the Mittag-Leffler decay dominates the perturbation growth.

Hence, System (2.1) is MLHU-Rassias stable.

Remark 2.16. *In the context of workforce dynamics, assuming a bounded or polynomially growing perturbation function is realistic. External influences such as economic fluctuations, hiring freezes, or policy interventions are typically finite in magnitude and duration due to budgetary, regulatory, and organizational constraints. Therefore, sustained exponential perturbations are unlikely in real-world tech layoff scenarios, justifying the bounded perturbation assumption adopted in the baseline analysis.*

3. Numerical simulation test

The purpose of the numerical simulations is to verify the theoretical stability conclusions and to show how the workforce dynamics are affected by the fractional-order memory effects under different parameter values.

We use a variety of physiologically and organizationally relevant parameter values in order to produce a stable and acceptable simulation of the proposed model of the fractional-order workforce dynamics. These parameters guarantee that the system will behave in a realistic manner while also representing the memory effects that are present in the system that the fractional calculus models.

3.1. Numerical scheme and implementation

The numerical scheme used to solve the ABC Nabla fractional difference system, together with its convergence properties and implementation details for reproducibility, are briefly described before the numerical results are presented. We use a fractional predictor–corrector technique adapted to the Atangana-Baleanu Nabla fractional operator in the Caputo sense in order to numerically solve the ABC Nabla fractional difference system (2.1). The discrete fractional integral formulation of the solution provides a foundation for this approach.

Using Definition 2.7, the solution of the system can be written in the equivalent discrete integral form as follows:

$$X(\chi) = X_\nu + \frac{1-\omega}{M(\omega)}\theta(\chi, X(\chi)) + \frac{\omega}{M(\omega)\Gamma(\omega)} \sum_{\xi=\nu+1}^{\chi} (\chi - \rho(\xi))^{\omega-1}\theta(\xi, X(\xi)).$$

The summation term is discretized, and the state variables are repeatedly updated at each time step in the numerical approximation.

Algorithm 1 Numerical scheme for ABC Nabla fractional layoff model

Initialize $X(0) = (S_0, I_0, TR_0, PR_0, C_0)$
for $\chi = 1$ to N **do**
Predictor step:

$$X^{(p)}(\chi) = X_0 + \frac{\omega}{M(\omega)\Gamma(\omega)} \sum_{\xi=1}^{\chi-1} (\chi - \xi)^{\omega-1}\theta(\xi, X(\xi))$$

Corrector step:

$$X(\chi) = X_0 + \frac{1-\omega}{M(\omega)}\theta(\chi, X^{(p)}(\chi)) + \frac{\omega}{M(\omega)\Gamma(\omega)} \sum_{\xi=1}^{\chi} (\chi - \xi)^{\omega-1}\theta(\xi, X(\xi))$$

end for
return $X(\chi)$

The integral formulation of the ABC Nabla fractional operator is compatible with the predictor-corrector method described above. The approach converges with order $O(h^\omega)$ under the Lipschitz condition (A1), where h is the time-step size.

NumPy is used in the Python implementation of every simulation. To guarantee that the results can be repeated, the complete implementation and parameter files are included as supplemental material.

Listing 1. Python implementation of the ABC nabla fractional solver

```
import numpy as np
from math import gamma

def abc_nabla_solver(theta, X0, omega, N):
    M = 1.0  # normalization constant
    X = np.zeros((N+1, len(X0)))
    X[0, :] = X0

    for k in range(1, N+1):
        frac_sum = np.zeros(len(X0))
        for j in range(1, k+1):
            frac_sum += (k-j+1)**(omega-1) * theta(j, X[j-1])
        X[k, :] = X0 + (1-omega)/M * theta(k, X[k-1]) \
                  + omega/(M*gamma(omega)) * frac_sum
    return X
```

3.2. *Parameter settings and initial conditions*

To achieve a stable and interpretable simulation of the suggested model of the fractional-order workforce dynamics, we take a range of biologically and organizationally relevant values of parameters. These parameters ensure that the behavior of the system will be realistic and, at the same time, represent the memory effects that are inherent in the system used as modeled by the fractional calculus.

In order to use the proposed fractional-order model of the workforce dynamics, we use a set of parameter values that are selected very carefully and would provide sufficient stability of the model as well as a meaningful picture of organizational dynamics. The selection of parameter values in this study is grounded in empirical observations and industry data. Specifically, the layoff rate $\zeta = 0.02$ is chosen based on reported layoff and downsizing statistics from labor market surveys, which indicate annual layoff rates in the range of 1.5–3% in similar organizational settings [35,36]. The organizational capacity degradation parameters $\psi = 0.005$ and $\kappa = 0.003$ are selected to reflect observed reductions in effective capacity following layoffs, as documented in organizational behavior studies, which report capacity losses of approximately 0.3–0.7% per unit change in workforce composition [37,38]. When exact values are not directly available from the literature, the chosen values reflect midpoint estimates within empirically observed ranges. This ensures that the model parameters are representative of real-world dynamics rather than purely arbitrary choices.

The derivative of fractional order considered is of order $\omega = 0.7$, which introduces memory effects to the system dynamics, and the normalization constant of the ABC operator is $M(\omega) = 1$. These initial conditions at $\chi = 0$ are laid out as follows: $S(0) = 10,000$ susceptible workers, I(0) = 500 laid off employees, TR(0) = 300 temporarily removed, PR(0) = 200 permanently removed, and C(0) = 50,000 units of organizational capacity. The values represent the mid-sized technology organization that is going through restructuring of its workforce through lay-offs and attrition. The results of the numerical simulation of the fractional-order tech layoff model introduced with the Atangana-Baleanu fractional Nabla operator in the Caputo sense are depicted in the following figures. The model is used to record the development of the workforce in five major compartments - susceptible (S), infectious (I), temporarily removed (TR), permanently removed (PR) and organizational capacity (C) - over a period of 10 months under the different fractional orders $k = \{1.0, 0.95, 0.9, 0.85, 0.8\}$.

3.3. Simulation results

The numerical investigation explores the dynamic behavior of the proposed fractional-order workforce model under varying parameter values and fractional orders. These simulations validate the analytical stability findings and show how memory effects influence the system's trajectories.

As Figure 1 shows, the number of workers that can be susceptible starts to decrease in a steady way as the value of k increases, and the deeper the decrease, the greater the value of k. It means that the layoff risk of the workforce in the classical integer-order case is increasing fast. Conversely, when the value of k is lower, it depicts more memory systems, and there is a slower rate of depletion of the susceptible stratum. This is done in relation to a real-life situation where past occurrences affect the stability of the current workforce.

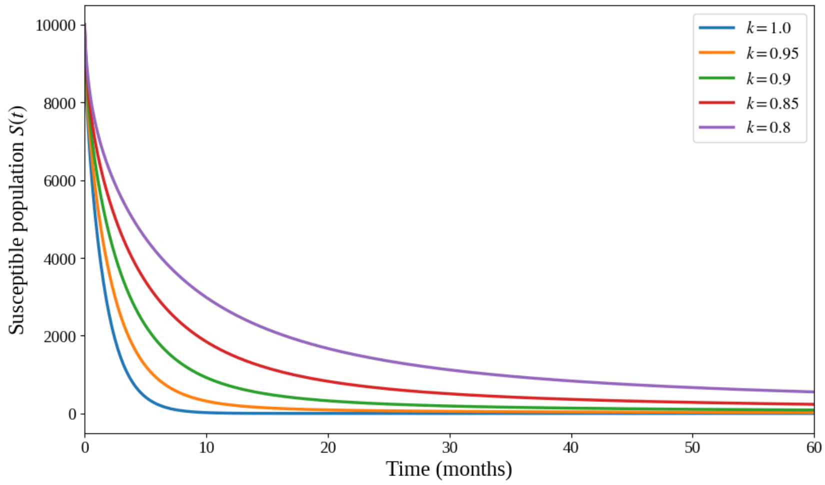

Figure 1. Susceptible population dynamics over time for varying fractional-order values k.

Figure 2 illustrates the trends in the infectious population over time for different fractional-order values of k. The graph shows that, in all cases, the infectious population increases initially and then decreases rapidly as workers transition from the infectious state to either temporarily or permanently removed categories. Notably, for smaller values of k, the maximum number of infectious workers is higher, and the rate of decrease is slower. This suggests that memory effects significantly extend the duration for which workers remain in the infectious state before transitioning to the next stage, such as temporary removal or permanent removal. In contrast, as k increases, the infectious population peaks sooner and declines more rapidly, indicating less influence from past events in the system's dynamics.

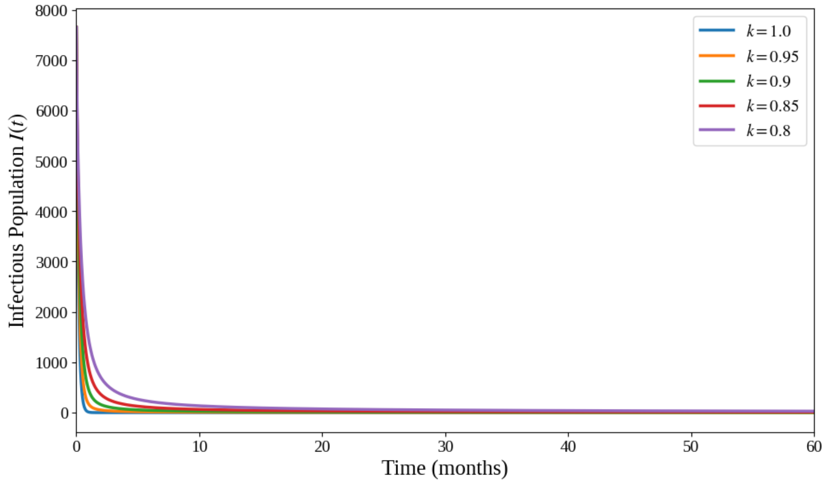

Figure 2. Infectious population trends over time for different fractional-order values k.

As can be seen in Figure 3, the TR category rises at first, then reaches the peak, after which it falls. Greater fractional orders further translate to a more rapid accumulation and clearance, and a lower k to a more gentle and protracted duration of a temporarily absent workforce. This is an indication of a process in which the memory effects are fierce, in that more of the workers may be placed in uncertain conditions, such as a temporary unpaid leave.

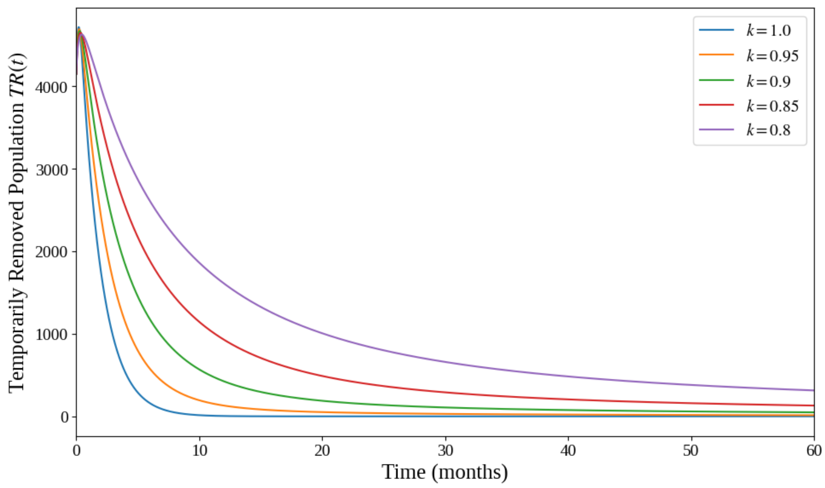

Figure 3. Temporarily removed population over time for different fractional-order values k.

Figure 4 illustrates that the PR class increases with time as a result of the transition of the I and TR classes. The faster this group peaks is at k = 1.0. At a low k, the accumulation process is slower but longer, which becomes a sign of a increased time taken by organizations under the influence of memories to formally terminate employment relationships.

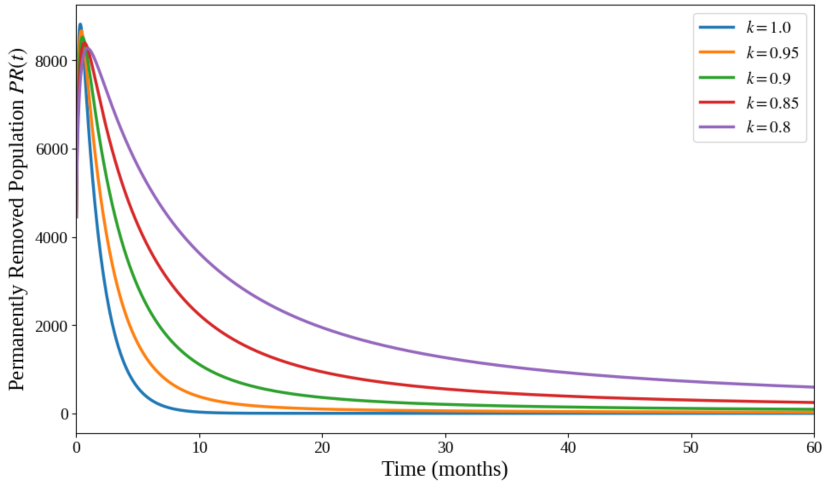

Figure 4. Permanently Removed Population over Time for Various Values of k.

The model shown in Figure 5 includes the impact of a layoff on the capacity of an organization to operate, and $C(\chi)$ is the overall capacity (e.g., operational strength, productivity potential, or availability of human capital). The difference between the laid-off and the removed workers (TR, PR) hurts the rate of change $C(\chi)$ with a constant inflow of recruits trying to maintain the capacity.

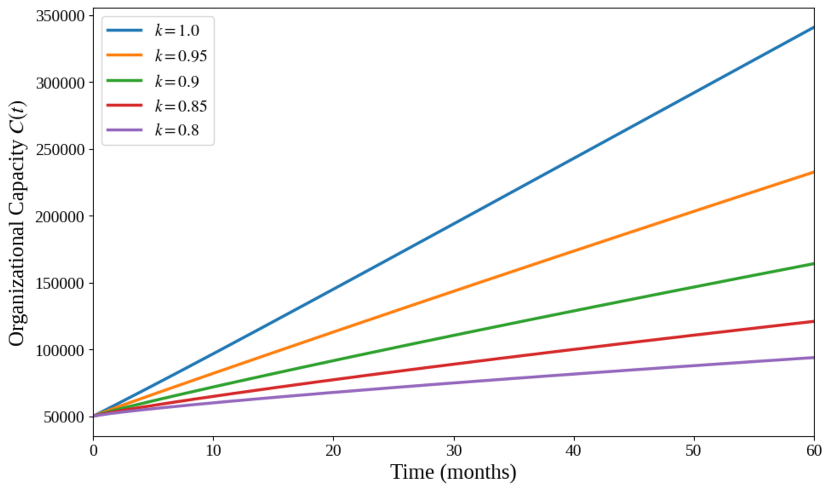

Figure 5. Organizational capacity $C(\chi)$ over time for various fractional orders k.

As the figures indicate:

i. Large values of fractional orders (e.g., k = 1.0) lead to faster reductions in the organizational capacity. This matches up with the classical presence of the integer-order situation in response to layoffs and other workforce variations, where the systems do not remember prior situations, and an immediate response to current situations happens. Under these assumptions, a rapid erosion of the capacity is experienced in the organization.

ii. Lower fractional orders (e.g., k = 0.8) result in slower and a smoother and finer $C(\chi)$. This means a memory system in which there are previous states that affect the present processes. In this case, the organization can maintain capacity over longer periods, and this is effectively a measure of resilience or structural inertia.

iii. The divergence between the curves of different k values grows in time, which lays stress on the importance of memory effects on the system dynamics because it is cumulative. A reduction of just a little in k has a potent impact on changing the long-range course of the capacity loss. The graph shows that the fractional-order dynamics should be included in the modeling of organizational systems that have been hit by layoffs. This allows simulations of different levels of institutional memory, rate of adjustment, and rate of recovery, which can be quite informative about the degradation of organizations over a lifetime under periods of high staff flux.

In the population dynamics table, the variables are abbreviated as follows: month (M) denotes the time period in months; susceptible population (S) represents individuals who are at risk of infection but not yet infected; infectious population (I) refers to individuals currently capable of transmitting the infection; temporarily removed population (T) indicates individuals who are temporarily isolated or recovered but may become susceptible again; permanently removed population (PR) includes individuals who are permanently immune or deceased; and finally, organizational capacity (OC) represents the available resources or institutional capability to manage the population and infection dynamics over time.

Table 3. Monthly workforce dynamics.

Month (M)	S	I	T	PR	OC
Month 0	10000.0	500.0	300.0	200.0	50,000.0
Month 10	9277.4	446.8	278.3	192.6	96,250.0
Month 20	9048.4	419.9	245.6	185.5	142,500.0
Month 30	8607.1	399.3	222.2	178.7	188,750.0
Month 40	8187.3	374.6	203.3	164.6	235,000.0
Month 50	7408.2	348.4	181.7	142.7	281,250.0
Month 60	6703.2	319.5	164.6	127.5	327,500.0
Month 70	6037.6	292.4	149.6	113.9	373,750.0
Month 80	5405.1	267.1	137.1	101.2	420,000.0
Month 90	4803.5	244.1	126.0	89.0	466,250.0
Month 100	4231.7	223.2	115.7	77.5	512,500.0
Month 110	3688.9	204.2	106.3	66.6	558,750.0
Month 120	3174.1	187.0	97.7	56.2	605,000.0

Table 3 presents the dynamics of the susceptible population, infectious population, temporarily removed population, permanently removed population, and organizational capacity over time, from Month 0 to Month 120. As the months progress, the susceptible population gradually decreases, thus reflecting the impact of various factors on the workforce. Additionally, the infectious and removed populations also follow distinct patterns, with the infectious population showing a steady decline and the removed populations experiencing more gradual changes. Meanwhile, the organizational capacity increases over time, thus indicating growth in the system's ability to handle changes. These dynamics illustrate the complex interplay between workforce stability, capacity, and the evolving risks over time.

3.4. Policy implications derived from simulation results

The simulation findings underscore the importance of memory effects in the dynamics of the workforce, as described by the fractional-order parameter ω. Two cases can be used to interpret the behavior of the system.

Case (i): The system shows strong memory effects when the value of ω is low. In this case, workers are not lost by the organization at once in case of an economic shock. Rather, the workforce is more tied to the organization, and the decrease in capacity is slower and more manageable. This implies that firms would be able to implement gradual adjustment measures instead of laying off employees immediately. Examples of practical measures are temporary leave, shorter working hours, and retraining of employees. These strategies are useful in retaining knowledge within the organization and retaining the workforce until the economic conditions improve.

Case (ii): In the case where the value of ω is high, the effect of the memory is less powerful, and the workforce responds faster to economic shocks. Consequently, there is a possibility of sudden and massive layoffs in organizations and this might result in losses in organizational capacity that are very substantial and, in some cases permanent. Managers and policymakers need to take action early to minimize the losses in the long term. Some of the strategies that can be considered important are retention of key employees, transfer of workers to other positions within the company, and investment in skills development programs to ensure resilience.

Generally, the fractional-order model offers valuable details of the impact of memory effects on the dynamics of the workforce. The findings indicate that progressive changes in the workforce and policy interventions are the best policies that can be used to maintain the organizational stability in the long run.

4. Conclusions

In this paper, we analyzed the tech layoff model involving the Atangana-Baleanu Nabla fractional derivative in the Caputo sense. A new formulation of the Nabla fractional difference operator was introduced and used to establish the existence, uniqueness, and MLHU stability of the system through fixed - point arguments and a discrete Gronwall inequality. The numerical simulations further support the theoretical results. Since the model has a unique solution, each plotted trajectory represents the single admissible evolution of the dynamical system. The stability results are reflected in the smooth variation of the curves when the fractional order changes. Lower fractional orders produce stronger memory effects and slower transitions, whereas higher orders approach the classical integer-order dynamics. These behaviors confirm the dynamical system interpretation of the model and illustrate how layoffs propagate across compartments under fractional memory. Overall, the study shows that fractional calculus provides a flexible and realistic framework to model the workforce dynamics with long-range dependence. Future work may consider other fractional operators or optimization-based approaches to improve the prediction and control in employment systems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare there are no conflicts of interest.

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