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Research article

A comparison of analytical solutions of nonlinear complex generalized Zakharov dynamical system for various definitions of the differential operator

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Abstract: This paper considers deriving new exact solutions of a nonlinear complex generalized Zakharov dynamical system for two different definitions of derivative operators called conformable and $M-$ truncated. The system models the spread of the Langmuir waves in ionized plasma. The extended rational $sine - cosine$ and $sinh - cosh$ methods are used to solve the considered system. The paper also includes a comparison between the solutions of the models containing separately conformable and $M-$ truncated derivatives. The solutions are compared in the $2D$ and $3D$ graphics. All computations and representations of the solutions are fulfilled with the help of Mathematica 12. The methods are efficient and easily computable, so they can be applied to get exact solutions of non-linear PDEs (or PDE systems) with the different types of derivatives.

Keywords: Zakharov dynamical system; extended rational sine-cosine method; extended rational sinh-cosh method exact solutions

1. Introduction

Researchers make great efforts to better model real-world problems. Fractional calculus, one of the important and fresh fields of mathematics, offers a better modeling opportunity. So, it has wide applications in various areas such as physics, biology, and chemistry, etc. There is no unique definition of the fractional derivative or integral in the literature. Some of the popular definitions of the fractional derivative are Riemann-Liouville [1, 2], Caputo [3], Caputo-Fabrizio [4], and Atangana-

Baleanu [5]. Besides, some scientists have introduced new definitions of differential operators such as conformable [6] and $M-$ truncated derivatives [7] in the last decade. Some works about the conformable derivative are new properties of conformable derivative [8], the geometric meaning of conformable derivative [9], price adjustment in market equilibrium [10], financial system analysis and hyperchaos detection [11], modeling neuron dynamics [12] and complex conformable derivative [13], etc. Regarding the conformable PDEs, there are many studies in the literature such as optical solution of Gerdjikov-Ivanov equation [14], numerical solution of Burgers' equation [15], wave solutions of Zakharov-Kuznetsov equation [16], exact solutions of space-time local fractal nonlinear evolution equations [17], Pochhammer-Chree equation [18], optical solutions of space-time nonlinear Schrodinger equation [19], modified KdV-Zakharov-Kuznetsov equation [20], Gardner equation [21], space-time Fokas-Lenells equation [22], deterministic and stochastic solutions of Schrodinger equation [23].

$M-$ truncated derivative, which can be thought of as generalizations of the conformable derivatives, is introduced by Sousa and Oliveira [7] in 2018. Some articles about the $M-$ truncated derivative can be listed as the longitudinal wave equation [24, 25], (2 + 1) dimensional Boussinesq dynamic model [26], Lakshmanan–Porsezian–Daniel equation [27], Gerdjikov-Ivanov equation [28], Schrodinger-Hirota equation [29], Sturm-Liouville problem [30], Hirota-Maccari system [31], Radhakrishnan–Kundu–Lakshmanan equation [31] and Biswas-Arsad model [32], comparative study on complex Ginzburg-Landau equation [33], and clinical medicine applications [34].

In this work, the extended rational $sine - cosine$ and $sinh - cosh$ methods [35–37] will be used to solve the non-linear complex generalized Zakharov dynamical system (NLCGZDS) [38]. The system is a modeling of the spread of the Langmuir waves in ionized plasma [39]. In order to obtain better modeling, the related system is discussed using different derivative operators called conformable and $M-$ truncated derivatives. We mainly aim to obtain novel exact solutions of the considered models to help further works in the different disciplines. Besides, we compare the obtained solutions to explain the behavior of the solutions.

The literature includes the studies on the classical NLCGZDS more. Some of them are Cauchy problem for the Zakharov system [40], quantum kinetics of Zakharov system [41], well-posedness of the system [42], hyperchaos investigation in the quantum Zakharov system [43]. The Zakharov system was solved by various methods such as time-splitting spectral method [44], extended trial equation method [45], exp-function method [46], local discontinuous Galerkin method [47], extended wave solutions of Klein-Gordon-Zakharov system [48] and also there is a comprehensive study on soliton solutions of Zakharov system [34].

The format of this article is organized as: in Section 2, some preliminaries to be used in subsequent parts are included. The mathematical analysis and the algorithm of the method are studied in Section 3. The governing model of considered PDE systems is dealt with in Section 4. In Section 5, the application of the method and the figures of the solutions of the considered equation are studied. The results and discussion can be found in Section 6. A conclusion is given in the final section.

2. Preliminaries

2.1. Conformable derivative

Definition 2.1. Let $g : [0, \infty) \to \mathbb{R}$ be a function. α order conformable derivative of $g(t)$ is defined as follows [6]:

$$\mathcal{D}_t^{\alpha}(g(t)) = \lim_{h \to 0} \frac{g(t + ht^{1-\alpha}) - g(t)}{h}, \tag{2.1}$$

where $\alpha \in (0, 1],\ t > 0$.

Theorem 2.2. *[6] Let $g(t)$ and $h(t)$ be α -differentiable functions for $\alpha \in (0, 1],\ t > 0$. Then,*

1) $\mathcal{D}_t^{\alpha}(cg(t) + dh(t)) = c\mathcal{D}_t^{\alpha} g(t) + d\mathcal{D}_t^{\alpha} h(t),$ for $c, d \in \mathbb{R}$,

2) $\mathcal{D}_t^{\alpha}(t^n) = nt^{n-\alpha},\ n \in \mathbb{R}$,

3) If $g(t) = c$ where c is a constant, then $\mathcal{D}_t^{\alpha}(c) = 0$,

4) $\mathcal{D}_t^{\alpha}(h(t)g(t)) = h(t)\mathcal{D}_t^{\alpha} g(t) + g(t)\mathcal{D}_t^{\alpha} h(t)$,

5) $\mathcal{D}_t^{\alpha}\left(\frac{g(t)}{h(t)}\right) = \frac{h(t)\mathcal{D}_t^{\alpha} g(t) - g(t)\mathcal{D}_t^{\alpha} h(t)}{h^2(t)}, \quad h(t) \neq 0$,

6) If the first derivative of $g(t)$ exists, then
$\mathcal{D}_t^{\alpha}(g(t)) = t^{1-\alpha}\frac{dg(t)}{dt}$.

2.2. $M-$ truncated derivative

Definition 2.3. The truncated Mittag-Leffler function is given as [7]:

$$_i\mathbb{E}_{\beta}(z) = \sum_{k=0}^{i} \frac{z^k}{\Gamma(\beta k + 1)}, \tag{2.2}$$

in which $\beta > 0$ and $z \in C$.

Definition 2.4. Let $f : [0, \infty) \to \mathbb{R}$ be a function, the $M-$ truncated derivative of f of order $\alpha \in (0, 1)$, w.r.t. t is defined by [7]

$$\mathscr{D}_{M,t}^{\alpha,\beta} g(t) = \lim_{\varepsilon \to 0} \frac{g\left(t + {}_i\mathbb{E}_{\beta}(\varepsilon t^{-\alpha})\right) - g(t)}{\varepsilon}, \tag{2.3}$$

where $\beta, t > 0$ and ${}_i\mathbb{E}_{\beta}(\cdot)$ is truncated Mittag-Leffler function.

Theorem 2.5. *Let $g(t)$ be α order differentiable function at $t_0 > 0$ with $\alpha \in (0, 1]$ and $\beta > 0$. Then, $g(t)$ is continuous at t_0 [7].*

Theorem 2.6. *[7] Let $0 < \alpha \leq 1, \beta > 0, p, q \in \mathbb{R}$ and assume that g, h is α -differentiable at a point $t > 0$. Then,*

1) $\mathscr{D}_{M,t}^{\alpha,\beta}(pg + qh)(t) = p\ \mathscr{D}_{M,t}^{\alpha,\beta} g(t) + q\ \mathscr{D}_{M,t}^{\alpha,\beta} h(t)$, where p, q are real constants,

2) $\mathscr{D}_{M,t}^{\alpha,\beta}(gh)(t) = g(t)\ \mathscr{D}_{M,t}^{\alpha,\beta} h(t) + h(t)\ \mathscr{D}_{M,t}^{\alpha,\beta} g(t)$,

3) $\mathscr{D}_{M,t}^{\alpha,\beta}\left(\frac{g}{h}\right)(t) = \frac{g(t)\ \mathscr{D}_{M,t}^{\alpha,\beta}h(t) - h(t)\ \mathscr{D}_{M,t}^{\alpha,\beta}g(t)}{h(t)^2}$,

4) $\mathscr{D}_{M,t}^{\alpha,\beta}\left(\frac{g}{h}\right)(t) = \frac{g(t)\ \mathscr{D}_{M,t}^{\alpha,\beta}h(t) - h(t)\ \mathscr{D}_{M,t}^{\alpha,\beta}g(t)}{h(t)^2}$,

5) *If g is differentiable, then* $\mathscr{D}_{M,t}^{\alpha,\beta}(g)(t) = \frac{t^{1-\alpha}}{\Gamma(\beta+1)}\frac{dg(t)}{dt}$.

3. Analysis of the method

i) Let's deal with the general form of the conformable PDE system and the wave transformation as follows:

$$F\left(\mathcal{D}_x^{\alpha}\Psi,\ \mathcal{D}_t^{\alpha}\Psi,\ \mathcal{D}_x^{\alpha}\mathcal{D}_t^{\alpha}\Psi,\ \mathcal{D}_t^{\alpha}\chi, \mathcal{D}_x^{\alpha}\chi, \ldots\right) = 0, \tag{3.1}$$

$$G\left(\mathcal{D}_x^{\alpha}\Psi,\ \mathcal{D}_t^{\alpha}\Psi,\ \mathcal{D}_x^{\alpha}\mathcal{D}_t^{\alpha}\Psi,\ \mathcal{D}_t^{\alpha}\chi, \mathcal{D}_x^{\alpha}\chi, \ldots\right) = 0, \tag{3.2}$$

and

$$\Psi(x,t) = Y(\xi)e^{i\phi},\ \xi = \frac{c_1x^{\alpha} + c_2t^{\alpha}}{\alpha},\ \phi = \frac{c_3x^{\alpha} + c_4t^{\alpha}}{\alpha},\ \chi(x,t) = Q(\xi). \tag{3.3}$$

ii) Consider the general form of the local $M-$ truncated PDE system and the wave transformation as follows:

$$F\left(\mathscr{D}_{M,x}^{\alpha,\beta}\Psi,\ \mathscr{D}_{M,t}^{\alpha,\beta}\Psi,\ \mathscr{D}_{M,x}^{\alpha,\beta}\mathscr{D}_{M,t}^{\alpha,\beta}\Psi,\ \mathscr{D}_{M,x}^{\alpha,\beta}\chi, \mathscr{D}_{M,t}^{\alpha,\beta}\chi, \ldots\right) = 0, \tag{3.4}$$

$$G\left(\mathscr{D}_{M,x}^{\alpha,\beta}\Psi,\ \mathscr{D}_{M,t}^{\alpha,\beta}\Psi,\ \mathscr{D}_{M,x}^{\alpha,\beta}\mathscr{D}_{M,t}^{\alpha,\beta}\Psi,\ \mathscr{D}_{M,x}^{\alpha,\beta}\chi, \mathscr{D}_{M,t}^{\alpha,\beta}\chi, \ldots\right) = 0, \tag{3.5}$$

and

$$\begin{gathered}\Psi(x,t) = Y(\xi)e^{i\phi}, \chi(x,t) = Q(\xi)\ , \\ \xi = \frac{\Gamma(\beta+1)(c_1x^{\alpha} + c_2t^{\alpha})}{\alpha}, \phi = \frac{\Gamma(\beta+1)(c_3x^{\alpha} + c_4t^{\alpha})}{\alpha},\end{gathered} \tag{3.6}$$

where $\Psi(x,t)$ and $\chi(x,t)$ are the unknown functions, $\mathcal{D}_*^{\alpha}$ and $\mathscr{D}_{M,*}^{\alpha,\beta}$ are the α order conformable and $M-$ truncated derivative operators with respect to $*$ (x or t), respectively.
Substituting the wave transformations to the PDE systems yields the following nonlinear ordinary differential equation systems:

$$K\left(Y, Y', Y'', \ldots, Q, Q', Q'', \ldots\right) = 0, \tag{3.7}$$

$$L\left(Y, Y', Y'', \ldots, Q, Q', Q'', \ldots\right) = 0, \tag{3.8}$$

where Y and Q are the unknown functions of ξ and the superscript indicates the derivative of the functions with respect to ξ.

3.1. Extended rational sine − cosine method

Step 1: Assume that the ODE system in Eq. (3.7) and Eq. (3.8) have the solutions as follows:

$$u(\psi) = \frac{\omega_0 \sin(\mu\psi)}{\omega_2 + \omega_1 \cos(\mu\psi)},\ \cos(\mu\psi) \neq -\frac{\omega_2}{\omega_1}, \tag{3.9}$$

or

$$u(\psi) = \frac{\omega_0 \cos(\mu\psi)}{\omega_2 + \omega_1 \sin(\mu\psi)}, \ \sin(\mu\psi) \neq -\frac{\omega_2}{\omega_1}. \tag{3.10}$$

where ω_0, ω_1, and ω_2 are parameters to be found in the next step. μ is the wave number to be also determined after substitution.

Step 2: Substitute the Eq. (3.9) or the Eq. (3.10) to the Eq. (3.7) and the Eq. (3.8). After collecting all terms with the same powers of $\cosh(\mu\psi)$ or $\sinh(\mu\psi)$, we derive a set of algebraic equations by equating all the coefficients of $\cosh(\mu\psi)$ or $\sinh(\mu\psi)$ to zero. In order to find the unknowns ($\omega_0, \omega_1, \omega_2$ and μ), the system can be solved by Maple, Mathematica, etc.

Step 3: Substitute the values of $\omega_0, \omega_1, \omega_2, \psi$ and μ into Eq. (3.9) or Eq. (3.10), the solutions of the ODE system in Eq. (3.7) and Eq. (3.8) can be found.

3.2. Extended rational sinh − cosh method

Step 1: Suppose that the ODE system in Eq. (3.7) and Eq. (3.8) have the solutions as follows:

$$u(\psi) = \frac{\omega_0 \sinh(\mu\psi)}{\omega_2 + \omega_1 \cosh(\mu\psi)}, \ \cosh(\mu\psi) \neq -\frac{\omega_2}{\omega_1}, \tag{3.11}$$

or

$$u(\psi) = \frac{\omega_0 \cosh(\mu\psi)}{\omega_2 + \omega_1 \sinh(\mu\psi)}, \ \sinh(\mu\psi) \neq -\frac{\omega_2}{\omega_1}, \tag{3.12}$$

where ω_0, ω_1, and ω_2 are parameters to be found in the next step. μ is the wave number to be also determined after substitution.

Step 2: Substitute the Eq. (3.11) or the Eq. (3.12) to the Eq. (3.7) and the Eq. (3.8). After collecting all terms with the same powers of $\cosh(\mu\psi)$ or $\sinh(\mu\psi)$, we derive a set of algebraic equations by equating all the coefficients of $\cosh(\mu\psi)$ or $\sinh(\mu\psi)$ to zero. In order to find the unknowns ($\omega_0, \omega_1, \omega_2$ and μ), the system can be solved by Maple, Mathematica, etc.

Step 3: Substitute the values of $\omega_0, \omega_1, \omega_2, \psi$ and μ into Eq. (3.11) or Eq. (3.12), the solutions of the ODE system in Eq. (3.7) and Eq. (3.8) can be found.

4. Governing models

In this section, we consider the NLCGZDS with respect to different definitions of derivatives [49]:

4.1. Model with conformable derivative

In conformable derivative, the considered equation might be written as [49]:

$$\begin{cases} i\, \mathcal{D}_t^{\alpha}\Psi + \Psi_{xx} - 2\delta|\Psi|^2\Psi + 2\chi\Psi = 0, \\ \mathcal{D}_t^{2\alpha}\chi - \chi_{xx} + \left(|\Psi|^2\right)_{xx} = 0. \end{cases} \tag{4.1}$$

For conformable derivative, the following wave transformations are employed:

$$\Psi(x,t) = Y(\xi)e^{i\phi}, \ \chi(x,t) = Q(\xi), \ \xi = x - \frac{2kt^{\alpha}}{\alpha}, \ \phi = \frac{ct^{\alpha}}{\alpha} + kx. \tag{4.2}$$

4.2. Model with M− truncated derivative

In $M-$ truncated derivative, the considered equation might be written as [49]:

$$\begin{cases} i\,{}_{i}\mathscr{D}_{M,t}^{\alpha,\beta}\Psi + \Psi_{xx} - 2\delta|\Psi|^2\Psi + 2\chi\Psi = 0, \\ {}_{i}\mathscr{D}_{M,t}^{2\alpha,\beta}\chi - \chi_{xx} + \left(|\Psi|^2\right)_{xx} = 0. \end{cases} \tag{4.3}$$

For $M-$ truncated derivative, the following wave transformations are employed:

$$\Psi(x,t) = Y(\xi)e^{i\phi},\ \chi(x,t) = Q(\xi),\ \xi = x - \frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha},\ \phi = \frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha} + kx. \tag{4.4}$$

4.3. Solutions of the governing models

Applying the wave transformations in Eq. (4.2) and Eq. (4.4) to the Eq. (4.1) and Eq. (4.3), respectively, we obtain the following ODE system:

$$\begin{cases} Y'' - \left(c+k^2\right)Y - 2\delta Y^3 + 2YQ = 0, \\ \left(4k^2-1\right)Q'' + \left(Y^2\right)'' = 0. \end{cases} \tag{4.5}$$

Integrating two times the second equation in the Eq. (4.5) and supposing the integration constant is zero, we get:

$$Q(\xi) = \frac{Y^2}{1-4k^2}. \tag{4.6}$$

Substituting $Q(\xi)$ to the first equation in the Eq. (4.5), we get:

$$Y'' - \left(c+k^2\right)Y + \left(\frac{2}{1-4k^2} - 2\delta\right)Y^3 = 0. \tag{4.7}$$

Balancing the highest order derivative term and the nonlinear term, we derive $N+2 = 3N \Rightarrow N = 1$.

5. Application

5.1. Conformable Zakharov system

5.1.1. Application of the extended rational *sine − cosine* method to the conformable Zakharov system

Assume that the Eq. (4.7) has the solution in the form:

$$Y(\xi) = \frac{\omega_0 \sin[\mu\xi]}{\omega_2 + \omega_1\cos[\mu\xi]}. \tag{5.1}$$

Substituting Eq. (5.1) into the Eq. (4.7) ODE to get the algebraic equations formed by collecting all terms comprising the same powers of $cos[\mu\xi]^m$.

$$\begin{aligned}
cos(\mu\xi)^2 :\quad & -4ck^2\omega_0\omega_1^2 + c\omega_0\omega_1^2 - 2\delta\omega_0^3 - 4k^4\omega_0\omega_1^2 + 8\delta k^2\omega_0^3 + k^2\omega_0\omega_1^2 + 2\omega_0^3 = 0, \\
cos(\mu\xi)^1 :\quad & -16ck^2\omega_0\omega_1\omega_2 + 4c\omega_0\omega_1\omega_2 - 16k^4\omega_0\omega_1\omega_2 + 8k^2\mu^2\omega_0\omega_1\omega_2 + 4k^2\omega_0\omega_1\omega_2 \\
& -2\mu^2\omega_0\omega_1\omega_2 = 0, \\
cos(\mu\xi)^0 :\quad & -4ck^2\omega_0\omega_1^2 - 16ck^2\omega_0\omega_2^2 + c\omega_0\omega_1^2 + 4c\omega_0\omega_2^2 + 6\delta\omega_0^3 - 4k^4\omega_0\omega_1^2 \\
& -16k^4\omega_0\omega_2^2 - 24\delta k^2\omega_0^3 + 32k^2\mu^2\omega_0\omega_1^2 - 16k^2\mu^2\omega_0\omega_2^2 + k^2\omega_0\omega_1^2 + 4k^2\omega_0\omega_2^2 \\
& -8\mu^2\omega_0\omega_1^2 + 4\mu^2\omega_0\omega_2^2 - 6\omega_0^3 = 0.
\end{aligned} \tag{5.2}$$

After solving the system of equations in the Eq. (5.2) via Mathematica, the following cases are derived:

Case 1:

$$\mu = \pm\frac{\sqrt{c+k^2}}{\sqrt{2}},\ \omega_0 = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\omega_1,\ \omega_1 = \omega_1,\ \omega_2 = 0. \tag{5.3}$$

Substituting the parameters in *Case 1* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_1(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\tan\left[\frac{\xi\sqrt{c+k^2}}{\sqrt{2}}\right]. \tag{5.4}$$

Considering the Eq. (5.4) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$\Psi_{11}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\tan\left[\frac{\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{c+k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.5}$$

$$\Psi_{12}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\tan\left[\frac{\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{c+k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.6}$$

$$\chi_{11}(x,t) = \chi_{12}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{\left(1-4k^2\right)\left(\delta\left(2-8k^2\right)-2\right)}\tan^2\left[\frac{\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)}{\sqrt{2}}\right]. \tag{5.7}$$

These solutions are corresponding to *trigonometric* soliton solutions in the literature and valid for $\left(c+k^2\right) > 0$.

Case 2:

$$\mu = \pm\sqrt{2}\sqrt{c+k^2}, \omega_0 = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\omega_1, \omega_2 = \omega_2, \omega_1 = \pm\omega_2. \tag{5.8}$$

Substituting the parameters in *Case 2* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_2(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\frac{\sin\left[\sqrt{2}\xi\sqrt{c+k^2}\right]}{\left[\omega_2\cos\left(\sqrt{2}\xi\sqrt{c+k^2}\right)+\omega_1\right]}. \tag{5.9}$$

Considering the Eq. (5.9) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$\Psi_{21}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\frac{\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{\left[1+\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right)\right]}, \tag{5.10}$$

$$\Psi_{22}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\frac{\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{\left[1+\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right)\right]}, \tag{5.11}$$

$$\Psi_{23}(x,t)=-\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\frac{\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{ct^{\alpha}}{\alpha}+kx\right)}}{\left[1-\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right)\right]}, \tag{5.12}$$

$$\Psi_{24}(x,t)=\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\frac{\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{ct^{\alpha}}{\alpha}+kx\right)}}{\left[1-\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right)\right]}, \tag{5.13}$$

$$\chi_{21}(x,t)=\chi_{22}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)(\delta(2-8k^2)-2)}\frac{\sin^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right]}{\left[1+\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right)\right]^2}, \tag{5.14}$$

$$\chi_{23}(x,t)=\chi_{24}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)(\delta(2-8k^2)-2)}\frac{\sin^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right]}{\left[1-\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right)\right]^2}. \tag{5.15}$$

These solutions are valid for $(c+k^2)>0$.

Suppose that solutions of the Eq. (4.7) is in the form:

$$Y(\xi)=\frac{\omega_0\cos[\mu\xi]}{\omega_2+\omega_1\sin[\mu\xi]}. \tag{5.16}$$

Substituting the Eq. (5.16) into the Eq. (4.7) to get the algebraic equations formed by collecting all terms comprising the same powers of $sin[\mu\xi]^m$.

$$\begin{aligned}
sin(\mu\xi)^2:\quad & 2\omega_0\omega_1^2ck^2-\frac{1}{2}\omega_0\omega_1^2c+\omega_0^3\delta+2\omega_0\omega_1^2k^4-4\omega_0^3\delta k^2-\frac{1}{2}\omega_0\omega_1^2k^2-\omega_0^3=0,\\
sin(\mu\xi)^1:\quad & -8\omega_0\omega_1\omega_2ck^2+2\omega_0\omega_1\omega_2c-8\omega_0\omega_1\omega_2k^4+4\omega_0\omega_1\omega_2k^2\mu^2+2\omega_0\omega_1\omega_2k^2\\
& -\omega_0\omega_1\omega_2\mu^2=0,\\
sin(\mu\xi)^0:\quad & -2\omega_0\omega_1^2ck^2-8\omega_0\omega_2^2ck^2+\frac{1}{2}\omega_0\omega_1^2c+2\omega_0\omega_2^2c+3\omega_0^3\delta-2\omega_0\omega_1^2k^4\\
& -8\omega_0\omega_2^2k^4-12\omega_0^3\delta k^2+16\omega_0\omega_1^2k^2\mu^2-8\omega_0\omega_2^2k^2\mu^2+\frac{1}{2}\omega_0\omega_1^2k^2\\
& +2\omega_0\omega_2^2k^2-4\omega_0\omega_1^2\mu^2+2\omega_0\omega_2^2\mu^2-3\omega_0^3=0.
\end{aligned} \tag{5.17}$$

After solving the system of equations in the Eq. (5.17) via Mathematica, the following cases are derived:

Case 3:

$$\mu=\pm\frac{\sqrt{c+k^2}}{\sqrt{2}},\omega_0=\pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}},\omega_1=\omega_1,\omega_2=0. \tag{5.18}$$

Substituting the parameters in *Case 3* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_3(\xi)=\pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\cot\left[\frac{\xi\sqrt{c+k^2}}{\sqrt{2}}\right]. \tag{5.19}$$

Considering the Eq. (5.19) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$\Psi_{31}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\cot\left[\frac{(x-\frac{2kt^\alpha}{\alpha})\sqrt{c+k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.20}$$

$$\Psi_{32}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\cot\left[\frac{(x-\frac{2kt^\alpha}{\alpha})\sqrt{c+k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.21}$$

$$\chi_{31}(x,t) = \chi_{32}(x,t) = \frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)(\delta(2-8k^2)-2)}\cot^2\left[\frac{\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)}{\sqrt{2}}\right]. \tag{5.22}$$

These solutions are corresponding to *trigonometric* soliton solutions in the literature and valid for $(c+k^2) > 0$.

Case 4:

$$\mu = \pm\sqrt{2}\sqrt{c+k^2}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}, \omega_2 = \omega_2, \omega_1 = \pm\omega_2. \tag{5.23}$$

Substituting the parameters in *Case 4* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_4(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\frac{\cos\left[\sqrt{2}\xi\sqrt{c+k^2}\right]}{\omega_2\pm\omega_1\sin\left[\sqrt{2}\xi\sqrt{c+k^2}\right]}. \tag{5.24}$$

Considering the Eq. (5.24) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$\Psi_{41}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\frac{\cos\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1+\sin\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]}, \tag{5.25}$$

$$\Psi_{42}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\frac{\cos\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1+\sin\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]}, \tag{5.26}$$

$$\Psi_{43}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\frac{\cos\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1-\sin\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]}, \tag{5.27}$$

$$\Psi_{44}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}\frac{\cos\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1-\sin\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{c+k^2}\right]}, \tag{5.28}$$

$$\chi_{41}(x,t) = \chi_{42}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)(\delta(2-8k^2)-2)}\frac{\cos^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]}{\left[1+\sin\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right)\right]^2}, \tag{5.29}$$

$$\chi_{43}(x,t) = \chi_{44}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)(\delta(2-8k^2)-2)}\frac{\cos^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]}{\left[1-\sin\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right)\right]^2}. \tag{5.30}$$

These solutions are valid for $(c+k^2) > 0$.

5.1.2. Application of the extended rational *sinh − cosh* method to the conformable Zakharov system

Suppose that solutions of the Eq. (4.7) is in the form:

$$Y(\xi) = \frac{\omega_0 \sinh[\mu\xi]}{\omega_2 + \omega_1 \cosh[\mu\xi]}. \tag{5.31}$$

Substituting the Eq. (5.31) into the Eq. (4.7) to get the algebraic equations formed by collecting all terms comprising the same powers of $cosh[\mu\xi]^m$.

$$\begin{aligned}
cosh(\mu\xi)^2: \quad & -4\omega_0\omega_1^2ck^2 + \omega_0\omega_1^2c + 2\omega_0^3\delta - 4\omega_0\omega_1^2k^4 - 8\omega_0^3\delta k^2 + \omega_0\omega_1^2k^2 - 2\omega_0^3,\\
cosh(\mu\xi)^1: \quad & -16\omega_0\omega_1\omega_2ck^2 + 4\omega_0\omega_1\omega_2c - 16\omega_0\omega_1\omega_2k^4 - 8\omega_0\omega_1\omega_2k^2\mu^2 + 4\omega_0\omega_1\omega_2k^2\\
& + 2\omega_0\omega_1\omega_2\mu^2 = 0,\\
cosh(\mu\xi)^0: \quad & -4\omega_0\omega_1^2ck^2 - 16\omega_0\omega_2^2ck^2 + \omega_0\omega_1^2c + 4\omega_0\omega_2^2c - 6\omega_0^3\delta - 4\omega_0\omega_1^2k^4 - 16\omega_0\omega_2^2k^4\\
& + 24\omega_0^3\delta k^2 - 32\omega_0\omega_1^2k^2\mu^2 + 16\omega_0\omega_2^2k^2\mu^2 + \omega_0\omega_1^2k^2 + 4\omega_0\omega_2^2k^2 + 8\omega_0\omega_1^2\mu^2\\
& - 4\omega_0\omega_2^2\mu^2 + 6\omega_0^3 = 0.
\end{aligned} \tag{5.32}$$

After solving the system of equations in the Eq. (5.32) via Mathematica, the following cases are derived:
Case 5:

$$\mu = \pm\frac{\sqrt{-c-k^2}}{\sqrt{2}}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}, \omega_1 = \omega_1, \omega_2 = 0. \tag{5.33}$$

Substituting the parameters in *Case 5* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_5(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\tanh\left[\frac{\xi\sqrt{-c-k^2}}{\sqrt{2}}\right]. \tag{5.34}$$

Considering the Eq. (5.34) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$\Psi_{51}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\tanh\left[\frac{(x-\frac{2kt^\alpha}{\alpha})\sqrt{-c-k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.35}$$

$$\Psi_{52}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\tanh\left[\frac{(x-\frac{2kt^\alpha}{\alpha})\sqrt{-c-k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.36}$$

$$\chi_{51}(x,t) = \chi_{52}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)(\delta(8k^2-2)+2)}\tanh^2\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)}{\sqrt{2}}\right]. \tag{5.37}$$

These solutions are corresponding to *dark* soliton solutions in the literature and valid for $(-c-k^2) > 0$ and
$\left(4k^2-1\right)\left(c+k^2\right)\left(\delta\left(8k^2-2\right)+2\right) < 0$.

Case 6:

$$\mu = \pm\sqrt{2}\sqrt{-c-k^2}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}, \omega_2 = \omega_2, \omega_1 = \pm\omega_2. \tag{5.38}$$

Substituting the parameters in *Case 6* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_6(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\frac{\sinh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]}{\omega_2 \pm \omega_1\cosh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]}. \tag{5.39}$$

Considering the Eq. (5.39) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$\Psi_{61}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\frac{\sinh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1+\cosh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.40}$$

$$\Psi_{62}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\frac{\sinh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1+\cosh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.41}$$

$$\Psi_{63}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\frac{\sinh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1-\cosh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.42}$$

$$\Psi_{64}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(8k^2-2)+2}}\frac{\sinh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1-\cosh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.43}$$

$$\chi_{61}(x,t) = \chi_{62}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)\left(\delta(8k^2-2)+2\right)}\frac{\sinh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]}{\left[1+\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]\right]^2}, \tag{5.44}$$

$$\chi_{63}(x,t) = \chi_{64}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)\left(\delta(8k^2-2)+2\right)}\frac{\sinh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]}{\left[1-\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]\right]^2}. \tag{5.45}$$

These solutions are valid for $\left(-c-k^2\right) > 0$.

Suppose that solutions of the Eq. (4.7) is in the form:

$$Y(\xi) = \frac{\omega_0\cosh[\mu\xi]}{\omega_2+\omega_1\sinh[\mu\xi]}. \tag{5.46}$$

Substituting the Eq. (5.46) into the Eq. (4.7) to get the algebraic equations formed by collecting all terms comprising the same powers of $sinh[\mu\xi]^m$.

$$\begin{aligned}
sinh(\mu\xi)^2 :& -4\omega_0\omega_1^2ck^2+\omega_0\omega_1^2c+2\omega_0^3\delta-4\omega_0\omega_1^2k^4-8\omega_0^3\delta k^2+\omega_0\omega_1^2k^2-2\omega_0^3,\\
sinh(\mu\xi)^1 :& -16\omega_0\omega_1\omega_2ck^2+4\omega_0\omega_1\omega_2c-16\omega_0\omega_1\omega_2k^4-8\omega_0\omega_1\omega_2k^2\mu^2+4\omega_0\omega_1\omega_2k^2+2\omega_0\omega_1\omega_2\mu^2=0,\\
sinh(\mu\xi)^0 :& -4\omega_0\omega_1^2ck^2-16\omega_0\omega_2^2ck^2+\omega_0\omega_1^2c+4\omega_0\omega_2^2c-6\omega_0^3\delta-4\omega_0\omega_1^2k^4-16\omega_0\omega_2^2k^4+24\omega_0^3\delta k^2\\
&-32\omega_0\omega_1^2k^2\mu^2+16\omega_0\omega_2^2k^2\mu^2+\omega_0\omega_1^2k^2+4\omega_0\omega_2^2k^2+8\omega_0\omega_1^2\mu^2-4\omega_0\omega_2^2\mu^2+6\omega_0^3=0.
\end{aligned} \tag{5.47}$$

After solving the system of equations in the Eq. (5.47) via Mathematica, the following cases are derived:

Case 7:

$$\mu = \pm\frac{\sqrt{-c-k^2}}{\sqrt{2}}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}, \omega_1 = \omega_1, \omega_2 = 0. \tag{5.48}$$

Considering the Eq. (5.49) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$Y_7(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\coth\left[\frac{\xi\sqrt{-c-k^2}}{\sqrt{2}}\right]. \tag{5.49}$$

Substituting the parameters in the *Case 7* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$\Psi_{71}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\coth\left[\frac{\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{-c-k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.50}$$

$$\Psi_{72}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\coth\left[\frac{\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{-c-k^2}}{\sqrt{2}}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}, \tag{5.51}$$

$$\chi_{71}(x,t) = \chi_{72}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{\left(1-4k^2\right)\left(\delta\left(8k^2-2\right)+2\right)}\coth^2\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)}{\sqrt{2}}\right]. \tag{5.52}$$

These solutions are corresponding to *singular* soliton solutions in the literature and valid for $(-c-k^2) > 0$.

Case 8:

$$\mu = \pm\sqrt{2}\sqrt{-c-k^2}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}, \omega_2 = \omega_2, \omega_1 = \pm\omega_2 i. \tag{5.53}$$

Substituting the parameters in *Case 8* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_8(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\frac{\cosh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]}{\omega_2\pm\omega_1 i\sinh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]}. \tag{5.54}$$

Considering the Eq. (5.54) and the Eq. (4.2), the following solutions of the conformable Zakharov system are obtained:

$$\Psi_{81}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\frac{\cosh\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1+i\sinh\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.55}$$

$$\Psi_{82}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\frac{\cosh\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1+i\sinh\left[\left(x-\frac{2kt^\alpha}{\alpha}\right)\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.56}$$

$$\Psi_{83}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\frac{\cosh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1-i\sinh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.57}$$

$$\Psi_{84}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}\frac{\cosh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]e^{i\left(\frac{ct^\alpha}{\alpha}+kx\right)}}{1-i\sinh\left[(x-\frac{2kt^\alpha}{\alpha})\sqrt{2}\sqrt{-c-k^2}\right]}, \tag{5.58}$$

$$\chi_{81}(x,t) = \chi_{82}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)}\frac{\cosh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]}{\left[1+i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]\right]^2}, \tag{5.59}$$

$$\chi_{83}(x,t) = \chi_{84}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)}\frac{\cosh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]}{\left[1-i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2kt^\alpha}{\alpha}\right)\right]\right]^2}. \tag{5.60}$$

These solutions are valid for $(-c-k^2) > 0$.

5.2. M− truncated Zakharov system

5.2.1. Application of the extended rational *sine − cosine* method to the *M*− truncated Zakharov system

Suppose that solutions of the Eq. (4.7) is in the form:

$$Y(\xi) = \frac{\omega_0 \sin[\mu\xi]}{\omega_2+\omega_1\cos[\mu\xi]}. \tag{5.61}$$

Substituting Eq. (5.61) into the Eq. (4.7) ODE to get the algebraic equations formed by collecting all terms comprising the same powers of $cos[\mu\xi]^m$.

$$\begin{aligned}
cos(\mu\xi)^2:\quad & -4\omega_0\omega_1^2ck^2+\omega_0\omega_1^2c-2\omega_0^3\delta+8\omega_0^3\delta k^2-4\omega_0\omega_1^2k^4+\omega_0\omega_1^2k^2+2\omega_0^3=0,\\
cos(\mu\xi)^1:\quad & -16\omega_0\omega_1\omega_2ck^2+4\omega_0\omega_1\omega_2c+8\omega_0\omega_1\omega_2k^2\mu^2-16\omega_0\omega_1\omega_2k^4+4\omega_0\omega_1\omega_2k^2\\
& -2\omega_0\omega_1\omega_2\mu^2=0,\\
cos(\mu\xi)^0:\quad & -4\omega_0\omega_1^2ck^2-16\omega_0\omega_2^2ck^2+\omega_0\omega_1^2c+4\omega_0\omega_2^2c+6\omega_0^3\delta-24\omega_0^3\delta k^2+32\omega_0\omega_1^2k^2\mu^2\\
& -16\omega_0\omega_2^2k^2\mu^2-4\omega_0\omega_1^2k^4-16\omega_0\omega_2^2k^4+\omega_0\omega_1^2k^2+4\omega_0\omega_2^2k^2-8\omega_0\omega_1^2\mu^2\\
& +4\omega_0\omega_2^2\mu^2-6\omega_0^3=0.
\end{aligned} \tag{5.62}$$

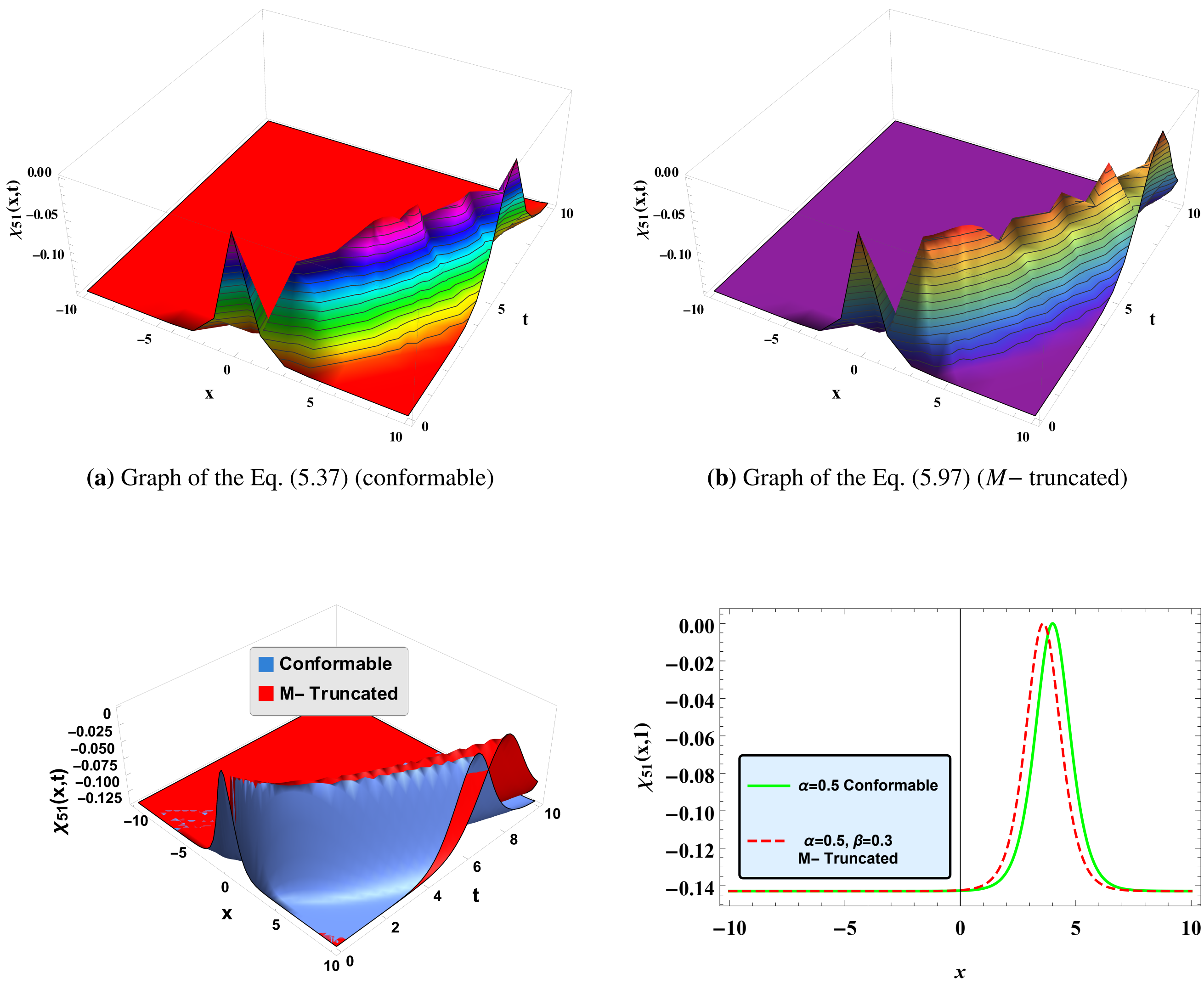
After solving the system of equations in the Eq. (5.62) via Mathematica, the following cases are derived:

Case 1:

$$\mu = \pm\frac{\sqrt{c+k^2}}{\sqrt{2}}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}, \omega_1 = \omega_1, \omega_2 = 0. \tag{5.63}$$

Substituting the parameters in *Case 1* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_1(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\tan\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(2-8k^2\right)-2}}. \tag{5.64}$$



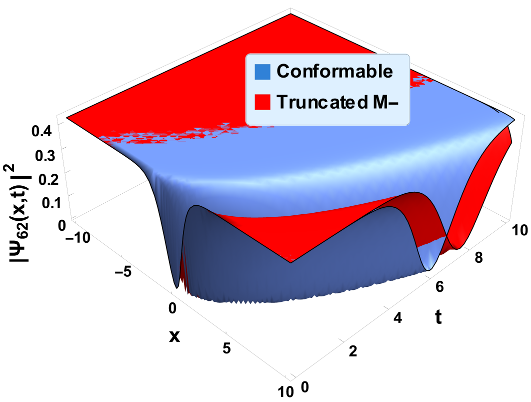
(a) Graph of the Eq. (5.37) (conformable)

(b) Graph of the Eq. (5.97) ($M-$ truncated)

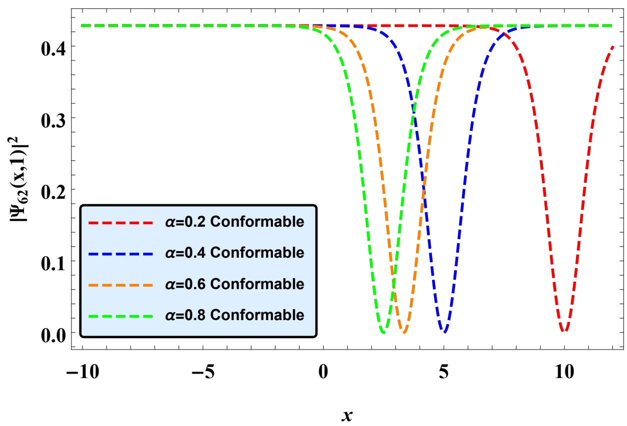
(c) Both conformable and $M-$ truncated

(d) Comparison between conformable and $M-$ truncated

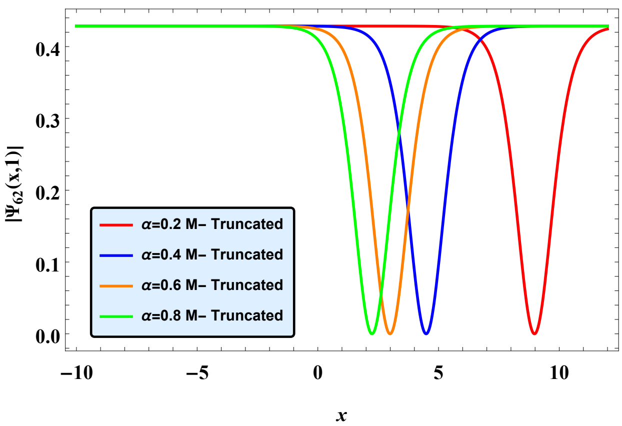
Figure 1. Solution comparison for the Eq. (5.37) and the Eq. (5.97) when $c = -3, k = 1, \delta = 2, \beta = 0.3$, and $\alpha = 0.5$.



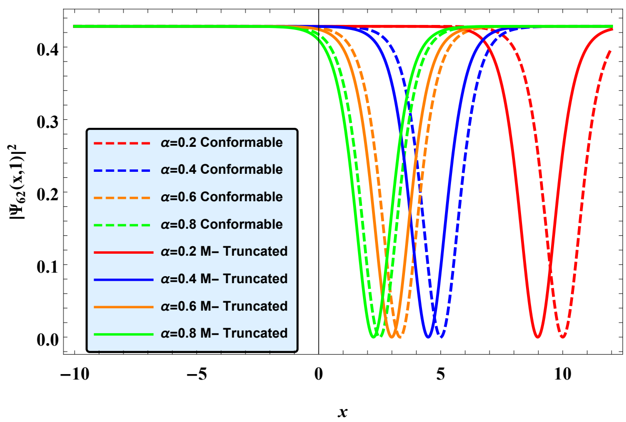
(a) 3*D* Comparison between conformable and $M-$ truncated for $\alpha = 0.5$



(b) Graph of the Eq. (5.41) for different α values and $\beta = 0.3$.

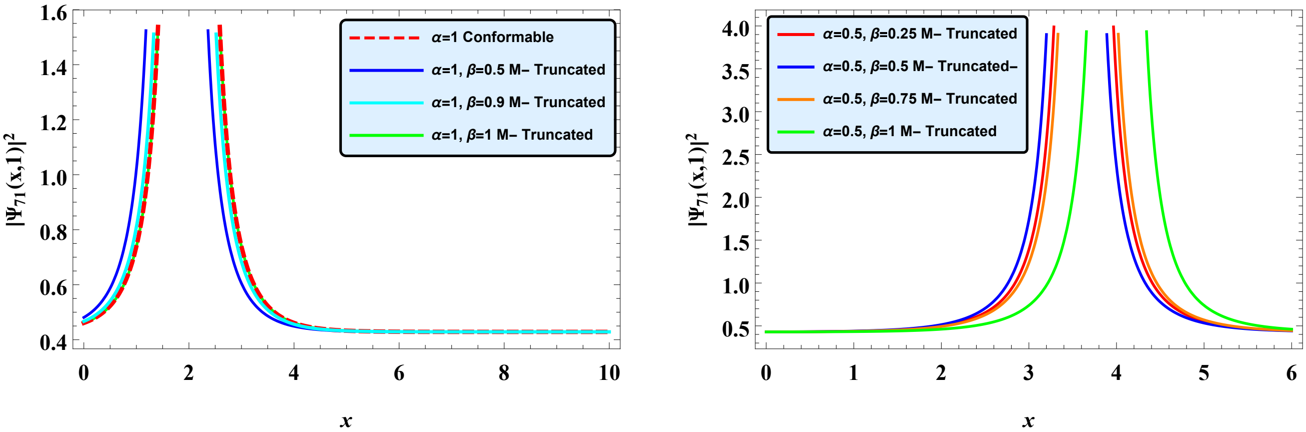


(c) Graph of Eq. (5.101) for different α values and $\beta = 0.3$.



(d) 2*D* comparison for different α values and $\beta = 0.3$.

Figure 2. Solution comparison for the Eq. (5.41) and the Eq. (5.101) when $c = -3, k = 1, \delta = 2$, and $\beta = 0.3$.



(a) $2D$ comparison between the Eq. (5.50) and the Eq. (5.110)

(b) Comparison of the graph of the Eq. (5.110) for $\alpha = 0.5$ and various β.

Figure 3. Solution comparison for $c = -3, k = 1$, and $\delta = 2$.

Considering the Eq. (5.64) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{11}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\tan\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(2-8k^2\right)-2}}e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}, \tag{5.65}$$

$$\Psi_{12}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\tan\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(2-8k^2\right)-2}}e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}, \tag{5.66}$$

$$\chi_{11}(x,t) = \chi_{12}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)\tan^2\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)}{\sqrt{2}}\right]}{(1-4k^2)\left(\delta\left(2-8k^2\right)-2\right)}. \tag{5.67}$$

These solutions are corresponding to *trigonometric* soliton solutions in the literature and valid for $\left(c+k^2\right) > 0$.

Case 2:

$$\mu = \pm\sqrt{2}\sqrt{c+k^2}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}, \omega_2 = \omega_2, \omega_1 = \pm\omega_2. \tag{5.68}$$

Substituting the parameters in *Case 2* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_2(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(2-8k^2\right)-2}}\frac{\sin\left[\sqrt{2}\xi\sqrt{c+k^2}\right]}{\left[\omega_1+\omega_2\cos\left(\sqrt{2}\xi\sqrt{c+k^2}\right)\right]}. \tag{5.69}$$

Considering the Eq. (5.69) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{21}(x,t)=\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}}{\sqrt{\delta(2-8k^2)-2}\left[1+\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right)\right]}, \tag{5.70}$$

$$\Psi_{22}(x,t)=-\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}}{\sqrt{\delta(2-8k^2)-2}\left[1+\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right)\right]}, \tag{5.71}$$

$$\Psi_{23}(x,t)=-\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}}{\sqrt{\delta(2-8k^2)-2}\left[1-\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right)\right]}, \tag{5.72}$$

$$\Psi_{24}(x,t)=\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}}{\sqrt{\delta(2-8k^2)-2}\left[1-\cos\left(\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right)\right]}, \tag{5.73}$$

$$\chi_{21}(x,t)=\chi_{22}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)}{(1-4k^2)(\delta(2-8k^2)-2)}\frac{\sin^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right]}{\left[1+\cos\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2kt^{\alpha}}{\alpha}\right)\right]\right]^2}, \tag{5.74}$$

$$\chi_{23}(x,t)=\chi_{24}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)\sin^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]}{(1-4k^2)(\delta(2-8k^2)-2)\left(1-\cos\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]\right)^2}. \tag{5.75}$$

These solutions are valid for $(c+k^2)>0$.

Suppose that solutions of the Eq. (4.7) is in the form:

$$Y(\xi)=\frac{\omega_0\cos[\mu\xi]}{\omega_2+\omega_1\sin[\mu\xi]}. \tag{5.76}$$

Substituting the Eq. (5.76) into the Eq. (4.7) to get the algebraic equations formed by collecting all terms comprising the same powers of $sin[\mu\xi]^m$.

$$\begin{aligned}
sin(\mu\xi)^2:\quad & 2\omega_0\omega_1^2ck^2-\frac{1}{2}\omega_0\omega_1^2c+\omega_0^3\delta-4\omega_0^3\delta k^2+2\omega_0\omega_1^2k^4-\frac{1}{2}\omega_0\omega_1^2k^2-\omega_0^3=0,\\
sin(\mu\xi)^1:\quad & -8\omega_0\omega_1\omega_2ck^2+2\omega_0\omega_1\omega_2c+4\omega_0\omega_1\omega_2k^2\mu^2-8\omega_0\omega_1\omega_2k^4+2\omega_0\omega_1\omega_2k^2\\
& -\omega_0\omega_1\omega_2\mu^2=0,\\
sin(\mu\xi)^0:\quad & -2\omega_0\omega_1^2ck^2-8\omega_0\omega_2^2ck^2+\frac{1}{2}\omega_0\omega_1^2c+2\omega_0\omega_2^2c+3\omega_0^3\delta-12\omega_0^3\delta k^2+16\omega_0\omega_1^2k^2\mu^2\\
& -8\omega_0\omega_2^2k^2\mu^2-2\omega_0\omega_1^2k^4-8\omega_0\omega_2^2k^4+\frac{1}{2}\omega_0\omega_1^2k^2+2\omega_0\omega_2^2k^2-4\omega_0\omega_1^2\mu^2\\
& +2\omega_0\omega_2^2\mu^2-3\omega_0^3=0.
\end{aligned} \tag{5.77}$$

After solving the system of equations in the Eq. (5.77) via Mathematica, the following cases are derived:

Case 3:

$$\mu = \pm\frac{\sqrt{c+k^2}}{\sqrt{2}}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}, \omega_1 = \omega_1, \omega_2 = 0. \tag{5.78}$$

Substituting the parameters in *Case 3* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_3(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cot\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta(2-8k^2)-2}}. \tag{5.79}$$

Considering the Eq. (5.79) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{31}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cot\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta(2-8k^2)-2}}e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}, \tag{5.80}$$

$$\Psi_{32}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cot\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta(2-8k^2)-2}}e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}, \tag{5.81}$$

$$\chi_{31}(x,t) = \chi_{32}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)\cot^2\left[\frac{\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{(1-4k^2)\left(\delta(2-8k^2)-2\right)}. \tag{5.82}$$

These solutions are corresponding to *trigonometric* soliton solutions in the literature and valid for $(c+k^2) > 0$.

Case 4:

$$\mu = \pm\sqrt{2}\sqrt{c+k^2}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta(2-8k^2)-2}}, \omega_2 = \omega_2, \omega_1 = \pm\omega_2. \tag{5.83}$$

Substituting the parameters in *Case 4* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_4(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cos\left[\sqrt{2}\xi\sqrt{c+k^2}\right]}{\sqrt{\delta(2-8k^2)-2}\left(\omega_1+\omega_2\sin\left[\sqrt{2}\xi\sqrt{c+k^2}\right]\right)}. \tag{5.84}$$

Considering the Eq. (5.84) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{41}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cos\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta(2-8k^2)-2}\left(\sin\left[1+\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.85}$$

$$\Psi_{42}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cos\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta(2-8k^2)-2}\left(\sin\left[1+\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.86}$$

$$\Psi_{43}(x,t)=-\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cos\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}}{\sqrt{\delta\left(2-8k^2\right)-2}\left(\sin\left[1-\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]\right)}, \tag{5.87}$$

$$\Psi_{44}(x,t)=\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\cos\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^{\alpha}}{\alpha}+kx\right)}}{\sqrt{\delta\left(2-8k^2\right)-2}\left(\sin\left[1-\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]\right)}, \tag{5.88}$$

$$\chi_{41}(x,t)=\chi_{42}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)\cos^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]}{(1-4k^2)\left(\delta\left(2-8k^2\right)-2\right)\left(1+\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]\right)^2}, \tag{5.89}$$

$$\chi_{43}(x,t)=\chi_{44}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)\cos^2\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]}{(1-4k^2)\left(\delta\left(2-8k^2\right)-2\right)\left(1-\sin\left[\sqrt{2}\sqrt{c+k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)\right]\right)^2}. \tag{5.90}$$

These solutions are valid for $(c+k^2)>0$.

5.2.2. Application of the extended rational *sinh* − *cosh* method to the $M-$ truncated Zakharov system

Suppose that solutions of the Eq. (4.7) is in the form:

$$Y(\xi)=\frac{\omega_0\sinh\left[\mu\,\xi\right]}{\omega_2+\omega_1\cosh\left[\mu\,\xi\right]}. \tag{5.91}$$

Substituting the Eq. (5.91) into the Eq. (4.7) to get the algebraic equations formed by collecting all terms comprising the same powers of $cosh[\mu\,\xi]^m$.

$$\begin{aligned}
cosh(\mu\,\xi)^2:\quad & -4\omega_0\omega_1^2ck^2+\omega_0\omega_1^2c+2\omega_0^3\delta-8\omega_0^3\delta k^2-4\omega_0\omega_1^2k^4+\omega_0\omega_1^2k^2-2\omega_0^3=0,\\
cosh(\mu\,\xi)^1:\quad & -16\omega_0\omega_1\omega_2ck^2+4\omega_0\omega_1\omega_2c-8\omega_0\omega_1\omega_2k^2\mu^2-16\omega_0\omega_1\omega_2k^4+4\omega_0\omega_1\omega_2k^2\\
& +2\omega_0\omega_1\omega_2\mu^2=0,\\
cosh(\mu\,\xi)^0:\quad & -4\omega_0\omega_1^2ck^2-16\omega_0\omega_2^2ck^2+\omega_0\omega_1^2c+4\omega_0\omega_2^2c-6\omega_0^3\delta+24\omega_0^3\delta k^2-32\omega_0\omega_1^2k^2\mu^2\\
& +16\omega_0\omega_2^2k^2\mu^2-4\omega_0\omega_1^2k^4-16\omega_0\omega_2^2k^4+\omega_0\omega_1^2k^2+4\omega_0\omega_2^2k^2+8\omega_0\omega_1^2\mu^2\\
& -4\omega_0\omega_2^2\mu^2+6\omega_0^3=0.
\end{aligned} \tag{5.92}$$

After solving the system of equations in the Eq. (5.92) via Mathematica, the following cases are derived:

Case 5:

$$\mu=\pm\frac{\sqrt{-c-k^2}}{\sqrt{2}},\omega_0=\pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}},\omega_1=\omega_1,\omega_2=0. \tag{5.93}$$

Substituting the parameters in *Case 5* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_5(\xi)=\pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\tanh\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^{\alpha}}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}}. \tag{5.94}$$

Considering the Eq. (5.94) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{51}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\tanh\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}}e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}, \tag{5.95}$$

$$\Psi_{52}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\tanh\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}}e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}, \tag{5.96}$$

$$\chi_{51}(x,t) = \chi_{52}(x,t) = -\frac{\left(4k^2-1\right)\left(c+k^2\right)\tanh^2\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)}. \tag{5.97}$$

These solutions are corresponding to *dark* soliton solutions in the literature and valid for $(-c-k^2) > 0$ and
$\left(4k^2-1\right)\left(c+k^2\right)\left(\delta\left(8k^2-2\right)+2\right) < 0.$

Case 6:

$$\mu = \pm\sqrt{2}\sqrt{-c-k^2}, \omega_0 = \pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}}, \omega_2 = \omega_2, \omega_1 = \pm\omega_2. \tag{5.98}$$

Substituting the parameters in *Case 6* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_6(\xi) = \pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sinh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}\left(\omega_1+\omega_2\cosh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]\right)}. \tag{5.99}$$

Considering the Eq. (5.99) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{61}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1+\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.100}$$

$$\Psi_{62}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1+\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.101}$$

$$\Psi_{63}(x,t) = -\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1-\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.102}$$

$$\Psi_{64}(x,t) = \frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1-\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.103}$$

$$\chi_{61}(x,t)=\chi_{62}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)\sinh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)\left(1+\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)^2}, \quad (5.104)$$

$$\chi_{63}(x,t)=\chi_{64}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)\sinh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)\left(1-\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)^2}. \quad (5.105)$$

These solutions are valid for $\left(-c-k^2\right)>0$.
Suppose that solutions of the Eq. (4.7) is in the form:

$$Y(\xi)=\frac{\omega_0\cosh\left[\mu\xi\right]}{\omega_2+\omega_1\sinh\left[\mu\xi\right]}. \quad (5.106)$$

Substituting the Eq. (5.106) into the Eq. (4.7) to get the algebraic equations formed by collecting all terms comprising the same powers of $sinh[\mu\xi]^m$.

$$\begin{aligned}
sinh(\mu\xi)^2 &: -2\omega_0\omega_1^2ck^2+\frac{1}{2}\omega_0\omega_1^2c+\omega_0^3\delta-4\omega_0^3\delta k^2-2\omega_0\omega_1^2k^4+\frac{1}{2}\omega_0\omega_1^2k^2-\omega_0^3=0,\\
sinh(\mu\xi)^1 &: -8\omega_0\omega_1\omega_2ck^2+2\omega_0\omega_1\omega_2c-4\omega_0\omega_1\omega_2k^2\mu^2-8\omega_0\omega_1\omega_2k^4+2\omega_0\omega_1\omega_2k^2+\omega_0\omega_1\omega_2\mu^2=0,\\
sinh(\mu\xi)^0 &: 2\omega_0\omega_1^2ck^2-8\omega_0\omega_2^2ck^2-\frac{1}{2}\omega_0\omega_1^2c+2\omega_0\omega_2^2c+3\omega_0^3\delta-12\omega_0^3\delta k^2+16\omega_0\omega_1^2k^2\mu^2+8\omega_0\omega_2^2k^2\mu^2\\
&\quad+2\omega_0\omega_1^2k^4-8\omega_0\omega_2^2k^4-\frac{1}{2}\omega_0\omega_1^2k^2+2\omega_0\omega_2^2k^2-4\omega_0\omega_1^2\mu^2-2\omega_0\omega_2^2\mu^2-3\omega_0^3=0.
\end{aligned} \quad (5.107)$$

After solving the system of equations in the Eq. (5.107) via Mathematica, the following cases are derived:
Case 7:

$$\mu=\pm\frac{\sqrt{-c-k^2}}{\sqrt{2}},\omega_0=\pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}},\omega_1=\omega_1,\omega_2=0. \quad (5.108)$$

Substituting the parameters in *Case 7* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_7(\xi)=\pm\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\coth\left[\frac{\xi\sqrt{-c-k^2}}{\sqrt{2}}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}}. \quad (5.109)$$

Considering the Eq. (5.109) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{71}(x,t)=\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\coth\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}}e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}, \quad (5.110)$$

$$\Psi_{72}(x,t)=-\frac{i\sqrt{4k^2-1}\sqrt{c+k^2}\coth\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}}e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}, \quad (5.111)$$

$$\chi_{71}(x,t)=\chi_{72}(x,t)=-\frac{\left(4k^2-1\right)\left(c+k^2\right)\coth^2\left[\frac{\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)}{\sqrt{2}}\right]}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)}. \tag{5.112}$$

These solutions are corresponding to *singular* soliton solutions in the literature and valid for $(-c-k^2)>0$.

Case 8:

$$\mu=\pm\sqrt{2}\sqrt{-c-k^2},\omega_0=\pm\frac{i\omega_1\sqrt{4k^2-1}\sqrt{c+k^2}}{\sqrt{\delta\left(8k^2-2\right)+2}},\omega_2=\omega_2,\omega_1=\pm\omega_2 i. \tag{5.113}$$

Substituting the parameters in *Case 8* to the Eq. (5.1), the solutions of Eq. (4.7) can be derived:

$$Y_8(\xi)=\pm\frac{\sqrt{4k^2-1}\sqrt{c+k^2}\cosh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]}{\sqrt{\delta\left(8k^2-2\right)+2}\left(\omega_1+i\omega_2\sinh\left[\sqrt{2}\xi\sqrt{-c-k^2}\right]\right)}. \tag{5.114}$$

Considering the Eq. (5.114) and the Eq. (4.4), the following solutions of the $M-$ truncated Zakharov system are obtained:

$$\Psi_{81}(x,t)=-\frac{\sqrt{4k^2-1}\sqrt{c+k^2}\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1+i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.115}$$

$$\Psi_{82}(x,t)=\frac{\sqrt{4k^2-1}\sqrt{c+k^2}\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1+i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.116}$$

$$\Psi_{83}(x,t)=\frac{\sqrt{4k^2-1}\sqrt{c+k^2}\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1-i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.117}$$

$$\Psi_{84}(x,t)-\frac{\sqrt{4k^2-1}\sqrt{c+k^2}\cosh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]e^{i\left(\frac{c\Gamma(\beta+1)t^\alpha}{\alpha}+kx\right)}}{\sqrt{\delta\left(8k^2-2\right)+2}\left(1-i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)}, \tag{5.118}$$

$$\chi_{81}(x,t)=\chi_{82}(x,t)=\frac{\left(4k^2-1\right)\left(c+k^2\right)\cosh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)\left(1+i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)^2}, \tag{5.119}$$

$$\chi_{83}(x,t)=\chi_{84}(x,t)=\frac{\left(4k^2-1\right)\left(c+k^2\right)\cosh^2\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]}{(1-4k^2)\left(\delta\left(8k^2-2\right)+2\right)\left(1-i\sinh\left[\sqrt{2}\sqrt{-c-k^2}\left(x-\frac{2k\Gamma(\beta+1)t^\alpha}{\alpha}\right)\right]\right)^2}. \tag{5.120}$$

These solutions are valid for $(-c-k^2)>0$.

6. Results and discussion

In this work, exact solutions of NLCGZDS for two different definitions of derivatives, conformable and $M-$ truncated derivatives, are studied by the extended rational $sine-cosine$ and $sinh-cosh$ method.
In Fig. 1, some plots of the solution $\chi_{51}(x,t)$ are demonstrated for $c=-3, k=1, \delta=2, \beta=0.3$, and $\alpha=0.5$. For example, in Fig. 1a and Fig. 1b, we depict 3D plots of Eq. (5.37) and Eq. (5.97), that are the solutions of the conformable and $M-$ truncated Zakharov system, respectively. In Fig. 1c, the solutions in Eq. (5.37) and Eq. (5.97) are compared on a single graph to show the difference between the solutions of the conformable and $M-$ truncated Zakharov system. In Fig. 1d, 2d comparison of Eq. (5.37) and Eq. (5.97) are given for $t=1, \alpha=0.5$ and $\beta=0.3$. We take $c=-3, k=1, \delta=2$, and $\beta=0.3$. in Fig. 2. In Fig. 2a, 3D plots of the Eq. (5.41) and the Eq. (5.101) which are some of the solutions of Zakharov system with conformable and $M-$ truncated derivatives, respectively, are compared. In Fig. 2b, 2D plot of the Eq. (5.41) is depicted for different values of α. 2D plot of the Eq. (5.101) is depicted for different values of α and fixed β in Fig. 2c. In Fig. 2d, we make a 2D comparison between the Eq. (5.41) and the Eq. (5.101) for some α and β values. In Fig. 3, we assume $c=-3, k=1$, and $\delta=2$. In Fig. 3a, it is showed that the solutions of the $M-$ truncated Zakharov system coincide with the solutions of conformable Zakharov system for a fixed value of α and $\beta=1$. In Fig. 3b, the 2D plots of the Eq. (5.110), one of the solutions of the considered PDE with $M-$ truncated derivative, is plotted for fixed $\alpha=0.3$ and various values of the β.
We have demonstrated the 3D and 2D plots to grasp clearly and comprehensively the physical properties of the constructed topological, singular solitons, periodic wave and singular periodic wave solutions, under the choice of the suitable values of parameters and different fractional values of α, the 3D and 2D are plotted. The perspective view of the topological soliton could be viewed in Fig. 1a and Fig. 1c under $\alpha=0.5$ for solutions Eq. (5.37) and Eq. (5.97). The perspective view of the singular soliton could be viewed in Fig. 1a under $\alpha=0.2$ for solutions Eq. (5.41) and Eq. (5.101). The propagation patterns of the wave for the topological, singular solitons and periodic wave solution along the x-axis for are depicted in the 2D plots through Fig. 1a,Fig. 1b and Fig. 1c with different labeled fractional values of α.

Furthermore, these outcomes have physical implications, such as the hyperbolic tangent appearing in the computation of magnetic moment and rapidity of special relativity, and the hyperbolic cotangent appearing in the Langevin function for magnetic polarization [50].

7. Conclusions

This paper investigates the analytical solutions of the NLCGZDS models with two types of derivatives called conformable and $M-$ truncated via the extended rational $sine-cosine$ and $sinh-cosh$. Using appropriate wave transformations, the PDE system is turned into an ODE system. The solutions of the ODE system are assumed in the forms of the extended rational $sine-cosine$ and $sinh-cosh$. A system of algebraic equations is derived by substituting the solutions to the ODE system, and by doing some calculations. When the system is solved, one finds the unknown coefficients in the rational form, and so, solutions of the PDE system. The novel solutions of NLCGZDS models including the different types of derivatives are derived to our best knowledge. The obtained solutions of conformable and $M-$

truncated system are compared in 2D and 3D figures to analyze the behavior of the two models. The technique is powerful and easily adaptable for other non-linear problems as well. It is expected that the new solutions of the considered system would be helpful for future papers in diverse science areas that use nonlinear partial differential equations nonlinear problems.

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Conflict of interest

The authors declare there is no conflicts of interest.

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