



Research article

An innovation decision model for a cross-competition system with spillovers

Guojun Leng^{1,2,*}, Naigang Hu³, Wei Wang³ and Peng Li²

¹ School of Electrical Engineering, Chengdu Aeronautic Polytechnic University, Chengdu 610100, China

² The State Key Laboratory of Electromechanical Integrated Manufacturing of High-Performance Electronic Equipments, Xidian University, Xi'an 710071, China

³ School of Mechano-Electronic Engineering, Xidian University, Xi'an 710071, China

* **Correspondence:** Email: guojun.leng@gmail.com; Tel: 028-8674-8423.

Abstract: We developed a game-theoretic model of an electronics manufacturing supply chain with cross-competition, in which each supplier trades with two common buyers simultaneously, and analyzed firms' innovation investment strategies with spillover effects. The model's structure and parameter space were motivated by key characteristics of the electronics manufacturing industry, such as rapid technological iteration, high spillover intensity, and cross-product competition. The results indicate that when the competition is weak, both process innovation and product innovation yield greater benefits to suppliers. However, fierce competition or increased investment scale can drive equilibrium shifts from innovations to non-innovations. Compared with product innovation, process innovation demonstrates greater resilience to competitive pressures in achieving dominance. At high spillover intensities, the benefits of product innovation can be completely negated, even in the presence of weak competition. This study constructs an innovation decision-making system for electronics manufacturing with cross-competition, providing guidance for innovation decisions in cross-competitive enterprises.

Keywords: decision model; product innovation; process innovation; spillover effects; electronics manufacturing; game theory

1. Introduction

To remain competitive in the global marketplace, firms must continue to innovate to enhance their competitive advantages [1]. Two widely adopted innovation strategies in the market are process

innovation (focusing on cost reductions) and product innovation (aiming at quality improvement) [2]. Process innovation involves the adoption of novel manufacturing technologies, optimized processes, and advanced production equipment, which collectively lower operational costs [3–5]. For instance, a door hinge supplier for General Motors has reduced manufacturing costs by 35% through design simplification [6]. Product innovation encompasses efforts to boost market demand by elevating product quality [7,8]. A prominent example is the electronics manufacturing industry, which consistently attracts consumers via technological advancements in computing power, wireless communication, and cybersecurity [9,10].

However, not all suppliers prefer to invest in innovations due to horizontal spillovers in competitive markets. When spillovers occur, firms cannot fully capture the benefits of their innovations because competitors can imitate or free-ride on these advancements at minimal or no cost [11–13]. This phenomenon is particularly prevalent in the manufacturing industry, where new technologies derived from process or product innovation are challenging to protect, inadvertently benefiting competitors [14,15]. A notable example is Apple's early fingerprint recognition technology, which has rapidly been replicated by rival smartphone suppliers due to spillover effects, subjecting Apple to intense competitive pressure. Therefore, once a technology is developed, restricting its dissemination or preventing others from accessing it becomes inherently difficult [16]. Firms in industries such as mobile phones, automotive manufacturing, and smart appliances frequently imitate competitors' innovations by acquiring the foundational technical knowledge and employing reverse engineering. Furthermore, the presence of cross-competitive markets amplifies spillovers, as competitors may obtain critical information through interactions with shared buyers.

This paper is motivated by the fact that although suppliers adopt different innovation strategies in response to changes in the market environment, previous studies have examined cross-competition and spillovers as isolated phenomena, neglecting their concurrent effects. However, a cross-competition structure formed by multiple suppliers with multiple buyers is prevalent in many industries, such as consumer electronics and computing devices, and the complexity of such transactions leads to increased spillovers, which further affect innovation decisions. In other words, prior work has yet to systematically investigate technological innovation strategies under conditions of cross-competition and technological spillovers. To address this gap, our research establishes an analytical framework to examine the innovation decision-making in a cross-competitive supply chain with spillovers, which provides guidance for suppliers' innovation decisions.

Based on the above observations and facts, we try to fill in the gaps by answering the following key questions: (i) Should suppliers in a cross-competitive market invest in process innovation or product innovation under spillover conditions? (ii) What roles do spillovers play in process/product innovation strategies of competing suppliers? (iii) How do process/product innovation strategies influence suppliers' decision-making and performance? (iv) How do factors such as the investment scale, the supply chain competition, and the innovation competition affect suppliers' innovation optimization decisions?

The contributions of this study are threefold. First, we develop a game-theoretic framework to analyze process and product innovation strategies under conditions of technological spillover and cross-competition. Such a quantitative study allows us to accurately express the threshold of suppliers' innovation, as well as the impact of the investment scale, spillover effects, supply chain competition, and innovation competition on decisions and profits. Second, we establish the boundary conditions under which suppliers achieve profitable process and product innovation despite

spillovers in a cross-competition supply chain. Furthermore, we demonstrate how critical parameters—including the investment scale, supply chain competition intensity, and innovation competition—determine equilibrium innovation strategies, thereby offering actionable guidelines for corporate R&D decision-making. Finally, by contrasting equilibrium boundaries between process innovation and product innovation, we deepen the theoretical understanding of their substitutive and complementary relationships.

The remainder of this paper is organized as follows: Section 2 reviews the relevant literature. Section 3 formalizes model assumptions. We analyze equilibrium conditions for process and product innovation adoptions in Sections 4 and 5, respectively. Section 6 uses numerical simulations to examine how innovation strategies affect product prices and innovation levels. Section 7 concludes the paper and provides future research directions.

2. Literature review

This section reviews three key research domains closely related to our study: supply chain cross-competition, innovation investment, and innovation spillover effects. By summarizing the progress and limitations of existing literature, this paper clarifies unresolved research gaps, highlights the innovations of this work, and provides a solid theoretical foundation for subsequent research.

2.1. Cross-competition

Studies on the cross-competition structure encompass two main aspects. The first involves comparing the cross-competition structure with the exclusive-competition structure. For instance, Trivedi [17], Wu and Mallik [18], and Cai et al. [19] examined price decisions and profits in supply chains characterized by exclusive competition and cross-competition. The second aspect focuses on cooperation within the cross-competition structure. Li et al. [20], Feng and Lu [21], and Li et al. [22] compared different contracts among supply chain members in a cross-competition supply chain. Li and Ai [23] investigated the equilibrium of choice between a non-innovating reselling format and a platform innovation format in a cross-competition supply chain. More recent studies have extended this line of research [16,24], but they continue to assume that suppliers are solely responsible for producing products without making active innovation investment decisions. In contrast, our study considers suppliers who not only produce products but also have the responsibility to invest in innovations. In other words, the scenarios considered in the above papers represent a special case within our model.

2.2. Innovation investment

We review related studies on innovation investment from three perspectives: First, from the perspective of competing suppliers and a single buyer: Li [24] and Li et al. [25] examined the impact of competition between two suppliers engaged in process innovation on a buyer and evaluated how the information structure affects process innovation. Additionally, Hu and Tadikamalla [26] explored when to introduce second-tier suppliers in cases where first-tier suppliers are involved in the new product development. Second, from the perspective of a single supplier and competing buyers: Hu et al. [27] discovered that competing buyers can induce suppliers to invest in both technologies and later

adopt the market-preferred technology; Cao and Chen [28] investigated demand information sharing strategies in a system consisting of a manufacturer and two buyers. Third, from the perspective of competing suppliers and competing buyers: Gupta and Loulou [12] studied whether sellers should choose an integrated or a decentralized structure for competing supply chains; Ha et al. [29] indicated that a supplier's process innovation may lead to demand information sharing in competing supply chains; and Xu et al. [30] examined innovation information sharing between two competing supply chains. Although these studies constructed models comprising two suppliers and two buyers, they did not allow cross-competition.

2.3. Spillover effects

Existing studies on innovation spillovers mainly focus on horizontal competition among peers and vertical cooperation within individual supply chains [31–33]. Some studies have examined competing supply chain scenarios. Gupta [3] constructed an exclusive competition model and revealed suppliers' incentives to form joint ventures or strategic alliances with competitors. Niu and Shen [18] investigated the impact of suppliers' innovation absorptive capacity on innovation decisions in two exclusive supply chains. Yu et al. [34] explored investment strategies for decarbonization innovation with technology spillovers in two exclusive supply chains. In addition, several studies have examined innovation decisions of common suppliers [35,36]. Differing from these studies, this paper jointly considers cross-competition and innovation spillovers to analyze suppliers' innovation investment decisions. Furthermore, we investigate how two typical innovation strategies, namely process innovation and product innovation, affect supply chain participants' operational decisions.

2.4. Research gaps and our contributions

Based on the above review, we identify three major research gaps. First, while cross-competition has been studied in the context of pricing and channel decisions, no prior work has examined innovation investment decisions under this structure. Second, although innovation investment and spillover effects have been extensively studied, existing research assumes exclusive supplier–buyer relationships rather than cross-competition. Third, comparative analyses of process versus product innovation under the joint influence of cross-competition and spillovers are entirely absent. To fill these gaps, our study makes three main contributions: (i) we develop a game-theoretic framework that jointly incorporates cross-competition, spillover effects, and innovation investment; (ii) we derive boundary conditions under which process and product innovation yield higher profits despite spillovers; and (iii) we compare the equilibrium boundaries of the two innovation types to deepen the theoretical understanding of their substitutive and complementary relationships.

3. The proposed model

We construct a stylized model with two suppliers and two buyers, and each supplier produces a single product for sale to two common buyers, forming the cross-trade, as shown in Figure 1. From the figure, it can be observed that competition exists between two competing suppliers and between two competing buyers, and each buyer simultaneously sells the products of two suppliers to customers. Our model explores a single-period game in which consumers are free to match a product with a buyer.

We use i , j , and ij as indices to denote the corresponding supplier, buyer, and combination, respectively, where $i = 1, 2$ and $j = 1, 2$. Faced with a cross-competitive marketplace, suppliers have two options: an innovation supplier (denoted as I) or a non-innovating supplier (denoted as T). Then, there are four competitive combinations: II , IT , TI , and TT .

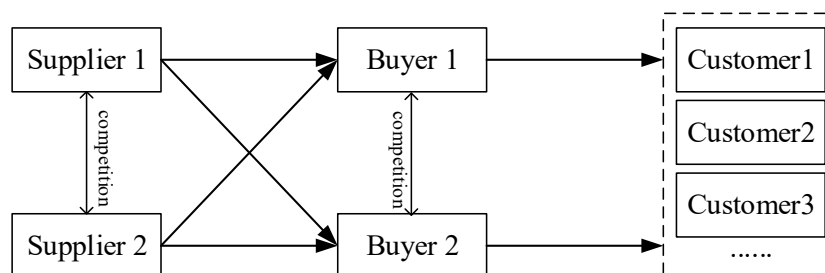


Figure 1. A cross-competition model with competing suppliers and competing buyers.

For non-innovating suppliers, the unit production cost is c . For innovation suppliers, in addition to the non-innovating unit production cost (c), they need to invest additional costs to innovate in order to reduce costs or improve the product quality (I). Following Gilbert et al. [37], Gupta [3], Ha et al. [29], and Hu and Tadikamalla [24], we set the cost of achieving A as an increasing convex function $I = kA^2$, where $A = \varepsilon_i, \tau_i$. ε_i is the unit cost reduction achieved by supplier i as a result of the process innovation level, and τ_i is the product innovation level of supplier i . $k > 0$ is the investment scale coefficient. The higher the level k , the greater the scale of the investment in innovations. In addition, we set the buyers' operating costs (e.g., manufacturing costs and distribution costs) to zero because they are constant.

The spillover effects of innovation are quite important in a competitive environment, especially when one supplier innovates and the other does not. We consider that if supplier i , who engages in innovations, can obtain process innovation level ε_i (i.e., the cost saving) or product innovation level τ_i (the increase in the demand) while paying the cost $k\varepsilon_i^2$ or $I = k\tau_i^2$, the other supplier $3 - i$ who does not adopt the innovation can attain a cost-saving ($v\varepsilon_i$) or an increase in the demand ($v\tau_i$) through spillovers without paying expenditures for related innovations. $v \in [0, 1]$ denotes the spillover level that represents the extent to which a supplier's investment in innovations benefits its competitors, i.e., the extent to which the innovator's technology is free-ridden. $v = 1$ means that the innovator's technology is completely spilled over, and the non-innovating supplier can improve to the same level of innovations without any cost investment. We assume that there are no spillovers when neither supplier innovates and that spillovers cancel each other out when both suppliers innovate. It is worth noting that there is a case where both suppliers are engaged in innovations and have different levels of innovation. In this case, the high-innovation supplier generates technological spillovers to the low-innovation supplier. Actually, the effect of such technological spillovers is very similar to the effect of technological spillovers from innovative to non-innovative suppliers. Therefore, to avoid redundancy, we only consider that spillovers cancel each other out when both suppliers innovate. This is a common representation in the study of spillovers and has been used in the literature [3,36,38].

The supplier i and buyer j form the supply chain ij , and we use q_{ij} to represent the demand for the supply chain ij . The product i 's total demand is $q_i = q_{ij} + q_{i,3-j}$. We use a to represent the potential market demand and p_{ij} to represent the retail price of the supply chain ij . We denote

ε_i and ε_{3-i} as the reduction in cost of the product i and the product $3-i$ due to process innovation, respectively; τ_i and τ_{3-i} denote the product innovation level of the product i and the product $3-i$, respectively. $b \in [0,1]$ describes the supply chain substitutability, which represents the intensity of supply chain competition. $\rho \in [0,1]$ denotes the intensity of i and $3-i$'s product innovation competition.

We denote $L = \{II, IT, TI, \text{ and } TT\}$ as the innovation choice combinations, where I and T represent the innovation strategy and the non-innovation strategy, respectively. $N = \{r, e\}$ is the innovation format, where r and e represent process innovation and product innovation, respectively. Referring to Ingene and Parry [39], we adopt the inverse demand function with process innovation (r) as Eq (1).

$$p_{ij}^{L-r} = a - q_{ij} - b(q_{i,3-j} + q_{3-i,j} + q_{3-i,3-j}), L = II, IT, TI, \text{ and } TT \quad (1)$$

The inverse demand functions with product innovation (e) are shown in Eqs (2)–(5).

$$p_{ij}^{II-e} = a - q_{ij} - bq_{i,3-j} - bq_{3-i,j} - bq_{3-i,3-j} + \tau_i - \rho\tau_{3-i}, \quad (2)$$

$$p_{1j}^{IT-e} = a - q_{ij} - bq_{i,3-j} - bq_{3-i,j} - bq_{3-i,3-j} + \tau_i, \quad (3)$$

$$p_{2j}^{IT-e} = a - q_{3-i,j} - bq_{i,3-j} - bq_{ij} - bq_{3-i,3-j} + v\tau_i, \quad (4)$$

$$p_{ij}^{TT-e} = a - q_{ij} - bq_{i,3-j} - bq_{3-i,j} - bq_{3-i,3-j}, \quad (5)$$

p_{ij}^{L-N} denotes the retail price of the supply chain ij for the innovation choice combination L with the innovation format N , where $L = \{II, IT, TI, \text{ and } TT\}$, $N = \{r, e\}$, $i = 1, 2$ and $j = 1, 2$. v denotes the spillover level. b denotes the substitutability of supply chains. ρ denotes the competition intensity of product innovation. ε_i and τ_i denote process and product innovation level, respectively.

We solve this problem using a two-stage game. First, we introduce the game sequence under process innovation. In the first stage, two suppliers, acting as the leaders of the supply chain, decide independently and simultaneously whether to innovate or not. If the suppliers do not innovate, they determine the wholesale price. If the suppliers innovate, they determine the wholesale price and the cost savings resulting from process innovation. In the second stage, the buyers determine their order quantities based on the suppliers' decisions.

On the other hand, the game sequence under product innovation is as follows: In the first stage, both suppliers, acting as the leaders of the supply chain, decide whether to invest in product innovation or not. If they do not innovate, the suppliers determine the wholesale price. If they innovate, the suppliers determine the wholesale price and the product innovation level. In the second stage, the buyers determine their order quantities according to the suppliers' decisions. In all of the above-mentioned decisions, the members aim to maximize their own profits. The notations used in our paper are shown in Table 1.

Table 1. Notations.

Symbols	Description
i	Subscript, the index of the supplier i or the product i , where $i = \{1, 2\}$
j	Subscript, the index of the buyer j , where $j = \{1, 2\}$
L	Superscript, the innovation choice combinations, $L = \{II, IT, TI, \text{ and } TT\}$, where I and T represent supplier choosing innovation and non-innovation, respectively
N	Superscript, the innovation format, $N = \{r, e\}$, where r and e represent process innovation and product innovation, respectively
a	The potential market size
b	The supply chains $\{ij\}$ substitutability, which represents the intensity of supply chain competition, where $i, j = \{1, 2\}$
k	The investment scale coefficient
ρ	The intensity of product innovation competition
v	The spillover level
c	The cost of a product without innovation
p_{ij}^{L-N}	The retail price of the supply chain ij for the innovation choice combination L with the innovation format N , where $N = \{r, e\}$
q_{ij}^{L-N}	The demand of the supply chain ij for the innovation choice combination L with the innovation format N , where $N = \{r, e\}$
w_{ij}^{L-N}	The wholesale price of the supply chain ij for the innovation choice combination L with the innovation format N , where $N = \{r, e\}$
ε_i^{L-N}	The process innovation level of product i for the innovation choice combination L
τ_i^{L-N}	The product innovation level of product i for the innovation choice combination L
π_{Sij}^{L-N}	The supplier's profits of the supply chain ij for the innovation choice combination L with the innovation format N , where $\pi_{S_i}^{L-N} = \pi_{S_{ij}}^{L-N} + \pi_{S_{i,3-j}}^{L-N}$
π_{Bij}^{L-N}	The buyer j 's profits for the innovation choice combination L with the innovation format N , where $\pi_{B_j}^{L-N} = \pi_{B_{ij}}^{L-N} + \pi_{B_{3-i,j}}^{L-N}$

4. Process innovation

4.1. Model solution

In the case of process innovation (r), the supplier sets both the wholesale price and the process innovation level, and the buyer offers the order quantity based on the supplier's decision. For the scenario without innovation, the supplier sets only the wholesale price, and the buyer then offers order quantities based on the wholesale price. We first present the optimal strategy for buyers, followed by an analysis of the optimal strategy for two suppliers. The buyer j 's profit is shown in Eq (6).

$$\pi_{B_j}^{L-r} = (p_{ij} - w_{ij})q_{ij} + (p_{3-i,j} - w_{3-i,j})q_{3-i,j}, \quad L = II, IT, TI, \text{ and } TT. \quad (6)$$

Now we analyze the optimal strategy of two suppliers. Let $\pi_{S_1}^{\hat{L}-r}$ be the profit of the supplier 1 for the innovation choice combination $\hat{L}$ with the process innovation (r), where $\hat{L} = II, IT$. In this case, the supplier 1's profit is as shown in Eq (7).

$$\pi_{S_1}^{\hat{L}-r} = (w_{11} - c + \varepsilon_1)q_{11} + (w_{12} - c + \varepsilon_1)q_{12} - k\varepsilon_1^2, \quad (7)$$

Let $\pi_{S_2}^{\hat{\hat{L}}-r}$ be the profit of the supplier 2 for the innovation choice combination $\hat{\hat{L}}$ with the process innovation, $\hat{\hat{L}} = IT, TT$. In this case, the supplier 2's profit is shown in Eq (8).

$$\pi_{S_2}^{\hat{\hat{L}}} = (w_{21} - c)q_{21} + (w_{22} - c)q_{22}. \quad (8)$$

Based on the game sequence and backward induction, the optimal decisions are summarized in Table A1. From the optimal decisions, we can derive the profits of supply chain players under the four combinations $\{II, IT, TI, \text{ and } TT\}$. In addition, the combinations IT and TI are symmetric, so we only present the results of the combination IT . Table A1 and the proofs can be found in the Appendix.

Lemma 1. The supplier 1's profit function $\pi_{S_1}^{\hat{L}-r}$ is jointly concave if and only if $k > 0.25$, where $\hat{L} = II, IT$.

The proof is provided in the Appendix.

4.2. Result analysis

In a competitive market, a supplier (supplier 1) has to solve an innovation choice problem in two scenarios, as follows. Case (i): If a competitor (supplier 2) does not adopt the process innovation, will supplier 1 adopt the innovation? This is equivalent to comparing the strategy combinations TT and IT . Case (ii): If the competitor (supplier 2) makes a process innovation, how does supplier 1 make a decision on innovations? This problem can be solved by comparing the strategy combinations TI and II .

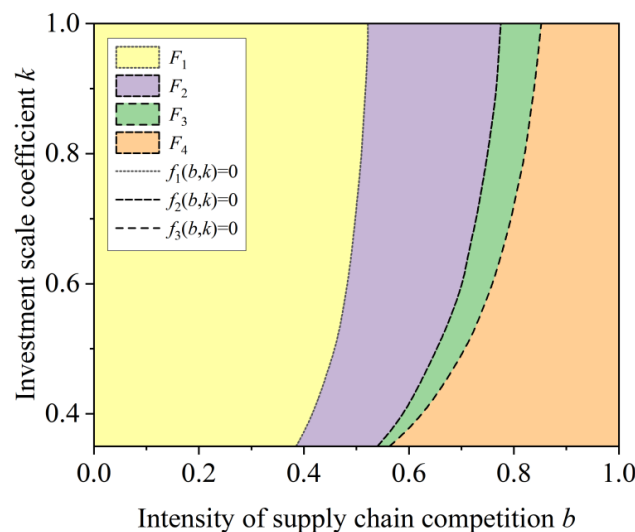


Figure 2. Influence of parameters b and k on the boundary value v_1 and v_2 .

To compare, we define v_1 as the boundary value of $\pi_{S_1}^{IT-r}$ and $\pi_{S_1}^{TT-r}$, and define v_2 as the boundary value of $\pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TI-r}$. Then, we set $f_1(b, k) = v_1$ and $f_2(b, k) = v_1 - 1$. $f_3(b, k) = v_1 - v_2$. Figure 2 shows the influence of the intensity of supply chain competition b and the investment scale coefficient k on the boundary value v_1 and v_2 . The three curves in the figure represent $f_1(b, k) = 0$, $f_2(b, k) = 0$, and $f_3(b, k) = 0$, and these curves divide the entire region into F_1, F_2, F_3 and F_4 . There exist

$$F_1 = \{(b, k) | f_1(b, k) > 0, f_2(b, k) > 0, f_3(b, k) < 0\},$$

$$F_2 = \{(b, k) | f_1(b, k) > 0, f_2(b, k) < 0, f_3(b, k) < 0\},$$

$$F_3 = \{(b, k) | f_1(b, k) > 0, f_2(b, k) < 0, f_3(b, k) > 0\},$$

$$F_4 = \{(b, k) | f_1(b, k) < 0, f_2(b, k) < 0, f_3(b, k) > 0\}.$$

First, we discuss Case (i), i.e., we compare combination TT and IT to address supplier 1's innovation decision when supplier 2 is a non-innovating supplier. We can obtain Proposition 1 as follows:

Proposition 1. In a cross-competition market, if supplier 2 adopts the strategy T ,

1) when the intensity of supply chain competition and the investment scale coefficient satisfy $(b, k) \in F_1$, or satisfy $(b, k) \in [F_2 \cup F_3]$ with a low spillover level simultaneously, process innovation is optimal for the supplier 1;

2) when the intensity of supply chain competition and the investment scale coefficient satisfy $(b, k) \in F_4$, or satisfy $(b, k) \in [F_2 \cup F_3]$ with a high spillover level simultaneously, not innovating is optimal for supplier 1.

Proposition 1 demonstrates that when supplier 2 chooses a non-innovation strategy, supplier 1's decision to engage in process innovation depends on three factors: (a) the intensity of the supply chain competition, (b) the spillover level, and (c) the investment scale. With weak supply chain competition, a process innovation strategy enables supplier 1 to achieve higher profits. However, as the intensity of supply chain competition increases, supplier 1 must increasingly allocate resources to competitive efforts. In this scenario, supplier 1 benefits from process innovation only if the spillover level is low ($1 > v > v_1$ and v_1 is provided in the Appendix), as spillover effects significantly diminish the value of process innovation. With extremely intense competition, supplier 1 can achieve higher profitability through non-innovations regardless of the spillover level.

Additionally, in comparison with supply chain competition and spillover effects, the investment scale coefficient plays a less significant role in influencing suppliers' innovation decisions. Although the increased investment scale coefficient may improve the benefits of process innovation strategies to some extent, these advantages gradually erode with rising competition intensity and the spillover level.

Second, we analyze Case (ii), i.e., comparing combinations II and TI to present supplier 1's innovation choice when supplier 2 engages in process innovation.

Proposition 2. In a cross-competition market, if the competitor (supplier 2) adopts strategy I , for supplier 1.

1) When the spillover level is low, process innovation is the optimal strategy.

2) When the spillover level is high, non-innovating production is the optimal strategy.

Proposition 2 indicates that, unlike scenarios where competitor 2 abstains from innovations, if competitor 2 adopts process innovation, supplier 1's strategic choice hinges solely on the spillover level, with the emergence of a critical spillover threshold v_2 (v_2 is provided in the Appendix). With

a low spillover level ($0 < v < v_2$), supplier 1 tends to adopt process innovation that can lead to higher profits. However, with a high spillover level ($v_2 < v < 1$), supplier 1 can benefit from spillover-driven cost reductions without direct investment.

It can be observed that the supplier 1's process innovation decision is also influenced by the intensity of the supply chain competition and the investment scale. However, these factors only modulate the spillover threshold, rather than directly altering supplier 1's decision. This contrasts with scenarios where competitors adopt the strategy T, where the competition intensity directly impacts strategic outcomes.

Finally, we discuss the symmetric decision offered by two suppliers, i.e., two suppliers simultaneously choose process innovation (II) or not (TT). From the expression $\pi_{S_i}^{II-r}$ and $\pi_{S_i}^{TT-r}$, it can be observed that if $(b, k) \in F_1$, $\pi_{S_i}^{II-r} > \pi_{S_i}^{TT-r}$; otherwise, $\pi_{S_i}^{II-r} < \pi_{S_i}^{TT-r}$.

4.3. Process innovation choice equilibrium

Based on the analytical results from Cases i and ii in Section 4.2, we can derive Proposition 3 as follows:

Proposition 3. In a cross-competition market, the Nash equilibrium of the process innovation choice game between two suppliers can be characterized as in Table 2.

Table 2. Process innovation choice equilibrium.

$(b, k) \in F_1$		$(b, k) \in F_2$		$(b, k) \in F_3$		$(b, k) \in F_4$	
$v \in [0, v_2]$	$v \in [v_2, 1]$	$v \in [0, v_2]$	$v \in [v_2, v_1]$	$v \in [v_1, 1]$	$v \in [0, v_2]$	$v \in [v_2, v_1]$	$v \in [v_1, 1]$
II	IT(TI)	II	IT(TI)	TT	II	TT	TT

Proposition 3 presents that the supplier's choice of process innovation in a cross-competition environment is related to the intensity of supply chain competition, the spillover level, and the investment scale. Additionally, it is highly dependent on the intensity of supply chain competition and the spillover level. When the supply chain competition is not intense and the spillover level is low, the supplier shifts from the non-innovation strategy to the process innovation strategy, and at that time, process innovation can increase the supplier's profit.

5. Product innovation

5.1. Equilibrium of the two-stage game

In this case, if the supplier engages in product innovation, it determines the wholesale price and the innovation level simultaneously. The buyer subsequently provides the order quantity based on the supplier's decisions. If the supplier chooses not to innovate, the supplier provides only the wholesale price, and the buyer then provides the order quantity based on the supplier's wholesale price. We first analyze the buyer's optimal strategy and then present the supplier's optimal strategy. Use $\pi_{B_j}^{L-e}$ as the profit of the buyer j in the scenario of L with product innovation (e), where $L = II, IT, TI$ and TT . q_{ij} and p_{ij} denote the demand and retail price of the supply chain ij , respectively, where $i = 1, 2$; $j = 1, 2$. Therefore, the buyer j 's profit is shown in Eq (9).

$$\pi_{B_j}^{L-e} = (p_{ij} - w_{ij})q_{ij} + (p_{3-i,j} - w_{3-i,j})q_{3-i,j}. \quad (9)$$

Now we analyze the optimal strategy of the supplier. Let $\pi_{S_1}^{\hat{L}-e}$ be the profit of supplier 1 in the scenario of $\hat{L}$ with product innovation (e), where $\hat{L} = II, IT$. In this case, the supplier 1's profit is shown in Eq (10).

$$\pi_{S_1}^{\hat{L}-e} = (w_{1j} - c)q_{1j} + (w_{1,3-j} - c)q_{1,3-j} - k\tau_1^2. \quad j = 1, 2 \quad (10)$$

Let $\pi_{S_2}^{\hat{L}-e}$ be the profit of supplier 2 in the scenario of $\hat{L}$ with product innovation (e), where $\hat{L} = IT, TT$. In this case, supplier 2's profit is shown in Eq (11).

$$\pi_{S_2}^{\hat{L}-e} = (w_{2j} - c)q_{2j} + (w_{2,3-j} - c)q_{2,3-j}. \quad j = 1, 2 \quad (11)$$

Based on the game sequence in Section 3 and backward induction, the optimal decisions and profits under the four arrangements $\{II, IT, TI, TT\}$ are summarized in Table A2. The contract arrangements IT and TI are symmetric; therefore, we only present the results of the arrangement IT . Table A2 and the proofs are provided in the Appendix.

Lemma 2. The supplier 1's profit function $\pi_{S_1}^{\hat{L}-e}$ is jointly concave if and only if $k > \frac{(b+3b\rho+2)^2}{(2b+1)(1-b)(b+2)}$, where $\hat{L} = II, IT$.

The proof is provided in the Appendix.

5.2. Influence of competition on product innovation decisions

We consider two types of competition in the market: the supply chain competition and the product innovation competition. In this subsection, we examine how members' product innovation decisions are affected by both types of competition. Suppliers' profits are also related to the investment scale coefficient k . We assume $k = 1/2$, which represents a moderate investment scale coefficient; most of the existing literature adopts this setting [40,41].

For better expression, we define $g_1^{v=0}(b, \rho) = \pi_{S_1}^{II-e} - \pi_{S_1}^{TT-e}$, $g_2^{v=0}(b, \rho) = \pi_{S_1}^{II-e} - \pi_{S_1}^{TI-e}$, and $g_3^{v=0}(b, \rho) = \pi_{S_1}^{IT-e} - \pi_{S_1}^{TT-e}$ for the case of no spillover effects, and $g_1^{v=1}(b, \rho) = \pi_{S_1}^{II-e} - \pi_{S_1}^{TT-e}$, $g_2^{v=1}(b, \rho) = \pi_{S_1}^{II-e} - \pi_{S_1}^{TI-e}$, and $g_3^{v=1}(b, \rho) = \pi_{S_1}^{IT-e} - \pi_{S_1}^{TT-e}$ for the case of the full spillover effects. Figure 3 shows the impact of the supply chain competition and the innovation competition on the product innovation choice. In Figure 3(a), three curves represent $\pi_{S_1}^{II-e} = \pi_{S_1}^{TT-e}$, $\pi_{S_1}^{II-e} = \pi_{S_1}^{TI-e}$, and $\pi_{S_1}^{IT-e} = \pi_{S_1}^{TT-e}$, as long as $v = 0$. From the figure, we notice that there exist four regions $G_1^{v=0}$, $G_2^{v=0}$, $G_3^{v=0}$, and $G_4^{v=0}$. Similarly, there exist five regions $G_1^{v=1}$, $G_2^{v=1}$, $G_3^{v=1}$, $G_4^{v=1}$, and $G_5^{v=1}$ for the case of full spillover effects, as shown in Figure 3(b). Therefore, we obtain Lemma 3 as follows.

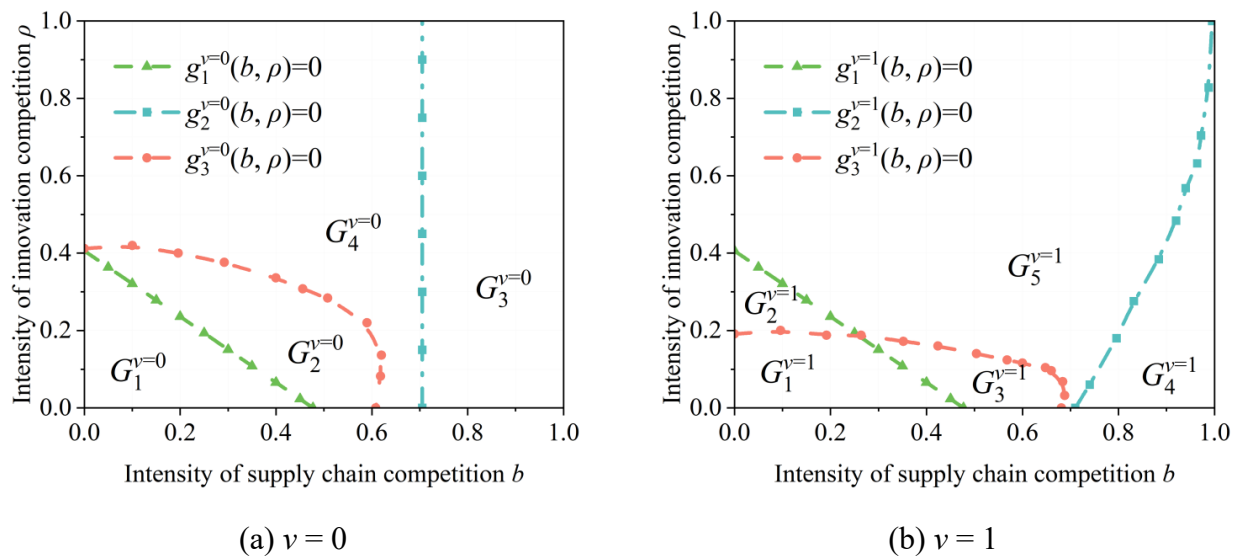


Figure 3. The impact of the supply chain competition and the innovation competition on the product innovation choice.

Lemma 3.

1) Without the spillovers ($v = 0$), there exist the following.

$(k, \rho) \in G_1^{v=0}$	$(k, \rho) \in G_2^{v=0}$	$(k, \rho) \in G_3^{v=0}$	$(k, \rho) \in G_4^{v=0}$
$\pi_{S_1}^{TT-e} < \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$
$\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$	$\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{IT-e}$	$\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$
$\pi_{S_1}^{TI-e} < \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TI-e} < \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TI-e} > \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TI-e} > \pi_{S_1}^{II-e}$

2) With the full spillovers ($v = 1$), there exist the following.

$(k, \rho) \in G_1^{v=1}$	$(k, \rho) \in G_2^{v=1}$	$(k, \rho) \in G_3^{v=1}$	$(k, \rho) \in G_4^{v=1}$	$(k, \rho) \in G_5^{v=1}$
$\pi_{S_1}^{TT-e} < \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TT-e} < \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$
$\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$	$\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$	$\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$	$\pi_{S_1}^{TT-e} > \pi_{S_1}^{IT-e}$	$\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$
$\pi_{S_1}^{TI-e} < \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TI-e} > \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TI-e} < \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TI-e} > \pi_{S_1}^{II-e}$	$\pi_{S_1}^{TI-e} > \pi_{S_1}^{II-e}$

Lemma 3 describes the innovation preferences under two scenarios: no spillovers and full product spillovers. We first analyze the case without spillovers. When two suppliers choose the same strategy, if both supply chain competition and innovation competition are weak (i.e., the region $G_1^{v=0}$ in Figure 3(a)), both suppliers' profits are higher with product innovation than non-innovation, i.e., $\pi_{S_1}^{TT-e} < \pi_{S_1}^{II-e}$. Otherwise, non-innovation becomes more advantageous for suppliers, i.e., $\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$. When the competitor engages in product innovation, the weak innovation competition combined with the weak/moderate supply chain competition (i.e., the region $G_1^{v=0} \cup G_2^{v=0}$ in Figure 3(a)) result in suppliers' higher profits under non-innovation, i.e., $\pi_{S_1}^{TI-e} < \pi_{S_1}^{II-e}$. Otherwise, non-innovation is optimal. Conversely, when the competitor adopts a non-innovation strategy, the supplier benefits from product innovation unless the product competitiveness is excessively high (i.e., the region $G_1^{v=0} \cup G_2^{v=0} \cup G_4^{v=0}$ in Figure 3(a)).

The case with full spillovers yields the same analytical results as the scenario without spillovers when suppliers adopt the identical strategy. That is, the region $G_1^{v=0}$ in Figure 3(a) coincides with the region $G_1^{v=1} \cup G_2^{v=1}$ in Figure 3(b). When the competitor engages in product innovation, the innovation advantage under full spillovers decreases significantly from the region $G_1^{v=0} \cup G_2^{v=0}$ to the region $G_3^{v=1}$. However, if the competitor abstains from innovations, the supplier's innovation advantage increases from the region $G_1^{v=0} \cup G_2^{v=0} \cup G_4^{v=0}$ in Figure 3(a) to $G_1^{v=1} \cup G_2^{v=1} \cup G_3^{v=1} \cup G_5^{v=1}$ in Figure 3(a). According to Lemma 3, we can derive the proposition as follows.

Proposition 4. In a cross-competition supply chain, the Nash equilibrium of the product innovation choice game between two suppliers can be characterized as:

1) Given no spillovers ($v = 0$),

$$\begin{cases} II, & \text{if } (k, \rho) \in G_1^{v=0}; \\ IT/TI, & \text{if } (k, \rho) \in [G_2^{v=0} \cup G_4^{v=0}] \\ TT, & \text{if } (k, \rho) \in G_3^{v=0}. \end{cases}$$

2) Given the full innovation ($v = 1$),

$$\begin{cases} II, & \text{if } (k, \rho) \in G_1^{v=1}; \\ IT / TI, & \text{if } (k, \rho) \in [G_2^{v=1} \cup G_3^{v=1} \cup G_5^{v=1}] \\ TT, & \text{if } (k, \rho) \in G_4^{v=1}. \end{cases}$$

Proposition 4 reveals that whether suppliers engage in product innovation depends on both the supply chain competition and the innovation competition. When the intensity of both the supply chain competition and the innovation competition is low (i.e., the region $G_1^{v=0}$ or $G_1^{v=1}$), product innovation is the only equilibrium and the optimal equilibrium. The region $G_1^{v=1}$ in Figure 3(b) is significantly smaller than the region $G_1^{v=0}$ in Figure 3(a), suggesting that the high spillover level reduces the willingness of suppliers to adopt product innovation.

When the supply chain competition is intense, not investing in product innovation is the optimal and only choice for suppliers, regardless of the level of innovation competition, if spillovers are not considered. This finding implies that supply chain competition plays a dominant role in the selection of product innovation. When spillovers are taken into account, spillover effects lead to a decrease in the probability under product innovation, i.e., $G_3^{v=0}$ in Figure 3(a) reduces to $G_4^{v=1}$ in Figure 3(b). That is, non-innovation becomes optimal for the suppliers if and only if the supply chain competition is strong but the innovation competition is weak. Furthermore, comparing regions $[G_2^{v=0} \cup G_4^{v=0}]$ and $[G_2^{v=1} \cup G_3^{v=1} \cup G_5^{v=1}]$ in Figure 3(b) reveals that the emergence of spillovers makes it easier for two suppliers to generate different innovation choices, i.e., IT or TI .

5.3. Influence of competition and investment scales on product innovation decisions

In this section, we study how members' product innovation decisions are affected by the investment cost. Since the supply chain competition and innovation effects are also key factors in innovation decisions, this section first considers the following extreme scenarios: 1) The supply chain competition b is the same as the innovation competition ρ (i.e., $b = \rho$) and there are no spillovers (i.e., $v = 0$); 2) the non-innovating supplier can enjoy the benefits of the innovating supplier's product innovation completely free of charge through spillovers (i.e., $v = 1$). For better expression, we define $h_1(b, k) = \pi_{S_1}^{II-e} - \pi_{S_1}^{TT-e}$, $h_2(b, k) = \pi_{S_1}^{IT-e} - \pi_{S_1}^{TI-e}$, and $h_3(b, k) = \pi_{S_1}^{II-e} - \pi_{S_1}^{TI-e}$.

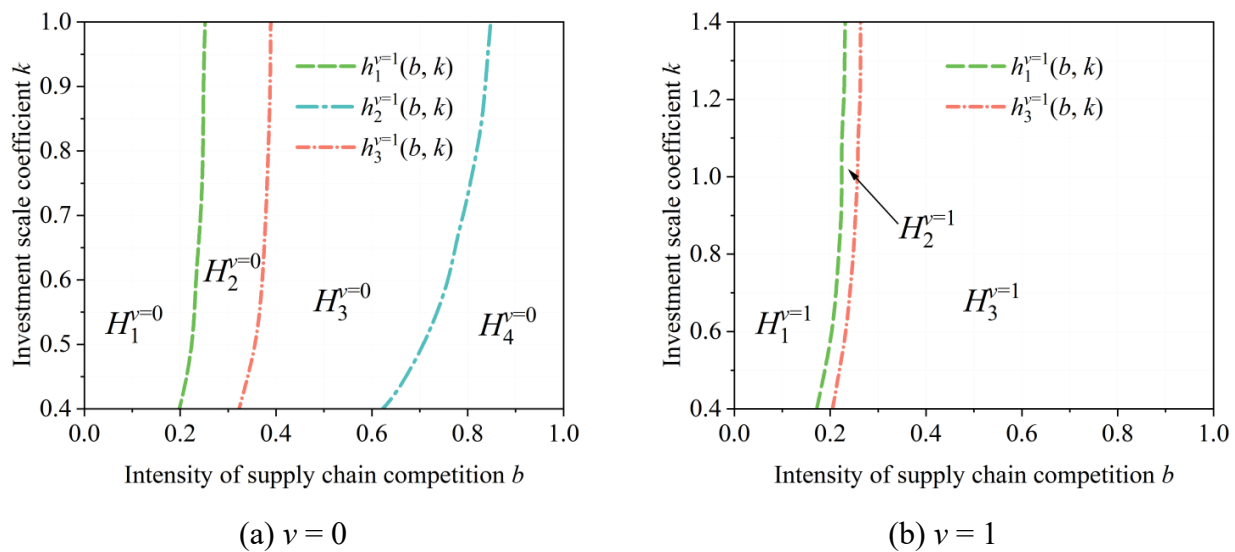


Figure 4. The impact of competition and investment scale coefficients on the product innovation choice.

Figure 4 illustrates that the investment scale coefficient and the competition intensity jointly influence the supplier's product innovation decisions. In the figure, the horizontal axis is the substitutability of supply chains b , and the vertical axis is the investment scale coefficient k . Three curves in Figure 4(a) mean $\pi_{S_1}^{II-e} = \pi_{S_1}^{TT-e}$, $\pi_{S_1}^{IT-e} = \pi_{S_1}^{TT-e}$, and $\pi_{S_1}^{II-e} = \pi_{S_1}^{TI-e}$ for $v = 0$, and there exist four regions $H_1^{v=0}$, $H_2^{v=0}$, $H_3^{v=0}$, and $H_4^{v=0}$ in Figure 4(a). From this figure, we notice that in the no-spillovers scenario, when the intensity of competition is very low, i.e., in the region $H_1^{v=0}$, the supplier achieves higher profits under product innovation compared to non-innovation, regardless of whether the competitor chooses to innovate or not, i.e., $\pi_{S_1}^{TT-e} < \pi_{S_1}^{II-e}$, $\pi_{S_1}^{TI-e} < \pi_{S_1}^{II-e}$, and $\pi_{S_1}^{TT-e} < \pi_{S_1}^{IT-e}$. On the contrary, given the high or moderately high competition intensity and low investment scale, the supplier is more profitable in the no-innovation scenario, i.e., $\pi_{S_1}^{TT-e} > \pi_{S_1}^{II-e}$, $\pi_{S_1}^{TI-e} > \pi_{S_1}^{II-e}$, and $\pi_{S_1}^{TT-e} > \pi_{S_1}^{IT-e}$.

The case with the full spillovers (Figure 4(b)) is analogous to the case with no spillovers (Figure 4(a)), except that the boundary of $\pi_{S_1}^{IT-e}$ and $\pi_{S_1}^{TT-e}$ is absent. That is, when the competitor does not innovate, the supplier is willing to innovate, regardless of the high or low competition and the investment scale coefficient, i.e., $\pi_{S_1}^{TI-e} > \pi_{S_1}^{II-e}$. As shown in Figure 4(a),(b), it can be observed that the increase in spillover effects may gradually diminish or even completely offset the advantages of product innovation.

Proposition 5. For two suppliers, there exist:

- 1) Given no spillovers ($v = 0$) of product innovation,
 - if $(k, b) \in H_1^{v=0}$, II dominates IT , TI , and TT ;
 - if $(k, b) \in [H_2^{v=0} \cup H_3^{v=0}]$, IT and TI dominate II and TT ; and
 - if $(k, b) \in H_4^{v=0}$, TT dominates IT , TI and II .
- 2) Given full spillovers ($v = 1$) of product innovation, IT and TI dominate II and TT .

Proposition 5 shows that with no spillovers ($v = 0$), product innovation is the optimal and only choice of the supplier in the region H_1 , while non-innovation is the optimal and only equilibrium for the supplier in region $H_4^{v=0}$ in Figure 4(a). As can be seen from Figure 4, the size of region $H_1^{v=0}$

(choosing product innovation) is primarily influenced by the competition intensity, whereas the region $H_4^{v=0}$ (choosing non-innovation) is strongly affected by both the competition intensity and the investment scale.

With full spillovers ($v = 1$), neither II nor TT is the final equilibrium. Under complete spillover conditions, the two suppliers would adopt asymmetric product innovation strategies, i.e., with one choosing non-innovation while the other chooses product innovation.

Comparing product innovation with process innovation, the results indicate that in the presence of cross-competition, suppliers are more likely to reach a dominant equilibrium with process innovation than with product innovation. (The equilibrium region changes from F_1 in Figure 2 to $G_1^{v=0}$ and $G_1^{v=1}$ in Figure 3 or $H_1^{v=0}$ in Figure 4.) In other words, process innovation is more likely to become a dominant equilibrium. Product innovation leads to suppliers generating different decisions, i.e., IT or TI .

6. Numerical analysis

In Sections 4 and 5, we examine the innovation preferences of competing suppliers. In this section, we analyze the effects of different innovation strategies on product prices and innovation levels from the perspectives of consumers, through numerical analyses.

Figures 5 and 6 compare product prices and innovation levels under four process innovation scenarios: (i) weak intensity of supply chain competition ($b = 0.25$) and low spillover level ($v = 0.25$), as shown in Figures 5(a) and 6(a); (ii) weak intensity of the supply chain competition ($b = 0.25$) and high spillover level ($v = 0.75$), as shown in Figures 5(b) and 6(b); (iii) fierce intensity of the supply chain competition ($b = 0.75$) and low spillover level ($v = 0.25$), as shown in Figures 5(c) and 6(c); (iv) fierce intensity of the supply chain competition ($b = 0.75$) and high spillover level ($v = 0.75$), as shown in Figures 5(d) and 6(d). In the figure, the horizontal axis represents the investment scale coefficient, and the vertical axis means retail prices. The curves represent retail prices in different combinations II , IT , TI , and TT for process innovation.

From Figure 5, it can be observed that the increased supply chain competition leads to lower product prices, which is beneficial for consumers, i.e., changing from (a) to (c) in Figure 5 and from (b) to (d) in Figure 5. Similarly, a higher spillover level also lowers the product price, shown as a change from (a) to (b) in Figure 5 and from (c) to (d) in Figure 5. Additionally, Figure 5 reveals that products under a no-innovation strategy consistently carry higher prices than those under process innovation, with the highest retail prices occurring when both suppliers abstain from innovations. When competitors do not innovate, suppliers adopting process innovation can achieve low optimal prices, even lower than in the case of II , as shown in (c) and (d) in Figure 5.

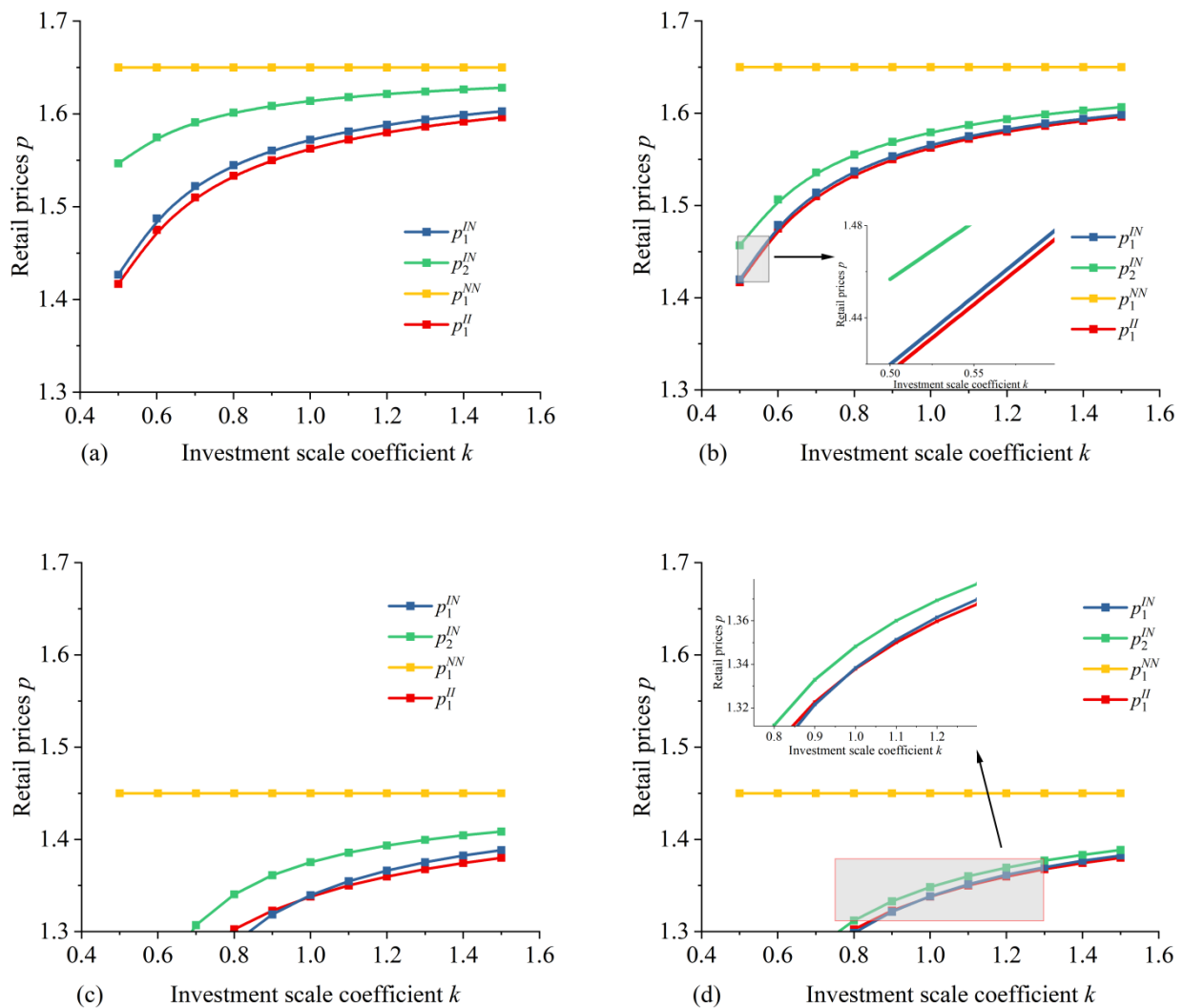


Figure 5. Retail prices for process innovation choice: (a) weak b and weak v ; (b) weak b and fierce v ; (c) fierce b and weak v ; (d) fierce b and fierce v .

Figure 6 describes the optimal process innovation levels under different scenarios. Comparing ε_1^{IT} and ε_1^{II} , it can be observed that the innovation level under a non-innovating competitor ε_1^{IT} is always higher than that under an innovating competitor ε_1^{II} . The gap between ε_1^{IT} and ε_1^{II} is the largest when the supply chain competition is intense and the level of technology spillover is low, as shown in Figure 6(c). As the level of technology spillover increases, the comparative advantage of ε_1^{IT} relative to ε_1^{II} decreases, as shown in Figure 6(d), but is still better than in the case of the weak supply chain competition, i.e., Figure 6(a),(b). In addition, k represents the investment scale in product innovation. Figure 6(a)–(d) also shows that firms become less innovative as k increases, i.e., as the investment scale coefficient of product innovation decreases.

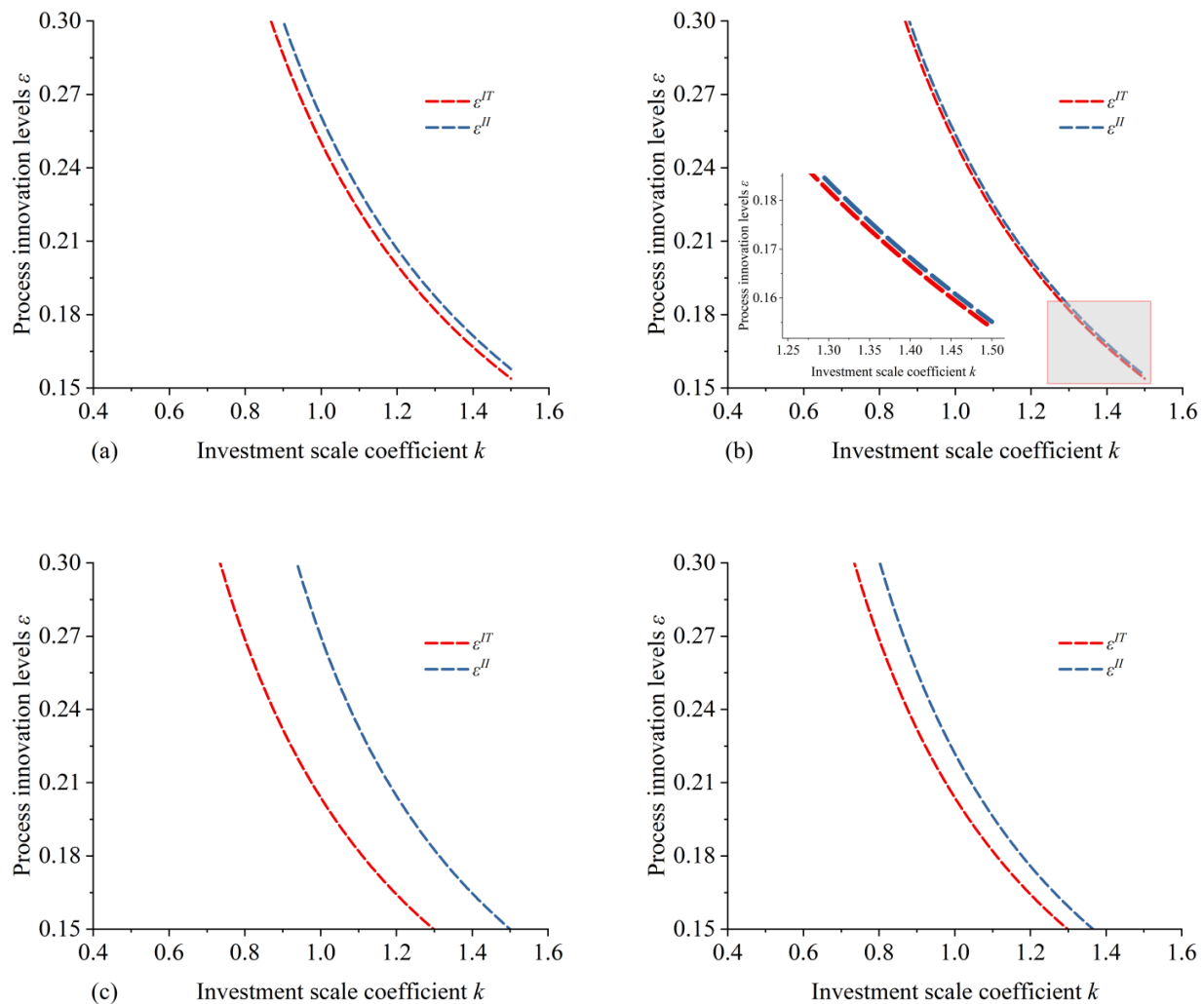


Figure 6. Process innovation levels for process innovation choice: (a) weak b and weak v ; (b) weak b and fierce v ; (c) fierce b and weak v ; (d) fierce b and fierce v .

Figures 5 and 6 indicate that in an environment with low competition and low spillovers, companies should still pursue process innovation. Although the competitive pressure is low, and technology spillovers are limited, process innovation can improve internal efficiency, reduce costs, strengthen risk resistance, reserve technologies for future competitions, and gradually build long-term advantages through low-cost trials and errors. In an environment marked by intense competition and high spillover level, companies can concentrate on process improvement (like simplifying production lines) or minor enhancements (such as improving range algorithms) to meet basic needs at a high value-for-money ratio, while collaborating with universities/research institutions to enhance their technological reserves.

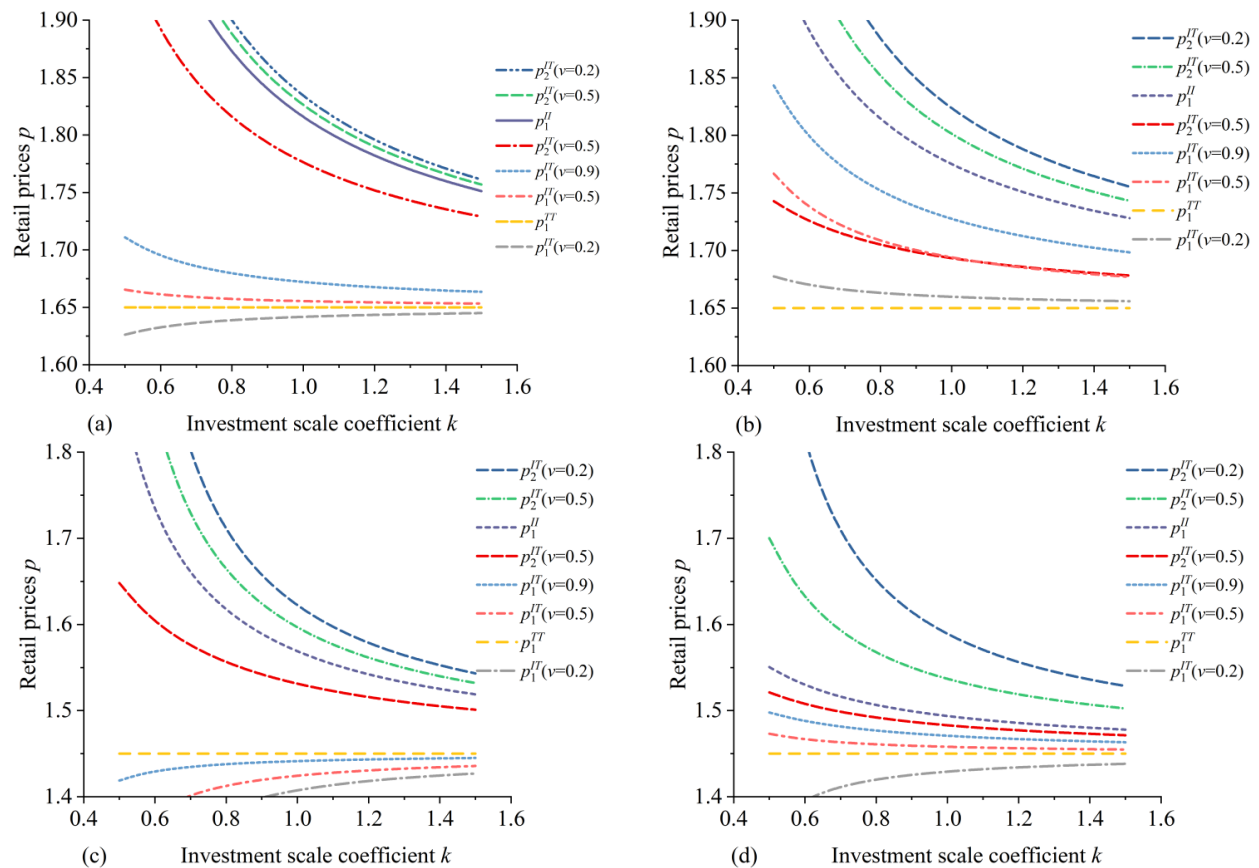


Figure 7. Retail prices for product innovation choice: (a) weak b and weak ρ ; (b) weak b and fierce ρ ; (c) fierce b and weak ρ ; (d) fierce b and fierce ρ .

Figure 7 illustrates retail prices under different competitive environments and different spillover levels. In the figure, the horizontal axis represents the investment scale coefficient, and the vertical axis means retail prices. The curves represent retail prices in different combinations II , $IT(TI)$, and TT for product innovation. We have the following observations. First, retail prices are simultaneously affected by the supply chain competition b , the product innovation competition ρ , and the technology spillover level v . Second, $p_2^{IT(v=0.2)}$, $p_2^{IT(v=0.5)}$, and $p_2^{IT(v=0.9)}$ in Figure 7(a)–(d) are higher than the retail price in other scenarios, which are in the first, second, and third positions, respectively. This implies that, regardless of the competitive environment, a competitor's investment in product innovation leads another non-innovating supplier to set a very high product price in order to maximize its own profit, which remains the case even when the spillover level is high. Third, the increased supply chain competition leads to lower retail prices, which benefits consumers and widens the gap between retail prices, as observed from Figure 7(a) to (c), and from Figure 7(b) to (d). Intense innovation competition also reduces retail prices, but to a lesser extent than in the case of the supply chain competition. An increase in spillover level also reduces the price gap between innovative and non-innovative products.

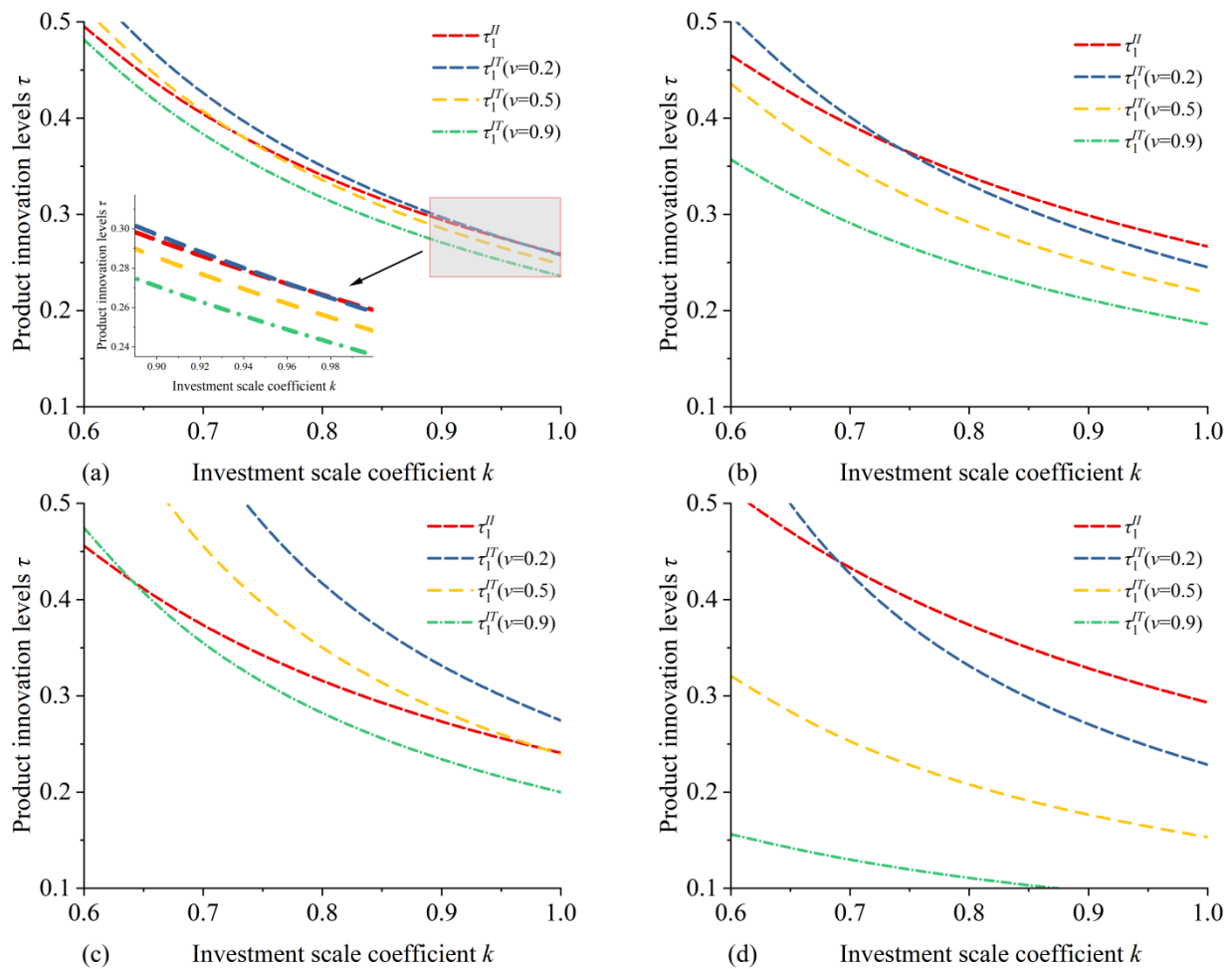


Figure 8. Product innovation levels for process innovation choice: (a) weak b and weak ρ ; (b) weak b and fierce ρ ; (c) fierce b and weak ρ ; (d) fierce b and fierce ρ .

Figure 8 illustrates product innovation levels under different competitive environments and different spillover levels. In the figure, the horizontal axis represents the investment scale coefficient, and the vertical axis means product innovation levels. The curves represent product innovation levels in different combinations II , $IT(TI)$, and TT . When suppliers invest in product innovation, a higher spillover level leads to a reduction in the level of product innovation, which consistently ranks the lowest compared to the other three scenarios, as depicted in Figure 8. However, it is noteworthy that intense innovation competition drives an increase in the innovation levels of both suppliers, as shown in Figure 8(b),(d). This outcome contrasts with the case of process innovation, where the optimal innovation level of strategy IT is higher than one of strategy II . This difference arises because, in product innovation, an increase in the innovation level directly stimulates consumer demand. The fierce innovation competition forces both suppliers to enhance their innovation levels to achieve higher profits. In contrast, when only one supplier engages in product innovation, there is no innovation competition.

Figures 7 and 8 reveal that in markets with weak competition and low spillovers, companies should focus their resources on developing unique product features to gain a first-mover advantage and capture new markets. For example, a smart home company could target less crowded markets,

avoid direct competition with big brands, and create home control devices with localized AI technologies. By designing products for specific user needs, they can break into smaller market segments and gain market shares. In a highly competitive environment with high technology spillovers, companies can leverage technology spillovers to rapidly absorb external technologies and drive product innovation. For example, Tesla maintains its market leadership in new energy vehicles by integrating cutting-edge technologies (e.g., batteries, autonomous driving).

7. Conclusions

This paper focuses on technological innovation strategies in competitive markets in the presence of spillovers. We develop a system with two competing suppliers, each of which sells products through two downstream buyers. Both suppliers choose to invest in innovation (process innovation or product innovation) or choose to adopt a non-innovation strategy. The non-innovating supplier can free-ride on the innovation outcomes of the innovating supplier to some extent, owing to technological spillover effects. Some of the main findings of this study are presented below.

First, when supply chain competition is weak, both process and product innovation yield greater benefits to suppliers. However, process innovation is more affected by spillovers than product innovation, i.e., under the high spillover level, its advantages disappear completely, whereas product innovation retains reduced but persistent benefits. Second, with intense competition and high spillovers, the no-innovation strategy becomes a dominant equilibrium relative to process innovation, though it does not completely outperform product innovation in achieving suppliers' profits optimization. Third, moderate horizontal competition leads to an asymmetric equilibrium where one supplier innovates while the other gives up. Fourth, increased spillovers and reduced investment scale coefficients weaken the advantages of process innovation and product innovation. Numerical analyses reveal that consumers have the opportunity to enjoy the lowest product price when both suppliers adopt process innovation and the highest product price when both suppliers adopt no innovation. Conversely, for product innovation, consumers benefit from the lowest prices only if two suppliers adopt different strategies.

While this paper contributes to the literature on technological innovations and provides managerial insights for policymakers, it has certain limitations. First, the model assumes homogeneous investment scales between suppliers. In reality, the firms will not have identical levels of management, technology, human resources, and capital, which may result in different innovation investment efficiencies for each firm. In future research, we will investigate the differentiated decisions of firms on whether to invest in innovation. Second, we do not consider suppliers facing the risk of innovation failure. The effect of the probability of innovation success in future research may yield some interesting results.

Based on our equilibrium analysis, we provide the following context-dependent strategies for managers operating in cross-competition markets with spillovers:

(i) Low competition intensity (b small) and low spillover level (v small): Firms should actively invest in both process and product innovation. Process innovation builds cost advantages, while product innovation captures market demand. The weak competitive pressure allows firms to fully appropriate innovation benefits.

(ii) High competition intensity (b large) and high spillover level (v large): Firms should avoid direct R&D investment. Instead, they should free-ride on competitors' innovations through technology scanning and reverse engineering, while focusing on complementary assets or process fine-tuning that

are difficult to imitate.

(iii) Moderate competition and spillover levels: An asymmetric equilibrium (one firm innovates, the other does not) may emerge. Firms should carefully assess their cost structure and competitor responses. The innovating firm gains a temporary advantage, while the non-innovating firm benefits from spillovers.

(iv) For product innovation specifically: When spillovers are high, the benefits of product innovation are significantly eroded. Firms should consider protecting intellectual property through patents, trade secrets, or focusing on features that are difficult to reverse-engineer. These recommendations are broadly applicable to consumer electronics, automotive components, home appliances, and smart devices—industries where cross-competition and rapid technology spillovers are prevalent.

We acknowledge that our study retains the traditional product/process innovation dichotomy, which does not capture business model innovation, organizational innovation, or ecosystem-based innovation. This is a deliberate simplification to maintain analytical tractability within the cross-competition context. Future research should extend our framework to incorporate these broader innovation types. Additionally, while we model process and product innovation separately, they often interact in practice. Future work could develop an integrated model that simultaneously endogenizes both innovation types to capture their synergies or trade-offs. Finally, our model assumes homogeneous investment scales between suppliers; future research could explore asymmetric investment efficiencies.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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Appendix A

Table A1. The decisions and profits for the combination II , $IT(TI)$, and TT with process innovation (r).

	II	IT	TT
q_{11}	$\frac{k(a-c)(2+b)}{14bk+8k-4b^2k-b-2}$	$\frac{2k(a-c)(1-b)(5b+4)(2+b)}{40kb^4+\theta_1+64k-16}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
q_{12}	$\frac{k(a-c)(2+b)}{14bk+8k-4b^2k-b-2}$	$\frac{2k(a-c)(1-b)(5b+4)(2+b)}{40kb^4+\theta_1+64k-16}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
q_{21}	$\frac{k(a-c)(2+b)}{14bk+8k-4b^2k-b-2}$	$\frac{(a-c)(2+b)\theta_5}{40kb^4+\theta_1+64k-16}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
q_{22}	$\frac{k(a-c)(2+b)}{14bk+8k-4b^2k-b-2}$	$\frac{(a-c)(2+b)\theta_5}{40kb^4+\theta_1+64k-16}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
ε_1	$\frac{(a-c)(2+b)}{14bk+8k-4b^2k-b-2}$	$\frac{2(a-c)(2+b)(1-b)(5b+4)}{40kb^4+\theta_1+64k-16}$	-----
ε_2	$\frac{(a-c)(2+b)}{14bk+8k-4b^2k-b-2}$	-----	-----
π_{s_1}	$\frac{k(a-c)^2(2+b)\theta_2}{(14bk+8k-4b^2k-b-2)^2}$	$\frac{4k(a-c)^2(2+b)(b+4-5b^2)^2\theta_4}{(40kb^4+\theta_1+64k-16)^2}$	$\frac{2(a-c)^2(b+2)(b-1)}{(4-b)^2(2b+1)}$
π_{s_2}	$\frac{k(a-c)^2(2+b)\theta_2}{(14bk+8k-4b^2k-b-2)^2}$	$\frac{8(a-c)^2(2+3b-3b^2-2b^3)\theta_5}{(40kb^4+\theta_1+64k-16)^2}$	$\frac{2(a-c)^2(b+2)(b-1)}{(4-b)^2(2b+1)}$
π_{B_1}	$\frac{2k^2(a-c)^2(2+b)(b^2+3b+2)}{(14bk+8k-4b^2k-b-2)^2}$	$\frac{(a-c)^2(b+2)^2\theta_3}{(40kb^4+\theta_1+64k-16)^2}$	$\frac{2(a-c)^2(b+1)(b+2)^2}{2(4+7b-2b^2)^2}$
π_{B_2}	$\frac{2k^2(a-c)^2(2+b)(b^2+3b+2)}{(14bk+8k-4b^2k-b-2)^2}$	$\frac{(a-c)^2(b+2)^2\theta_3}{(40kb^4+\theta_1+64k-16)^2}$	$\frac{2(a-c)^2(b+1)(b+2)^2}{2(4+7b-2b^2)^2}$

$$\theta_1 = (3v - 148k + 7)b^3 + (12v - 84k + 6)b^2 + (128k + 12v - 24)b,$$

$$\theta_2 = 8bk + 8k - 16b^2k - b - 2$$

$$\theta_3 = 8(1+b)(-5b^2+b+4)^2k^2 - 4(v-1)(5b^4+14b^3+3b^2-14b-8)k + (b+2)^2(v-1)^2$$

$$\theta_4 = (8bk + 8k - 16b^2k - b - 2)$$

$$\theta_5 = (2bk + 2v + bv + 8k - 10b^2k - b - 2)^2$$

Table A2. The decisions and profits for the combination II , $IT(TI)$, and TT with product innovation (e).

	II	IT	TT
q_{11}	$\frac{k(a-c)(2+b)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{2k(a-c)(b+2)(1-b)(5b+4)}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
q_{12}	$\frac{k(a-c)(2+b)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{2k(a-c)(b+2)(1-b)(5b+4)}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
q_{21}	$\frac{k(a-c)(2+b)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{(a-c)(b+2)[(1-v\rho)\mu_8 + 2k(b-1)(5b+4)]}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
q_{22}	$\frac{k(a-c)(2+b)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{(a-c)(b+2)[(1-v\rho)\mu_8 + 2k(b-1)(5b+4)]}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{(a-c)(b+2)}{2(4-b)(2b+1)}$
w_{11}	$\frac{2(2b+1)k[2a(1-b) + c(b+2)] - c\mu_2(1-\rho)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{8ak\mu_5(b-1) + 4ck\mu_5(b+2) + [\mu_4 - 3bvp(2+b)]\mu_8c}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{2a+2c+bc-2ab}{4-b}$
w_{12}	$\frac{2(2b+1)k[2a(1-b) + c(b+2)] - c\mu_2(1-\rho)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{8ak\mu_5(b-1) + 4ck\mu_5(b+2) + [\mu_4 - 3bvp(2+b)]\mu_8c}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{2a+2c+bc-2ab}{4-b}$
w_{21}	$\frac{2(2b+1)k[2a(1-b) + c(b+2)] - c\mu_2(1-\rho)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{8ak\mu_5(b-1) + a\mu_7 + 4ck\mu_5(b+2) + \mu_6\mu_{10}c}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{2a+2c+bc-2ab}{4-b}$
w_{22}	$\frac{2(2b+1)k[2a(1-b) + c(b+2)] - c\mu_2(1-\rho)}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{8ak\mu_5(b-1) + a\mu_7 + 4ck\mu_5(b+2) + \mu_6\mu_{10}c}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	$\frac{2a+2c+bc-2ab}{4-b}$
τ_1	$\frac{k(a-c)\mu_2}{2k\mu_1 - \mu_2(1-\rho)}$	$\frac{2(a-c)(b-1)(5b+4)\mu_8}{(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5}$	—
τ_2	$\frac{k(a-c)\mu_2}{2k\mu_1 - \mu_2(1-\rho)}$	—	—
m_1	$\frac{k(a-c)^2[u_2^2(4-b) + 8ku_1(b+2)(1-b)]}{[2k\mu_1 - \mu_2(1-\rho)]^2(4-b)}$	$\frac{4k(a-c)^2(5b^2 - b - 4)^2[8k(2b+1)(b+2)(1-b) - \mu_8^2]}{[(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5]^2}$	$\frac{2(a-c)^2(b+2)(1-b)}{(4-b)^2(2b+1)}$
m_2	$\frac{k(a-c)^2[u_2^2(4-b) + 8ku_1(b+2)(1-b)]}{[2k\mu_1 - \mu_2(1-\rho)]^2(4-b)}$	$\frac{4k(a-c)^2(5b^2 - b - 4)^2[8k(2b+1)(b+2)(1-b) - \mu_8^2]}{[(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5]^2}$	$\frac{2(a-c)^2(b+2)(1-b)}{(4-b)^2(2b+1)}$
r_1	$\frac{2k^2(a-c)^2(b+1)(b+2)^2}{[2k\mu_1 - \mu_2(1-\rho)]^2}$	$\frac{(a-c)^2(b+2)^2[2k(1-b^2)(5b+4) - (1-v\rho)\mu_8]^2 + \mu_9}{[(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5]^2}$	$\frac{(a-c)^2(b+1)(b+2)^2}{2u_1^2}$
r_2	$\frac{2k^2(a-c)^2(b+1)(b+2)^2}{[2k\mu_1 - \mu_2(1-\rho)]^2}$	$\frac{(a-c)^2(b+2)^2[2k(1-b^2)(5b+4) - (1-v\rho)\mu_8]^2 + \mu_9}{[(b+2)(\mu_3 - 9v^2\rho^2b^2) - 18vb\rho\mu_4 + 4k(4-b)\mu_5]^2}$	$\frac{(a-c)^2(b+1)(b+2)^2}{2u_1^2}$

$$\mu_1 = 4 + 7b - 2b^2, \mu_2 = 3b\rho + b + 2, \mu_3 = b^2 - 2b - 2, \mu_4 = 7b^2 - 8b - 8, \mu_5 = (1-b)(5b+4)(2b+1), \mu_6 = 4b + 4\rho v + b^2 - 2bvp - 11b^2vp + 4, \mu_7 = (2b+1)(b-1)(v\rho-1)(3bvp-b-2), \mu_8 = 3bvp-b-2, \mu_9 = 4k^2(5b+4)^2(1-b)^2(1-b^2)(a-c)^2(b+2)^2.$$

Proof of Table A1:

First, for the combination II with process innovation (r), take the first-order optimal condition with q_{ij} and q_{3-ij} for Eq (6),

$$\frac{\partial \pi_{Bj}^{II-r}}{\partial q_{ij}} = a - 2q_{ij} - w_{ij} - bq_{i,3-j} - 2bq_{3-i,j} - bq_{3-i,3-j} = 0, \quad (A1)$$

$$\frac{\partial \pi_{Bj}^{II-r}}{\partial q_{3-i,j}} = a - 2q_{3-i,j} - w_{3-i,j} - bq_{3-i,3-j} - 2bq_{i,j} - bq_{i,3-j} = 0, \quad (A2)$$

The Hessian matrix is:

$$H = \begin{bmatrix} -2 & -2b \\ -2b & -2 \end{bmatrix}, \text{ and } |H| = 4 - 4b^2 > 0.$$

Therefore, we can show that the Hessian matrix H is a diagonally dominant matrix, which guarantees joint concavity of the profit function $\pi_{B_j}^{II-r}(q_{ij}, q_{3-i,j})$. Solving (A1) and (A2), we can obtain

$$q_{ij} = a/2 - w_{ij}/2 - bq_{i,3-j}/2 - bq_{3-i,j}/2 - bq_{3-i,3-j}/2, \quad (\text{A3})$$

$$q_{3-i,j} = a/2 - w_{3-i,j}/2 - bq_{3-i,3-j}/2 - bq_{i,j} - bq_{i,3-j}/2. \quad (\text{A4})$$

With (A3) and (A4), we have the reaction function of the order quantity as follows:

$$q_{ij} = \frac{2a - 2w_{ij} - 2ab - 2bw_{ij} + bw_{i,3-j} + 2bw_{3-i,j} + bw_{3-i,3-j} + b^2w_{ij} - b^2w_{i,3-j} + b^2w_{3-i,j} - b^2w_{3-i,3-j}}{4(2b+1)(1-b)}, \quad (\text{A5})$$

The supplier i uses the reaction function of the order quantity (A5) in profit function Eq (11) and takes the first derivatives w.r.t. $w_{ij}, w_{i,3-j}$ and ε_i :

$$\frac{\partial \pi_{S_i}^{II-r}}{\partial w_{ij}} = \frac{2(a+c-ab) - (2+b)\varepsilon_i + bc + 2(b^2 - 2 - 2b)w_{ij} + 2b(1-b)w_{i,3-j} + b(2+b)w_{3-i,j} + b(1-b)w_{3-i,3-j}}{4(1+b-2b^2)} = 0, \quad (\text{A6})$$

$$\frac{\partial \pi_{S_i}^{II-r}}{\partial w_{i,3-j}} = \frac{2(a+c-ab) - (2+b)\varepsilon_i + bc + 2(b^2 - 2 - 2b)w_{i,3-j} + 2b(1-b)w_{ij} + b(2+b)w_{3-i,3-j} + b(1-b)w_{3-i,j}}{4(1+b-2b^2)} = 0, \quad (\text{A7})$$

$$\frac{\partial \pi_{S_i}^{II-r}}{\partial \varepsilon_i} = \frac{4a - 2w_{ij} - 2w_{i,3-j} - 4ab - bw_{ij} - bw_{i,3-j} + 3bw_{3-i,j} + 3bw_{3-i,3-j} + 8k\varepsilon_i(2b^2 - b - 1)}{4(1+b-2b^2)} = 0, \quad (\text{A8})$$

The Hessian matrix is:

$$H = \begin{bmatrix} \frac{2+2b-b^2}{2(2b+1)(b-1)} & \frac{b}{4b+2} & \frac{-(b+2)}{4(1+b-2b^2)} \\ \frac{b}{4b+2} & \frac{2+2b-b^2}{2(2b+1)(b-1)} & \frac{-(b+2)}{4(1+b-2b^2)} \\ \frac{-(b+2)}{4(1+b-2b^2)} & \frac{-(b+2)}{4(1+b-2b^2)} & -2k \end{bmatrix}, \text{ and } |H| = \frac{(4-b^2)(b-8k-8bk+16b^2k+2)}{16(1-b)^3(2b+1)^2} > 0, \text{ if and only}$$

if $k > \frac{b+2}{8+8b-16b^2}$. Due to $\max(\frac{b+2}{8+8b-16b^2}) = 0.25$, we set $k > 0.25$, which satisfies $|H| > 0$ and does not increase the complexity of the formula.

Therefore, the Hessian matrix H is a diagonally dominant matrix, which guarantees the joint concavity of the profit function $\pi_{S_i}^{II-r}$. Solving (A6) and (A8), we can obtain

$$w_{ij} = \frac{2(a+c-ab) - (2+b)\varepsilon_i + bc + 2b(1-b)w_{i,3-j} + b(2+b)w_{3-i,j} + b(1-b)w_{3-i,3-j}}{4 + 4b - 2b^2}, \quad (\text{A9})$$

$$w_{i,3-j} = \frac{2(a+c-ab) - (2+b)\varepsilon_i + bc + 2b(1-b)w_{ij} + b(2+b)w_{3-i,3-j} + b(1-b)w_{3-i,j}}{4 + 4b - 2b^2}, \quad (\text{A10})$$

$$\varepsilon_i = \frac{4a - 2w_{ij} - 2w_{i,3-j} - 4ab - bw_{ij} - bw_{i,3-j} + 3bw_{3-i,j} + 3bw_{3-i,3-j}}{8k(1+b-2b^2)}, \quad (\text{A11})$$

With (A9)–(A11), we can derive the equilibrium wholesale price and process innovation level as $w_{ij}^{II-r} = \frac{a(4bk+4k-8b^2k-b-2)+2kc(2b+1)(2+b)}{14bk+8k-4b^2k-b-2}$ and $\varepsilon_i^{II-r} = \frac{(a-c)(2+b)}{14bk+8k-4b^2k-b-2}$. Substituting w_{ij}^{II-r} and ε_i^{II-r} into Eq (A5), we get equilibrium order quantities as shown in Table 2.

Second, for the combination TT with process innovation(r), supplier i can get the buyer j 's reaction function as (A5), then uses it in profit function Eq (8), and takes the first derivatives w.r.t. w_{ij} and $w_{i,3-j}$:

$$\frac{\partial \pi_{S_i}^{TT-r}}{\partial w_{ij}} = \frac{2(a+c-ab)+bc+2(b^2-2-2b)w_{ij}+2b(1-b)w_{i,3-j}+b(2+b)w_{3-i,j}+b(1-b)w_{3-i,3-j}}{4+4b-2b^2} = 0, \quad (A12)$$

$$\frac{\partial \pi_{S_i}^{TT-r}}{\partial w_{i,3-j}} = \frac{2(a+c-ab)+bc+2(b^2-2-2b)w_{i,3-j}+2b(1-b)w_{ij}+b(2+b)w_{3-i,3-j}+b(1-b)w_{3-i,j}}{4(1+b-2b^2)} = 0. \quad (A13)$$

The Hessian matrix is:

$$H = \begin{bmatrix} \frac{2+2b-b^2}{2(2b+1)(b-1)} & \frac{-2b-b^2}{2(2b+1)(b-1)} \\ \frac{-2b-b^2}{2(2b+1)(b-1)} & \frac{2+2b-b^2}{2(2b+1)(b-1)} \end{bmatrix}, \text{ and } |H| = \frac{1+b}{2(2b+1)(1-b)} > 0.$$

Therefore, the Hessian matrix H is a diagonally dominant matrix, which guarantees the joint concavity of the profit function $\pi_{S_i}^{TT-r}$. Solving (A9) and (A10), we can obtain

$$w_{ij} = \frac{2(a+c-ab)+bc+2b(1-b)w_{i,3-j}+b(2+b)w_{3-i,j}+b(1-b)w_{3-i,3-j}}{4+4b-2b^2}, \quad (A14)$$

$$w_{i,3-j} = \frac{2(a+c-ab)+bc+2b(1-b)w_{ij}+b(2+b)w_{3-i,3-j}+b(1-b)w_{3-i,j}}{4+4b-2b^2}. \quad (A15)$$

With (A11) and (A12), we can derive the equilibrium wholesale price as $w_{ij}^{TT-r} = \frac{2a+2c-2ab+bc}{4-b}$.

Substituting w_{ij}^{TT-r} into Eq (A5), we get equilibrium order quantities as shown in Table 2.

Third, for the combination IT with process innovation (r), supplier 1 uses the reaction function of order quantities (A5) in profit function Eq (7) and solves them, which can derive Eqs (A9) and (A10); supplier 2 uses the reaction function of order quantities (A5) in profit function Eq (12) and solves them, which can derive Eqs (A14) and (A15). With (A9), (A10), (A14), and (A15), we can obtain the equilibrium wholesale price as shown in Table 2. Substituting w_{ij}^{TT-r} into Eqs (A5) and (A11), we get equilibrium order quantities and innovation level as shown in Table 2.

Based on the equilibrium decisions above, we can obtain the members' profits for the combination $\{II, IT, TI, TT\}$ with process innovation (r), which is shown in Table 2. The proof of Table A1 is completed.

Proof of Table A2

The proof of Table A2 is similar to Table A1, and the detail is omitted.

Proof of Lemma 1

From the proof of Table A1, we can derive Lemma 1.

Proof of Proposition 1

From expression $\pi_{S_1}^{IT-r}$ and $\pi_{S_1}^{TT-r}$, we get that if $v < v_1$, then $\pi_{S_1}^{IT-r} > \pi_{S_1}^{TT-r}$; if $v > v_1$, then $\pi_{S_1}^{IT-r} < \pi_{S_1}^{TT-r}$, where $v_1 = \frac{4k(b-1)(4-b)(4+5b)(2b+1)+(4-b)(4+5b)\theta_{15}+(8b-7b^2+8)(b+2)}{3b(b+2)^2}$ and $\theta_{15} = [2k(8bk + 8k - 16b^2k - b - 2)(1 - b)(1 + 2b)]^{1/2}$. According to the definitions of F_1 , F_2 , and F_3 , we can derive that if $(b, k) \in F_1$, then $v_1 > 1$; if $(b, k) \in [F_2 \cup F_3]$, then $0 < v_1 < 1$; if $(b, k) \in F_4$, then $v_1 < 0$.

Therefore, 1) if $(b, k) \in F_1$, or (2) if $(b, k) \in F_2$ and $0 < v < v_1$, then $\pi_{S_1}^{IT-r} > \pi_{S_1}^{TT-r}$, which denotes that the process innovation is optimal for supplier 1. If (1) $(b, k) \in [F_2 \cup F_3]$, $0 < v_1 < 1$, or (2) if $(b, k) \in F_4$, then $\pi_{S_1}^{IT-r} < \pi_{S_1}^{TT-r}$, which shows that non-innovating production is optimal for supplier 1. The proof of Proposition 1 is completed.

Proof of Proposition 2

From the expressions of $\pi_{S_1}^{TI-r}$ and $\pi_{S_1}^{II-r}$, we can get

$$v_2 = \frac{32k^3\theta_{16}-12k^2\theta_{17}-3k\theta_{18}-8(b+2)^3(b-1)(2b+1)-4\theta_{19}}{8k^2(b-1)(2b+1)(7b^2-8b-8)(b^2-20b-8)+k\theta_{20}+8(b-1)(2b+1)(b+2)^2},$$

$$\theta_{16} = (5b + 4)(4 - b)(7b^2 - 8b - 8)(b - 1)^2(2b + 1)^2,$$

$$\theta_{17} = (83b^4 - 266b^3 - 120b^2 + 256b + 128)(b - 1)(2b + 1)(b + 2),$$

$$\theta_{18} = (103b^4 - 202b^3 - 120b^2 + 128b + 64)(b + 2)^2,$$

$$\theta_{19} = (8bk + 8k - 16b^2k - b - 2)^{3/2}(b + 2 + 7b^2 - 8b - 8)(5b + 4)(4kb + 2k)^{1/2},$$

$$\theta_{20} = (119b^4 - 548b^3 - 132b^2 + 352b + 128)(b + 2).$$

Then, if $0 < v < v_2$, then $\pi_{S_1}^{TI-r} \leq \pi_{S_1}^{II-r}$; if $v_2 < v < 1$, then $\pi_{S_1}^{TI-r} > \pi_{S_1}^{II-r}$. The proof of Proposition 2 is completed.

Proof of Proposition 3

From the expressions of $\pi_{S_1}^{IT-r}$ and $\pi_{S_1}^{II-r}$, we can derive that if $v = 0$, then $\pi_{S_1}^{IT-r} = \pi_{S_1}^{II-r}$. v_1 is the boundary value of $\pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TI-r}$; therefore, $\pi_{S_1}^{TT-r} = \pi_{S_1}^{II-r} = \pi_{S_1}^{IT-r}$ as long as $v_1 = 0$. According to Proposition 1, we can derive that if $(b, k) \in F_1$, then $\pi_{S_1}^{IT-r} > \pi_{S_1}^{TT-r}$; otherwise, $\pi_{S_1}^{IT-r} < \pi_{S_1}^{TT-r}$. Therefore, it can be found that if $(b, k) \in F_1$, then $\pi_{S_1}^{TT-r} < \pi_{S_1}^{II-r}$; otherwise, $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$.

From Propositions 1 and 2, we can get that

- 1) when $(b, k) \in F_1$, if $0 < v < v_2$, then $\pi_{S_1}^{TT-r} < \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} < \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT} < \pi_{S_1}^{II}$; if $v_2 < v < 1$, then $\pi_{S_1}^{TT-r} < \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} > \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} < \pi_{S_1}^{II-r}$.
- 2) when $(b, k) \in F_{2-1}$, if $0 < v < v_2$, then $\pi_{S_1}^{TT-r} < \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} < \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$; if $v_2 < v < v_1$, then $\pi_{S_1}^{TT-r} < \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} > \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$; if $v_1 < v < 1$, then $\pi_{S_1}^{TT-r} > \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} > \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$.
- 3) when $(b, k) \in F_{2-2}$, if $0 < v < v_1$, then $\pi_{S_1}^{TT-r} < \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} < \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$; if $v_1 < v < v_2$, then $\pi_{S_1}^{TT-r} > \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} < \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$; if $v_2 < v < 1$, then $\pi_{S_1}^{TT-r} > \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} > \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$.
- 4) when $(b, k) \in F_3$, if $0 < v < v_2$, then $\pi_{S_1}^{TT-r} > \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} < \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$; if $v_2 < v < 1$, then $\pi_{S_1}^{TT-r} > \pi_{S_1}^{IT-r}, \pi_{S_1}^{TI-r} > \pi_{S_1}^{II-r}$ and $\pi_{S_1}^{TT-r} > \pi_{S_1}^{II-r}$.

Therefore, we can derive Proposition 3. The proof of Proposition 3 is completed.

Proof of Corollary 1

From the region F_1 to the regions F_2 and F_3 in Figure 3, this means that the intensity of the channel competition is increasing. According to Proposition 4, it can be found that the supplier gradually shifts from process innovation to non-innovation from the region F_1 to the regions F_2 and F_3 . Therefore, we can get Corollary 1(1). The proof of Corollary 1(2) and Corollary 1(3) is similar to Corollary 1(1), and the detail is omitted. The proof of Corollary 1 is completed.

Proof of Lemmas 2 and 3

The proof of Lemmas 2 and 3 is similar to Lemma 1, and the detail is omitted.

Proof of Propositions 4 and 5

The proof of Propositions 4 and 5 is similar to Propositions 1–3, and the detail is omitted.



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