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Research article

Chaos control in discrete fractional systems with variable order: analysis and numerical simulations

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Abstract: This study investigates how to control chaos in discrete-time systems that have variable fractional orders. We examine a general type of discrete fractional-order equations where the order changes with time, and we show that these systems can become chaotic when certain parameters are chosen. To address this, we develop and apply tailored control techniques to suppress chaos and achieve system stabilization. Using detailed numerical simulations, we confirm that the suggested control method works effectively in two example cases. Our findings underscore that chaos control in variable fractional-order systems provides significant flexibility in modulating dynamic behavior, offering valuable insights into the broader applicability of these methods in discrete fractional-order systems.

Keywords: chaos; control; Lorenz system; sine-based memristor map; discrete systems; fractional order; variable order

1. Introduction

Discrete fractional calculus has recently become a key method for describing systems with memory effects, spatial interactions, and nonlocal dynamics. Unlike classical integer-order calculus, fractional-order operators incorporate historical system states into their formulations, making them particularly suitable for describing processes where past behavior significantly influences the future dynamics. This property has led to widespread applications in physics, biology, economics, and engineering [1–8]. Notably, fractional-order models have been successfully employed in studying anomalous diffusion, viscoelastic materials, biological signal processing, and financial systems with memory effects [9–12]. Among the diverse phenomena explored using fractional calculus, chaos stands out as a critical area of research due to its profound implications for systems' stability, predictability, and control.

Chaotic systems are highly sensitive to their starting conditions and may seem random, even though they follow specific rules. These systems are found in many natural and engineering applications. Exploring chaos in fractional-order systems is particularly important because the nonlocal properties of fractional operators make the system's behavior more complex. Discrete fractional calculus provides a more refined approach to analyzing chaotic behavior, as it allows for the precise characterization of memory-dependent dynamics in discrete-time systems. This is especially relevant in digital control systems, computational biology, and networked systems where continuous-time approximations may not suffice.

A significant advancement in this field is the introduction of variable fractional-order calculus, where the fractional order is no longer constant but varies with time or system state [12]. This generalization enhances modeling flexibility, enabling the description of systems where the degree of memory dependence changes dynamically, such as adaptive biological processes, evolving neural networks, or self-regulating mechanical systems. In discrete settings, variable-order models are particularly valuable because they can capture sudden transitions, adaptive memory effects, and multiscale dynamics that fixed-order models cannot represent. The ability to adjust the fractional order in response to the system's behavior makes variable-order calculus a powerful tool for studying stability, bifurcations, and chaos in nonlinear systems.

Stabilizing chaotic systems and designing effective control strategies remain fundamental challenges in nonlinear dynamics. Uncontrolled chaos can lead to undesirable instabilities in engineering applications, such as power grids, communication networks, and robotic systems. Various control techniques, including feedback linearization, adaptive control, and synchronization schemes, have been developed to mitigate chaotic behavior. However, applying these methods to discrete fractional-order systems with variable orders introduces additional complexities due to the interplay among memory effects, nonlinearity, and order variability. Understanding how variable fractional orders influence controllability and stability is thus essential for developing robust control frameworks for modern dynamic systems.

This study focuses on a broad type of systems where the order can change over time. These systems are flexible models that help us study chaos, multiple stable states, and bifurcations. These systems exhibit a rich spectrum of dynamic behaviors, from periodic oscillations to fully developed chaos, depending on the parameter choices and order variations. By systematically analyzing the impact of variable fractional orders on a system's stability, we aim to derive novel control strategies that can suppress chaos and stabilize the desired trajectories. Our research helps deepen the understanding of how fractional-order systems behave in discrete time and shows how variable-order calculus can support progress in science and engineering fields.

The implications of this research extend beyond theoretical developments, offering practical insights for controlling complex systems in fields such as secure communications, biomedical engineering, and autonomous robotics. By bridging the gap between variable fractional-order calculus and chaos control, this study paves the way for future innovations in adaptive and memory-dependent system design.

Several recent works related to fractional-order control and synchronization, such as projective synchronization in fractional reaction–diffusion systems, Mittag–Leffler synchronization of delayed fractional memristor neural networks, and sliding mode control of discrete fractional chaotic systems, provide valuable perspectives. These studies further highlight the relevance of fractional-order models in modern control theory.

In contrast to existing works on fixed-order fractional difference systems, the present study investigates stabilization under a time-varying fractional order. The dependence of the fractional operator on time introduces additional technical challenges in stability and control design. The results derived here explicitly account for these variations and establish stability conditions that remain valid for any order trajectory within the prescribed bounds.

Variable-order fractional operators allow the memory effect of a system to evolve over time. Unlike fixed-order models, variable-order systems can capture changes in the internal dynamics, adaptation mechanisms, and time-dependent memory behavior. This added flexibility provides a richer mathematical framework and enables the study of how varying memory influences stability, control performance, and chaotic dynamics.

This manuscript is organized as follows. Section 2 presents the main preliminaries of discrete fractional calculus, with an emphasis on the variable-order Caputo operator. Section 3 introduces the general formulation of the variable-order discrete fractional system together with the proposed control strategy. In Section 4, we analyze the variable-order discrete Lorenz-type system and demonstrate the effect of the control technique through numerical simulations. Section 5 is devoted to the variable order sine-based memristor map, where the efficiency of the control method is further verified. Finally, Section 6 provides the concluding remarks.

2. Preliminaries

We begin by introducing the key definitions and properties of discrete fractional calculus, with an emphasis on the variable-order case.

Definition 1. *[13] Given $\varrho > 0$, and $\omega : \mathbb{N}_a \to \mathbb{R}$, we have*

$$\Delta_a^{-\varrho}\omega(t) = \frac{1}{\Gamma(\varrho)}\sum_{s=a}^{t-\varrho}(t-s-1)^{(\varrho-1)}\omega(s), \text{ for } t \in \mathbb{N}_{a+\varrho}, \tag{2.1}$$

is the fractional sum of ω, where $\mathbb{N}_a = \{a, a+1, a+2, \cdots\}$ and $t^{(\varrho)} = \frac{\Gamma(t+1)}{\Gamma(t+1-\varrho)}$.

Definition 2. *[13] The Caputo fractional difference of order ϱ is*

$${}^{C}\Delta_a^{\varrho}\omega(t) = \Delta_a^{-(1-\varrho)}\Delta\omega(t), \forall t \in \mathbb{N}_{a+1-\varrho}, 0 < \varrho < 1. \tag{2.2}$$

Definition 3. *[12] Let $\varrho : \mathbb{Z} \to (0, 1]$, the variable-order Caputo fractional difference discrete is defined by*

$${}^{C}\Delta^{\varrho(t)}\omega\left(t+1-\varrho(t)\right) = \omega(t+1) + \sum_{s=0}^{t}(-1)^{t-s+1}\binom{\varrho(t-s)}{t-s+1}\omega(s) + (-1)^{t}\binom{\varrho(t)-1}{t+1}\omega(0), \tag{2.3}$$

where $\omega : \mathbb{N}_0 \to \mathbb{R}$.

Let

$$^{C}\Delta^{\varrho(t)}\omega\left(t+1-\varrho(t)\right) = A\omega\left(t\right), t \in \mathbb{N}_0, \tag{2.4}$$

where $^{C}\Delta^{\varrho(t)}\omega = (^{C}\Delta^{\varrho(t)}\omega_1, ^{C}\Delta^{\varrho(t)}\omega_2, \cdots, ^{C}\Delta^{\varrho(t)}\omega_n)$, $\omega(0) = \omega_0 \in \mathbb{R}^n$, $\varrho(\cdot) : \mathbb{N}_0 \to [0,1]$, and $A \in \mathbb{R}^{n\times n}$.

Theorem 1. *[12] Let*

$$\rho := \max\left\{\left(1 - \frac{\varrho_{\inf}}{\varrho_{\sup}} + \varrho_{\inf} - \varrho(0)\right), \left(\frac{\varrho_{\sup}}{\varrho_{\inf}} - \varrho_{\sup} - 1 + \varrho(0)\right)\right\}, \tag{2.5}$$

where

$$\varrho_{\inf} = \inf \varrho\left(t\right), \varrho_{\sup} = \sup \varrho\left(t\right), \tag{2.6}$$

and

$$d\left(\lambda, \mathbb{C}\backslash S^{\varrho(0)}\right) = \inf\left\{|\lambda - z|, z \in \mathbb{C}\backslash S^{\varrho(0)}\right\}. \tag{2.7}$$

If

$$d\left(\lambda, \mathbb{C}\backslash S^{\varrho(0)}\right) > \rho, \tag{2.8}$$

for any eigenvalues λ *of A, then the system (2.4) is asymptotically stable where*

$$S^{\varrho(0)} = \left\{z \in \mathbb{C} : |z| < \left(2cos\frac{|argz| - \pi}{2 - \varrho(0)}\right) \text{ and } |argz| > \frac{\varrho(0)\pi}{2}\right\}.$$

The stability conditions obtained in this work depend only on the bounds of the time-varying order, (that is, on the interval $[\rho_{\inf}, \rho_{\sup}]$) rather than on the rate at which $\rho(t)$ varies. Therefore, the system remains stable even when the order changes rapidly, provided that $\rho(t)$ stays within the prescribed interval.

3. System definition and control approach

This section introduces a general class of discrete fractional systems with variable order and discusses the formulation used for control design. We decompose the system into its linear and nonlinear components, which allows a systematic application of linear feedback control enhanced by nonlinear compensation terms. The objective is to establish a general framework for stabilizing variable-order discrete fractional systems.

3.1. System definition

This study explores a broad category of discrete fractional systems with variable order, offering a flexible framework for modeling complex dynamics in nonlinear systems. The evolution of the fractional order follows a prescribed function that determines how the memory of the system changes over time. This rule defines whether the order increases, decreases, or oscillates, enabling the analysis of its influence on the system's stability. Let

$$^{C}\Delta^{\varrho(t)}\omega\left(t+1-\varrho(t)\right) = A\omega\left(t\right) + g\left(\omega\left(t\right)\right), t \in \mathbb{N}_0, \tag{3.1}$$

where $g : \mathbb{R}^n \to \mathbb{R}^n$. Equation (3.1) expresses the dynamics of the variable fractional-order system in a compact form. The term $A\omega(t)$ corresponds to the linear part of the system, while $g(\omega(t))$ represents all nonlinear contributions satisfying $\|g(\omega(t))\| = o(\|\omega(t)\|)$ near the origin. This decomposition separates the system into linear and nonlinear components, which is essential for defining the control law. The operator ${}^{C}\Delta^{\varrho(t)}$ denotes the Caputo variable fractional-order difference, ensuring that the current state depends not only on the present but also on the past values with a memory effect modulated by $\varrho(t)$.

3.2. Control approach

Now we present a control strategy to stabilize the variable-order system and mitigate chaotic behavior. The main goal is to construct a control input $C(t)$ that ensures convergence to a desired trajectory or equilibrium point, thereby effectively suppressing chaos and achieving stabilization. The system with control can be represented as:

$$ {}^{C}\Delta^{\varrho(t)}\omega(t+1-\varrho(t)) = A\omega(t) + g(\omega(t)) + C(t), \tag{3.2} $$

where $C(t)$ is the control input to be determined.

We propose a feedback control strategy that combines linear and non-linear components. The control input $C(t)$ is designed as

$$ C(t) = -K\omega(t) - g(\omega(t)). \tag{3.3} $$

$K\omega(t)$ represents a feedback gain matrix designed to stabilize the linear component of the system, $\breve{g}(\omega(t))$ is an estimate or compensation term for the non-linear part $g(\omega(t))$, and K is chosen so that the eigenvalues of $A-K$ lie within the stability region of the system. The nonlinear compensation term $\breve{g}(\omega(t))$ is designed to cancel or mitigate the effects of nonlinearity $g(\omega(t))$, ensuring that the system's dynamics are dominated by the stabilized linear part.

In this section, we present the key theoretical result, which establishes sufficient conditions for controlling discrete fractional-order systems with a variable order. Consider the controlled system in the (3.2).

Theorem 2. *Suppose the control law is given as follows:*

$$ C(t) = -K\omega(t) - g(\omega(t)), \tag{3.4} $$

Let K be chosen such that for every eigenvalue λ of the matrix $(A-K)$, the following condition holds:

$$ d\left(\lambda, \mathbb{C}\backslash S^{\varrho(0)}\right) = \inf\left\{|\lambda - z|, z \in \mathbb{C}\backslash S^{\varrho(0)}\right\}. \tag{3.5} $$

Then the controlled system (3.2) *becomes asymptotically stable.*

Proof. Under the control of (3.3) and (3.2) becomes

$$ {}^{C}\Delta^{\varrho(t)}\omega(t+1-\varrho(t)) = (A-K)\omega(t). \tag{3.6} $$

By assumption, the matrix K is selected so that for every eigenvalue λ of $(A-K)$, the following condition holds:

$$ d\left(\lambda, \mathbb{C}\backslash S^{\varrho(0)}\right) = \inf\left\{|\lambda - z|, z \in \mathbb{C}\backslash S^{\varrho(0)}\right\}. \tag{3.7} $$

This ensures that all eigenvalues of $(A-K)$ lie strictly within the stability region for discrete fractional-order systems. Then, according to Theorem 1, (3.2) is asymptotically stable.

The proposed control method requires knowledge of the system's linear part and an estimate of the nonlinear term. In addition, the control design assumes that the variable order remains within known bounds. The method may require adjustment when dealing with systems subject to strong uncertainties or unknown disturbances.

4. Control of chaotic behavior in a variable-order discrete Lorenz system

In this section, we analyze a two-dimensional discrete Lorenz-type system under a variable fractional-order operator, focusing on its chaotic behavior and the stabilizing effect of the proposed control strategy. This system demonstrates complex behaviors such as chaos, bifurcations, and transient states, influenced by its variable-order parameters. By analyzing how variations in the fractional order affect the system's dynamics, we aim to develop strategies for stabilizing chaotic behavior and improving controllability. This advances the understanding of variable-order discrete systems, supporting their potential applications in secure communications, optimization, and computational modeling. In this section, we analyze a two-dimensional discrete Lorenz-type system under a variable fractional-order operator, focusing on its chaotic behavior and the stabilizing effect of the proposed control strategy. This system demonstrates complex behaviors such as chaos, bifurcations, and transient states, influenced by its variable-order parameters. By analyzing how variations in the fractional order affect the system's dynamics, we aim to develop strategies for stabilizing chaotic behavior and improving controllability. This advances the understanding of variable-order discrete systems, supporting their potential applications in secure communications, optimization, and computational modeling.

Consider the following discrete-time system, as introduced in [14]:

$$\begin{cases} \Delta\omega_1(t) = ab\omega_1(t) - b\omega_1(t)\,\omega_2(t), \\ \Delta\omega_2(t) = b\left(\omega_1^2(t) - \omega_2^2(t)\right), \end{cases} \tag{4.1}$$

for specific parameter values $\omega_1(0) = 0.19$, $\omega_2(0) = 0.31$, $a = 0.99$, *and* $b = 0.72$. The dynamic of this system has been previously analyzed in the discrete fractional-order case in [15].

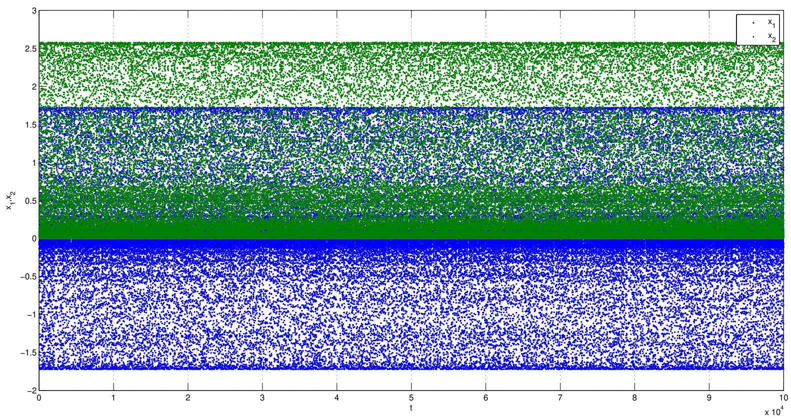

Figure 1. Numerical simulations of System (4.1) clearly indicate chaotic dynamics under the considered parameter values.

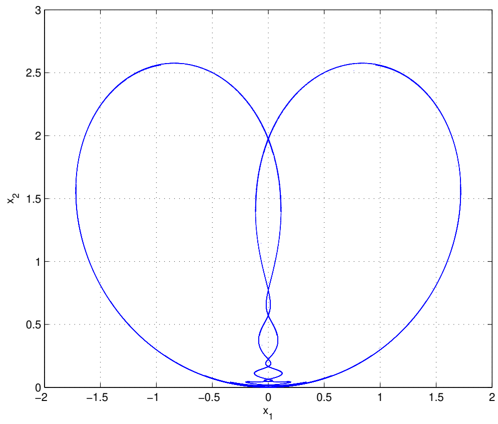

Figure 2. The phase space of System (4.1) shows the chaotic attractor.

Figures 1 and 2 illustrate the uncontrolled chaotic behavior of the system.

We now extend our analysis to a fractional discrete model with variable order, expressed as follows:

$$\begin{cases} {}^{C}\Delta^{\varrho(t)}\omega_1 (t+1-\varrho(t)) = ab\omega_1 (t) - b\omega_1 (t)\,\omega_2 (t)\,, \\ {}^{C}\Delta^{\varrho(t)}\omega_2 (t+1-\varrho(t)) = b\left(\omega_1^2 (t) - \omega_2^2 (t)\right), \end{cases} \tag{4.2}$$

where $\varrho(t) = 0.98 + 0.01\cos(t)$. The chosen form of the variable order ($\varrho(t)$ provides a smooth periodic variation in the memory effect of the system. This type of fluctuation is commonly used to analyze how small oscillations in the fractional order influence the chaotic behavior of nonlinear discrete systems. It also allows us to assess the robustness of the proposed control method under time-varying memory characteristics.

System (4.1) incorporates the variable-order fractional operator, allowing the memory effect of the system to vary over time. This enhances modeling flexibility and enables the analysis of how adaptive memory influences chaotic behavior, which cannot be captured by the fixed-order system (4.1). Such adaptability is useful in practical applications involving systems with evolving internal dynamics.

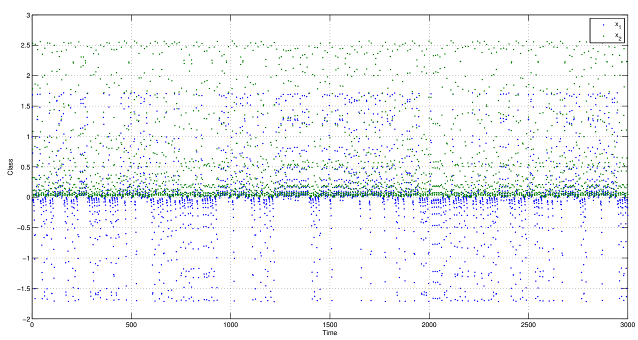

Figure 3. A numerical simulation of System (4.3) shows the chaotic behavior of the system.

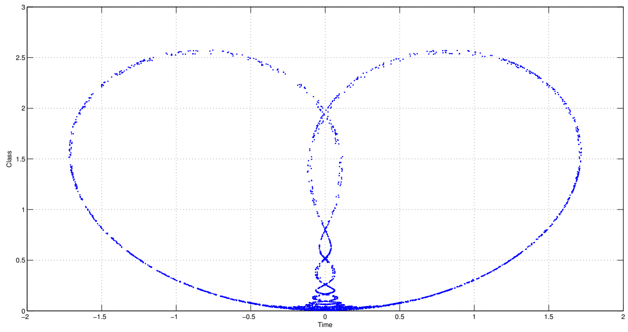

Figure 4. The phase space of System (4.3) shows the chaotic attractor.

To stabilize the chaotic dynamics, we introduce a control strategy, modifying the system as follows:

$$\begin{cases} {}^{C}\Delta^{\varrho(t)}\omega_1\left(t+1-\varrho(t)\right) = ab\omega_1\left(t\right) - b\omega_1\left(t\right)\omega_2\left(t\right) + C_1\left(t\right), \\ {}^{C}\Delta^{\varrho(t)}\omega_2\left(t+1-\varrho(t)\right) = b\left(\omega_1^2\left(t\right) - \omega_2^2\left(t\right)\right) + C_2\left(t\right). \end{cases} \tag{4.3}$$

We also apply

$$\begin{cases} C_1\left(t\right) = \lambda_1\omega_1\left(t\right) + b\omega_1\left(t\right)\omega_2\left(t\right), \\ C_2\left(t\right) = -b\left(\omega_1^2\left(t\right) - \omega_2^2\left(t\right)\right) + \lambda_2\omega_2\left(t\right), \end{cases} \tag{4.4}$$

where $\lambda_1 = 0.94, \lambda_2 = -0.23$. The system reduces to the following stabilized linear form:

$$\begin{cases} {}^{C}\Delta^{\varrho(t)}\omega_1\left(t+1-\varrho(t)\right) = \left(\lambda_1 + ab\right)\omega_1\left(t\right), \\ {}^{C}\Delta^{\varrho(t)}\omega_2\left(t+1-\varrho(t)\right) = \lambda_2\omega_2\left(t\right), \end{cases} \tag{4.5}$$

with ($\rho = 0.02$), the eigenvalues of the controlled system fall within the required stability region described in Theorem 1. This confirms that the control gains have been properly selected and that the controlled system is asymptotically stable.

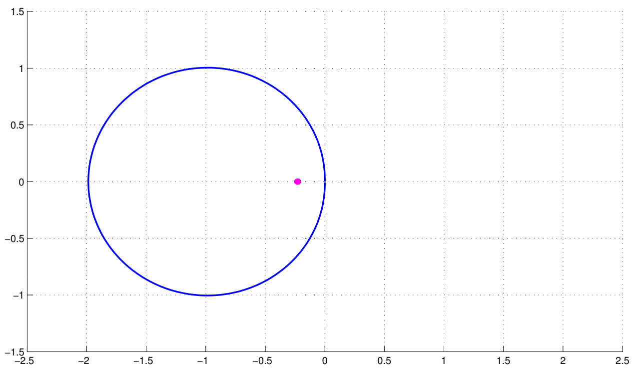

Figure 5. The eigenvalues' location and the disks, with the center at the eigenvalues and radius ρ.

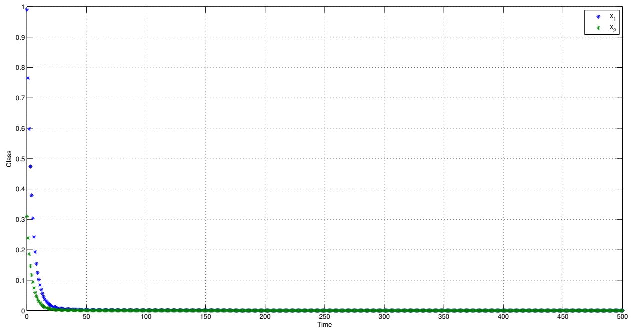

Figure 6. A numerical simulation of System (4.5) shows the stability of the system.

Figures 3–6 present the controlled dynamics and confirm the stabilization results.

5. Chaos control in a variable-order sine-based memristor map

We now examine the three-dimensional sine-based memristor map in this section, an essential dynamical model for exploring chaotic behavior in nonlinear systems. This model exhibits rich dynamic properties, including chaotic behavior, multi stability, and periodic attractors. By analyzing the impact of variable fractional orders on this system, we aim to understand how to control chaos and achieve stability in complex dynamical models. This contributes to advancing fractional-order computation applications in science and engineering.

Initially, assume the following system, as mentioned in [16], as follows:

$$\begin{cases} \Delta\omega_1(t) = a_1 \sin(\omega_2(t)) + m\omega_2(t)\sin(\omega_3(t)), \\ \Delta\omega_2(t) = a_2 \sin(\omega_1(t))\sin(\omega_2(t)), \\ \Delta\omega_3(t) = k\omega_3(t), \end{cases} \tag{5.1}$$

for the values $\omega_1(0) = -1.9$, $\omega_2(0) = 0.9$, $\omega_3(0) = 0.09$, $a_1 = 1.49$, $a_2 = 3.79, m = 0.1$ and $k = 0.01$. In the discrete fractional case, the dynamic behavior of this system has been examined with both commensurate and incommensurate orders in [16].

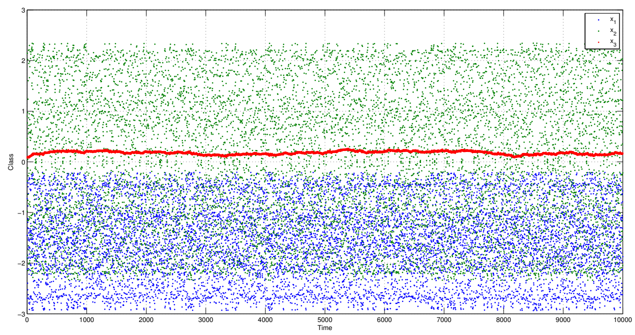

Figure 7. A numerical simulation of System (5.1) shows the chaotic behavior of the system.

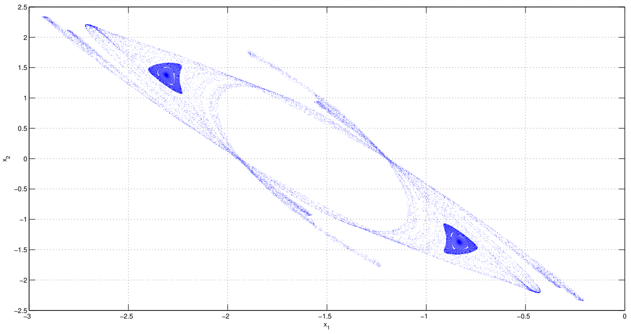

Figure 8. The phase space of System (5.1) shows the chaotic attractor.

Figures 7 and 8 display the chaotic attractor of the memristor map.

We now examine the discrete fractional model with a variable order, which is

$$\begin{cases} {}^{C}\Delta^{\varrho(t)}\omega_1(t+1-\varrho(t)) = a_1 \sin(\omega_2(t)) + m\omega_2(t)\sin(\omega_3(t)), \\ {}^{C}\Delta^{\varrho(t)}\omega_2(t+1-\varrho(t)) = a_2 \sin(\omega_1(t))\sin(\omega_2(t)), \\ {}^{C}\Delta^{\varrho(t)}\omega_3(t+1-\varrho(t)) = k\omega_3(t). \end{cases} \tag{5.2}$$

The variable-order definition in this section, ($\varrho(t) = 0.97 + 0.02\cos\left(\frac{\pi}{2}(1-t)\right)$), is selected to introduce a smooth adaptive memory effect. Such forms are frequently used in variable-order modeling to capture gradual transitions in the system's memory, particularly in nonlinear memristive structures.

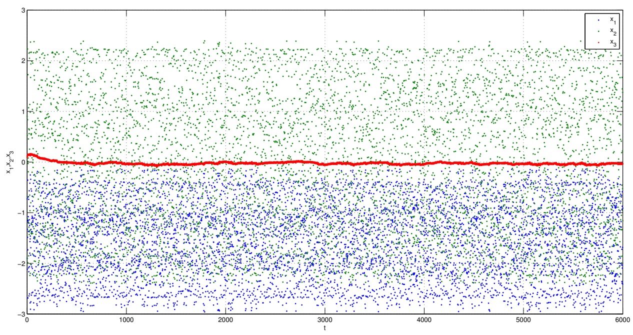

Figure 9. A numerical simulation of System (5.2) shows the chaotic behavior.

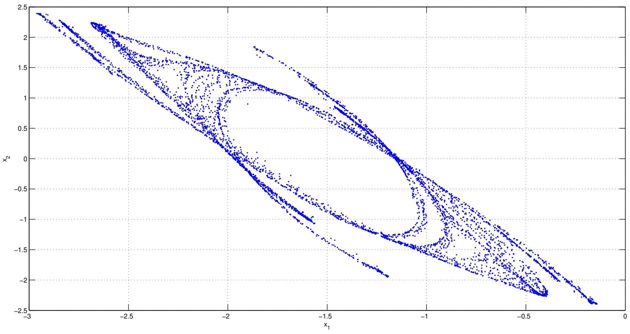

Figure 10. The phase space of System (5.2) shows the attractor.

The corresponding controlled system is

$$\begin{cases} {}^{C}\Delta^{\varrho(t)}\omega_1(t+1-\varrho(t)) = a_1\sin(\omega_2(t)) + m\omega_2(t)\sin(\omega_3(t)) + C_1(t), \\ {}^{C}\Delta^{\varrho(t)}\omega_2(t+1-\varrho(t)) = a_2\sin(\omega_1(t))\sin(\omega_2(t)) + C_2(t), \\ {}^{C}\Delta^{\varrho(t)}\omega_3(t+1-\varrho(t)) = k\omega_3(t) + C_3(t). \end{cases} \tag{5.3}$$

By applying the control law

$$\begin{cases} C_1(t) = -a_1\sin(\omega_2(t)) - m\omega_2(t)\sin(\omega_3(t)) + a_1\omega_2(t) + \lambda_1\omega_1(t), \\ C_2(t) = -a_2\sin(\omega_1(t))\sin(\omega_2(t)) + \lambda_2\omega_2(t), \\ C_3(t) = (\lambda_3 - k)\,\omega_3(t), \end{cases} \tag{5.4}$$

where $\lambda_1 = \lambda_2 = \lambda_3 = -2.0203 * 10^{-2}$, and the system becomes

$$\begin{cases} {}^{C}\Delta^{\varrho(t)}\omega_1(t+1-\varrho(t)) = \lambda_1\omega_1(t) + a_1\omega_2(t), \\ {}^{C}\Delta^{\varrho(t)}\omega_2(t+1-\varrho(t)) = \lambda_2\omega_2(t), \\ {}^{C}\Delta^{\varrho(t)}\omega_3(t+1-\varrho(t)) = \lambda_3\omega_3(t). \end{cases} \tag{5.5}$$

We obtain ($\rho = 2.0203 \times 10^{-2}$). With this value, all eigenvalues of the controlled system lie inside the stability region specified by Theorem 1, thereby verifying the asymptotic stability of the system.

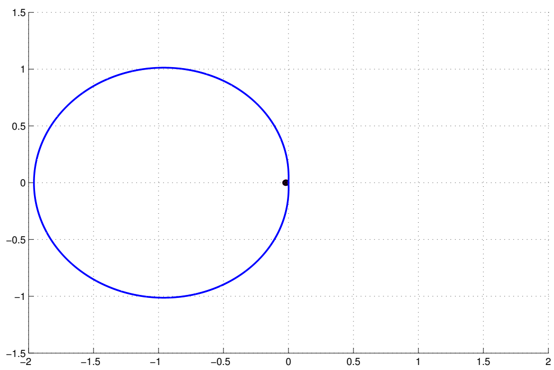

Figure 11. The location of eigenvalues and disks whose center eigenvalues and radius is ρ.

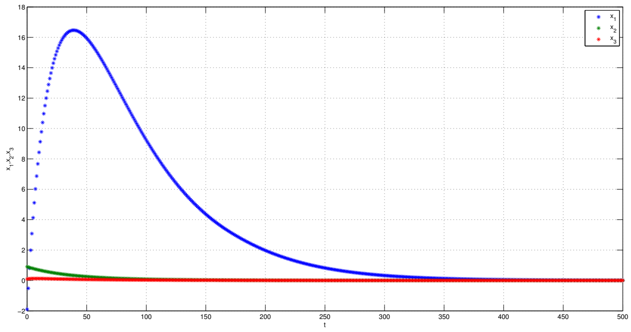

Figure 12. A numerical simulation of System 5.5 shows the stability of the system.

Figures 9–12 show the effect of the proposed control and verify the stability conditions.

6. Conclusions and future outlook

In this study, we have analyzed the dynamics of discrete fractional systems with variable orders, focusing on a particular emphasis on the 2D variable-order Lorenz discrete system and the variable fractional-order memristor map. Through rigorous theoretical analysis, we established the stability conditions for these systems and verified them using appropriate control strategies. The numerical results presented in Sections 4 and 5 confirm the effectiveness of the proposed control strategy in stabilizing variable-order discrete fractional systems, including the Lorenz-type system and the sine-based memristor map. The simulations demonstrate how variations in the fractional order influence the dynamics and how the designed control ensures asymptotic stability. These observations underline the relevance of variable-order modeling in capturing complex behaviors in nonlinear systems. Numerical simulations in MATLAB further corroborated our results, demonstrating that the systems' eigenvalues fulfill the required stability criteria, thereby ensuring controlled and predictable behavior. Our results demonstrate the effectiveness of variable fractional-order models in capturing complex dynamical phenomena and suggest their broader applicability in engineering and computational sciences. This study not only contributes to the theoretical understanding of such systems but also provides a foundation for future research on chaos control, secure communications, and memristor-based applications. Future studies should focus on incommensurate variable-order systems, examining their stability, chaotic dynamics, and control strategies. This line of research could unlock new possibilities in engineering applications, particularly in secure communications and information protection. Moreover, investigating their role in cryptographic systems and advanced control mechanisms may lead to innovative developments in cybersecurity and complex system design.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there are no conflicts of interest.

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