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Mathematical and sociological investigation of two-region migration model in bigeometric calculus

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Abstract: The two-region migration model is an important tool for analyzing population movements and evaluating their economic, social, and environmental impacts. This model mathematically examines the reasons and consequences of individuals that migrate from one region to another, thereby shedding light on issues such as labor force distribution, income differences, urbanization dynamics, and regional development. This migration model, which has a significant impact on classical analyses, is defined in bigeometric calculus, and solved by the bigeometric Laplace transform. In addition, the numerical patterns and curves presented by mathematical modeling in these two migration analyses are interpreted within the framework of sociological theories, thus demonstrating that the phenomenon of migration is not only quantitative but is also too complex to be limited to socially constructed numerical data and is closely linked to the historical process.

Keywords: multiplicative calculus; two region migration model; migration theories

1. Introduction

Migration is a phenomenon as old as human history. A series of demographic, economic, sociocultural, and psychological problems affect the nature, pattern, and direction of voluntary human migration, while forced migrations occur as a result of civil war, political and ethnic persecution, famine, and environmental disasters. Considering the reasons for this spatial mobility, migration is a multifaceted term that includes all types of voluntary and forced movements of a population [1]. Migration can be defined as a form of spatial mobility that an individual carries out from the region of origin where they have a certain minimum residence period to a destination where they aim to reside for a similar minimum time period. In line with this definition, migration is distinguished from other types of spatial mobility such as daily commuting and tourism [2]. Increasing social and economic inequalities and resulting unfulfilled life expectations cause millions, even billions, of people to migrate around the world. The reasons why people migrate have been theorized and studied for decades, and scientific literature has identified several fundamental dimensions of the reasons for migration, including economic, political, social, cultural, demographic, and ecological factors [3]. When we look at the diversity of definitions, migration is a very complex phenomenon with many dimensions. Immigrants can move within their own country and between countries. This spatial mobility can be short-term, permanent, forced, or voluntary. Migration is not static, on the contrary, it is a widespread and growing phenomenon. People tend to migrate from places where there is war, where economic opportunities are unequally distributed across borders, or where there are great inequalities in the worldwide living standards [4]. Migration and types of migration, which have been classified in numerous ways, are explained in this study by taking Petersen's classification into account. Petersen sought the basic reasons that underlie the push-pull factors related to migration. He emphasized that a phenomenon that constitutes a push factor for migration over a certain period of time can turn into a pull factor after some time. Considering individual and class differences, Peterson proposed five types of migration by taking the push-and-pull factors of the migration phenomenon. These types of migrations are primitive, forced, directed, free, and mass [5].

• **Primitive migration** is a migration that occurs as a result of an ecological push. This type of migration expresses a movement that results from the inability of humans to cope with natural forces, and the activating factor is ecological pressure.

• **In forced migrations**, the activating factor is the state or a functionally equivalent social institution. Forced migration represents the movement of a part of society by the power that has power in a country.

• **Directed migration** is when a community has the initiative to migrate. Although it is similar to forced migration, it differs from it in that individuals can use their own initiative to move.

• **In free migration**, there is no compelling situation or driving force applied to individuals, communities, and societies. In free migration, the migrant makes his/her own migration decision in a definite and determined manner and acts accordingly. This type of migration does not describe large mass migrations, but rather personal migrations that proceed with individual preferences.

• As the most obvious and distinguishing feature from other migrations, **mass migration** represents a collective phenomenon. This type of migration expresses a situation that progresses in parallel with technological development. Petersen states that mass migration evolved with the development of transportation routes and opportunities in the world. In other words, mass migration represents the migration of a large number of people from one place to another with the wide opportunities provided by transportation as a result of technological developments.

In this study, migration will be discussed by a mathematical model and mathematically compared with two different calculi. Many migration models have been developed in the literature. Two-region migration model is one of these mathematical methods to understand dynamics of migration. Then, the developed model will be evaluated based on theoretical frameworks deemed appropriate for a sociological understanding. In evaluating the two-region migration model within a sociological context, both the neoclassical migration theory, which explains individual decision-making processes based on economic considerations, and the network theory, which grounds migration in its reproduction through social networks, are considered within a sociological framework to analyze the multidimensional nature of migration.

According to the neoclassical migration theory, factors that drive migration are geographical differences in the labor supply and demand and wage differences between labor-rich and capital-rich countries. In this respect, the central argument of the neoclassical approach focuses on wages. This theory predicts that there is a linear relationship between wage differences and migration flows under the assumption of full employment. In extended neoclassical models, the determinant of the migration decision is the expected returns rather than current earnings, while the key variable is earnings based on employment probability [6]. In this context, the neoclassical theory focuses on the migration costs and considers migration as a decision taken by individuals to maximize their income [7]. In line with the explanations of the theory in question, the main reason for migration is wage differences. These differences direct people to migrate from low-wage regions to high-wage regions. The basis of this spatial mobility is to improve the economic situation.

The network theory of migration can be expressed as follows. Migration networks are sets of personal relationships that connect migrants, former migrants, and non-migrants in the regions of migration and emigration through kinship, friendship, and common community origin ties. These networks increase the likelihood of international mobility because they reduce the costs and risks of migration and increase the expected net return from migration. Network connections create a type of social capital that people can use to find work in foreign countries. When the number of migrants reaches a critical threshold, expanding networks reduce the costs and risks of migration, which increases the likelihood of migration and leads to more people migrating. Thus, as the wave of migration grows, networks expand and this process continues. Over time, this migration trend spreads to wider segments of society and leads to the migration of larger masses in the countries of migration [7]. This situation can also occur between two cities in the same country.

There are many different and important studies in literature on mathematical migration models. In terms of the relationship between demography and migration, Rogers [8] introduced a mathematical model based on matrix algebra that analyzed population changes between regions that are affected by factors of birth, death, and migration rates. Rogers and Ledent [9] presented a mathematical method that demonsrated demographic changes in areas in the future influenced by many parameters. Keyfitz [10] studied the evolution of migration between urban and rural regions using a pair of differential equations, under condition that the natural increase and migration rates are fixed. In terms of the relationship between economy and migration, Todaro [11] developed a model to examine the interaction of migration with socio-economic factors such as the urban-rural real income differential and the possibility of obtaining a urban job. These studies show that the economic motivations of immigrants and socio-economic inequalities have a powerful impact on migration dynamics in high and low welfare regions. In recent decades, many researchers have contributed to this subject. For instance, Camacho [12] provided a structure to comprehend natural and economic factors that affect migration, population agglomerations, and their environmental consequences. Harding and Neamtu [13] constructed and analyzed a model of unemployment based on a dynamic system by considering the condition that job searches are open to both native and migrant workers. Volpert et al. [14] developed a model to analyze the interaction of human migration and wealth distribution that consisted of a system of equations for the population density, conventional diffusion terms for the wealth distribution and cross diffusion terms explaining human migration identified by a wealth gradient and wealth transition determined by human migration.

Now, we will briefly describe the non-Newtonian analyses and multiplicative calculi that will define the two-region migration model and form the basis of study. Novel calculus has been established through the use of different arithmetic operations. Grossman and Katz presented non-Newtonian calculus between 1967 and 1970 [15]. They introduced classical calculus, harmonic calculus, geometric calculus, and quadratic calculus in 1967, and anageometric calculus, bigeometric calculus, anaharmonic calculus, biharmonic calculus, anaquadratic calculus, and biquadratic calculus in 1970. They utilized the adjective "non-Newtonian" to discern these novel calculus from classical calculus. The best-known non-Newtonian calculus are geometric multiplicative calculus and bigeometric multiplicative calculus. Apart from these, there are several implementations of both calculus in the literature, such as sigmoidal, anasigmoidal, bisigmoidal, parabolic, anaparabolic, and biparabolic [16–19]. Multiplicative calculus has implementations in various areas such as applied mathematics, physics, economics, and engineering. Elasticity in economics, interest rates, blood viscosity, biology, computer science containing artificial intelligence, and image processing, differential equations, and the probability theory [20–22] can be given as examples of these fields. Bashirov et al. [23] put forward the fundamental ideas of multiplicative calculus and gave some implementations. Kadak and Özlük [24] introduced the general form of the Runge-Kutta method for ordinary differential equations in multiplicative calculus. Yalçın and Çelik [25] defined the multiplicative Laplace transform. Boruah and Hazarika [26] studied on bigeometric integral calculus. They defined some basic properties of bigeometric calculus and its applications in numerical analyses [27]. Boruah et al. [28] examined the solvability of bigeometric differential equations by numerical methods. Yalçın and Dedetürk [29] studied multiplicative linear differential equations. Kaymak and Yalçın [30] introduced a bigeometric version of the Laplace transform. Although there are many types of multiplicative analyses, the structure of a bigeometric multiplicative analysis was most clearly presented by Georgiev and Zennir in 2022 [31]. One of the important and key concept in this study is the Laplace transform. This transform is an extremely important tool in classical analyses because it reduces the solution of differential equations to a simpler form. More specifically, in the analysis of linear systems, complex differential equations in the time domain are usually transformed into algebraic equations by the Laplace transform, which greatly simplifies the solution process. All these properties are of critical importance in many fields such as engineering, physics, and economics. Due to this importance, the bigeometric Laplace transform is strongly used in the solution of the two-region migration model.

2. Preliminaries

In this section, we present basic operations and concepts in bigeometric calculus, such as fundamental arithmetic operations, bigeometric derivative and bigeometric integral formulas, the bigeometric Laplace transform, and the two-region migration model in classical calculus.

2.1. Fundamentals of bigeometric calculus

In the following, the basic definitions and theorems about bigeometric calculus are expressed.

Definition 2.1. *[30] Exponential sets of numbers are given as follows:*

- *Set of multiplicative real numbers:* $\mathbb{R}_\star = (0, \infty)$*;*
- *Set of multiplicative positive real numbers:* $\mathbb{R}_\star^+ = (1, \infty)$ *; and*

- *Set of multiplicative negative real numbers:* $\mathbb{R}_\star^- = (0,1)$

Definition 2.2. *[31] Multiplicative addition, subtraction, multiplication, division, and other basic operations on* $\mathbb{R}_\star = (0,\infty)$ *are listed as below:*

$$\begin{array}{ll} a +_\star b = ab & a^{k\star} = a^{(\log a)^k} \\ a -_\star b = a/b & {}_\star\sqrt{a} = e^{(\log a)^{\frac{1}{2}}} \\ a \cdot_\star b = e^{\log a \log b} & a^{-1\star} = e^{\frac{1}{\log a}} \\ a/_\star b = e^{\frac{\log a}{\log b}} & a \cdot_\star e = a \\ a \cdot_\star a = a^{\log a} & e^n \cdot_\star a = a^n \end{array}$$

where $a, b \in \mathbb{R}_\star$. *Moreover,* $0_\star = 1$, $1_\star = e$ *and* $k_\star = e^k$.

Definition 2.3. *[30]* $|\cdot|_\star : \mathbb{R}_\star \to \mathbb{R}_\star^+$ *is defined by the following*

$$|\cdot|_\star = e^{|\ln x|}.$$

Definition 2.4. *[30, 31] The multiplicative absolute value of* $x \in \mathbb{R}_\star$ *is described as follows:*

$$|x|_\star = \begin{cases} x & \text{if } x \geq 1, \\ \frac{1}{x} & \text{if } x \leq 1. \end{cases}$$

Definition 2.5. *[30] Let* $\mathcal{A}$ *be a subset of* $\mathbb{R}_\star$. *Then,* $f_\star : \mathcal{A} \to \mathbb{R}_\star$ *is called a bigeometric function.*

Definition 2.6. *[18, 26] Suppose that* $f : \mathcal{A} \subseteq \mathbb{R}_\star \to \mathbb{R}_\star$ *is a bigeometric function and* α *is an accumulation point of* $\mathcal{A}$ *as to the multiplicative neighborhood.* f *has a limit in the bigeometric sense at the point* $\alpha \in \mathbb{R}_\star$ *and it is a singular multiplicative number* $M \in \mathbb{R}_\star$ *if and only if there is a multiplicative number* $\delta = \delta(\varepsilon) > 1$ *for all* $\varepsilon > 1$ *such that* $f(x) \in (M -_\star \varepsilon, M +_\star \varepsilon) = \left(\frac{M}{\varepsilon}, M \cdot \varepsilon\right)$ *whenever* $x \in (\alpha -_\star \delta, \alpha +_\star \delta) \setminus \{\alpha\} = \left(\frac{\alpha}{\delta}, \alpha \cdot \delta\right) \setminus \{\alpha\}$. *The bigeometric limit of* $f : \mathcal{A} \subseteq \mathbb{R}_\star \to \mathbb{R}_\star$ *is given by the following:*

$$^\star \lim_{x \to \alpha} f(x) = M.$$

Definition 2.7. *[30] Assume that* $f : \mathcal{A} \subseteq \mathbb{R}_\star \to \mathbb{R}_\star$ *is a bigeometric function and* $\alpha \in \mathcal{A}$. *The bigeometric right and bigeometric left side limits of* f *at the point* a *are described for* $\varepsilon > 1$, *respectively, as follows:*

$$\begin{aligned} f(a^{+_\star}) &= {}^\star \lim_{\varepsilon \to 1} f(\varepsilon \cdot a), \\ f(a^{-_\star}) &= {}^\star \lim_{\varepsilon \to 1} f\left(\frac{a}{\varepsilon}\right). \end{aligned}$$

If f *has a bigeometric limit at the point* a, *then the following equality is valid:*

$$^\star \lim_{x \to a} f(x) = f(a^{+_\star}) = f(a^{-_\star}).$$

Definition 2.8. *[30] Assume that* $f : \mathcal{A} \subseteq \mathbb{R}_\star \to \mathbb{R}_\star$ *is a bigeometric function and* $a \in \mathcal{A}$. f *is bigeometric continuous at the point* $a \in \mathcal{A}$ *if there is a multiplicative number* $\delta = \delta(\varepsilon) > 1$ *for all* $\varepsilon > 1$ *such that* $|f(x) -_\star f(a)|_\star < \varepsilon$ *whenever* $|x -_\star a|_\star < \delta$. *If* $f(x)$ *is continuous at the point* $x = a$, *then we obtain the following:*

$$^\star \lim\nolimits_{x \to a} f(x) = f(a).$$

Definition 2.9. *[31] The first order multiplicative derivative in the bigeometric sense of $f \in C^1(A)$ at $x \in A \subseteq \mathbb{R}_\star$ is defined by the following:*

$$f^\star(x) = {}^\star\lim_{h\to 0_\star}\left(f(x +_\star h) -_\star f(x)\right)/_\star h.$$

This formula can also be expressed as follows:

$$f^\star(x) = e^{\frac{xf'(x)}{f(x)}} \in C^1_\star(A)\,. \tag{2.1}$$

Here, $C^1_\star(A)$ is the space of all functions that are continuous on A and have continuous first order bigeometric multiplicative derivatives on A.

Corollary 2.10. *[31] Let $f, g \in C^1_\star(A)$ and $a, b \in \mathbb{R}_\star$. Some features of the first order bigeometric multiplicative derivative are listed below:*

- $(a \cdot_\star f)^\star(x) = a \cdot_\star f^\star(x)$,
- $(af)^\star = f^\star(x)$,
- $(f +_\star g)^\star(x) = f^\star(x) +_\star g^\star(x)$,
- $(f -_\star g)^\star(x) = f^\star(x) -_\star g^\star(x)$,
- $(f \cdot_\star g)^\star(x) = f^\star(x) \cdot_\star g(x) +_\star f(x) \cdot_\star g^\star(x)$,
- $(f/_\star g)^\star(x) = \left(f^\star(x) \cdot_\star g(x) -_\star f(x) \cdot_\star g^\star(x)\right)/_\star (g(x))^{2_\star}$,
- $(f \circ g)^\star(x) = \left(f^\star(g(x))\right) \cdot_\star g^\star(x)$,

where $x \in A$.

Definition 2.11. *[31] Let $A \subseteq \mathbb{R}_\star$, $f \in C^k_\star(A)$, and $k \in \mathbb{N}$. Then, the $k-th$ order bigeometric multiplicative derivative formula is described as follows:*

$$f^{\star(k)}(x) = \left(f^{\star(k-1)}\right)^\star(x)\,,\ x \in A.$$

Definition 2.12. *[31] Let $A \subseteq \mathbb{R}_\star$, $f \in C^k_\star(A)$, $k \in \mathbb{N}$, and $k \geq 2$. Then, the bigeometric multiplicative differential of $f(x)$ is defined as follows:*

$$d_\star f(x) = e^{d\left(\log f(x)\right)} = e^{\frac{f'(x)}{f(x)}dx},$$

and

$$f^\star(x) = d_\star f(x)/_\star d_\star x,$$

where $x \in A$. By the aforementioned definition, the following is obtained:

$$d_\star f(x) = f^\star(x) \cdot_\star d_\star x.$$

Definition 2.13. *[31] Assume that $a, b \in \mathbb{R}_\star$ and $f \in C(\mathbb{R}_\star)$. Then, the bigeometric indefinite integral can be represented as follows:*

$$\int_\star f(x) \cdot_\star d_\star x = e^{\int \frac{1}{x}\log f(x)dx},\ x \in \mathbb{R}_\star.$$

The bigeometric Cauchy integral is defined as follows:

$$\int_{\star a}^{b} f(x) \cdot_{\star} d_{\star}x = e^{\int_a^b \frac{1}{x} \log f(x)dx}.$$

For $x \in \mathbb{R}_{\star}$, the following holds:

$$d_{\star}\left(\int_{\star a}^{b} f(s) \cdot_{\star} d_{\star}s\right) /_{\star} d_{\star}x = f(x) \in \mathbb{R}_{\star}.$$

Furthermore, the following equality is satisfied:

$$\int_{\star a}^{b} f^{\star}(s) \cdot_{\star} d_{\star}s = f(b) -_{\star} f(a).$$

Corollary 2.14. *[31] Assume that $a, b, c \in \mathbb{R}_{\star}$ and $f, g \in C_{\star}(\mathbb{R}_{\star})$. Then, the following features hold:*

- $\int_{\star} (a \cdot_{\star} f(x) +_{\star} b \cdot_{\star} g(x)) \cdot_{\star} d_{\star}x = a \cdot_{\star} \int_{\star} f(x) \cdot_{\star} d_{\star}x +_{\star} b \cdot_{\star} \int_{\star} g(x) \cdot_{\star} d_{\star}x$;
- $\int_{\star} (a \cdot_{\star} f(x) -_{\star} b \cdot_{\star} g(x)) \cdot_{\star} d_{\star}x = a \cdot_{\star} \int_{\star} f(x) \cdot_{\star} d_{\star}x -_{\star} b \cdot_{\star} \int_{\star} g(x) \cdot_{\star} d_{\star}x$;
- $\int_{\star a}^{a} f(x) \cdot_{\star} d_{\star}x = 0_{\star}$;
- $\int_{\star a}^{b} f(x) \cdot_{\star} d_{\star}x = -_{\star} \int_{\star b}^{a} f(x) \cdot_{\star} d_{\star}x$; *and*
- $\int_{\star a}^{b} f(x) \cdot_{\star} d_{\star}x = \int_{\star a}^{c} f(x) \cdot_{\star} d_{\star}x +_{\star} \int_{\star c}^{b} f(x) \cdot_{\star} d_{\star}x$.

2.2. Laplace transform in bgeometric calculus

In this section, the bigeometric Laplace transform and its properties are explained.

Definition 2.15. *[30] Suppose that $f : \mathbb{R}_{\star}^{+} \to \mathbb{R}_{\star}$ is bigeometric continuous on $[a, b] \subset \mathbb{R}_{\star}^{+}$ for each multiplicative positive real number $b \geq a$. The bigeometric limit*

$$^{\star}\lim_{b\to\infty} \int_{\star a}^{b} f^{\star}(t) \cdot_{\star} d_{\star}t$$

is named as a first type improper bigeometric integral of f on $[a, \infty)$ and is given by

$$\int_{\star a}^{\infty} f^{\star}(t) \cdot_{\star} d_{\star}t.$$

If the bigeometric limit exists and equals to a multiplicative number $M \in \mathbb{R}_{\star}$, then the improper bigeometric integral has bigeometric convergence. Otherwise, improper bigeometric integral is considered as divergent, when the bigeometric limit doesn't exist or equals to either ∞ or 0.

Definition 2.16. *[30] Assume that $f : \mathbb{R}_{\star}^{+} \to \mathbb{R}_{\star}$ is a bigeometric function. Then, the bigeometric Laplace transform of $f(t)$ is described by the following:*

$$\mathcal{L}_{\star}\{f(t)\} = F_{\star}(s) = \int_{\star 1}^{\infty} f(t) \cdot_{\star} e^{-_{\star} s \cdot_{\star} t} d_{\star}t, \tag{2.2}$$

where $F_{\star}(s) : \mathbb{R}_{\star} \to \mathbb{R}_{\star}$ is a bigeometric function.

Lemma 2.17. *[30] Suppose that $f:\mathbb{R}_\star^+\to\mathbb{R}_\star$ is a bigeometric function. By substituting $t=e^\tau$ in the bigeometric Laplace transform of $f(t)$, the following is obtained:*

$$\mathcal{L}_\star\{f(t)\}=e^{\int_0^\infty \ln f(e^\tau)\cdot s^{-\tau}\cdot\tau d\tau}.$$

Lemma 2.18. *[30] Let $f:(0,\infty)\to\mathbb{R}$ be a function and $F(\sigma)$ is classical Laplace transform of this function, that is, $\mathcal{L}\{f(\tau)\}=F(\sigma)$. Then, the following equality holds for $t=e^\tau$ and $s=e^\sigma$:*

$$\mathcal{L}_\star\left\{e^{f(\ln t)}\right\}=e^{F(\ln s)}.$$

In Table 1, the classical and bigeometric Laplace transforms of some common functions are presented.

Table 1. Classical and Bigeometric Laplace Transforms of some common functions.

$f(\tau)$	$\mathcal{L}\{f(\tau)\}=F(\sigma)$	$e^{f(\ln t)}$	$\mathcal{L}_\star\left\{e^{f(\ln t)}\right\}=e^{F(\ln s)}$
0	$\mathcal{L}\{0\}=0$	1	$\mathcal{L}_\star\{1\}=1$
1	$\mathcal{L}\{1\}=1/\sigma$	e	$\mathcal{L}_\star\{e\}=e^{\frac{1}{\ln s}}$
$\ln a,\ (a>0)$	$\mathcal{L}\left\{\frac{\ln a}{\sigma}\right\}$	$a,\ (a\in\mathbb{R}_\star)$	$\mathcal{L}_\star\{a\}=a^{\frac{1}{\ln s}}$
e^τ	$\mathcal{L}\{e^\tau\}=\frac{1}{\sigma-1},\ (\sigma>1)$	e^t	$\mathcal{L}_\star\{e^t\}=e^{\frac{1}{\ln s-1}}$
$e^{a\tau}$	$\mathcal{L}\{e^{a\tau}\}=\frac{1}{\sigma-a},\ (\sigma>1)$	$e^{t^a},\ (a\in\mathbb{R})$	$\mathcal{L}_\star\left\{e^{t^a}\right\}=e^{\frac{1}{\ln s-a}}$
$\tau^n,\ (n\in\mathbb{N})$	$\mathcal{L}\{\tau^n\}=\frac{n!}{\sigma^{n+1}}$	$e^{(\ln t)^n}$	$\mathcal{L}_\star\left\{e^{(\ln t)^n}\right\}=e^{\frac{n!}{(\ln s)^{n+1}}}$
$\cos(a\tau),\ (a\in\mathbb{R})$	$\mathcal{L}\{\cos(a\tau)\}=\frac{\sigma}{\sigma^2+a^2}$	$e^{\cos(\ln(at))},\ (a\in\mathbb{R})$	$\mathcal{L}_\star\left\{e^{\cos(\ln(at))}\right\}=e^{\frac{\ln s}{(\ln s)^2+a^2}}$
$\sin(a\tau),\ (a\in\mathbb{R})$	$\mathcal{L}\{\sin(a\tau)\}=\frac{1}{\sigma^2+a^2}$	$e^{\sin(\ln(at))},\ (a\in\mathbb{R})$	$\mathcal{L}_\star\left\{e^{\sin(\ln(at))}\right\}=e^{\frac{1}{(\ln s)^2+a^2}}$

Definition 2.19. *[30] Suppose that $f:\mathbb{R}_\star^+\to\mathbb{R}_\star$ is a bigeometric function. For $t_0\in\mathbb{R}_\star^+$, $K\in\mathbb{R}_\star$, $\alpha\in\mathbb{R}_\star$, and $t>t_0$, the equality $|f(t)|_\star\le K^{\left(\alpha^{\ln t}\right)}$ is satisfied. For $t>t_0$, f is called to be of $\alpha-$bigeometric exponential order.*

Definition 2.20. *[30] Assume that a piecewise bigeometric continuous function $f:\mathbb{R}_\star^+\to\mathbb{R}_\star$ is an $\alpha-$bigeometric multiplicative order for $t>t_0$. Then, $\mathcal{L}_\star\{f(t)\}$ exists for $s>\alpha$.*

Theorem 2.21. *[30] The bigeometric Laplace transform is multiplicative linear. In other words, assume that m_1, m_2 are arbitrary real constants and $f_1(t)$, $f_2(t)$ are two bigeometric functions that have bigeometric Laplace transforms. Then, the following equality holds:*

$$\mathcal{L}_\star\{[f_1(t)]^{m_1}\cdot[f_2(t)]^{m_2}\}=\{\mathcal{L}_\star[f_1(t)]\}^{m_1}\cdot\{\mathcal{L}_\star[f_2(t)]\}^{m_2}$$

Theorem 2.22. *[30] Suppose that $f:\mathbb{R}_\star^+\to\mathbb{R}_\star$ is a bigeometric function and $F_\star(s)=\mathcal{L}_\star\{f(t)\}$ holds. Then, the following equality is achieved:*

$$\mathcal{L}_\star\left\{f(t)^{\left(a^{\ln t}\right)}\right\}=F_\star\left(\frac{s}{a}\right)$$

for $a\in\mathbb{R}_\star$.

Theorem 2.23. *[30] The bigeometric Laplace transform of the first-order bigeometric derivative is denoted by the following:*

$$\mathcal{L}_{\star}\{f^{\star}(t)\} = [F_{\star}(s)]^{\ln s} \cdot [f(1)]^{-1}.$$

Theorem 2.24. *[30] The bigeometric Laplace transform of the n^{th}−order bigeometric derivative is represented by the following:*

$$\mathcal{L}_{\star}\left\{f^{\star(n)}(t)\right\} = [F_{\star}(s)]^{(\ln s)^{n}} \cdot \left[\prod_{k=1}^{n}\left(f^{\star(k-1)}(1)\right)^{(\ln s)^{n-k}}\right]^{-1}.$$

Definition 2.25. *[30] Let $F_{\star} : \mathbb{R}_{\star} \to \mathbb{R}_{\star}$ be a bigeometric function and $f : \mathbb{R}_{\star}^{+} \to \mathbb{R}_{\star}$ be a bigeometric piecewise continuous function, which is a bigeometric multiplicative order given as follows: $\mathcal{L}_{\star}\{f(t)\} = F_{\star}(s)$. Then, $f(t)$ is the bigeometric inverse Laplace transform of $F_{\star}(s)$ and it is demonstrated by $f(t) = \mathcal{L}_{\star}^{-1}\{F_{\star}(s)\}$.*

Theorem 2.26. *[30] The bigeometric inverse Laplace transform is also bigeometrically linear. In other words, let m_1, m_2 be arbitrary constant exponents and $f_1(t)$, $f_2(t)$ be two continuous functions that have the bigeometric Laplace transforms $L_{\star}\{f_1(t)\} = F_1$ and $\mathcal{L}_{\star}\{f_2(t)\} = F_2$, respectively. Then, the following equality is satisfied:*

$$\mathcal{L}_{\star}^{-1}\left\{F_1^{m_1} \cdot F_2^{m_2}\right\} = \mathcal{L}_{\star}^{-1}\{F_1\}^{m_1} \cdot \mathcal{L}_{\star}^{-1}\{F_2\}^{m_2}.$$

2.3. Two-region migration model in classical calculus

In this section, we explain the two-region migration model in classical calculus. In addition, one example related to it with a figure and numerical values are presented. Assume that there are two regions A and B which have the populations P_A and P_B, respectively. In addition, m_{AB} and m_{BA} are considered the rate of migration from A to B and the rate of migration from B to A, respectively. Here, the birth and death rates of populations in A and B are ignored [10, 32, 33]. Then, under these conditions, the two-region migration model is given by the following system:

$$\frac{dP_A}{dt} = m_{BA}P_B(t) - m_{AB}P_A(t), \tag{2.3}$$

$$\frac{dP_B}{dt} = m_{AB}P_A(t) - m_{BA}P_B(t), \tag{2.4}$$

where $P_A(t)$ and $P_B(t)$ represent populations of A and B at time t, respectively.

Theorem 2.27. *The solutions of the two region migration systems (2.3) and (2.4) are*

$$P_A(t) = \frac{Cm_{BA}}{m_{AB}+m_{BA}} + \left(P_A(0) - \frac{Cm_{BA}}{m_{AB}+m_{BA}}\right)e^{-(m_{AB}+m_{BA})t},$$

and

$$P_B(t) = \frac{Cm_{AB}}{m_{AB}+m_{BA}} + \left(P_B(0) - \frac{Cm_{AB}}{m_{AB}+m_{BA}}\right)e^{-(m_{AB}+m_{BA})t},$$

respectively, where C is total population.

Proof. Since $(P_A(t) + P_B(t))' = 0$, the total population is a constant C. By using $P_B(t) = C - P_A(t)$, Eq (2.3) can be rewritten as follows:

$$\frac{dP_A}{dt} = -m_{BA}P_A(t) + m_{AB}(C - P_A(t)) = m_{AB}C - (m_{AB} + m_{BA})P_A(t).$$

The solution of this equation is given by

$$P_A(t) = \frac{Cm_{BA}}{m_{AB} + m_{BA}} + \left(P_A(0) - \frac{Cm_{BA}}{m_{AB} + m_{BA}}\right)e^{-(m_{AB}+m_{BA})t},$$

and as $t \to \infty$, we obtain $P_A(\infty) = \frac{Cm_{BA}}{m_{AB}+m_{BA}}$. Then,

$$\begin{aligned} P_B(t) &= C - P_A(t) \\ &= \frac{Cm_{AB}}{m_{AB} + m_{BA}} + \left(P_B(0) - \frac{Cm_{AB}}{m_{AB} + m_{BA}}\right)e^{-(m_{AB}+m_{BA})t}, \end{aligned}$$

and as $t \to \infty$, we obtain $P_B(\infty) = \frac{Cm_{AB}}{m_{AB}+m_{BA}}$. □

Remark 2.28. *There are many models in literature to find m_{AB} for the two region migration model. Here, one of these models is the gravity model. This model is expressed as $m_{AB} = \frac{kP_AP_B}{d^2}$. Here, k is a constant to be calibrated, and d is distance between the regions A and B. In this formula, m_{AB} and m_{BA} have the same value. However, in real applications, the migration rates in both directions are often not the same. To overcome this problem, the gravity model can be modified as $m_{AB} = \frac{kP_AP_B}{d^2F_{AB}}$. Here, F_{AB}, or the "coefficient of distance friction", represents how migration is made more difficult by obstacles other than geographic distance. Unlike physical distance, it includes cultural, legal, and economic barriers. Its calculation is not direct, but is usually derived based on indicators or calibrated by regression. F_{AB} is calculated with the following formula: $F_{AB} = \frac{1}{1+H_{AB}}$, where $H_{AB} = \sum_{i=1}^{n} w_i s_i$ is the total barrier score between the regions A and B. Here, w_i is the coefficient between 0 and 1 that determines the relative impact of each obstacle on the total score, and s_i is the numerical value that corresponds to each obstacle type. Here, if the barrier is weak, then it takes the value 0; if it is medium, then it takes the value 0.5; and if it is strong, then it takes the value 1. For example, let's calculate the F_{AB} value for countries A and B, where there are 5 barriers to migration between these two countries (Language difference-$s_1 = 1$, Religious difference-$s_2 = 1$, Visa requirement-$s_3 = 1$, Cultural proximity-$s_4 = 0.5$, and Historical connection-$s_5 = 0.5$). If each barrier has equal weight, then $w_i = 0.2$. When these values are written in the formula, $H_{AB} = 0.8$ and $F_{AB} = 0.55$ are obtained.*

Similarly, m_{BA} can be defined. In Example 2.29, m_{AB} and m_{BA} were arbitrarily obtained using this modified model [34, 35].

Example 2.29. *Let $P_A(t)$ and $P_B(t)$ be populations at time t of the regions A and B under the conditions $P_A(0) = 1000$, $P_B(0) = 500$. Additionally, the migration rates are selected as $m_{AB} = 0.01$ and $m_{BA} = 0.005$. Then, Figure 1 and Table 2 will explain the change of populations P_A and P_B obtained using usual the two-region migration model for the given values.*

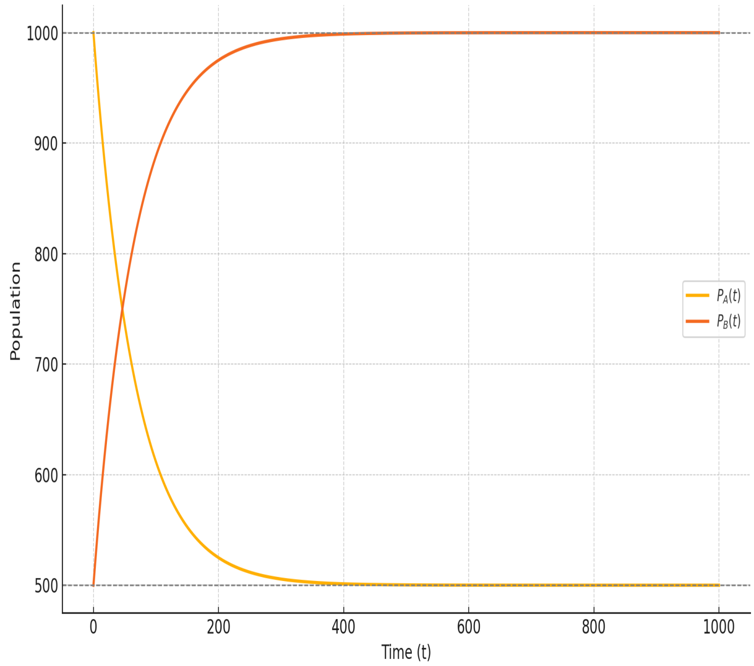

Figure 1. Populations of the Regions A and B in classical calculus.

Table 2. $P_A(t)$, $P_B(t)$ and $P_A(t) + P_B(t)$ values as t progresses.

t **(Time)**	$P_A(t)$	$P_B(t)$	$P_A(t) + P_B(t)$
0.0	1000.0	500.0	1500.0
10.1	929.71	570.29	1500.0
30.3	817.38	682.62	1500.0
46.2	750.04	749.96	1500.0
70.71	673.12	826.88	1500.0
111.11	594.44	905.56	1500.0
212.12	520.76	979.24	1500.0
333.33	503.37	996.63	1500.0
414.14	501.0	999.0	1500.0
606.06	500.06	999.94	1500.0
1000.0	500.0	1000.0	1500.0

Figure 1 and Table 2 show the data of the migration model created with a classical analysis. The migration movement showed a sudden increase at starting point, but this increase decreased after a while and became stationary. Here, the migration flow presented by the classical model appears to reach equilibrium much more quickly than the bigeometric multiplicative model. The migration flow in this model can be explained by the assumption that the migration process is driven by economic motivations and may decline as the supply-demand balance is established, as predicted by the neoclassical migration theory. According to the theory, the migration movement between countries and cities is directed on the axis of wage differences between the regions, sending and receiving migrants, and a balance of labor supply and demand. This spatial mobility stops when economic equality is achieved. Therefore, it can be said that migration here is based on an individual and economic basis. If a person compares the cost of migration (moving, etc.) and desired gain (earning a higher income, having better living conditions, etc.), then they decide to migrate if the desired gain is at a higher level. In this context, the neoclassical migration theory argues that individuals migrate to regions where they can earn higher incomes as a result of their rational decisions. Based on this idea, in the migration model calculated by classical analysis, the initial intensive migration movement can be explained by factors such as job opportunities and unemployment rates, wage differences, the search for economic balance, maximizing the quality of life, migration costs, expectations and migration being a rational decision. Figure 1 shows that after a certain period of time, the migration wave came to a halt and created a certain balance. When this situation is considered within the framework of the neoclassical theory, it shows that wage and employment differences have disappeared and migration is no longer an economic necessity. The classical analysis model can be seen as the mathematical model that best represents the neoclassical theory, which examines migration processes solely based on economic foundations.

3. Main results

In this section, we introduce bigeometric two-region migration model. First, the equations of the model are defined with bigeometric calculus. Thereafter, the bigeometric two-region migration model is solved by the bigeometric Laplace transform. Finally, we present one example with a figure and numerical values in a table to clarify the matter.

Suppose that there exist two regions A and B that have the populations P_A and P_B. Additionally, we consider m_{AB} and m_{BA} as the rate of migration from A to B and the rate of migration from B to A, respectively. Moreover, the birth and death rates of populations in A and B are ignored. Then, the equations of the bigeometric two-region migration model are defined as follows:

$$P_A^{\star}(t) = m_{BA} \cdot_{\star} P_B(t) -_{\star} m_{AB} \cdot_{\star} P_A(t), \tag{3.1}$$
$$P_B^{\star}(t) = m_{AB} \cdot_{\star} P_A(t) -_{\star} m_{BA} \cdot_{\star} P_B(t). \tag{3.2}$$

Theorem 3.1. *The solutions of the bigeometric two region migration models (3.1) and (3.2) are*

$$P_A(t) = P_B \left(\frac{P_A}{P_B}\right)^{\frac{1}{m_{AB}m_{BA}\ln t}}, \tag{3.3}$$

and

$$P_B(t) = P_A \left(\frac{P_B}{P_A}\right)^{\frac{1}{m_{AB}m_{BA}\ln t}}, \tag{3.4}$$

where $P_A(1) = P_A, P_B(1) = P_B$.

Proof. To obtain the solutions for Eqs (3.1) and (3.2), we rewrite the equations as follows:

$$P_A^{\star}(t)\, e^{\ln m_{AB} \cdot \ln P_A(t)} e^{-\ln m_{BA} \cdot \ln P_B(t)} = 1, \tag{3.5}$$
$$P_B^{\star}(t)\, e^{\ln m_{BA} \cdot \ln P_B(t)} e^{-\ln m_{AB} \cdot \ln P_A(t)} = 1. \tag{3.6}$$

First, we solve Eq (3.5) as follows. Using (3.1) and (3.2), the following is obtained:

$$P_A^{\star}(t) +_{\star} P_B^{\star}(t) = 0_{\star}. \tag{3.7}$$

We get the following equalities by taking the bigeometric integral of (3.7):

$$P_A(t) +_\star P_B(t) = C_1\star$$

or

$$P_A(t) \cdot P_B(t) = C_\star. \tag{3.8}$$

Thereafter, the following is written using (2.2) and (3.5):

$$\mathcal{L}_\star\left\{P_A^\star(t)\, e^{\ln m_{AB}\cdot \ln P_A(t)} e^{-\ln m_{BA}\cdot \ln P_B(t)}\right\} = \mathcal{L}_\star\{1\}.$$

If we proceed with the calculations, then the following is achieved:

$$\mathcal{L}_\star\{P_A^\star(t)\}\, \mathcal{L}_\star\{P_A(t)\}^{\ln m_{AB}}\, \mathcal{L}_\star\{P_B(t)\}^{-\ln m_{BA}} = 1. \tag{3.9}$$

By substituting $P_B(t)$ into $C_\star \cdot P_A^{-1}(t)$ and changing $C_\star$ with $C^{\frac{\ln(m_{AB}m_{BA})}{\ln m_{BA}}}$ in (3.9), we obtain the following:

$$\mathcal{L}_\star\{P_A^\star(t)\}\, \mathcal{L}_\star\{P_A(t)\}^{\ln m_{AB}}\, \mathcal{L}_\star\{P_A(t)\}^{\ln m_{BA}}\, \mathcal{L}_\star\{C\}^{-\ln(m_{BA}m_{AB})} = 1. \tag{3.10}$$

By using Theorem 2.23 and taking the bigeometric Laplace transforms of functions in (3.10), the following equality is acquired:

$$[P_{A\star}(s)]^{\ln s}\, P_A^{-1}(1)\, P_{A\star}(s)^{\ln m_{AB}}\, P_{A\star}(s)^{\ln m_{BA}}\, C^{-\frac{\ln(m_{AB}m_{BA})}{\ln s}} = 1. \tag{3.11}$$

The equalities of (3.11) can be rewritten as follows:

$$P_{A\star}(s) = C^{\frac{\ln(m_{AB}m_{BA})}{\ln s(\ln s + \ln(m_{AB}m_{BA}))}}\, P_A(1)^{\frac{1}{\ln s + \ln(m_{AB}m_{BA})}}.$$

or

$$P_{A\star}(s) = C^{\frac{1}{\ln s}}\, C^{-\frac{1}{\ln s + \ln(m_{AB}m_{BA})}}\, P_A(1)^{\frac{1}{\ln s + \ln(m_{AB}m_{BA})}}. \tag{3.12}$$

By applying the inverse bigeometric Laplace transform to both side of (3.12) and using (2.22), the following equality is obtained:

$$\begin{aligned}
\mathcal{L}_\star^{-1}\{P_{A\star}(s)\} &= \mathcal{L}_\star^{-1}\left\{C^{\frac{1}{\ln s}}\, C^{-\frac{1}{\ln s + \ln(m_{AB}m_{BA})}}\, P_A(1)^{\frac{1}{\ln s + \ln(m_{AB}m_{BA})}}\right\} \\
&= \mathcal{L}_\star^{-1}\left\{C^{\frac{1}{\ln s}}\right\} \mathcal{L}_\star^{-1}\left\{C^{-\frac{1}{\ln s + \ln(m_{AB}m_{BA})}}\right\} \mathcal{L}_\star^{-1}\left\{P_A(1)^{\frac{1}{\ln s + \ln(m_{AB}m_{BA})}}\right\} \\
&= C \cdot \frac{P_A(1)^{\frac{1}{m_{AB}m_{BA}\ln t}}}{C^{\frac{1}{m_{AB}m_{BA}\ln t}}}
\end{aligned}$$

If we select the initial conditions as

$$P_A(1) = P_A,\, P_B(1) = P_B$$

and change C with P_B, then the following solution for (3.1) is achieved:

$$P_A(t) = P_B\left(\frac{P_A}{P_B}\right)^{\frac{1}{m_{AB}m_{BA}\ln t}}. \tag{3.13}$$

If we apply the aforementioned calculations to (3.2), the following solution of the equation is obtained:

$$P_B(t) = P_A\left(\frac{P_B}{P_A}\right)^{\frac{1}{m_{AB}m_{BA}\ln t}}. \tag{3.14}$$

By taking the limits of (3.13) and (3.14) as $t \to \infty$, we obtain

$$P_A(\infty) = P_B(1) = P_B$$

and

$$P_B(\infty) = P_A(1) = P_A,$$

respectively. □

Remark 3.2. *The expression m_{AB} will be defined using the bigeometric multiplicative analysis structure. In bigeometric calculus, the gravity migration rate is given by the following:*

$$m_{AB} = \frac{k \cdot_\star P_A \cdot_\star P_B}{d^{*2}}.$$

The generalized gravity migration rate m_{AB} that we will use in our study is calculated with following formula in the bigeometric multiplicative analysis:

$$m_{AB} = \frac{k \cdot_\star P_A \cdot_\star P_B}{d^{*2} \cdot_\star F_{AB}}.$$

Similarly, F_{AB} is the correction factor that affects migration from region A to region B, and includes average wages in the regions, quality of life indices, and employment rates. m_{BA} the value can be similarly established in the bigometric multiplicative analysis. In both formulas above, k is a calibration constant.

In Example 3.3, the m_{AB} and m_{BA} values were calculated using the aforementioned bigeometric generalized gravity migration rate formula.

Example 3.3. *Assume that $P_A(t)$ and $P_B(t)$ are populations at time t of A and B under conditions of $P_A(1) = 1000$, $P_B(1) = 500$, respectively, which represents the initial population of the two regions. Additionally, the migration rates are selected as $m_{AB} = e^{0.01}$ and $m_{BA} = e^{0.005}$. Then, Figure 2 and Table 3 are obtained using the two-region migration model on bigeometric calculus.*

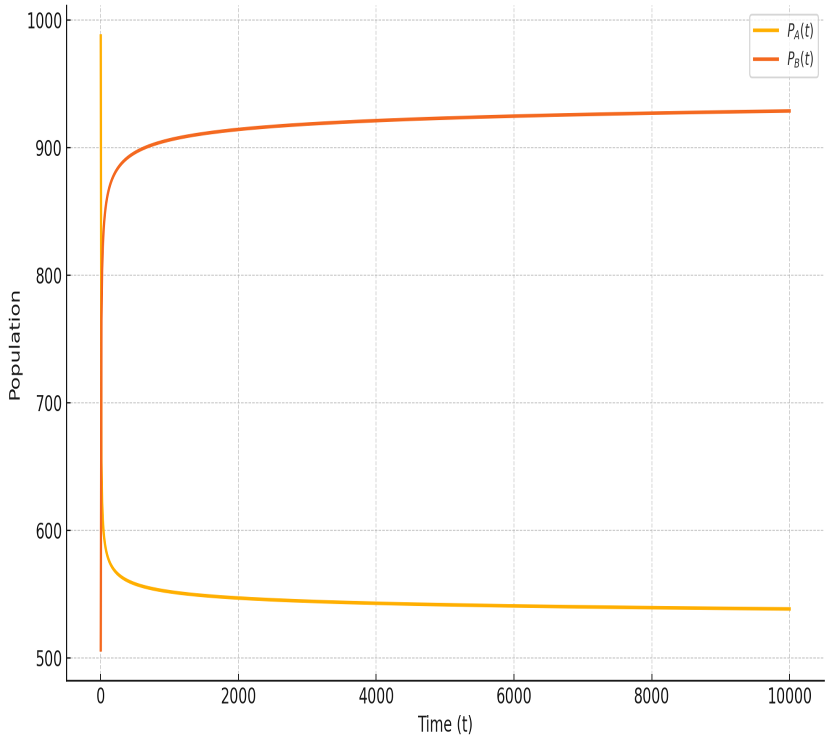

Figure 2. Populations for the regions *A* and *B* in bigeometric calculus.

Table 3. $P_A(t)$, $P_B(t)$ and $P_A(t).P_B(t)$ values as t progress.

***t* (Time)**	$P_A(t)$	$P_B(t)$	$P_A(t) \cdot P_B(t)$
2.59	1024.67	487.96	500000.0
22.63	622.36	803.4	500000.0
102.78	579.41	862.95	500000.0
202.98	568.57	879.4	500000.0
864.26	553.13	903.95	500000.0
1124.77	551.04	907.38	500000.0
1385.27	549.5	909.92	500000.0
1846.17	547.52	913.21	500000.0
2026.52	546.91	914.22	500000.0
2166.79	546.48	914.94	500000.0

In Figure 2 and Table 3 obtained by the bigeometric multiplicative analysis, the migration movement suddenly increased at the begining and continued for a long time compared to the classical analysis, and then decreased and came to a halt after some time. This second model reflects the relational dynamics of migration over time. The fact that the migration wave did not end quickly has been evaluated within the sociological framework of the network theory. What is important in the network migration theory, it is important that the migration spreads itself into an ongoing process. According to the theory, individuals who step into the migration process establish a settled order in the places they go to and gain a certain economic and social status, and then open the door to new migration waves, thus creating the ground for new migrations. The existence of migration networks provides important opportunities in terms of information and guidance, as well as provides economic and psychological support for new migrants. These opportunities created by migrant networks make the migration process less risky and more predictable for new migrants and make migration continuous, thus leading to a longer period of migration. This situation is clearly seen in Figure 2. Initially, migration increased rapidly and continued for a long time before decreasing and finally coming to a standstill. In this context, the temporal trends presented by the mathematical model are sociologically interpreted through the self-reproducing structure of migration networks. The stagnation or slowdown of the migration process suggests numerous factors, including the possibility that the established network of relationships may eventually reach saturation, the absence of new migration pressures in migrant-sending regions, changes in integration policies, the lack of a need for migration by the new generation, and so on. These and similar factors can lead to a slowdown in network-based migration flows. From the perspective of sociological migration theories, it is impossible to predict with certainty that migration will cease completely. This is because migration requires a multidimensional field of interpretation beyond mathematical models, shaped not only by economic or demographic processes, but also by political, cultural, emotional, and historical processes.

In fact, the sociological theories discussed in the study can be explained with a concrete example. It is possible to concretize neoclassical and network migration theories through the Türkiye-Germany migration movement. The migration movement from Türkiye to Germany can be examined in two basic periods: 1960–1980 and after 1980. While the first period between 1960–1980, can be explained with the basic assumptions of the neoclassical migration theory, the migration dynamics after 1980 can be evaluated within the framework of the network theory. After World War II, Western European countries needed a large-scale workforce in order to support their rapidly growing economies. In this context, Germany signed a labor force agreement with Türkiye on October 30, 1961 and invited workers from Türkiye to its country. This agreement marked the beginning of labor force mobility between the two countries, and an intensive labor migration from Türkiye to Germany began in the 1960s. Most of the workers who initially went to Germany as "guest workers" planned to work for a temporary period in order to increase their economic development and welfare. However, over time, the Turkish community in Germany moved towards permanent settlement, and family reunifications accelerated this migration flow from the 1970s onwards [36].

In this context, the migration process from Türkiye to Germany transformed into a self-sustaining social movement over time, as predicted by the network theory. As Kerobyan [37] emphasizes, once the migration of the Turkish began, it became a self-sustaining process and continued to expand through migration networks, independent of the initial reasons and policy changes in Germany. This situation reveals that migration is shaped not only by economic factors but also by social ties and migrant networks.

Considering all these developments, in the 1960s, there was a labor shortage in Germany and unemployment and low wages in Türkiye. In this case, the migration movement that took place was a migration movement based on individual and economic rationality, which the neoclassical migration theory emphasized, and in which the cost-benefit calculations were prioritized. Individuals who migrated made a rational cost-benefit analysis in order to reach more economically advantageous conditions and made the decision to migrate. Thus, this migration, which was directed from Türkiye, where the labor supply was high, to Germany, where the labor demand was high, took place within the framework of market mechanisms and constituted a typical example of achieving the economic balance predicted by the neoclassical theory. Since the 1980s, the Türkiye-Germany migration has become more complex not only for economic reasons but also due to family reunification, social solidarity, cultural belonging, and the influence of second-generation immigrants. For this reason, the post-1980 period is explained by the network theory rather than the neoclassical theory. This theory, in particular, provides a strong infrastructure in terms of explaining how the migration movement became sustainable and chained. After 1980, migrations took place through the social networks of the Turkish who had previously settled in Germany. Migration is shaped not by individuals but by kinship, fellow countrymen and community ties. What sustains migration is no longer wage differences but social ties. Castles [38] states that once migration movements begin, they become self-sustaining social processes. This situation is an indication that migration is no longer driven by individual decisions, but by established communities, social networks, and kinship ties that continue migration, and clearly proves that migration creates its own internal dynamics.

Therefore, while the migration movement between Türkiye and Germany before 1980 can be explained by the two-region migration model in a classical analysis, the migration movement between Türkiye and Germany after 1980 can be explained by the bigeometric multiplicative two-region migration model.

4. Conclusions

In this study, the two-region migration model was solved with classical and bigeometric multiplicative analyses and compared mathematically and sociologically. Then, the situation was concretized using some numerical examples with Figures and Tables. Figures 1 and 2 modeled the same migration event with different mathematical analysis methods. Figure 1 was created with a classical analysis and showed that the migration process reaches equilibrium at a certain point and remains constant. Figure 2 was created with a bigeometric multiplicative analysis and showed that the migration movement has a longer-term, and slowly decreasing dynamic. While the classical analysis considers migration as a shorter-term, predictable, and equilibrium-reaching process, the bigeometric multiplicative analysis shows that migration is a long-term process affected by social and cultural factors. Since migration movements in today's world are shaped not only by economic but also social, political, and environmental dynamics, it can be said that the bigeometric multiplicative model explains migration processes more comprehensively and realistically. In the examples considered in both analyses, the sum of the populations of the two regions and the product of the populations of these two regions remained constant. The migration models generated by both classical and bigeometric multiplicative analyses do not claim to reflect social reality on their own. These data offer clues about the structural dynamics behind migration only when interpreted through sociological theories. In this context, the mathematical model outputs were evaluated as an object of interpretation. To understand the social implications of these models, a sociological perspective is necessary. All Figures in this study were drawn with Python, version 3.12.5.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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