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Research article

H_∞ quantized control for networked systems subject to stochastic deception attacks: A dynamic event-triggered scheme

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Abstract: This paper is devoted to studying dynamic event-triggered quantized H_∞ control for networked control systems (NCSs) with stochastic deception attacks. To save limited system resources, a dynamic event-triggered scheme is offered, in which a new triggering error is introduced. A lower trigger frequency can be obtained by appropriately adjusting the triggering error. Then, considering the conventional deception attack, accumulated dynamic cyber-attack, and dynamic event-triggered scheme, a new quantized control model is constructed, and the stochastic deception attack is described by two independent Bernoulli distributed variables. Moreover, a new H_∞ performance criterion is given by using a customized Lyapunov-Krasovskii functional (LKF), and a novel controller design approach is derived based on the criterion. Finally, some simulations are listed to verify the validity of the derived methods.

Keywords: H_∞ performance; event-triggered scheme; cyber-attacks; quantization; networked control systems

1. Introduction

For two decades, networked control systems (NCSs) have been successfully applied in unmanned aerial vehicles, mobile communications, industrial automation, smart grids, and so on [1–3]. Data is transmitted via network among the actuator, controller, and sensor in NCSs. Up to now, a rich body of outstanding research works have been reported for NCSs [4]. For example, the paper [5] studied event-triggered output feedback H_∞ control for NCSs. The paper [6] investigated H_∞ static output-feedback control for NCSs subject to Markov packet dropout, and the resilient controller design and synthesis for NCSs with denial-of-service (DoS) attacks were investigated in [7] via an adaptive event-triggered strategy, and the paper [8] studied a novel practical prescribed-time control approach for nonlinear systems subject to state constraints.

In general, the time-triggered scheme (TTS) paves an easy way to the controller analysis and design. However, it tends to generate redundant or unnecessary sampled data packets, which can lead to the waste of system resources. To realize the efficient utilization of network resources, the event-triggered scheme (ETS) was introduced [9, 10]. Now, researchers have focused extensively on event-triggered controller design for NCSs [11–13]. For example, the paper [14] was concerned with event-triggered H_∞ control for NCSs with packet loss and communication delay by the static ETS. The output feedback $\mathcal{L}_\infty$ load frequency control of networked power systems was studied in [15] by an adaptive ETS. The leader-following consensus for linear multi-agent systems was considered in [16] by a dynamic ETS. Recently, the paper [17] addressed periodic event-triggered dynamic output feedback control for NCSs, and periodic event-triggered control for NCSs subject to external disturbance was investigated in [18]. The paper [19] focused on the practical finite-time synchronization for master-slave Lur'e nonlinear systems with performance constraint and time-varying actuator faults via the memory-based quantized dynamic event-triggered control. Note that the signal quantization is also needful in NCSs, which aims to reduce the size of the data [20]. For example, quantized output feedback for continuous-time switched systems with time-delay was studied in [21].

Now, the security issue of NCSs has received broad interest, and cyber-attacks are mainly classified into deception attacks and DoS attacks [22]. Until now, researchers have focused extensively on cyber-attacks [23–25]. For example, co-design of a dynamic ETS and resilient observer-based control with aperiodic DoS attacks was investigated in [26]. The paper [27] was concerned with stochastic event-triggered H_∞ control for NCSs under DoS attacks, and the event-triggered impulsive control problem of NCSs was studied in [28] under random cyber-attacks. Recently, adaptive event-triggered control for networked interconnected systems under cyber-attacks was done in [29], and the paper [30] was concerned with the asynchronous sliding mode control for networked hidden stochastic jump systems with cyber-attacks.

Recently, event-triggered H_∞ control of NCSs with stochastic deception attacks has been addressed in [31, 32]. Based on the dynamic ETS, the paper [31] investigated H_∞ control of NCSs with stochastic deception attacks. By the LKF theory, some sufficient conditions were obtained to guarantee the closed-loop singular system is asymptotically stable, regular, and impulse free with an H_∞ performance, and a new controller design method was derived. The paper [32] investigated dynamic event-triggered H_∞ control of NCSs under stochastic deception attacks. By the LKF method, an H_∞ performance condition was obtained, and dynamic ETS parameters and controller gain were co-designed.

Based on the above discussion, we found that in the research of deception attacks, the accumulated dynamic cyber-attack is often ignored, and only conventional deception attacks such as those which satisfy the Lipschitz condition are considered. At the same time, with the continuous improvement of the ETS, there is still room for further research on how to balance the control performance of the system and the utilization rate of network resources. To our knowledge, few scholars have investigated event-triggered quantized control for NCSs with stochastic deception attacks. Based on these, the dynamic event-triggered quantized control for NCSs is studied with stochastic cyber-attacks. Compared with the recently reported works [31, 32], the contributions are highlighted in following aspects. 1) A more advanced dynamic ETS is proposed, in which a new triggering error is introduced. A lower trigger frequency can be obtained by appropriately adjusting the error parameter. Moreover, it can reduce to the traditional static or dynamic one. 2) A closed-loop quantized control model is given for NCSs with the conventional deception attack, accumulated dynamic cyber-attack, and dynamic ETS. 3) Based on the stability analysis, a reformative event-triggered H_∞ controller design method is obtained.

Notations: $\mathscr{E}\{\cdot\}$ is the expectation; $Pb\{B\}$ is the probability of event B that happens; $G > 0$ shows G is the positive-definite matrix; $diag\{\cdot\}$ means block-diagonal matrix. $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$, and $\mathbb{S}_n^+$ stand for the n-dimensional Euclidean space, the set of $n \times m$ real matrices, and the set of $n \times n$ positive-definite matrices, respectively; $col\{a, b\} = \left[a^T, b^T\right]^T$.

2. Problem formulation

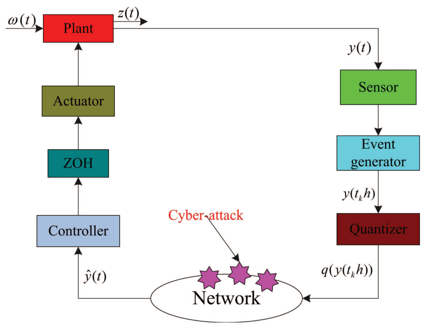

Figure 1. Event-triggered networked control.

In Figure 1, the plant is described by

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{D}\omega(t) \\ y(t) = Cx(t) \\ z(t) = \mathcal{L}x(t) \end{cases} \tag{2.1}$$

where $\mathcal{A} \in \mathbb{R}^{n\times n}$, $\mathcal{B} \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{m\times n}$, $\mathcal{D} \in \mathbb{R}^{n\times r}$, and $\mathcal{L} \in \mathbb{R}^{q\times n}$; $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{bmatrix}^T \in \mathbb{R}^n$ is the system state; $y(t) = \begin{bmatrix} y_1(t) & y_2(t) & \cdots & y_m(t) \end{bmatrix}^T \in \mathbb{R}^m$ is the measured output; $\omega(t) \in \mathbb{R}^r$ is the disturbance input; $z(t) \in \mathbb{R}^q$ is the controlled output; and $u(t) \in \mathbb{R}^p$ is the control input.

The ETS is given as follows:

$$e_y^T(lh)\Omega_1 e_y(lh) \le \sigma y_\rho^T(t_{k_l}h)\Omega_2 y_\rho(t_{k_l}h) + \frac{1}{\alpha}\eta(t_{k_l}h) \tag{2.2}$$

where the triggering error

$$e_y(lh) = y(t_k h) - y_\rho(t_{k_l}h), \ t_{k_l}h = t_k h + lh$$

and $y_\rho(t_{k_l}h) = \rho y(t_{k_l}h) + (1-\rho)y(t_k h)$ with $\rho \in [0, 1]$ and $l \in \mathbb{N}$; $\Omega_k > 0$ $(k = 1, 2)$ are the weighting matrices; $y(t_{k_l}h)$ and $y(t_k h)$ are the currently sampled signal and the last transmitted signal, respectively; and the variable $\eta(t)$ satisfies

$$\dot{\eta}(t) = -\beta\eta(t) + \sigma y_\rho^T(t_{k_l}h)\Omega_2 y_\rho(t_{k_l}h) - e_y^T(lh)\Omega_1 e_y(lh) \tag{2.3}$$

where $\beta > 0$ and $\eta_0 > 0$.

Remark 1. *Compared with the ETSs in [31–33], the dynamic ETS (2.2) with (2.3) is more effective. First, a new triggering error $e_y(lh) = y(t_k h) - y_\rho(t_{k_l} h)$ is offered, and when $\rho = 1$, it can reduce to the general one $e(t) = y(t_k h) - y(t_{k_l} h)$. Based on this, when the sampled data have a rapid change arising from the external disturbance, spurious triggering events may decrease. Second, more parameters are used to adjust the ETS (2.2) with (2.3), such as σ, ρ, α, β, Ω_1, and Ω_2. Third, the variable $\eta(t)$ can be adjusted as the system changes instead of a preset constant.*

Remark 2. *When $\eta(t) \to 0$, $\Omega_1 = \Omega_2$, and $\rho = 1$, the ETS (2.2) can reduce to the static ones in [34–36]. So, the static ETSs in [34–36] are the special case of ETS (2.2). When $\Omega_1 = \Omega_2$, $y = x$, and $\rho = 1$, the ETS (2.2) can reduce to the state-based dynamic ETS in [31]. So, the dynamic ETS in [31] can also be the special case of ETS (2.2). Moreover, when $\Omega_1 = \Omega_2$, the ETS (2.2) can reduce to the dynamic ETS in [33].*

Remark 3. *Refer to the paper [37]. Since the event-triggered condition is only tested at the periodic moment, the minimum value of the time interval of the adjacent event-triggered moment is the sampling period h, which can directly exclude Zeno behavior.*

The quantizer is presented as

$$q(\cdot) = [q_1(\cdot), q_2(\cdot), \cdots, q_m(\cdot)]^T$$

The rigorous definition of $q_i(\cdot)$ is described by

$$q_i(y_i(t_k h)) = \begin{cases} u_i^{(l)} & \frac{1}{1+w_i} u_i^{(l)} < y_i(t_k h) \le \frac{1}{1-w_i} u_i^{(l)},\ y_i(t_k h) > 0 \\ 0 & y_i(t_k h) = 0 \\ -q_i(-y_i(t_k h)) & y_i(t_k h) < 0 \end{cases}$$

with the quantized levels set

$$\{\pm u_i^{(l)} | u_i^{(l)} = (d_i)^l u_i^{(0)}, l = 0, \pm 1, \pm 2, \cdots\} \cup \{0\}, d_i \in (0,1), u_i^{(0)} > 0$$

where $w_i = \frac{1-d_i}{1+d_i}$ $(i = 1, \cdots, m)$, d_i and $u_i^{(0)}$ denote the quantizer density and initial quantization, respectively. From [38], we have

$$q(y(t_k h)) = y(t_k h) + h(y(t_k h))$$

where

$$h(y(t_k h)) = \begin{bmatrix} h_1(y_1(t_k h)) & h_2(y_2(t_k h)) & \cdots & h_m(y_m(t_k h)) \end{bmatrix}^T$$

with

$$-w_i[y_i(t_k h)]^2 \le y_i(t_k h) h_i(y_i(t_k h)) \le w_i [y_i(t_k h)]^2 \ \ (i = 1, \cdots, m) \tag{2.4}$$

Assume that after experiencing the network-induced delay τ_k, $q(y(t_k h))$ arrives the controller side, and the communication network is under cyber-attacks. It is assumed that τ_k satisfies $\underline{\tau} \le \tau_k \le \overline{\tau}$ $(k = 1, 2, \cdots)$, where $\underline{\tau}$ and $\overline{\tau}$ are constants. For $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$, the controller input $\widehat{y}(t)$ is expressed as follows:

$$\widehat{y}(t) = \delta(t_k) q(y(t_k h)) + \kappa(t_k) C f(x(t - \nu(t))) + \lambda(t_k) C \int_{t-\theta(t)}^{t} g(x(q)) dq$$

where $\kappa(t_k) = [1-\delta(t_k)]\varphi(t_k)$, $\lambda(t_k) = [1-\delta(t_k)][1-\varphi(t_k)]$ with $\delta(t_k) \in \{0,1\}$, and $\varphi(t_k) \in \{0,1\}$; $Pb\{\delta(t_k)=1\} = \bar{\delta}$, $Pb\{\delta(t_k)=0\} = 1-\bar{\delta}$, $Pb\{\varphi(t_k)=1\} = \bar{\varphi}$, and $Pb\{\varphi(t_k)=0\} = 1-\bar{\varphi}$; $f(\cdot)$ and $g(\cdot)$ are the cyber-attacks, $\nu(t) \in (0, \nu_M]$, $\theta(t) \in (0, \theta_M]$.

Assumption 1. *[39] Deception attacks $f(\cdot)$ and $g(\cdot)$ are continuous and satisfy*

$$p_j^- \leq \frac{f_j(v_1) - f_j(v_2)}{v_1 - v_2} \leq p_j^+,\ q_j^- \leq \frac{g_j(v_1) - g_j(v_2)}{v_1 - v_2} \leq q_j^+\ (v_1 \neq v_2) \tag{2.5}$$

where q_j^-, q_j^+, p_j^-, p_j^+ $(j = 1, \cdots, n)$ are constants, and $f(0) = g(0) = 0$.

The interval $[t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$ can be divided as

$$[t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) = \cup_{d=0}^{d_k}\chi_{t_k}^d,\ \ d_k = t_{k+1} - t_k - 1$$

where $\chi_{t_k}^d = \left[t_kh + dh + \tau_k^d, t_kh + dh + h + \tau_k^{d+1}\right)$ with $\tau_k^0 = \tau_k$ and $\tau_k^{d_k+1} = \tau_{k+1}$. For $t \in \chi_{t_k}^d$, we define $\tau(t) = t - t_kh - dh$ and $e(t) = \rho y(t_kh) - \rho y(t_kh + dh)$. Clearly,

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M,\quad y(t_kh) = \rho^{-1}e(t) + y(t - \tau(t))$$

where $\tau_m := \underline{\tau}$ and $\tau_M := \overline{\tau} + h$. So, for $t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$, the dynamic ETS (2.2) with (2.3) can be rewritten as

$$e^T(t)\Omega_1 e(t) \leq \sigma\left[\varrho e(t) + y(t-\tau(t))\right]^T \Omega_2 \left[\varrho e(t) + y(t-\tau(t))\right] + \frac{1}{\alpha}\eta(t-\tau(t)) \tag{2.6}$$

where $\varrho = \rho^{-1} - 1$, and

$$\dot{\eta}(t) = -\beta\eta(t) + \sigma\left[\varrho e(t) + y(t-\tau(t))\right]^T \Omega_2 \left[\varrho e(t) + y(t-\tau(t))\right] - e^T(t)\Omega_1 e(t) \tag{2.7}$$

In Figure 1, the controller is

$$u(t) = K\widehat{y}(t) \tag{2.8}$$

where K is the controller gain.

Substituting (2.8) into (2.1), we obtain the following system subject to (2.4), (2.5) and (2.6):

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \bar{\kappa}\mathcal{B}KCf(x(t-\nu(t))) + \bar{\lambda}\mathcal{B}KC\int_{t-\theta(t)}^{t} g(x(r))dr + \mathcal{D}\omega(t) \\ \qquad +\bar{\delta}\mathcal{B}K\left[Cx(t-\tau(t)) + \rho^{-1}e(t) + h(y(t_kh))\right] \\ \qquad + \left[\delta(t_k) - \bar{\delta}\right]\mathcal{B}K\left[Cx(t-\tau(t)) + \rho^{-1}e(t) + h(y(t_kh))\right] \\ \qquad + \left[\kappa(t_k) - \bar{\kappa}\right]\mathcal{B}KCf(x(t-\nu(t))) + \left[\lambda(t_k) - \bar{\lambda}\right]\mathcal{B}KC\int_{t-\theta(t)}^{t} g(x(r))dr \\ z(t) = \mathcal{L}x(t) \end{cases} \tag{2.9}$$

where $t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$, $\bar{\kappa} = (1-\bar{\delta})\bar{\varphi}$, and $\bar{\lambda} = (1-\bar{\delta})(1-\bar{\varphi})$.

Remark 4. *Compared with the papers [32, 34], the event-triggered H_∞ quantized control for NCSs is considered with the conventional deception attack, accumulated dynamic cyber-attack, and dynamic ETS. In this paper, two types of cyber-attacks $f(x)$ and $g(x)$ are considered. The former is assumed as a conventional nonlinear function, while the latter is an accumulated dynamic cyber-attack that is modeled as an integral function. The accumulated dynamic cyber-attack, as a kind of deception attack, is rarely considered.*

For theoretical analysis, we offer two lemmas.

Lemma 1. *[40] For scalar $0 \leq \mu(t) \leq \mu_M$, and matrices $W \in \mathbb{S}_n^+$, $U \in \mathbb{R}^{n\times n}$ satisfying $\left[\begin{smallmatrix} W & * \\ U & W \end{smallmatrix}\right] \geq 0$ and integral function $\{\dot{x}(r) \| r \in [-\mu_M, 0]\}$, we have*

$$-\mu_M \int_{t-\mu_M}^{t} \dot{x}^T(r) W \dot{x}(r)\, dr \leq \psi^T(t) \Lambda \psi(t)$$

where

$$\psi(t) = \begin{bmatrix} x(t) \\ x(t-\mu(t)) \\ x(t-\mu_M) \end{bmatrix}, \quad \Lambda = \begin{bmatrix} -W & * & * \\ W-U & -2W+U+U^T & * \\ U & W-U & -W \end{bmatrix}$$

Lemma 2. *[41] For a scalar $\beta \in (0, 1)$, vectors δ_1 and δ_2, and matrices $W \in \mathbb{S}_n^+$, $U \in \mathbb{R}^{n\times n}$ satisfying $\left[\begin{smallmatrix} W & U \\ * & W \end{smallmatrix}\right] \geq 0$, the following inequality holds:*

$$\frac{1}{\beta}\delta_1^T W \delta_1 + \frac{1}{1-\beta}\delta_2^T W \delta_2 \geq \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}^T \begin{bmatrix} W & U \\ * & W \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$$

3. Main Results

In this paper, we will design a controller such that 1) the system (2.9) is mean-square asymptotically stable for $\omega(t) = 0$; 2) for $\omega(t) \neq 0 \in \mathcal{L}_2[0, \infty)$, $z(t)$ satisfies

$$\|z(t)\|_{\mathscr{E}_2} < \gamma \|\omega(t)\|_2$$

under zero initial condition.

Theorem 1. *For parameters γ, $\bar{\varphi}$, δ, ρ, σ, α, β, τ_M, ν_M, θ_M, $\bar{\delta}$, and K, if there exist matrices P, Q_ℓ, R_ℓ, Z_ℓ ($\ell = 1, 2, 3$), $U \in \mathbb{S}_n^+$, diagonal matrices $F_{b\ell} \geq 0$, $F_{d\ell} \geq 0$, $H_{b\ell} \geq 0$, $H_{d\ell} \geq 0$, $G_{b\ell} \geq 0$, $G_{d\ell} \geq 0$ ($\ell = 1, 2, 3$), $\Omega_1 \geq 0$, $\Omega_2 \geq 0$, $\widehat{\mathcal{D}} > 0$, and appropriate dimensioned matrices N_1, N_2, N_3, and X such that*

$$\begin{bmatrix} \Xi & * & * & * \\ \mathcal{D}^T P e_1 & -\gamma^2 I & * & * \\ \Upsilon & \digamma & \Lambda & * \\ \Gamma & 0 & 0 & \Lambda \end{bmatrix} < 0 \tag{3.1}$$

$$\begin{bmatrix} U & X \\ * & U \end{bmatrix} \geq 0, \ \begin{bmatrix} Q_\ell & * \\ N_\ell & Q_\ell \end{bmatrix} \geq 0 \ \ (\ell = 1, 2, 3) \tag{3.2}$$

where

$$\begin{aligned}
\Xi =& \varXi_1 + \varXi_2 + \varXi_3 + e_1^T \mathcal{L}^T \mathcal{L} e_1 \\
\varXi_1 =& Sym\left\{e_1^T \left(P\mathcal{A}e_1 + \bar{\delta}P\mathcal{B}K\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \bar{\kappa}P\mathcal{B}KCe_9 + \bar{\lambda}P\mathcal{B}KCe_{14}\right)\right\} \\
\varXi_2 =& e_1^T (R_1 + R_2 + R_3) e_1 - e_3^T R_1 e_3 - e_5^T R_2 e_5 - e_7^T R_3 e_7 \\
&+ e_3^T (\theta_\tau Z_3 - \tau_\nu Z_1) e_3 + e_5^T (\tau_\nu Z_1 - \nu_\theta Z_2) e_5 + e_7^T (\nu_\theta Z_2 - \theta_\tau Z_3) e_7 \\
&+ \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}^T \Psi_1 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_4 \\ e_5 \end{bmatrix}^T \Psi_2 \begin{bmatrix} e_1 \\ e_4 \\ e_5 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_6 \\ e_7 \end{bmatrix}^T \Psi_3 \begin{bmatrix} e_1 \\ e_6 \\ e_7 \end{bmatrix} \\
&+ (1 + \alpha\beta)\left[(Ce_2 + \varrho e_{16})^T \sigma\Omega_2 (Ce_2 + \varrho e_{16})\right] \\
&- (1 + \alpha\beta)e_{16}^T \Omega_1 e_{16} + \nu_M^2 e_{11}^T U e_{11} - \begin{bmatrix} e_{14} \\ e_{15} \end{bmatrix}^T \begin{bmatrix} U & X \\ * & U \end{bmatrix} \begin{bmatrix} e_{14} \\ e_{15} \end{bmatrix}
\end{aligned}$$

$$
\begin{aligned}
&+\left(\rho^{-1}e_{16}+Ce_2\right)^T W^T\widehat{\mathcal{D}}W\left(\rho^{-1}e_{16}+Ce_2\right)-e_{17}^T\widehat{\mathcal{D}}e_{17}\\
\Xi_3 =& Sym\{\left(K_pe_1-e_8\right)^T F_{b1}\left(e_8-K_me_1\right)\}\\
&+\sum_{i=2}^{3} Sym\{\left(K_pe_{i+2}-e_{i+7}\right)^T F_{bi}\left(e_{i+7}-K_me_{i+2}\right)\}\\
&+e_1^T\widehat{K}G_{b1}\widehat{K}e_1-e_8^TG_{b1}e_8+\sum_{i=2}^{3}\left\{e_{i+2}^T\widehat{K}G_{bi}\widehat{K}e_{i+2}-e_{i+7}^TG_{bi}e_{i+7}\right\}\\
&+Sym\{\left[K_p(e_1-e_4)-(e_8-e_9)\right]^T H_{b1}\left[(e_8-e_9)-K_m(e_1-e_4)\right]\}\\
&+Sym\{\left[K_p(e_4-e_5)-(e_9-e_{10})\right]^T H_{b2}\left[(e_9-e_{10})-K_m(e_4-e_5)\right]\}\\
&+Sym\{\left[K_p(e_1-e_5)-(e_8-e_{10})\right]^T H_{b3}\left[(e_8-e_{10})-K_m(e_1-e_5)\right]\}\\
&+Sym\{\left(L_pe_1-e_{11}\right)^T F_{d1}\left(e_{11}-L_me_1\right)\}\\
&+\sum_{i=2}^{3} Sym\{\left(L_pe_{i+4}-e_{i+10}\right)^T F_{di}\left(e_{i+10}-L_me_{i+4}\right)\}\\
&+e_1^TLG_{d1}Le_1-e_{11}^TG_{d1}e_{11}+\sum_{i=2}^{3}\left\{e_{i+4}^TLG_{di}Le_{i+4}-e_{i+10}^TG_{di}e_{i+10}\right\}\\
&+Sym\{\left[L_p(e_1-e_6)-(e_{11}-e_{12})\right]^T H_{d1}\left[(e_{11}-e_{12})-L_m(e_1-e_6)\right]\}\\
&+Sym\{\left[L_p(e_6-e_7)-(e_{12}-e_{13})\right]^T H_{d2}\left[(e_{12}-e_{13})-L_m(e_6-e_7)\right]\}\\
&+Sym\{\left[L_p(e_1-e_7)-(e_{11}-e_{13})\right]^T H_{d3}\left[(e_{11}-e_{13})-L_m(e_1-e_7)\right]\}\\
\Psi_\ell =& \begin{bmatrix} -Q_\ell & * & * \\ Q_\ell-N_\ell & -2Q_\ell+N_\ell+N_\ell^T & * \\ N_\ell & Q_\ell-N_\ell & -Q_\ell \end{bmatrix} \quad (\ell=1,2,3)\\
\Upsilon =& \left[\tau_M\Upsilon_1^T \quad \nu_M\Upsilon_2^T \quad \theta_M\Upsilon_3^T\right]^T,\ \Gamma=\left[\tau_M\Gamma_1^T \quad \nu_M\Gamma_2^T \quad \theta_M\Gamma_3^T\right]^T\\
\Upsilon_i =& Q_i\mathcal{A}e_1+Q_i\mathcal{B}K\left(\bar{\delta}\left(Ce_2+\rho^{-1}e_{16}+e_{17}\right)+\bar{\kappa}Ce_9+\bar{\lambda}Ce_{14}\right)\ (i=1,2,3)\\
\Gamma_\ell =& Q_\ell\mathcal{B}K\left(\mu_1\left(Ce_2+\rho^{-1}e_{16}+e_{17}\right)+\mu_2Ce_9+\mu_3Ce_{14}\right)\ (\ell=1,2,3)\\
F =& \left[\tau_M(Q_1\mathcal{D})^T \quad \nu_M(Q_2\mathcal{D})^T \quad \theta_M(Q_3\mathcal{D})^T\right]^T,\ \Lambda=diag\{-Q_1,-Q_2,-Q_3\}\\
e_\ell =& \left[0_{n\times(\ell-1)n} \quad I_{n\times n} \quad 0_{n\times(17-\ell)n}\right]\ (\ell=1,\cdots,17)\\
\tau_\nu =& \tau_M-\nu_M,\ \ \nu_\theta=\nu_M-\theta_M,\ \ \theta_\tau=\theta_M-\tau_M\\
\mu_1 =& \sqrt{\bar{\delta}(1-\bar{\delta})},\quad \mu_2=\sqrt{\bar{\kappa}(1-\bar{\kappa})},\quad \mu_3=\sqrt{\bar{\lambda}(1-\bar{\lambda})}
\end{aligned}
$$

then the system (2.9) is mean-square asymptotically stable.

Proof. The LKF candidate is

$$
\mathcal{V}(t)=\sum_{i=1}^{5}\mathcal{V}_i(t) \tag{3.3}
$$

where

$$
\begin{aligned}
\mathcal{V}_1(t) =& x^T(t)Px(t) \\
\mathcal{V}_2(t) =& \int_{t-\tau_M}^{t} x^T(q)R_1x(q)dq + \int_{t-\nu_M}^{t} x^T(q)R_2x(q)dq + \int_{t-\theta_M}^{t} x^T(q)R_3x(q)dq \\
\mathcal{V}_3(t) =& \tau_\nu \int_{t-\tau_M}^{t-\nu_M} x^T(q)Z_1x(q)dq + \nu_\theta \int_{t-\nu_M}^{t-\theta_M} x^T(q)Z_2x(q)dq \\
&+ \theta_\tau \int_{t-\theta_M}^{t-\tau_M} x^T(q)Z_3x(q)dq \\
\mathcal{V}_4(t) =& \tau_M \int_{t-\tau_M}^{t} \int_{s}^{t} \dot{x}^T(p)Q_1\dot{x}(p)dpds + \nu_M \int_{t-\nu_M}^{t} \int_{s}^{t} \dot{x}^T(p)Q_2\dot{x}(p)dpds \\
&+ \theta_M \int_{t-\theta_M}^{t} \int_{s}^{t} \dot{x}^T(p)Q_3\dot{x}(p)dpds \\
\mathcal{V}_5(t) =& \theta_M \int_{t-\theta_M}^{t} \int_{s}^{t} g^T(x(q))Ug(x(q))dqds
\end{aligned}
$$

From $P > 0$, $U > 0$, $R_\ell > 0$, $Z_\ell > 0$, $Q_\ell > 0$ ($\ell = 1, 2, 3$), we can get $\mathcal{V}(t) > 0$. For convenience, we define

$$
\begin{aligned}
\chi_1(t) &= col\{x(t) \;\; x(t-\tau(t)) \;\; x(t-\tau_M) \;\; x(t-\nu(t)) \;\; x(t-\nu_M)\} \\
\chi_2(t) &= col\{x(t-\theta(t)) \;\; x(t-\theta_M) \;\; f(x(t))\} \\
\chi_3(t) &= col\{f(x(t-\nu(t))) \;\; f(x(t-\nu_M)) \;\; g(x(t)) \;\; g(x(t-\theta(t))) \;\; g(x(t-\theta_M))\} \\
\chi_4(t) &= col\left\{\int_{t-\theta(t)}^{t} g(x(r))dr \quad \int_{t-\theta_M}^{t-\theta(t)} g(x(r))dr \quad e(t) \quad h(y(t_kh))\right\} \\
\xi(t) &= col\{\chi_1(t) \;\; \chi_2(t) \;\; \chi_3(t) \;\; \chi_4(t)\}
\end{aligned}
$$

Computing $\dot{\mathcal{V}}(t)$, we obtain

$$
\dot{\mathcal{V}}_1(t) = 2x^T(t)P\dot{x}(t) \tag{3.4}
$$

$$
\begin{aligned}
\dot{\mathcal{V}}_2(t) =& x^T(t)\left(R_1 + R_2 + R_3\right)x(t) - x^T(t-\tau_M)R_1x(t-\tau_M) \\
&- x^T(t-\nu_M)R_2x(t-\nu_M) - x^T(t-\theta_M)R_3x(t-\theta_M)
\end{aligned} \tag{3.5}
$$

$$
\begin{aligned}
\dot{\mathcal{V}}_3(t) =& x^T(t-\tau_M)\left(\theta_\tau Z_3 - \tau_\nu Z_1\right)x(t-\tau_M) \\
&+ x^T(t-\nu_M)\left(\tau_\nu Z_1 - d_\nu Z_2\right)x(t-\nu_M) \\
&+ x^T(t-\theta_M)\left(\nu_\theta Z_2 - \theta_\tau Z_3\right)x(t-\theta_M)
\end{aligned} \tag{3.6}
$$

$$
\begin{aligned}
\dot{\mathcal{V}}_4(t) =& \dot{x}^T(t)\left(\tau_M^2Q_1 + \nu_M^2Q_2 + \theta_M^2Q_3\right)\dot{x}(t) \\
&- \tau_M \int_{t-\tau_M}^{t} \dot{x}^T(w)Q_1\dot{x}(w)dw - \nu_M \int_{t-\nu_M}^{t} \dot{x}^T(w)Q_2\dot{x}(w)dw \\
&- \theta_M \int_{t-\theta_M}^{t} \dot{x}^T(w)Q_3\dot{x}(w)dw \\
\dot{\mathcal{V}}_5(t) =& \theta_M^2g^T(x(t))Ug(x(t)) - \theta_M \int_{t-\theta_M}^{t} g^T(x(p))Ug(x(p))dp
\end{aligned}
$$

By Lemma 1, there exist N_ℓ ($\ell = 1,2,3$) satisfying (3.2) such that

$$-\tau_M \int_{t-\tau_M}^{t} \dot{x}^T(w)Q_1\dot{x}(w)dw \leq \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau_M) \end{bmatrix}^T \Psi_1 \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau_M) \end{bmatrix} \tag{3.7}$$

$$-\nu_M \int_{t-\nu_M}^{t} \dot{x}^T(w)Q_2\dot{x}(w)dw \leq \begin{bmatrix} x(t) \\ x(t-\nu(t)) \\ x(t-\nu_M) \end{bmatrix}^T \Psi_2 \begin{bmatrix} x(t) \\ x(t-\nu(t)) \\ x(t-\nu_M) \end{bmatrix} \tag{3.8}$$

$$-\theta_M \int_{t-\theta_M}^{t} \dot{x}^T(w)Q_3\dot{x}(w)dw \leq \begin{bmatrix} x(t) \\ x(t-\theta(t)) \\ x(t-\theta_M) \end{bmatrix}^T \Psi_3 \begin{bmatrix} x(t) \\ x(t-\theta(t)) \\ x(t-\theta_M) \end{bmatrix} \tag{3.9}$$

By Lemma 2, we have

$$\begin{aligned} & -\theta_M \int_{t-\theta_M}^{t} g^T(x(p))Ug(x(p))dp \\ \leq & -\begin{bmatrix} \int_{t-\theta(t)}^{t} g(x(p))dp \\ \int_{t-\theta_M}^{t-\theta(t)} g(x(p))dp \end{bmatrix}^T \begin{bmatrix} U & X \\ * & U \end{bmatrix} \begin{bmatrix} \int_{t-\theta(t)}^{t} g(x(p))dp \\ \int_{t-\theta_M}^{t-\theta(t)} g(x(p))dp \end{bmatrix} \end{aligned} \tag{3.10}$$

From (2.4), we obtain

$$\begin{aligned} 0 \leq & -[h(y(t_kh)) + Wy(t_kh)]^T \widehat{\mathfrak{D}}[h(y(t_kh)) - Wy(t_kh)] \\ = & \left(\rho^{-1}e(t) + Cx(t-\tau(t))\right)^T W^T \widehat{\mathfrak{D}} W \left(\rho^{-1}e(t) + Cx(t-\tau(t))\right) \\ & - h^T(y(t_kh))\widehat{\mathfrak{D}}h(y(t_kh)) \end{aligned} \tag{3.11}$$

Based on the inequality (2.5), we obtain

$$\begin{aligned} \widetilde{f}_{1\ell}(w) &:= 2\Big(K_p x(w) - f(x(w))\Big)^T F_{b\ell}\left(f(x(w)) - K_m x(w)\right) \geq 0 \\ \widetilde{g}_{1\ell}(w) &:= x^T(w)\widehat{K}G_{b\ell}\widehat{K}x(w) - f^T(x(w))G_{b\ell}f(x(w)) \geq 0 \\ \widetilde{h}_{1\ell}(r_1,r_2) &:= 2\Big[K_p\left(x(r_1) - x(r_2)\right) - \left(f(x(r_1)) - f(x(r_2))\right)\Big]^T H_{b\ell} \\ &\quad \times \left[\left(f(x(r_1)) - f(x(r_2))\right) - K_m\left(x(r_1) - x(r_2)\right)\right] \geq 0 \\ \widetilde{f}_{2\ell}(w) &:= 2\Big(L_p x(w) - g(x(w))\Big)^T F_{d\ell}\left(g(x(w)) - L_m x(w)\right) \geq 0 \\ \widetilde{g}_{2\ell}(w) &:= x^T(w)LG_{d\ell}Lx(w) - g^T(x(w))G_{d\ell}g(x(w)) \geq 0 \\ \widetilde{h}_{2\ell}(r_1,r_2) &:= 2\Big[L_p\left(x(r_1) - x(r_2)\right) - \left(g(x(r_1)) - g(x(r_2))\right)\Big]^T H_{d\ell} \\ &\quad \times \left[\left(g(x(r_1)) - g(x(r_2))\right) - L_m\left(x(r_1) - x(r_2)\right)\right] \geq 0 \end{aligned}$$

where $F_{b\ell}$, $F_{d\ell}$, $G_{b\ell}$, $G_{d\ell}$, $H_{b\ell}$, $H_{d\ell}$ ($\ell = 1,2,3$) are n-dimensioned diagonal matrices, and

$$\begin{aligned} K_m &= diag\{p_1^-, p_2^-, \cdots, p_n^-\}, \qquad K_p = diag\{p_1^+, p_2^+, \cdots, p_n^+\} \\ \widehat{K} &= diag\{p_1, p_2, \cdots, p_n\}, \qquad p_\ell = \max\left\{\left\|p_\ell^+\right\|, \left\|p_\ell^-\right\|\right\} (\ell = 1, \cdots, n) \end{aligned}$$

$$L_m = diag\left\{q_1^-, q_2^-, \cdots, q_n^-\right\}, \qquad L_p = diag\left\{q_1^+, q_2^+, \cdots, q_n^+\right\}$$
$$L = diag\left\{q_1, q_2, \cdots, q_n\right\}, \qquad q_\ell = \max\left\{\left\|q_\ell^+\right\|, \left\|q_\ell^-\right\|\right\}(\ell = 1, \cdots, n)$$

Thus, we obtain

$$\begin{aligned}
&\widetilde{f}_{11}(t) + \widetilde{f}_{12}(t-\nu(t)) + \widetilde{f}_{13}(t-\nu_M) + \widetilde{g}_{11}(t) + \widetilde{g}_{12}(t-\nu(t)) + \widetilde{g}_{13}(t-\nu_M)\\
&+\widetilde{h}_{11}(t, t-\nu(t)) + \widetilde{h}_{12}(t-\nu(t), t-\nu_M) + \widetilde{h}_{13}(t, t-\nu_M)\\
&+\widetilde{f}_{21}(t) + \widetilde{f}_{22}(t-\theta(t)) + \widetilde{f}_{23}(t-\theta_M) + \widetilde{g}_{21}(t) + \widetilde{g}_{22}(t-\theta(t)) + \widetilde{g}_{23}(t-\theta_M)\\
&+\widetilde{h}_{21}(t, t-\theta(t)) + \widetilde{h}_{22}(t-\theta(t), t-\theta_M) + \widetilde{h}_{23}(t, t-\theta_M) = \xi^T(t)\Xi_3\xi(t) \geq 0
\end{aligned} \tag{3.12}$$

From (2.6), we have

$$\begin{aligned}
0 = &-\beta\eta(t-\tau(t)) + [\varrho e(t) + y(t-\tau(t))]^T \sigma\Omega_2 [\varrho e(t) + y(t-\tau(t))] - e^T(t)\Omega_1 e(t)\\
&\leq (1+\alpha\beta)\left[(\varrho e(t) + Cx(t-\tau(t)))^T \sigma\Omega_2 (\varrho e(t) + Cx(t-\tau(t))) - e^T(t)\Omega_1 e(t)\right]
\end{aligned} \tag{3.13}$$

Now, using (3.4)–(3.12) together with (3.13) yields

$$\dot{\mathcal{V}}(t) \leq \dot{x}^T(t)Q\dot{x}(t) + \xi^T(t)(\Xi_2 + \Xi_3)\xi(t) + 2x^T(t)P\dot{x}(t)$$

where $Q = \tau_M^2 Q_1 + \nu_M^2 Q_2 + \theta_M^2 Q_3$.

Let $\omega(t) = 0$. Taking the expectation of $\dot{\mathcal{V}}(t)$, we obtain

$$\mathscr{E}\left\{\dot{\mathcal{V}}(t)\right\} \leq \mathscr{E}\left\{\dot{x}^T(t)Q\dot{x}(t)\right\} + \mathscr{E}\left\{\xi^T(t)(\Xi_2 + \Xi_3)\xi(t)\right\} + \mathscr{E}\left\{2x^T(t)P\dot{x}(t)\right\} \tag{3.14}$$

Note that

$$\mathscr{E}\left\{2x^T(t)P\dot{x}(t)\right\} = 2\xi^T(t)e_1^T P\Pi_0\xi(t) = \xi^T(t)\Xi_1\xi(t) \tag{3.15}$$
$$\mathscr{E}\left\{\dot{x}^T(t)Q\dot{x}(t)\right\} = \mathscr{E}\left\{\xi^T(t)\left(\Pi_0^T Q\Pi_0 + \Pi_1^T Q\Pi_1\right)\xi(t)\right\} \tag{3.16}$$

where

$$\Pi_0 = \mathcal{A}e_1 + \bar{\delta}\mathcal{B}K\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \bar{\kappa}\mathcal{B}KCe_9 + \bar{\lambda}\mathcal{B}KCe_{14}$$
$$\Pi_1 = \mu_1\mathcal{B}K\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \mu_2\mathcal{B}KCe_9 + \mu_3\mathcal{B}KCe_{14}$$

From (3.14)–(3.16), we have

$$\begin{aligned}
\mathscr{E}\left\{\dot{\mathcal{V}}(t)\right\} \leq &\mathscr{E}\left\{\xi^T(t)\left(\Xi_1 + \Xi_2 + \Xi_3 + \Pi_0^T Q\Pi_0 + \Pi_1^T Q\Pi_1\right)\xi(t)\right\}\\
= &\mathscr{E}\left\{\xi^T(t)\left(\Xi_1 + \Xi_2 + \Xi_3 - \Upsilon_1^T\Lambda^{-1}\Upsilon_1 - \Upsilon_2^T\Lambda^{-1}\Upsilon_2\right)\xi(t)\right\}
\end{aligned}$$

From (3.1), we can get

$$\mathscr{E}\{\dot{\mathcal{V}}(t)\} < 0$$

When $\omega(t) \neq 0$,

$$\mathscr{E}\left\{2x^T(t)P\dot{x}(t)\right\} = \xi^T(t)\Xi_1\xi(t) + 2\xi^T(t)e_1^T P\mathcal{D}\omega(t) \tag{3.17}$$

and

$$\mathscr{E}\left\{\dot{x}^T(t)Q\dot{x}(t)\right\} = (\Pi_0\xi(t) + \mathcal{D}\omega(t))^T Q(\Pi_0\xi(t) + \mathcal{D}\omega(t)) + \xi^T(t)\Pi_1^T Q\Pi_1\xi(t) \tag{3.18}$$

By (2.1), we get

$$z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) = \xi^T(t)e_1^T\mathcal{L}^T\mathcal{L}e_1\xi(t) - \gamma^2\omega^T(t)\omega(t) \tag{3.19}$$

Based on (3.17)–(3.19) together with (3.14), we have

$$\begin{aligned}
&\mathscr{E}\left\{\dot{\mathcal{V}}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right\}\\
\leq&\mathscr{E}\Big\{\widehat{\chi}^T(t)\begin{bmatrix}\Xi & * \\ \mathcal{D}^T Pe_1 & -\gamma^2 I\end{bmatrix}\widehat{\chi}(t)\\
&-\widehat{\chi}^T(t)\Big(\begin{bmatrix}\Upsilon_1 & F\end{bmatrix}^T\Lambda^{-1}\begin{bmatrix}\Upsilon_1 & F\end{bmatrix} + \begin{bmatrix}\Upsilon_2 & 0\end{bmatrix}^T\Lambda^{-1}\begin{bmatrix}\Upsilon_2 & 0\end{bmatrix}\Big)\widehat{\chi}(t)\Big\}
\end{aligned} \tag{3.20}$$

where $\widehat{\chi}(t) = col\{\xi(t) \;\; \omega(t)\}$. So, under zero initial condition, we get

$$\mathscr{E}\left\{\int_0^\infty \dot{\mathcal{V}}(\ell)d\ell\right\} = 0$$

By (3.20) and (3.1), we have

$$\mathscr{E}\left\{\int_0^\infty \left[z^T(\ell)z(\ell) - \gamma^2\omega^T(\ell)\omega(\ell)\right]d\ell\right\} < 0$$

which means $\|z(t)\|_{\mathscr{E}_2} < \gamma\|\omega(t)\|_2$. □

Remark 5. *Compared with recently reported works [31, 32], we use more system information. First, the information of quantization error $h(y(t_kh))$ is used. Second, the cyber-attack functions $g(x(t))$, $f(x(t))$, and their ramifications*

$$\begin{aligned}
&g(x(t-\nu(t))), g(x(t-\nu_M)), f(x(t-d(t))), f(x(t-d_M))\\
&\int_{t-\nu(t)}^{t} g(x(s))ds, \int_{t-\nu_M}^{t-\nu(t)} g(x(s))ds
\end{aligned}$$

are all utilized. Finally, the dynamic variable $\eta(t)$ is employed based on the ETS (2.2).

By Theorem 1, we will give H_∞ controller design for NCSs with deception attacks.

Theorem 2. *For parameters γ, $\bar{\delta}$, δ, $\bar{\varphi}$, α, τ_M, ν_M, θ_M, β, ρ, σ, θ_1, and θ_2, if there exist matrices P, Q_ℓ, R_ℓ, Z_ℓ ($\ell = 1, 2, 3$), $U \in \mathbb{S}_n^+$, appropriate dimensioned diagonal matrices $F_{b\ell} \geq 0$, $F_{d\ell} \geq 0$, $H_{b\ell} \geq 0$, $H_{d\ell} \geq 0$, $G_{b\ell} \geq 0$, $G_{d\ell} \geq 0$ ($\ell = 1, 2, 3$), $\Omega_1 \geq 0$, $\Omega_2 \geq 0$, $\widehat{\mathcal{D}} > 0$, and matrices N_1, N_2, N_3, X, M, and Y such that (3.2) and the following LMI hold:*

$$\begin{bmatrix}
\widetilde{\Xi} & e_1^T P\mathcal{D} & \widetilde{\Upsilon}^T & \widetilde{\Gamma}^T & e_1^T(P\mathcal{B}-\mathcal{B}M) & \Theta_1 & 0 & \Theta_2\\
* & -\gamma^2 I & F^T & 0 & 0 & 0 & 0 & 0\\
* & * & \Lambda & 0 & \Delta & 0 & 0 & 0\\
* & * & * & \Lambda & 0 & 0 & \Delta & 0\\
* & * & * & * & \theta_1 I-2M & 0 & 0 & 0\\
* & * & * & * & * & -\theta_1 I & 0 & 0\\
* & * & * & * & * & * & \theta_2 I-2M & 0\\
* & * & * & * & * & * & * & -\theta_2 I
\end{bmatrix} < 0 \tag{3.21}$$

where

$$\begin{aligned}
\widetilde{\Xi} &= \widetilde{\Xi}_1 + \Xi_2 + \Xi_3 + e_1^T \mathcal{L}^T \mathcal{L} e_1 \\
\widetilde{\Xi}_1 &= Sym\left\{e_1^T \left(P\mathcal{A}e_1 + \bar{\delta}\mathcal{B}Y\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \bar{\kappa}\mathcal{B}YCe_9 + \bar{\lambda}\mathcal{B}YCe_{14}\right)\right\} \\
\widetilde{\Upsilon} &= \begin{bmatrix} \tau_M\widetilde{\Upsilon}_1^T & \nu_M\widetilde{\Upsilon}_2^T & \theta_M\widetilde{\Upsilon}_3^T \end{bmatrix}^T, \widetilde{\Gamma} = \begin{bmatrix} \tau_M\widetilde{\Gamma}_1^T & \nu_M\widetilde{\Gamma}_1^T & \theta_M\widetilde{\Gamma}_1^T \end{bmatrix}^T \\
\widetilde{\Upsilon}_\ell &= Q_\ell\mathcal{A}e_1 + \mathcal{B}Y\left(\bar{\delta}\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \bar{\kappa}Ce_9 + \bar{\lambda}Ce_{14}\right) \ (\ell = 1, 2, 3) \\
\widetilde{\Gamma}_1 &= \mathcal{B}Y\left(\mu_1\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \mu_2Ce_9 + \mu_3Ce_{14}\right) \\
\Theta_1 &= \left[\bar{\delta}\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \bar{\kappa}Ce_9 + \bar{\lambda}Ce_{14}\right]^T Y^T \\
\Theta_2 &= \left[\mu_1\left(Ce_2 + \rho^{-1}e_{16} + e_{17}\right) + \mu_2Ce_9 + \mu_3Ce_{14}\right]^T Y^T \\
\Delta &= \begin{bmatrix} \tau_M(Q_1\mathcal{B} - \mathcal{B}M)^T & \nu_M(Q_2\mathcal{B} - \mathcal{B}M)^T & \theta_M(Q_3\mathcal{B} - \mathcal{B}M)^T \end{bmatrix}^T
\end{aligned}$$

where F, Λ, Ξ_2, and Ξ_3 are defined as in Theorem 1. Then the closed-loop system (2.9) with the controller gain $K = M^{-1}Y$ is mean-square asymptotically stable.

Proof. From (3.21) and the inequality $-M^T\theta^{-1}M \leq \theta I - 2M$, we have

$$\begin{bmatrix}
\widetilde{\Xi} & e_1^T P\mathcal{D} & \widetilde{\Upsilon}^T & \widetilde{\Gamma}^T & e_1^T(P\mathcal{B}-\mathcal{B}M) & \Theta_1 & 0 & \Theta_2 \\
* & -\gamma^2 I & F^T & 0 & 0 & 0 & 0 & 0 \\
* & * & \Lambda & 0 & \Delta & 0 & 0 & 0 \\
* & * & * & \Lambda & 0 & 0 & \Delta & 0 \\
* & * & * & * & -M^T\theta_1^{-1}M & 0 & 0 & 0 \\
* & * & * & * & * & -\theta_1 I & 0 & 0 \\
* & * & * & * & * & * & -M^T\theta_2^{-1}M & 0 \\
* & * & * & * & * & * & * & -\theta_2 I
\end{bmatrix} < 0 \tag{3.22}$$

By using the Schur-complement to (3.22), one can obtain that

$$\begin{bmatrix}
\widetilde{\Xi} & e_1^T P\mathcal{D} & \widetilde{\Upsilon}^T & \widetilde{\Gamma}^T \\
* & -\gamma^2 I & F^T & 0 \\
* & * & \Lambda & 0 \\
* & * & * & \Lambda
\end{bmatrix} + \theta_1 a_1 a_1^T + \theta_1^{-1} b_1 b_1^T + \theta_2 a_2 a_2^T + \theta_2^{-1} b_2 b_2^T < 0$$

where

$$\begin{aligned}
a_1 &= \begin{bmatrix} (P\mathcal{B} - \mathcal{B}M)^T e_1 & 0 & \Delta^T & 0 \end{bmatrix}^T M^{-1}, \ b_1 = \begin{bmatrix} \Theta_1^T & 0 & 0 & 0 \end{bmatrix}^T \\
a_2 &= \begin{bmatrix} 0 & 0 & 0 & \Delta^T \end{bmatrix}^T M^{-1}, \ b_2 = \begin{bmatrix} \Theta_2^T & 0 & 0 & 0 \end{bmatrix}^T
\end{aligned}$$

Furthermore, in view of the inequality $ab^T + ba^T \leq \theta aa^T + \theta^{-1}bb^T$, we have

$$\begin{bmatrix}
\widetilde{\Xi} & e_1^T P\mathcal{D} & \widetilde{\Upsilon}^T & \widetilde{\Gamma}^T \\
* & -\gamma^2 I & F^T & 0 \\
* & * & \Lambda & 0 \\
* & * & * & \Lambda
\end{bmatrix} + a_1 b_1^T + b_1 a_1^T + a_2 b_2^T + b_2 a_2^T < 0$$

which is (3.1). From Theorem 1, the system (2.9) under (2.4)–(2.6) is mean-square asymptotically stable. □

Remark 6. *Notice that there are some effective methods to deal with coupling nonlinear terms [9,34,40]. Among the fruitful methods, the methods in [9,34,40] assumed that $\mathcal{B}$ must be a full column rank matrix to solve nonlinear term $P\mathcal{B}K$. However, in general, matrix $\mathcal{B}$ is not always a full column rank matrix in many practical systems. So, the mentioned methods are no longer valid. In Theorem 2, we do not assume that $\mathcal{B}$ is a full column rank matrix.*

4. Numerical examples

Here, we list two examples to show the validity of the derived results.

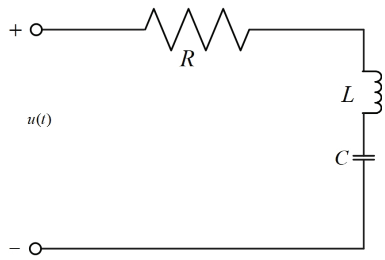

Figure 2. RLC series circuit.

Example 1. Consider the RLC series circuit system (2.1) in Figure 2 with the following parameters [32]:

$$\mathcal{A} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix},\ \mathcal{B} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix},\ \mathcal{D} = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}, \mathcal{L} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \omega(t) = e^{-0.2t}\sin 0.5t$$

where $L = 1H$, $C = 1F$, and $R = 1\Omega$. The cyber-attacks are assumed as

$$f(x) = \begin{bmatrix} -tanh(0.5x_2) \\ -tanh(0.3x_1) \end{bmatrix}, g(x) = \begin{bmatrix} -tanh(0.03x_1) \\ -tanh(0.06x_2) \end{bmatrix}$$

and the quantizer densities $d_1 = 9/11$, and $d_2 = 2/3$. We can obtain $K_m = diag\{0, 0\}$, $K_p = diag\{0.3, 0.5\}$, $L_p = diag\{0.03, 0.06\}$, and $L_m = diag\{0, 0\}$.

Set $\gamma = 1$, $\theta_1 = \theta_2 = 1$, $\bar{\delta} = 0.5$, $\bar{\varphi} = 0.5$, $\nu_M = 0.4$, $\tau_M = 0.3$, $\theta_M = 0.4$, $\delta = 0.5$, $\alpha = 5, \beta = 0.1$, $\rho = 0.3$, $h = 0.1$, and $\sigma = 0.01$. By Theorem 2, we have

$$P = \begin{bmatrix} 1.2019 & -0.2693 \\ -0.2693 & 0.9885 \end{bmatrix},\ Y = \begin{bmatrix} 0.1342 \\ 0.0370 \end{bmatrix}^T,\ M = 1.1353$$
$$\Omega_1 = \begin{bmatrix} 0.9816 & 0.1228 \\ 0.1228 & 0.9557 \end{bmatrix},\ \Omega_2 = \begin{bmatrix} 0.8283 & -0.0120 \\ -0.0120 & 0.6512 \end{bmatrix}$$

Then the controller gain is

$$K = \begin{bmatrix} 0.1182 & 0.0326 \end{bmatrix}$$

For $x(0) = \begin{bmatrix} 6 & -2 \end{bmatrix}^T$, Figure 3 shows the state trajectory of the system (2.9). It is not difficult to see that the state trajectory of the system (2.9) is convergent to zero, which shows the system (2.9) is stochastic asymptotically stable.

For $\eta(0) = 0.0001$ and $x(0) = \begin{bmatrix} 6 & -2 \end{bmatrix}^T$, Figure 4 shows the event-triggered instants and release intervals. From Figure 4, during the period [0, 40s], only 55 times are triggered. So, we can obtain that

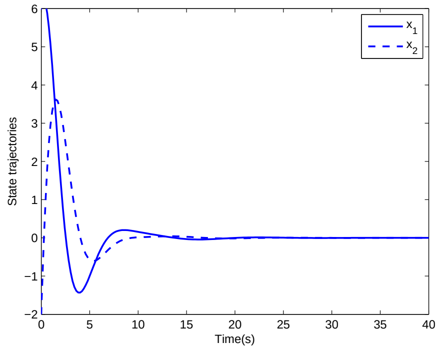

Figure 3. State trajectory of NCSs.

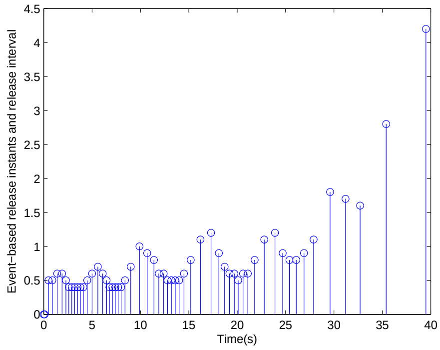

Figure 4. The diagram of ETS.

the average trigger rate is 13.75%, which is less than the average trigger rate of 15.78% in [32]. In this sense, Theorem 2 is more effective than the method in [32]. Moreover, we can also obtain the desired result by adjusting the parameter ρ.

Example 2. Consider an NCS (2.1) with parameters

$$\mathcal{A} = \begin{bmatrix} -1.9 & 0 & 0 \\ -0.2 & -1.1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & -0.6 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}$$

$$\mathcal{L} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} -1.1 \\ -1.2 \\ 0.9 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, \omega(t) = e^{-0.3t}$$

The cyber-attacks are assumed as

$$f(x) = \begin{bmatrix} tanh(0.04x_1) \\ tanh(0.04x_2) \\ tanh(0.04x_3) \end{bmatrix}, g(x) = \begin{bmatrix} tanh(0.03x_1) \\ tanh(0.06x_2) \\ tanh(0.03x_3) \end{bmatrix}$$

and quantizer densities $d_1 = 9/11$, $d_2 = 2/3$, and $d_3 = 9/11$. Clearly, we have $K_m = diag\{0, 0, 0\}$, $K_p = diag\{0.04, 0.04, 0.04\}$, $L_m = diag\{0, 0, 0\}$, and $L_p = diag\{0.03, 0.06, 0.03\}$.

Set $\gamma = 1$, $\theta_1 = \theta_2 = 1$, $\bar{\delta} = 0.5$, $\bar{\varphi} = 0.5$, $\nu_M = 0.4$, $\tau_M = 0.3$, $\theta_M = 0.4$, $\delta = 0.5$, $\alpha = 5$, $\beta = 0.1$, $\rho = 0.5$, $\sigma = 0.01$, and $\eta_0 = 0.0001$. For different h, the number of triggered data during [0,50s] are listed in Table 1.

Table 1. The number of triggered data.

h	0.1	0.2	0.5
ETS in [36]	91	85	62
ETS in [31]	56	46	40
This paper	45	22	19

From Table 1, it is clear that the number of triggered data for various h are far less than those in [31] and [36], which means our ETS is more advanced.

Set $h = 0.1$, $\gamma = 1$, $\theta_1 = \theta_2 = 1$, $\bar{\delta} = 0.5$, $\bar{\varphi} = 0.5$, $\nu_M = 0.4$, $\tau_M = 0.3$, $\theta_M = 0.4$, $\alpha = 5$, $\delta = 0.5$, $\beta = 0.1$, $\rho = 0.5$, and $\sigma = 0.01$. By Theorem 2, we can get

$$P = \begin{bmatrix} 3.9509 & -0.4407 & 0.0462 \\ -0.4407 & 4.6739 & 0.2956 \\ 0.0462 & 0.2956 & 4.1812 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.0271 \\ -0.0376 \\ -0.2474 \end{bmatrix}^T, \quad M = 3.3924$$

$$\Omega_1 = \begin{bmatrix} 3.4005 & 0.2000 & 0.1730 \\ 0.2000 & 3.5989 & 0.2123 \\ 0.1730 & 0.2123 & 3.4867 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} 3.8230 & -0.0657 & -0.0176 \\ -0.0657 & 3.8690 & -0.0152 \\ -0.0176 & -0.0152 & 3.8749 \end{bmatrix}$$

Then the controller gain $K = \begin{bmatrix} 0.0080 & -0.0111 & -0.0729 \end{bmatrix}$.

For $x(0) = \begin{bmatrix} -0.5 & 0.3 & -0.5 \end{bmatrix}^T$, Figure 5 shows the state trajectory of the system (2.9). It is not difficult to see that the state trajectory of system (2.9) is convergent to zero, which denotes the system (2.9) is mean-square asymptotically stable.

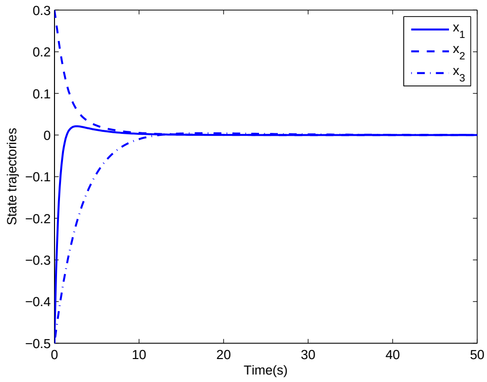

Figure 5. State trajectory of NCSs.

Choose $\eta(0) = 10^{-6}$. Figure 6 shows the event-triggered instants and release intervals. As seen from Figure 6, during period [0, 50 s], the number of transmitted data is 45. So, the average trigger rate is 9%, and more system resources can be saved.

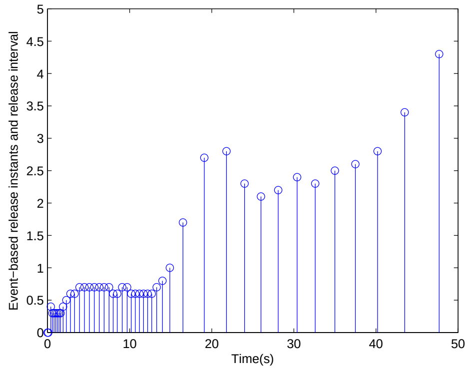

Figure 6. Event-triggered instants.

5. Conclusions

The problem of dynamic event-triggered control for NCSs with deception attacks has been studied. With the dynamic ETS and quantized mechanism being introduced, a closed-loop control model was formulated subject to stochastic deception attacks. By the LKF approach, an H_∞ performance condition was derived, and the event-triggered quantized H_∞ controller with deception attacks was designed. Finally, two examples were given to show the reliability of the derived methods.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflicts of interest.

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