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Research article

An interval-valued cooperative game method with power asymmetry for benefit allocation in marine fishery insurance

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Abstract: Cooperative benefit allocation becomes difficult when coalition worths are imprecise and decision rights are asymmetric. This study develops a power-weighted interval-valued core framework that integrates asymmetric voting power with satisfaction-based interval coalition rationality. Individual voting power is derived from coalition criticality in a weighted voting game and aggregated at the coalition level through a least-squares rule, while the resulting max-min allocation model is solved using a bisection procedure based on monotonic feasibility. In a stylized, normalized three-player example motivated by marine fishery insurance, one optimal allocation generated by the proposed model satisfies the adopted efficiency and coalitional rationality conditions, whereas a selected power-adjusted interval Shapley benchmark does not under the same parameterization. Parameter sensitivity analysis further illustrates how voting weights, decision thresholds, and the grand coalition worth interval are associated with changes in power indices and the distribution of allocation uncertainty. The numerical results provide methodological illustrations under a controlled parameterization and demonstrate the feasibility and computational implementation of the proposed solution procedure.

Keywords: cooperative game; interval-valued core; power asymmetry; interval ranking; marine fishery insurance

1. Introduction

Financial system reforms emphasize the orientation of finance serving the real economy and promoting inclusive development. As a crucial institutional arrangement for safeguarding the marine food supply system and supporting the marine economy, marine fishery insurance plays an important role in risk diversification and industry stabilization. It also contributes to inclusive financial services and shared economic growth. In practice, China's fishery mutual insurance system has established a risk protection network characterized by tripartite collaboration among the government, insurance suppliers (primarily the China Fishery Mutual Insurance Association and the China Fishery Mutual Insurance Corporation, hereinafter referred to as mutual insurance institutions), and fishermen. This system forms a multiple-player game process in which the interests of all players (or participants) are interdependent and mutually constraining. Specifically, the government invests fiscal resources through premium subsidies and the purchase of services, dedicated to building a safety net for production to maximize social welfare. Mutual insurance institutions possess dual attributes of policy implementation and market operation, pursuing a balance of revenue and expenditure as well as sustainable development under fiscal constraints. Fishermen, as demanders, seek to transfer risks at a reasonable cost to safeguard their returns. However, constrained by the specificity of fishery risks, the complexity of the institutional environment, and the rise of emerging fields such as deep-sea aquaculture, this game relationship faces severe challenges. Specifically, the uncertainty of fishery risks and information asymmetry make it difficult to accurately determine the cooperative benefits. For instance, in complex game environments, players often represent payoffs using approximate ranges rather than crisp values [1,2], leading to ambiguous expectations regarding cooperative returns. Furthermore, the divergent interests and unequal bargaining power among the government, institutions, and fishermen significantly affect benefit distribution and the stability of the cooperative alliance.

Cooperative game theory provides a theoretical framework for analyzing interest distribution and the stability of cooperative alliances. As noted by Zhu and Tuo [3], the "cooperation and administrative support" model adopted by China Fishery Mutual Insurance Association achieves a positive-sum game relationship involving the government, the association, and its members, thereby strongly validating the applicability of this theory in the field of fishery insurance. Since Borch [4] introduced cooperative game theory into insurance market analysis in the 1960s, relevant research has evolved from basic theoretical exploration to application in complex scenarios. Early studies primarily focused on cost allocation [5,6] and contract negotiation [7] in deterministic environments. With the increasing complexity of risk environments, scholars have shifted their attention to game equilibria under uncertainty, introducing methods such as stochastic games to investigate risk allocation [8] and catastrophe risks [9], thereby providing insights for decision-making involving fuzzy information. According to cooperative game theory, the core condition for coalition formation is "super-additivity", which demands that the total return of the coalition exceeds the sum of the returns of members acting alone. Through marine fishery insurance cooperation, the government maximizes social welfare, mutual insurance institutions achieve sustainable development, and fishermen secure income guarantees, thereby realizing a mutually beneficial outcome for all players.

However, current research generally assumes that payoff functions are either precise values or follow specific stochastic distributions. Due to the characteristics of marine fishery insurance, such as high risk and uncertainty in industry information, traditional precise games or stochastic games requiring known distributions are often inapplicable. In the absence of precise statistical data, the characteristic function values of coalitions often cannot be accurately measured, resulting in a lack of credibility in interest distribution schemes. In response to this limitation, the theory of fuzzy cooperative games has emerged. This theory can be traced back to the concept of "ideal set function" implicitly used by Aumann and Shapley [10]. Subsequently, Aubin [11] formally proposed the concept of fuzzy coalitions and fuzzy games, using fuzzy sets to describe the participation degree of decision-making players, laying the foundation for dealing with uncertainty. A recent comprehensive survey by Özcan and Weber [12] systematically synthesizes the theoretical developments and methodological advances in cooperative games under uncertainty, highlighting how fuzzy and interval representations extend classical game theory to handle imprecise payoff information. Regarding solution concepts, scholars have conducted extensive explorations. In terms of set solutions, Aubin [13] first proposed the Aubin core. Since then, concepts such as the dominant core [14], generalized core [15], and stable set [16] have been successively proposed. Nishizaki and Sakawa [17] further demonstrated the non-emptiness of the core in fuzzy linear production games. In practice, players (or decision-makers) often rely on empirical data and expert judgments to estimate the lower and upper bounds of expected payoffs, naturally representing them as intervals. In such models where coalitions' values are expressed with intervals, which are often called interval-valued cooperative games for short, Mallozzi et al. [18] extended the interval representation framework to fuzzy environments and proposed the fuzzy interval core (F-core). Luo et al. [19] further introduced the fuzzy central core for fuzzy cooperative games under Zadeh algebraic orders, establishing a generalized solution framework that extends classical core concepts and encompasses existing fuzzy core solutions. In contrast to set solutions, single-valued solutions aim to determine a unique payoff vector for each player. Butnariu [20] first defined the Shapley value function of fuzzy cooperative games, continuing the idea of marginal contribution distribution. Subsequent scholars have expanded this from different perspectives. Tsurumi et al. [21] reset axiomatic conditions, endowing the fuzzy Shapley value with good properties such as monotonicity and continuity. Branzei et al. [14] proposed an egalitarian solution based on the principle of egalitarianism and introduced a compromise value utilizing the path solution covering. More recently, Mallozzi and Vidal-Puga [22] defined and characterized an efficient extension of the Shapley value for games with fuzzy characteristic functions. Additionally, for games with coalition structures and incomplete information, Liu et al. [23] proposed the interval Owen-augmenting value by evaluating the unknown worths of infeasible coalitions through feasible ones, extending the Owen value to interval settings. Li [24,25] also carried out fruitful research in the field of fuzzy cooperative games. In particular, Hong and Li [26] introduced the concept of the satisfaction degree of interval numbers and constructed a nonlinear programming model for maximizing satisfaction, thereby proposing a solution method for interval-valued cooperative games. In summary, introducing interval-valued fuzzy cooperative game methods provides a promising framework for solving interest distribution problems under ambiguous and uncertain environments. More broadly, recent studies on interval-valued fuzzy approaches have demonstrated their effectiveness in representing and processing uncertain information in complex decision-making scenarios [27–30].

Furthermore, current literature often assumes that game players are equal, neglecting the significant power asymmetry structure in the marine fishery insurance market. In fact, in complex decision-making environments involving multiple stakeholders, participants often hold asymmetric status and weights due to their divergent roles and expertise [31]. Specifically, the current cooperative game of marine fishery insurance has essentially formed a pattern guided by government policies, with fishery mutual insurance institutions as the main operating players and fishermen participating together. The government holds dominance over funds and policies, and mutual insurance institutions possess an advantage in professional information, whereas fishermen, despite having a collective negotiation mechanism, remain in a relatively weak position. This is a typical government-guided cooperative game, characterized by status asymmetry among players. The leading player (government) has strong comprehensive strength and weak dependence on the cooperation mechanism, while ordinary players (fishermen) have weak strength and strong dependence. This asymmetric structure profoundly affects the stability of the coalition and the fairness of interest distribution. Ignoring this reality, traditional solution concepts based on symmetry assumptions will struggle to guide practice.

To address the issue of unequal status among players, game theory introduces power index methodologies. For instance, in games characterized by winning coalitions, such as voting elections, the Shapley-Shubik power index [32] and the Banzhaf power index [33] are widely used to quantify the influence and bargaining power of decision-making players within a coalition. Although Gardner [34] noted that these indices are essentially specific solutions based on the concept of transferable utility, the fundamental principle of quantifying player power offers a vital theoretical reference for resolving benefit distribution challenges caused by status asymmetry in marine fishery insurance.

Existing studies have developed fuzzy or interval cooperative games to represent payoff imprecision and voting power indices to represent unequal influence, but these two dimensions are often treated separately. This study addresses this gap by developing a power-weighted interval-valued core framework in which power asymmetry and interval coalition rationality are incorporated within a unified allocation model. The methodological contributions are threefold. First, individual voting power is derived from coalition criticality in a weighted voting game, and a least-squares aggregation rule is introduced as a representative measure of coalition level power. Second, the adopted interval coalition rationality relations are converted into satisfaction-based crisp constraints and embedded in a max-min model that maximizes the least power-weighted coalition satisfaction. Third, for the interval relation domain considered, monotonic feasibility with respect to the satisfaction level is established and exploited to construct a bisection-based solution procedure, with each fixed level feasibility problem formulated as a linear program. The methodological contribution therefore lies in the integration of power representation, interval rationality, and computational solution within a single internally consistent framework, rather than in any one of these components in isolation. A stylized, normalized three-player example in the field of marine fishery insurance, a benchmark comparison, and a parameter sensitivity analysis are used to illustrate these mechanisms and examine their behavior under alternative parameter settings.

Beyond these specific contributions, the proposed framework possesses broader methodological relevance. By formalizing stability conditions under interval uncertainty, it offers new theoretical insights for mathematicians studying cooperative games with asymmetric structures. From a computational perspective, the model's monotone feasibility structure provides a robust basis for algorithm implementation, facilitating the development of scalable decision-support systems. Furthermore, the method offers an interpretable allocation procedure for operations research practitioners dealing with collaborative settings where coalition worths are imprecise and decision rights are unequal, thereby extending the applicability of classical game theory to complex real-world scenarios.

The rest of this study is organized as follows. Section 2 introduces interval arithmetic, interval ranking satisfaction degrees, individual voting power indices, and the least-squares aggregation rule for coalition level power. Section 3 formulates the satisfaction-based max-min model for the power-weighted interval-valued core and develops the bisection-based solution procedure for the stated interval relation domain. Section 4 presents the stylized, normalized three-player example, compares the proposed allocation with a selected power-adjusted interval Shapley benchmark, and conducts a parameter sensitivity analysis for voting weights, decision thresholds, and the grand coalition worth interval. Section 5 concludes this study and discusses limitations and future research.

2. Arithmetic operations over intervals and calculation of coalition power index

2.1. Interval numbers and their operation

Let R be the set of real numbers. An interval may be expressed as $\hat{a}=[\underline{a},\ \overline{a}]=\{a\,|\,\underline{a}\le a\le\overline{a},\ \underline{a}\in\mathrm{R},\ \overline{a}\in\mathrm{R}\}$, where $\underline{a}$ and $\overline{a}$ are called the lower and upper bounds of the interval $\hat{a}$, respectively. If $\underline{a}=\overline{a}$, then $\hat{a}=[\underline{a},\ \overline{a}]$ is reduced to a real number a, where $a=\underline{a}=\overline{a}$. Alternatively, an interval $\hat{a}$ may be expressed in midpoint-radius form as $\hat{a}=<m(\hat{a}),\ w(\hat{a})>$, where $m(\hat{a})=(\underline{a}+\overline{a})/2$ and $w(\hat{a})=(\overline{a}-\underline{a})/2$ are the midpoint and radius of the interval $\hat{a}$, respectively. The set of intervals in the real number set R is denoted by $I(\mathrm{R})$.

For any intervals $\hat{a}=[\underline{a},\ \overline{a}]$ and $\hat{b}=[\underline{b},\ \overline{b}]$, their operations are defined as follows:

$$\hat{a}+\hat{b}=[\underline{a},\ \overline{a}]+[\underline{b},\ \overline{b}]=[\underline{a}+\underline{b},\ \overline{a}+\overline{b}], \tag{1}$$

$$\gamma\hat{a}=\gamma[\underline{a},\ \overline{a}]=\begin{cases}[\gamma\underline{a},\ \gamma\overline{a}], & \text{if } \gamma\ge 0,\\ [\gamma\overline{a},\ \gamma\underline{a}], & \text{if } \gamma<0.\end{cases} \tag{2}$$

Using the midpoint-radius form, the above operations can be concisely rewritten as

$$\hat{a}+\hat{b}=<m(\hat{a})+m(\hat{b}),\ w(\hat{a})+w(\hat{b})>, \tag{3}$$

$$\gamma\hat{a}=<\gamma m(\hat{a}),\ |\gamma|\,w(\hat{a})>. \tag{4}$$

Traditional ranking methods for intervals, such as those by Moore [35] and Ishibuchi and Tanaka [36] (the lower-upper ranking method, LR method), are relatively strict as they primarily consider strict relationships (e.g., intersection or being strictly greater) rather than the inclusion and/or overlap relations. To overcome this limitation, the statement "the interval $\hat{a}$ is not greater than the interval $\hat{b}$" is regarded as a fuzzy relation denoted by $\hat{a}\le_I\hat{b}$, taking full account of inclusion and overlap relations [26,37].

Definition 2.1 Let $\hat{a}=[\underline{a},\ \overline{a}]$ and $\hat{b}=[\underline{b},\ \overline{b}]$ be two intervals. The premise $\hat{a}\le_I\hat{b}$ is regarded as a fuzzy set, whose membership function is defined as follows:

$$\varphi(\hat{a}\le_I\hat{b})=\begin{cases}1, & \text{if } \overline{a}<\underline{b},\\ 1^-, & \text{if } \underline{a}<\underline{b}\le\overline{a}<\overline{b},\\ \dfrac{\overline{b}-\overline{a}}{2(w(\hat{b})-w(\hat{a}))}, & \text{if } \underline{b}\le\underline{a}\le\overline{a}\le\overline{b} \text{ and } w(\hat{b})>w(\hat{a}),\\ 0.5, & \text{if } w(\hat{a})=w(\hat{b}) \text{ and } \underline{a}=\underline{b},\end{cases} \tag{5}$$

where "1^-" is a fuzzy number being less than 1, which linguistically indicates the fact that the interval $\hat{a}$ is weakly not greater than the interval $\hat{b}$. The symbol "$\le_I$" is an interval-valued version of the order relation "$\le$" in R and has the linguistic interpretation "essentially not greater than". The symbols "$\ge_I$" and "$=_I$" are similarly explained [37]. Obviously, this fuzzy ranking index (the satisfactory degree) $\varphi(\hat{a}\le_I\hat{b})$ ranges from 0 to 1, representing the extent to which the premise $\hat{a}\le_I\hat{b}$ is accepted.

Definition 2.2 Let $\hat{a}=[\underline{a},\ \overline{a}]$ and $\hat{b}=[\underline{b},\ \overline{b}]$ be two intervals. The premise $\hat{a}\ge_I\hat{b}$ is regarded as a fuzzy set, whose membership function is defined as $\varphi(\hat{a}\ge_I\hat{b})=1-\varphi(\hat{a}\le_I\hat{b})$, i.e.,

$$\varphi(\hat{a} \geq_I \hat{b}) = \begin{cases} 0, & \text{if } \overline{a} < \underline{b}, \\ 0^+, & \text{if } \underline{a} < \underline{b} \leq \overline{a} < \overline{b}, \\ \dfrac{\underline{a} - \underline{b}}{2(w(\hat{b}) - w(\hat{a}))}, & \text{if } \underline{b} \leq \underline{a} \leq \overline{a} \leq \overline{b} \text{ and } w(\hat{b}) > w(\hat{a}), \\ 0.5, & \text{if } w(\hat{a}) = w(\hat{b}) \text{ and } \underline{a} = \underline{b}, \end{cases} \quad (6)$$

where "0^+" is a fuzzy number being greater than 0. The equality relation "$=_I$" is defined as $\hat{a} =_I \hat{b}$ if and only if $\hat{a} \geq_I \hat{b}$ and $\hat{a} \leq_I \hat{b}$. Definitions 2.1 and 2.2 may provide quantitative methods to determine the exact degree of satisfaction for ranking two intervals. Subsequently, the satisfactory degree φ is used to define satisfactory crisp equivalent forms of interval-valued inequality relations.

2.2. Coalition power indices for interval-valued cooperative games with power asymmetry

Due to information incompleteness and uncertainty, coalition payoffs in real cooperative games with power asymmetry are usually expressed as intervals, resulting in interval-valued cooperative games with power asymmetry. Such a game in coalitional form is defined as an ordered triple $< N, \hat{\upsilon}, \boldsymbol{\lambda} >$, where $N = \{1, 2, \ldots, n\}$ is the set of players, $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ is the vector of power indices in which λ_i represents the power index of player i, and $\hat{\upsilon} : 2^N \to I(\mathrm{R})$ is the characteristic function. This function assigns to each coalition $S \subseteq N$ a closed interval $\hat{\upsilon}(S) \in I(\mathrm{R})$, satisfying the condition $\hat{\upsilon}(\varnothing) = [0, 0]$. For each coalition S, the worth interval $\hat{\upsilon}(S)$ is denoted by $[\underline{\upsilon}(S), \overline{\upsilon}(S)]$. Notably, if all worth intervals degenerate, i.e., $\underline{\upsilon}(S) = \overline{\upsilon}(S)$, the game reduces to a classical cooperative game with power asymmetry.

In cooperative game theory, a player i is considered critical to a coalition if their joining transforms a losing coalition into a winning one S, or vice versa. Classic measurement methods for individual power indices primarily λ_i include the Shapley-Shubik index [32]:

$$\lambda_i = \sum \frac{(|S| - 1)!(n - |S|)!}{n!} \quad (i = 1, 2, \ldots, n), \quad (7)$$

and the Banzhaf index [33]:

$$\lambda_i = \frac{Q_i}{\sum_{i=1}^{n} Q_i,} \quad (i = 1, 2, \ldots, n), \quad (8)$$

where Q_i denotes the number of winning coalitions in which player i is critical.

Both indices quantify a player's expected contribution, laying the foundation for coalition power measurement.

Definition 2.3 For any interval-valued cooperative game with power asymmetry $(N, \hat{\upsilon}, \boldsymbol{\lambda})$, the power index λ_S of coalition S is defined as the solution to the following least-squares optimization problem:

$$\min_{\lambda \in \mathrm{R}} \{ \sum_{i \in S} (\lambda - \lambda_i)^2 \}. \quad (9)$$

Taking the derivative with respect to λ and setting it to zero yields

$$\lambda_S = \frac{\sum_{i \in S} \lambda_i}{|S|}, \tag{10}$$

where $|S|$ denotes the number of players in the coalition S. This indicates that the coalition power index λ_S is the arithmetic mean of the individual power index λ_i within the coalition S. This aggregation rule follows directly from the least-squares criterion, since the arithmetic mean minimizes the sum of squared deviations between the coalition level index and individual power indices. In the context of cooperative games, this averaging mechanism serves as a representative aggregation rule. Therefore, λ_S is adopted as a coalition level aggregation measure for the subsequent benefit allocation model.

3. Interval-valued cores and solution method for interval-valued cooperative games with power asymmetry

3.1. Interval-valued cores for interval-valued cooperative games with power asymmetry

Let $\hat{x}_i = [\underline{x}_i, \overline{x}_i] \in I(\mathrm{R})$ be an interval-valued payoff of the player $i \in N$ and $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ be an interval-valued payoff vector of n players. According to the interval addition [35], for each $S \in 2^N \setminus \{\varnothing\}$, the total interval payoff is defined as

$$\sum_{i \in S} \hat{x}_i = [\sum_{i \in S} \underline{x}_i, \sum_{i \in S} \overline{x}_i] \in I(\mathrm{R}).$$

In the following, we define an interval-type solution concept for interval-valued cooperative games $\hat{\upsilon} \in IG^n$. For conciseness, $\hat{\upsilon}(\{i\})$ and $\hat{\upsilon}(\{i, j\})$ are rewritten as $\hat{\upsilon}(i)$ and $\hat{\upsilon}(i, j)$, respectively. The interval-valued imputation set $I(\hat{\upsilon})$ of the game $\hat{\upsilon}$ is defined as follows [38]:

$$I(\hat{\upsilon}) = \{\hat{\mathbf{x}} \in (I(\mathrm{R}))^n \,\Big|\, \sum_{i \in N} \hat{x}_i =_I \hat{\upsilon}(N),\ \hat{x}_i \geq_I \hat{\upsilon}(i) \text{ for all } i \in N\},$$

where the condition $\sum_{i \in N} \hat{x}_i =_I \hat{\upsilon}(N)$ represents efficiency, indicating that the sum of all players' imputations equals the value of the grand coalition N. The condition $\hat{x}_i \geq_I \hat{\upsilon}(i)$ represents individual rationality, implying that each player's imputation should not be less than their stand-alone payoff.

If the interval-valued cooperative game is essential, players benefit from cooperation, and the interval-valued imputation set $I(\hat{\upsilon})$ exists (i.e., nonempty). Conversely, if the game is inessential, the imputation set $I(\hat{\upsilon})$ is a singleton, given by $I(\hat{\upsilon}) = \{(\hat{\upsilon}(1), \hat{\upsilon}(2), \ldots, \hat{\upsilon}(n))\}$. In this case, the interval-valued core coincides with this single imputation. Therefore, this study focuses on essential games where the imputation set is nonempty and typically contains infinite elements.

Moreover, the interval-valued imputation set is convex. For any $\hat{x}' \in I(\hat{\upsilon})$, $\hat{x}'' \in I(\hat{\upsilon})$, and $\theta \in [0, 1]$, the convex combination $\theta \hat{x}' + (1-\theta)\hat{x}''$ satisfies both efficiency and individual rationality, namely

$$\begin{aligned}\sum_{i \in N} [\theta \hat{x}_i' + (1-\theta)\hat{x}_i''] &= \theta \sum_{i \in N} \hat{x}_i' + (1-\theta) \sum_{i \in N} \hat{x}_i'' \\ &=_I \theta \hat{\upsilon}(N) + (1-\theta)\hat{\upsilon}(N) \\ &=_I \hat{\upsilon}(N),\end{aligned}$$

and

$$\theta \hat{x}_i' + (1-\theta)\hat{x}_i'' \geq_I \theta \hat{\upsilon}(i) + (1-\theta)\hat{\upsilon}(i) =_I \hat{\upsilon}(i).$$

Thus, $\theta\hat{x}_i^{'}+(1-\theta)\hat{x}_i^{''}\in I(\hat{\upsilon})$, proving that $I(\hat{\upsilon})$ is convex.

The interval-valued core $C(\hat{\upsilon})$ of the game $\hat{\upsilon}$ is defined as

$$C(\hat{\upsilon})=\{\hat{\mathbf{x}}\in(I(\mathrm{R}))^n \left| \sum_{i\in N}\hat{x}_i=_I\hat{\upsilon}(N),\ \sum_{i\in S}\hat{x}_i\geq_I\hat{\upsilon}(S)\text{ for all } \mathrm{S}\subset N\right.\},$$

where $\sum_{i\in N}\hat{x}_i=_I\hat{\upsilon}(N)$ is the efficiency condition and $\sum_{i\in S,\mathrm{S}\neq N}\hat{x}_i\geq_I\hat{\upsilon}(S)$ ($S\in 2^N\setminus\{\varnothing\}$) represents the stability condition. Clearly, due to the fact that an individual player may be regarded as a coalition S, i.e., $S=\{i\}$, then $\hat{x}_i\geq_I\hat{\upsilon}(i)$ is derived from $\sum_{i\in S,\mathrm{S}\neq N}\hat{x}_i\geq_I\hat{\upsilon}(S)$.

Theoretically, the interval-valued core $C(\hat{\upsilon})$ can be computed by solving the following system of interval inequalities:

$$\begin{cases}\sum_{i\in S}\hat{x}_i\geq_I\hat{\upsilon}(S)\ \ (S\subset N),\\ \sum_{i\in N}\hat{x}_i=_I\hat{\upsilon}(N),\\ \overline{x}_i\geq\underline{x}_i\ \ (i=1,2,\ \ldots,\ n).\end{cases}\tag{11}$$

However, solving Eq (11) is challenging due to the involvement of interval comparison operators and the complexity arising from power asymmetry among coalitions. In the next section, we focus on developing a method to solve Eq (11).

3.2. Satisfaction-based max-min formulation of the power-weighted interval-valued core

Using the satisfactory degree φ given above, interval-valued coalition rationality relations can be transformed into satisfaction-based crisp constraints. These equivalent formulations are then incorporated into the auxiliary max-min programming model for the power-weighted interval-valued core.

Let $\alpha\in[0,\ 1]$ denote the satisfactory degree of the interval-valued inequality constraint which may be satisfied. For the situations $\overline{a}\leq\overline{b}$, $\underline{a}\geq\underline{b}$, and $w(\hat{b})>w(\hat{a})$, according to Definition 2.1, a satisfactory crisp equivalent form of the interval-valued inequality constraint $\hat{a}\leq_I\hat{b}$ is defined as follows:

$$\begin{cases}\overline{a}\leq\overline{b},\\ \underline{a}\geq\underline{b},\\ \varphi(\hat{a}\leq_I\hat{b})\geq\alpha,\end{cases}\tag{12}$$

which can be further written as the following system of inequalities:

$$\begin{cases}\overline{a}\leq\overline{b},\\ \underline{a}\geq\underline{b},\\ (\overline{b}-\overline{a})/[2(w(\hat{b})-w(\hat{a}))]\geq\alpha.\end{cases}\tag{13}$$

It is easy to see from Eq (13) that $w(\hat{b})>w(\hat{a})$ is due to $\overline{a}\leq\overline{b}$ and $\alpha\in[0,\ 1]$.

Similarly, based on Definition 2.2, a satisfaction-based crisp equivalent constraint of the interval-valued inequality constraint $\hat{a}\geq_I\hat{b}$ is defined as follows:

$$\begin{cases} \bar{a} \le \bar{b}, \\ \underline{a} \ge \underline{b}, \\ \varphi(\hat{a} \ge_I \hat{b}) \ge \alpha, \end{cases} \tag{14}$$

which can be further written as

$$\begin{cases} \bar{a} \le \bar{b}, \\ \underline{a} \ge \underline{b}, \\ (\underline{a} - \underline{b}) / [2(w(\hat{b}) - w(\hat{a}))] \ge \alpha. \end{cases} \tag{15}$$

It should be noted that the satisfactory crisp equivalent formulation in Eqs (14) and (15) corresponds to the case $w(\hat{b}) > w(\hat{a})$ in Definition 2.2. This condition characterizes the particular branch of the interval-ranking relation adopted in the subsequent formulation.

Analogously, for other situations such as $\bar{a} < \underline{b}$, $\underline{a} < \underline{b} \le \bar{a} < \bar{b}$ or $w(\hat{a}) = w(\hat{b})$ and $\underline{a} = \underline{b}$, we can respectively obtain the satisfaction-based crisp equivalent constraints of the interval-valued inequality constraint $\hat{a} \le_I \hat{b}$ according to Definition 2.1 (omitted).

Subsequently, we apply the satisfactory crisp equivalent formulation in Eqs (14) and (15) to establish the auxiliary nonlinear programming model for Eq (11). In this application, $\hat{a}$ and $\hat{b}$ in Eqs (14) and (15) respectively correspond to the interval-valued allocation $\sum_{i \in S, S \ne N} \hat{x}_i$ of the coalition S and its interval-valued coalition worth $\hat{\upsilon}(S)$. Accordingly, the particular branch of Definition 2.2 adopted in Eqs (14) and (15) requires $w(\hat{\upsilon}(S)) > w(\sum_{i \in S, S \ne N} \hat{x}_i)$.

Therefore, the following satisfaction-based max-min formulation, i.e., Eqs (16)–(18), are derived for the case in which the coalition worth interval is wider than the corresponding allocation interval.

For any coalition $S \subset N$, let $\alpha_S = \varphi(\sum_{i \in S} \hat{x}_i \ge_I \hat{\upsilon}(S))$ denote the satisfactory degree of the interval-valued inequality $\sum_{i \in S} \hat{x}_i \ge_I \hat{\upsilon}(S)$, which may be satisfied.

Considering the differing power indices of coalitions (players) and the relationships $\sum_{i \in S} \underline{x}_i \ge \underline{\upsilon}(S)$ and $\sum_{i \in S} \bar{x}_i \le \bar{\upsilon}(S)$ ($S \subset N$), we construct the satisfaction-based max-min formulation of the power-weighted interval-valued core based on the above crisp reformulations and the discussion regarding power index variables and coalition satisfaction. The model is formulated as follows:

$$\begin{aligned} & \text{maximize} \{ \min_{S \subset N} \{ \lambda_S \alpha_S \} \} \\ & \text{subject to} \begin{cases} \sum_{i \in S} \underline{x}_i \ge \underline{\upsilon}(S) \quad (S \subset N), \\ \sum_{i \in S} \bar{x}_i \le \bar{\upsilon}(S) \quad (S \subset N), \\ \alpha_S = \varphi(\sum_{i \in S} \hat{x}_i \ge_I \hat{\upsilon}(S)) \quad (S \subset N), \\ \sum_{i \in N} \bar{x}_i = \bar{\upsilon}(N), \\ \sum_{i \in N} \underline{x}_i = \underline{\upsilon}(N), \\ \bar{x}_i \ge \underline{x}_i \ (i = 1,\ 2,\ \ldots,\ n). \end{cases} \end{aligned} \tag{16}$$

Let $\beta = \min_{S \subset N} \{\lambda_S \alpha_S\}$, then $\lambda_S \alpha_S \ge \beta$ and $0 \le \beta \le 1$. Thereby, according to Definition 2.2, Eq (16) can be rewritten as the following mathematical programming model:

$$\text{maximize}\{\beta\}$$
$$\text{subject to}\begin{cases} \sum_{i \in S} \underline{x}_i \ge \underline{\upsilon}(S) \ \ (S \subset N), \\ \sum_{i \in S} \overline{x}_i \le \overline{\upsilon}(S) \ \ (S \subset N), \\ \dfrac{\lambda_S (\sum_{i \in S} \underline{x}_i - \underline{\upsilon}(S))}{2(w(\hat{\upsilon}(S) - w(\sum_{i \in S} \hat{x}_i))} \ge \beta \quad (S \subset N), \\ \sum_{i \in N} \overline{x}_i = \overline{\upsilon}(N), \\ \sum_{i \in N} \underline{x}_i = \underline{\upsilon}(N), \\ \overline{x}_i \ge \underline{x}_i \ (i=1,\ 2,\ \ldots,\ n), \\ 0 \le \beta \le 1, \end{cases} \tag{17}$$

which may be rewritten as the following nonlinear programming model:

$$\text{maximize}\{\beta\}$$
$$\text{subject to}\begin{cases} \sum_{i \in S} \underline{x}_i \ge \underline{\upsilon}(S) \ \ (S \subset N), \\ \sum_{i \in S} \overline{x}_i \le \overline{\upsilon}(S) \ \ (S \subset N), \\ (\lambda_S - \beta)\sum_{i \in S} \underline{x}_i + \beta \sum_{i \in S} \overline{x}_i \ge (\lambda_S - \beta)\underline{\upsilon}(S) + \beta \overline{\upsilon}(S) \ \ (S \subset N), \\ \sum_{i \in N} \overline{x}_i = \overline{\upsilon}(N), \\ \sum_{i \in N} \underline{x}_i = \underline{\upsilon}(N), \\ \overline{x}_i \ge \underline{x}_i \ (i=1,\ 2,\ \ldots,\ n), \\ 0 \le \beta \le 1, \end{cases} \tag{18}$$

where β, $\overline{x}_i$, and $\underline{x}_i$ ($i=1,\ 2,\ \ldots,\ n$) are decision variables.

By assigning the coalition power index λ_S and solving Eq (18) by the bisection method and algorithms detailed in the following section, we can obtain the solution to this nonlinear programming model, denoted by $(\beta^*,\ \hat{\mathbf{x}}^*)$.

It is noted that the satisfactory crisp equivalent formulation derived above is limited to the interval relation case specified in Eqs (14) and (15), rather than covering all cases in Definition 2.2. In interval-valued cooperative games, a core allocation requires coalition rationality for every proper coalition and efficiency for the grand coalition. Interval relations with a satisfactory degree of 0 or 0^+ ($\sum_{i \in S} \overline{x}_i < \underline{\upsilon}(S)$ or $\sum_{i \in S} \underline{x}_i < \underline{\upsilon}(S) \le \sum_{i \in S} \overline{x}_i < \overline{\upsilon}(S)$) violate this rationality and are thus excluded. Although exact equality ($w(\sum_{i \in S} \hat{x}_i) = w(\hat{\upsilon}(S))$ and $\sum_{i \in S} \underline{x}_i = \underline{\upsilon}(S)$) for a particular coalition is mathematically admissible, simultaneous exact equality for all nonempty proper coalitions, together with grand coalition efficiency, implies that no positive cooperative surplus is generated. This corresponds to an inessential or additive interval-valued game and therefore falls outside the essential game setting considered in this study. Accordingly, the proposed optimization model is restricted to the derived interval relation case under the assumption of a positive cooperative surplus.

Several special cases of the proposed optimization model warrant mention for practical applications. First, if all coalition power indices entering the objective are identical and positive, the objective of maximizing the power-weighted satisfaction reduces, up to a positive constant, to maximizing the minimum coalition satisfaction, thereby recovering an unweighted interval-satisfaction formulation. Second, in scenarios where cooperation is restricted to a prescribed family $F \subset 2^N$ due to institutional or structural constraints, coalition rationality conditions need only be imposed for $S \in F$, which significantly reduces the problem size. Third, regarding the functional form of the payoff, a separable extension may replace the summation $\sum_{i \in S} x_i$ with $\sum_{i \in S} g_i(x_i)$. For a fixed satisfaction level, affine functions g_i preserve the linear structure of the feasibility subproblem, whereas suitable convex specifications may yield a convex subproblem, depending on the form and direction of the resulting constraints. Finally, when all coalition worth and allocation intervals degenerate to singleton values, the underlying game becomes a classical crisp cooperative game. However, since the present satisfactory degree transformation is designed for intervals with positive width, this degenerate limit requires a separate crisp formulation rather than direct substitution into Eq (18).

3.3. Bisection-based solution procedure

Using the bisection method [39], we can obtain the global optimal solution of Eq (18), denoted by $(\beta^*, \hat{\mathbf{x}}^*)$. Here, β^* represents the maximum attainable minimum power-weighted satisfaction level among all nonempty proper coalitions (the optimal power-weighted satisfaction level), and $\hat{\mathbf{x}}^*$ is the corresponding element of the interval-valued core $C(\hat{\upsilon})$. Obviously, if $\beta^* = 1$, we obtain an element of the interval-valued core where the optimal value β^* reaches its upper bound of 1.

It should be noted that the optimal value of β is determined endogenously by the proposed optimization model rather than being externally specified. Since β represents the minimum power-weighted satisfaction level among all nonempty proper coalitions, its attainable value is jointly affected by coalition power indices and feasible coalition satisfaction levels.

For a fixed $\beta \in [0, 1]$, let $\Omega(\beta)$ denote the feasible set of Eq (18). Equation (18) is nonlinear when β and the allocation variables are optimized simultaneously. However, once β is fixed, it reduces to a linear programming problem with respect to the allocation variables. Under the interval relation conditions used to derive Eqs (17) and (18), the β dependent constraints are equivalent to $\lambda_S \alpha_S \geq \beta$ for every nonempty proper coalition $S \subset N$.

Therefore, for any $0 \leq \beta_1 \leq \beta_2 \leq 1$, every solution feasible at β_2 is also feasible at β_1, and hence $\Omega(\beta_2) \subseteq \Omega(\beta_1)$. Thus, feasibility is monotone in β. Feasibility at a given value implies feasibility at every smaller value, whereas infeasibility implies infeasibility at every larger value. If Eq (18) is feasible at $\beta = 1$, then $\beta^* = 1$. If it is infeasible at $\beta = 0$, no feasible solution exists. Otherwise, the bisection procedure maintains a feasible lower bound $\underline{\beta}_t$ and an infeasible upper bound $\overline{\beta}_t$ satisfying $\underline{\beta}_t \leq \beta^* \leq \overline{\beta}_t$.

At each iteration, feasibility at the midpoint determines which half-interval still contains β^*. Since the interval length is halved at every iteration, $\overline{\beta}_t - \underline{\beta}_t = 2^{-t}$ for the initial interval $[0, 1]$. Consequently, if $m_0 \geq -\ln \varepsilon / \ln 2$, the final interval length does not exceed ε. The feasible allocation associated with the final lower bound is therefore an ε-optimal solution of Eq (18), while the midpoint of the final interval estimates β^* with an absolute error not exceeding $\varepsilon/2$.

The bisection procedure and algorithm used to estimate the global optimal solution of Eq (18) at a given precision $\varepsilon \in (0, 1]$ (hereby the number of iterations is a positive integer m_0, which satisfies $m_0 \geq -\ln\varepsilon/\ln 2$) are summarized as follows:

Step 1: Set $t = 0$. Test $\bar{\beta}_t = 1$. In this case, the nonlinear programming problem (Eq (18)) is transformed into a linear programming problem. Solve Eq (18) with $\bar{\beta}_t = 1$ using LINGO software (or the simplex method). If a feasible solution $\hat{\mathbf{x}}_t^*$ exists, then $\beta^* = \bar{\beta}_t = 1$ is the optimal value of the objective function of Eq (18), and $\hat{\mathbf{x}}^* = \hat{\mathbf{x}}_t^*$ is a corresponding interval-valued core $C(\hat{\upsilon})$ achieving the optimal power-weighted satisfaction level. The algorithm stops. Otherwise, if no feasible solution exists, proceed to Step 2.

Step 2: Test $\underline{\beta}_t = 0$. Solve Eq (18) using LINGO software (or the simplex method). If no feasible solution exists, this implies that the linear programming problem (and thus Eq (18)) has no solutions, then the algorithm stops. Conversely, if a feasible solution $\hat{\mathbf{x}}_t^*$ is obtained, it is determined that the optimal value of the objective function lies strictly between 0 and 1, i.e., $\beta^* \in [0, 1)$. Proceed to Step 3.

Step 3: Apply the bisection method. Let the search interval be $\hat{\beta}_t = [\underline{\beta}_t, \bar{\beta}_t]$. Calculate the midpoint $m(\hat{\beta}_t) = (\underline{\beta}_t + \bar{\beta}_t)/2 = (0+1)/2 = 0.5$. Solve Eq (18) with $\beta = 0.5$ using LINGO software (or the simplex method). If no feasible solution exists, the optimal value falls into the range between the lower bound $\underline{\beta}_t$ and the midpoint $m(\hat{\beta}_t)$, i.e., $\beta^* \in [\underline{\beta}_t, m(\hat{\beta}_t)) = [0, 0.5)$. The interval $\hat{\beta}_t$ is narrowed. Update the upper bound and set $\bar{\beta}_{t+1} = m(\hat{\beta}_t) = 0.5$ and $\underline{\beta}_{t+1} = \underline{\beta}_t = 0$. Proceed to Step 4. Conversely, if a feasible solution is obtained, the optimal value falls into the range between the midpoint $m(\hat{\beta}_t)$ and the upper bound $\bar{\beta}_t$, i.e., $\beta^* \in [m(\hat{\beta}_t), \bar{\beta}_t) = [0.5, 1)$. The interval $\hat{\beta}_t$ is also narrowed. Update the lower bound and set $\underline{\beta}_{t+1} = m(\hat{\beta}_t) = 0.5$ and $\bar{\beta}_{t+1} = \bar{\beta}_t = 1$. Proceed to Step 4.

Step 4: Set $t := t+1$. Repeat Step 3 within the new narrow interval $\hat{\beta}_t = [\underline{\beta}_t, \bar{\beta}_t)$ until the m_0-th iteration. Then, proceed to Step 5.

Step 5: After the m_0-th iteration, the final bounds satisfy $\bar{\beta}_{m_0} - \underline{\beta}_{m_0} \leq \varepsilon$. Let $\beta_{mid} = (\underline{\beta}_{m_0} + \bar{\beta}_{m_0})/2$. Then, β_{mid} is used as a numerical estimate of the global optimal objective value β^*, with an absolute error not exceeding $\varepsilon/2$. The feasible allocation retained at the final lower bound $\underline{\beta}_{m_0}$ is an ε-optimal solution of Eq (18). The corresponding $\hat{\mathbf{x}}^* = \hat{\mathbf{x}}_{m_0}^*$ is an element of the interval-valued core $C(\hat{\upsilon})$.

The proposed nonlinear solution method transforms the problem into a series of linear programming subproblems via the bisection method, which can be efficiently solved using LINGO software (or the simplex method). The number of bisection iterations depends solely on the desired precision rather than the number of players, requiring only a logarithmic number of steps. Although the number of constraints in the linear programming model grows combinatorially with the number of players (i.e., $2^n - 1$), usually the simplex method can obtain the optimal solution of the derived linear programming model in a finite number of iterations despite the increased number of constraints. Even if the worst happens, the linear programming models are theoretically polynomial-time solvable. Therefore, even in multi-agent regional frameworks with an expanded player set, the proposed method remains computationally feasible and usually can obtain the optimal solutions of the derived linear programming model in a finite number of iterations.

4. Numerical example analysis

The purpose of this section is to illustrate the computational procedure and examine whether the resulting allocations satisfy the adopted efficiency and coalitional rationality conditions under alternative parameter settings. The example is a stylized numerical experiment based on the institutional roles of three participants in a government-led marine fishery mutual insurance setting. All coalition worths are expressed in normalized units and constitute a controlled parameterization for methodological illustration.

4.1. Case setting and parameter construction

This subsection establishes the baseline parameterization used to illustrate the proposed interval-valued core solution procedure. The asymmetric decision structure is represented by a weighted voting game, in which the voting weights encode the ordinal institutional influence of the government, the mutual insurance institution, and the fishermen. Coalition worths are represented by interval-valued payoffs and constructed using a common midpoint-radius specification. The baseline parameters are selected to preserve the institutional ordering of the participants, distinguish coalition-specific uncertainty, and generate a non-degenerate interval-valued core problem. The subsequent parameter sensitivity analysis examines how the outputs vary under alternative voting weights, decision threshold, interval width and coalition worth interval settings.

4.1.1. Derivation of voting weights and decision threshold

The player set is $N=\{1,\ 2,\ 3\}$, where Player 1 is the government, Player 2 is the mutual insurance institution, and Player 3 is the fishermen. The baseline decision structure is represented by the weighted voting game $[q;\ W_1,\ W_2,\ W_3]=[4;\ 3,\ 2,\ 1]$. The weights 3, 2, and 1 are the smallest consecutive positive integers used to encode the assumed ordinal ranking of institutional influence: government > mutual insurance institution > fishermen. This coding preserves the intended power asymmetry while avoiding an unsupported interpretation of the weights as direct empirical measures or final power indices. The institutional basis of the ranking is summarized in Table 1. The government is assigned the highest voting level because it provides policy authority, fiscal support, and regulatory coordination. The mutual insurance institution is assigned the intermediate level because it is responsible for underwriting, claims settlement, operational implementation, and information processing. The fishermen are assigned the basic participation level because they contribute premiums, bear production risks, and exercise participation and collective negotiation rights.

The total voting weight is $W=3+2+1=6$. Major decisions are assumed to require a strict majority, where the threshold is set as $q=\lfloor W/2\rfloor+1$. Therefore, the decision threshold is $q=\lfloor 6/2\rfloor+1=4$. This rule ensures that no individual player can pass a decision alone in the baseline case. It also separates nominal voting weights from effective power, where the latter is calculated from whether a player is critical to transforming a winning coalition into a losing coalition.

Table 1. Institutional basis of the voting weights.

Player	Main institutional role in the cooperation	Ordinal influence level	Voting weight
Government	Policy authority, fiscal support, supervision, and cross-party coordination	High	3
Mutual insurance institution	Underwriting, claims settlement, operational implementation, and information processing	Intermediate	2
Fishermen	Premium contribution, risk-bearing participation, and collective representation	Basic	1

4.1.2. Construction of coalition payoff intervals

For each coalition $S \subseteq N$, its payoff is represented by the interval $\hat{\upsilon}(S) = [\underline{\upsilon}(S),\ \bar{\upsilon}(S)] = [m_S - w_S,\ m_S + w_S]$ as introduced in Section 2, where m_S denotes the benchmark cooperative payoff and w_S denotes the uncertainty radius. This midpoint-radius specification provides a common and reproducible rule for generating all coalition payoff intervals.

The baseline values are constructed according to three design principles. First, the three bilateral coalitions are assigned the common midpoint $m_S = 26$. This holds their benchmark payoff scale constant and allows the numerical analysis to isolate the effects of power asymmetry and heterogeneous uncertainty. Second, the radii 2, 4, and 6 are used as equally spaced low-, medium-, and high-uncertainty levels. These levels are assigned according to the risk-sharing structure of each coalition. Third, the grand coalition is assigned a higher midpoint and a small radius to capture the additional surplus and uncertainty-reduction effects generated by full three-party cooperation.

Specifically, coalition $\{1,\ 2\}$ is assigned the medium radius $w_{\{1,\ 2\}} = 4$. It combines policy and fiscal support with professional insurance operation, but the absence of direct participation by fishermen may create implementation and behavioral uncertainty. Coalition $\{1,\ 3\}$ is assigned the low radius $w_{\{1,\ 3\}} = 2$ because direct policy support and beneficiary participation provide a relatively stable cooperation link, although the coalition lacks the institution's full operational capacity. Coalition $\{2,\ 3\}$ is assigned the high radius $w_{\{2,\ 3\}} = 6$ because it lacks the government's fiscal and policy backstop and is therefore more exposed to disaster and claims volatility.

The grand coalition is assigned the benchmark midpoint $m_N = 42$ and the radius $w_N = 2$. The midpoint represents the additional surplus created by combining policy support, professional operation, and fishermen's participation, while the small radius represents the stabilizing effects of full participation, risk pooling, and coordination. This choice also ensures that the grand coalition has an unambiguous synergy advantage in the baseline setting, because its lower bound 40 exceeds the upper bound of every bilateral coalition. Singleton values are set to $[0,\ 0]$ because no player can independently generate the cooperative insurance surplus modeled in this example.

Thus, the interval-valued cooperative game is specified by $\hat{\upsilon}(1,\ 2) = [22,\ 30]$, $\hat{\upsilon}(1,\ 3) = [24,\ 28]$, $\hat{\upsilon}(2,\ 3) = [20,\ 32]$, $\hat{\upsilon}(1,\ 2,\ 3) = [40,\ 44]$, and $\hat{\upsilon}(1) = \hat{\upsilon}(2) = \hat{\upsilon}(3) = [0,\ 0]$.

The assigned radii are scenario assumptions used to create ordered levels of coalition worth uncertainty. The grand coalition specification deliberately embeds a positive synergy margin because its lower bound exceeds the upper bounds of the bilateral coalitions in the baseline setting. The subsequent parameter sensitivity analysis examines how the model outputs change when these assumptions are varied.

Table 2. Construction of the coalition payoff intervals.

Coalition S	Midpoint m_S	Radius w_S	Payoff interval $\hat{\upsilon}(S)$	Design interpretation
$\{1\}$, $\{2\}$, $\{3\}$	0	0	$[0,\ 0]$	No standalone cooperative surplus
$\{1,\ 2\}$	26	4	$[22,\ 30]$	Policy/fiscal support plus professional operation; medium uncertainty
$\{1,\ 3\}$	26	2	$[24,\ 28]$	Direct policy-participation link; relatively narrow uncertainty
$\{2,\ 3\}$	26	6	$[20,\ 32]$	No public fiscal backstop; highest exposure to claims volatility
$N=\{1,\ 2,\ 3\}$	42	2	$[40,\ 44]$	Full synergy, risk pooling, and lower relative uncertainty

4.2. Computational results obtained by the proposed method

4.2.1. Calculate player power indices

Using the normalized Banzhaf index [33], we count the winning coalitions in which each player is critical. And based on the case data, one might intuitively assume that the power ratio of the three players corresponds to their voting weights of $3:\ 2:\ 1$. However, analyzing their respective power indices requires identifying the "critical player" in the winning coalition. For this problem, the winning coalitions are $\{1,\ 2\}$, $\{1,\ 3\}$, and $\{1,\ 2,\ 3\}$. Player 1 is a critical player in all these coalitions, resulting in a power count of 3. Player 2 is critical only in the coalition $\{1,\ 2\}$, resulting in a power count of 1. Similarly, Player 3 is critical only in the coalition $\{1,\ 3\}$, resulting in a power count of 1. Therefore, the normalized vector of powers indices for the three players is $\boldsymbol{\lambda}=(0.6,\ 0.2,\ 0.2)^{\mathrm{T}}$. Consequently, the effective power ratio is $3:\ 1:\ 1$, rather than the nominal voting weight ratio $3:\ 2:\ 1$. This distinction illustrates why power must be calculated from coalition criticality rather than inferred directly from assigned votes.

4.2.2. Calculate coalition power indices

According to Eq (10), we can obtain coalition power indices as follows:

$$\lambda_{\{1,\,2\}}=\frac{\lambda_1+\lambda_2}{|\{1,\ 2\}|}=\frac{0.6+0.2}{2}=0.4\,,$$

$$\lambda_{\{1,\,3\}}=\frac{\lambda_1+\lambda_3}{|\{1,\ 3\}|}=\frac{0.6+0.2}{2}=0.4\,,$$

$$\lambda_{\{2,\,3\}}=\frac{\lambda_2+\lambda_3}{|\{2,\ 3\}|}=\frac{0.2+0.2}{2}=0.2\,.$$

4.2.3. Construct nonlinear programming model

According to Eq (18), the nonlinear programming model can be constructed as follows:

$$\text{maximize}\{\beta\}$$

$$\text{subject to } \begin{cases} \underline{x}_1+\underline{x}_2 \geq 22, \\ \overline{x}_1+\overline{x}_2 \leq 30, \\ (0.4-\beta)(\underline{x}_1+\underline{x}_2)+\beta(\overline{x}_1+\overline{x}_2) \geq 22(0.4-\beta)+30\beta, \\ \underline{x}_1+\underline{x}_3 \geq 24, \\ \overline{x}_1+\overline{x}_3 \leq 28, \\ (0.4-\beta)(\underline{x}_1+\underline{x}_3)+\beta(\overline{x}_1+\overline{x}_3) \geq 24(0.4-\beta)+28\beta, \\ \underline{x}_2+\underline{x}_3 \geq 20, \\ \overline{x}_2+\overline{x}_3 \leq 32, \\ (0.2-\beta)(\underline{x}_2+\underline{x}_3)+\beta(\overline{x}_2+\overline{x}_3) \geq 20(0.2-\beta)+32\beta, \\ \overline{x}_1+\overline{x}_2+\overline{x}_3 = 44, \\ \underline{x}_1+\underline{x}_2+\underline{x}_3 = 40, \\ \overline{x}_i \geq \underline{x}_i \ (i=1,\ 2,\ 3), \\ \underline{x}_i \geq 0 \ (i=1,\ 2,\ 3), \\ 0 \leq \beta \leq 1, \end{cases} \tag{19}$$

where β, $\overline{x}_i$, and $\underline{x}_i$ ($i=1,\ 2,\ 3$) are decision variables.

We can solve Eq (19) using the bisection method and the algorithm described previously. First, set $t=0$ and test $\overline{\beta}_t=1$. This nonlinear programming problem (i.e., Eq (19)) is transformed into a linear programming problem. Solving Eq (19) with $\beta=\overline{\beta}_t=1$ using LINGO software (or the simplex method) yields no feasible solution. Second, setting $\underline{\beta}_t=0$, we obtain a feasible solution $\hat{\mathbf{x}}_t^*$. This confirms that the optimal value is between 0 and 1, i.e., $\beta^*\in[0,\ 1)$. Next, calculate the midpoint $m(\hat{\beta}_t)=(\underline{\beta}_t+\overline{\beta}_t)/2=(0+1)/2=0.5$. Solving Eq (19) with $\beta=m(\hat{\beta}_t)=0.5$ using LINGO software (or the simplex method) also yields no feasible solution. The optimal value falls into the range between the lower bound $\underline{\beta}_t$ and the midpoint $m(\hat{\beta}_t)$, i.e., $\beta^*\in[\underline{\beta}_t,\ m(\hat{\beta}_t))=[0,\ 0.5)$. Thereby, the interval $\hat{\beta}_t$ is narrowed. Update the interval $\hat{\beta}_t$ and let $t:=t+1$, $\overline{\beta}_{t+1}=m(\hat{\beta}_t)=0.5$, and $\underline{\beta}_{t+1}=\underline{\beta}_t=0$. Repeat above procedures in the new smaller interval $\hat{\beta}_t=[\underline{\beta}_t,\ \overline{\beta}_t)$ until the $m_0=16$-th iterations. The range of β is narrowed to $\beta\in[0.19998169,\ 0.20001221)$. Therefore, the optimal power-weighted satisfaction level of Eq (19) at the given precision is estimated as $\beta^*=0.2$. One corresponding optimal allocation returned by the solver is $\hat{x}_1^*=[12,\ 12]$, $\hat{x}_2^*=[16,\ 16]$, and $\hat{x}_3^*=[12,\ 16]$. Since the interval-valued core is a set-valued solution concept, the proposed optimization model aims at identifying one optimal allocation from the interval-valued core that maximizes the minimum power-weighted satisfaction level. Other allocations achieving the same optimal objective value may also exist. The obtained allocation $\hat{\mathbf{x}}^*$ satisfies the individual rationality, coalition rationality, and efficiency constraints of the interval-valued core $C(\hat{\upsilon})$. Hence, the interval-valued core is nonempty, indicating that a stable payoff allocation exists under which no player or proper coalition has an incentive to deviate from the grand coalition.

It is easy to see that the obtained value $\beta^*=0.2$ is determined by the optimization procedure under the given coalition structure and parameter settings. In this numerical example, although the coalition power indices provide the upper bound of the power-weighted satisfaction level, the final attainable value is further restricted by the coalition rationality constraints and feasible allocation intervals. Therefore, $\beta^*=0.2$ represents the highest balanced satisfaction level achievable among all coalitions under the given cooperative structure. From a cooperative management perspective, the derived solution maximizes the weighted satisfaction of the least-satisfied coalition. This approach balances coalition power, payoff acceptability, and coalition stability.

4.3. Comparison with a selected power-adjusted interval Shapley benchmark

To provide a benchmark comparison, we apply the interval Shapley allocation procedure of Yu and Zhang [40] together with the power-index adjustment described below. This procedure is hereafter referred to as the selected power-adjusted interval Shapley benchmark.

4.3.1. Preliminary Shapley value

The preliminary Shapley value $\hat{x}_i^{'}$ for player i is calculated as

$$\hat{x}_i^{'} = \sum_{i \in S,\, S \subseteq N} \frac{(n-|s|)!(|s|-1)!}{n!} (\hat{\upsilon}(S) - \hat{\upsilon}(S \setminus \{i\})), \tag{20}$$

where the interval subtraction follows the rule $\hat{a} - \hat{b} = [\underline{a} - \overline{b},\ \overline{a} - \underline{b}]$.

4.3.2. Determine initial allocation weights

Using the preliminary Shapley value $\hat{x}_i^{'}$, the initial allocation weights for the upper and lower bounds are separately calculated as follows:

$$\bar{\mathbf{Q}} = (\bar{Q}_1,\ \bar{Q}_2,\ \ldots,\ \bar{Q}_n)^{\mathrm{T}} = \frac{(\bar{x}_1^{'},\ \bar{x}_2^{'},\ \ldots,\ \bar{x}_n^{'})^{\mathrm{T}}}{\sum_{i=1}^{n} \bar{x}_i^{'}}, \tag{21}$$

$$\underline{\mathbf{Q}} = (\underline{Q}_1,\ \underline{Q}_2,\ \ldots,\ \underline{Q}_n)^{\mathrm{T}} = \frac{(\underline{x}_1^{'},\ \underline{x}_2^{'},\ \ldots,\ \underline{x}_n^{'})^{\mathrm{T}}}{\sum_{i=1}^{n} \underline{x}_i^{'}}, \tag{22}$$

where $\bar{\mathbf{Q}}$ and $\underline{\mathbf{Q}}$ represent initial allocation weights for the interval-valued game with power asymmetry.

4.3.3. Calculate player power indices

Considering power asymmetry, the influence $\boldsymbol{\lambda}_i$ of each player is separately calculated according to Eq (8).

4.3.4. Calculate adjusted allocation weights

The allocation weights from Eqs (21) and (22) are adjusted using each player's power index $\boldsymbol{\lambda}_i$:

$$\bar{\mathbf{Q}}^{*} = (\bar{Q}_1^{*},\ \bar{Q}_2^{*},\ \ldots,\ \bar{Q}_n^{*})^{\mathrm{T}} = \frac{(\bar{Q}_1\lambda_1,\ \bar{Q}_2\lambda_2,\ \ldots,\ \bar{Q}_n\lambda_n)^{\mathrm{T}}}{\sum_{i=1}^{n} \bar{Q}_i\lambda_i}, \tag{23}$$

$$\underline{\mathbf{Q}}^* = (\underline{Q}_1^*,\ \underline{Q}_2^*,\ \ldots,\ \underline{Q}_n^*)^{\mathrm{T}} = \frac{(\underline{Q}_1\lambda_1,\ \underline{Q}_2\lambda_2,\ \ldots,\ \underline{Q}_n\lambda_n)^{\mathrm{T}}}{\sum_{i=1}^{n}\underline{Q}_i\lambda_i}. \tag{24}$$

4.3.5. Calculate the selected benchmark allocation

The final selected power-adjusted interval Shapley benchmark allocation is determined by applying the adjusted weights in Eqs (23) and (24) to the payoff intervals:

$$\bar{\mathbf{x}}^* = (\bar{x}_1^*,\ \bar{x}_2^*,\ \ldots,\ \bar{x}_n^*)^{\mathrm{T}} = (\bar{Q}_1^*,\ \bar{Q}_2^*,\ \ldots,\ \bar{Q}_n^*)^{\mathrm{T}}\sum_{i=1}^{n}\bar{x}_i^{'}, \tag{25}$$

$$\underline{\mathbf{x}}^* = (\underline{x}_1^*,\ \underline{x}_2^*,\ \ldots,\ \underline{x}_n^*)^{\mathrm{T}} = (\underline{Q}_1^*,\ \underline{Q}_2^*,\ \ldots,\ \underline{Q}_n^*)^{\mathrm{T}}\sum_{i=1}^{n}\underline{x}_i^{'}. \tag{26}$$

Now, apply this method to the numerical example for comparative analysis. The game possesses the required properties of interval efficiency, symmetry, and super-additivity. Using Eq (20), the initial interval Shapley value for Player 1 is calculated in Table 3.

According to Table 3, the initial interval Shapley value for Player 1 is $[10.33,\ 17.67]$. Similarly, the values for Players 2 and 3 are calculated as intervals $[11,\ 17]$ and $[10.67,\ 17.33]$, respectively.

Table 3. Initial interval Shapley values for players.

S	$\{1\}$	$\{1, 2\}$	$\{1, 3\}$	$\{1, 2, 3\}$
$\hat{\upsilon}(S)$	$[0,\ 0]$	$[22,\ 30]$	$[24,\ 28]$	$[40,\ 44]$
$\hat{\upsilon}(S\setminus\{1\})$	$[0,\ 0]$	$[0,\ 0]$	$[0,\ 0]$	$[20,\ 32]$
$\{\hat{\upsilon}(S)-\hat{\upsilon}(S\setminus\{1\})\}$	$[0,\ 0]$	$[22,\ 30]$	$[24,\ 28]$	$[8,\ 24]$
$\frac{(3-\lvert s\rvert)!(\lvert s\rvert-1)!}{3!}$	$2/6$	$1/6$	$1/6$	$2/6$
$\hat{x}_1^{'}$			$[10.33,\ 17.67]$	

Second, according to Eqs (21) and (22), the initial allocation weights are

$$\bar{\mathbf{Q}} = (\bar{Q}_1,\ \bar{Q}_2,\ \bar{Q}_3)^{\mathrm{T}} = \frac{(\bar{x}_1^{'},\ \bar{x}_2^{'},\ \bar{x}_3^{'})^{\mathrm{T}}}{\sum_{i=1}^{3}\bar{x}_i^{'}} = (0.3398,\ 0.3269,\ 0.3333)^{\mathrm{T}}, \tag{27}$$

$$\underline{\mathbf{Q}} = (\underline{Q}_1,\ \underline{Q}_2,\ \underline{Q}_3)^{\mathrm{T}} = \frac{(\underline{x}_1^{'},\ \underline{x}_2^{'},\ \underline{x}_3^{'})^{\mathrm{T}}}{\sum_{i=1}^{3}\underline{x}_i^{'}} = (0.3228,\ 0.3438,\ 0.3334)^{\mathrm{T}}. \tag{28}$$

Third, the power indices of the players are calculated as $\boldsymbol{\lambda} = (0.6,\ 0.2,\ 0.2)^{\mathrm{T}}$.

Fourth, adjust the allocation weights $\mathbf{Q}^*$ for Eqs (27) and (28) according to each player's power index $\boldsymbol{\lambda}$:

$$\bar{\mathbf{Q}}^* = (\bar{Q}_1^*,\ \bar{Q}_2^*,\ \bar{Q}_3^*)^{\mathrm{T}} = \frac{(\bar{Q}_1\lambda_1,\ \bar{Q}_2\lambda_2,\ \bar{Q}_3\lambda_3)^{\mathrm{T}}}{\sum_{i=1}^{3}\bar{Q}_i\lambda_i} = (0.6069,\ 0.1946,\ 0.1985)^{\mathrm{T}}, \tag{29}$$

$$\underline{\mathbf{Q}}^* = (\underline{Q}_1^*,\ \underline{Q}_2^*,\ \underline{Q}_3^*)^{\mathrm{T}} = \frac{(\underline{Q}_1\lambda_1,\ \underline{Q}_2\lambda_2,\ \underline{Q}_3\lambda_3)^{\mathrm{T}}}{\sum_{i=1}^{3}\underline{Q}_i\lambda_i} = (0.5885,\ 0.2089,\ 0.2026)^{\mathrm{T}}. \tag{30}$$

Finally, the selected power-adjusted interval Shapley benchmark allocation is calculated using the adjusted allocation weights in Eqs (29) and (30):

$$\bar{\mathbf{x}}^* = (\bar{x}_1^*,\ \bar{x}_2^*,\ \bar{x}_3^*)^{\mathrm{T}} = (\bar{Q}_1^*,\ \bar{Q}_2^*,\ \bar{Q}_3^*)^{\mathrm{T}}\sum_{i=1}^{3}\bar{x}_i' = (31.56,\ 10.12,\ 10.32)^{\mathrm{T}},$$

$$\underline{\mathbf{x}}^* = (\underline{x}_1^*,\ \underline{x}_2^*,\ \underline{x}_3^*)^{\mathrm{T}} = (\underline{Q}_1^*,\ \underline{Q}_2^*,\ \underline{Q}_3^*)^{\mathrm{T}}\sum_{i=1}^{3}\underline{x}_i' = (18.83,\ 6.68,\ 6.49)^{\mathrm{T}},$$

which highlight Player 1's dominance, increasing its allocation from the initial interval $[10.33,\ 17.67]$ to the interval $[18.83,\ 31.56]$, while the allocations for Players 2 and 3 decrease. This reflects the principle of distribution according to contribution and status.

However, under the selected benchmark, interval arithmetic, and parameterization, the resulting allocation does not satisfy the efficiency and coalitional rationality requirements of the interval-valued core under the adopted parameterization. According to the coalitional rationality principle, the sum of allocation for Players 2 and 3 is $[6.68,\ 10.12]+[6.49,\ 10.32]=[13.17, 20.44]$. Compare this to the coalition payoff interval $\hat{\upsilon}(2,\ 3)=[20,\ 32]$ and the satisfaction degree $\varphi([13.17, 20.44]\geq_I [20, 32])=0^+$. This indicates that Players 2 and 3 have minimal satisfaction regarding the coalition inequality, violating stability. Furthermore, the sum of all allocations is $[18.83,\ 31.56]+[6.68,\ 10.12]+[6.49,\ 10.32]=_I[32, 52]$, which is not equal to the grand coalition value $\hat{\upsilon}(1,\ 2,\ 3)=[40,\ 44]$. Obviously, applying the selected interval Shapley-based benchmark to the stylized parameterization yields allocations that violate both efficiency and coalitional rationality. Specifically, the sum of the benchmark allocations $[32,\ 52]$ does not equal the grand coalition worth $[40,\ 44]$, and the coalition $\{2,\ 3\}$ receives less than its coalition worth under the adopted interval arithmetic. These deviations highlight the difference between a general allocation rule and a stability-oriented interval-valued core solution. The violation of coalitional rationality indicates that the basic risk protection interests of the mutual institution and fishermen could be compromised, implying a theoretical risk of coalition instability or withdrawal by vulnerable players. Similarly, the violation of efficiency implies a potential financial shortfall, requiring external financial adjustments to sustain the mutual insurance mechanism. While stylized, these numerical observations yield a valuable theoretical insight. Balancing power asymmetry with payoff satisfaction is essential for long-term cooperative viability, and absolute power dominance alone may not guarantee stable cooperation without adequate stakeholder satisfaction. This static comparative insight motivates the subsequent sensitivity analysis, which examines how formal voting weights and coalition worth intervals affect power indices and the distribution of allocation uncertainty within the model.

4.4. Parameter sensitivity analysis

The parameter sensitivity analysis re-solves the proposed model under alternative parameter settings, and the resulting comparative-statics patterns illustrate how voting rules and coalition worth intervals influence the allocation outcomes. Accordingly, the sensitivity analysis is conducted based on the representative optimal allocation obtained from the proposed optimization procedure. The reported allocation changes should therefore be interpreted as variations of this representative optimal solution under different parameter settings, rather than a complete characterization of all possible optimal allocations.

4.4.1. Sensitivity analysis for voting weights

Variations in voting weights induce significant shifts in both individual and coalition power indices, and these shifts consequently propagate into alterations in the interval-valued core allocations, as illustrated in Figures 1 and 2. When the government's assigned voting weight reaches configurations such as $4:2:1$ or $5:2:1$, the stylized model exhibits what may be termed a scale diseconomy threshold, where concentrated resource input heavily skews the cooperative game structure. Under such conditions, the government's power index surges while the formal influence of the institution and fishermen diminishes significantly. Additionally, a weak coalition collapse manifests as the combined grassroots capacity of subordinate players falls below the decision threshold. This centralized structure restricts negotiation flexibility. Furthermore, fishermen may experience a lower-bound collapse where their worst-case payoff guarantees recede, illustrating that extreme fiscal dominance can compress baseline protections for weaker participants.

Conversely, when weights are modified so that any two players can form a winning coalition, power indices converge toward equilibrium, for example, around 0.333 in scenarios like $1:2:1$ or $3:2:2$. This theoretical indispensability is powerfully evidenced by real-world institutional shifts. For instance, when the resource manager's direct regulatory power was severely restricted in the British Columbia groundfish fishery, the system's stability relied heavily on the bottom-up coalition of fishers and the internalization of market pressure generated by Environmental Non-Governmental Organizations (ENGOs), which effectively re-balanced the power structure and prevented systemic collapse [41]. This reflects an absolute democratic state in terms of voting leverage. Under this balanced state, the model reveals a structural redistribution of payoff uncertainty, where the dominant player absorbs systemic volatility through an interval allocation while subordinate players receive secure, deterministic payoffs. While these dynamics stem from a simplified parameter configuration, they emphasize that policymakers must carefully calibrate subsidy intensity to avoid over-concentration. This approach helps preserve stakeholder vitality and prevents potential systemic instability in broader cooperative networks.

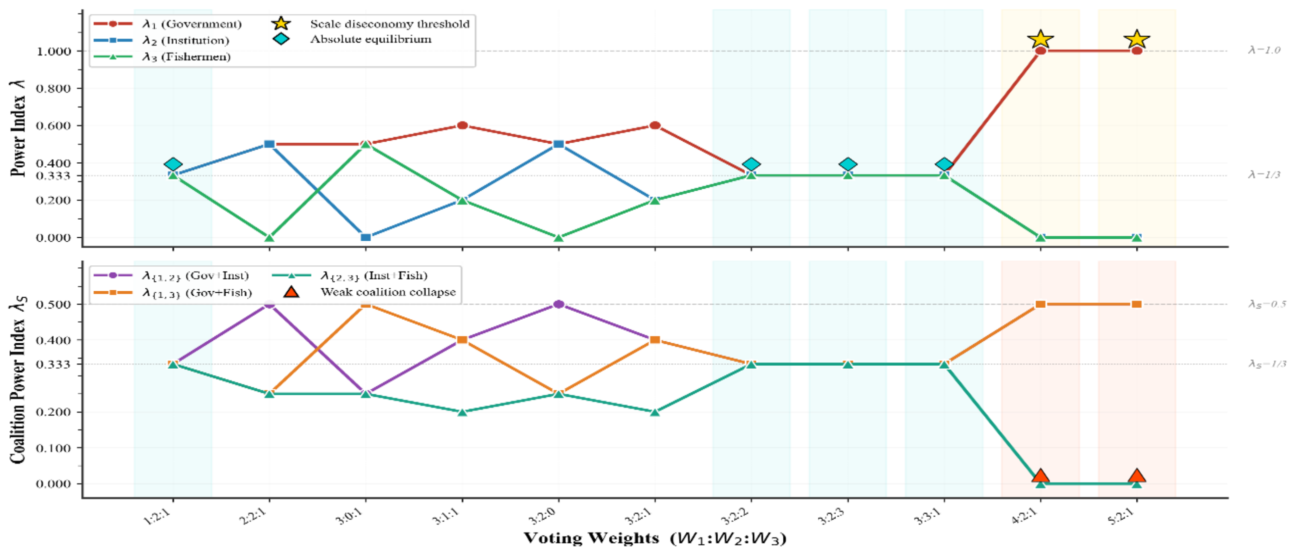

Figure 1. Sensitivity of power indices to voting weights.

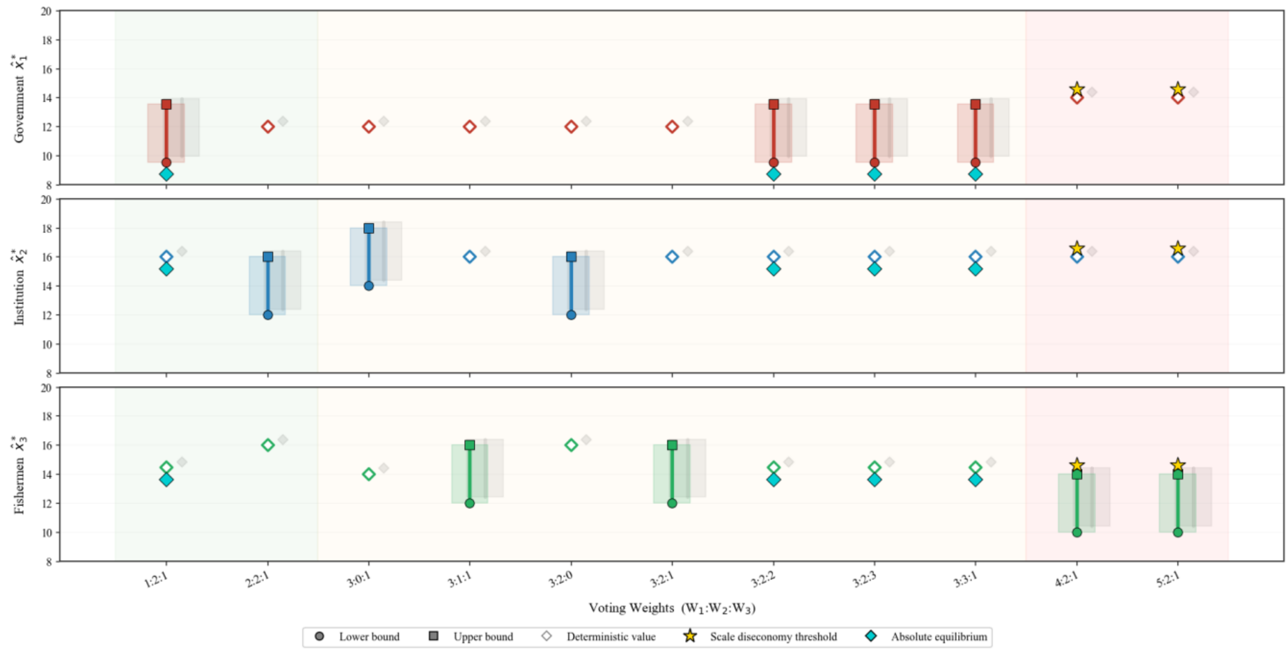

Figure 2. Sensitivity of interval-valued core allocations to voting weights.

4.4.2. Sensitivity analysis for decision threshold variation

Variations in the decision threshold fundamentally reshape the strategic landscape of the stylized model, and they directly alter both the power indices and the interval-valued core allocations, as illustrated in Figures 3 and 4. It should be noted that the decision threshold must satisfy the feasibility condition $q \le W$, where W denotes the total voting weight. Therefore, threshold settings exceeding the total voting weight do not represent feasible voting rules because no coalition can become the winning condition. In Figure 3, such settings are retained only as boundary scenarios to illustrate the loss of feasible winning coalitions and the resulting collapse of the voting mechanism, rather than as valid cooperative game configurations. Lowering the decision threshold, for instance, to 3 votes, makes it significantly easier for a single player to dominate the decision-making process. This shifts the scale diseconomy threshold leftward in the model space, implying that lower decision hurdles can lead to rapid centralized control even under moderate administrative intervention. Consequently, the government's power index approaches 1.0, while secondary players experience marginalization and potential lower-bound collapse in their core allocations if consensus requirements are overly lax.

Conversely, elevating the decision threshold to higher levels, such as 5 votes, introduces severe structural rigidities. When the threshold exceeds the total voting weight, no winning coalition exists, and the corresponding voting structure loses practical decision-making meaning. Therefore, these cases are interpreted as infeasible boundary conditions rather than valid voting games. This leads to a decision deadlock where coalition leverage vanishes. As the threshold varies, alternative configurations such as 3 : 1 : 1 or 2 : 2 : 1 permit multiple winning coalitions, and this aligns power indices toward a balanced distribution known as the absolute democratic state. Within this regime, the dominant entity absorbs interval uncertainty to shield subordinate partners. However, excessively high consensus requirements can trigger a weak coalition collapse, where grassroots entities lack sufficient voting weight to form viable counterweights and are forced into passive dependency. In summary, within the confines of this numerical simulation, the decision threshold dictates the delicate trade-off between administrative efficiency and market autonomy. The results highlight that uncalibrated decision rules risk either administrative over-concentration or institutional paralysis.

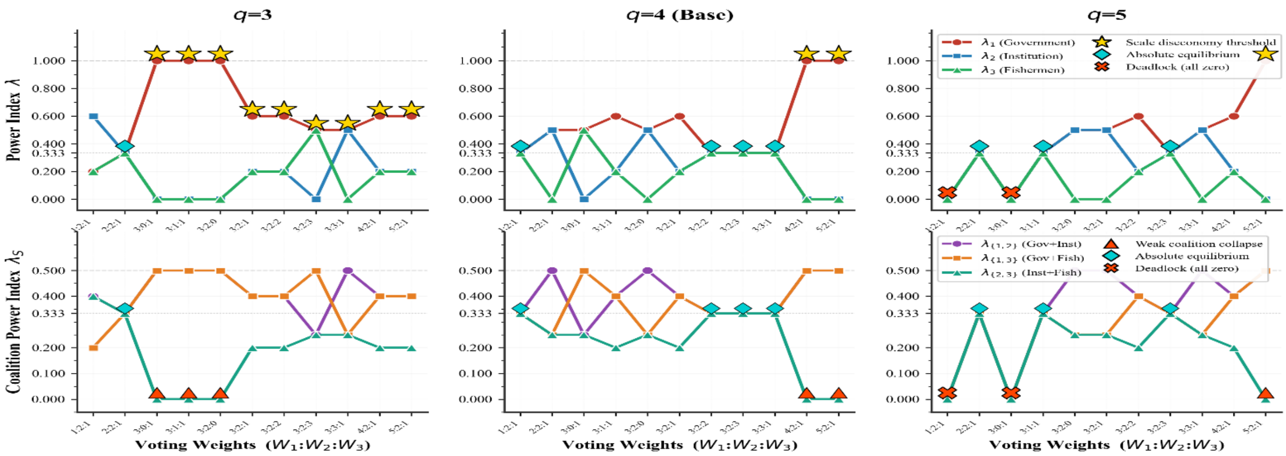

Figure 3. Sensitivity of power indices to decision thresholds including infeasible threshold scenarios for boundary analysis.

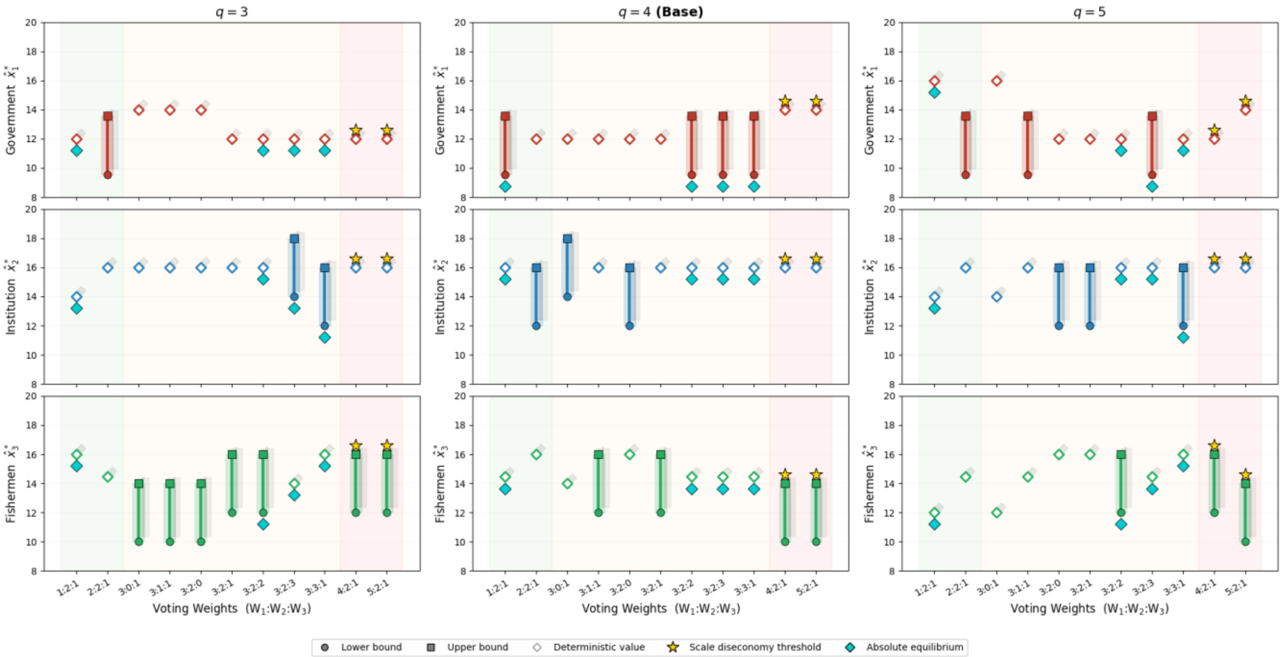

Figure 4. Sensitivity of interval-valued core allocations to decision thresholds.

4.4.3. Sensitivity analysis for the grand coalition interval width and coalition values

Variations in the grand coalition interval width and coalition values fundamentally reshape the interval-valued core allocations, and these variations are illustrated in Figure 5. The interval width essentially reflects the systemic uncertainty inherent in the stylized marine fishery insurance cooperation, where a wider interval signifies greater ambiguity regarding potential risk protection benefits. When this uncertainty is entirely eliminated and the width shrinks to zero, all players receive deterministic allocations, effectively causing the interval to collapse to a single point. Conversely, as the interval width expands to represent heightened market volatility, the burden of uncertainty shifts among the subordinate players. While the government consistently secures a deterministic value to maintain its fiscal stability, the institution is forced to absorb the expanded interval volatility, and this leaves fishermen with a guaranteed deterministic payoff. This suggests that under expanded interval width representing heightened environmental volatility, intermediate market-oriented institutions may structurally absorb a larger share of payoff uncertainty.

Furthermore, variations in the grand coalition value represent the total size of the cooperative benefit pool. These variations demonstrate that expanding total returns allows all participants to secure enhanced payoffs. In contrast, a contracting benefit pool triggers a lower-bound collapse for the operating institution, forcing it into strict cost-control constraints. Interpreted through the lens of this stylized framework, these findings underscore that expanding the overall cooperative surplus and mitigating source-level risk uncertainty is theoretically preferable to relying solely on administrative reallocation. This approach helps avoid operational distress for mutual insurance providers within the model parameters.

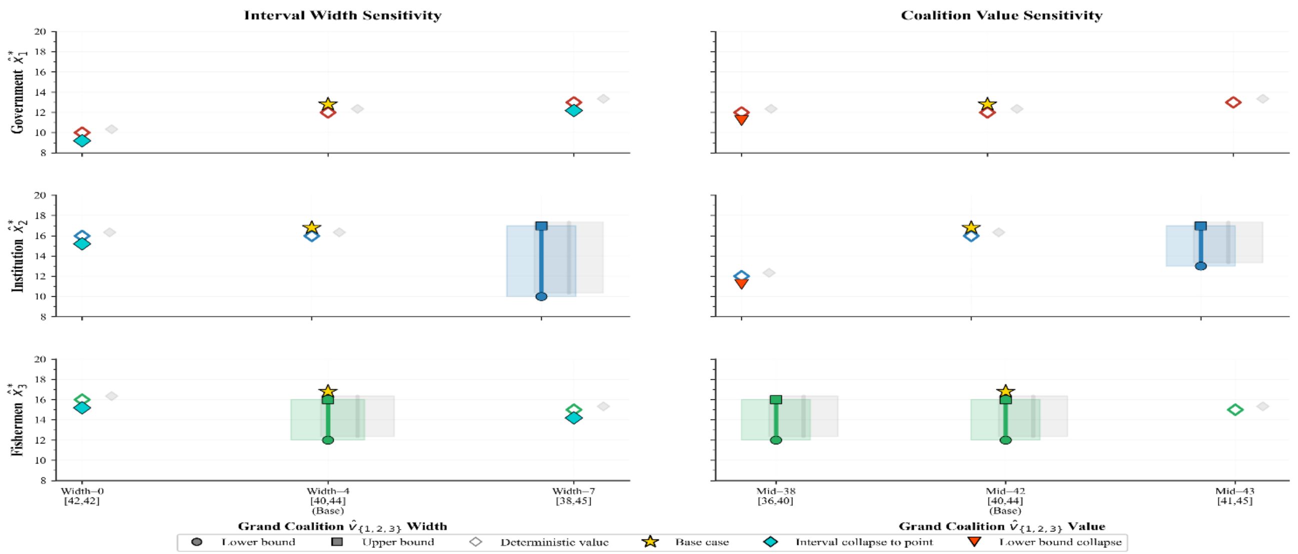

Figure 5. Sensitivity of interval-valued core allocations to the grand coalition interval width and coalition values.

5. Conclusions

This study addresses cooperative benefit allocation under the joint presence of imprecise coalitions' worths and asymmetric decision rights. By integrating coalition-based power representation with interval coalition rationality in a power-weighted core framework, the proposed approach provides a way to evaluate allocation stability without requiring coalition worths to be specified as precise values. The formulation also yields a tractable solution structure for the interval relation domain considered.

The stylized, normalized three-player example illustrates several features of the proposed framework. Under the baseline parameterization, one optimal allocation obtained from the model satisfies the adopted efficiency and coalitional rationality conditions, while the selected power-adjusted interval Shapley benchmark does not satisfy the same conditions under the same interval arithmetic and parameter setting. The sensitivity analysis further indicates that voting weights, decision thresholds, and the grand coalition worth interval can alter both the distribution of power and the way allocation uncertainty is distributed among players. In particular, some parameter configurations exhibit regime changes that may be described as a scale-diseconomy threshold, balanced power states, or weakened coalition viability. These numerical patterns are provided as examples under the controlled parameterization, demonstrating the feasibility and computational implementation of the proposed solution procedure.

Several mathematical questions remain open. First, existence and uniqueness conditions for the power-weighted interval-valued core require further characterization. Second, restricted coalition families and games with a larger number of players deserve separate analysis, where constraint generation, decomposition, and parallel feasibility tests may be more appropriate than explicit enumeration of all coalitions. Alternative fairness and stability benchmarks, such as the interval Shapley value, Owen values, the nucleolus, and the τ-value, could also be investigated and compared in terms of efficiency, rationality properties, and computational tractability under the same interval data [42–45].

When sufficiently rich empirical data become available, coalition worth midpoints and radii may be estimated rather than specified as controlled parameters. Nonlinear and interaction effects can be captured by multivariate adaptive regression splines [46], while generalized partial linear models combine interpretable parametric components with flexible nonparametric terms [47]. Furthermore, a dynamic extension could represent coalition worths as state-dependent stochastic processes and examine intertemporal stability using stochastic games, robust optimization, or stochastic control [48–50]. Future research may also integrate machine-learning-based scenario generation and constraint-generation algorithms, while ensuring that data-driven components retain an interpretable link between estimated coalition worths and institutional power. Behavioral factors such as risk perception, bounded rationality, and negotiation delay may be incorporated when suitable longitudinal or experimental data are available. These extensions represent promising directions for future research.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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