Research article

A cooperative control of wireless power transfer system using two transmitters

Takuya Hirata[1]**, Ichijo Hodaka**[1,*]**and Lunde Ardhenta**[2]

[1] Faculty of Engineering, University of Miyazaki, 1-1 Gakuen Kibanadai-nishi, Miyazaki, 889-2192, Japan

[2] Department of Materials and Informatics, Interdisciplinary Graduate School of Agriculture and Engineering, 1-1 Gakuen Kibanadai-nishi, Miyazaki, 889-2192, Japan

* **Correspondence:** Email: hijhodaka@cc.miyazaki-u.ac.jp.

Abstract: A wireless power transfer system with a single transmitter fed by a voltage source and a single receiver with a load is widely studied to increase load power. A system with two transmitters is also studied, especially by controlling currents through transmitter coils to maximize load power, which means current sources to drive transmitter coils are assumed instead of the voltage source. This paper presents a method for adjusting voltage amplitude and phase of each voltage source embedded in two transmitters to maximize load power. The maximized load power is also presented in an explicit form based on circuit constants of the system. As a result, advantages of the two transmitters system over the single transmitter system are revealed from the perspective of efficient power delivery.

1. Introduction

A wireless power transfer (WPT) system enables power delivery without electric wires and physical contacts, which is expected to be applied in various fields. For instance, power delivery for medical implants [1, 2] and electric vehicles (EVs) [3–8] have been studied. Review papers such as [6] and [7] especially prove that many studies of the system for EVs exist, which also indicates an active application field of the system. One of reasons why EVs are the active application field of the system is that EVs are transportation without emitting air pollutants in operation, which are expected to play a key role in reducing environmental impact. There are still challenges for widespread use of EVs. For example, charging spots are relatively few, and charging takes a relatively long time [9]. In addition, increasing battery size to extend driving range of the EVs cause heavy weight of EVs, which may

increase damage to paved roads [10]. If the embedded WPT systems in roads can charge EVs in operation, it leads to reduced charging time and increase of the weight. Since the WPT system is considered a promising technology to solve the challenges, many studies have been conducted. In particular, [8] presents a tuning method for wireless charging of EVs in operation with multiple coils for reducing electromagnetic interference and enhancing magnetic coupling compared to a single coil configuration. Since the coil configurations such as the number of coils, placement etc. affect power delivery, many types of the configurations have been proposed [3]. [11] has reported pentagon coils achieve high energy transfer efficiency under coil misalignment conditions compared with other coil shapes such as square and Double-D coils. In addition, [5] uses not only multiple coils but also multiple voltage sources. This configuration contributes to improving reliability of the system compared to a single voltage source configuration. In the case of medical implants, supplying power to them inside the human body through wires and contacts requires surgical intervention, which may involve risks. Therefore it is desirable to transfer power without the need of physical connections, that is, it is better that the medical implant has the WPT system. [1] and [2] are focused on reducing the size of the WPT system for implantation inside the body since the size of the medical implant should also be taken into consideration to avoid interference with other organs. [1] proposes an impedance matching method without additional components, while [2] introduces single-turn printed coils in the system and proposes the optimization procedure involving self and mutual inductances, parasitic resistance and capacitance of the coils. The WPT systems are not only expected to be applied in the above situations where it is hard to transfer power with such connections but also to enhance user convenience because of eliminating the need to plug and unplug cables in the case of charging devices such as EVs during parking and mobile phones. [12] considers the case of charging a mobile battery using the WPT system.

It is found that the system with two transmitters and a single receiver has certain advantages over the system with a single transmitter in practical usages, and studies for the system with two or more transmitters and/or two receivers have been conducted. For example, the system with two transmitters can improve power transfer efficiency. [13] uses two transmitters are utilized for power delivery to a single receiver and demonstrates a successful case of improving the power transfer efficiency compared to a single transmitter and receiver by changing current phase of each transmitter coil. [14] presents a criterion for selecting one or two transmitters configuration based on magnetic coupling coefficients between each transmitter coil and the receiver coil to improve the power transfer efficiency. In more than two multiple transmitters, assuming that magnetic couplings among transmitter coils are neglected, a method for adjusting voltage magnitude and voltage phase of each voltage source in the transmitters to improve the power transfer efficiency is presented in an explicit form [15]. In addition, [16] presents a load optimization method for maximizing the power transfer efficiency in a system with two or more transmitters and the single receiver. In a system with two transmitters and receivers, to improve the power transfer efficiency, [17] presents approximate solutions to a voltage ratio of voltage sources and load resistance in the case when magnetic couplings between transmitter coils and between receiver coils are neglected.

The system with two transmitters has more components compared to the single transmitter, which means that it needs to adjust more components to deliver high power. For instance, if two transmitters each have a voltage source, and we need to select voltage amplitude and phase of each voltage source. This is because some selections of the voltage amplitude and phase may cause zero receiver current and power delivery due to cancellation of magnetic field at the receiver coil [18]. Moreover, although the

power delivery does not drop to zero, it does not guarantee high power delivery. A proper selection of the voltage amplitude and phase for high power delivery depends on a circuit configuration including a relative position of the coils, and it is not trivial. To handle this situation, in the system, [18] presents a method for adjusting voltage amplitude and phase of each voltage source to maximize or minimize load power. For minimizing the load power, the method is expressed in an explicit form, and for maximizing the load power, it provides a proper phase selection. However, in a proper amplitude selection for maximizing the load power, it states only that we should adjust the maximum available amplitude of each voltage source. To maximize the load power, we should consider a voltage amplitude ratio between voltage sources. In [19], it presents the method for maximizing the load power via maximizing a current through a receiver coil in a receiver in which all components are connected in series by controlling currents through transmitter coils. However, further calculations are needed to pull the required currents from voltage sources, and thus, it does not directly express how to select the voltage amplitude and phase. In the system with multiple transmitters and receivers, [20] presents a method for maximizing currents through receiver coils by adjusting voltages and currents of transmitter coils. However, it also needs further calculations to find selection of the voltage amplitude and phase of each voltage source. Thus, a method for directly determining voltage amplitude and phase of each voltage source to maximize load power has not been clearly presented in a system with multiple transmitters and/or receivers.

In this paper, we propose a method for maximizing load power via adjusting the root mean square (RMS) value of voltage amplitude and phase of each voltage source in the system with two transmitters and a single receiver, which is expressed in an explicit form based on the circuit constants of the system. To measure the effectiveness of the proposed method, numerical simulations are performed by LTspice for two cases that adjust the voltage with and without the proposed method. This paper also aims to clarify advantages of the system compared with a conventional single transmitter fed by a voltage source.

2. Maximizing load power with two transmitters

We consider the WPT system with two transmitters and a single receiver shown in Figure 1 called a Multi Input Single Output (MISO) configuration. L_1, L_2, and L_3 denote self inductances of coils 1, 2, and 3 respectively. M_{pq}'s denote mutual inductance between coil p and q, where $p \neq q$, and $p, q \in \{1, 2, 3\}$. C_1, C_2, and C_3 denote capacitances of capacitors 1, 2, and 3 respectively. R_L denotes a pure resistive load. V_1 and V_2 denote the voltages of the voltage sources which generate sinusoidal voltages with the same angular frequency ω in phasor domain receptively. R_{O1} and R_{O2} denote internal resistances of the voltage sources respectively. I_1, I_2, and I_3 denote circuit currents respectively. We call an electric power of load R_L load power P_L. In this paper, our goal is to find a method for maximizing the load power P_L via adjusting voltages V_1 and V_2, and we take the following approach: as a first step, we express the load power P_L by circuit constants and the voltages V_1 and V_2. As a second step, we clarify the relationships between V_1 and V_2 and circuit constants based on the load power P_L obtained the previous step. Finally, we find how V_1 and V_2 should be set based on circuit parameters to maximize the load power P_L. We first represent the behavior of the WPT system by circuit constants and the voltages V_1 and V_2 to derive the load power P_L expressed in circuit constants
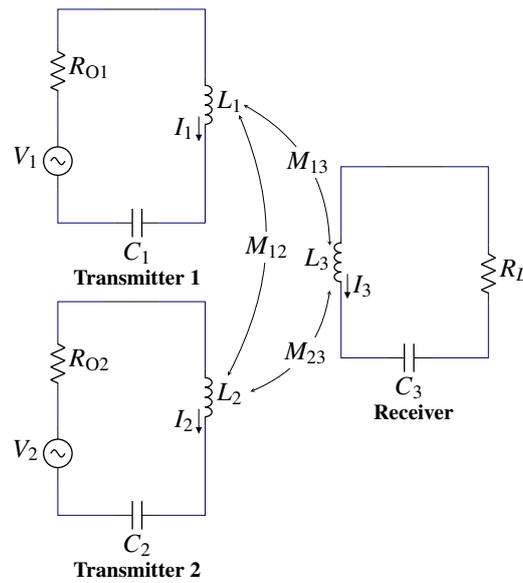
Figure 1. The WPT system with two transmitters and a single receiver.

and the voltages. The behavior of the WPT system is expressed in

$$
\begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix} = \begin{bmatrix} z_1 & j\omega M_{12} & j\omega M_{13} \\ j\omega M_{12} & z_2 & j\omega M_{23} \\ j\omega M_{13} & j\omega M_{23} & z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}
\tag{2.1}
$$

where $z_1 = R_{O1} + j(\omega L_1 - 1/(\omega C_1))$, $z_2 = R_{O2} + j(\omega L_2 - 1/(\omega C_2))$, $z_3 = R_L + j(\omega L_3 - 1/(\omega C_3))$ and j denotes an imaginary unit [18]. In the WPT system, the load power P_L is expressed in $P_L = R_L|I_3|^2$. Therefore, we first solve Eq. (2.1) for the current I_3 by Cramer's rule, and then we can express the load power P_L in the following form based on circuit constants with the voltages V_1 and V_2:

$$
P_L = \frac{R_L}{\alpha} |\beta_1 V_1 + \beta_2 V_2|^2
\tag{2.2}
$$

where α, β_1, and β_2 correspond to Eqs. (2.3), (2.4) and (2.5) respectively, and det denotes determinant.

$$
\alpha = \left| \det \begin{bmatrix} z_1 & j\omega M_{12} & j\omega M_{13} \\ j\omega M_{12} & z_2 & j\omega M_{23} \\ j\omega M_{13} & j\omega M_{23} & z_3 \end{bmatrix} \right|^2,
\tag{2.3}
$$

$$
\beta_1 = \omega(M_{13}(-jR_{O2} + \omega L_2 - 1/(\omega C_2)) - M_{12}M_{23}\omega),
\tag{2.4}
$$

$$
\beta_2 = \omega(M_{23}(-jR_{O1} + \omega L_1 - 1/(\omega C_1)) - M_{12}M_{13}\omega)
\tag{2.5}
$$

We note that α, β_1, and β_2 do not include the voltages V_1 and V_2, which are represented by circuit constants.

We consider a method to maximize load power P_L by adjusting the voltages V_1 and V_2 of the voltage sources. In linear algebra, it is well known that a symbolic solution exists for the problem for maximizing P_L expressed as shown in Eq. (2.2) by V_1 and V_2 under a constraint that $|V_1|^2 + |V_2|^2$ is constant, and therefore, to configure an optimization problem, we impose a constraint $|V_1|^2 + |V_2|^2 = g^2$

where g is any positive constant. In sum, for a given $g > 0$, we consider the optimization problem that is

$$\max_{V_1,\ V_2 \in \mathbb{C}} R_L |I_3|^2$$
$$\text{subject to Eq. (2.1)}, \ |V_1|^2 + |V_2|^2 = g^2. \tag{2.6}$$

The solution to the optimization problem Eq. (2.6) is

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{g e^{j\theta}}{\sqrt{1 + |d|^2}} \begin{bmatrix} d \\ 1 \end{bmatrix}, \tag{2.7}$$

where θ is any real number and d is expressed in Eq. (2.8):

$$d = \frac{M_{13} (\omega L_2 - 1/(\omega C_2)) + j M_{13} R_{O2} - \omega M_{12} M_{23}}{M_{23} (\omega L_1 - 1/(\omega C_1)) + j M_{23} R_{O1} - \omega M_{12} M_{13}}, \tag{2.8}$$

and then, if we use the voltages given by Eq. (2.7) for the voltage sources, the load power P_L reaches its maximum value, and the maximum value of the load power $P_{L\max}$ is represented in Eq. (2.9):

$$P_{L\max} = \frac{a_2 \omega^4 + a_1 \omega^3 + a_0 \omega^2}{b_5 \omega^6 + b_4 \omega^5 + b_3 \omega^4 + b_2 \omega^3 + b_1 \omega^2 + b_0} R_L g^2, \tag{2.9}$$

where a_i's and b_i's are given in Eqs. (2.10)-(2.18) with u_i's in Eqs. (2.19)-(2.21).

$$a_0 = C_2 \left(M_{23}^2 \left(R_{O1}^2 + u_1^2 \right) + M_{13}^2 \left(R_{O2}^2 + u_2^2 \right) \right)$$
$$+ 2 M_{12} M_{13} M_{23} \tag{2.10}$$

$$a_1 = -2 C_2 M_{12} M_{13} M_{23} u_1 \tag{2.11}$$

$$a_2 = C_2 M_{12} \left(M_{12} (M_{13}^2 + M_{23}^2) - 2 L_2 M_{13} M_{23} \right) \tag{2.12}$$

$$b_0 = C_2 \left(R_L^2 + u_3^2 \right) \left(R_{O1}^2 + u_1^2 \right) \left(R_{O2}^2 + u_2^2 \right) \tag{2.13}$$

$$b_1 = -2 C_2 (-(M_{12}^2 (R_L^2 + u_3^2)(R_{O1} R_{O2} - u_1 u_2))$$
$$+ M_{13}^2 (R_{O2}^2 + u_2^2)(u_1 u_3 - R_L R_{O1})$$
$$- M_{23}^2 (R_{O1}^2 + u_1^2)(R_L R_{O2} - u_2 u_3)) \tag{2.14}$$

$$b_2 = -4 C_2 M_{12} M_{13} M_{23} (R_L (u_2 R_{O1} + u_1 R_{O2})$$
$$+ u_3 (R_{O1} R_{O2} - u_1 u_2)) \tag{2.15}$$

$$b_3 = C_2 (2 M_{12}^2 (M_{23}^2 (R_L R_{O1} + u_1 u_3)$$
$$+ M_{13}^2 (R_L R_{O2} + u_2 u_3)) + M_{12}^4 (R_L^2 + u_3^2)$$
$$+ 2 M_{13}^2 M_{23}^2 (R_{O1} R_{O2} + u_1 u_2)$$
$$+ M_{23}^4 (R_{O1}^2 + u_1^2) + M_{13}^4 (R_{O2}^2 + u_2^2)) \tag{2.16}$$

$$b_4 = -4 C_2 M_{12} M_{13} M_{23} \left(M_{12}^2 u_3 + M_{23}^2 u_1 + M_{13}^2 u_2 \right) \tag{2.17}$$

$$b_5 = 4 C_2 M_{12}^2 M_{13}^2 M_{23}^2 \tag{2.18}$$

$$u_1 = \omega L_1 - \frac{1}{\omega C_1} \tag{2.19}$$

$$u_2 = \omega L_2 - \frac{1}{\omega C_2} \tag{2.20}$$

$$u_3 = \omega L_3 - \frac{1}{\omega C_3} \tag{2.21}$$

We note that any solution to the optimization problem Eq. (2.6) is represented in Eq. (2.7). Thus, if we set the voltages of voltage sources to Eq. (2.7) for a given $g > 0$ and a real number θ, the load power P_L reaches $P_{L\text{max}}$. θ plays a role as an initial phase of each voltage. If θ is changed, all voltages and currents in the circuits are simultaneously shifted by θ/ω in the time domain. If a real number is chosen as θ, we set V_1 and V_2 to Eq. (2.7) with θ and find the load power P_L, and the same load power P_L is obtained even if another real number is chosen as θ. Thus, it is enough to choose a real number as a representative value for θ. In the proposed method below, we set $\theta = 0$, as a representative value for θ. g is a scaling factor to adjust load power to the required value. For a given g, under the constraint $|V_1|^2 + |V_2|^2 = g^2$ represented by the voltage amplitude of each voltage source, the load power P_L reaches its maximum value $P_{L\text{max}}$ if we set the voltages of voltage sources to Eq. (2.7). Hence, if the load power P_L does not satisfy the required value, we need to readjust g until the load power P_L satisfies it.

We state an outline of proof for the solution Eq. (2.7) to the optimization problem Eq. (2.6) in appendix A.

The proposed method
We consider the WPT system with two transmitters and a single receiver shown in Figure 1. To maximize load power $P_L = R_L |I_3|^2$ by adjusting the voltages V_1 and V_2 in phasor domain, we should select V_1 and V_2 as following procedures:

1. Let the values of L_1, L_2, C_1, C_2, M_{12}, M_{13}, M_{23}, ω, R_{O1}, R_{O2}, and R_L be given.
2. Substitute the values into Eq. (2.8) to obtain d.
3. Substitute $g = 1$ and $\theta = 0$ into Eq. (2.7) to obtain the V_1 and V_2.
4. Substitute V_1 and V_2 into the load power $P_L = R_L |I_3|^2$ by Eq. (2.1).
5. Use $\sqrt{P_{\text{req}}/P_L}\,V_1$, $\sqrt{P_{\text{req}}/P_L}\,V_2$ as the voltages to scale the load power just to a required load power P_{req} (if necessary).

The proposed method implies that to maximize load power P_L, the voltages of voltage sources V_1 and V_2 should maintain a ratio $V_1 : V_2 = d : 1$ and adjust voltage magnitude by g to satisfy required power delivery. $V_1 : V_2 = d : 1$ means that both phase condition which is $\arg V_1 - \arg V_2 = \arg d$ and voltage magnitude condition which is $|V_1|/|V_2| = |d|$ are needed to maximize P_L. The conventional method to maximize P_L in [18] considers essentially only the phase condition, although the constraint of that method is different from ours. That is, the voltage magnitude of each voltage source should be adjusted by maintaining the ratio $|V_1| : |V_2| = |d| : 1$ scaled by g as mentioned in Introduction.

3. Effectiveness of the proposed method and advantages of two transmitters

To measure effectiveness of the proposed method, and to clarify advantages of WPT system with two transmitters, we perform numerical simulations with two scenarios:
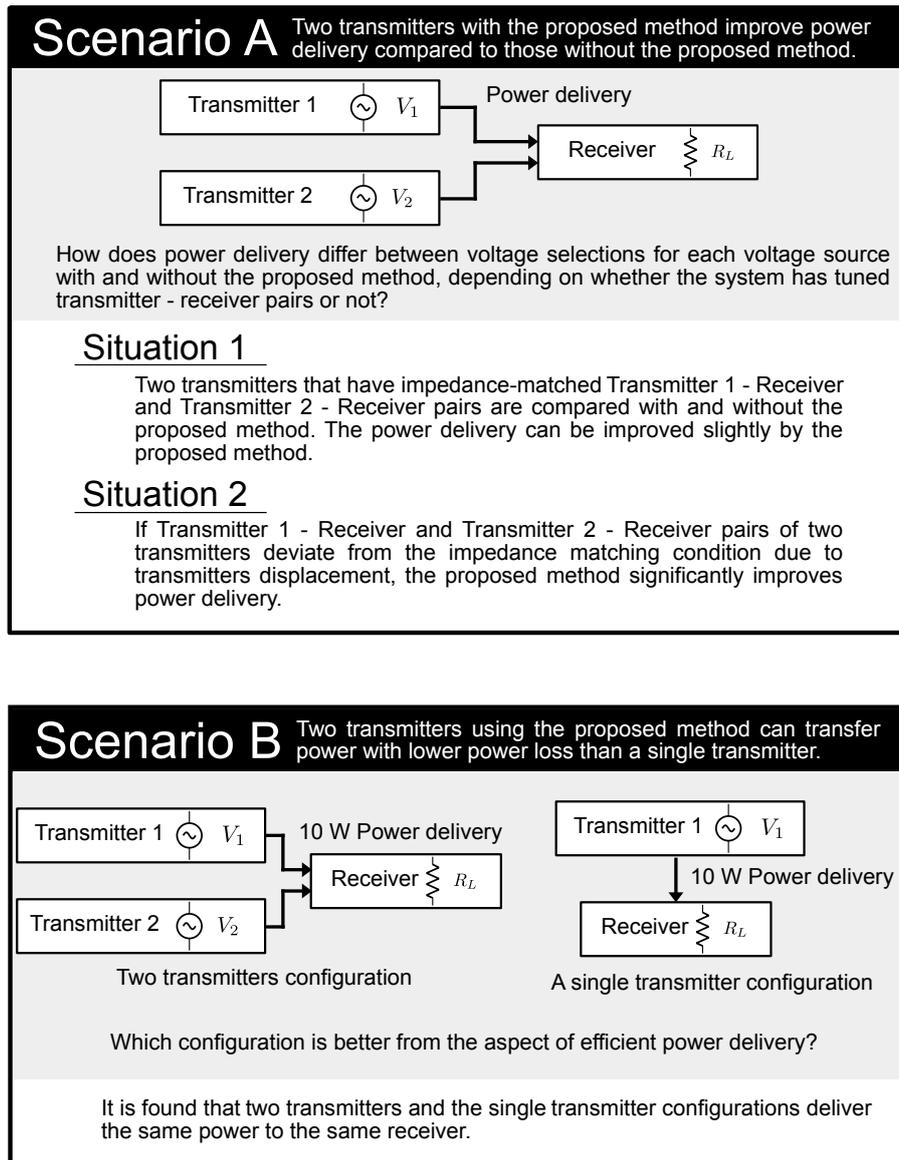
Figure 2. The overview of our Scenario A and B to investigate improvement of the WPT system using the proposed method.

- Scenario A:

  We measure the effectiveness of the proposed method with a WPT system involving two transmitters and a single receiver (i.e. a MISO configuration).

- Scenario B:

  We clarify characteristics of a WPT system involving a single transmitter and receiver system called a Single Input Single Output (SISO) configuration, and a WPT system with a MISO configuration by comparison between both systems. From the comparison, we clarify advantages of the WPT system with the MISO configuration.

Figure 2 provides the overview of our Scenario A and B.

In Scenario A, we consider two situations:

- Situation 1:

  We use a WPT system in a MISO configuration, which is shown in Figure 1 with circuit constants shown in Table 1. The pair of Transmitter 1 and Receiver, as well as the pair of Transmitter 2 and Receiver can transfer power as SISO configurations. Two SISO configured systems are merged into one WPT system forming the MISO configuration that would be expected to achieve higher power transfer than the SISO configurations. As mentioned in Introduction, the SISO configurations often are optimized using impedance matching to obtain high power delivery. Therefore, in this situation, impedance matching applies to both SISO configurations.

  In the WPT system with the MISO configuration, the numerical simulations are performed by LTspice to find load power for the two cases: one is using the proposed method to adjust the RMS value of voltage amplitude and phase of each voltage source. The other is not using the proposed method to decide the voltage amplitude and phase. For instance, as a first attempt, suppose we choose $V_1 = V_2 = 1$ V as the selection of each the voltage amplitude and phase in phasor domain. The selection is a simple choice to consider whether the WPT system can transfer more power with two transmitters than a single receiver or not.

  However if we select the voltage amplitude and phase by using the proposed method for the same WPT system, it would be expected that the WPT system can transfer more power than the case of voltage selections with $V_1 = V_2 = 1$ V. Since the first attempt is chosen $V_1 = V_2 = 1$ V, if the voltage selection is performed with the proposed method, we first calculate $|V_1|^2 + |V_2|^2 = 2$, and then, from comparison with the constraint in Eq. (2.6), we find $g = \sqrt{2}$. We choose $\theta = 0$ since the load power can be maximized as well as the case when θ is another real number. The voltage selection is found with the proposed method using $g = \sqrt{2}$ and $\theta = 0$, which are $V_1 \simeq 1.095$ V and $V_2 \simeq 894.4$ mV in phasor domain, and we find that voltage phase of V_1 is extremely close to zero radian in this case.

  We perform numerical simulations using LTspice with AC analysis, and simulation models that include above configurations are shown in Figure 3. We note that all parameters shown in Figure 3 are displayed with four significant figures, and the voltage magnitude and phase of each voltage source are displayed as voltage amplitude and degrees respectively. The mutual inductances are also displayed as coupling coefficients respectively.

- Situation 2:

  It is possible that the Transmitter 1 or 2 is moved due to placement reasons. Therefore, assuming that the Transmitter 2 moves toward the Transmitter 1 and Receiver, it causes that M_{12} and M_{23} increase 1% from the corresponding values in Table 1. Since the M_{12} and M_{23} change, Transmitter 2 - Receiver pair goes out of impedance matching condition, and it may lead to cause degrade power delivery than Situation 1.

  If the proposed method can enhance power delivery in Situation 2, since we do not add any extra components such as additional impedance matching circuit to internal or external system, it may be a better choice. In this situation, as well as Situation 1, we perform numerical simulations using voltage selections with/without the proposed method by LTspice. For voltage selections without the proposed method, we use $V_1 = V_2 = 1$ V as a first attempt as well as Situation 1. Therefore, for voltage selections with the proposed method using $g = \sqrt{2}$ and $\theta = 0$ which are derived by the same procedures in the case of Situation1 since the voltage selections without the
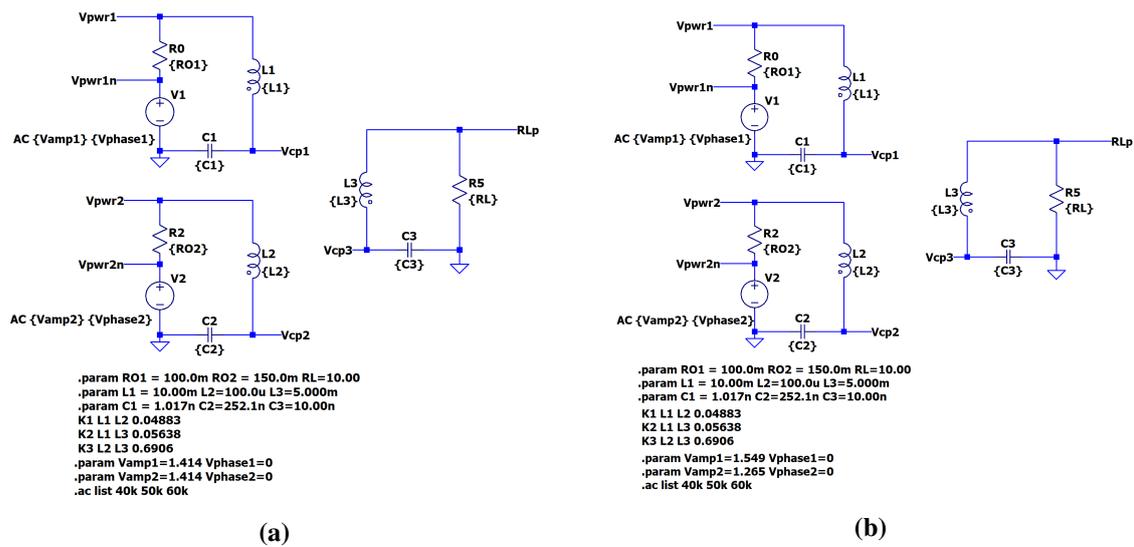
Figure 3. The simulation models and conditions in Situation 1: (a) without the proposed method (b) with the proposed method.

proposed method are $V_1 = V_2 = 1$ V, we use $V_1 \simeq 0.4995 + j1.257$ V and $V_2 \simeq 411.9$ mV in phasor domain. We note that V_1 has an imaginary part, which means the phase difference between V_1 and V_2 exists. The 1 % increased values to M_{12} and M_{23} are $M_{12} \simeq 49.32$ μH and $M_{23} \simeq 493.2$ μH respectively.

If Transmitter 1 is moved and M_{12} and M_{13} increase by 1% from the corresponding values in Table 1. Transmitter 1 - Receiver pair goes out of impedance matching condition. In this situation, we also compare the load power of the system with and without the proposed method. Similarly, suppose we use $V_1 = V_2 = 1$ V for the voltage selections without the proposed method, voltage selections with the proposed method using $g = \sqrt{2}$ and $\theta = 0$ which are derived by the same procedures as the case when Transmitter 2 is moved, which are $V_1 \simeq 0.2169 - j0.5459$ V and $V_2 \simeq 1.286$ V in phasor domain. The 1 % increased values to M_{12} and M_{13} are $M_{12} \simeq 49.32$ μH and $M_{13} \simeq 402.7$ μH respectively.

We perform numerical simulations using LTspice with AC analysis as well as Situation 1, and simulation models for the case when Transmitter 2 is moved are shown in Figure 4 and the case when Transmitter 1 is moved are shown in Figure 5. We also note that all parameters shown are displayed with four significant figures, and the voltage magnitude and phase of each voltage source are displayed as voltage amplitude and degrees respectively. The mutual inductances are also displayed as coupling coefficients respectively.

In sum, we set $V_1 = V_2 = 1$ V as the voltage selections without the proposed method, and we set $V_1 \simeq 1.095$ V and $V_2 \simeq 894.4$ mV as voltage selections with the proposed method, which is the first situation in Situation 1 of Scenario A. As Situation 2 of Scenario A, if Transmitter 2 is moved, the system is the same as Situation 1 except using $M_{12} \simeq 49.32$ μH and $M_{23} \simeq 493.2$ μH, and we set $V_1 = V_2 = 1$ V as the voltage selections without the proposed method, and we set $V_1 \simeq 0.4995 + j1.257$ V and $V_2 \simeq 411.9$ mV as voltage selections with the proposed method. Similarly, if Transmitter 1 is moved, M_{12} and M_{13} are changed to $M_{12} \simeq 49.32$ μH and $M_{13} \simeq 402.7$ μH respectively. We
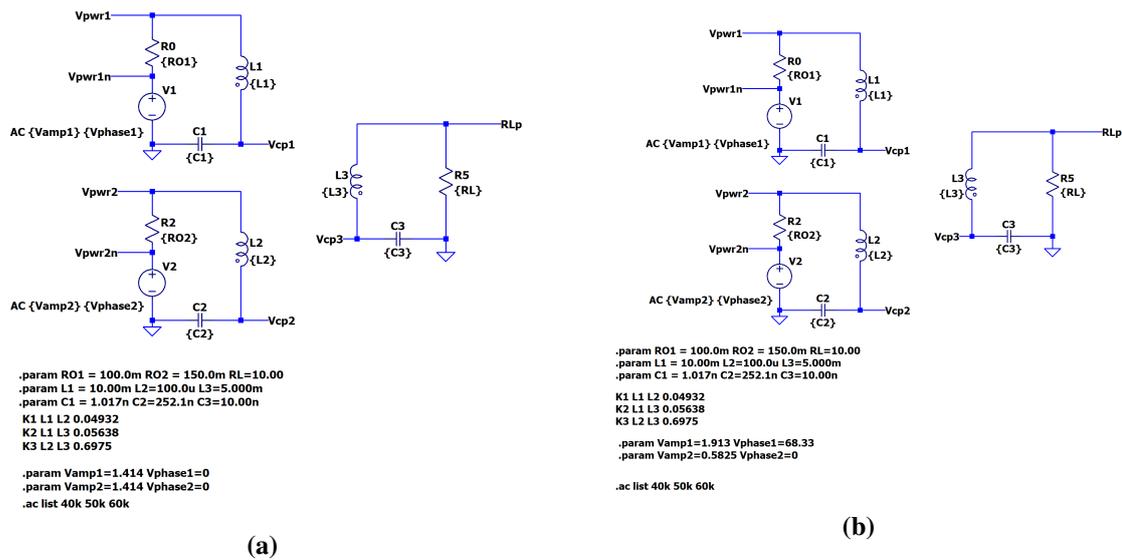
Figure 4. The simulation models and conditions in Situation 2 (if Transmitter 2 is moved): (a) without the proposed method (b) with the proposed method.

use $V_1 \simeq 0.2169 - j545.9\,\text{mV}$ and $V_2 \simeq 1.286\,\text{V}$ in phasor domain as the voltage selections with the proposed method. We use these numerical simulation conditions to measure the effectiveness of the proposed method.

Scenario B clarifies characteristics of both SISO configured and MISO configured WPT systems using the same WPT systems in Scenario A under 10 W as the required load power from perspective of power dissipation in the internal resistance of each voltage source, currents through circuits and each required RMS value of voltage amplitude. The reason why we choose 10 W as the required load power is that, if we consider wireless charging of mobile devices as one of the possible applications of the proposed method, such mobile devices typically comply with the Qi standard and are rated on the order of 10 W. Numerical simulations are performed for both SISO configured and MISO configured WPT systems by LTspice, and we note that the SISO configurations have two types; that is Transmitter 1 and Receiver, and Transmitter 2 and Receiver. If we simulate Transmitter 1 and Receiver, Transmitter 2 is eliminated from the circuit. It is also that Transmitter 1 is eliminated as we simulate Transmitter 2 and Receiver. Since we perform numerical simulations in the case of 10 W as required load power, we set $V_1 \simeq 2.000\,\text{V}$ and $V_2 \simeq 2.449\,\text{V}$ in the SISO configurations. To derive these voltages, we find the load power of each SISO configuration as $V_1 = V_2 = 1$ [V]. For Transmitter 1 - Receiver pair, the load power $P_{L1} = 2.5$ W, and for Transmitter 2 - Receiver pair equals to the load power $P_{L2} \simeq 1.667\,\text{W}$. In the configuration, each load power is proportional to the square of the voltage amplitude. Thus, each voltage to satisfy $P_{L1} = P_{L2} = 10$ W, and we solve $10 = 2.500|V_1|^2$ and $10 = 1.667|V_2|^2$ for V_1 and V_2. We have $V_1 \simeq \pm 2.000\,\text{V}$ and $V_2 \simeq \pm 2.449\,\text{V}$ as the solutions. The sign part of each solution denotes an initial phase, which does not contribute to load power. Therefore, we use $V_1 \simeq 2.000\,\text{V}$ and $V_2 \simeq 2.449\,\text{V}$. We set also $V_1 \simeq 1.800\,\text{V}$ and $V_2 \simeq 1.470\,\text{V}$ in the MISO configuration using the proposed method. The voltages in the MISO configuration can be derived by the following procedure of the proposed method:
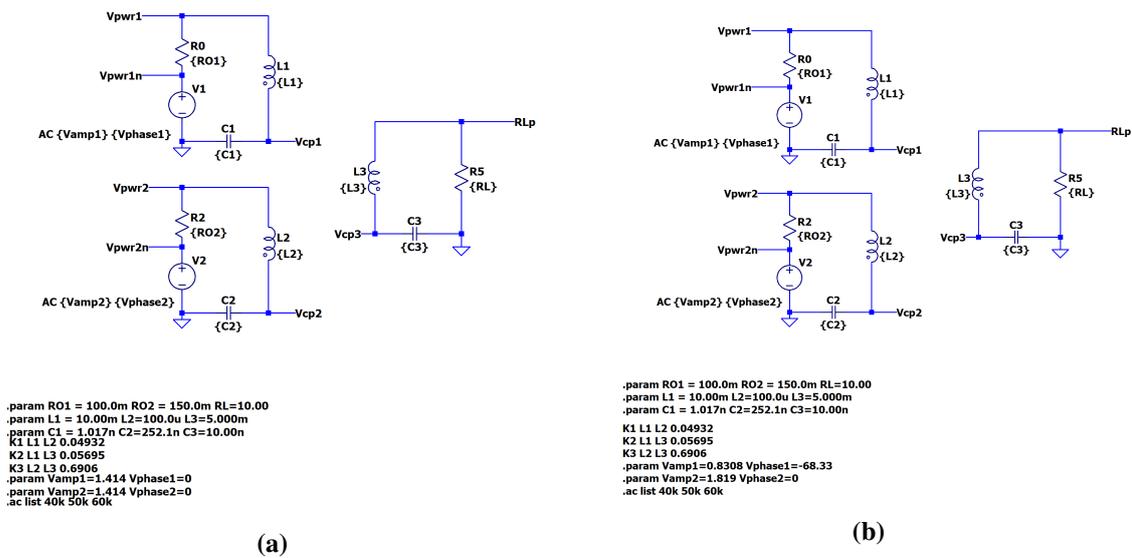
.param RO1 = 100.0m RO2 = 150.0m RL=10.00
.param L1 = 10.00m L2=100.0u L3=5.000m
.param C1 = 1.017n C2=252.1n C3=10.00n
K1 L1 L2 0.04932
K2 L1 L3 0.05695
K3 L2 L3 0.6906
.param Vamp1=1.414 Vphase1=0
.param Vamp2=1.414 Vphase2=0
.ac list 40k 50k 60k

(a)

.param RO1 = 100.0m RO2 = 150.0m RL=10.00
.param L1 = 10.00m L2=100.0u L3=5.000m
.param C1 = 1.017n C2=252.1n C3=10.00n
K1 L1 L2 0.04932
K2 L1 L3 0.05695
K3 L2 L3 0.6906
.param Vamp1=0.8308 Vphase1=-68.33
.param Vamp2=1.819 Vphase2=0
.ac list 40k 50k 60k

(b)

Figure 5. The simulation models and conditions in Situation 2 (if Transmitter 1 is moved): (a) without the proposed method (b) with the proposed method.

1. We set parameters such as L_1, L_2, C_1, C_2, M_{12}, M_{13}, M_{23}, ω, R_{O1}, R_{O2}, and R_L to Table 1.
2. We find d by Eq. (2.8) with the values: $d \simeq 1.225$.
3. We find V_1 and V_2 by Eq. (2.7): $V_1 \simeq \pm\, 774.6\,\mathrm{mV}$ and $V_2 \simeq \pm\, 632.5\,\mathrm{mV}$
4. We calculate P_L with the V_1 and V_2: $P_L \simeq 1.852\,\mathrm{W}$
5. We use $\sqrt{P_{\mathrm{req}}/P_L}\,V_1$, $\sqrt{P_{\mathrm{req}}/P_L}\,V_2$ with $P_{\mathrm{req}} = 10$ as the voltages to reach $P_L = 10$ W: $V_1 \simeq 1.800\,\mathrm{V}$ and $V_2 \simeq 1.470\,\mathrm{V}$

We note that these choices of all V_1 and V_2 are in phasor domain. In the case of our numerical simulation for the MISO configuration, we find that the voltage phase of V_1 is extremely close to zero radian. We note that we show only power of the internal resistance and load in the case of Transmitter 1 and Receiver as a result of power calculation by the numerical simulation in SISO configurations. This is because the power is the same between in the case of Transmitter 1 and Receiver, and in the case of Transmitter 2 and Receiver since Transmitter 1 as well as Transmitter 2 is impedance-matched to Receiver.

Table 1. WPT system parameters.

Parameter	Quantity	Parameter	Quantity
R_{O1}	100.0 mΩ	L_1	10.00 mH
C_1	1.017 nF	M_{13}	398.7 μH
R_{O2}	150.0 mΩ	L_2	100.0 μH
C_2	252.1 nF	M_{23}	488.3 μH
R_L	10.00 Ω	L_3	5.000 mH
C_3	10.00 nF	M_{12}	48.83 μH
ω	314.2 krad/s		

We also perform numerical simulations using LTspice with AC analysis, and simulation models that

include above configurations shown in Figure 6. As in Scenario A, we note that all parameters shown in Figure 6 are displayed with four significant figures, and the voltage magnitude and phase of each voltage source are displayed as voltage amplitude and degrees respectively, and the mutual inductances are also displayed as coupling coefficients.
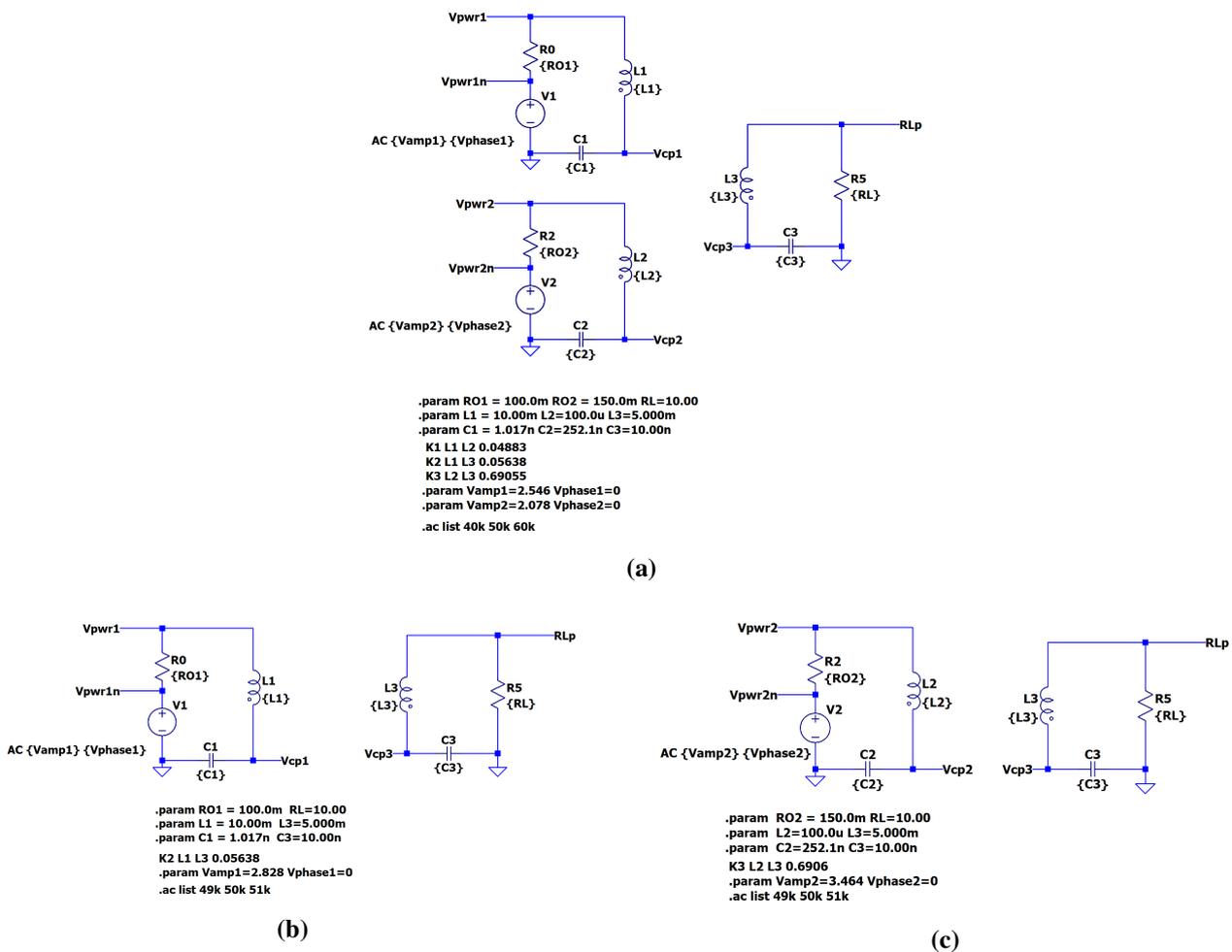

Figure 6. The simulation models and conditions in Scenario B: (a) MISO configuration (b) SISO configuration (Transmitter 1 - Receiver pair) (c) SISO configuration (Transmitter 2 - Receiver pair).

3.1. Scenario A: the effectiveness of proposed method

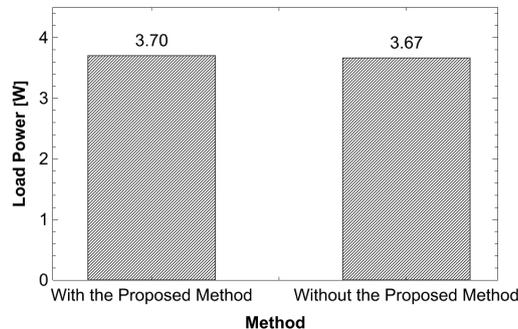### 3.1.1. Situation 1: a pair of a transmitter and receiver is tuned by impedance matching

Figure 7. The load power in Situation 1 with and without the proposed method.

It is found that if the WPT system is configured with well tuned transmitters and a receiver such that the Transmitter 1 - Receiver and Transmitter 2 - Receiver pairs are tuned by impedance matching respectively, the proposed method slightly improves load power. The result of numerical simulation for Situation 1 is shown in Figure 7, and we find that in the case of using the proposed method, the load power increases by approximately 1.01 times to the case of not using the proposed method as the voltage selections of voltage sources. In Situation 1, the proposed method can improve the load power and does not provide significant improvement. We note that in Situation 1, the WPT system has the pair that is performed by impedance matching of transmitter and receiver such as the pair of Transmitter 1 and Receiver and the pair of Transmitter 2 and Receiver. For Transmitter 1 and Receiver, the input impedance $Z_{in1} := V_{in1}/I_1$ in Figure 8(a) is impedance-matched to R_{O1}. Similarly, for Transmitter 2 and Receiver, the input impedance $Z_{in2} := V_{in2}/I_2$ in Figure 8(b) is impedance-matched to R_{O2}. The circuit parameters including the result of impedance matching are shown in Table 1. In the WPT system, it is already tuned well to achieve high power delivery by impedance matching, and therefore, it may be only a small space to improve the power delivery.
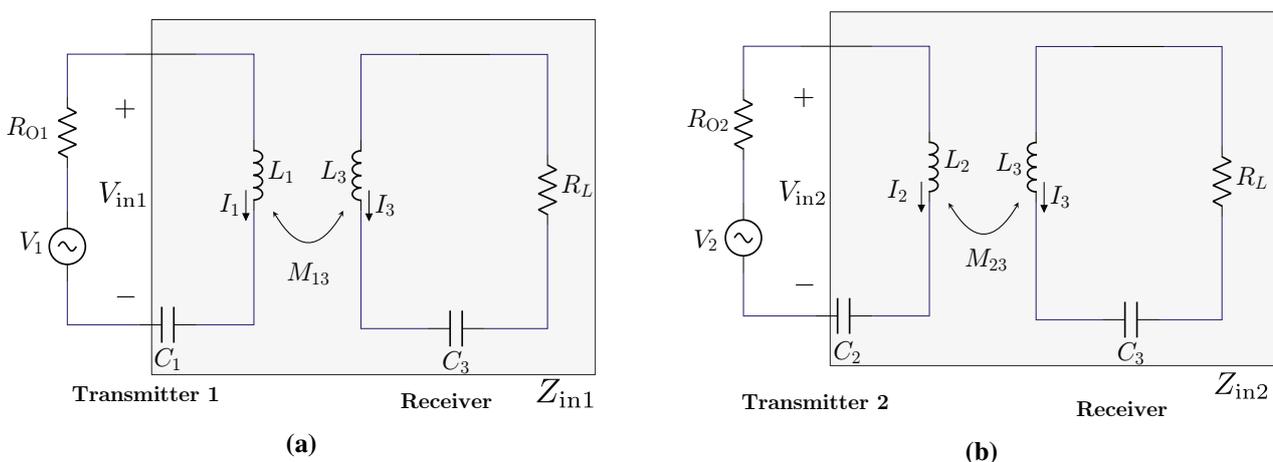

Figure 8. The input impedance parts Z_{in1} and Z_{in2} for impedance matching: (a)Transmitter 1 - Receiver pair, (b) Transmitter 2 - Receiver pair.

3.1.2. Situation 2: variation of M_{12} and M_{23}

In contrast to the WPT system is configured with well tuned transmitters and receivers such as Situation A, if the WPT system is not, the proposed method clearly improves the load power. The result of numerical simulation for Situation 2 is shown in Figure 9. We also find that in the case of using the proposed method, the load power increases by approximately 1.66 times to the case of not using the proposed method as the voltage selections. In Situation 2, the proposed method can significantly improve the load power. This result indicates that the changes in position of Transmitter 2 degrade the power delivery, and however it can improve the power delivery by the voltage selections of voltage sources with the proposed method if the pair of Transmitter 2 and Receiver goes out of impedance matching condition. It only adjusts voltages of the voltage sources to improve higher power and does not require external compensation circuits.
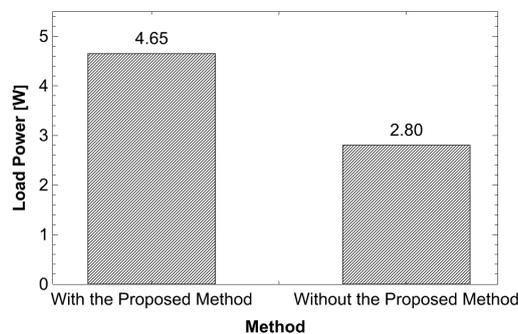

Figure 9. The load power comparison in Situation 2 with and without the proposed method if Transmitter 2 is moved.

3.1.3. Situation 2: variation of M_{12} and M_{13}

We also find the system using the proposed method can transfer more power to the load not only when Transmitter 2 is moved but also when Transmitter 1 is moved. The result of numerical simulation for Situation 2 when Transmitter 1 is moved is shown in Figure 10. It is found that in the case of using the proposed method, the load power increases by approximately 1.56 times to the case of not using the proposed method as the voltage selections.
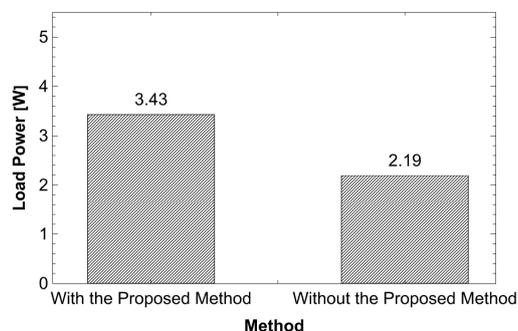

Figure 10. The load power comparison in Situation 2 with and without the proposed method if Transmitter 1 is moved.

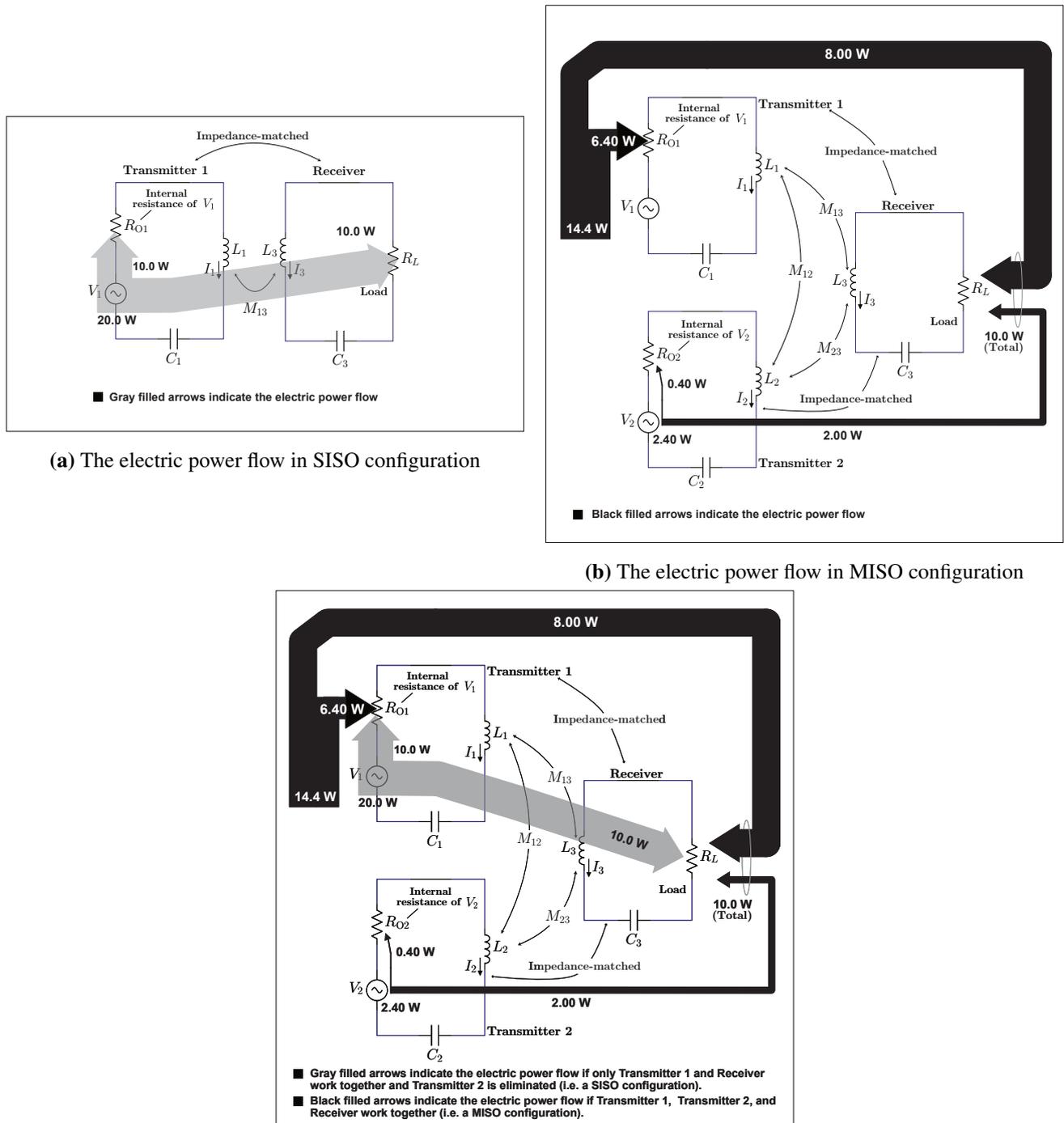*3.2. Scenario B: the comparison between SISO and MISO configurations*

(a) The electric power flow in SISO configuration

(b) The electric power flow in MISO configuration

(c) The summary of each electric power flow in SISO and MISO configurations

Figure 11. SISO vs MISO: power dissipation at internal resistance under 10 W consumption at load.

Table 2. The required RMS value of voltage amplitude to reach 10 W power at the load and the currents through the circuits with SISO and MISO configurations in Scenario B.

Circuit configuration	Magnitudes of each voltage and current
Transmitter 1 & Receiver (SISO configuration)	$\lvert V_1 \rvert$ = 2.00 Vrms $\lvert I_1 \rvert$ = 10.0 Arms $\lvert I_2 \rvert$ = 1.00 Arms
Transmitter 2 & Receiver (SISO configuration)	$\lvert V_2 \rvert$ = 2.45 Vrms $\lvert I_2 \rvert$ = 8.16 Arms $\lvert I_3 \rvert$ = 1.00 Arms
Transmitter 1, 2 & Receiver (MISO configuration)	$\lvert V_1 \rvert$ = 1.80 Vrms $\lvert V_2 \rvert$ = 1.47 Vrms $\lvert I_1 \rvert$ = 8.00 Arms $\lvert I_2 \rvert$ = 1.63 Arms $\lvert I_3 \rvert$ = 1.00 Arms

Figure 11 and Table 2 show the numerical simulation results for Scenario B. The MISO configuration has lower power dissipation at internal resistance of each voltage source under the condition that the same power is delivered to the same load as an advantage over the SISO configuration by tracking electric power flow in each configuration. In Figure 11(a), the gray filled arrows indicate the electric power flows as Transmitter 1 transfers 10 W power to a load in Receiver if Transmitter 2 is eliminated (i.e. the WPT system works in the SISO configuration). The voltage source V_1 supplies 20 W power, and half of the power is consumed at the internal resistance of the voltage source V_1. The rest of the power reaches the load. These results show that Transmitter 1 and Receiver are under impedance matching conditions. In Figure 11(b), the black filled arrows also indicate the electric power flows Transmitter 1 and 2 transfer totally 10 W power to the load in the Receiver. We note that Transmitter 1 and Receiver are the same as the SISO configuration and we add Transmitter 2 that is impedance-matched to Receiver to them as the MISO configuration. The voltage source V_1 supplies 14.4 W power, and 6.4 W is consumed at the internal resistance of the voltage source V_1. The rest of the power reaches the load. The voltage source V_2 also supplies 2.4 W power, and 0.4 W is consumed at the internal resistance of the voltage source V_2. The rest of the power reaches the load. We summarize the electric power flow between each configuration in Figure 11(c). If the same power delivery to the same load R_L, it shows clearly that power dissipation at each internal resistance in the MISO configuration is lower than the power dissipation in the SISO configuration. Especially, the MISO configuration reduces the power dissipation by approximately 3.2 W compared to the SISO configuration. In practical applications, the power dissipation in internal resistance of each voltage source wastes energy as heat, and therefore, it is desirable that the power dissipation is close to zero. As load power demand increases, it also increases the power dissipation and heat. Therefore, it needs further efforts to thermal design, which may lead to complex system design. At this point, the MISO configuration is a better choice than the SISO configuration.

Table 2 provides a summary of the circuit currents $\lvert I_1 \rvert$, $\lvert I_2 \rvert$, and $\lvert I_3 \rvert$ obtained by the numerical

simulations in Scenario B and the required RMS value of voltage amplitude of each voltage source to reach 10 W at a load among configurations. It is clearly found that the RMS value of currents $|I_1|$ and $|I_2|$ through the transmitters are lower in the MISO configuration compared to the SISO configurations. Especially, it can be found that the current $|I_2|$ reduces significantly by approximately 6.53 A. In the MISO configuration, the required RMS value of voltage amplitude in each transmitter is lower than the required RMS value of voltage amplitude in SISO configurations. Especially, the RMS value of voltage amplitude $|V_2|$ in Transmitter 2 reduces by approximately 1 V. These results indicate that the system can transfer power with margin of spare power or saving component cost in each transmitter since it allows for suppression of required rating of each voltage and current.

4. Conclusions

In this paper, we have proposed the method for maximizing load power to adjust independently voltage amplitude and phase of each voltage in the WPT system with two transmitters and a single receiver. The method clearly optimizes each voltage amplitude and phase based on circuit constants in the WPT system.

We have confirmed by numerical simulation as well as mathematical proof that the proposed method improves power delivery in the WPT system. While it is a little improvement of power delivery in the case of the system that has a pair of a transmitter and receiver under impedance matching condition, it is significant improvement of power delivery in the case of the system that has a pair of a transmitter and receiver under out of impedance matching condition. The proposed method has an advantage because it can only adjust each voltage amplitude and phase without any external circuits and replacing physical components for the improvement.

We have also performed the numerical simulation for the cases of two transmitters with a single receiver called the MISO configuration and a single transmitter with a single receiver called the SISO configuration. As the result of the numerical simulation, it can be found that the MISO configuration has advantages which are lower magnitude of the required voltage of each voltage source, magnitude of each current through the WPT system circuits, and the power dissipation in each voltage source compared to SISO configurations under the same load power. Therefore, from the perspective of an efficient power delivery and margin of spare power, the WPT system with the MISO configuration would be a better choice than the SISO configuration. This result implies that power delivery performance of the conventional WPT system with the SISO configuration can be improved by an addition of a new transmitter to the system without any modifications such as replacing internal physical components and changing circuit topology of the conventional system.

Author contributions

Use of Generative-AI tools declaration

Conflict of interest

The authors declare no conflicts of interest associated with this paper.

References

1. Masuda S, Hirose T, Akihara Y, Kuroki N, Numa M, Hashimoto M (2017) Impedance matching in magnetic-coupling-resonance wireless power transfer for small implantable devices. In *2017 IEEE Wireless Power Transfer Conference (WPTC)*, Taipei, Taiwan, 1–3. https://doi.org/10.1109/WPT. 2017.7953839

2. Cheng Y, Su X, Xuan D, Wang L, Liu Y, Ghovanloo M, et al. (2017) Modeling and optimization of single-turn printed coils for powering biomedical implants. In *2017 IEEE Wireless Power Transfer Conference (WPTC)*, Taipei, Taiwan, 1–3. https://doi.org/10.1109/WPT.2017.7953863

3. Liu C, Jiang C, Qiu C (2017) Overview of coil designs for wireless charging of electric vehicle. In *2017 IEEE PELS Workshop on Emerging Technologies: Wireless Power Transfer (WoW)*, Chongqing, China, 1–6. https://doi.org/10.1109/WoW.2017.7959389

4. Lee JY, Han BM (2015) A Bidirectional Wireless Power Transfer EV Charger Using Self-Resonant PWM. *IEEE T Power Electr* 30: 1784–1787. https://doi.org/10.1109/TPEL.2014.2346255

5. Wu X, Shen Y, Xiao J, Mo Y, Wu N, Chen S (2023) Design of MISO Wireless Power Transfer System for Electric Vehicle in Optimal Efficiency. In *2023 International Conference on Electronics and Devices, Computational Science (ICEDCS)*, Marseille, France, 7–13. https://doi.org/10.1109/ ICEDCS60513.2023.00008

6. Panchal C, Stegen S, Lu J (2018) Review of static and dynamic wireless electric vehicle charging system. *Engineering Science and Technology, an International Journal* 21: 922–937. https://doi. org/10.1016/j.jestch.2018.06.015

7. Yadav A, Bera TK (2025) A critical review of electromagnetic coil assembly design and optimization for wireless power transfer in electric vehicles: Technical insights. *Renew Sust Energy Rev* 223: 115944. https://doi.org/10.1016/j.rser.2025.115944

8. Zhou S, and Chris Mi C (2016) Multi-Paralleled LCC Reactive Power Compensation Networks and Their Tuning Method for Electric Vehicle Dynamic Wireless Charging. *IEEE T Ind Electron* 63: 6546–6556. https://doi.org/10.1109/TIE.2015.2512236

9. He X, Hu Y (2023) Optimal Mileage of Electric Vehicles Considering Range Anxiety and Charging Times. *World Electric Vehicle Journal* 14: 21. https://doi.org/10.3390/wevj14010021

10. Gkyrtis K (2025) The Impact of Weight Distribution in Heavy Battery Electric Vehicles on Pavement Performance: A Preliminary Study. *World Electric Vehicle Journal* 16: 520. https: //doi.org/10.3390/wevj16090520

11. Debnath T, Majumder S, De K (2025) High-efficiency coil design for wireless power transfer: Mitigating misalignment challenges in transportation. *Energy* 325: 135929. https://doi.org/10. 1016/j.energy.2025.135929

12. Akbar SR, Kurniawan W, Basuki A, Budi AS, Prasetio BH (2025) Wireless Power Transfer Optimization with a Minimalist Single-Capacitor Design for Battery Charging. *Energies* 18: 1574. https://doi.org/10.3390/en18071574

13. Oodachi N, Ogawa K, Kudo H, Shoki H, Obayashi S, Morooka T (2011) Efficiency improvement of wireless power transfer via magnetic resonance using transmission coil array. In *2011 IEEE International Symposium on Antennas and Propagation (APSURSI)*, Spokane, WA, USA, 1707–1710. https://doi.org/10.1109/APS.2011.5996636

14. Huh S, Ahn D (2018) Two-Transmitter Wireless Power Transfer with Optimal Activation and Current Selection of Transmitters. *IEEE T Power Electr* 33: 4957–4967. https://doi.org/10.1109/TPEL.2017.2725281

15. Lee H, Boo S, Kim G, Lee B (2020) Optimization of Excitation Magnitudes and Phases for Maximum Efficiencies in a MISO Wireless Power Transfer System. *J Electromagn Eng Sci* 20: 16–22. https://doi.org/10.26866/jees.2020.20.1.16

16. Lang HD, Ludwig A, Sarris CD (2014) Convex Optimization of Wireless Power Transfer Systems With Multiple Transmitters. *IEEE T Antenn Propag* 62: 4623–4636. https://doi.org/10.1109/TAP.2014.2330584

17. Lee H, Lee B (2020) Investigation of MIMO Wireless Power Transfer Efficiency in Optimization Techniques. In *2020 IEEE International Symposium on Antennas and Propagation and North American Radio Science Meeting*, Montreal, QC, Canada, 1417–1418. https://doi.org/10.1109/IEEECONF35879.2020.9330339

18. Waters BH, Mahoney BJ, Ranganathan V, Smith JR (2015) Power Delivery and Leakage Field Control Using an Adaptive Phased Array Wireless Power System. *IEEE T Power Electr* 30: 6298–6309. https://doi.org/10.1109/TPEL.2015.2406673

19. Jadidian J, Katabi D (2014) Magnetic MIMO: How to charge your phone in your pocket. In *Proceedings of the 20th Annual International Conference on Mobile Computing and Networking*, Maui, Hawaii, USA, 495–506. https://doi.org/10.1145/2639108.2639130

20. Shi L, Kabelac Z, Katabi D, Perreault D (2015) Wireless Power Hotspot that Charges All of Your Devices. In *Proceedings of the 21st Annual International Conference on Mobile Computing and Networking*, Paris France, 2–13. https://doi.org/10.1145/2789168.2790092

Appendix

A. An outline of proof for the solution to the optimization problem

An outline of proof for the solution Eq. (2.7) to the optimization problem Eq. (2.6) is as follows. We note that $\cdot^T$, $\cdot^*$, and $\bar{\cdot}$ denote transpose, Hermitian transpose, and complex conjugate respectively. u_1, u_2, and u_3 correspond to Eqs. (2.19), (2.20), and (2.21) respectively.

Let $x = [V_1 \ V_2]^T$ and let A be the Hermitian matrix consisting of

$$A = \begin{bmatrix} |q_1|^2 & \bar{q}_1 q_2 \\ q_1 \bar{q}_2 & |q_2|^2 \end{bmatrix} \tag{A.1}$$

where

$$q_1 = \omega(M_{13}(-\mathrm{j}R_{O2} + u_2) - M_{12}M_{23}\omega), \tag{A.2}$$

$$q_2 = \omega(M_{23}(-\mathrm{j}R_{O1} + u_1) - M_{12}M_{13}\omega). \tag{A.3}$$

The eigenvalues λ_1 and λ_2 of A are

$$\lambda_1 = 0 \tag{A.4}$$

$$\lambda_2 = |q_1|^2 + |q_2|^2 \tag{A.5}$$

which are non-negative values respectively. Therefore A is a positive-semidefinite Hermitian matrix. The load power $P_L = R_L|I_3|^2$ can be expressed in

$$P_L = R_L \frac{x^*Ax}{f_5\omega^6 + f_4\omega^5 + f_3\omega^4 + f_2\omega^3 + f_1\omega^2 + f_0}, \tag{A.6}$$

where f_i's are given in Eqs.(A.7)-(A.12).

$$f_5 = 4M_{12}^2M_{13}^2M_{23}^2, \tag{A.7}$$

$$f_4 = -4M_{12}M_{13}M_{23}(M_{23}^2u_1 + M_{13}^2u_2 + M_{12}^2u_3), \tag{A.8}$$

$$\begin{aligned} f_3 &= M_{23}^4(R_{O1}^2 + u_1^2) + 2M_{13}^2M_{23}^2(R_{O1}R_{O2} + u_1u_2) \\ &\quad + M_{13}^4(R_{O2}^2 + u_2^2) + M_{12}^4(R_L^2 + u_3^2) \\ &\quad + 2M_{12}^2(M_{23}^2(R_LR_{O1} + u_1u_3) + M_{13}^2(R_LR_{O2} + u_2u_3)), \end{aligned} \tag{A.9}$$

$$\begin{aligned} f_2 &= -4M_{12}M_{13}M_{23}(R_LR_{O2}u_1 + R_LR_{O1}u_2 \\ &\quad + R_{O1}R_{O2}u_3 - u_1u_2u_3), \end{aligned} \tag{A.10}$$

$$\begin{aligned} f_1 &= 2(M_{13}^2(R_{O2}^2 + u_2^2)(R_LR_{O1} - u_1u_3) \\ &\quad + M_{23}^2(R_{O1}^2 + u_1^2)(R_LR_{O2} - u_2u_3) \\ &\quad + M_{12}^2(R_{O1}R_{O2} - u_1u_2)(R_L^2 + u_3^2)), \end{aligned} \tag{A.11}$$

$$f_0 = (R_{O1}^2 + u_1^2)(R_{O2}^2 + u_2^2)(R_L^2 + u_3^2). \tag{A.12}$$

We note that the voltages V_1 and V_2 are only included in a numerator of P_L Eq. (A.6), and it is expressed in the Hermitian form x^*Ax scaled by the load resistance R_L. It is well known in linear algebra that a procedure for finding x that maximizes x^*Ax under the constraint $x^*x = g^2$ is an eigenvalue problem. We will investigate the maximum of x^*Ax under the constraint $|V_1|^2 + |V_2|^2 = g^2$. Let s_1 and s_2 be the unit eigenvectors corresponding to the eigenvalues λ_1 and λ_2 represented in Eqs (A.4) and (A.5) respectively, and then p_1 and p_2 are expressed in

$$s_1 = \frac{e^{j\phi_1}}{\sqrt{1 + |q_2/q_1|^2}} \begin{bmatrix} -q_2/q_1 \\ 1 \end{bmatrix}, \tag{A.13}$$

$$s_2 = \frac{e^{j\phi_2}}{\sqrt{1 + |\bar{q}_1/\bar{q}_2|^2}} \begin{bmatrix} \bar{q}_1/\bar{q}_2 \\ 1 \end{bmatrix}, \tag{A.14}$$

where ϕ_1 and ϕ_2 are arbitrary real numbers. Since in general the unit eigenvectors of a complex matrix have degree of freedom for phases, Eqs. (A.13) and (A.14) have terms of $e^{j\phi_1}$ and $e^{j\phi_2}$. Let $S = [s_1 \ s_2]$, and then we find $S^*S = SS^* = I$, where I is the identity matrix, and thus S is the unitary matrix by its definition. Furthermore, we have

$$S^*AS = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}. \tag{A.15}$$

We consider variable transformation such that

$$x = Sy \tag{A.16}$$

where $y = [y_1 \ y_2]^T$ and y_1 and y_2 are complex numbers respectively and then, with $|y_1|^2 \geq 0$, $|y_2|^2 \geq 0$, $\lambda_1 \leq \lambda_2$ and Eqs. (A.15) and (A.16),

$$x^*Ax = y^*S^*ASy = \lambda_1|y_1|^2 + \lambda_2|y_2|^2 \leq \lambda_2(|y_1|^2 + |y_2|^2). \tag{A.17}$$

We note that $y^*y = |y_1|^2 + |y_2|^2$ and $y^*y = (P^*x)^*y = x^*x = g^2$ by the its definition. Thus with Eq.(A.17), we find

$$x^*Ax \leq g^2\lambda_2. \tag{A.18}$$

Eq. (A.18) implies that the maximum value of x^*Ax is $g^2\lambda_2$. We now find x that maximizes x^*Ax. We need to find x under the equality in Eq. (A.18) holds and by using Eq. (A.4), we obtain

$$|y_2|^2 = g^2, \tag{A.19}$$

$$|y_1|^2 + |y_2|^2 = g^2, \tag{A.20}$$

where the last equation comes from $y^*y = x^*x = g^2$. To find x, we solve Eqs. (A.19) and (A.20) for y_1 and y_2, and then we find x by using Eq. (A.16). From (A.19) and (A.20), we find $y_1 = 0$. Let y_2 be represented in the exponential form, i.e., $y_2 = re^{j\phi_3}$ where $r \geq 0$ and ϕ_3 is an arbitrary real number. From Eq. (A.19), we find $r = g$ since $g > 0$ and $r \geq 0$ by the definitions. Thus we obtain $y = [0 \ ge^{j\phi_3}]^T$ and then, for x, we calculate Eq. (A.16), and we finally

$$x = \frac{ge^{j\theta}}{\sqrt{1 + |d|^2}} \begin{bmatrix} d \\ 1 \end{bmatrix} \tag{A.21}$$

where $\theta = \phi_2 + \phi_3$ and $d = \bar{q}_1/\bar{q}_2$. This Eq. (A.21) corresponds to Eq. (2.7). If we set x to Eq. (A.21), then x^*Ax reaches its maximum value $g^2\lambda_2$, and the load power represented in Eq. (A.6) reaches its maximum value represented in Eq. (2.9). A solution to this problem is that we should set x to a unit eigenvector multiplied by $ge^{j\theta}$, corresponding to the maximum eigenvalue of A. □