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Research article

Prescribed-time stabilization of uncertain nonlinear impulsive systems with multiple delays

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Abstract: Researchers have reported on prescribed-time stability (PTS) for delay systems, but they do not take into account the significant impulses phenomenon and focus on only single delay. To tackle these two aspects, we aimed to investigate PTS issues for nonlinear impulsive systems with multiple time-varying delays and uncertainties. By the Lyapunov-Krasovskii functional method, PTS criteria were established for multi-delay systems. Specifically speaking, an adaptive control strategy was proposed for multi-delay systems to achieve PTS through the integrated design of time-varying delay compensation and uncertainty handling, which could ensure system states and control inputs to precisely converge to the origin within the prescribed time frame. Furthermore, the proposed method significantly enhanced the system's robustness against time-varying delays and uncertainties, thus overcoming the limitations of traditional methods in terms of convergence time and disturbance rejection capability. Finally, a simulation result was given to verify the feasibility and effectiveness of the proposed method.

Keywords: uncertain impulsive systems; multiple delays; Lyapunov-Krasovskii functional; prescribed time stability; settling time

1. Introduction

Prescribed-time stability (PTS) has attracted considerable attention due to its capability of achieving system stabilization within a precisely specified time frame. PTS represents an advancement from finite-time stability (FtTS) and fixed-time stability (FxTS). On FtTS, systems with large initial conditions require longer adjustment and recovery periods [1]. When applied to time-sensitive systems, FtTS may lead to output fluctuations, oscillations, or loss of control, compromising system safety and reliability. To address this issue, Polyakov [2] proposed FxTS, where the convergence time is bounded and independent of initial conditions. Compared to FtTS, FxTS guarantees initialization-free stabilization time and offers a more flexible design approach [3,4]. However, although the upper bound of stabilization time of FxTS is independent of initial conditions, it remains influenced by design parameters and often suffers from time overestimation, resulting in imprecise system performance characterization. To overcome these limitations, Song et al. [5] pioneered the issue on PTS. The distinctive feature of prescribed-time control is its ability to regulate system states to the origin within any specified time [6]. In other words, regardless of initial conditions or control parameters, the stabilization time can be arbitrarily predetermined according to practical requirements. Song et al. [7] comprehensively reviewed the latest developments on PTS. For stochastic nonlinear strict-feedback systems, Li and Krstic [8,9] solved PTS by introducing time-varying controllers and developing a novel non-scaling backstepping design scheme. Zhang and Xiang [10] addressed prescribed-time optimal switching and control for switched systems. Researchers have made progress in addressing PTS for nonlinear systems with uncertainties, such as [11–13]. As a discrete control strategy, impulsive control achieves effective regulation through instantaneous state jumps and has demonstrated excellent performance [14–16]. To achieve PTS, impulsive control can significantly improve response speed and reduce energy consumption. The PTS of impulsive piecewise smooth systems was examined by Li et al. [17] via the Lyapunov theory and set-valued analysis. Feng et al. [18] developed two feedback controllers without time-varying piecewise-smooth functions to achieve prescribed-time synchronization for impulsive piecewise-smooth dynamical networks. Mapui and Mukhopadhyay [19] studied the PTS problem of time-varying impulsive nonlinear systems by rationally designing time-varying gain functions and stabilizing impulsive sequences. Zhang et al. [20] proposed mode-dependent event-triggered mechanisms to investigate the PTS problem for nonlinear impulsive switched systems. Nevertheless, the above research on PTS is relatively scarce for systems with impulses and uncertainties. Hence, enriching PTS theory for nonlinear systems remains challenging due to the comprehensive effects of impulses phenomenon and uncertainties, which is our first research motivation.

Delays are ubiquitous in engineering systems, such as networked systems [21,22], power systems [23], and multi-agent systems [24], and often lead to performance degradation or instability. Therefore, it is essential to study the stability of delay systems. Feng et al. [25] investigated general decay stability for nonlinear, non-autonomous hybrid neutral stochastic delay systems. Zeng et al. [26] achieved the exponential stability of linear time-varying delay systems by modeling it as switched systems (increasing/decreasing modes). Yang and Sun [27] proposed delay-independent/dependent finite-time control strategies for nonlinear delay systems. Xu et al. [28] achieved FxTS for high-order nonlinear stochastic delay systems. Furthermore, PTS for delay systems has been reported [29,30]. Ning et al. [29] developed PTS for delay systems using prescribed-time tuning functions and Lyapunov-Krasovskii functional methods. Ning et al. [30] proposed a prescribed-time output feedback control strategy based on a dual-gain-function reduced-order observer for nonlinear time-delay systems, achieving PTS. Although effective control strategies on PTS of delay systems have been investigated [29,30], these works focus on systems with single delay. It is noted that multi-delay phenomenon is more prevalent and technically significant in practical engineering applications, and that the stability for multi-delay systems has always been a significant topic. For instance, Hua et al. [31] achieved global PTS for uncertain multi-delay systems by constructing a dynamic state feedback controller. Targui et al. [32] proposed a chained observer design method incorporating proportional-integral compensation terms for Lipschitz multi-delay systems. Hernández-González et al. [33] proposed a robust flight observer design method based on a cascaded structure for multi-delay payload transport systems. Wang et al. [34] examined uncertain systems with multiple incommensurate delays. Wu et al. [35] proposed a feedback control law for discrete-time linear systems with both input delay and multiple state delays. Until now, research on PTS for multi-delay systems remains relatively limited and urgently requires further in-depth exploration. Given the prevalent practical significance of multi-delay characteristics, investigating PTS for such multi-delay systems constitutes an essential research issue, which forms our second research motivation.

In contrast to delay systems, impulsive delay systems (IDSs) incorporating impulse effects demonstrate significant advantages. A series of achievements have been made in the field of nonlinear IDSs. For instance, Samy et al. [36] solved the problems of global asymptotic stability and synchronization for multi-agent systems with multi-delay and impulsive perturbations. Wang et al. [37] investigated input/output-to-state FtTS of switched IDSs with two asynchronous switching phenomena. Wu and Li [38] proposed Lyapunov-based sufficient conditions on FtTS for nonlinear systems with delayed impulses. Palanisamy et al. [39] established a unified framework for FtTS analysis of discontinuous fractional-order neural network systems with delays and state-dependent delayed impulses, employing Lyapunov-Razumikhin methods and the Filippov mapping theory. Wang et al. [40] investigated FxTS of nonlinear systems with destabilizing delayed impulses, independent of delay magnitudes. Muni and George [41] investigated a class of semilinear impulsive control systems with multi-delay and established sufficient conditions for controllability. Liu et al. [42] established a novel impulsive differential inequality to prove exponential stability of highly nonlinear impulsive coupled networks with multi-delay. However, it is noteworthy that, so far, research on PTS of IDSs remains unexplored, particularly for the more challenging case of nonlinear impulsive systems with multi-delay. Hence, investigating the PTS of uncertain nonlinear impulsive systems with multi-delay holds significant theoretical innovation value, which is the third research motivation built on the two above motivations.

Inspired by the aforesaid three motivations, for the significant factors of impulse phenomenon and multi-delay, we investigate the PTS problem for a class of uncertain nonlinear impulsive systems with multiple time-varying delays. To the best of our knowledge, this is the first work on PTS of impulsive systems with multi-delay. The main work of this paper can be summarized as follows:

(i) Based on the Lyapunov-Krasovskii (L-K) functional approach, we establish and rigorously prove PTS criteria incorporating multiple time-varying delays, parameter uncertainties, and impulsive effects.

(ii) It is proven that the controller can enable the system states and control inputs to synchronously converge to the origin within the prescribed time.

(iii) Compared with other work [12], our work highlights the mixed significant factors of impulsive phenomenon and multi-delay.

The structure of this paper is as follows: In Section II, we introduce fundamental notations, definitions, and lemmas. In Section III, we present the PTS theorem for nonlinear uncertain delay impulsive systems. In Section IV, we prove that the proposed controller can achieve PTS. In Section V, we validate the theoretical results through numerical examples. In Section VI, we conclude the paper.

2. Materials and methods

Notations. Throughout this article, refer to the set of positive integers as Z^+, the set of non-negative real numbers as R^+, and the set of all real numbers as R. Denote R^n as the n-dimensional real space and $R_{\geq t_0}$ as the set of real numbers greater than or equal to t_0. $\|\cdot\|$ represents the norm of a vector. For a function, parameters are omitted. For example, $\wp$ or $\wp(\cdot)$ can stand for $\wp(t)$. For $\forall a \leq b$, $C([a, b], R^n)$ is the space of continuous functions mapping from $[a, b]$ to R^n, and Y is an open subset of $C([a, b], R^n)$ that encompasses the origin. A function $\alpha: R^+ \to R^+$ is called a class $\mathcal{K}$ function with the properties of continuity, strictly increasing, and $\alpha(0) = 0$. It is said to be a class $\mathcal{K}_\infty$ function if α is a class $\mathcal{K}$ function and $\alpha(\mathbb{s}) \to +\infty$ as $\mathbb{s} \to +\infty$. $\lambda_{max}(B)$ means the maximum eigenvalue of matrix B.

Consider a nonlinear impulsive delay system with uncertain parameter (IDSU, for short)

$$\begin{cases} \dot{z}(t) = \wp\big(t, z(t), z\big(t - b_1(t)\big), \cdots, z\big(t - b_m(t)\big), u, \xi\big), t \neq t_k, t \geq t_0 \\ z(t) = q\big(z(t^-)\big), \quad t = t_k \\ z(\mathbb{s}) = x(\mathbb{s}), \quad \mathbb{s} \in [t_0 - \epsilon, t_0] \end{cases} \tag{1}$$

where $z(t) \in R^n$ is state vector, $u \in R^r$ is control parameter vector, and $\xi \in R^c$ is an uncertain parameter vector. $\dot{z}$ denotes the upper right-hand derivative of z. $\{t_k, k \in Z^+\}$ is the set of impulse instants with $t_0 < t_1 < \cdots < t_k \to +\infty$ as $k \to +\infty$. For $\forall t \geq t_0$, $z_{b_i} = z\big(t - b_i(t)\big)$, $b_i(t): R_{\geq t_0} \to [0, \epsilon]$ represents time-varying delay; $\wp(\cdot): R_{\geq t_0} \times \underbrace{R^n \times \cdots \times R^n}_{m+1} \times R^r \times R^c \to R^n$ and $q(\cdot): R^n \to R^n$ with $\wp(t, 0, z_{b_1}, \cdots, z_{b_m}, u, \xi) = 0$, $q(0) = 0$ and other conditions hold such that, for any initial value $x(\mathbb{s})$, the solution $z(t)$ of system (1) exists and is piecewise continuous on $t \neq t_k$. $x(\mathbb{s}) \in C([t_0 - \epsilon, t_0], R^n)$ is a continuous function. Therefore, $z = 0$ is the equilibrium point of system (1). In addition, $z(t)$ of system (1) is assumed to be right-continuous with left limitation.

Definition 1 [5]: For a given constant $M_p > 0$, system (1) is said to be globally PTS, if

1) $\lim_{t \to (t_0 + M_p)^-} z(t) = 0$, and $z(t) = 0$, $\forall t \geq t_0 + M_p$.

2) For $\forall x(\mathbb{s}) \in C([t_0 - \epsilon, t_0], R^n)$, a controller $u(t)$ can be designed to ensure the asymptotic stability of system (1).

Definition 2 [29]: A continuously function $\phi(t)$ is classified as a prescribed-time adjustment (M_p-PTA) function, if $\phi(t) > 0$ and $\lim_{t \to (t_0 + M_p)^-} \int_{t_0}^{t} \phi(\zeta) d\zeta = +\infty$, $\forall t \in [t_0, t_0 + M_p)$.

Remark 1: Many functions satisfy the requirements of the M_p-PTA criteria, such as $\phi_1(t) = \frac{M_p^{a+r}}{(M_p - t)^{a+r}}$ (initialized at $t_0 = 0$ with positive integers a and r) in [9,43], $-\phi_2(t) = \frac{h}{M_p - t}$ (where $h > 0$) in [44]. The class of admissible scaling functions includes, yet extends beyond, rational forms like $\frac{a}{(M_p - t)^r}$ and transcendental variants such as $e^{\frac{o}{(M_p - t)^v}}$ for $o > 0$, $v \geq 1$ and $\frac{1}{ln(1 + M_p - t)}$, $\phi(t) = \left(1 - \frac{t}{M_p}\right)^{-\varkappa}$, $\forall \varkappa > 0$ ect.

Definition 3 [45]: Consider a locally Lipschitz continuous function $V: R^n \to R^+$. The upper-right Dini derivative V with respect to the trajectories of system (1) is given by:

$$\dot{V}(t) = \limsup_{\hbar \to 0^+} \frac{1}{\hbar}\left[V\left(z(t) + \hbar \wp(z(t))\right) - V(z(t))\right].$$

Definition 4: $V(t)$ is said to be a Lyapunov-Krasovskii functional for system (1), if it is locally Lipschitz continuous, radially unbounded, and there exist continuous differentiable functions $V_1(\cdot): R^n \to R_{\geq 0}$, $V_2(\cdot): R^m \to R_{\geq 0}$, $V_3(\cdot): R_{\geq 0} \times \underbrace{[t_0 - \epsilon, +\infty] \times \cdots \times [t_0 - \epsilon, +\infty]}_{m} \to R_{\geq 0}$ such that

$$V(\cdot) = V_1(\cdot) + V_2(\cdot) + V_3(\cdot),$$

$$\dot{V}(t) \leq -\Phi_1(t) V_1(z(t)) + \Phi_2(t), t \neq t_k,$$

$$V_1(q(z)) \leq \beta V_1(z),$$

with appropriate continuous functions $\Phi_1(t), \Phi_2(t)$, and constant β, in which $\dot{V}(t)$ is the upper right hand Dini derivative along system (1).

Essentially, Definition 4 is defined similarly to that in [46].

Lemma 1 [47]: Let $l(y) \geq 0$ be a continuous function on $[f, g)$ with a singularity at g. If $j \in (0, 1)$ exists for $\lim_{y \to g^-} (g - y)^j l(y) = c \neq \infty$, then $\int_f^g l(y)dy = \lim_{w \to g^-} \int_f^w l(y)dy < \infty$, where c is a positive constant.

Assumption 1: Function $\wp(\cdot)$ is piecewise continuous in t and locally Lipschitz continuous in z and z_{b_i}. That is, for any compact sets $\mathcal{D}_z \subset R^n$ and $\mathcal{D}_{z_{b_i}} \subset R^n$, there exists a constant $\mathcal{L}_\wp > 0$ such that, for $\forall u, \xi$

$$\begin{aligned}&\left\|\wp(t, z, z_{b_1}, \cdots, z_{b_m}, u, \xi) - \wp(t, z', z_{b_1}', \cdots, z_{b_m}', u, \xi)\right\| \\ &\leq \mathcal{L}_\wp\left(\|z - z'\| + \|z_{b_1} - z_{b_1}'\| + \cdots + \|z_{b_m} - z_{b_m}'\|\right).\end{aligned}$$

Furthermore, $q(\cdot)$ is continuous in z.

Assumption 2: Delays $b_i(t): R_{\geq t_0} \to [0, \epsilon]$ are measurable and satisfies $\forall t \geq t_0$, $0 \leq b_i(t) \leq \epsilon < \infty$, $\dot{b}_i(t) \leq \bar{b} < 1$, $i = 1, 2, \cdots, m$, with unknown nonnegative constants ϵ and $\bar{b}$.

Lemma 2 [48]: It is assumed that Assumptions 1 and 2 hold if there are locally integrable functions $D(\xi)$, $M(z)$, $N_i(z)$, and $Z_i(z_{b_i})$, and constant $E > 0$, such that

$$\left\|\wp(z, z_{b_1}, z_{b_2}, \cdots, z_{b_m}, \xi)\right\| \leq D(\xi)\left(M(z)\|z\| + \sum_{i=1}^{m} N_i(z) Z_i(z_{b_i})\|z_{b_i}\|\right)$$

and

$$\left\|q(z(t_n^-))\right\|^2 \leq E z^2(t_n^-),$$

then the unique global solution $z(t)$ exists for IDSU (1) on $t \geq t_0$.

Here, we mainly focus on the PTS issue of IDSU (1) rather than the existence of solution. Assumptions 1 and 2 and Lemma 2 are given to guarantee the wholeness of this paper. In what follows, the existence of solution of IDSU (1) will not be discussed.

3. Main results

Here, we formulate the necessary conditions ensuring the PTS of system (1). Moreover, define $\Xi \in R^m$ as an uncertain parameter vector that relates to ξ and $\bar{b}$, $\hat{\Xi}$ as its estimation, then estimation error $\tilde{\Xi} = \Xi - \hat{\Xi}$.

Lemma 3: Let $M_p > 0$, $z(t)$ and $\hat{\Xi}(t)$ are bounded for $t \in [t_0, t_0 + M_p)$, if there exist $\mathcal{K}_\infty$-class functions α_1, α_2, Lyapunov-Krasovskii functional $W(t)$and continuous differentiable functions $W_1(\cdot): R^n \to R_{\geq 0}$, $W_2(\cdot): R^m \to R_{\geq 0}$ and $W_3(\cdot): R_{\geq 0} \times \underbrace{[t_0 - \epsilon, +\infty] \times \cdots \times [t_0 - \epsilon, +\infty]}_{m} \to R_{\geq 0}$ such that:

$$W(t) = W_1(z(t)) + W_2\left(\tilde{\Xi}(t)\right) + W_3(t, t - b_1(t), \cdots, t - b_m(t)), \tag{2}$$

$$\alpha_1(\|z(t)\|) \leq W_1(z(t)), \tag{3}$$

$$\alpha_2\left(\|\tilde{\Xi}(t)\|\right) \leq W_2\left(\tilde{\Xi}(t)\right), \tag{4}$$

$$\lim_{t \to (t_0 + M_p)^-} \left((t_0 + M_p) - t\right)^j \phi(t)\varphi(t) = \bar{\varphi} \neq +\infty, \tag{5}$$

$$\dot{W}(t) \leq -\sigma\Phi(t)W_1(z(t)) + \delta\Phi(t)\psi(t), t \neq t_k, \tag{6}$$

$$W_1(q(z)) \leq \beta W_1(z), \tag{7}$$

where function $\Phi(t) = \phi(t)$, $\psi(t) = \varphi(t)$, $t \in [t_0, t_0 + M_p)$, and $\Phi(t) = \varepsilon$, $\psi(t) = 0$, $t \in [t_0 + M_p, +\infty)$, $\phi(t)$ is the M_p-PTA function, $\varphi(t)$ is positive continuous function, constants $\varepsilon > 0$, $\sigma > 0$, $\delta \geq 0$, $\bar{\varphi} \geq 0$, $j \in (0, 1)$ and $\beta \in (0, 1)$.

Proof: For $t \in [t_0, t_0 + M_p)$, when $t = t_k$, substituting (7) into (2), yield

$$\begin{aligned} W(t_n) &\leq \beta W_1(z(t_n^-)) + W_2\left(\tilde{\Xi}(t_n)\right) + W_3(t_n, t_n - b_1(t_n), \cdots, t_n - b_m(t_n)) \\ &\leq \beta W_1(z(t_n^-)) + W_2\left(\tilde{\Xi}(t_n^-)\right) + W_3(t_n^-, t_n^- - b_1(t_n^-), \cdots, t_n^- - b_m(t_n^-)) \\ &\leq W(t_n^-). \end{aligned} \tag{8}$$

When $t \neq t_k$, From (6) and $\Phi(t) > 0$ obtain

$$\dot{W}(t) \leq \delta\Phi(t)\psi(t). \, t \neq t_k$$

When $t \in [t_0, t_1)$, according to the comparison lemma, it can be calculated that

$$W(t) \leq W(t_0) + \int_{t_0}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta,$$

and

$$W(t_1^-) \leq W(t_0) + \int_{t_0}^{t_1^-} \delta\Phi(\zeta)\psi(\zeta)d\zeta\,. \tag{9}$$

Similarly, when $t \in [t_1, t_2)$, it can be obtained that

$$W(t) \leq W(t_1) + \int_{t_1}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta\,.$$

From (8) and (9), we can deduce that

$$\begin{aligned} W(t) &\leq W(t_1^-) + \int_{t_1}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta \leq W(t_0) \\ &\leq W(t_0) + \int_{t_0}^{t_1^-} \delta\Phi(\zeta)\psi(\zeta)d\zeta + \int_{t_1}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta \\ &\leq W(t_0) + \int_{t_0}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta\,. \end{aligned}$$

When $t \in [t_n, t_{n+1})$, with $t_{n+1} \leq t_0 + M_p$, by mathematical induction, one gets

$$W(t) \leq W(t_0) + \int_{t_0}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta\,.$$

Then, when $t \in \left[t_0, t_0 + M_p\right)$, it holds that,

$$W(t) \leq W(t_0) + \int_{t_0}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta\,. \tag{10}$$

Since, $\psi(t) > 0$ and $\delta \geq 0$, it follows that,

$$\int_{t_0}^{t} \delta\Phi(\zeta)\psi(\zeta)d\zeta < \int_{t_0}^{t_0+M_p} \delta\Phi(\zeta)\psi(\zeta)d\zeta\,.$$

Thus, one can derive

$$W(t) \leq W(t_0) + \int_{t_0}^{t_0+M_p} \delta\Phi(\zeta)\psi(\zeta)d\zeta\,.$$

According to Lemma 1 and (5), the improper integral $\int_{t_0}^{t_0+M_p} \delta\Phi(\zeta)\psi(\zeta)d\zeta < \infty$, i.e., there is $\Lambda > 0$ with

$$\int_{t_0}^{t_0+M_p} \delta\Phi(\zeta)\psi(\zeta)d\zeta = \Lambda.$$

From this, one can obtain

$$W(t) \leq W(t_0) + \Lambda.$$

Therefore, $W(t)$ is bounded for $t \in [t_0, t_0 + M_p)$. $W_1(z(t))$ and $W_2\left(\tilde{\Xi}(t)\right)$ are also bounded for $t \in [t_0, t_0 + M_p)$, satisfying

$$W_1(z(t)) \le W(t) \le W(t_0) + \Lambda,$$

$$W_2\left(\tilde{\Xi}(t)\right) \le W(t) \le W(t_0) + \Lambda.$$

That is, from (3) and (4)

$$\|z(t)\| \le \alpha_1^{-1}\left(W_1(z(t))\right) \le \alpha_1^{-1}(W(t_0) + \Lambda), \tag{11}$$

$$\left\|\tilde{\Xi}(t)\right\| \le \alpha_2^{-1}\left(W_2\left(\tilde{\Xi}(t)\right)\right) \le \alpha_2^{-1}(W(t_0) + \Lambda). \tag{12}$$

Thus, $z(t)$ and $\tilde{\Xi}(t)$ are bounded for $t \in [t_0, t_0 + M_p)$. Since $\tilde{\Xi} = \Xi - \hat{\Xi}$, $\hat{\Xi}$ is also bounded for $t \in [t_0, t_0 + M_p)$.

Remark 2: Since controller $u(t)$ depends on the values of $z(t)$ and $\hat{\Xi}(t)$, the boundedness of $z(t)$ and $\hat{\Xi}(t)$ ensures the boundedness of $u(t)$. Otherwise, $u(t)$ may grow infinitely.

Theorem 1: Let $M_p > 0$, system (1) is globally PTS w.r.t. $t_0 + M_p$, if all the conditions of Lemma 3 are satisfied.

Proof: To gain a comprehensive understanding, the proof is divided into two parts. The first part is to prove that when $t \in [t_0, t_0 + M_p)$, $\lim_{t \to (t_0 + M_p)^-} z(t) = 0$. The second part is to prove that for $t \in [t_0 + M_p, +\infty)$, $z(t) \equiv 0$. For given $M_p > 0$, this is an integer d with $t_d \le t_0 + M_p < t_{d+1}$.

Part 1: $t \in [t_0, t_0 + M_p)$. From (2) and (6), one obtains

$$\dot{W}(t) \le -\sigma\phi(t)W(t) + \sigma\phi(t)\bar{W}(t) + \delta\phi(t)\varphi(t), t \ne t_k$$

where $\bar{W}(t) = W_2(t) + W_3(t)$.

By applying the comparison lemma, for $t \in [t_0, t_1)$, the following inequality holds

$$W(t) \le W(t_0)e^{-\sigma\int_{t_0}^{t}\phi(\zeta)d\zeta} + e^{-\sigma\int_{t_0}^{t}\phi(\zeta)d\zeta}\int_{t_0}^{t}\left(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\right)e^{\sigma\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta\,.$$

Assuming the inequality persists for $t \in [t_{n-1}, t_n)$ with $\forall n \le d-1$, that is

$$W(t) \le W(t_0)e^{-\sigma\int_{t_0}^{t}\phi(\zeta)d\zeta} + e^{-\sigma\int_{t_0}^{t}\phi(\zeta)d\zeta}\int_{t_0}^{t}\left(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\right)e^{\sigma\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta\,.$$

Then, at $t = t_n^-$, one has

$$\begin{aligned} W(t_n^-) \le{}& W(t_0)e^{-\sigma\int_{t_0}^{t_n^-}\phi(\zeta)d\zeta} \\ &+ e^{-\sigma\int_{t_0}^{t_n^-}\phi(\zeta)d\zeta}\int_{t_0}^{t_n^-}\left(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\right)e^{\sigma\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta\,. \end{aligned} \tag{13}$$

For $t \in [t_n, t_{n+1})$, with $t_{n+1} \le t_0 + M_p$, one gets

$$W(t) \le W(t_n) e^{-\sigma \int_{t_n}^{t} \phi(\zeta) d\zeta} + e^{-\sigma \int_{t_n}^{t} \phi(\zeta) d\zeta} \int_{t_n}^{t} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_n}^{\zeta} \phi(\tau) d\tau} d\zeta \,. \tag{14}$$

By substituting (13) into (8), it derives

$$W(t_n) \le W(t_0) e^{-\sigma \int_{t_0}^{t_n} \phi(\zeta) d\zeta} + e^{-\sigma \int_{t_0}^{t_n} \phi(\zeta) d\zeta} \int_{t_0}^{t_n} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta \,. \tag{15}$$

Furthermore, by substituting (15) into (14), one obtains

$$\begin{aligned}
W(t) &\le W(t_0) e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} + e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} \int_{t_0}^{t_n} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta \\
&\quad + e^{-\sigma \int_{t_n}^{t} \phi(\zeta) d\zeta} \int_{t_n}^{t} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_n}^{\zeta} \phi(\tau) d\tau} d\zeta \\
&\le W(t_0) e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} + e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} \int_{t_0}^{t_n} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta \\
&\quad + e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} e^{\sigma \int_{t_0}^{t_n} \phi(\zeta) d\zeta} \int_{t_n}^{t} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_n}^{\zeta} \phi(\tau) d\tau} d\zeta \\
&\le W(t_0) e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} + e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} \int_{t_0}^{t_n} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta \\
&\quad + e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} \int_{t_n}^{t} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta \\
&\le W(t_0) e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} + e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} \int_{t_0}^{t} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta \,.
\end{aligned}$$

That is, for $t \in \big[t_0, t_0 + M_p\big)$, it holds that

$$W(t) \le W(t_0) e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} + e^{-\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta} \int_{t_0}^{t} \big(\sigma\phi(\zeta)\bar{W}(\zeta) + \delta\phi(\zeta)\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta \,. \tag{16}$$

By Definition 2, together with L'Hôpital's Rule, one can prove that

$$\begin{aligned}
\lim_{t \to (t_0 + M_p)^-} W(t) &= \lim_{t \to (t_0 + M_p)^-} \Big(W_1\big(z(t)\big) + \bar{W}(t)\Big) \\
&\le \lim_{t \to (t_0 + M_p)^-} \frac{\int_{t_0}^{t} \phi(\zeta)\big(\sigma\bar{W}(\zeta) + \delta\varphi(\zeta)\big) e^{\sigma \int_{t_0}^{\zeta} \phi(\tau) d\tau} d\zeta}{e^{\sigma \int_{t_0}^{t} \phi(\zeta) d\zeta}} \\
&= \lim_{t \to (t_0 + M_p)^-} \Big(\bar{W}(t) + \frac{\delta\varphi(t)}{\sigma}\Big).
\end{aligned} \tag{17}$$

According to (5) and Lemma 1, it can be known that $\int_{t_0}^{t_0 + M_p} \phi(t)\varphi(t) dt < \infty$. Next, we show that $\lim_{t \to (t_0 + M_p)^-} \varphi(t) = 0$. If this is not sure, then $\lim_{t \to (t_0 + M_p)^-} \varphi(t) \ne 0$.
Then, there exists $0 < \widetilde{M_p}$, such that, $\forall t \in \big[t_0 + \widetilde{M_p}, t_0 + M_p\big)$

$$\varphi(t) \ge \mathbb{C}, \mathbb{C} > 0.$$

Hence,

$$\int_{t_0+\widetilde{M_p}}^{t_0+M_p} \phi(t)\varphi(t)dt \geq \mathbb{C}\int_{t_0+\widetilde{M_p}}^{t_0+M_p} \phi(t)dt.$$

From $\int_{t_0}^{t_0+M_p} \phi(t)\varphi(t)dt < \infty$, it follows that $\int_{t_0+\widetilde{M_p}}^{t_0+M_p} \phi(t)\varphi(t)dt < \infty$, which implies $\int_{t_0+\widetilde{M_p}}^{t_0+M_p} \phi(t)dt < \infty$, which contradicts Definition 2. Hence, the required assertion holds.

Thus, it follows that from (17)

$$\lim_{t\to(t_0+M_p)^-} W(t) \leq \lim_{t\to(t_0+M_p)^-} \overline{W}(t).$$

Hence,

$$\lim_{t\to(t_0+M_p)^-} W_1\big(z(t)\big) = 0. \tag{18}$$

From (11), it is straightforward to conclude that

$$\lim_{t\to(t_0+M_p)^-} z(t) = 0.$$

Part 2: $t \in \big[t_0 + M_p, +\infty\big)$. When $t_0 + M_p \neq t_d$, based on the continuity of solution on $[t_d, t_{d+1})$, one has $z\big(t_0 + M_p\big) = 0$. When $t_0 + M_p = t_d$, from (1) and (18), we have $\lim_{t\to(t_0+M_p)} W_1\big(z(t)\big) = \lim_{t\to(t_0+M_p)} W_1\Big(q\big(z(t^-)\big)\Big) = \lim_{t\to(t_0+M_p)^-} W_1\Big(q\big(z(t)\big)\Big) \leq \lim_{t\to(t_0+M_p)^-} \beta W_1\big(z(t)\big) \leq 0$, then $z\big(t_0 + M_p\big) = 0$. Since the equilibrium point of system (1) is $z = 0$, from $\wp(t, 0, z_{b_1}, \cdots, z_{b_m}, u, \xi) = 0$ and $q(0) = 0$, one has $z(t) \equiv 0$ for $\forall t \in \big[t_0 + M_p, +\infty\big)$.

Therefore, combining part 1 and part 2, system (1) is globally PTS w.r.t. $t_0 + M_p$.

Remark 3: Similarly to [49], when an impulsive system reaches 0, it will not deviate from 0 even if it is subjected to the next impulsive action.

Remark 4: Compared to [29], this study highlights impulse phenomenon and extends the delay from a single term to multiple terms. The addition of impulse signals can improve the initial response speed and enhance the system's anti-interference capability. Moreover, multi-delay enables a more accurate representation of the system's dynamic behavior, thereby improving control performance and stability. Hence, our Theorem 1 represents an extension of the earlier result in [29].

Remark 5: Compared with the input/output-to-state FtTS in [50], we consider the PTS of nonlinear impulsive systems with multi-delay, which has a clearly prescribed time boundary and can accurately predict the stabilization time, thereby significantly improving the system performance and making improvements to the results in [50].

Remark 6: For PTS, we introduce uncertain parameters into impulsive systems based on the research in [45]. The introduction of uncertain parameters enables the system to effectively address internal errors and external disturbances. In other words, we extend the research of [45], enhancing the system's robustness while maintaining the prescribed-time convergence property.

Remark 7: 1) In Lemma 3, condition (2) defines Lyapunov-Krasovskii (L-K) functional $W(t)$ consisting of three components: $W_1\big(z(t)\big)$, $W_2\left(\tilde{\Xi}(t)\right)$ and $W_3\big(t, t - b_1(t), \cdots, t - b_m(t)\big)$. Here,

$W_1(z(t))$ is Lyapunov function related to current state $z(t)$, $W_2\left(\tilde{\hat{\Xi}}(t)\right)$ is Lyapunov function associated with error $\hat{\Xi}(t)$, $W_3(t, t-b_1(t), \cdots, t-b_m(t))$ represents Lyapunov function accounting for multi-delayed states $z(t-b_i(t))$. Compared with [12], L-K functional $W_3(t, t-b_1(t), \cdots, t-b_m(t))$ is added.

2) In Lemma 3, condition (3) and (4) provide bounds for $W_1(z(t))$ and $W_2\left(\tilde{\hat{\Xi}}(t)\right)$ using $\mathcal{K}_\infty$-class functions α_1 and α_2, ensuring the positive definiteness. Furthermore, this guarantees the boundedness of $z(t)$ and $\hat{\Xi}(t)$.

3) In Lemma 3, condition (5) ensures that improper integral $\int_{t_0}^{t_0+M_p} \delta\Phi(\zeta)\psi(\zeta)d\zeta < \infty$, i.e., there is $\Lambda > 0$ with $W(t) \le W(t_0) + \Lambda$, thereby guaranteeing that $W(t)$ is bounded for $t \in [t_0, t_0 + M_p)$. In contrast to the special function $\Phi(\zeta)$ in [29], $\Phi(\zeta)$ in this study is designed to be non-fixed, thereby relaxing the original constraints of [29] and enhancing the system's adaptability.

4) In Lemma 3, condition (6) guarantees global PTS while addressing both time delays and uncertainties. Specifically speaking, negative term $-\sigma\Phi(t)W_1(z(t))$ enforces convergence of system states to 0 within M_p. During $t \in [t_0, t_0 + M_p)$, $\Phi(t)$ serves as an unbounded function that drives $W(t)$ to converge rapidly as $t \to (t_0 + M_p)^-$. When $t \in [t_0 + M_p, +\infty)$, $\Phi(t)$ becomes a positive constant, ensuring the system remains at the origin. Positive term $\delta\Phi(t)\psi(t)$ acts as a disturbance compensation term, demonstrating the system's robustness against external disturbances while maintaining stability. Here, nonlinear Lyapunov inequality (6) with a positive linear term $\delta\Phi(t)\psi(t)$ has a wider application range than that only with a negative linear term in [12].

5) In Lemma 3, condition (7) describes the impulsive function that adjusts system dynamics through instantaneous state jumps. In this work, $\beta \in (0, 1)$ is designed to enforce faster convergence of system states toward the equilibrium point.

4. Application

Consider an IDSU

$$\begin{cases} \dot{z}(t) = u + P\big(z(t), z(t-b_1(t)), \cdots, z(t-b_m(t)), \xi\big), & t \ne t_k, t \ge t_0 \\ z(t) = Q\big(z(t^-)\big), & t = t_k \\ z(\mathbb{s}) = x(\mathbb{s}), & \mathbb{s} \in [t_0 - \epsilon, t_0] \end{cases} \tag{19}$$

where $z(t) \in R^n$, $u \in R^r$ and $\xi \in R^c$ are state variable, controller and uncertain parameter vector respectively; $P(\cdot): \underbrace{R^n \times \cdots \times R^n}_{m+1} \times R^c \to R^n$ and $Q(\cdot): R^n \to R^n$ are continuous with $P(0, z_{b_1}, \cdots, z_{b_m}, \xi) = 0$ and $Q(0) = 0$. $\dot{z}(t)$, $b_i(t)$, t_k, and $x(\mathbb{s})$ are defined in system (1).

In this section, we apply our Theorem 1 in Section 3 to prescribed-time stabilize IDSU (19) with uncertain parameters. Before the main assertion, we first give two assumptions below.

Assumption 3: Function $P(\cdot)$ of system (19) satisfies

$$\left\|P(z, z_{b_1}, z_{b_2}, \cdots, z_{b_m}, \xi)\right\| \le X(\xi)\left(\Upsilon(z)\|z\| + \sum_{i=1}^{m} H_i(z)\varrho_i(z_{b_i})\left\|z_{b_i}\right\|\right),$$

where $X(\xi): R^c \to R_{\ge 0}$, $\Upsilon(z): R^n \to R_{\ge 0}$, $H_i(z): R^n \to R_{\ge 0}$, with $H_i(0) = 0$ and $\varrho_i(z): R^n \to R_{\ge t_0}$ are continuous, $i = 1, 2, \cdots, m$.

Assumption 4: Function $Q(\cdot)$ of system (19) satisfies

$$\left\|Q\left(z(t_n^-)\right)\right\|^2 \leq \beta z^2(t_n^-).$$

For uncertain parameters $\bar{b}$, define

$$\Xi = \max\left\{X(\xi), X^2(\xi), \frac{1}{1-\bar{b}}\right\} \in R.$$

Theorem 2: For given $t_0 + M_p > 0$, under Assumptions 3 and 4, if the control input $u(t)$ and adaptive law $\dot{\hat{\Xi}}$ are given by

$$u(t) = -\Phi(t)z - \hat{\Xi}\Upsilon(z)z - \sum_{i=1}^{m}\frac{\hat{\Xi}}{2}H_i^2(z)z - \sum_{i=1}^{m}\frac{\hat{\Xi}}{2}\varrho_i^2(z)z + \Phi(t)\psi(t)\frac{1}{\|z^T\|+\omega}, \tag{20}$$

$$\dot{\hat{\Xi}} = \Upsilon(z)z^Tz + \sum_{i=1}^{m}\frac{1}{2}H_i^2(z)z^Tz + \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2(z)z^Tz, \tag{21}$$

where functions $\Phi(t) = \phi(t)$, $\psi(t) = \varphi(t)$, $t \in [t_0, t_0 + M_p)$, and $\Phi(t) = \varepsilon$, $\psi(t) = 0$, $t \in [t_0 + M_p, +\infty)$, constants $\omega > 0$, $\varepsilon > 0$, and $\phi(t)$ is the M_p-PTA function satisfying condition (5), and for any continuous function $K(t)$,

$$\lim_{t\to(t_0+M_p)^-}\phi(t)e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta} = 0, \tag{22}$$

$$\lim_{t\to(t_0+M_p)^-}\phi(t)e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta}\int_{t_0}^{t}K(\zeta)e^{\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta = \lim_{t\to(t_0+M_p)^-}-K(t), \tag{23}$$

then system (19) is globally PTS w.r.t. $t_0 + M_p$.

Proof. The proof procedure consists of two parts: $t \in [t_0, t_0 + M_p)$ and $t \in [t_0 + M_p, +\infty)$.

Part 1: $t \in [t_0, t_0 + M_p)$. Suppose

$$W_1(z(t)) = \frac{1}{2}z^T(t)z(t).$$

Then, condition (3) holds with $\alpha_1(\|z(t)\|) = \frac{1}{2}\|z(t)\|^2$, and

$$\dot{W}_1 = z^T\left(u + P\left(z, z_{b_1}, \cdots, z_{b_m}, \xi\right)\right).$$

From Assumption 3 and Young's inequality, we obtain

$$\begin{aligned} z^TP\left(z, z_{b_1}, \cdots, z_{b_m}, \xi\right) &\leq X(\xi)\Upsilon(z)z^Tz + \sum_{i=1}^{m}\frac{1}{2}X^2(\xi)H_i^2(z)z^Tz + \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2\left(z_{b_i}\right)z_{b_i}^Tz_{b_i} \\ &\leq \Xi\Upsilon(z)z^Tz + \sum_{i=1}^{m}\frac{\Xi}{2}H_i^2(z)z^Tz + \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2\left(z_{b_i}\right)z_{b_i}^Tz_{b_i}. \end{aligned} \tag{24}$$

By (20) and (24), we derive

$$\dot{W}_1 \le z^T\left(-\phi(t)z - \hat{\Xi}\Upsilon(z)z - \sum_{i=1}^{m}\frac{\hat{\Xi}}{2}H_i^2(z)z - \sum_{i=1}^{m}\frac{\hat{\Xi}}{2}\varrho_i^2(z)z + \phi(t)\varphi(t)\frac{1}{\|z^T\|+\omega}\right)$$
$$+\Xi\Upsilon(z)z^Tz + \sum_{i=1}^{m}\frac{\Xi}{2}H_i^2(z)z^Tz + \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2(z_{b_i}){z_{b_i}}^T z_{b_i}.$$

According to $\tilde{\Xi}(t) = \Xi - \hat{\Xi}(t)$, we further simplify the expression as

$$\dot{W}_1 \le z^T\left(-\phi(t)z + \tilde{\Xi}\Upsilon(z)z + \sum_{i=1}^{m}\frac{\tilde{\Xi}}{2}H_i^2(z)z - \sum_{i=1}^{m}\frac{\Xi}{2}\varrho_i^2(z)z + \phi(t)\varphi(t)\frac{1}{\|z^T\|+\omega}\right)$$
$$+\sum_{i=1}^{m}\frac{1}{2}\varrho_i^2(z_{b_i}){z_{b_i}}^T z_{b_i}.$$

Suppose

$$W_2\left(\tilde{\Xi}(t)\right) = \frac{1}{2}\tilde{\Xi}^2(t).$$

Then, condition (4) holds with $\alpha_2\left(\|\tilde{\Xi}(\mathrm{t})\|\right) = \frac{1}{2}\left\|\tilde{\Xi}(\mathrm{t})\right\|^2$, and

$$\dot{W}_2 = -\tilde{\Xi}\dot{\hat{\Xi}}.$$

Suppose

$$W_3\big(t, t-b_1(t), \cdots, t-b_m(t)\big) = \sum_{i=1}^{m}\frac{1}{2(1-\bar{b})}\int_{t-b_i(t)}^{t}\varrho_i^2\big(z(\zeta)\big)z^T(\zeta)z(\zeta)d\zeta.$$

Then,

$$\dot{W}_3 = \sum_{i=1}^{m}\frac{1}{2(1-\bar{b})}\varrho_i^2(z)z^Tz - \sum_{i=1}^{m}\frac{1-\dot{b}_i(t)}{2(1-\bar{b})}\varrho_i^2(z_{b_i}){z_{b_i}}^T z_{b_i}$$
$$\le \sum_{i=1}^{m}\frac{\Xi}{2}\varrho_i^2(z)z^Tz - \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2(z_{b_i}){z_{b_i}}^T z_{b_i}.$$

Further let

$$W(t) = W_1\big(z(t)\big) + W_2\left(\tilde{\Xi}(t)\right) + W_3\big(t, t-b_1(t), \cdots, t-b_m(t)\big).$$

Then,

$$\begin{aligned}\dot{W}(t) \le\ & z^T\left(-\phi(t)z + \tilde{\Xi}\Upsilon(z)z + \sum_{i=1}^{m}\frac{\tilde{\Xi}}{2}H_i^2(z)z - \sum_{i=1}^{m}\frac{\hat{\Xi}}{2}\varrho_i^2(z)z + \phi(t)\varphi(t)\frac{1}{\|z^T\|+\omega}\right)\\ & + \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2(z_{b_i})z_{b_i}{}^T z_{b_i} - \tilde{\Xi}\dot{\hat{\Xi}} + \sum_{i=1}^{m}\frac{\Xi}{2}\varrho_i^2(z)z^Tz - \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2(z_{b_i})z_{b_i}{}^T z_{b_i}\\ \le\ & z^T\left(-\phi(t)z + \tilde{\Xi}\Upsilon(z)z + \sum_{i=1}^{m}\frac{\tilde{\Xi}}{2}H_i^2(z)z + \sum_{i=1}^{m}\frac{\tilde{\Xi}}{2}\varrho_i^2(z)z + \phi(t)\varphi(t)\frac{1}{\|z^T\|+\omega}\right) - \tilde{\Xi}\dot{\hat{\Xi}} \qquad (25)\\ \le\ & z^T\left(-\phi(t)z + \phi(t)\varphi(t)\frac{1}{\|z^T\|+\omega}\right)\\ & -\tilde{\Xi}\left(\dot{\hat{\Xi}} - \Upsilon(z)z^Tz - \sum_{i=1}^{m}\frac{1}{2}H_i^2(z)z^Tz - \sum_{i=1}^{m}\frac{1}{2}\varrho_i^2(z)z^Tz\right).\end{aligned}$$

Substituting (21) into (25), we obtain

$$\begin{aligned}\dot{W}(t) &\le z^T\left(-\phi(t)z + \phi(t)\varphi(t)\frac{1}{\|z^T\|+\omega}\right)\\ &\le -2\phi(t)W_1(z(t)) + \phi(t)\varphi(t).\end{aligned} \tag{26}$$

On the other hand, in accordance to system (19), one can deduce that under feedback control $u(t)$ and Assumption 4, for any $z(t) \in R^n$,

$$\begin{aligned}W_1\left(Q(z(t_n^-))\right) &= \frac{1}{2}\left(Q(z(t_n^-))\right)^T Q(z(t_n^-))\\ &\le \frac{1}{2}\left\|Q(z(t_n^-))\right\|^2\\ &\le \frac{1}{2}\beta z^2(t_n^-)\\ &= \beta W_1(z(t_n^-)).\end{aligned} \tag{27}$$

Thus, applying Theorem 1, it follows that $z(t)$ and $\hat{\Xi}(t)$ are bounded on $t \in [t_0, t_0 + M_p)$, and $\lim_{t\to(t_0+M_p)^-} W_1(z(t)) = 0$, $\lim_{t\to(t_0+M_p)^-} z(t) = 0$.

It is usually required that the control input $u(t)$ cannot tend to infinity, as actuators in practical applications have limited energy supply and cannot generate infinite power to drive the system. Moreover, to ensure that system stabilizes within the desired time, $u(t)$ must converge to zero as $t \to (t_0 + M_p)^-$. Therefore, when designing the control strategy, it is essential to appropriately select M_p-PTA function to ensure that $u(t)$ remains bounded and ultimately converges to zero.

Next, we prove $u(t)$ is bounded on $[t_0, t_0 + M_p)$. It follows that if $\lim_{t\to(t_0+M_p)^-} u(t) = 0$, then $u(t)$ is bounded on $(t_0 + M_p{}', t_0 + M_p)$ for $M_p{}' \in (t_0, M_p)$. From the definition of $u(t)$, it is known that $u(t)$ is bounded on $[t_0, t_0 + M_p{}']$. Therefore, it suffices to prove $\lim_{t\to(t_0+M_p)^-} u(t) = 0$.

Substituting (20) into (19), we obtain

$$\dot{z}(t) = -\phi(t)z + \Omega(t),\ t \ne t_k \tag{28}$$

where $\Omega(t) = -\hat{\Xi}\Upsilon(z)z - \sum_{i=1}^{m}\frac{\hat{\Xi}}{2}H_i^2(z)z - \sum_{i=1}^{m}\frac{\hat{\Xi}}{2}\varrho_i^2(z)z + \phi(t)\varphi(t)\frac{1}{\|z^T\|+\omega} + P(z, z_{b_1}, \cdots, z_{b_m}, \xi)$.

Since $\lim_{t\to(t_0+M_p)^-} z(t) = 0$, $\hat{\Xi}$ is bounded, and $\Upsilon(z)$, $H_i(z)$, $\varrho_i(z)$ and $P(\cdot)$ are also bounded, we can deduce that $\lim_{t\to(t_0+M_p)^-} \Omega(t) = 0$.

According to Lemma 1, from (28), similarly to (16), we obtain

$$z(t)=z(t_0)e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta}+e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta}\int_{t_0}^{t}\Omega(\zeta)e^{\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta\,.$$

Additionally, from (22) and (23), we obtain

$$\begin{aligned}\lim_{t\to(t_0+M_p)^-}u(t)&=\lim_{t\to(t_0+M_p)^-}-\phi(t)z(t)\\&=\lim_{t\to(t_0+M_p)^-}-\phi(t)z(t_0)e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta}-\phi(t)e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta}\int_{t_0}^{t}\Omega(\zeta)e^{\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta\\&=\lim_{t\to(t_0+M_p)^-}-\Omega(t)\\&=0.\end{aligned}\tag{29}$$

Here, $u(t)$ converge to zero as $t\to\left(t_0+M_p\right)^-$.Therefore, $u(t)$ is bounded on $t\in\left[t_0,\ t_0+M_p\right)$.

Part 2: $t\in\left[M_p,\ +\infty\right)$. The control input $u(t)$ is

$$u(t)=-\varepsilon z-\hat{\Xi}\Upsilon(z)z-\sum_{i=1}^{m}\frac{\hat{\Xi}}{2}H_i^2(z)z-\sum_{i=1}^{m}\frac{\hat{\Xi}}{2}\varrho_i^2(z)z\,.$$

Similarly to (26), it is easy to calculate

$$\dot{W}(t)\leq-2\varepsilon W_1\big(z(t)\big).\ t\neq t_k$$

By Theorem 1, we have $z(t)\equiv0,\ \forall t\in\left[t_0+M_p,\ +\infty\right)$. From (29), one can obtain that $u\left(t_0+M_p\right)=0$. Further, we have $u(t)=0,\ \forall t\in\left[t_0+M_p,\ +\infty\right)$. Hence, controller $u(t)$ is bounded on $[t_0,\ +\infty)$ and converges to zero as $t\to\left(t_0+M_p\right)^-$.

Therefore, combining parts 1 and 2, system (19) is globally PTS.

Corollary 1: Assume all conditions of Theorem 2 hold except for condition (22) and (23), and (5) is replaced by

$$\phi(t)=\frac{\mathscr{v}}{\left(t_0+M_p-t\right)^{\mathscr{w}}},\tag{30}$$

$$\varphi(t)=\frac{1}{\left(\phi(t)\right)^{\varpi}},\tag{31}$$

where constants $\mathscr{v}>0,\ \mathscr{w}\geq2,\ \varpi$ satisfy $\mathscr{w}(\varpi-1)+j>0$ for a $j\in(0,1)$, under control input (20) and adaptive law (21), system (19) is globally PTS w.r.t. t_0+M_p.

Proof. From (30) and (31), by L'Hôpital's rule,

$$\lim_{t\to(t_0+M_p)^-}\phi(t)e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta} = \lim_{t\to(t_0+M_p)^-}\frac{\frac{\mathcal{v}}{(t_0+M_p-t)^{\mathcal{w}}}}{e^{\int_{t_0}^{t}\frac{\mathcal{v}}{(t_0+M_p-\zeta)^{\mathcal{w}}}d\zeta}}$$

$$= \lim_{t\to(t_0+M_p)^-}\frac{\frac{\mathcal{v}\mathcal{w}}{(t_0+M_p-t)^{\mathcal{w}+1}}}{\frac{\mathcal{v}}{(t_0+M_p-t)^{\mathcal{w}}}e^{\int_{t_0}^{t}\frac{\mathcal{v}}{(t_0+M_p-\zeta)^{\mathcal{w}}}d\zeta}}$$

$$= \lim_{t\to(t_0+M_p)^-}\frac{\frac{\mathcal{w}}{t_0+M_p-t}}{e^{\int_{t_0}^{t}\frac{\mathcal{v}}{(t_0+M_p-\zeta)^{\mathcal{w}}}d\zeta}}$$

$$= \lim_{t\to(t_0+M_p)^-}\frac{\mathcal{w}}{\frac{\mathcal{v}}{(t_0+M_p-t)^{\mathcal{w}-2}}e^{\int_{t_0}^{t}\frac{\mathcal{v}}{(t_0+M_p-\zeta)^{\mathcal{w}}}d\zeta}}$$

$$= 0,$$

which implies that condition (22) holds. Furthermore,

$$\lim_{t\to(t_0+M_p)^-}\phi(t)e^{-\int_{t_0}^{t}\phi(\zeta)d\zeta}\int_{t_0}^{t}K(\zeta)e^{\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta$$

$$= \lim_{t\to(t_0+M_p)^-}\frac{\int_{t_0}^{t}K(\zeta)e^{\int_{t_0}^{\zeta}\phi(\tau)d\tau}d\zeta}{\frac{e^{\int_{t_0}^{t}\phi(\zeta)d\zeta}}{\phi(t)}}$$

$$= \lim_{t\to(t_0+M_p)^-}\frac{K(t)e^{\int_{t_0}^{t}\phi(\zeta)d\zeta}}{\frac{\phi^2(t)e^{\int_{t_0}^{t}\phi(\zeta)d\zeta}+e^{\int_{t_0}^{t}\phi(\zeta)d\zeta}\phi'(t)}{\phi^2(t)}}$$

$$= \lim_{t\to(t_0+M_p)^-}\frac{K(t)}{\left(1-\frac{\mathcal{v}\mathcal{w}}{(t_0+M_p-t)^{\mathcal{w}+1}}\cdot\frac{(t_0+M_p-t)^{2\mathcal{w}}}{\mathcal{v}^2}\right)}$$

$$= \lim_{t\to(t_0+M_p)^-}\frac{K(t)}{\left(1-\frac{\mathcal{w}(t_0+M_p-t)^{\mathcal{w}-1}}{\mathcal{v}}\right)}$$

$$= \lim_{t\to(t_0+M_p)^-}K(t),$$

which implies that condition (23) holds. Moreover,

$$\lim_{t\to(t_0+M_p)^-}\left(\left(t_0+M_p\right)-t\right)^j\phi(t)\varphi(t)=\lim_{t\to(t_0+M_p)^-}\left(\left(t_0+M_p\right)-t\right)^j\phi(t)\frac{1}{\left(\phi(t)\right)^{\varpi}}$$

$$=\lim_{t\to(t_0+M_p)^-}\left(\left(t_0+M_p\right)-t\right)^j\left(\frac{\vartheta}{\left(t_0+M_p-t\right)^{w}}\right)^{1-\varpi}$$

$$=\lim_{t\to(t_0+M_p)^-}\frac{\left(t_0+M_p-t\right)^{w(\varpi-1)+j}}{(\vartheta)^{\varpi-1}},$$

which implies that condition (5) holds.

Thus, by Theorem 2, system (19) is globally PTS w.r.t. t_0+M_p.

Remark 8: The design motivation of controller $u(t)$ is primarily to ensure that the system state converges to zero within a preset time. Term $-\Phi(t)z$ provides a time-varying gain through time adjustment function $\Phi(t)$, establishing a time-constrained framework for the overall control strategy to guarantee rapid convergence of system. Term $-\hat{\Xi}\Upsilon(z)z$ compensates for the influence of nonlinear uncertainty $X(\xi)\Upsilon(z)z$. Term $-\sum_{i=1}^{m}\frac{\hat{\Xi}}{2}H_i^2(z)z$ compensates for the effect of nonlinear uncertainty $\sum_{i=1}^{m}\frac{1}{2}X^2(\xi)H_i^2(z)z$. Term $-\sum_{i=1}^{m}\frac{\hat{\Xi}}{2}\varrho_i^2(z)z$ compensates for time-delay term $\sum_{i=1}^{m}\frac{1}{2}\varrho_i^2\left(z_{b_i}\right)z_{b_i}$. These compensation terms can offset the effects caused by multi-delay of system (19), effectively improving the dynamic response performance of system. The dynamic regulation term $\Phi(t)\psi(t)\frac{1}{\|z^T\|+\omega}$ is derived from $\delta\Phi(t)\psi(t)$ in the L-K functional. Compared with the controller in [31], our proposed controller (20) incorporates a dynamic regulation term that enables real-time adjustment based on system states to enhance robustness, along with multiple nonlinear compensation terms to improve uncertainty handling adaptability.

Remark 9: This paper and [30] are extensions of [29]. The innovation of this paper is primarily reflected in the introduction of multi-delay and impulse phenomena into system model, while the main contribution of [29] lies in its skillful integration of dynamic gain and time-varying gain functions to simplify the output feedback controller design through a non-scaling framework. Although our paper and [29] have their distinctive features, the innovative controller design methodology in [29] is particularly worthy to us.

Remark 10: In this study, we investigate a class of nonlinear IDSUs by incorporating impulsive phenomena and employing a single time-varying gain control strategy. While the dual time-varying gain approach in [31] demonstrates more robust handling ability of uncertainties through dynamic gain adjustment, the novelty of our work lies in integrating impulsive phenomena into the research framework. It should be noted that if the impulsive element is removed from consideration, our control method may show inferior performance compared to [31] when dealing with system uncertainties. In future research, we plan to introduce the dual time-varying gain strategy to strengthen system's robustness and adaptability in complex environments.

Here, an example is presented to verify the effectiveness of the proposed results.

5. Simulation results

Here, an example is presented to verify the effectiveness of the proposed results. Consider an IDSU

$$\begin{cases} \dot{z}(t) = u(t) + P\big(z(t), z\big(t - b_1(t)\big), z\big(t - b_2(t)\big), \xi\big), & t \neq t_k, t \geq 0 \\ z(t) = Q\big(z(t^-)\big), & t = t_k \\ z(\mathbb{s}) = x(\mathbb{s}), & \mathbb{s} \in [-\epsilon, 0] \end{cases} \tag{32}$$

with $P\big(z, z_{b_1}, z_{b_2}, \xi\big) = \xi_1 z^2 + \sum_{i=1}^{2} \xi_{i+1}|z|\big(z_{b_i}\big)^2$, $Q\big(z(t^-)\big) = Az(t^-)$, $A = 0.6$, $t_k = 0.1k$, $k = 1, 2 \cdots$, $b_1(t) = 0.1(1 + \sin(t))$, and $b_2(t) = 0.4(1 + \sin(0.5t))$.

Suppose uncertain parameters $\xi_1 = 0.8$, $\xi_2 = 0.8$, and $\xi_3 = 1.0$, with Assumption 3 holding for $X(\xi) = \sum_{i=1}^{2}|\xi_i|$, $\Upsilon\big(z(t)\big) = |z(t)|$, $H_i\big(z(t)\big) = |z(t)|$ and $\varrho_i\left(z_{b_i}(t)\right) = \left|z_{b_i}(t)\right|$. Thus, $b_1(t) \leq 0.2$, $b_2(t) \leq 0.8$ and its derivatives $\dot{b}_1(t) = 0.1\cos(t) \leq \bar{b} = 0.2$, $\dot{b}_2(t) = 0.2\cos(t) \leq \bar{b} = 0.2$. We set initial value $x(\mathbb{s}) = 2$, $\mathbb{s} \in [-0.8, 0]$, and $M_p = 2, 3, 5$ as the desired prescribed times. Without controller implementation, system (32) is unstable, as illustrated in Figure 1.

To achieve PTS of system (32), the controller $u(t)$ and adaptive law $\dot{\hat{\Xi}}$ are selected as

$$u(t) = -\Phi(t)z(t) - \hat{\Xi}|z(t)|z(t) - 2\hat{\Xi}|z(t)|^2 z(t) + \Phi(t)\psi(t)\frac{1}{|z(t)| + \omega}, \tag{33}$$

$$\dot{\hat{\Xi}} = |z(t)|z^2(t) + 2|z(t)|^2 z^2(t), \tag{34}$$

with function $\Phi(t) = \phi(t)$, $\psi(t) = \varphi(t)$, $t \in \big[0, M_p\big)$ and $\Phi(t) = \varepsilon$, $\psi(t) = 0$, $t \in \big[M_p, +\infty\big)$, $\omega = 0.05$, where $\phi(t) = \frac{\mathcal{v}}{(M_p - t)^{\mathcal{w}}}$, $\varphi(t) = \frac{1}{(\phi(t))^{\varpi}}$ and constants $\varepsilon = 1$, $\mathcal{v} = 10$, $\mathcal{w} = 2$, and $\varpi = 1.5$.

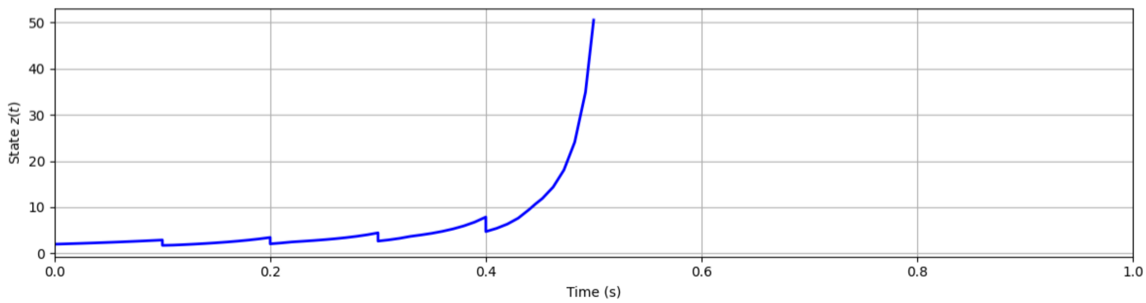

Figure 1. Uncontrolled state response $\boldsymbol{z(t)}$ of system (32).

With controller (33) and adaptive law (34), system (32) achieves stabilization before $M_p = 2, 3, 5$, shown in Figure 2. Furthermore, Figures 3–5 give the responses of controller (33), parameter estimation $\hat{\Xi}$, and parameter estimation error $\tilde{\Xi}$, respectively. Figure 2 labels the exact convergence time, and Figure 3 indicates the positive and negative control peaks.

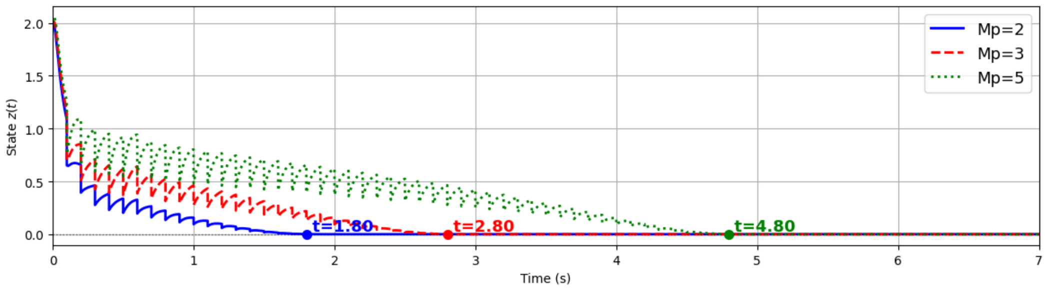

Figure 2. Responses of $\boldsymbol{z(t)}$ of system (32) under different $\boldsymbol{M_p}$ and controller with $\boldsymbol{\upsilon = 10}$, $\boldsymbol{w = 2}$, and $\boldsymbol{\varpi = 1.5}$.

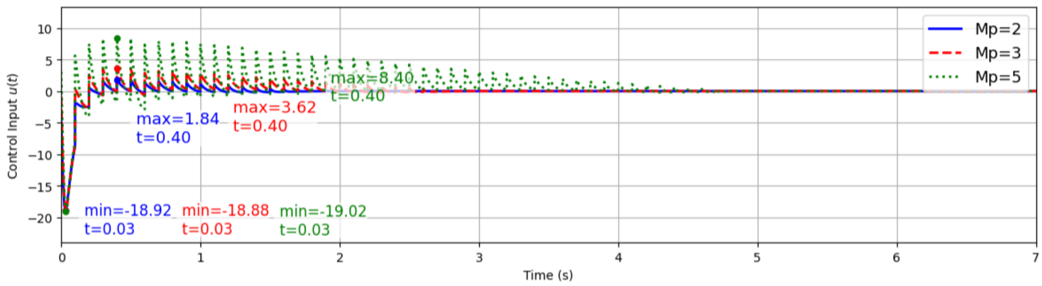

Figure 3. Responses of $\boldsymbol{u(t)}$ under different $\boldsymbol{M_p}$ with $\boldsymbol{\upsilon = 10}$, $\boldsymbol{w = 2}$, and $\boldsymbol{\varpi = 1.5}$.

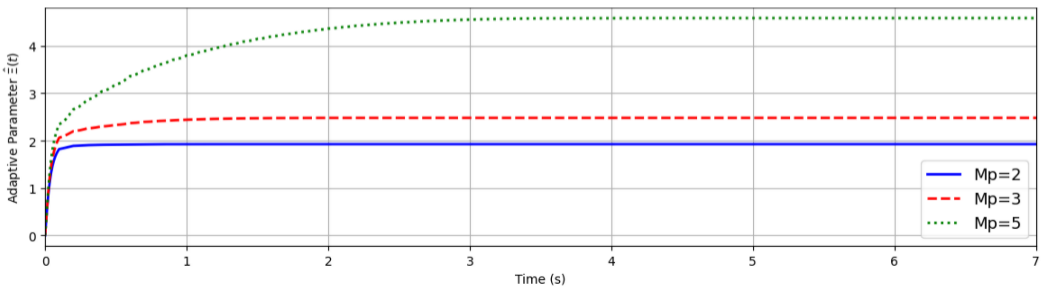

Figure 4. Responses of $\widehat{\boldsymbol{\Xi}}$ under different $\boldsymbol{M_p}$.

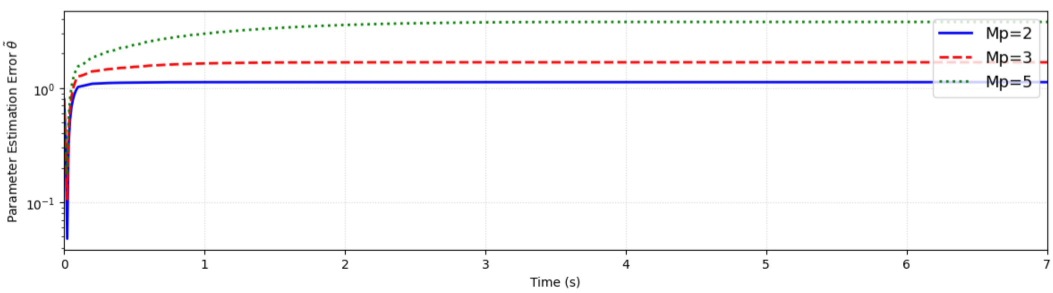

Figure 5. Responses of $\widetilde{\boldsymbol{\Xi}}$ under different $\boldsymbol{M_p}$.

When only changing original $\mathscr{v}$, $\mathscr{w}$, and ϖ to $\mathscr{v} = 5$, $\mathscr{w} = 3$, and $\varpi = 1.2$ while keeping other conditions unchanged, the response of system (32) is shown in Figure 6. From this, we can see that although system (32) converges to zero within M_p, and that the increase of $\mathscr{w}$, the decrease of $\mathscr{v}$, and ϖ result in a slower convergence rate.

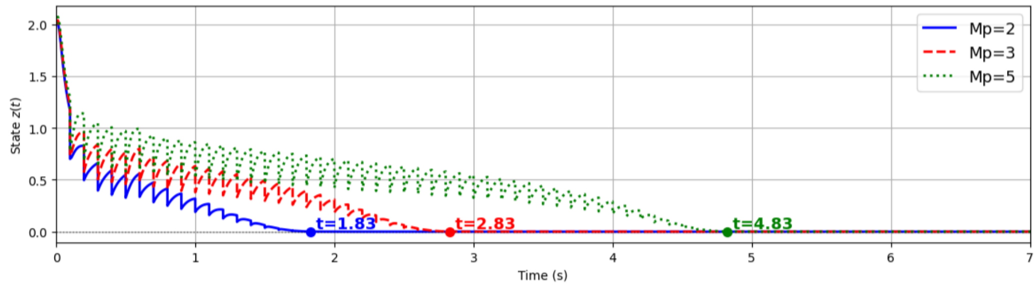

Figure 6. Responses of $\boldsymbol{z(t)}$ of system (32) under different $\boldsymbol{M_p}$ and controller (33) with $\boldsymbol{\mathscr{v} = 5}$, $\boldsymbol{\mathscr{w} = 3}$, and $\boldsymbol{\varpi = 1.2}$.

When the uncertain parameters are increased to $\xi_1 = 2$, $\xi_2 = 3$, and $\xi_3 = 4$ while keeping other conditions unchanged, the response of system (32) is shown in Figure 7. It can be seen that despite the increase on uncertain parameters, system (32) can converge to zero within M_p.

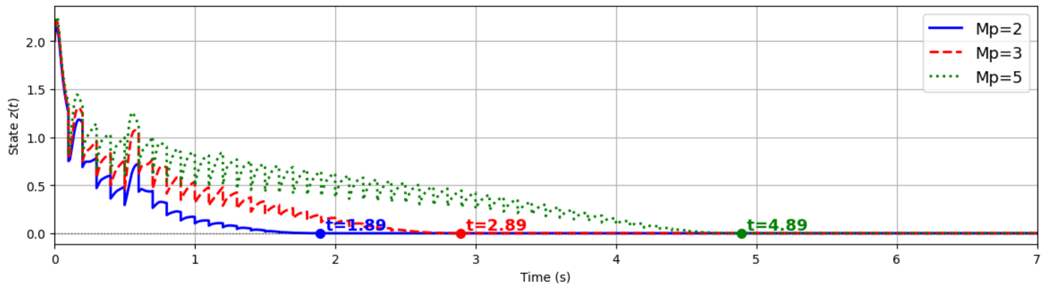

Figure 7. Responses of $\boldsymbol{z(t)}$ of system (32) under different $\boldsymbol{M_p}$ and $\boldsymbol{P(\cdot)}$ with $\boldsymbol{\xi_1 = 2}$, $\boldsymbol{\xi_2 = 3}$, and $\boldsymbol{\xi_3 = 4}$.

6. Conclusions

In this paper, we study the PTS for an IDSU, where the convergence time can be independent of the system's initial values, and the control parameters can be arbitrarily specified. Based on the L-K functional method and a PTA function, an adaptive feedback controller is designed, and sufficient criteria are established to ensure that the system states and control inputs converge within the preset time. Future research can be extended to the non-differentiable time-delay systems and dual time varying gain control strategies.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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