



Research article

Resilient decentralized event-triggered control for interconnected power systems under aperiodic DoS attacks

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Abstract: This work investigated the decentralized load frequency control (LFC) problem for multi-area power systems with renewable energy sources under aperiodic denial-of-service (DoS) attacks. A decentralized resilient event-triggered mechanism (DRETM) was proposed, in which a standard output-based ETM governs transmissions during DoS-free intervals and a forced recovery transmission is introduced after each DoS-active interval, enabling prompt closed-loop recovery while explicitly accounting for network transmission delays. A decentralized dynamic output-feedback (DOF) switched controller was developed without centralized state collection. Sufficient conditions are derived for local exponential stability and weighted $\mathcal{H}_\infty$ attenuation under aperiodic DoS attacks. The neighboring-area frequency terms were recognised as internal physical interconnection inputs rather than independent disturbances. After the decentralized controller gains were recovered, the original physical feedback was explicitly restored and the complete two-area closed loop was verified by a fixed-gain joint stability condition with two independent equivalent-delay channels.

Keywords: dynamic output-feedback control; aperiodic DoS attacks; resilient event-triggered mechanism; decentralized load frequency control

1. Introduction

The frequency of a power system is a key indicator of secure and stable operation. With the growth of electricity demand and renewable generation, modern power systems have become increasingly interconnected across multiple control areas [1], which improves power-balancing flexibility but also increases regulation complexity. Load frequency control (LFC) regulates active power generation to restore system frequency and tie-line power following disturbances [2]. Accordingly, robust $\mathcal{H}_\infty$ control [3], model predictive control [4], fault-tolerant control [5], and data-driven control [6] have been investigated for frequency regulation.

Modern LFC increasingly relies on communication networks for real-time information exchange,

which also exposes frequency regulation to network delays, packet loss, and cyber attacks [7, 8]. Denial-of-service (DoS) attacks [9], in particular, may interrupt sensor-to-controller measurement transmission and degrade closed-loop performance. During prolonged DoS-active intervals, controllers may receive no fresh measurements, which can be particularly detrimental during large load changes [10]. Existing resilient LFC studies have described communication disruptions using random packet-loss models [11], periodic DoS attacks [12, 13], and energy-limited DoS processes [14, 15].

To reduce communication burden while maintaining closed-loop regulation under network constraints, event-triggered mechanisms have subsequently been widely investigated in networked and resilient control systems. Event-triggered frequency regulation has also been investigated from decentralized, secure, and output-feedback perspectives, including decentralised LFC schemes [16, 17], secure event-triggered regulation under DoS attacks [2], and observer-based output-feedback control with communication delays [18]. Among these studies, periodic event-triggered consensus under switching topology was studied for stochastic multi-agent systems in [19], while a memory-based adaptive ETM was developed for secure synchronization of Markovian jump neural networks under DoS attacks in [20]. For networked LFC, Shen et al. [21] proposed a switching-like event-triggered sliding-mode strategy under energy-limited DoS attacks. These studies demonstrate periodic, adaptive, and switching-like triggering strategies, but address different subsets of the present problem. In particular, they do not simultaneously incorporate decentralized output-feedback LFC, bounded time-varying sampled-data delays, aperiodic DoS frequency/duration constraints, and explicit post-DoS recovery within one switched framework. The present DRETm therefore combines local output-based triggering during DoS-free intervals with a forced recovery transmission after each DoS-active interval, while the associated mode-dependent Lyapunov–Krasovskii functional (LKF) separately characterizes the DoS-free and DoS-active dynamics.

Table 1. Comparison of representative event-triggered mechanisms, control structures, and Lyapunov frameworks.

Reference	Event-triggering mechanism	Controller / control structure	Forced recovery	LKF construction
[19]	Periodic event-triggering	Consensus control protocol	N/A	Lyapunov-based analysis
	Observed-error criterion			Mean-square consensus framework
[20]	Memory-based adaptive triggering	Synchronization controller	No explicit recovery	Unified LKF
	Adaptive threshold			Loop functional
[21]	Switching-like event-triggering	Sliding-mode controller	No explicit recovery	Lyapunov-based analysis
	acknowledgment character (ACK)-based dual-threshold logic			Conventional framework
This paper	Decentralized output-based DRETm	Decentralized dynamic output-feedback controller	Post-DoS forced recovery	Mode-dependent piecewise LKF
	Local output-error criterion			Independent dual-delay joint LKF

Among the closely related studies, Zhong et al. [22] investigated adaptive event-triggered proportional-integral-derivative (PID) load frequency control under aperiodic DoS attacks, whereas Liu et al. [23] developed event-triggered H_∞ load frequency control under hybrid cyber attacks, while resilient distributed coordination of multi-area power systems under hybrid attacks was studied in [24]. More recently, Liu et al. developed a data-driven resilient predictive-control framework based on pre-collected input–output trajectories under DoS attacks [25]. In a separate study, a deadbeat controller and resilient transmission protocol were co-designed for quantized communication under DoS attacks [26]. Compared with these studies, the present framework differs in five aspects. First, the attack model

focuses on DoS-induced switching of the sensor-to-controller (S-C) channel while explicitly retaining bounded physical and sampled-data delays. Second, the communication architecture is decentralized and output-based, with a forced measurement transmission introduced immediately after DoS recovery. Third, the proposed controller is a switched dynamic output-feedback controller, rather than the PID, data-driven predictive, or deadbeat-based structures considered in the above studies. Fourth, the proposed synthesis is model-based and requires the subsystem matrices at the design stage, in contrast to the trajectory-based data-driven formulation in [25], while the deadbeat-based transmission protocol in [26] addresses a different controller/communication architecture. Finally, the analysis establishes local exponential stability and weighted H_∞ disturbance attenuation for each subsystem with the inter-area coupling retained as an open physical interconnection port, followed by fixed-gain exponential-stability verification of the restored physical two-area closed loop.

Against this background, the above observations motivate a resilient decentralized LFC framework integrating communication recovery, delayed output feedback, and interconnection-aware stability analysis. The main contributions are summarized as follows.

1) A unified decentralized resilient event-triggered framework is developed for delayed S-C communication under aperiodic DoS attacks, integrating local output-based triggering and post-DoS forced recovery transmission within the same communication model.

2) A decentralized switched dynamic output-feedback controller is designed with a mode-dependent piecewise Lyapunov–Krasovskii functional, yielding sufficient conditions for local exponential stability and weighted H_∞ attenuation under bounded sampled-data delays.

3) The physical inter-area coupling is explicitly retained in the stability analysis. After controller synthesis, the complete two-area closed loop is restored and verified through a fixed-gain joint stability condition with two independent equivalent-delay channels.

Notations. $\mathbb{N}$ and $\mathbb{N}_0 = \{0, 1, 2, \dots\}$ denote the sets of positive and nonnegative integers, respectively. I and 0 denote identity and zero matrices of compatible dimensions. The symbol $*$ denotes the symmetric counterpart in a block matrix, $\text{diag}\{\cdot\}$ denotes a block-diagonal matrix, and $\text{He}(X) = X + X^T$. The notation $\text{col}\{x_1, \dots, x_n\}$ denotes vertical stacking and $\text{Im}(R_p)$ denotes the range of R_p .

2. Model description and preliminaries

2.1. System framework

The structure of the i -th area networked LFC system is presented in Figure 1, where the communication network is affected by aperiodic DoS attacks. The variables and parameters of the i -th area LFC system are defined in Table 2. The dynamic model of a multi-area power system considering the decentralized LFC problem can be expressed as follows [27]:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + F_i \omega_i(t) \\ y_i(t) = C_i x_i(t), \end{cases} \quad (2.1)$$

where

$$x_i(t) = \begin{bmatrix} \Delta f_i(t) & \Delta P_{tie-i}(t) & \Delta P_{Si}(t) & \Delta P_{mi}(t) & \Delta P_{vi}(t) & \Delta P_{Wi}(t) \end{bmatrix}^T,$$

$$d_i(t) = \begin{bmatrix} \Delta P_{di}(t) & \Delta P_{solar-i}(t) & \Delta P_{wind-i}(t) \end{bmatrix}^T, \quad v_i(t) = \sum_{j=1, j \neq i}^N T_{ij} \Delta f_j(t), \quad \omega_i(t) = \begin{bmatrix} d_i^T(t) & v_i(t) \end{bmatrix}^T,$$

$$y_i(t) = \begin{bmatrix} \beta_i \Delta f_i(t) & \Delta P_{tie-i}(t) \end{bmatrix}^T,$$

$$A_i = \begin{bmatrix} -\frac{D_i}{M_i} & -\frac{1}{M_i} & \frac{1}{M_i} & \frac{1}{M_i} & 0 & \frac{1}{M_i} \\ 2\pi \sum_{j=1, j \neq i}^N T_{ij} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{SLi}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{chi}} & \frac{1}{T_{chi}} & 0 \\ -\frac{1}{R_i T_{gi}} & 0 & 0 & 0 & -\frac{1}{T_{gi}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{Wi}} \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{T_{gi}} \\ 0 \end{bmatrix}, C_i = \begin{bmatrix} \beta_i & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T, F_i = \begin{bmatrix} -\frac{1}{M_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -2\pi \\ 0 & \frac{1}{T_{SLi}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T_{Wi}} & 0 \end{bmatrix}.$$

Table 2. Definitions of the variables and parameters.

Notation	Name
$\Delta f_i(t)$	Frequency deviation
$\Delta P_{mi}(t)$	Generator mechanical-power deviation
$\Delta P_{vi}(t)$	Valve-position deviation
$\Delta P_{di}(t)$	Load-power deviation
$\Delta P_{Wi}(t)$	Wind-power deviation
$\Delta P_{Si}(t)$	Solar-power deviation
$\Delta P_{tie-i}(t)$	Tie-line power deviation
$\Delta P_{solar-i}(t)$	Solar disturbance
$\Delta P_{wind-i}(t)$	Wind disturbance
M_i	Rotational inertia of generators
D_i	Damping factor for generators
T_{gi}	Governor time constant
T_{chi}	Turbine time constant
T_{Wi}	Wind-turbine time constant
T_{SLi}	Solar-power time constant
R_i	Speed droop coefficient
β_i	Frequency bias factor
T_{ij}	Tie-line synchronizing coefficient

The vector $d_i(t)$ collects the exogenous load and renewable-generation disturbances acting on Area i . Following the standard multi-area tie-line model in [16], $v_i(t) = \sum_{j=1, j \neq i}^N T_{ij} \Delta f_j(t)$ is generated by the neighboring-area frequency states. Accordingly, in the present analysis, $v_i(t)$ is treated as an internal physical interconnection input rather than an independent exogenous disturbance. For compact local analysis,

is termed the generalized local input. The physical feedback relation associated with v_i is restored after the decentralized controller gains have been recovered.

Thus, v_i is only the neighboring-frequency component of the complete tie-line equation and cannot be interpreted independently of the neighboring-area dynamics when stability of the complete interconnection is considered.

Following [28], an aperiodic DoS model is adopted to describe intermittent interruptions of the communication channel:

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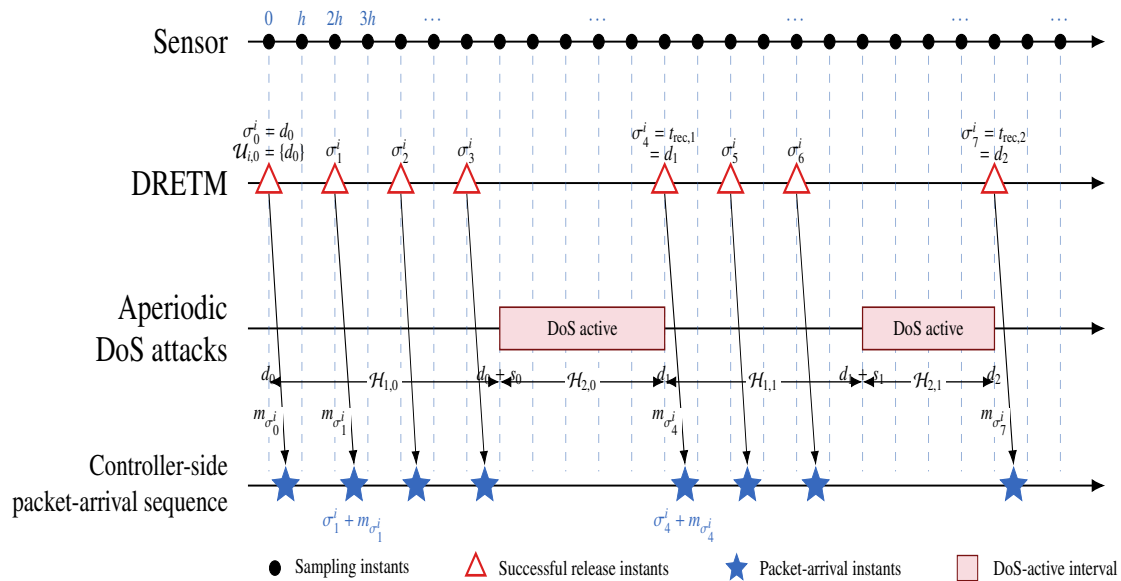


Figure 2. Sampled data transfer timing diagram under aperiodic DoS attacks.

Let $\{d_n\}_{n \in \mathbb{N}_0}$ be the sequence of cycle start times, and let s_n denote the DoS-free duration of the n -th cycle, satisfying $0 = d_0$ and $d_n < d_n + s_n \leq d_{n+1}$ for $n \in \mathbb{N}_0$. The interval $[d_n, d_n + s_n)$ is DoS-free, whereas $[d_n + s_n, d_{n+1})$ is DoS-active.

The DoS attacks considered in this paper are assumed to affect only the remote sensor-to-controller (S-C) measurement channel, consistent with channel-specific DoS models in the literature [29]. The local controller-to-actuator (C-A) link is assumed to remain available during DoS-active intervals. Hence, when the S-C channel is blocked, no fresh measurement is received by the controller, while the DOF controller continues to evolve according to its internal state and the resulting control input $u_i(t)$ can still be applied through the local C-A link.

For $\mathbf{t} \geq \mathbf{s}_* \geq 0$, let $\Phi(\mathbf{s}_*, \mathbf{t})$ denote the number of DoS attack initiations within $[\mathbf{s}_*, \mathbf{t})$, and let $\bar{\Gamma}(\mathbf{s}_*, \mathbf{t})$ denote the set of DoS-active instants over the same interval. Following [30], the attack frequency and cumulative DoS-active duration are constrained as follows.

Assumption 1 (DoS frequency): There exist $\kappa_1 \geq 0$ and $\mathcal{T}_{D1} > 0$ such that

$$\Phi(\mathbf{s}_*, \mathbf{t}) \leq \kappa_1 + \frac{\mathbf{t} - \mathbf{s}_*}{\mathcal{T}_{D1}}.$$

Assumption 2 (DoS duration): There exist $\kappa_2 \geq 0$ and $\mathcal{T}_{D2} > 1$ such that

$$|\bar{\Gamma}(\mathbf{s}_*, \mathbf{t})| \leq \kappa_2 + \frac{\mathbf{t} - \mathbf{s}_*}{\mathcal{T}_{D2}}.$$

Here $|\bar{\Gamma}(\mathbf{s}_*, \mathbf{t})|$ denotes the total duration (Lebesgue measure) of DoS-attack active intervals within $[\mathbf{s}_*, \mathbf{t})$, i.e., the cumulative time during which communication is blocked.

To reduce unnecessary transmissions, a standard ETM is used to determine when the most recently sampled output measurement $y_i(l_k^i h)$ is transmitted to the controller. Let $h > 0$ denote the sampling period. Define $t_k^i \in \mathbb{N}$ as the index of the k -th event-triggered transmission instant for the i -th area, so

that the actual transmission time is $t_k^i h$ (seconds). Let $l_k^i \in \mathbb{N}$ denote the index of the current sampling instant satisfying $t_k^i \leq l_k^i < t_{k+1}^i$, so that the current sampled output is $y_i(l_k^i h)$. The general decentralized event-triggering conditions are as follows:

$$t_{k+1}^i h = t_k^i h + \min\{lh | \mathcal{F}_i[y_i(t_k^i h), e_i(t)] \geq 0\}, \quad (2.3)$$

with

$$\begin{aligned} \mathcal{F}_i[y_i(t_k^i h), e_i(t)] &= e_i^T(t) T_i e_i(t) - \delta_i y_i^T(t_k^i h) T_i y_i(t_k^i h), \\ e_i(t) &= y_i(l_k^i h) - y_i(t_k^i h), \quad l_k^i h = t_k^i h + lh, \end{aligned}$$

where $T_i > 0$ for the i -th area; δ_i is the local event-triggering threshold; and $l \in \mathbb{N}$, $l_k^i h \in (t_k^i h, t_{k+1}^i h]$, $t_k^i > 0$, and $t_k^i \in \mathbb{N}$.

Due to the aperiodic DoS attack (2.2), intermittent interruptions occur in the data transmission within the communication network. Therefore, the event-triggering condition (2.3) alone cannot describe the complete transmission sequence over alternating DoS-free and DoS-active intervals. To address this issue, we propose a DRETm, which leads to the following triggered conditions:

$$\mathcal{T}_{i,n} = \{t_r^i h : \mathcal{F}_i[\cdot] \geq 0, t_r^i h \in \mathcal{H}_{1,n}, t_r^i \in \mathbb{N}\} \cup \mathcal{U}_{i,n}, \quad (2.4)$$

where $\mathcal{H}_{1,n} = [d_n, d_n + s_n)$ denotes the n -th DoS-free interval, and

$$\mathcal{U}_{i,0} = \{d_0\}, \quad \mathcal{U}_{i,n} = \{t_{\text{rec},n}\}, \quad t_{\text{rec},n} = d_n, \quad n \geq 1.$$

Here, the packet released at d_0 initializes the controller-side held measurement, while $t_{\text{rec},n}$ denotes the forced-recovery transmission issued when the S-C channel becomes available at the beginning of the n -th DoS-free interval. Let $\mathcal{T}_i = \bigcup_{n \in \mathbb{N}_0} \mathcal{T}_{i,n}$ and let σ_k^i denote its k -th element in chronological order. Both ordinary event-triggered packets and forced-recovery packets are transmitted through the same physical S-C delay channel before updating the controller-side held measurement. Figure 2 illustrates the resulting transmission sequence under aperiodic DoS attacks.

Whenever a forced-recovery packet is released at $t_{\text{rec},n}$, its sampled output becomes the sensor-side reference for subsequent ordinary event-trigger evaluations. The triggering error is therefore reset for the subsequent tests, while the controller-side held measurement is updated only after the packet's physical transmission delay. Accordingly, the most recent element $\sigma_k^i \in \mathcal{T}_i$ serves as the reference release instant in the sampled-data representation below.

2.3. Design of decentralized dynamic output feedback controllers and the formulation of switched systems

A decentralized switched dynamic output-feedback controller is adopted in the following form [27]:

$$\begin{cases} \dot{x}_{ci}(t) = \mathcal{A}_i^1 x_{ci}(t) + \mathcal{B}_i^1 x_{ci}(s_{k,p}^i) + C_i^1 y_i^c(t), & t \in \Omega_p^i \cap \mathcal{H}_{1,n}, \\ \dot{x}_{ci}(t) = \mathcal{A}_i^2 x_{ci}(t), & t \in \mathcal{H}_{2,n}, \\ u_i(t) = \mathcal{D}_i^j x_{ci}(t), & j \in \{1, 2\}, \end{cases} \quad (2.5)$$

where $\mathcal{A}_i^j$ and $\mathcal{D}_i^j$, $j \in \{1, 2\}$, denote the mode-dependent controller matrices, while $\mathcal{B}_i^1$ and C_i^1 appear only in the DoS-free mode. For $t \in \Omega_p^i$, $s_{k,p}^i$ denotes the corresponding local sampling instant and

$x_{ci}(s_{k,p}^i)$ is locally available without transmission through the S-C channel. During DoS-active intervals, the controller receives no fresh measurement and evolves only according to its internal state. The subintervals Ω_p^i and the associated sampling instants $s_{k,p}^i$ are defined explicitly in (2.9) and (2.10).

Let $y_i^c(t)$ denote the controller-side measurement signal, distinguished from the physical plant output $y_i(t) = C_i x_i(t)$. During DoS-free intervals, it is updated upon successful packet arrivals and held between consecutive arrivals, whereas no S-C measurement is available to the controller during DoS-active intervals. Thus,

$$y_i^c(t) = \begin{cases} y_i(\sigma_k^i), & t \in \Omega_k^i \cap \mathcal{H}_{1,n}, \\ 0, & t \in \mathcal{H}_{2,n}. \end{cases} \quad (2.6)$$

Before the initialization packet first arrives, i.e., for $0 \leq t < \sigma_0^i + m_{\sigma_0^i}$ with $\sigma_0^i = d_0 = 0$, the controller uses a prescribed initial held value $y_{i,0}^c$. In the reported simulations, $y_{i,0}^c = 0$; the packet released at d_0 replaces this value after its physical transmission delay. The successfully transmitted packets are assumed to preserve their arrival order,

$$\sigma_{k+1}^i + m_{\sigma_{k+1}^i} > \sigma_k^i + m_{\sigma_k^i}, \quad (2.7)$$

so that

$$\Omega_k^i = [\sigma_k^i + m_{\sigma_k^i}, \sigma_{k+1}^i + m_{\sigma_{k+1}^i}) \quad (2.8)$$

is well-defined. A forced-recovery packet released at d_n updates $y_i^c(t)$ only after its physical transmission delay; until then, the preceding successfully received sample remains held.

σ_k^i and σ_{k+1}^i denote two consecutive successful packet-release instants of Area i . Let p_M be the number of ordinary sampling instants strictly between them, and denote these instants in chronological order by $s_{k,p}^i$. Hence,

$$\sigma_k^i = s_{k,0}^i < s_{k,1}^i < \dots < s_{k,p_M}^i < s_{k,p_M+1}^i = \sigma_{k+1}^i, \quad 0 < s_{k,p+1}^i - s_{k,p}^i \leq h. \quad (2.9)$$

To obtain the sampled-data time-delay representation, the interval between the two consecutive successful packet arrivals is partitioned as

$$\Omega_k^i = \bigcup_{p=0}^{p_M} \Omega_p^i, \quad \Omega_p^i = [s_{k,p}^i + \Lambda_{k,p}^i, s_{k,p+1}^i + \Lambda_{k,p+1}^i), \quad (2.10)$$

where the arrival offsets are defined by

$$\Lambda_{k,p}^i = \begin{cases} m_{\sigma_k^i}, & p = 0, 1, \dots, p_M, \\ m_{\sigma_{k+1}^i}, & p = p_M + 1. \end{cases} \quad (2.11)$$

Here $m_{\sigma_k^i}$ denotes the physical transmission delay of the packet released at σ_k^i . The partition associates each subinterval Ω_p^i with one sampling or release instant $s_{k,p}^i$, allowing the held measurement to be expressed as a delayed sampled output plus the event-triggering error. Define

$$\tau_k^i(t) = t - s_{k,p}^i, \quad e_k^i(t) = y_i(s_{k,p}^i) - y_i(\sigma_k^i), \quad t \in \Omega_p^i. \quad (2.12)$$

Assume that the physical packet delay satisfies

$$0 < \underline{m}_i \leq m_{\sigma_k^i} \leq \bar{m}_i. \quad (2.13)$$

The partition boundaries are assumed to preserve their chronological order,

$$s_{k,p}^i + \Lambda_{k,p}^i < s_{k,p+1}^i + \Lambda_{k,p+1}^i, \quad p = 0, 1, \dots, p_M, \quad (2.14)$$

so that every Ω_p^i is well-defined. For $t \in \Omega_p^i$,

$$\underline{m}_i \leq \Lambda_{k,p}^i \leq \tau_k^i(t) < (s_{k,p+1}^i - s_{k,p}^i) + \Lambda_{k,p+1}^i \leq h + \overline{m}_i. \quad (2.15)$$

The above construction also gives an intuitive interpretation of the subintervals and the resulting delay bounds. Within each subinterval Ω_p^i , the reference sampling instant $s_{k,p}^i$ remains fixed, so that $\tau_k^i(t) = t - s_{k,p}^i$ represents the age of the corresponding sampled information at time t . As indicated by (2.15), this equivalent sampled-data delay is jointly determined by the sampling interval and the physical packet-transmission delay. Hence, common bounds ϕ_m and ϕ_M are introduced to cover all admissible values of $\tau_k^i(t)$ over the subintervals and areas; they are not the bounds of the physical packet delay $m_{\sigma_k^i}$ itself. Specifically,

$$0 < \phi_m \leq \tau_k^i(t) \leq \phi_M, \quad t \in \Omega_p^i \cap \mathcal{H}_{1,n}. \quad (2.16)$$

Therefore, during the DoS-free mode,

$$y_i^c(t) = y_i(t - \tau_k^i(t)) - e_k^i(t), \quad t \in \Omega_p^i \cap \mathcal{H}_{1,n}. \quad (2.17)$$

By the definition in (2.12), the locally sampled controller state satisfies

$$x_{ci}(s_{k,p}^i) = x_{ci}(t - \tau_k^i(t)). \quad (2.18)$$

Hence, in the augmented switched model, the delayed blocks $C_i^1 C_i$ and $\mathcal{B}_i^1$ originate from the held plant measurement and the locally sampled controller state, respectively.

Accordingly, define the augmented state

$$\hat{x}_i(t - \tau_k^i(t)) = \text{col}\{x_i(t - \tau_k^i(t)), x_{ci}(t - \tau_k^i(t))\}. \quad (2.19)$$

Combining the plant, DRETm, and decentralized DOF controller yields the following switched system, where the mode is determined by the DoS state and the equivalent sampled-data delay appears only in the DoS-free subsystem:

$$\begin{cases} \Sigma_1 : \begin{cases} \dot{\hat{x}}_i(t) = \mathbf{A}_{1i} \hat{x}_i(t) + \mathbf{A}_{2i} \hat{x}_i(t - \tau_k^i(t)) + \mathbf{A}_{3i} e_k^i(t) + \mathbf{A}_{4i} \omega_i(t) \\ y_i(t) = C_i E_i \hat{x}_i(t), t \in \Omega_p^i \cap \mathcal{H}_{1,n}, \end{cases} \\ \Sigma_2 : \begin{cases} \dot{\hat{x}}_i(t) = \mathbf{B}_{1i} \hat{x}_i(t) + \mathbf{B}_{4i} \omega_i(t) \\ y_i(t) = C_i E_i \hat{x}_i(t), t \in \mathcal{H}_{2,n}, \\ \hat{x}_i(t) = \vartheta_i(t), \quad t \in [-\phi_M, 0], \end{cases} \end{cases} \quad (2.20)$$

where $\vartheta_i(\cdot) \in C([- \phi_M, 0], \mathbb{R}^{12})$ is a prescribed continuous initial function specifying the state history of the augmented system over the delay interval. The system matrices are

$$\hat{x}_i(t) = \begin{bmatrix} x_i(t) \\ x_{ci}(t) \end{bmatrix}, \mathbf{A}_{1i} = \begin{bmatrix} A_i & B_i \mathcal{D}_i^1 \\ 0 & \mathcal{A}_i^1 \end{bmatrix}, \mathbf{A}_{2i} = \begin{bmatrix} 0 & 0 \\ C_i^1 C_i & \mathcal{B}_i^1 \end{bmatrix}, \mathbf{A}_{3i} = \begin{bmatrix} 0 \\ -C_i^1 \end{bmatrix}, \mathbf{A}_{4i} = \begin{bmatrix} F_i \\ 0 \end{bmatrix},$$

$$E_i = \begin{bmatrix} I \\ 0 \end{bmatrix}^T, \mathbf{B}_{1i} = \begin{bmatrix} A_i & B_i \mathcal{D}_i^2 \\ 0 & \mathcal{A}_i^2 \end{bmatrix}, \mathbf{B}_{4i} = \begin{bmatrix} F_i \\ 0 \end{bmatrix}.$$

Since the S-C channel is blocked in the DoS-active mode, no delayed or live S-C measurement term is present in Σ_2 .

3. Main results

In this section, exponential stability and weighted H_∞ attenuation are first established for the area-wise switched subsystems with the neighboring-area interaction retained as an open physical interconnection port. An linear matrix inequality (LMI)-based synthesis and recovery procedure is then developed for the decentralized DOF controller gains. Finally, the physical inter-area feedback is restored, and the complete two-area closed loop with the recovered fixed gains is verified through a joint stability condition.

3.1. Local stability and weighted attenuation

Motivated by weighted L_2 -gain and H_∞ performance formulations for switched time-delay systems [31, 32], the following local weighted attenuation notion is adopted.

Definition 1. For the positive mode-dependent weight $\rho_i(t)$ specified in Theorem 1, the local subsystem of Area i is said to achieve a weighted H_∞ attenuation level $\gamma_i > 0$ if, under zero initial conditions,

$$\int_0^T \rho_i(t) y_i^T(t) y_i(t) dt \leq \gamma_i^2 \int_0^T \rho_i(t) \omega_i^T(t) \omega_i(t) dt, \quad \forall T > 0, \quad (3.1)$$

for every generalized local input satisfying

$$\int_0^\infty \rho_i(t) \omega_i^T(t) \omega_i(t) dt < \infty.$$

Here, $\omega_i(t) = \text{col}\{d_i(t), v_i(t)\}$ is the generalized local input defined in Section 2.1. This performance statement applies only to the local subsystem with $v_i(t)$ retained as an open physical interconnection port; it does not constitute an H_∞ performance guarantee for the complete interconnected system.

Theorem 1. Consider the local switched closed-loop subsystem of Area i . Let $\gamma_i > 0$ be the prescribed weighted attenuation level. The positive mode-dependent weight $\rho_i(t)$ satisfies $\dot{\rho}_i = 2\alpha_{1i}\rho_i$ during DoS-free intervals and $\dot{\rho}_i = -2\alpha_{2i}\rho_i$ during DoS-active intervals, with the switching updates $\rho_i(t^+) = \rho_i(t^-)/[\mu_{1i}e^{2(\alpha_{1i}+\alpha_{2i})\phi_M}]$ at DoS-free-to-DoS-active transitions and $\rho_i(t^+) = \rho_i(t^-)/\mu_{2i}$ at the reverse transitions.

For scalars $0 < \delta_i < 1$, $\phi_M > \phi_m > 0$, $h > 0$, $\mathcal{T}_{D1} > 0$, $\mathcal{T}_{D2} > 1$, $\alpha_{ji} > 0$, $\mu_{ji} > 1$ ($j = 1, 2$), suppose that

$$\lambda_i = 2\alpha_{1i} - \frac{2(\alpha_{1i} + \alpha_{2i})\phi_M + \ln(\mu_{1i}\mu_{2i})}{\mathcal{T}_{D1}} - \frac{2(\alpha_{1i} + \alpha_{2i})}{\mathcal{T}_{D2}} > 0. \quad (3.2)$$

Here, α_{1i} and α_{2i} denote the DoS-free decay rate and the admissible DoS-active growth rate, respectively, while μ_{1i} and μ_{2i} are the switching comparison factors. The exponential factor $e^{2(\alpha_{1i}+\alpha_{2i})\phi_M}$ accounts for the worst-case kernel ratio over the history interval $[t - \phi_M, t]$, so no condition $\phi_M \leq h$ is required.

Suppose there exist positive-definite matrices $T_i, P_{ji}, Q_{ji}, R_{ji}, Z_{ji}, W_{ji}$ and matrices S_{ji} of appropriate dimensions such that, for $j = 1, 2$,

$$P_{1i} \leq \mu_{2i}P_{2i}, \quad P_{2i} \leq \mu_{1i}e^{2(\alpha_{1i}+\alpha_{2i})\phi_M}P_{1i}, \quad (3.3)$$

$$\zeta_{1i} \leq \mu_{2i}\zeta_{2i}, \quad \zeta_{2i} \leq \mu_{1i}\zeta_{1i}, \quad \zeta \in \{W, Q, R, Z\}, \quad (3.4)$$

$$\begin{bmatrix} Z_{ji} & S_{ji} \\ * & Z_{ji} \end{bmatrix} > 0, \quad j \in \{1, 2\}, \quad (3.5)$$

$$\Pi_{1i} = \begin{bmatrix} \Pi_{11i} & \Pi_{12i} \\ * & \Pi_{22i} \end{bmatrix} < 0, \quad (3.6)$$

$$\Upsilon_{1i} = \begin{bmatrix} \Upsilon_{11i} & \Upsilon_{12i} \\ * & \Upsilon_{22i} \end{bmatrix} < 0, \quad (3.7)$$

where the explicit block entries of Π_{11i} , Π_{12i} , Π_{22i} , Υ_{11i} , Υ_{12i} , and Υ_{22i} , together with the definitions of $\Xi_{\ell mi}$, $\Psi_{\ell mi}$, and $k_{\ell i}$, are given in Appendix A. Then the local disturbance-free switched closed-loop system (2.20), with $d_i(t) = 0$ and $v_i(t) = 0$, is exponentially stable. Moreover, every generalized local input with finite weighted energy satisfies the local weighted $\mathcal{H}_\infty$ attenuation level γ_i in the sense of Definition 1. This theorem establishes only the local open-port properties; stability of the complete interconnected system is addressed separately in Theorem 3.

Proof. Define

$$V_i(t) = \begin{cases} V_{1i}(t), & t \in \Omega_p^i \cap \mathcal{H}_{1,n}, \\ V_{2i}(t), & t \in \mathcal{H}_{2,n}, \end{cases} \quad (3.8)$$

where

$$\begin{aligned} V_{ji}(t) &= \hat{x}_i^T(t) P_{ji} \hat{x}_i(t) + \int_{t-\phi_m}^t \hat{x}_i^T(s) e^{(-1)^j 2\alpha_{ji}(t-s)} W_{ji} \hat{x}_i(s) ds + \int_{t-\phi_M}^{t-\phi_m} \hat{x}_i^T(s) e^{(-1)^j 2\alpha_{ji}(t-s)} Q_{ji} \hat{x}_i(s) ds + H_{ji}(t), \\ H_{ji}(t) &= \phi_m \int_{-\phi_m}^0 \int_{t+\theta}^t \hat{x}_i^T(s) e^{(-1)^j 2\alpha_{ji}(t-s)} R_{ji} \hat{x}_i(s) ds d\theta + (\phi_M - \phi_m) \int_{-\phi_M}^{-\phi_m} \int_{t+\theta}^t \hat{x}_i^T(s) e^{(-1)^j 2\alpha_{ji}(t-s)} Z_{ji} \hat{x}_i(s) ds d\theta. \end{aligned}$$

Differentiating (3.8), applying the event-triggering condition, as well as Jensen's and reciprocally convex inequalities to the two delay intervals, and using the Schur complement give the DoS-free dissipation inequality

$$\dot{V}_{1i}(t) \leq -2\alpha_{1i} V_{1i}(t) + \xi_{1i}^T(t) \Pi_{1i} \xi_{1i}(t) - y_i^T(t) y_i(t) + \gamma_i^2 \omega_i^T(t) \omega_i(t), \quad (3.9)$$

where

$$\xi_{1i}^T(t) = [\hat{x}_i(t), \hat{x}_i(t - \phi_m), \hat{x}_i(t - \tau_k^i(t)), \hat{x}_i(t - \phi_M), e_k^i(t), \omega_i(t)].$$

The same calculation for the DoS-active mode gives

$$\dot{V}_{2i}(t) \leq 2\alpha_{2i} V_{2i}(t) + \xi_{2i}^T(t) \Upsilon_{1i} \xi_{2i}(t) - y_i^T(t) y_i(t) + \gamma_i^2 \omega_i^T(t) \omega_i(t), \quad (3.10)$$

where

$$\xi_{2i}^T(t) = [\hat{x}_i(t), \hat{x}_i(t - \phi_m), \hat{x}_i(t - \theta_i(t)), \hat{x}_i(t - \phi_M), \omega_i(t)], \quad \theta_i(t) \in [\phi_m, \phi_M].$$

Here, $\theta_i(t) \in [\phi_m, \phi_M]$ is an auxiliary history point used only in the LKF estimate and does not represent an S-C communication delay. For $\omega_i = 0$, the two inequalities provide phase-wise decay/growth rates $-2\alpha_{1i}$ and $2\alpha_{2i}$. Combining them with the jump bounds (3.3) and (3.4) over all completed phases gives

$$\begin{aligned} V_i(t) &\leq (\mu_{1i} \mu_{2i})^{\Phi(0,t)} e^{-2\alpha_{1i}(t-|\bar{\Gamma}(0,t)|)} e^{2\alpha_{2i}|\bar{\Gamma}(0,t)|} \\ &\quad \times e^{2(\alpha_{1i} + \alpha_{2i})\phi_M \Phi(0,t)} V_i(0). \end{aligned} \quad (3.11)$$

Applying Assumptions 1 and 2 gives

$$\Phi(0, t) \leq \kappa_1 + \frac{t}{\mathcal{T}_{D1}}, \quad |\bar{\Gamma}(0, t)| \leq \kappa_2 + \frac{t}{\mathcal{T}_{D2}}.$$

Substituting these bounds into (3.11) yields

$$V_i(t) \leq C_i e^{-\lambda_i t} V_i(0), \quad (3.12)$$

for some $C_i > 0$. Since $P_{ji} > 0$ and $\lambda_i > 0$, this proves the exponential stability of the local disturbance-free subsystem.

For $\omega_i \neq 0$, define the weighted storage function

$$\mathcal{V}_i(t) := \rho_i(t) V_i(t),$$

where $V_i = V_{1i}$ in the DoS-free mode and $V_i = V_{2i}$ in the DoS-active mode. The local weight satisfies $\dot{\rho}_i = 2\alpha_{1i}\rho_i$ in the DoS-free mode. Hence, using (3.9),

$$\begin{aligned} \frac{d}{dt}[\rho_i V_{1i}] &= \dot{\rho}_i V_{1i} + \rho_i \dot{V}_{1i} \\ &\leq 2\alpha_{1i}\rho_i V_{1i} + \rho_i[-2\alpha_{1i}V_{1i} - y_i^T y_i + \gamma_i^2 \omega_i^T \omega_i] \\ &= -\rho_i y_i^T y_i + \gamma_i^2 \rho_i \omega_i^T \omega_i. \end{aligned}$$

In the DoS-active mode, choose $\dot{\rho}_i = -2\alpha_{2i}\rho_i$. Equation (3.10) gives

$$\begin{aligned} \frac{d}{dt}[\rho_i V_{2i}] &= \dot{\rho}_i V_{2i} + \rho_i \dot{V}_{2i} \\ &\leq -2\alpha_{2i}\rho_i V_{2i} + \rho_i[2\alpha_{2i}V_{2i} - y_i^T y_i + \gamma_i^2 \omega_i^T \omega_i] \\ &= -\rho_i y_i^T y_i + \gamma_i^2 \rho_i \omega_i^T \omega_i. \end{aligned}$$

At a DoS-free-to-DoS-active transition, the storage comparison inequalities imply

$$V_{2i}(t^+) \leq \mu_{1i} e^{2(\alpha_{1i} + \alpha_{2i})\phi_M} V_{1i}(t^-).$$

Set

$$\rho_i(t^+) = \frac{\rho_i(t^-)}{\mu_{1i} e^{2(\alpha_{1i} + \alpha_{2i})\phi_M}},$$

so that $\mathcal{V}_i(t^+) \leq \mathcal{V}_i(t^-)$. At a DoS-active-to-DoS-free transition,

$$V_{1i}(t^+) \leq \mu_{2i} V_{2i}(t^-), \quad \rho_i(t^+) = \frac{\rho_i(t^-)}{\mu_{2i}},$$

which again gives $\mathcal{V}_i(t^+) \leq \mathcal{V}_i(t^-)$. Integrating the flow inequalities over all mode intervals and summing them with these nonincreasing jump relations yields

$$\mathcal{V}_i(T) - \mathcal{V}_i(0) + \int_0^T \rho_i(t) y_i^T(t) y_i(t) dt \leq \gamma_i^2 \int_0^T \rho_i(t) \omega_i^T(t) \omega_i(t) dt.$$

Under zero initial conditions, $\mathcal{V}_i(0) = 0$; since $\mathcal{V}_i(T) \geq 0$, dropping the terminal term gives exactly the weighted attenuation inequality in Definition 1. $\square$

Remark 1. Condition (3.2) characterizes the net Lyapunov decay margin over alternating DoS-free and DoS-active intervals. The term $2\alpha_{1i}$ represents the Lyapunov decay rate available during DoS-free operation. The term $[2(\alpha_{1i} + \alpha_{2i})\phi_M + \ln(\mu_{1i}\mu_{2i})]/T_{D1}$ accounts for the penalty associated with repeated DoS activations, including the worst-case history-kernel effect and the cross-mode Lyapunov comparisons, while $2(\alpha_{1i} + \alpha_{2i})/T_{D2}$ reflects the cumulative effect of the admissible Lyapunov growth during DoS-active intervals. Hence, $\lambda_i > 0$ requires the decay accumulated during DoS-free operation to dominate the combined frequency- and duration-related DoS penalties, thereby guaranteeing a strictly decreasing exponential envelope for the Lyapunov functional. Moreover, larger T_{D1} and T_{D2} correspond to less-frequent DoS activations and a smaller admissible fraction of DoS-active time, respectively, and therefore increase the available stability margin.

3.2. Decentralized DOF controller synthesis

Theorem 2. Let the weighted attenuation level $\gamma_i > 0$ be given. For prescribed scalars $0 < \phi_m < \phi_M$, $\epsilon_1 > 0$, $\epsilon_2 > 0$, $0 < \delta_i < 1$, $h > 0$, $\mathcal{T}_{D1} > 0$, $\mathcal{T}_{D2} > 1$, $\alpha_{ji} > 0$, and $\mu_{ji} > 1$, $j \in \{1, 2\}$, consider the switched augmented system (2.20).

Suppose there exist positive-definite matrices T_i , $\bar{Q}_{ji}$, $\bar{R}_{ji}$, $\bar{Z}_{ji}$, and $\bar{W}_{ji}$, symmetric matrices $X_{ji} = X_{ji}^T$ and $Y_{ji} = Y_{ji}^T$, and matrices $\bar{S}_{ji}$, M_{1i} , M_{2i} , M_{3i} , M_{4i} , U_{b1i} , and U_{b4i} of appropriate dimensions such that, for $j = 1, 2$,

$$N_{ji} = \begin{bmatrix} X_{ji} & I \\ I & Y_{ji} \end{bmatrix} > 0, \quad (3.13)$$

$$\begin{bmatrix} \bar{Z}_{1i} & \bar{S}_{1i} \\ * & \bar{Z}_{1i} \end{bmatrix} > 0, \quad (3.14)$$

$$\begin{bmatrix} \bar{Z}_{2i} & \bar{S}_{2i} \\ * & \bar{Z}_{2i} \end{bmatrix} > 0, \quad (3.15)$$

$$\bar{\Pi}_{1i} = \begin{bmatrix} \bar{\Pi}_{11i} & \bar{\Pi}_{12i} \\ * & \bar{\Pi}_{22i} \end{bmatrix} < 0, \quad (3.16)$$

$$\bar{\Upsilon}_{1i} = \begin{bmatrix} \bar{\Upsilon}_{11i} & \bar{\Upsilon}_{12i} \\ * & \bar{\Upsilon}_{22i} \end{bmatrix} < 0. \quad (3.17)$$

Here, X_{ji} and Y_{ji} are mode-dependent decision variables, and no common- X/Y restriction is imposed. The explicit entries of $\bar{\Pi}_{1i}$ and $\bar{\Upsilon}_{1i}$, together with the auxiliary matrices used in their compact representation, are given in Appendix B.

For any feasible solution of (3.13)–(3.17), choose nonsingular algebraic recovery factors $\mathcal{G}_{ji}$ of compatible dimensions. The DoS-free controller matrices are recovered as

$$\begin{cases} \mathcal{A}_i^1 = \mathcal{G}_{1i}^{-T} (M_{4i} - Y_{1i}A_iX_{1i} - Y_{1i}B_iM_{1i})(I - Y_{1i}X_{1i})^{-1}\mathcal{G}_{1i}^T \\ \mathcal{B}_i^1 = \mathcal{G}_{1i}^{-T} (M_{3i} - M_{2i}C_iX_{1i})(I - Y_{1i}X_{1i})^{-1}\mathcal{G}_{1i}^T \\ \mathcal{C}_i^1 = \mathcal{G}_{1i}^{-T} M_{2i} \\ \mathcal{D}_i^1 = M_{1i}(I - Y_{1i}X_{1i})^{-1}\mathcal{G}_{1i}^T, \end{cases} \quad (3.18)$$

whereas the DoS-active controller matrices are recovered as

$$\begin{cases} \mathcal{A}_i^2 = \mathcal{G}_{2i}^{-T}(U_{b4i} - Y_{2i}A_iX_{2i} - Y_{2i}B_iU_{b1i})(I - Y_{2i}X_{2i})^{-1}\mathcal{G}_{2i}^T \\ \mathcal{D}_i^2 = U_{b1i}(I - Y_{2i}X_{2i})^{-1}\mathcal{G}_{2i}^T. \end{cases} \quad (3.19)$$

No $\mathcal{B}_i^2$ or $\mathcal{C}_i^2$ is recovered, because the DoS-active subsystem contains no delayed or live S-C measurement input. Using the same nonsingular algebraic recovery factors $\mathcal{G}_{ji}$, construct

$$L_{ji} = \begin{bmatrix} X_{ji} & I \\ \mathcal{G}_{ji}^{-T}(I - Y_{ji}X_{ji}) & 0 \end{bmatrix}.$$

The original-coordinate Lyapunov matrices are then reconstructed through

$$P_{ji} = L_{ji}^{-T} N_{ji} L_{ji}^{-1}$$

and

$$\zeta_{ji} = L_{ji}^{-T} \bar{\zeta}_{ji} L_{ji}^{-1}, \quad \zeta \in \{W, Q, R, Z, S\}.$$

The following original-coordinate inequalities are checked after controller and matrix recovery:

$$P_{1i} \leq \mu_{2i} P_{2i}, \quad P_{2i} \leq \mu_{1i} e^{2(\alpha_{1i} + \alpha_{2i})\phi_M} P_{1i}, \quad (3.20)$$

$$\zeta_{1i} \leq \mu_{2i} \zeta_{2i}, \quad \zeta_{2i} \leq \mu_{1i} \zeta_{1i}, \quad \zeta \in \{W, Q, R, Z\}. \quad (3.21)$$

Only (3.13)–(3.17) constitute the convex synthesis problem. Conditions (3.20) and (3.21), together with the controller-recovery identities and the DoS balance condition $\lambda_i > 0$ in (3.2), are verified after reconstruction. If these conditions are satisfied, the recovered decentralized DOF controller fulfils the local stability and weighted attenuation conditions of Theorem 1. Stability of the complete physical interconnection is subsequently verified in Theorem 3.

Proof. Define the congruence-transformed Lyapunov matrices $\bar{Q}_{ji} = L_{ji}^T Q_{ji} L_{ji}$, $\bar{W}_{ji} = L_{ji}^T W_{ji} L_{ji}$, $\bar{Z}_{ji} = L_{ji}^T Z_{ji} L_{ji}$, $\bar{R}_{ji} = L_{ji}^T R_{ji} L_{ji}$, $\bar{S}_{ji} = L_{ji}^T S_{ji} L_{ji}$, $j = 1, 2$, and define the block-diagonal congruence matrices

$$\mathcal{L}_{1i} = \text{diag}\{L_{1i}, L_{1i}, L_{1i}, L_{1i}, I, I, \mathbf{V}_{1i}, \mathbf{V}_{1i}, I, I\}, \quad \mathcal{L}_{2i} = \text{diag}\{L_{2i}, L_{2i}, L_{2i}, I, I, \mathbf{V}_{2i}, \mathbf{V}_{2i}, I\}.$$

Here, we define, for $j = 1, 2$,

$$P_{ji} = \begin{bmatrix} Y_{ji} & \mathcal{G}_{ji}^T \\ \mathcal{G}_{ji} & \mathcal{G}_{ji}(Y_{ji} - X_{ji}^{-1})^{-1}\mathcal{G}_{ji}^T \end{bmatrix}, L_{ji} = \begin{bmatrix} X_{ji} & I \\ \mathcal{G}_{ji}^{-T}(I - Y_{ji}X_{ji}) & 0 \end{bmatrix}, \mathbf{V}_{ji} = \begin{bmatrix} I & Y_{ji} \\ 0 & \mathcal{G}_{ji} \end{bmatrix}, N_{ji} = \begin{bmatrix} X_{ji} & I \\ I & Y_{ji} \end{bmatrix}, \quad (3.22)$$

where $\mathcal{G}_{ji}$ is an arbitrary nonsingular algebraic recovery factor of suitable dimension and may be selected independently for each mode. The recovered matrices $\mathcal{A}_i^1, \mathcal{B}_i^1, \mathcal{C}_i^1, \mathcal{D}_i^1, \mathcal{A}_i^2, \mathcal{D}_i^2$ are nevertheless defined in the single fixed controller-state ordering x_{ci} of (2.5). The factors $\mathcal{G}_{ji}$ do not represent online coordinate changes and introduce neither a state reset nor a mode-dependent rescaling at switching instants.

Since $N_{ji} > 0$, the Schur complement gives $X_{ji} > 0$ and $Y_{ji} - X_{ji}^{-1} > 0$. Hence

$$Y_{ji}X_{ji} - I = (Y_{ji} - X_{ji}^{-1})X_{ji}$$

is nonsingular, and therefore $I - Y_{ji}X_{ji}$ is nonsingular. Thus, the recovery formulas (3.18) and (3.19) are well-defined.

Direct block multiplication gives

$$P_{ji}L_{ji} = \mathbf{V}_{ji}, \quad L_{ji}^T P_{ji} L_{ji} = N_{ji}.$$

Equivalently, $P_{ji} = \mathbf{V}_{ji}L_{ji}^{-1} = L_{ji}^{-T}\mathbf{V}_{ji}^T$ and $P_{ji}^{-1} = L_{ji}\mathbf{V}_{ji}^{-1} = \mathbf{V}_{ji}^{-T}L_{ji}^T$. Since $N_{ji} > 0$ and L_{ji} is nonsingular, $P_{ji} = L_{ji}^{-T}N_{ji}L_{ji}^{-1} > 0$ and is symmetric.

The controller-dependent bilinear products are replaced by the following transformed decision variables:

$$\begin{cases} M_{1i} = \mathcal{D}_i^1 \mathcal{G}_{1i}^{-T} (I - Y_{1i}X_{1i}) \\ M_{2i} = \mathcal{G}_{1i}^T \mathcal{C}_i^1 \\ M_{3i} = M_{2i}C_iX_{1i} + \mathcal{G}_{1i}^T \mathcal{B}_i^1 \mathcal{G}_{1i}^{-T} (I - Y_{1i}X_{1i}) \\ M_{4i} = Y_{1i}A_iX_{1i} + Y_{1i}B_iM_{1i} + \mathcal{G}_{1i}^T \mathcal{A}_i^1 \mathcal{G}_{1i}^{-T} (I - Y_{1i}X_{1i}) \end{cases}$$

and

$$\begin{cases} U_{b1i} = \mathcal{D}_i^2 \mathcal{G}_{2i}^{-T} (I - Y_{2i}X_{2i}) \\ U_{b4i} = Y_{2i}A_iX_{2i} + Y_{2i}B_iU_{b1i} + \mathcal{G}_{2i}^T \mathcal{A}_i^2 \mathcal{G}_{2i}^{-T} (I - Y_{2i}X_{2i}). \end{cases}$$

Solving these identities for the controller matrices gives the recovery formulas (3.18) and (3.19). In particular, the definition of U_{b1i} gives exactly the recovery formula for $\mathcal{D}_i^2$, so the two expressions are algebraically consistent.

Premultiplying and postmultiplying the original inequalities of Theorem 1 by the corresponding congruence matrices and applying the above substitutions remove the controller-dependent bilinear products. The auxiliary matrices $\Gamma_{\ell i}$ and $\Omega_{\ell i}$ used to write the transformed blocks compactly are defined in Appendix B.

The inverse-matrix blocks are bounded using the following affine inequalities:

$$\begin{aligned} -N_{ji}\bar{R}_{ji}^{-1}N_{ji} &\leq -2\epsilon_1 N_{ji} + \epsilon_1^2 \bar{R}_{ji} \leq -\epsilon_1 N_{ji} + \epsilon_1^2 \bar{R}_{ji}, \\ -N_{ji}\bar{Z}_{ji}^{-1}N_{ji} &\leq -2\epsilon_2 N_{ji} + \epsilon_2^2 \bar{Z}_{ji} \leq -\epsilon_2 N_{ji} + \epsilon_2^2 \bar{Z}_{ji}, \\ -T_i^{-1} &\leq T_i - 2I. \end{aligned}$$

The first chain follows from

$$(N_{ji} - \epsilon_1 \bar{R}_{ji})^T \bar{R}_{ji}^{-1} (N_{ji} - \epsilon_1 \bar{R}_{ji}) \geq 0,$$

and the second follows analogously with $(\epsilon_2, \bar{Z}_{ji})$. Since $\epsilon_\ell N_{ji} \geq 0$, replacing the coefficient $-2\epsilon_\ell N_{ji}$ by $-\epsilon_\ell N_{ji}$ gives the valid but more conservative affine upper bounds used in $\bar{\Pi}_{22i}$ and $\bar{\Upsilon}_{22i}$.

The third bound follows from $(T_i - I)^T T_i^{-1} (T_i - I) \geq 0$, which is equivalent to $T_i + T_i^{-1} - 2I \geq 0$. Multiplication by the nonnegative scalar δ_i transforms the original block $-\delta_i T_i^{-1}$ in Theorem 1 into the conservative affine upper bound $\delta_i(T_i - 2I)$ used in $\bar{\Pi}_{22i}$.

Substituting these affine upper bounds into the congruence-transformed inequalities yields the convex LMIs (3.16)–(3.17), whose explicit block entries are given in Appendix B. Hence, feasibility of (3.13)–(3.17), followed by successful controller reconstruction and verification of (3.20) and (3.21), and $\lambda_i > 0$, yields the controller stated in the theorem. $\square$

3.3. Stability verification of the physically interconnected system

After the local decentralized controller gains have been recovered and verified by Theorem 2, the physical inter-area feedback is restored as

$$v_1(t) = T_{12}\Delta f_2(t), \quad v_2(t) = T_{21}\Delta f_1(t).$$

Hence, the neighboring-area frequency deviations become internal states of the complete closed loop rather than independent exogenous inputs.

For the reciprocal single tie-line model, $\Delta P_{tie-2}(t) = -\Delta P_{tie-1}(t)$. Thus, the 12 local plant coordinates are restricted to the physical invariant manifold

$$\mathcal{M} = \{x_{12} \in \mathbb{R}^{12} : \Delta P_{tie-1} + \Delta P_{tie-2} = 0\}.$$

For joint verification, we introduce the 11-dimensional minimal physical coordinate

$$x_p = \text{col}\{\Delta f_1, \Delta P_{tie,12}, \Delta P_{s1}, \Delta P_{m1}, \Delta P_{v1}, \Delta P_{w1}, \Delta f_2, \Delta P_{s2}, \Delta P_{m2}, \Delta P_{v2}, \Delta P_{w2}\} \in \mathbb{R}^{11},$$

such that

$$x_{12} = R_p x_p, \quad \text{Im}(R_p) = \mathcal{M},$$

where R_p is given explicitly in Appendix C. With the two six-dimensional controller states, define

$$\chi(t) = \text{col}\{x_p(t), x_{c1}(t), x_{c2}(t)\} \in \mathbb{R}^{23}.$$

The delay system is supplied with a continuous initial history

$$\chi(\theta) = \varphi_J(\theta), \quad \theta \in [-\phi_M, 0],$$

whose plant component satisfies $R_p x_p(\theta) \in \mathcal{M}$ throughout the history interval. The two areas retain independent equivalent sampled-data delays,

$$\tau_r(t) \in [\phi_m, \phi_M], \quad r = 1, 2,$$

and no equality constraint between $\tau_1(t)$ and $\tau_2(t)$ is imposed. For compactness in the joint model, $e_r(t)$ denotes the active local error $e_k^r(t)$ on the current DoS-free subinterval of Area r , $r = 1, 2$. The fixed-gain joint closed-loop model is

$$\begin{aligned} \Sigma_J^1 : \quad \dot{\chi}(t) &= A_{J1}\chi(t) + A_{Jd,1}\chi(t - \tau_1(t)) + A_{Jd,2}\chi(t - \tau_2(t)) \\ &\quad + E_{J1}e_1(t) + E_{J2}e_2(t) + B_{Jw}w(t), \\ \Sigma_J^2 : \quad \dot{\chi}(t) &= A_{J2}\chi(t) + B_{Jw}w(t), \end{aligned}$$

where $w(t) = \text{col}\{d_1(t), d_2(t)\}$ contains all exogenous disturbance channels, while the physical inter-area feedback is embedded in A_{J1} and A_{J2} . The recovered decentralized controller gains are kept fixed in the joint model, and no controller variable is re-optimized at this stage. In Σ_J^2 , no delayed or live S-C measurement channel or event-error term is present. The complete fixed-gain matrices are given in Appendix C.

Theorem 3. Consider the fixed-gain joint closed-loop system $\Sigma_J^1 - \Sigma_J^2$ defined above, with two independent equivalent delays $\tau_r(t) \in [\phi_m, \phi_M]$, $r = 1, 2$. Suppose there exist scalars $\alpha_{1J}, \alpha_{2J} > 0$, $\mu_{1J}, \mu_{2J} > 1$, matrices

$$P_{Jq} > 0, \quad W_{Jq,r}, Q_{Jq,r}, R_{Jq,r}, Z_{Jq,r} > 0,$$

and $S_{Jq,r}$ such that

$$\begin{bmatrix} Z_{Jq,r} & S_{Jq,r} \\ S_{Jq,r}^T & Z_{Jq,r} \end{bmatrix} > 0, \quad q, r \in \{1, 2\}.$$

The strict joint LMIs

$$\widehat{\Theta}_{J1} < 0, \quad \widehat{\Theta}_{J2} < 0$$

hold, where their explicit block entries are given in Appendix D. The cross-mode comparisons satisfy

$$P_{J1} \leq \mu_{2J} P_{J2}, \quad P_{J2} \leq \mu_{1J} e^{2(\alpha_{1J} + \alpha_{2J})\phi_M} P_{J1},$$

and, for $X \in \{W, Q, R, Z\}$ and $r = 1, 2$,

$$X_{J1,r} \leq \mu_{2J} X_{J2,r}, \quad X_{J2,r} \leq \mu_{1J} X_{J1,r}.$$

If Assumptions 1 and 2 hold and

$$\lambda_J = 2\alpha_{1J} - \frac{2(\alpha_{1J} + \alpha_{2J})\phi_M + \ln(\mu_{1J}\mu_{2J})}{\mathcal{T}_{D1}} - \frac{2(\alpha_{1J} + \alpha_{2J})}{\mathcal{T}_{D2}} > 0, \quad (3.23)$$

then, for $w(t) = 0$, the complete physical two-area interconnected closed-loop system is exponentially stable for every continuous physically admissible initial history whose plant component lies in $\mathcal{M}$ on $[-\phi_M, 0]$.

Proof. For the reciprocal tie-line model,

$$\begin{aligned} \dot{P}_{tie-1}(t) &= 2\pi T_{12}(\Delta f_1(t) - \Delta f_2(t)), \\ \dot{P}_{tie-2}(t) &= 2\pi T_{21}(\Delta f_2(t) - \Delta f_1(t)). \end{aligned}$$

Since $T_{12} = T_{21}$,

$$\dot{P}_{tie-1}(t) + \dot{P}_{tie-2}(t) = 0.$$

Hence, $\mathcal{M}$ is invariant and x_p provides an equivalent parameterization of every physically admissible trajectory through $x_{12} = R_p x_p$.

Substituting $v_1 = T_{12}\Delta f_2$ and $v_2 = T_{21}\Delta f_1$ into the two local closed-loop models yields Σ_J^1 and Σ_J^2 . Therefore, the neighboring-area frequency deviations are internal components of $\chi(t)$, whereas $w(t) = \text{col}\{d_1(t), d_2(t)\}$ contains all exogenous disturbance channels.

To establish stability of the fixed-gain joint system, consider the mode-dependent, sum-separable, two-delay Lyapunov–Krasovskii functional

$$V_{Jq}(t) = \chi^T(t) P_{Jq} \chi(t) + \sum_{r=1}^2 V_{Jq,r}^d(t), \quad q \in \{1, 2\},$$

where

$$\begin{aligned}
 V_{Jq,r}^d(t) = & \int_{t-\phi_m}^t e^{\sigma_q 2\alpha_{qJ}(t-s)} \chi^T(s) W_{Jq,r} \chi(s) ds \\
 & + \int_{t-\phi_M}^{t-\phi_m} e^{\sigma_q 2\alpha_{qJ}(t-s)} \chi^T(s) Q_{Jq,r} \chi(s) ds \\
 & + \phi_m \int_{-\phi_m}^0 \int_{t+\theta}^t e^{\sigma_q 2\alpha_{qJ}(t-s)} \dot{\chi}^T(s) R_{Jq,r} \dot{\chi}(s) ds d\theta \\
 & + (\phi_M - \phi_m) \int_{-\phi_M}^{-\phi_m} \int_{t+\theta}^t e^{\sigma_q 2\alpha_{qJ}(t-s)} \dot{\chi}^T(s) Z_{Jq,r} \dot{\chi}(s) ds d\theta,
 \end{aligned}$$

with $\sigma_1 = -1$ and $\sigma_2 = 1$. The sum over $r = 1, 2$ assigns an independent delay-dependent functional to each equivalent-delay channel, so no common-delay assumption is introduced.

Differentiating V_{Jq} along the two joint modes, applying the reciprocally convex estimates to the two delay channels, and incorporating the two local event-triggering inequalities through the S-procedure give

$$\begin{aligned}
 \dot{V}_{J1}(t) & \leq -2\alpha_{1J} V_{J1}(t) + \zeta_{J1}^T(t) \widehat{\Theta}_{J1} \zeta_{J1}(t), \\
 \dot{V}_{J2}(t) & \leq 2\alpha_{2J} V_{J2}(t) + \zeta_{J2}^T(t) \widehat{\Theta}_{J2} \zeta_{J2}(t),
 \end{aligned}$$

where the exact stacked vectors and complete block entries are given in Appendix D. The strict joint LMIs therefore imply, for $w = 0$,

$$\dot{V}_{J1}(t) \leq -2\alpha_{1J} V_{J1}(t), \quad \dot{V}_{J2}(t) \leq 2\alpha_{2J} V_{J2}(t).$$

The cross-mode comparisons in the theorem imply the Lyapunov jump bounds

$$V_{J1}(t^+) \leq \mu_{2J} V_{J2}(t^-)$$

at a DoS-active-to-DoS-free transition, and

$$V_{J2}(t^+) \leq \mu_{1J} e^{2(\alpha_{1J} + \alpha_{2J})\phi_M} V_{J1}(t^-)$$

at a DoS-free-to-DoS-active transition. Let $V_J(t)$ denote the active mode-dependent functional. Iterating the phase-wise flow and jump estimates gives

$$\begin{aligned}
 V_J(t) & \leq (\mu_{1J} \mu_{2J})^{\Phi(0,t)} e^{-2\alpha_{1J}(t - |\bar{\Gamma}(0,t)|)} e^{2\alpha_{2J}|\bar{\Gamma}(0,t)|} \\
 & \quad \times e^{2(\alpha_{1J} + \alpha_{2J})\phi_M \Phi(0,t)} V_J(0).
 \end{aligned}$$

By Assumptions 1 and 2,

$$\Phi(0, t) \leq \kappa_1 + \frac{t}{\mathcal{T}_{D1}}, \quad |\bar{\Gamma}(0, t)| \leq \kappa_2 + \frac{t}{\mathcal{T}_{D2}}.$$

Substitution into the preceding inequality and the use of (3.23) yield

$$V_J(t) \leq C_J e^{-\lambda_J t} V_J(0), \quad C_J > 0.$$

Since $P_{Jq} > 0$ and $\lambda_J > 0$, $\chi(t)$ converges to zero exponentially. Since $x_{12} = R_p x_p$ on the invariant manifold $\mathcal{M}$, the original physical two-area interconnected closed loop is exponentially stable. $\square$

4. Numerical simulation

4.1. Performance evaluation of the proposed method

Numerical simulations are conducted on a two-area interconnected power system whose parameters are adopted from [18, 24].

Table 3. System parameters for the two-area interconnected power system simulation.

Parameter	Description	Area 1	Area 2	Unit
M_i	Generator inertia constant	10	8	s
D_i	Load damping coefficient	0.6	0.9	p.u./Hz
T_{gi}	Governor time constant	0.2	0.3	s
T_{chi}	Turbine time constant	0.5	0.6	s
T_{Wi}	Wind turbine time constant	1.5	1.5	s
T_{SLi}	Solar photovoltaic time constant	1.3	1.3	s
R_i	Speed droop coefficient	0.05	0.0625	Hz/p.u.MW
β_i	Frequency bias factor ($D_i + 1/R_i$)	20.6	16.9	p.u.MW/Hz
$T_{12} = T_{21}$	Tie-line synchronizing coefficient	0.1968		p.u.MW/Hz

For each reported scenario, a fixed realization of the aperiodic DoS sequence is generated using the specified DoS-free and DoS-active duration ranges and is verified to satisfy Assumptions 1 and 2 over the corresponding simulation horizon. The DoS-free and DoS-active durations are sampled independently and uniformly from [1.2, 2.8] s and [0.35, 1.00] s, respectively, with $\mathcal{H}_{2,n} = [d_n + s_n, d_{n+1})$. The local design parameters are $\alpha_{11} = \alpha_{12} = 0.009$, $\alpha_{21} = \alpha_{22} = 0.0015$, $\epsilon_1 = \epsilon_2 = 0.10$, $\delta_1 = \delta_2 = 0.1$, $\mu_{11} = \mu_{12} = 1.01$, $\mu_{21} = \mu_{22} = 1.02$, $\mathcal{T}_{D1} = \mathcal{T}_{D2} = 3$, $\kappa_1 = \kappa_2 = 8$, $h = 0.02$ s, and $[\phi_m, \phi_M] = [0.01, 0.04]$ s. The physical packet delay is sampled in [0.01, 0.019] s, so that $h + \bar{m}_i = 0.039 < \phi_M$. Substitution into (3.2) gives $\lambda_1 = \lambda_2 = 8.023473 \times 10^{-4} > 0$. The fixed-gain joint verification uses $\alpha_{1J} = 0.0087$, $\alpha_{2J} = 0.0003$, $\mu_{1J} = 1.01$, and $\mu_{2J} = 1.02$, giving $\lambda_J = 1.242347 \times 10^{-3} > 0$. The local synthesis conditions and the fixed-gain joint-stability conditions are feasible for the reported parameter set, and the recovered controller gains are listed in Appendix E.

The standard simulation is conducted over 50 s with an integration step of 0.001 s. The physical S-C packet delays of the two areas are independently sampled from [0.01, 0.019] s. The corresponding attack sequence is shown in Figure 3.

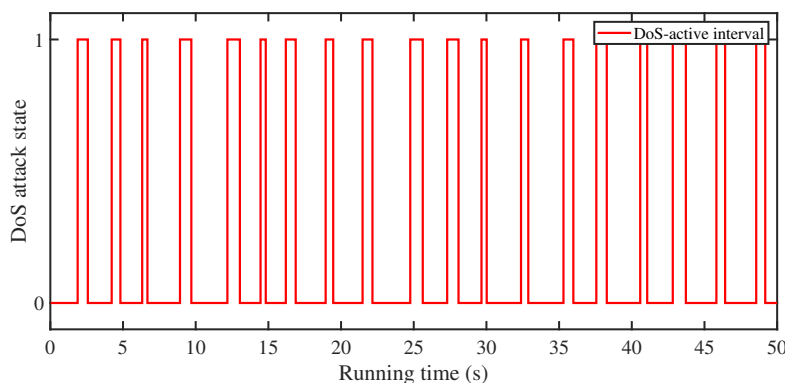


Figure 3. Aperiodic DoS attack signal.

The quantitative performance measures considered are $\Delta f_{\text{peak},i} = \max_t |\Delta f_i(t)|$; the settling time, defined as the first time after which $|\Delta f_i(t)| \leq 5 \times 10^{-4}$ Hz remains satisfied; and the mean successful transmission count $\bar{N}_{\text{tr}} = (N_{\text{tr},1} + N_{\text{tr},2})/2$, including forced recovery packets and excluding initialization and DoS-blocked trigger intents. The peak frequency deviations are 0.028203 and 0.030657 Hz, the settling times are 16.48 and 10.94 s, and the transmission counts are 148 and 104, giving $\bar{N}_{\text{tr}} = 126$.

For the simulation results, the area control error is defined as

$$ACE_i(t) = \beta_i \Delta f_i(t) + \Delta P_{\text{tie}-i}(t).$$

Figure 4 reports the first component $y_{i,1}^c(t)$ of the controller-side held measurement vector for the two areas.

Figures 5–7 show the physical responses, Figure 4 shows the controller-side held measurements, and Figures 8 and 9 show successful transmission intervals. The frequency deviations, tie-line power deviation, and area control errors exhibit damped transient responses and converge toward their equilibrium values despite the intermittent loss of S-C measurements, indicating that the recovered decentralized controller maintains closed-loop regulation under the prescribed DoS and delay conditions. The opposite directional tie-line responses in Figure 6 also remain consistent with the physical relation $\Delta P_{\text{tie}-2} = -\Delta P_{\text{tie}-1}$. Successful transmissions are more frequent during the initial transient state and become less frequent as the closed-loop response settles, apart from forced-recovery transmissions.

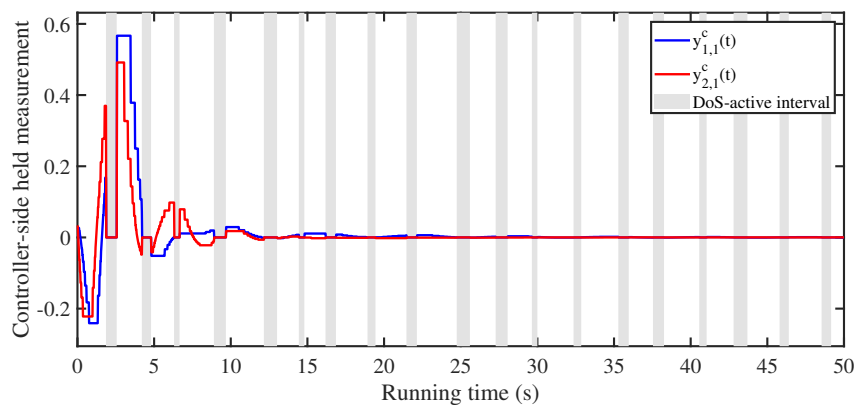


Figure 4. Controller-side held measurements $y_{1,1}^c(t)$ and $y_{2,1}^c(t)$ under aperiodic DoS attacks.

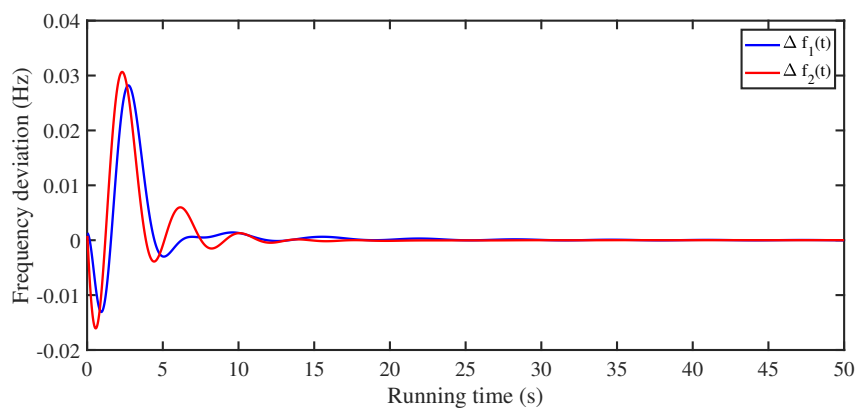


Figure 5. Frequency deviation Δf under the DRETM.

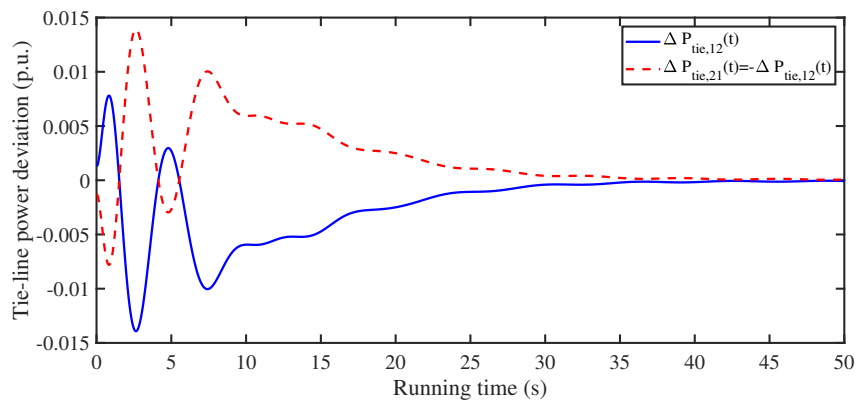


Figure 6. Two directional representations of the single physical tie-line power deviation on $\mathcal{M}$.

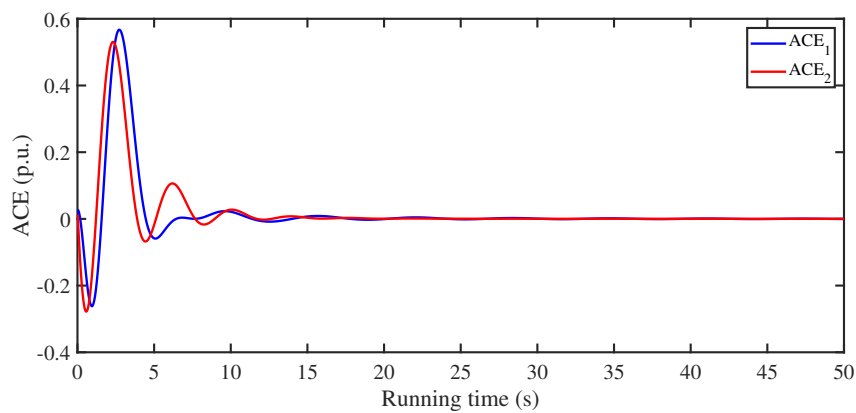


Figure 7. Area control errors ACE_i under the DRETM.

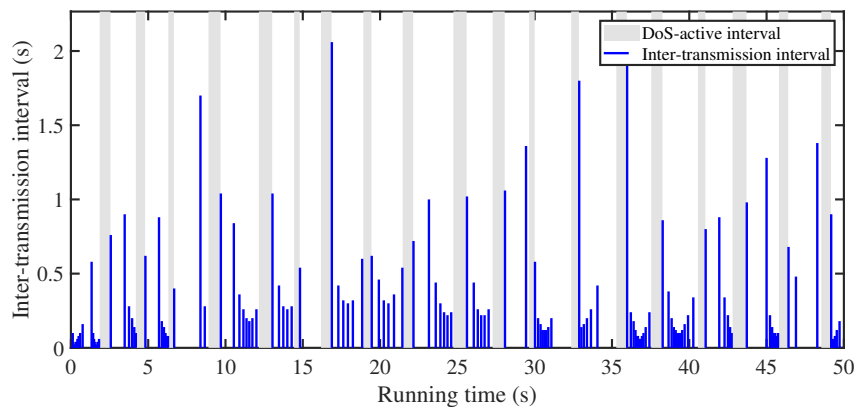


Figure 8. Successful transmission intervals in Area 1 under the DRETM.

A 100-s multi-step load-disturbance test is further conducted to examine the closed-loop response over a longer horizon. Load changes are applied to the two areas under both DoS-active and DoS-free communication conditions. As shown in Figure 10, the frequency deviations remain bounded after each load variation and gradually recover toward the equilibrium after the disturbances are removed, demonstrating sustained regulation capability under repeated load changes and intermittent communication interruptions.

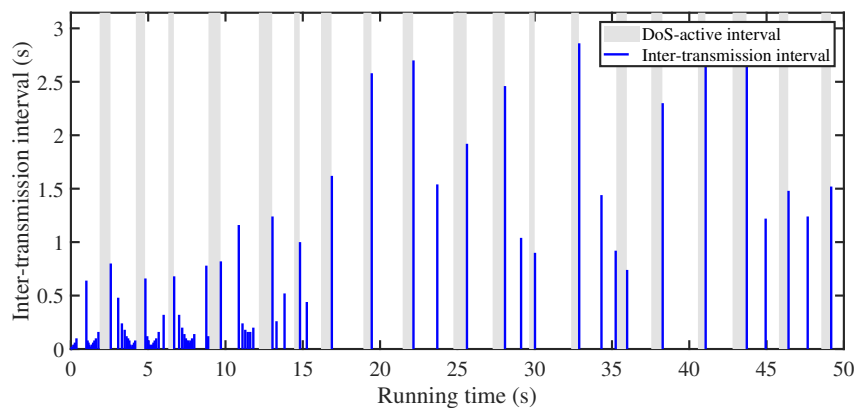


Figure 9. Successful transmission intervals in Area 2 under the DRETM.

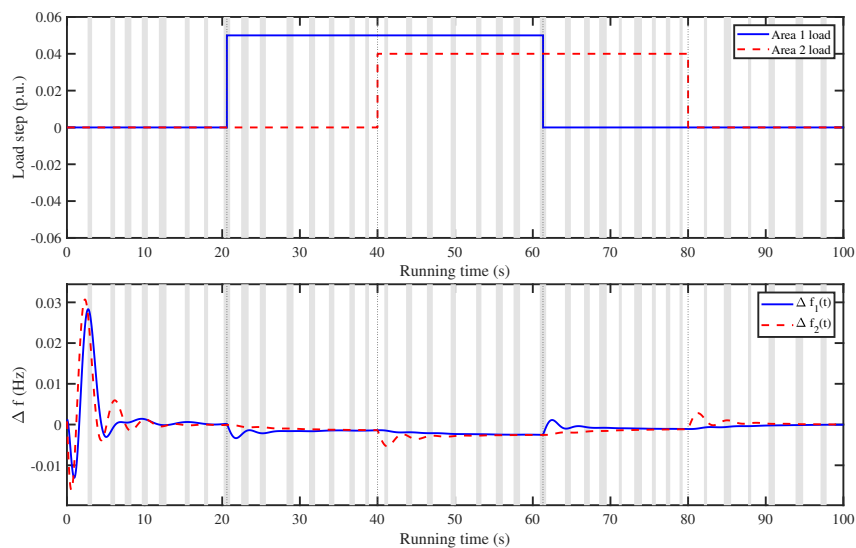


Figure 10. Load-step schedule and frequency responses over 100 s; shaded bands denote DoS-active intervals.

4.2. Comparison with an adapted ET-SMC baseline

A representative event-triggered sliding-mode control (ET-SMC) method adapted from Shen et al. [21] is selected as the numerical baseline. Its original switching-like event-triggering and sliding-mode control structure is retained, while the controller gains are redesigned for the common two-area benchmark. For a fair comparison, both methods are evaluated under the same physical plant, initial conditions, disturbance profile, DoS sequence, and packet-delay environment. The proposed method employs decentralized dynamic output feedback with post-DoS forced recovery, whereas the adapted baseline retains its original event-triggered sliding-mode architecture.

As shown in Figure 11 and Table 4, the proposed method reduces the maximum frequency deviations from 0.065982 Hz to 0.028203 Hz in Area 1 and from 0.055533 Hz to 0.030657 Hz in Area 2. The corresponding settling times decrease from 21.80 s to 16.48 s and from 12.28 s to 10.94 s, respectively. The comparison therefore demonstrates improved frequency-regulation performance of the proposed method under the considered common attack-and-delay environment, as reflected by the reduced peak

frequency deviations and settling times.

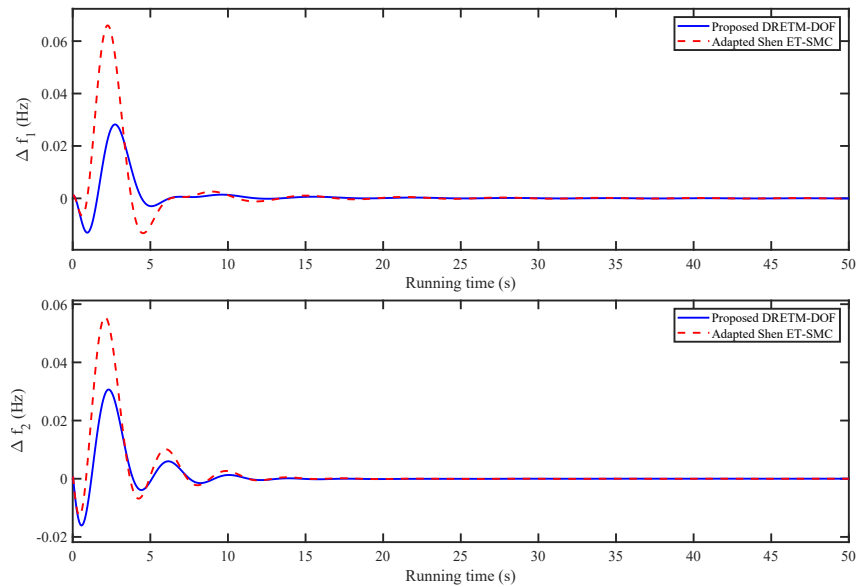


Figure 11. Frequency responses of the proposed method and the adapted ET-SMC baseline under the common S-C DoS and packet-delay environment.

Table 4. Quantitative comparison with the adapted ET-SMC baseline.

Method	Area	Maximum $ \Delta f $ (Hz)	Settling time (s)
Proposed DRETm-DOF	Area 1	0.028203	16.48
Proposed DRETm-DOF	Area 2	0.030657	10.94
Adapted Shen ET-SMC	Area 1	0.065982	21.80
Adapted Shen ET-SMC	Area 2	0.055533	12.28

5. Conclusions

This paper developed a DRETm and decentralized DOF switched controller for multi-area LFC under aperiodic DoS attacks and bounded sampled-data delays. The local mode-dependent Lyapunov–Krasovskii analysis established exponential stability and weighted H_∞ attenuation. An LMI-based synthesis and recovery procedure separated the convex synthesis conditions from controller recovery and the subsequent algebraic, switching and DoS-balance checks. After gain recovery, the neighboring-area feedback was restored explicitly, and fixed-gain joint Lyapunov–Krasovskii conditions verified the stability of the considered physical two-area interconnected system with two independent equivalent-delay channels. Numerical studies, including the longer-horizon multi-step test and the adapted ET-SMC comparison, further demonstrated effective frequency regulation under the considered DoS and packet-delay conditions.

The method remains model-based, was developed for the linearized LFC setting, and assumes a common global DoS pattern. The present joint verification was developed for the considered two-area system with a single physical tie-line; extension to larger heterogeneous multi-area networks will increase the dimension and offline computational burden of the joint verification LMIs. Future work

will consider scalable synthesis for unknown dynamics, asynchronous or regional DoS processes, and combined DoS/false-data-injection attacks.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there are no conflicts of interest.

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Appendix

Appendix A: Explicit LMI blocks for Theorem 1

The explicit entries of the compact LMIs in Theorem 1 are collected below.

$$\Pi_{11i} = \begin{bmatrix} \Xi_{11i} & e^{-2\alpha_{1i}\phi_m} R_{1i} & P_{1i} \mathbf{A}_{2i} & 0 & P_{1i} \mathbf{A}_{3i} & P_{1i} \mathbf{A}_{4i} \\ * & \Xi_{22i} & \Xi_{23i} & e^{-2\alpha_{1i}(\phi_M - \phi_m)} S_{1i} & 0 & 0 \\ * & * & \Xi_{33i} & \Xi_{34i} & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & 0 \\ * & * & * & * & -T_i & 0 \\ * & * & * & * & * & -\gamma_i^2 I \end{bmatrix},$$

$$\Pi_{12i} = \begin{bmatrix} k_{1i} \phi_m \mathbf{A}_{1i}^T & k_{2i}(\phi_M - \phi_m) \mathbf{A}_{1i}^T & (C_i E_i)^T & 0 \\ 0 & 0 & 0 & 0 \\ k_{1i} \phi_m \mathbf{A}_{2i}^T & k_{2i}(\phi_M - \phi_m) \mathbf{A}_{2i}^T & 0 & \delta_i (C_i E_i)^T \\ 0 & 0 & 0 & 0 \\ k_{1i} \phi_m \mathbf{A}_{3i}^T & k_{2i}(\phi_M - \phi_m) \mathbf{A}_{3i}^T & 0 & -\delta_i I \\ k_{1i} \phi_m \mathbf{A}_{4i}^T & k_{2i}(\phi_M - \phi_m) \mathbf{A}_{4i}^T & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{11i} = \begin{bmatrix} \Psi_{11i} & e^{2\alpha_{2i}\phi_m} R_{2i} & 0 & 0 & P_{2i} \mathbf{B}_{4i} \\ * & \Psi_{22i} & \Psi_{23i} & e^{2\alpha_{2i}(\phi_M - \phi_m)} S_{2i} & 0 \\ * & * & \Psi_{33i} & \Psi_{34i} & 0 \\ * & * & * & \Psi_{44i} & 0 \\ * & * & * & * & -\gamma_i^2 I \end{bmatrix},$$

$$\Upsilon_{12i} = \begin{bmatrix} k_{3i} \phi_m \mathbf{B}_{1i}^T & k_{4i}(\phi_M - \phi_m) \mathbf{B}_{1i}^T & (C_i E_i)^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_{3i} \phi_m \mathbf{B}_{4i}^T & k_{4i}(\phi_M - \phi_m) \mathbf{B}_{4i}^T & 0 \end{bmatrix},$$

$$\Pi_{22i} = \text{diag}\{-R_{1i}^{-1}, -Z_{1i}^{-1}, -I, -\delta_i T_i^{-1}\}, \quad \Upsilon_{22i} = \text{diag}\{-R_{2i}^{-1}, -Z_{2i}^{-1}, -I\},$$

$$\Xi_{11i} = (\mathbf{A}_{1i})^T P_{1i} + P_{1i} \mathbf{A}_{1i} + W_{1i} - e^{-2\alpha_{1i}\phi_m} R_{1i} + 2\alpha_{1i} P_{1i},$$

$$\Xi_{22i} = e^{-2\alpha_{1i}\phi_m} (Q_{1i} - W_{1i}) - e^{-2\alpha_{1i}\phi_m} R_{1i} - e^{-2\alpha_{1i}(\phi_M - \phi_m)} Z_{1i},$$

$$\Xi_{23i} = e^{-2\alpha_{1i}(\phi_M - \phi_m)} (Z_{1i} - S_{1i}), \quad \Xi_{33i} = e^{-2\alpha_{1i}(\phi_M - \phi_m)} (-2Z_{1i} + S_{1i} + S_{1i}^T),$$

$$\Xi_{34i} = e^{-2\alpha_{1i}(\phi_M - \phi_m)} (Z_{1i} - S_{1i}), \quad \Xi_{44i} = -e^{-2\alpha_{1i}(\phi_M - \phi_m)} Z_{1i} - e^{-2\alpha_{1i}\phi_M} Q_{1i},$$

$$\Psi_{11i} = (\mathbf{B}_{1i})^T P_{2i} + P_{2i} \mathbf{B}_{1i} + W_{2i} - e^{2\alpha_{2i}\phi_m} R_{2i} - 2\alpha_{2i} P_{2i},$$

$$\Psi_{22i} = e^{2\alpha_{2i}\phi_m} (Q_{2i} - W_{2i}) - e^{2\alpha_{2i}\phi_m} R_{2i} - e^{2\alpha_{2i}(\phi_M - \phi_m)} Z_{2i},$$

$$\Psi_{23i} = e^{2\alpha_{2i}(\phi_M - \phi_m)} (Z_{2i} - S_{2i}), \quad \Psi_{33i} = e^{2\alpha_{2i}(\phi_M - \phi_m)} (-2Z_{2i} + S_{2i} + S_{2i}^T),$$

$$\Psi_{34i} = e^{2\alpha_{2i}(\phi_M - \phi_m)} (Z_{2i} - S_{2i}), \quad \Psi_{44i} = -e^{2\alpha_{2i}(\phi_M - \phi_m)} Z_{2i} - e^{2\alpha_{2i}\phi_M} Q_{2i},$$

$$k_{1i} = \sqrt{e^{-2\alpha_{1i}\phi_m}}, \quad k_{2i} = \sqrt{e^{-2\alpha_{1i}(\phi_M - \phi_m)}}, \quad k_{3i} = \sqrt{e^{2\alpha_{2i}\phi_m}}, \quad k_{4i} = \sqrt{e^{2\alpha_{2i}(\phi_M - \phi_m)}}.$$

Appendix B: Explicit transformed LMI blocks for Theorem 2

For compact representation of the congruence-transformed LMIs, we define

$$\begin{aligned}\Gamma_{0i} &= \begin{bmatrix} A_i X_{1i} + B_i M_{1i} & A_i \\ M_{4i} & Y_{1i} A_i \end{bmatrix}, \quad \Gamma_{1i} = \begin{bmatrix} 0 & 0 \\ M_{3i} & M_{2i} C_i \end{bmatrix}, \\ \Gamma_{2i} &= \begin{bmatrix} 0 \\ -M_{2i} \end{bmatrix}, \quad \Gamma_{3i} = \begin{bmatrix} F_i \\ Y_{1i} F_i \end{bmatrix}, \quad \Gamma_{4i} = \begin{bmatrix} (C_i X_{1i})^T \\ C_i^T \end{bmatrix}^T\end{aligned}$$

and

$$\begin{aligned}\Omega_{0i} &= \begin{bmatrix} A_i X_{2i} + B_i \widehat{U}_{1i} & A_i \\ \widehat{U}_{4i} & Y_{2i} A_i \end{bmatrix}, \\ \Omega_{1i} &= 0, \quad \Omega_{3i} = \begin{bmatrix} F_i \\ Y_{2i} F_i \end{bmatrix}, \quad \Omega_{4i} = \begin{bmatrix} (C_i X_{2i})^T \\ C_i^T \end{bmatrix}^T.\end{aligned}$$

The complete transformed LMI entries are listed below. They contain no additional decision variables beyond those introduced in Theorem 2.

$$\begin{aligned}\bar{\Pi}_{11i} &= \begin{bmatrix} \bar{\Xi}_{11i} & \bar{R}_{1i} & \Gamma_{1i} & 0 & \Gamma_{2i} & \Gamma_{3i} \\ * & \bar{\Xi}_{22i} & \bar{\Xi}_{23i} & \bar{S}_{1i} & 0 & 0 \\ * & * & \bar{\Xi}_{33i} & \bar{\Xi}_{34i} & 0 & 0 \\ * & * & * & \bar{\Xi}_{44i} & 0 & 0 \\ * & * & * & * & -T_i & 0 \\ * & * & * & * & * & -\gamma_i^2 I \end{bmatrix}, \\ \bar{\Pi}_{12i} &= \begin{bmatrix} k_{1i} \phi_m \Gamma_{0i}^T & k_{2i} (\phi_M - \phi_m) \Gamma_{0i}^T & \Gamma_{4i}^T & 0 \\ 0 & 0 & 0 & 0 \\ k_{1i} \phi_m \Gamma_{1i}^T & k_{2i} (\phi_M - \phi_m) \Gamma_{1i}^T & 0 & \delta_i \Gamma_{4i}^T \\ 0 & 0 & 0 & 0 \\ k_{1i} \phi_m \Gamma_{2i}^T & k_{2i} (\phi_M - \phi_m) \Gamma_{2i}^T & 0 & -\delta_i I \\ k_{1i} \phi_m \Gamma_{3i}^T & k_{2i} (\phi_M - \phi_m) \Gamma_{3i}^T & 0 & 0 \end{bmatrix}, \\ \bar{\Upsilon}_{11i} &= \begin{bmatrix} \bar{\Psi}_{11i} & \bar{R}_{2i} & \Omega_{1i} & 0 & \Omega_{3i} \\ * & \bar{\Psi}_{22i} & \bar{\Psi}_{23i} & \bar{S}_{2i} & 0 \\ * & * & \bar{\Psi}_{33i} & \bar{\Psi}_{34i} & 0 \\ * & * & * & \bar{\Psi}_{44i} & 0 \\ * & * & * & * & -\gamma_i^2 I \end{bmatrix}, \\ \bar{\Upsilon}_{12i} &= \begin{bmatrix} k_{3i} \phi_m \Omega_{0i}^T & k_{4i} (\phi_M - \phi_m) \Omega_{0i}^T & \Omega_{4i}^T \\ 0 & 0 & 0 \\ k_{3i} \phi_m \Omega_{1i}^T & k_{4i} (\phi_M - \phi_m) \Omega_{1i}^T & 0 \\ 0 & 0 & 0 \\ k_{3i} \phi_m \Omega_{3i}^T & k_{4i} (\phi_M - \phi_m) \Omega_{3i}^T & 0 \end{bmatrix},\end{aligned}$$

$$\bar{\Pi}_{22i} = \text{diag}\{-\epsilon_1 N_{1i} + \epsilon_1^2 \bar{R}_{1i}, -\epsilon_2 N_{1i} + \epsilon_2^2 \bar{Z}_{1i}, -I, \delta_i (T_i - 2I)\},$$

$$\bar{\Upsilon}_{22i} = \text{diag}\{-\epsilon_1 N_{2i} + \epsilon_1^2 \bar{R}_{2i}, -\epsilon_2 N_{2i} + \epsilon_2^2 \bar{Z}_{2i}, -I\},$$

$$\begin{aligned}
\bar{\Xi}_{11i} &= \Gamma_{0i}^T + \Gamma_{0i} + \bar{W}_{1i} - e^{-2\alpha_{1i}\phi_m} \bar{R}_{1i} + 2\alpha_{1i}N_{1i}, \\
\bar{\Xi}_{22i} &= e^{-2\alpha_{1i}\phi_m} (\bar{Q}_{1i} - \bar{W}_{1i}) - e^{-2\alpha_{1i}\phi_m} \bar{R}_{1i} - e^{-2\alpha_{1i}(\phi_M - \phi_m)} \bar{Z}_{1i}, \\
\bar{\Xi}_{23i} &= e^{-2\alpha_{1i}(\phi_M - \phi_m)} (\bar{Z}_{1i} - \bar{S}_{1i}), \quad \bar{\Xi}_{33i} = e^{-2\alpha_{1i}(\phi_M - \phi_m)} (-2\bar{Z}_{1i} + \bar{S}_{1i} + \bar{S}_{1i}^T), \\
\bar{\Xi}_{34i} &= e^{-2\alpha_{1i}(\phi_M - \phi_m)} (\bar{Z}_{1i} - \bar{S}_{1i}), \quad \bar{\Xi}_{44i} = -e^{-2\alpha_{1i}(\phi_M - \phi_m)} \bar{Z}_{1i} - e^{-2\alpha_{1i}\phi_M} \bar{Q}_{1i}, \\
\bar{\Psi}_{11i} &= \Omega_{0i}^T + \Omega_{0i} + \bar{W}_{2i} - e^{2\alpha_{2i}\phi_m} \bar{R}_{2i} - 2\alpha_{2i}N_{2i}, \\
\bar{\Psi}_{22i} &= e^{2\alpha_{2i}\phi_m} (\bar{Q}_{2i} - \bar{W}_{2i}) - e^{2\alpha_{2i}\phi_m} \bar{R}_{2i} - e^{2\alpha_{2i}(\phi_M - \phi_m)} \bar{Z}_{2i}, \\
\bar{\Psi}_{23i} &= e^{2\alpha_{2i}(\phi_M - \phi_m)} (\bar{Z}_{2i} - \bar{S}_{2i}), \quad \bar{\Psi}_{33i} = e^{2\alpha_{2i}(\phi_M - \phi_m)} (-2\bar{Z}_{2i} + \bar{S}_{2i} + \bar{S}_{2i}^T), \\
\bar{\Psi}_{34i} &= e^{2\alpha_{2i}(\phi_M - \phi_m)} (\bar{Z}_{2i} - \bar{S}_{2i}), \quad \bar{\Psi}_{44i} = -e^{2\alpha_{2i}(\phi_M - \phi_m)} \bar{Z}_{2i} - e^{2\alpha_{2i}\phi_M} \bar{Q}_{2i}.
\end{aligned}$$

Appendix C: Physical-coordinate reconstruction and fixed-gain joint matrices

C.1 Physical-coordinate reconstruction

Let

$$x_i = \text{col}\{\Delta f_i, \Delta P_{tie-i}, \Delta P_{Si}, \Delta P_{mi}, \Delta P_{vi}, \Delta P_{Wi}\} \in \mathbb{R}^6.$$

The local area-wise stack is

$$x_{12} = \text{col}\{x_1, x_2\} \in \mathbb{R}^{12}.$$

Under the reciprocal single-tie-line relation

$$\Delta P_{tie-2} = -\Delta P_{tie-1},$$

we introduce the 11-dimensional physical coordinate

$$\begin{aligned}
x_p &= \text{col}\{\Delta f_1, \Delta P_{tie,12}, \Delta P_{S1}, \Delta P_{m1}, \Delta P_{v1}, \Delta P_{W1}, \Delta f_2, \\
&\quad \Delta P_{S2}, \Delta P_{m2}, \Delta P_{v2}, \Delta P_{W2}\} \in \mathbb{R}^{11}.
\end{aligned}$$

The area-wise stack is reconstructed through

$$x_{12} = R_p x_p, \quad R_p = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix},$$

where

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The reconstruction matrix R_p parameterizes the physical tie-line manifold:

$$\text{Im}(R_p) = \mathcal{M}, \quad \mathcal{M} = \{x_{12} \in \mathbb{R}^{12} : \Delta P_{tie-1} + \Delta P_{tie-2} = 0\}.$$

C.2 Fixed-gain joint matrices

Let $a_{i,k}^T$ denote row k of the local plant matrix A_i , and let $e_k^{(11)}$ be column k of I_{11} . The physical 11-state plant matrix is

$$A_p = \text{col}\{a_{1,1}^T E_1, 2\pi T_{12}(e_1^{(11)} - e_7^{(11)})^T, a_{1,3}^T E_1, a_{1,4}^T E_1, a_{1,5}^T E_1, a_{1,6}^T E_1, a_{2,1}^T E_2, a_{2,3}^T E_2, a_{2,4}^T E_2, a_{2,5}^T E_2, a_{2,6}^T E_2\}.$$

Define the remaining reduced plant blocks by

$$b_{p1} = \frac{e_5^{(11)}}{T_{g1}}, \quad b_{p2} = \frac{e_{10}^{(11)}}{T_{g2}}, \quad C_{p1} = C_1 E_1, \quad C_{p2} = C_2 E_2,$$

$$F_{p1} = \begin{bmatrix} -e_1^{(11)}/M_1 & e_3^{(11)}/T_{SL1} & e_6^{(11)}/T_{W1} \end{bmatrix},$$

$$F_{p2} = \begin{bmatrix} -e_7^{(11)}/M_2 & e_8^{(11)}/T_{SL2} & e_{11}^{(11)}/T_{W2} \end{bmatrix}.$$

With the two six-dimensional controller states, we set

$$\chi = \text{col}\{x_p, x_{c1}, x_{c2}\} \in \mathbb{R}^{23}, \quad w = \text{col}\{d_1, d_2\} \in \mathbb{R}^6,$$

where

$$d_i = \text{col}\{\Delta P_{di}, \Delta P_{solar-i}, \Delta P_{wind-i}\}$$

contains the three exogenous disturbance components of Area i .

With the controller gains recovered from Theorem 2 kept fixed, we define

$$A_{J1} = \begin{bmatrix} A_p & b_{p1}\mathcal{D}_1^1 & b_{p2}\mathcal{D}_2^1 \\ 0 & \mathcal{A}_1^1 & 0 \\ 0 & 0 & \mathcal{A}_2^1 \end{bmatrix}, \quad A_{J2} = \begin{bmatrix} A_p & b_{p1}\mathcal{D}_1^2 & b_{p2}\mathcal{D}_2^2 \\ 0 & \mathcal{A}_1^2 & 0 \\ 0 & 0 & \mathcal{A}_2^2 \end{bmatrix},$$

$$A_{Jd,1} = \begin{bmatrix} 0 & 0 & 0 \\ C_1^1 C_{p1} & \mathcal{B}_1^1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_{Jd,2} = \begin{bmatrix} 0 & 0 & 0 \\ C_2^1 C_{p2} & 0 & \mathcal{B}_2^1 \end{bmatrix},$$

$$E_{J1} = \text{col}\{0_{11 \times 2}, -C_1^1, 0_{6 \times 2}\}, \quad E_{J2} = \text{col}\{0_{11 \times 2}, 0_{6 \times 2}, -C_2^1\},$$

$$E_J = \begin{bmatrix} E_{J1} & E_{J2} \end{bmatrix}, \quad B_{Jw} = \begin{bmatrix} F_{p1} & F_{p2} \\ 0_{12 \times 3} & 0_{12 \times 3} \end{bmatrix}.$$

Here E_{J1} and E_{J2} embed the two local triggering-error channels into the joint coordinate. The fixed-gain joint-switched realization is

$$\begin{aligned} \Sigma_J^1: \quad \dot{\chi}(t) &= A_{J1}\chi(t) + A_{Jd,1}\chi(t - \tau_1(t)) + A_{Jd,2}\chi(t - \tau_2(t)) \\ &\quad + E_{J1}e_1(t) + E_{J2}e_2(t) + B_{Jw}w(t), \\ \Sigma_J^2: \quad \dot{\chi}(t) &= A_{J2}\chi(t) + B_{Jw}w(t), \end{aligned}$$

with independent channels $\tau_r(t) \in [\phi_m, \phi_M]$, $r = 1, 2$.

For the event-error embedding used in Appendix D, define

$$C_{e,1} = \begin{bmatrix} C_{p1} & 0 & 0 \end{bmatrix}, \quad C_{e,2} = \begin{bmatrix} C_{p2} & 0 & 0 \end{bmatrix}.$$

The joint S-procedure uses $V_r = \varrho_r T_r > 0$ with $\varrho_r > 0$ and $r = 1, 2$. The joint matrices and event-error definitions above are used directly in the independent two-delay Lyapunov–Krasovskii stability conditions of Appendix D.

Appendix D: Explicit independent dual-delay joint LMI conditions

The joint Lyapunov–Krasovskii functional is defined in the proof of Theorem 3. This appendix collects the stacked vectors, auxiliary matrices, reciprocally convex terms, event-error embeddings, and explicit LMI blocks used in the joint derivative inequalities.

D.1 Stacked-vector ordering and auxiliary matrices

For the DoS-free mode, we define

$$\chi_{m,r} = \chi(t - \phi_m), \quad \chi_{d,r} = \chi(t - \tau_r(t)), \quad \chi_{M,r} = \chi(t - \phi_M), \quad e = \text{col}\{e_1, e_2\}.$$

For the DoS-active mode, we introduce

$$\chi_{\theta,r} = \chi(t - \theta_r(t)), \quad \theta_r(t) \in [\phi_m, \phi_M],$$

where $\theta_r(t)$ is an auxiliary history point used only in the LKF estimate and does not represent an S-C communication delay. Define the stacked vectors

$$\zeta_{J1} = \text{col}\{\chi, \chi_{m,1}, \chi_{d,1}, \chi_{M,1}, \chi_{m,2}, \chi_{d,2}, \chi_{M,2}, e\},$$

$$\zeta_{J2} = \text{col}\{\chi, \chi_{m,1}, \chi_{\theta,1}, \chi_{M,1}, \chi_{m,2}, \chi_{\theta,2}, \chi_{M,2}\}.$$

The block label d_r below continues to denote the middle historical position of channel r : it corresponds to $\chi_{d,r}$ in mode 1 and $\chi_{\theta,r}$ in mode 2. Thus, this notation change does not alter the block ordering or LMI dimensions. Define

$$\begin{aligned} a_{1m} &= e^{-2\alpha_{1J}\phi_m}, & a_{1M} &= e^{-2\alpha_{1J}\phi_M}, & a_{1d} &= e^{-2\alpha_{1J}(\phi_M - \phi_m)}, \\ a_{2m} &= e^{2\alpha_{2J}\phi_m}, & a_{2M} &= e^{2\alpha_{2J}\phi_M}, & a_{2d} &= e^{2\alpha_{2J}(\phi_M - \phi_m)}. \end{aligned}$$

Let $m_r = 2 + 3(r - 1)$, $d_r = m_r + 1$, and $M_r = m_r + 2$.

D.2 Explicit joint LMI blocks

Under the positivity and reciprocally convex conditions stated in Theorem 3, we write $\Theta_{Jq}^0 = [\Theta_{Jq}^{0,ab}]$ in the corresponding block order; omitted blocks are zero and lower-triangular blocks are transposes. The complete nonzero DoS-free base blocks are

$$\begin{aligned} \Theta_{J1}^{0,11} &= \text{He}(P_{J1}A_{J1}) + 2\alpha_{1J}P_{J1} + \sum_{r=1}^2 (W_{J1,r} - a_{1m}R_{J1,r}), \\ \Theta_{J1}^{0,1m_r} &= a_{1m}R_{J1,r}, & \Theta_{J1}^{0,1d_r} &= P_{J1}A_{Jd,r}, \\ \Theta_{J1}^{0,m_r m_r} &= a_{1m}(Q_{J1,r} - W_{J1,r}) - a_{1m}R_{J1,r} - a_{1d}Z_{J1,r}, \\ \Theta_{J1}^{0,m_r d_r} &= a_{1d}(Z_{J1,r} - S_{J1,r}), & \Theta_{J1}^{0,m_r M_r} &= a_{1d}S_{J1,r}, \\ \Theta_{J1}^{0,d_r d_r} &= a_{1d}(-2Z_{J1,r} + S_{J1,r} + S_{J1,r}^T), \\ \Theta_{J1}^{0,d_r M_r} &= a_{1d}(Z_{J1,r} - S_{J1,r}), & \Theta_{J1}^{0,M_r M_r} &= -a_{1d}Z_{J1,r} - a_{1M}Q_{J1,r}, \\ \Theta_{J1}^{0,1e} &= P_{J1}E_J, & \Theta_{J1}^{0,ee} &= -\text{diag}(V_1, V_2). \end{aligned}$$

The complete nonzero DoS-active base blocks are

$$\begin{aligned}
 \Theta_{J_2}^{0,11} &= \text{He}(P_{J_2} A_{J_2}) - 2\alpha_{2J} P_{J_2} + \sum_{r=1}^2 (W_{J_2,r} - a_{2m} R_{J_2,r}), \\
 \Theta_{J_2}^{0,m_r m_r} &= a_{2m} (Q_{J_2,r} - W_{J_2,r}) - a_{2m} R_{J_2,r} - a_{2d} Z_{J_2,r}, \\
 \Theta_{J_2}^{0,m_r d_r} &= a_{2d} (Z_{J_2,r} - S_{J_2,r}), & \Theta_{J_2}^{0,m_r M_r} &= a_{2d} S_{J_2,r}, \\
 \Theta_{J_2}^{0,d_r d_r} &= a_{2d} (-2Z_{J_2,r} + S_{J_2,r} + S_{J_2,r}^T), \\
 \Theta_{J_2}^{0,d_r M_r} &= a_{2d} (Z_{J_2,r} - S_{J_2,r}), & \Theta_{J_2}^{0,M_r M_r} &= -a_{2d} Z_{J_2,r} - a_{2M} Q_{J_2,r}.
 \end{aligned}$$

In the same block orders, define

$$\begin{aligned}
 \mathcal{D}_1 &= \text{col}\{A_{J_1}^T, 0, A_{J_d,1}^T, 0, 0, A_{J_d,2}^T, 0, E_J^T\}, \\
 \mathcal{D}_2 &= \text{col}\{A_{J_2}^T, 0, 0, 0, 0, 0, 0\}.
 \end{aligned}$$

Define

$$\mathcal{E}_{e,1} = \text{col}\{I_2, 0_{2 \times 2}\}, \quad \mathcal{E}_{e,2} = \text{col}\{0_{2 \times 2}, I_2\}.$$

For each r , the event-embedding matrix $\mathcal{G}_r$ has only two nonzero block rows,

$$[\mathcal{G}_r]_{d_r} = \delta_r C_{e,r}^T, \quad [\mathcal{G}_r]_e = -\delta_r \mathcal{E}_{e,r}.$$

Here $V_r = \varrho_r T_r > 0$ with $\varrho_r > 0$, as defined in Appendix C. The complete joint stability LMIs are

$$\begin{aligned}
 \widehat{\Theta}_{J_1} &= \Theta_{J_1}^0 + \sum_{r=1}^2 \left[\phi_m^2 \mathcal{D}_1 R_{J_1,r} \mathcal{D}_1^T + (\phi_M - \phi_m)^2 \mathcal{D}_1 Z_{J_1,r} \mathcal{D}_1^T \right] \\
 &\quad + \sum_{r=1}^2 \delta_r^{-1} \mathcal{G}_r V_r \mathcal{G}_r^T < 0, \\
 \widehat{\Theta}_{J_2} &= \Theta_{J_2}^0 + \sum_{r=1}^2 \left[\phi_m^2 \mathcal{D}_2 R_{J_2,r} \mathcal{D}_2^T + (\phi_M - \phi_m)^2 \mathcal{D}_2 Z_{J_2,r} \mathcal{D}_2^T \right] < 0.
 \end{aligned}$$

These block definitions yield the derivative inequalities used in the proof of Theorem 3.

Appendix E: Controller gains

The recovered controller parameters are given below. The synthesis and online trigger use the common threshold $\delta_i = 0.10$.

$$\begin{aligned}
 T_1 &= \begin{bmatrix} 0.54287844 & 0.3151161 \\ 0.3151161 & 1.0344951 \end{bmatrix} \\
 T_2 &= \begin{bmatrix} 0.53947608 & 0.34452678 \\ 0.34452678 & 0.97821568 \end{bmatrix}
 \end{aligned}$$

$$\mathcal{A}_1^1 = \begin{bmatrix} 7.0664365 & -20.009875 & 5.8967769 & 5.1520234 & 6.2000489 & 5.0628145 \\ 112.22416 & -290.14737 & 84.111795 & 66.6241 & 83.440206 & 64.905014 \\ 7.1347863 & -18.376715 & 4.5697807 & 4.219526 & 5.3203217 & 4.1330312 \\ 146.23991 & -376.92834 & 109.36216 & 85.369619 & 110.93649 & 85.888683 \\ 240.94986 & -620.13478 & 180.21151 & 141.81616 & 175.41618 & 136.64932 \\ 7.683407 & -19.796783 & 5.7507255 & 4.5399138 & 5.7167911 & 3.7725241 \end{bmatrix}$$

$$\mathcal{B}_1^1 = \begin{bmatrix} 0.0044990655 & -0.011428989 & 0.0034589877 & 0.0033367054 & 0.0029888535 & 0.0026669524 \\ 0.00014103076 & 0.020136114 & -0.0060363606 & -0.007593978 & -0.0044526938 & -0.0046441375 \\ 0.0013351752 & -0.0018425039 & 0.00055907768 & 0.0003934065 & 0.00054875844 & 0.00043108532 \\ 0.060640594 & -0.059449343 & 0.018119657 & 0.0087296385 & 0.019559791 & 0.013981269 \\ 0.095311107 & -0.076153614 & 0.023318727 & 0.0073463129 & 0.026864753 & 0.018009727 \\ 0.001658128 & -0.0021523957 & 0.00065371762 & 0.00043816552 & 0.00065116153 & 0.0005041552 \end{bmatrix}$$

$$C_1^1 = \begin{bmatrix} -0.022163066 & -0.0090035879 \\ 0.071658175 & -0.044838923 \\ -0.00026119719 & -0.00019897959 \\ 0.07704803 & 0.047276962 \\ 0.16763825 & 0.11129951 \\ 8.4801759e-05 & 2.9149588e-05 \end{bmatrix}$$

$$\mathcal{D}_1^1 = \begin{bmatrix} -5.7171966 & 14.294227 & -4.142705 & -3.2375454 & -4.1816576 & -3.222234 \end{bmatrix}$$

$$\mathcal{A}_1^2 = \begin{bmatrix} 7.0603191 & -20.041216 & 5.9058564 & 5.1639774 & 6.2071489 & 5.0699993 \\ 111.83126 & -290.20416 & 84.122568 & 66.71543 & 83.423283 & 64.915943 \\ 7.114471 & -18.386487 & 4.5723353 & 4.2265972 & 5.3211026 & 4.1351663 \\ 145.81686 & -377.11868 & 109.41152 & 85.512443 & 110.94955 & 85.930154 \\ 240.23765 & -620.42164 & 180.2848 & 142.04505 & 175.43008 & 136.71145 \\ 7.6615693 & -19.807345 & 5.7534883 & 4.5475351 & 5.7176445 & 3.7748326 \end{bmatrix}$$

$$\mathcal{D}_1^2 = \begin{bmatrix} -5.7001937 & 14.300197 & -4.1441975 & -3.2427038 & -4.1817762 & -3.2235133 \end{bmatrix}$$

$\mathcal{B}_1^2, C_1^2$ are structurally absent.

$$\mathcal{A}_2^1 = \begin{bmatrix} 14.157102 & -28.558546 & 12.130946 & 4.9506401 & 15.103341 & 11.484766 \\ 516.77069 & -991.61085 & 417.58835 & 146.16724 & 531.05057 & 390.1891 \\ 19.98403 & -38.342401 & 15.386286 & 5.6608051 & 20.594293 & 15.132149 \\ 566.38459 & -1086.6126 & 457.77033 & 159.05094 & 583.78162 & 428.58996 \\ 754.43593 & -1445.9608 & 609.33069 & 212.38697 & 774.04736 & 568.77051 \\ 21.836318 & -41.897244 & 17.651849 & 6.1745981 & 22.499379 & 15.860193 \end{bmatrix}$$

$$\mathcal{B}_2^1 = \begin{bmatrix} 0.0012089651 & -0.0087166216 & 0.0036931121 & 0.0025133965 & 0.0038689788 & 0.0034270492 \\ 0.043467123 & -0.022949248 & 0.010421591 & -0.0022595521 & 0.016046834 & 0.00948353 \\ 0.0015552522 & -0.0021727248 & 0.00094041916 & 0.00033874311 & 0.0011541571 & 0.00086773322 \\ 0.10858159 & -0.10982308 & 0.048070382 & 0.010614298 & 0.062636984 & 0.044211282 \\ 0.1453033 & -0.13449409 & 0.059099301 & 0.010387197 & 0.078434096 & 0.054291135 \\ 0.0020991006 & -0.0027728525 & 0.0012023529 & 0.00040832524 & 0.0014888163 & 0.00110882 \end{bmatrix}$$

$$C_2^1 = \begin{bmatrix} -0.029104507 & -0.014069321 \\ 0.12622185 & 0.010169186 \\ -0.00048383114 & -0.00041762772 \\ 0.12378692 & 0.084683061 \\ 0.20617366 & 0.14782558 \\ -0.00011769652 & -0.00015754221 \end{bmatrix}$$

$$\mathcal{D}_2^1 = \begin{bmatrix} -15.533336 & 29.484748 & -12.415627 & -4.2998993 & -15.879195 & -11.634522 \end{bmatrix}$$

$$\mathcal{A}_2^2 = \begin{bmatrix} 14.164645 & -28.608833 & 12.151863 & 4.9628886 & 15.127333 & 11.504507 \\ 516.23001 & -992.32538 & 417.86643 & 146.44202 & 531.32867 & 390.4615 \\ 19.966921 & -38.375271 & 15.399304 & 5.6722117 & 20.607852 & 15.144773 \\ 565.88792 & -1087.5239 & 458.13062 & 159.37097 & 584.15523 & 428.93973 \\ 753.7598 & -1447.1489 & 609.79959 & 212.8088 & 774.53162 & 569.2262 \\ 21.817779 & -41.933338 & 17.666151 & 6.1870857 & 22.514293 & 15.874059 \end{bmatrix}$$

$$\mathcal{D}_2^2 = \begin{bmatrix} -15.518728 & 29.508126 & -12.424814 & -4.3083937 & -15.888601 & -11.643471 \end{bmatrix}$$

$\mathcal{B}_2^2$ and C_2^2 are structurally absent.



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