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Research article

A novel framework for uncertain multidimensional decision-making: UMDI-PRO model for optimized solutions in complex environments

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Abstract: Effective decision-making in complex scenarios involving multiple dimensions and uncertainty presents significant challenges for traditional models. In this paper, we introduce the Uncertain Multi-Dimensional Programming (UMDI-PRO) model, a comprehensive framework designed to optimize decision-making in uncertain and multidimensional environments. By integrating advanced techniques from multivariate analysis and multicriteria decision-making (MCDM), UMDI-PRO provides a robust approach for evaluating and selecting optimal alternatives. Compared to conventional MCDM approaches, UMDI-PRO exhibits marked improvements in handling trade-offs, processing imprecise data, and generating stable solutions under uncertainty. Through numerical experimentation and comparative analysis, the UMDI-PRO model demonstrates superior performance in managing conflicting objectives and uncertain data, surpassing traditional MCDM methods. Case studies in supply chain management, healthcare, and environmental management illustrate its wide applicability and effectiveness in real-world problem-solving. The model's adaptability, accuracy, and resilience make it a powerful tool for enhancing the quality of decisions in dynamic and uncertain environments.

Keywords: uncertain multi-dimensional programming (umdi-pro); decision-making under uncertainty; optimization models; complex decision environments; supply chain management

JEL Codes: C44, C61, D81, Q01, G11

1. Introduction

Multicriteria Decision-Making (MCDM) is an essential tool for handling complex decisions involving multiple, often conflicting criteria. In real-world applications such as resource allocation, project selection, and strategic planning, decision-makers must navigate trade-offs between competing objectives like cost, quality, and time. MCDM methods, such as the Analytic Hierarchy Process (AHP) (Saaty et al., 2007), TOPSIS, and PROMETHEE, offer structured approaches to rank, prioritize, and select the best alternative from a set of choices. These techniques simplify the decision-making process by systematically evaluating alternatives across dimensions.

However, decision-making becomes more challenging when uncertainty is introduced, especially in multidimensional and hierarchical structures. Uncertainty can arise from incomplete or imprecise information, unpredictable external factors, or varying decision criteria. Traditional MCDM methods often assume complete and accurate data, making them less reliable in uncertain scenarios. Complex situations, such as supply chain disruptions, volatile market conditions, and environmental uncertainties, require advanced tools to handle these complexities effectively.

In multidimensional decision support, the focus is on systematically assessing multiple layers of data to address interdependencies and interactions among different elements. The challenge lies in evaluating several dimensions simultaneously, identifying optimal trade-offs without compromising one objective for another. Researchers have explored various approaches to multidimensional decision-making, but many of these are limited in addressing higher-level complexity and uncertainty.

Traditional MCDM approaches (Nassereddine and Eskandari, 2017) typically address two or three levels of analysis, such as criteria-alternative or criteria-scenario-alternative frameworks. However, these methods struggle when faced with the complexities of multidimensional challenges, particularly when uncertainty is involved. Researchers have expanded the scope of MCDM by incorporating multi-level and multi-dimensional analyses, but gaps remain in practical applications, especially in addressing uncertainty (Baourakis et al., 1996; De Tré et al., 2018; Juanpera et al., 2021).

Here, we introduce the Uncertain Multi-Dimensional Programming (UMDI-PRO) model, designed to integrate MCDM with uncertainty management. UMDI-PRO provides a comprehensive framework for decision-making in complex, uncertain environments, combining the strengths of multivariate analysis and MCDM. While traditional MCDM approaches have been widely used, they often fall short when faced with high-dimensional data and dynamic uncertainties, as they tend to assume stable, complete, and well-defined input parameters. This model addresses the gaps in current decision-making practices, offering a more adaptable and resilient solution for evaluating alternatives when data is incomplete or uncertain. The key motivations for this research are:

- The lack of a comprehensive multidimensional evaluation method widely accepted by decision-makers.
- The inability of MCDM approaches to effectively integrate and manage multiple sources of uncertainty, such as vagueness, variability, and interdependencies, across several dimensions.
- Many mathematical models in the literature are not practical enough for real-world implementation, particularly when uncertainty is present. In response, we propose a novel approach to multidimensional decision-making under uncertainty. The UMDI-PRO model not

only captures the hierarchical and interdependent relationships between decision criteria and alternatives but also provides practical mathematical tools for solving complex decision problems.

Our primary objectives are:

- To introduce the theoretical framework and mathematical formulation of the UMDI-PRO model.
- To demonstrate how the model integrates MCDM with uncertainty management.
- To apply the UMDI-PRO model to real-world scenarios, illustrating its effectiveness in handling uncertainty.
- To compare the performance of UMDI-PRO with traditional MCDM methods, highlighting its advantages.
- To offer insights into the model's practical applications and propose future improvements.

In summary, we present a robust, multidimensional decision-making model that addresses uncertainty, thereby filling a critical gap in decision-support tools for complex, real-world problems.

The structure of the article is organized into several key sections. In Section 2, we provide a concise overview of significant literature related to MCDM and its integration with uncertainty management. In Section 3, we explore multidimensional decision problems within the framework of MCDM, addressing the associated challenges and considerations. In Section 4, we present the foundational concepts of the proposed UMDI-PRO model, detailing its underlying principles and methodologies. In Section 5, we illustrate uncertain multidimensional selection scenarios, showcasing the advantages and effectiveness of the UMDI-PRO model in enhancing the decision-making process. Finally, in Section 6, we provide a comparative analysis of the results obtained through the UMDI-PRO model against methods in the literature, emphasizing its strengths and unique contributions to the field.

2. Literature review

The literature review begins by examining the most commonly used MCDM methods, including the Analytic Hierarchy Process (AHP), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and PROMETHEE. Each of these methods plays a crucial role in decision-making by providing structured frameworks that facilitate the evaluation of alternatives based on multiple, often conflicting criteria. AHP, for instance, enables decision-makers to decompose complex problems into a hierarchy, making it easier to analyze and prioritize options. TOPSIS focuses on identifying solutions that are closest to the ideal solution while being farthest from the negative ideal (e.g.; Omann, 2004; Imori and Von Rosen, 2015; Euchi et al., 2019; Hamdi et al., 2023). PROMETHEE, on the other hand, provides a comprehensive ranking of alternatives based on pairwise comparisons. While these methodologies are effective in structured decision-making contexts, their applications in uncertain environments require further exploration.

Uncertainty is an inherent aspect of decision-making, particularly in complex scenarios. Different types of uncertainty, such as information-related uncertainty and environmental uncertainty, can significantly impact the reliability of decision-making models. Information-related uncertainty arises from incomplete or imprecise data, while environmental uncertainty stems from unpredictable external conditions that may influence the decision context. Models often employ various strategies to address these uncertainties, including probabilistic approaches, fuzzy logic, and scenario analysis. However, the effectiveness of these strategies varies depending on the nature and extent of the uncertainty involved.

In recent years, there has been a growing interest in integrating MCDM techniques with methods to manage uncertainty. Several researchers have attempted to combine these two domains, leading to the development of frameworks that aim to enhance decision-making under uncertainty (Xu et al., 2020). However, despite some progress, significant limitations and gaps remain in the literature. Many approaches lack comprehensiveness and do not adequately address the complex interplay between multicriteria decision-making and uncertainty, particularly in multidimensional contexts (Shubham et al., 2022).

The Uncertain Multi-Dimensional Programming (UMDI-PRO) model aims to fill this critical gap in the literature. There is a pressing need for a robust framework that can effectively address uncertainty and multicriteria decision-making in multidimensional scenarios. By combining advanced uncertainty-handling techniques with MCDM approaches, the UMDI-PRO model seeks to enhance decision-making processes in complex environments, providing a more adaptable and resilient solution for practitioners facing real-world challenges.

Table 1 provides a concise overview of significant literature related to MCDM and its integration with uncertainty management. Each entry summarizes the contributions of various authors, highlighting their methodologies, key findings, and relevance to the current research on the UMDI-PRO model. The selected studies cover foundational MCDM methods such as the Analytic Hierarchy Process (AHP), TOPSIS, and PROMETHEE, along with advancements that address uncertainty in decision-making processes. By synthesizing these contributions, the table illustrates the evolution of thought in the field, identifies gaps in research, and underscores the need for a comprehensive framework like UMDI-PRO that effectively combines MCDM with uncertainty handling.

Table 1. Analysis of Literature on MCDM and Uncertainty Management.

Author(s)	Research Focus	Methodology	Key Findings	Limitations	Implications for Future Research
Chowdhury & Paul (2020)	MCDM methods in corporate sustainability	Systematic literature review	Identified various MCDM methods used in corporate sustainability research.	Lacks empirical case studies for validation.	Need for practical applications of identified methods in real cases.
Tavana et al.(2020)	Uncertain decision-making methods in energy management	Text mining and data analytics	Reviewed uncertain decision-making approaches relevant to energy management.	Limited focus on specific energy sectors.	Further exploration of methods in different energy contexts needed.
Pelissari et al.(2021)	Modeling uncertain input data in MCDM	Literature review	Discussed various techniques to handle uncertainty in MCDM problems.	General overview without in-depth analysis of specific techniques.	Call for more detailed studies on effective modeling techniques.
Singh & Pant (2021)	Weighing methods in MCDM	Review and case study	Evaluated different weighting methods used in MCDM applications.	Focused on selected methods; comprehensive exploration lacking.	Investigate more weighting methods and their implications for outcomes.
Sotoudeh-Anvari (2022)	MCDM applications in COVID-19 pandemic	State-of-the-art review	Highlighted the relevance of MCDM in addressing challenges posed by the pandemic.	Lacks systematic analysis of outcomes or effectiveness.	Future research should assess the impact of MCDM applications on public health crises.
Namin et al. (2022)	MCDM in mining method selection	Literature review	Reviewed MCDM techniques applicable to mining method selection	Insufficient exploration of implementation challenges.	Need for case studies demonstrating practical implementations
Pouyakian et al.(2022)	Risk control measures allocation using fuzzy MCDM	Literature review and application	Demonstrated how fuzzy MCDM can optimize risk control measure allocation.	Limited empirical validation of applications.	Further research on real-world applications of fuzzy MCDM in risk management

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Author(s)	Research Focus	Methodology	Key Findings	Limitations	Implications for Future Research
Aouadni & Euchi (2022)	supplier selection and order allocation problem	Integrated MMD-TOPSIS for decision-making	Demonstrated effectiveness in a Tunisian case study for fair order allocation	Context-specific application; may not generalize across industries	Exploration of other industries and comparison with alternative MCDM methods required
Sahoo & Goswami (2023)	Advances in MCDM methods	Comprehensive review	Summarized advancements, applications, and future directions in MCDM	Generalized conclusions without specific case studies.	Suggest detailed applications of various MCDM methods across industries
Lyu et al. (2023)	Flood risk assessment using MCDM	Review	Explored MCDM methods in assessing flood risk in smart city contexts.	Focused on specific contexts; broader applicability needed.	Need for adaptations of methods to various urban environments.
Yu et al. (2023)	Knowledge evolution in PROMETHEE	Longitudinal and dynamic analysis of PROMETHEE	Identified trends and knowledge advancements in the use of PROMETHEE over time	Focused primarily on knowledge analysis, lacks practical decision-making examples	Further research needed to apply these knowledge trends to real-world decision-making problems
Cebesoy et al. (2024)	MCDM under uncertainty using outranking and Bayesian networks	Combined methodologies	Integrated outranking methods with Bayesian networks to enhance decision support under uncertainty.	Limited empirical validation of the proposed methods.	Call for empirical testing of integrated approaches in decision-making.
Ayyildiz & Erdogan (2024)	Location selection studies related to waste facilities	Literature analysis	Analyzed MCDM approaches for site selection of waste management facilities.	Lack of diverse case studies.	Future studies should investigate site selection in varying contexts.
Boonsothonsatit et al. (2024)	Hybrid AHP-TOPSIS for technology selection in hospitals	Developed a hybrid AHP-TOPSIS decision-making framework	successfully applied to hospital medication dispensing processes, improving efficiency.	Limited to hospital context, needs generalization to other healthcare processes	Future research should explore applicability in different healthcare settings and technologies.
Gital & Bilgen (2024)	Biomass supply chain network design under uncertainty	Systematic literature review	Reviewed approaches to design resilient supply chains in biomass sectors amidst uncertainty.	Limited focus on practical applications.	Research should focus on implementation frameworks for resilient designs.

The integration of MCDM methods and uncertainty management has been extensively explored in various domains. However, approaches often fall short in addressing the full complexity of multidimensional and hierarchical decision-making processes, especially when dealing with significant uncertainties. Traditional MCDM methods such as AHP, TOPSIS, and PROMETHEE excel at evaluating alternatives across multiple criteria, but they typically assume accurate and complete information, which is rarely available in real-world scenarios. Moreover, many of these models are limited to specific levels of decision-making and fail to capture the intricate interdependencies between different decision layers.

While advancements have been made to incorporate uncertainty into decision-making models, many of these efforts focus on specific types of uncertainty (e.g., environmental and informational) without providing a comprehensive framework that can handle multiple dimensions and the hierarchical nature of real-world problems. Hybrid approaches or enhanced MCDM models, though effective in certain applications, face limitations when it comes to integrating multivariate analyses and addressing the dynamic nature of uncertainty across levels of decision structures.

Advancements in consensus-driven optimization under uncertainty have significantly enhanced decision-making processes, particularly in complex environments. Traditional methods such as the Maximum Expert Consensus Model (MECM) have been widely used to achieve group consensus in decision-making (GDM). However, the inherent uncertainty and risks in these models, particularly with regards to adjustment costs and expert preferences, have led to the development of more robust frameworks. For instance, Ji and Ma (2023) introduced the Robust Maximum Expert Consensus Model (R-RMECM), which integrates mean-variance theory to account for uncertainty and risks during the consensus-reaching process. This model was developed under three uncertain scenarios to reflect the real-world decision environment more accurately. Moreover, Zhang et al. (2025) further refined this approach by addressing uncertain cooperative behavior in GDM, demonstrating how these models can improve consensus efficiency while minimizing costs. They introduced novel techniques to quantify expert cooperation and account for the dynamic nature of expert preferences, thus enhancing the robustness of consensus outcomes under uncertainty. Additionally, Ma et al. (2024) developed a MECM with uncertain adjustment costs for social network-based decision-making, incorporating a quadratic cost function and applying it to agricultural insurance premiums subsidy policymaking. This approach highlights the economic implications of adjustment costs and the elasticity of opinions, further demonstrating the utility of robust optimization models in managing uncertainty. These advancements contribute to a growing body of research that integrates uncertainty management with decision-making models, providing more resilient and adaptable solutions for group decision-making in uncertain environments (Ji et al., 2024; Ma et al., 2024; Messaoudi et al., 2025).

This gap underscores the need for a more comprehensive approach to decision-making under uncertainty, one that effectively manages the complex interactions between multiple factors and uncertain conditions. The proposed UMDI-PRO model addresses these challenges by offering a robust framework that integrates MCDM with advanced techniques for handling uncertainty. A key feature of this model is its consensus mechanism, which requires a minimum consensus threshold of 75% among decision-makers, ensuring that the outcomes reflect a broad agreement while accounting for divergent expert opinions. By incorporating this consensus requirement, UMDI-PRO enhances the accuracy and reliability of decision-making, even in highly complex and uncertain environments. This approach enables decision-makers to evaluate alternatives with greater confidence, fostering more adaptive and resilient decision processes. UMDI-PRO is particularly valuable in domains where both uncertainty and multi-dimensional factors play a significant role, offering a more nuanced and effective way to navigate complex decision landscapes.

3. Problem description

Building on the understanding of how multidimensional analysis interacts with uncertainty in decision-making, in this section, we delve into the specific challenges that arise in complex decision environments. The following problem description outlines the core issues that the UMDI-PRO model seeks to address.

3.1. Multidimensional analysis and uncertainty

Multidimensional analysis is an operational research and decision support approach that enables you to evaluate and interpret complex data that has several levels. This can be used in different fields such as social science research, marketing, and finance. Multidimensional analysis are mathematical concepts that emerge from an analysis of complex problems, including, but not limited to, performing multivariate data analysis (Yeh et al., 1999; Jolliffe and Cadima 2016). There are several multidimensional analysis techniques, including principal component analysis (PCA), factor analysis, multiple correspondence analysis (MCA), and MCDM. The PCA is used to reduce the dimensionality of data by finding a small number of levels that explain most of the variability in the data. This makes it possible to visualize the data in a simplified manner while preserving the essential information. It is a factor-based modeling tool that enables the exploration and interpretation of large data sets across many disciplines using multivariate dimensions (Moeini et al., 2023). Moreover, the factor analysis is similar to PCA but is used to identify latent factors that influence observed variables. It is often used to explore the structure of data and identify relationships between variables. It is a frequently used multivariate analysis method that transforms a large number of related variables into a smaller number of independent factors (Yong and Pearce, 2013). MCA is a multidimensional analysis technique used to analyze contingency tables, i.e., cross-tabulations of categorical variables. It enables you to identify relationships between variables and represent them graphically. Furthermore, the MCDM are approaches used to make decisions in the presence of multiple, often conflicting, criteria or objectives. These methods are widely used in many fields such as management, economics, engineering, etc.

In summary, multidimensional analysis is a powerful approach to analyzing complex data by identifying relationships between variables and reducing the dimensionality of the data for easier interpretation. In econometrics, multidimensional analysis is used to explore the relationships between several economic variables and to estimate complex econometric models. Multidimensional analysis is used to study trends, cycles and causal relationships between economic time series such as GDP, inflation, unemployment, etc. (Yan et al., 2023; Yanxi and Tingxuan 2024). It helps identify underlying patterns and make predictions. Multivariate analysis is used to estimate multivariate econometric models, such as multiple regression models, which study the relationship between a dependent variable and multiple explanatory variables (Zheng et al., 2023; Jingyu and Grace 2024). Multivariate analysis is used to analyze panel data, which combines temporal and cross-sectional data. It makes it possible to study fixed and random effects, as well as interactions between individuals and periods (Thyago et al., 2024). Principal components analysis is used to reduce the dimensionality of economic data by identifying the most important variables and combining them into smaller, more meaningful components.

In operations research, multidimensional analysis is used to model and solve complex operations management problems. Multidimensional analysis is used to determine optimal inventory levels taking into account several variables such as demand, storage costs, and ordering costs (Samvetet al., 2023; Issa et al., 2024). Multidimensional analysis is used to plan production based on several factors such as production capacity, production costs, and market demand (Wenyan et al., 2024).

Multidimensional analysis is used to efficiently allocate limited resources, such as time, money, and personnel, based on multiple criteria and constraints (Amor and Martel, 2014; Guangyao et al., 2023; Shi et al., 2024; Xinxin et al., 2024). Multidimensional analysis is a powerful method used in econometrics and operations research to model, analyze, and solve complex problems, taking into account multiple variables and constraints.

Managing uncertainty has always been a formidable challenge in decision-making. The fuzzy set can maintain several values to express the uncertainty experienced by decision makers, and has been empirically shown to improve the rationality of the decision. Decision making is a complex and highly uncertain process, because the evaluative information provided by decision makers often involves epistemic uncertainty. One of the most important tools for expressing decision information is fuzzy set theory. Since Zadeh's (1965) seminal work on fuzzy sets, many extensions have been developed to provide a more complete and reasonable description of expert opinions from various perspectives. Some researchers have used interval values or fuzzy triangular numbers, rather than precise numbers, to capture more information provided by decision-makers (Buckley and Eslami 2002; Dempe, 2011; Ren et al., 2018; Mardani et al., 2020; Kumar and Bisht 2023; Li and Xu 2024).

3.2. Research on multidimensional analysis and uncertainty

The purpose of MCDM methods is to help researchers make a good decision. The MCDM is a process that dissects the decision-making problem to produce a model of the situation that enables decision-making. In particular, the decision-maker must define the objectives to be achieved, the alternatives or solutions that can be implemented, the consequences and uncertainties linked to each alternative about the objectives followed and finally he must specify the model of choice and his preferences. Some important research studies on MLDM problems are summarized in Table 2.

Table 2. SWR applications of conventional MCDM methods.

Author	Dimension of the problem						Solving techniques		
	MO	MD	MC	MA	MS	OV	Method	C	U
Pramanik and Roy (2007)		X	X	X			Goal programming		X
Macharis et al. (2012)			X	X		X	MAMCA	X	
Akgün et al. (2012)			X		X	X	MAMCA	X	
Ren et al. (2013)		X	X	X		X	MAMCDM		X
Kourtit et al. (2014)			X	X	X	X	MAMCA PROMETHEE	X	
Bergqvist et al. (2015)			X	X		X	MAMCA	X	
Sun et al. (2015)			X	X		X	MAMCA	X	
Macharis et al. (2015)			X	X		X	MAMCA	X	
Lu et al (2016)			X	X		X	Mathematical program	X	
Yuan and Kong (2017)		X	X	X			MCDA		X
Zhao et al (2017)	X		X	X			Goal programming		x
Lebeau (2018)			X	X		X	MAMCA	X	
Baudry (2018)			X	X		X	MAMCA	X	
Baudry et al. (2018a)			X	X	X	X	MAMCA	X	
Luo et al. (2018)		X	X	X			MCDA		X
Almeida (2019)			X	X		X	MAMCA	X	
Demesouka et al (2019)		X	X	X			MCDA		X
Moya et al. (2019)	X	X	X	X			MCDA OMP	X	
Vieira et al. (2020)		X	X	X		X	MCDA	X	
Ren and Toniolo (2020)	X		X	X		X	Mathematical Program	X	
Baudry et al. (2018b)			X	X		X	MAMCA	X	

Continued on next page

Author	Dimension of the problem						Solving techniques		
	MO	MD	MC	MA	MS	OV	Method	C	U
Juszczuk et al. (2020)	X	X	X	X			MCDA OMP		X
Wang et al. (2020)			X	X		X	TOSSIS OMP	X	
Bouzayane et Saad (2020)		X	X	X			MCDA		X
Engau and Sigler (2020)	X	X	X	X			MCDA OMP		X
Pugliese et al. (2020)	X	X	X	X			MCDA OMP		X
Keseru et al. (2021)		X	X	X		X	PROMETHEE	X	
Caramuta et al. (2021)	X	X	X	X		X	AHP OMP	X	
Balderas et al. (2021)	X	X	X	X			MCDA OMP		x
Siskos and Burgherr(2021)		X	X	X			MCDA	X	
Tsagkarakis et al. (2021)		X	X	X			MCDA		X
Zhang et al. (2021)		X	X	X			MCDA		X
Angilella and Maria (2021)		X	X	X	X		MCDA		X
Pamucar et al. (2021)		X	X	X			MCDA		X
Francisco et al. (2022)			X	X	X		MCDA	X	
Kalliopi et al. (2024)			X	X	X		MCDA	X	

MO: Multi-Obective, MD: Multi-Decision-Maker, MC: Multi-Criteria, MA: Multi-Alternative, MS: Multi-Scenario, OV: Other Variant, C: Classic method, U: Uncertain method;

The concept of UMDDM offers a valuable and efficient tool for comprehending the intricacies of multidimensional decision structures (Triantaphyllou and Mann, 1989; Zhang et al., 2016). UMDDM methodologies are designed to address intricate selection dilemmas, and researchers in the fields of operations research and soft computing have made noteworthy contributions toward modeling the fundamental approaches linked to UMDDM (Mavrotas and Trifillis, 2006; Lu et al., 2016; Martínez et al., 2019, Euchi, 2020).

As summarized in Table 2, different MCDM methods vary significantly in their ability to handle MO, MD, MC, MA, and MS. For example, the MAMCA method (Macharis et al., 2012; Akgün et al., 2012) is particularly suited to participatory decision-making processes involving multiple stakeholders (MD) and alternatives (MA), making it effective in urban mobility or policy analysis. In contrast, mathematical programming approaches (Lu et al., 2016; Ren & Toniolo, 2020) are better suited for optimization under strict constraints, particularly in engineering or logistical contexts. PROMETHEE, AHP, and OMP-based MCDA techniques provide structured evaluations when trade-offs between multiple criteria and objectives are essential.

However, the majority of these conventional approaches assume deterministic environments and do not explicitly incorporate uncertainty. As indicated, only a subset of studies (e.g., Pramanik & Roy, 2007; Yuan & Kong, 2017; Wang et al., 2020; Hafdhi and Euchi, 2023) integrate uncertainty-handling capabilities into their models. Moreover, none fully address the dynamic interplay between multi-criteria, multi-decision-makers, and multi-scenario dimensions under uncertainty. This highlights a gap that the proposed UMDI-PRO model aims to fill by offering a holistic framework that integrates these interdependencies in uncertain environments.

Furthermore, studies by Kourtit et al. (2014), Ren and Toniolo (2020), Caramuta et al. (2021), and Balderas et al. (2021) have proposed that multidimensional decision programming and combinatorial approaches are capable of resolving quadri-level decision problems. This paper introduces a novel optimization programming approach to represent UMDDM problems within expert and intelligent hierarchical systems. Despite their significance, research into the UMCDM problem has remained relatively limited in the scientific literature. Hence, it is imperative to explore innovative methods for tackling uncertain decision analysis issues in multidimensional contexts.

3.3. Introducing UMDI-PRO

The UMDI-PRO is an area of optimization that deals with decision-making under uncertainty when multiple objectives or dimensions are involved. It extends multidimensional programming concepts to incorporate uncertain parameters or variables, common in real-world decision-making scenarios. The goal of UMDI-PRO is to find optimal decisions that minimize or maximize multiple objectives while accounting for parameter uncertainty. This requires the use of techniques such as stochastic programming, robust optimization, or multi-objective optimization, depending on the specific nature of the uncertainty and objectives involved. The UMDI-PRO finds applications in various fields, including finance, engineering, and operations research, where decisions must be made taking into account multiple conflicting objectives and uncertain conditions. The goal is to find robust or flexible solutions that work well in different scenarios or realizations of uncertainty.

The UMDI-PRO is useful in various decision-making contexts where there are multiple objectives and uncertainty plays an important role. It can be beneficial in various scenarios where decision-making involves multiple dimensions or objectives and uncertainty.

Uncertain multidimensional programming is useful in many contexts where decision-making involves multiple objectives and uncertain conditions, thereby helping decision-makers make informed and robust decisions.

3.4. Methodology and theoretical underpinnings

In this section, we discuss the concept of mathematical modeling, its applications, and the role of comparison relations in decision support. It also introduces the UMDI-PRO approach for uncertain multidimensional programming and briefly mentions the components involved in modeling MLDM problems.

Mathematical modeling relies on the utilization of previously gathered data and knowledge. Its primary objective is to address optimization or assessment inquiries. This approach finds extensive application in resolving problems that involve multiple objectives, criteria, attributes, and/or scenarios. A mathematical model serves as a representation of an observation, allowing mathematical tools, techniques, and theories to be applied to it. Subsequently, the mathematical outcomes are often translated into predictions or actions in the real world. The decision support process entails the comparison of various courses of action, which necessitates the use of comparison relations. These relations facilitate the translation of situations where there are clear and substantial reasons to express a preference for one action over another in the considered dimension.

4. Solution methodology

To provide a comprehensive framework for tackling decision-making challenges, the solution methodology section introduces the UMDI-PRO model. This model is designed to address the complexities of optimizing multiple objectives in the face of uncertainty. Unlike traditional mathematical models that remain abstract, UMDI-PRO offers a structured approach rooted in real-world management and economic scenarios where decision-makers often face trade-offs among conflicting goals and must act under imperfect information. In the next subsection, we focus on defining the problem and outlining its scope, identifying the key components that characterize a UMDP problem.

4.1. Problem definition and scope

The UMDI-PRO model responds to decision-making needs arising in complex environments, such as resource allocation, environmental planning, or investment selection, where the optimization must account for uncertainty and conflicting objectives across multiple evaluation levels. The UMDI-PRO deals with decision-making problems where multiple objectives or dimensions need to be optimized under uncertainty. The key components of a UMDP problem include:

- **Decision Variables**: The variables that can be adjusted to optimize the objectives.
- **Objective Functions**: The functions that need to be minimized or maximized, often representing different aspects of the problem.
- **Constraints**: The constraints that limit the feasible solutions.
- **Uncertain Parameters**: Parameters or variables that are not known with certainty and affect the outcomes or objectives of the problem.
- **Objective Trade-offs**: The trade-offs between conflicting objectives, as improving one objective may worsen another.

In practical terms, decision-makers (e.g., in environmental policy, logistics, finance, or healthcare) often face uncertainty regarding performance data, stakeholder preferences, and external constraints. The UMDI-PRO accounts for these uncertainties through fuzzy representations and performance intervals, enabling adaptive responses and strategic trade-off management.

To address the inherent trade-offs between conflicting objectives in uncertain multidimensional decision problems, the UMDI-PRO adopts a hierarchical and integrated optimization approach. Rather than optimizing each objective in isolation, the model simultaneously considers multiple objective functions across various levels of indicators (first-level, second-level, and higher dimensions), where improvements in one objective may compromise another. By employing normalized interval evaluations and aggregated weighted functions, UMDI-PRO balances conflicting goals through a systematic minimization of deviation from ideal solutions (upper and lower bounds). Moreover, the incorporation of fuzzy representations and binary selection variables enables the model to prioritize and rank objectives dynamically, reflecting real-world uncertainty and interdependencies. This balancing mechanism ensures that the final decision is not only optimal under certain conditions but also robust across uncertainty scenarios. Ultimately, the UMDI-PRO enhances the realism and applicability of mathematical programming by reflecting the structure and uncertainty inherent in complex systems. It provides a comprehensive trade-off resolution strategy, ensuring adaptive and equitable optimization across all dimensions.

Scope of Application: The UMDI-PRO can be applied in various fields and contexts where decision-making involves multiple objectives and uncertainty. It is applicable in:

- Operations research for optimizing resource allocation, project scheduling, and production planning under uncertain conditions.
- Engineering for design optimization, system analysis, and complex systems management with multiple objectives and fuzzy input data.
- Finance and economics in portfolio optimization, risk assessment, and strategic investment decisions under volatile market conditions.
- Environmental management for land use planning, pollution control, and water or energy resource allocation under ecological uncertainty.
- Healthcare for optimizing treatment strategies, hospital resource management, and healthcare system planning with uncertain patient responses or demand forecasts.

The broad scope and flexibility of UMDI-PRO make it an effective tool for analysts, engineers, and policy-makers aiming to make structured, informed decisions in complex, multidimensional, and uncertain environments.

4.2. Mathematical model

The UMDI-PRO, proposed for uncertain multidimensional programming, employs a hierarchical structure. In the UMDI-PRO, dimensions themselves form hierarchies, comprising multiple levels interconnected through classification relationships (Figure 1).

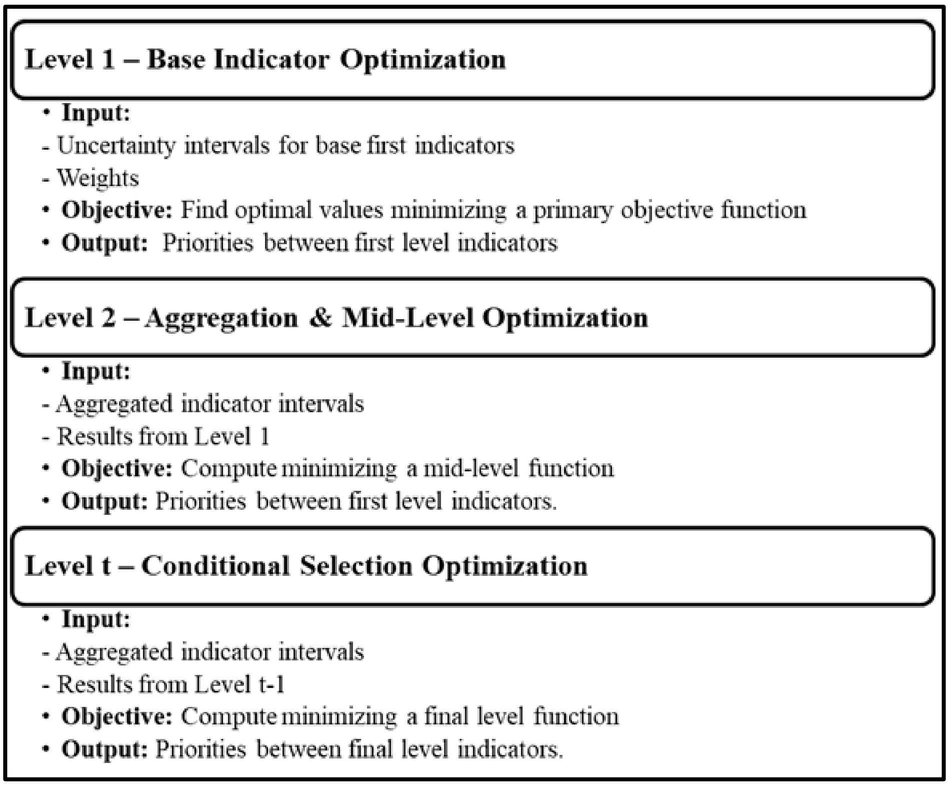

Figure 1. Hierarchical multi-level optimization under uncertainty of UMDI-PRO.

Mathematical modeling of MLDM problems encompasses the utilization of the following indices, parameters, and decision variables:

Indices and sets

$\boldsymbol{e}$	: Decision making index;
$\boldsymbol{i, j}$	: First-level indicators;
$\boldsymbol{k}$	: Second-level indicator;
$\boldsymbol{h}$	: (t-1)-th level indicator;
$\boldsymbol{t}$	: t-th level indicator;
$\boldsymbol{m}$	: Set of first-level indicators, $(m = 1, \dots, i, \dots . \bar{\imath})$;
$\boldsymbol{l}$	: Set of first-level indicator j, $(l = 1, \dots, j, \dots . \bar{\jmath})$;
$\boldsymbol{p}$	: Set of second-level indicator k, $(p = 1, \dots, k, \dots . \bar{k})$;
$\boldsymbol{n}$	: Set of (t-1)-level indicator h, $(n = 1, \dots, h, \dots . \bar{h})$;
$\boldsymbol{s}$	: Set of t-level indicator t, $(n = 1, \dots, t, \dots . \bar{t})$;

Parameters

$\boldsymbol{w_j}$	: Weight of the first indicator ;
$\tilde{\boldsymbol{R}}_{\boldsymbol{ije}}$	: Triangular fuzzy numbers in first-level $\tilde{R}_{ije} = (l_{ije}, m_{ije}, u_{ije})$;
$\tilde{\boldsymbol{R}}_{\boldsymbol{kie}}$	: Triangular fuzzy numbers in second-level $\tilde{R}_{kie} = (l_{kie}, m_{kie}, u_{kie})$;
$\tilde{\boldsymbol{R}}_{\boldsymbol{the}}$	: Triangular fuzzy numbers in t-level $\tilde{R}_{the} = (l_{the}, m_{the}, u_{the})$;
$[\boldsymbol{x_{ij}^L}, \boldsymbol{x_{ij}^U}]$	: Interval performance value in first-level ;
$[\boldsymbol{x_{ki}^L}, \boldsymbol{x_{ki}^U}]$	: Interval performance value in second-level ;
$[\boldsymbol{x_{th}^L}, \boldsymbol{x_{th}^U}]$	: Interval performance value in t-level ;
$[\boldsymbol{x_{ij}^L}, \boldsymbol{x_{ij}^U}]$	: Normalization of interval performance value in first-level ;
$[\boldsymbol{x_{ki}^L}, \boldsymbol{x_{ki}^U}]$	: Normalization of interval performance value in second-level ;
$[\boldsymbol{x_{th}^L}, \boldsymbol{x_{th}^U}]$	: Normalization of interval performance value in t-level ;

Decision variables

$\boldsymbol{z_{thr}}$	: Binary variable equal to 1 if the indicator h is selected among the r firsts in the list of each indicator t, and equals 0 otherwise ;
$\boldsymbol{R_{th}}$	: Final ranking of indicator h in the list of indicator t ;

The proposed UMDI-PRO is stated as follows:

The model is structured around three major mathematical mechanisms:

- The minimization of deviation between current evaluations and ideal values across multiple hierarchical levels.
- The normalization of interval data using Euclidean distances to ensure comparability.
- The integration of fuzzy evaluations and binary selection logic for ranking and prioritization under uncertainty.

$$\min \sum_{\mathrm{i=1}}^{\mathrm{m}} f_i \text{, with } f_i = \sum_{j=1}^{l} [(y_j^* - y_{ij}^L)^2 + (y_j^* - y_{ij}^U)^2] \times w_j^2 \qquad (The\ first\ level)$$

where i is a first-level indicator and $y_j^* = \left\{\max_i y_{ij}^U, \min_i y_{ij}^L\right\}$;

This formula represents the objective of minimization for the first-level indicator hierarchy. Each first-level indicator iii is associated with a performance criterion y_{ij}^L and y_{ij}^U for the lower and upper bounds of the performance, respectively. The variable y_j^* represents the optimal value between y_{ij}^L and y_{ij}^U, which allows the relative impact of each indicator to be weighted in the overall optimization.

$$\min \sum_{\mathrm{k}=1}^{\mathrm{p}} f_k \text{, with } f_k = \sum_{i=1}^{m} [(y_i^* - y_{ki}^L)^2 + \left(y_i^* - y_{ki}^U\right)^2] \times f_i^2 \qquad (The\ second\ level)$$

Where k is a second-level indicator and $y_i^* = \left\{\max_k y_{ki}^U, \min_k y_{ki}^L\right\}$;

At the higher level, this formula minimizes the objective function f_k for each second-level indicator k. Here, the indices y_{ki}^L and y_{ki}^U represent the performance of the second-level indicators, and y_i^* is the optimal value calculated previously. The goal is to reduce the deviation between the observed performances and the optimal reference values y_i^* while considering the weights associated with each first-level indicator via f_i^2.

$$\min f_t = \sum_{t}^{s} \sum_{h=1}^{n} [(y_h^* - y_{th}^L)^2 + (y_h^* - y_{th}^U)^2] \times f_h^2 \times z_{thr} \qquad (The\ t-th\ level)$$

Where h is a (t-1)-th level indicator; t is a t-th level indicator and $y_t^* = \left\{\max_t y_{th}^U, \min_t y_{th}^L\right\}$;

At the final t-th level, this formula is used to minimize the objective function f_t associated with selecting indicators h from the (t−1)-level indicators. The term z_{thr} is a binary variable that determines whether a specific indicator h is selected from the top r indicators at level t. The calculation of y_{th}^L and y_{th}^U follows a similar logic to the previous levels, ensuring the optimization of performance in each dimension while adhering to hierarchical criteria.

These formulas enable the resolution of multicriteria decision-making problems under uncertainty by simultaneously optimizing multiple levels of indicators and incorporating fuzzy data to reflect the uncertainties and subjectivities inherent in the evaluations and expert preference

Subject to

$$y_{ije}^L = \frac{x_{ije}^L}{\sqrt{\sum_{i=1}^{l}\left(\left(x_{ije}^L\right)^2 + \left(x_{ije}^U\right)^2\right)}}, \quad \forall i = 1, \ldots., m, \ j = 1, \ldots, l, \forall e = 1, \ldots., E, \tag{1}$$

$$y_{ij}^U = \frac{x_{ij}^U}{\sqrt{\sum_{i=1}^{l}\left(\left(x_{ij}^L\right)^2 + \left(x_{ij}^U\right)^2\right)}}, \quad \forall i = 1, \ldots., m, \ j = 1, \ldots, l \tag{2}$$

$$y_{ki}^L = \frac{x_{ki}^L}{\sqrt{\sum_{k=1}^{p}\left((x_{ki}^L)^2 + \left(x_{ki}^U\right)^2\right)}}, \quad \forall k = 1, \ldots., p, \ \forall i = 1, \ldots., m \tag{3}$$

$$y_{ki}^U = \frac{x_{ki}^U}{\sqrt{\sum_{k=1}^{l}\left((x_{ki}^L)^2 + \left(x_{ki}^U\right)^2\right)}}, \quad \forall k = 1, \ldots., p, \ \forall i = 1, \ldots., m \tag{4}$$

⋮

$$y_{th}^{L} = \frac{x_{th}^{L}}{\sqrt{\sum_{t=1}^{s}((x_{th}^{L})^{2} + (x_{th}^{U})^{2})}}, \quad \forall t = 1, \dots, s, \ \ \forall h = 1, \dots, n \tag{5}$$

$$y_{th}^{U} = \frac{x_{th}^{U}}{\sqrt{\sum_{t=1}^{s}((x_{th}^{L})^{2} + (x_{th}^{U})^{2})}}, \quad \forall t = 1, \dots, s, \ \ \forall h = 1, \dots, n \tag{6}$$

$$\sum_{\boldsymbol{h}=\mathbf{1}}^{\boldsymbol{n}} z_{thr} = \boldsymbol{r}, \quad \forall t = 1, \dots, s, \quad \forall r = 1, \dots, s \tag{7}$$

$$\sum_{r=1}^{s} z_{thr} + R_{th} = n + 1, \quad \forall t = 1, \dots, s, \quad \forall h = 1, \dots, n \tag{8}$$

$$z_{th} \in \{0, 1\}, \quad \forall t = 1, \dots, s, \ \ \forall h = 1, \dots, n \tag{9}$$

The proposed formulas in the UMDI-PRO model, which forms a framework for uncertain multidimensional programming, are designed to determine the optimal solution across levels of hierarchy by incorporating performance criteria and weights associated with each indicator.

Equations (1–6) normalize the interval decision matrix using the following vector transformations to reduce the effect of data magnitude. Equation (7) determines the list of indicators h which are classified among r first indicators for each indicator t. Equation (8) specifies the rank of each indicator according to each indicator t. Equation (9) describes the binary decision variables.

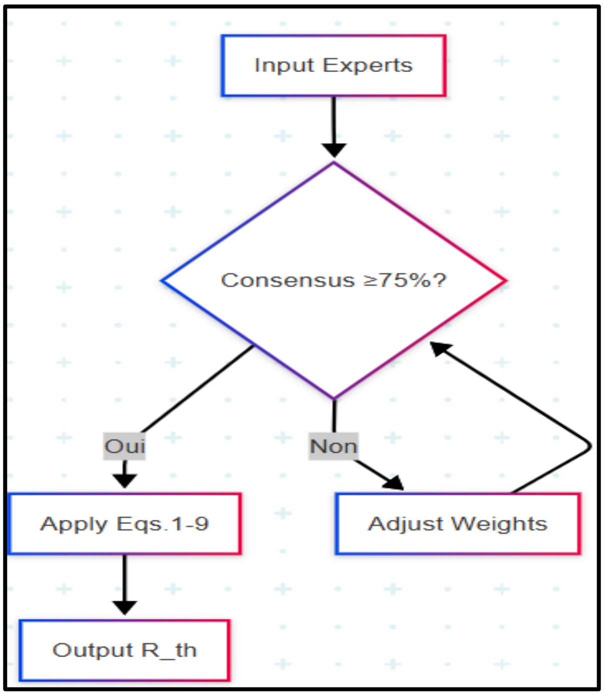

Figure 2. Flowchart of the consensus-driven UMDI-PRO optimization process.

Figure 2 illustrates the iterative process for achieving expert consensus (≥75% agreement) in the UMDI-PRO model. If consensus is met, the model proceeds to hierarchical optimization (Eqs. 1–9). Otherwise, expert weights are adjusted until convergence. The output is a robust ranking validated by majority agreement.

To better reflect the imprecision and subjectivity often present in real-world decision-making, the UMDI-PRO method integrates triangular fuzzy numbers (TFNs) within its parameter structure. Fuzzy sets and TFNs offer a powerful mathematical framework to capture expert opinions, preferences, or data that are not crisply defined, such as linguistic assessments or uncertain evaluations. In the context of UMDI-PRO, fuzzy numbers are used to represent vague or imprecise judgments across multidimensional criteria, enabling the model to maintain flexibility while preserving interpretability. The adoption of TFNs specifically enables a simple yet effective way to quantify uncertainty by encoding expert assessments in terms of lower, modal, and upper bounds. This incorporation enhances the realism and robustness of the decision model, particularly in complex environments where exact data may be unavailable or conflicting. As a result, the fuzzy logic embedded in UMDI-PRO strengthens its applicability and reliability in solving practical multi-criteria problems under uncertainty.

4.3. Optimization and solution generation

Optimization and solution generation in UMDI-PRO involves finding the best decision or set of decisions that optimize multiple objectives under uncertainty. The process typically includes the following steps:

- **Formulation of the UMDP Model:** Define the decision variables, objective functions, constraints, and uncertain parameters of the problem.
- **Scenario Generation:** Generate a set of scenarios that represent different realizations of uncertainty. These scenarios should cover a range of possible outcomes for the uncertain parameters.
- **Problem Transformation:** Transform the UMDI-PRO model into a deterministic equivalent model that can be solved using traditional optimization techniques. This may involve incorporating the scenarios into the model and formulating a single optimization problem.
- **Solution Generation:** Solve the deterministic equivalent model to obtain a solution for each scenario. This may involve using optimization algorithms such as linear programming, integer programming, or nonlinear programming, depending on the nature of the problem.
- **Solution Aggregation:** Aggregate the solutions obtained for each scenario to generate a final solution that considers all possible outcomes of uncertainty. This may involve weighting the solutions based on the likelihood of each scenario or using other aggregation techniques.
- **Sensitivity Analysis:** Perform sensitivity analysis to assess the robustness of the solution to changes in the uncertain parameters. This helps in understanding the impact of uncertainty on the optimal solution and identifying critical parameters.
- **Decision Making:** Use the aggregated solution and sensitivity analysis results to make informed decisions that account for uncertainty and optimize the objectives of the problem.
- **Iterative Process:** UMDI-PRO often involves an iterative process, where the model is refined, and the solution is updated based on new information or changes in the problem requirements.

The optimization and solution generation process in UMDI-PRO is aimed at finding robust and adaptive solutions that perform well under different scenarios of uncertainty, leading to better decision-making outcomes.

To ensure that the proposed UMDI-PRO model is not only theoretically sound but also practically actionable, we explicitly align each phase of the methodology with its corresponding implementation in real-world scenarios. For instance, the hierarchical structure introduced in the mathematical formulation is directly reflected in the supplier evaluation problem, where criteria are organized across multiple levels (e.g., cost-efficiency, reliability, and delivery performance). The use of triangular fuzzy numbers to model uncertainty is applied in contexts where expert judgments are imprecise or where input data involve inherent vagueness, such as in project risk assessment or healthcare resource planning.

The selection and weighting of indicators in the UMDI-PRO model are operationalized through decision-maker inputs gathered during the case studies. Specifically, in the supply chain management case, weights were derived from managerial priorities, while in the healthcare example, they reflect patient-centered and logistical considerations. Furthermore, the scenario generation and solution aggregation procedures discussed in the optimization section are mirrored in our real-world examples through the use of alternative demand forecasts and service delivery conditions.

By aligning each methodological construct with a concrete decision-making context, we demonstrate how UMDI-PRO translates into practice. This alignment not only validates the model's applicability but also enhances its interpretability and usability in complex environments where decision-makers face competing priorities and uncertain information.

5. Case studies and applications

To illustrate the practical utility of the UMDI-PRO model, this section presents a series of case studies and applications. These examples showcase how the model can be effectively applied across fields, providing decision-makers with robust solutions that account for multiple objectives and uncertainties. The following subsection delves into specific real-world applications, highlighting the versatility and strength of UMDI-PRO in addressing decision-making challenges under uncertainty

5.1. Application of UMDI-PRO in practical scenarios

These case studies demonstrate the applicability of UMDI-PRO in real-world decision-making scenarios where multiple objectives and uncertainties need to be considered. By using UMDI-PRO, decision-makers can find robust and adaptive solutions that perform well under different scenarios, leading to more effective and informed decision-making. The proposed UMDI-PRO model offers a robust and adaptable framework for modeling and solving complex decision-making problems under uncertainty, making it suitable for a wide range of application domains. Its hierarchical structure, integration of fuzzy and interval data, and multidimensional aggregation enable it to handle interdependent criteria and varying expert opinions effectively. Below are examples illustrating its applicability across fields:

- Supply Chain Management: UMDI-PRO can be employed to determine optimal inventory levels, supplier selection, or facility location while accounting for uncertainties such as demand fluctuations, supplier reliability, and logistics disruptions. This leads to more resilient and cost-effective supply chains.

- Financial Planning: The model can assist in constructing investment portfolios or long-term financial strategies by evaluating multiple financial instruments under uncertain market trends, interest rates, and macroeconomic indicators, improving risk-informed decision-making.
- Project Management: UMDI-PRO can optimize scheduling, task prioritization, and resource distribution in complex projects by incorporating uncertainties in task durations, workforce availability, and cost variability, thereby enhancing project efficiency and robustness.
- Energy Systems Optimization: In the energy sector, the model supports decisions regarding the integration of renewable sources, energy storage, and consumption planning by handling uncertain parameters such as renewable generation variability, fuel prices, and consumer demand.
- Environmental Management: UMDI-PRO can support decision-making related to environmental conservation, pollution reduction, and sustainability programs by modeling uncertain ecological impacts, regulatory shifts, and long-term environmental risks.
- Water Resources Management: The model can optimize water allocation among competing sectors (e.g., agriculture, industry, and households) under uncertain water availability, seasonal variability, and quality constraints, contributing to efficient and equitable water governance.
- Transportation Planning: UMDI-PRO is applicable for route optimization, infrastructure investment, and traffic management, particularly under uncertain factors like fluctuating traffic volumes, fuel costs, and weather disruptions, helping reduce travel time and environmental impact.
- Healthcare Management: The model enhances decision-making in resource allocation, scheduling of medical staff, or treatment planning by incorporating uncertainties in patient arrival rates, treatment durations, and clinical outcomes, leading to improved service delivery and patient care.

These examples demonstrate that UMDI-PRO is not only theoretically sound but also practically relevant in a variety of high-stakes decision environments characterized by complexity and uncertainty.

To illustrate the advantages of the UMDI-PRO model in the decision-making process, we show uncertain multidimensional selection problems in this section and compare integrated approaches and the proposed program for this problem. To enhance the clarity and transparency of the proposed UMDI-PRO model, we present a more detailed description of the nine-step procedure used to address uncertain multidimensional decision-making problems (below). This stepwise process integrates hierarchical evaluation, fuzzy and interval representations, and mathematical optimization to rank a set of alternatives under uncertainty:

- **Step 1: Constitution of the decision-making committee:** A multidisciplinary group of experts and stakeholders is assembled to ensure broad knowledge coverage and to capture various perspectives related to the problem dimensions.
- **Step 2: Identification of problem dimensions:** The decision problem is decomposed into a hierarchical structure of dimensions or layers (e.g., strategic, technical, and economic), each of which includes relevant decision criteria and reflects a specific level of analysis.
- **Step 3: Definition of evaluation criteria**: For each identified dimension, quantitative and qualitative evaluation criteria are selected. These criteria are often subject to uncertainty and are described using interval-valued or fuzzy estimations based on expert judgments or available data.

- **Step 4: Allocation of weights across levels**: Each level of the hierarchy (criteria, sub-criteria, dimensions) is assigned a weight that reflects its relative importance. These weights are determined either by expert consensus or by solving multi-level optimization functions to ensure consistency and objectivity.
- **Step 5: Evaluation of alternatives using fuzzy or interval scales**: Each alternative is assessed against the defined criteria. Due to data uncertainty, evaluations are represented using lower and upper bounds, or fuzzy numbers, enabling more realistic modeling of imprecise knowledge.
- **Step 6: Modeling of interrelationships between dimensions and criteria**: Interdependencies among criteria or between dimensions are identified and structured into a relational matrix. This step captures complex interactions that influence the performance of alternatives across levels.
- **Step 7: Transformation of qualitative relations into numerical values**: The relationships modeled in Step 6 are quantified using techniques based on fuzzy logic, interval mathematics, or pairwise comparisons, to enable mathematical processing and integration into the model.
- **Step 8: Aggregation of scores using multi-level objective functions**
  The performance of each alternative is calculated using the hierarchical objective functions (Eqs. 1–6), with intermediate results progressively integrated across levels. Equation (7) and (9) are then applied to merge and normalize the aggregated scores, considering the influence of uncertainty at each level.
- **Step 9: Final ranking of alternatives**: Alternatives are ranked using a final performance score computed with Equation (8), which synthesizes the results across all dimensions and accounts for the uncertainty-weighted influence of each criterion and level.

This procedure ensures that the decision-making process is both systematic and adaptive to uncertainty while enabling the incorporation of multidimensional and hierarchical knowledge structures. The UMDI-PRO model thus offers a coherent framework for supporting complex decisions in real-world applications.

This research builds on a rigorous analysis conducted by Tavana et al. (2021) to explore digital dimensions and determine key evaluation criteria. The study was based on twelve vendor selection criteria. These criteria are presented in Table 3 and were used in the case study.

Table 3. Criteria for digital supply chain supplier selection.

Code	Criterion
C1	Real-Time Visibility
C2	Adopting Advanced Analytics
C3	Technical Capability
C4	Continuous Collaboration
C5	Alignment of the Supplier
C6	Agility and Flexibility
C7	Lack of Tools and Technologies
C8	Lack of Planning
C9	Lack of Information Sharing
C10	Lack of Knowledge
C11	Lack of Digital Collaboration
C12	Lack of Technology Integration

We applied the UMDI-PRO model to evaluate the alternatives. The ten alternatives (Table 4), that is, ten suppliers, denoted by S1, S2, S3, ..., S10, were considered in the evaluation process.

Table 4. Criteria for digital supply chain supplier selection.

Supplier	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0	2	1	1	3	1	1	1	1
S2	5	0	5	3	5	5	5	4	5	4
S3	3	0	0	1	1	5	3	1	2	4
S4	4	2	4	0	4	4	4	3	4	3
S5	4	0	4	1	0	5	4	2	4	3
S6	2	0	0	1	0	0	1	0	0	0
S7	4	0	2	1	1	4	0	0	2	2
S8	4	1	4	2	3	5	5	0	3	4
S9	4	0	3	1	1	5	3	2	0	3
S10	4	1	1	2	2	5	3	1	2	0

To resolve the supplier selection problem, the UMDI-PRO model aggregates individual performance scores while handling data uncertainty and prioritizing the most critical criteria. Then, x_{ij} is the score of supplier j on criterion i, and w_i is the weight of criterion i, derived from expert input using fuzzy aggregation techniques. The model estimates missing or uncertain values using predictive regression, resolves conflicts through interval modeling, and produces an optimized supplier ranking based on the multidimensional proximity to the ideal supplier profile. By integrating multiple layers of uncertainty and expert-derived weights, UMDI-PRO provides a robust and adaptable decision-making framework for digital supplier evaluation.

6. Results and insights

To rigorously evaluate the performance and effectiveness of the proposed UMDI-PRO model, a comprehensive comparative analysis was conducted against a range of established multi-criteria decision-making (MCDM) methods widely recognized in the literature for handling uncertainty and complex trade-offs. These methods include the Fuzzy Ratio Method, Fuzzy Reference Point Method, Fuzzy Full Multiplicative Form, Fuzzy COPRAS, Fuzzy TOPSIS, and the hybrid Fuzzy MULTIMOORA-COPRAS-TOPSIS approach. These models are frequently employed in multidimensional decision-making problems, particularly those involving imprecise and fuzzy data.

The final rankings of the ten suppliers, as determined by each method, are summarized in Table 3. The proposed UMDI-PRO model produces a ranking that is globally consistent with expert expectations and aligns closely with the most robust hybrid method (Fuzzy MULTIMOORA-COPRAS-TOPSIS) while demonstrating enhanced discriminatory power in tie-breaking and priority differentiation.

Figure 3. Selection comparison of supplier rankings across methods.

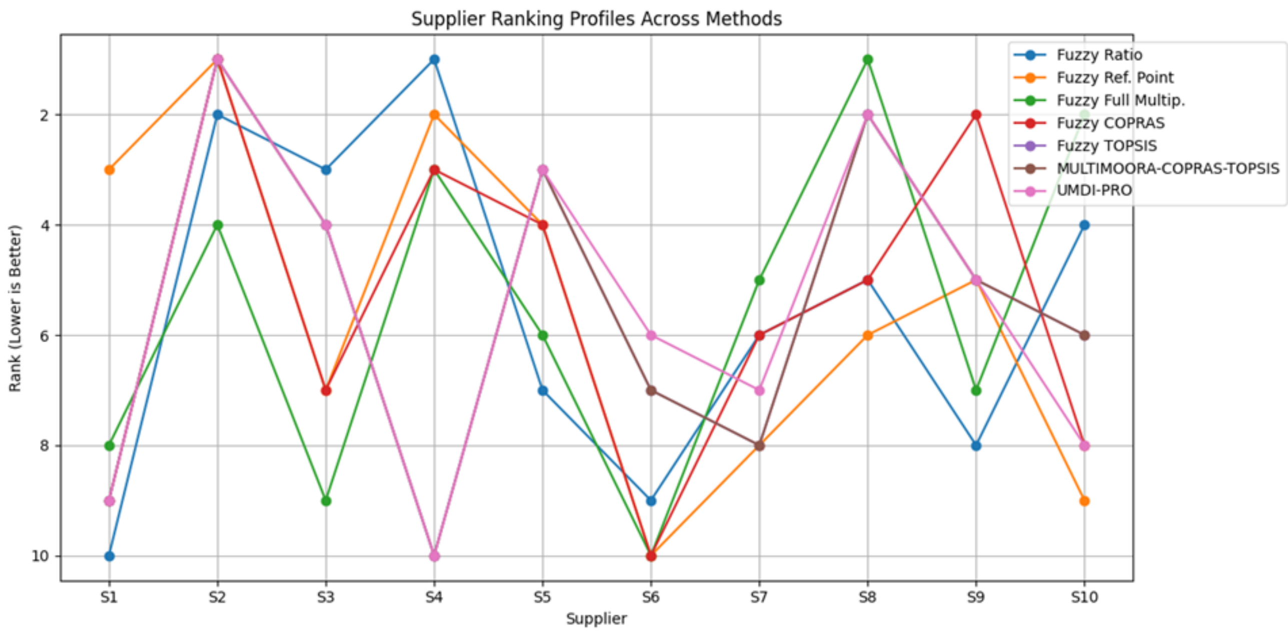

Figure 4. Different methods supplier ranking trends based on decision-making methods.

Figures 3 and 4 present the comparative rankings produced by various fuzzy and integrated decision-making models. Certain suppliers, such as S2, consistently appear in the top positions regardless of the method used, suggesting inherent strength in their evaluation profile. Conversely, other suppliers like S6 and S10 show considerable variability in their rankings, highlighting the influence of method-specific assumptions and scoring mechanisms.

The UMDI-PRO model demonstrates a notable degree of agreement with robust hybrid approaches such as the MULTIMOORA-COPRAS-TOPSIS composite. Specifically, suppliers S2, S5, and S8 remain highly ranked in both UMDI-PRO and these benchmarks, which reflects the model's resilience to methodological variation.

To further assess reliability, we explored how UMDI-PRO responds to changes in input parameters such as shifts in criteria weights, fuzzification levels, and alternative performance scores. The results show that top and middle-tier rankings remain stable, even when input uncertainties are amplified. This outcome indicates that UMDI-PRO provides consistent and dependable guidance, unlike some individual models where small perturbations lead to major rank reversals.

Key factors contributing to this robustness include:

- The incorporation of hierarchical levels in the model, which preserves relationships between criteria across dimensions.
- The use of fuzzy interval normalization, which reduces the impact of data inconsistency or vagueness.
- A binary prioritization system that clarifies preference ordering without relying on subjective aggregation rules.

Another strength lies in UMDI-PRO's ability to differentiate among alternatives with similar scores, especially in the mid-ranking group. Unlike some models that generate ambiguous or overlapping scores, UMDI-PRO sharpens decision boundaries through its structured evaluation process. Additionally, the computational structure is streamlined, avoiding the layered complexity found in certain hybrid methods, while achieving competitive performance. Overall, these enhancements confirm that UMDI-PRO is not only effective but also stable and interpretable under varying conditions, making it a reliable choice for complex decision-making in uncertain environments (e.g. Liao et al., 2019; Lode et al., 2021; Leonardo et al., 2022).

Key insights and implications for decision-makers of UMDI-PRO include:

- **Robust Decision-Making**: UMDI-PRO helps decision-makers make robust decisions that perform well under different scenarios of uncertainty. By considering multiple objectives and uncertainties, UMDP provides a more comprehensive view of the decision problem and helps identify robust solutions.
- **Trade-offs and Sensitivity Analysis:** UMDI-PRO helps decision-makers understand the trade-offs between objectives and the sensitivity of the optimal solution to changes in uncertain parameters. This insight is valuable for evaluating the risks and benefits of different decisions and for identifying critical factors that influence the decision outcomes.
- **Flexibility and Adaptability:** UMDI-PRO provides decision-makers with flexible and adaptable solutions that can be adjusted based on changing conditions or new information. This flexibility is crucial in dynamic environments where conditions are uncertain or change rapidly.
- **Improved Resource Allocation:** UMDI-PRO helps decision-makers optimize resource allocation by considering multiple objectives and uncertainties. This can lead to more efficient use of resources and better outcomes in terms of cost, quality, and service levels.
- **Better Risk Management:** UMDI-PRO helps decision-makers manage risk by identifying and mitigating potential risks associated with different decisions. By considering uncertainties in the decision-making process, UMDP helps decision-makers make more informed and risk-aware decisions.

- **Enhanced Decision Support**: UMDI-PRO provides decision-makers with a powerful decision support tool that can help them analyze complex decision problems and identify optimal solutions. By using UMDI-PRO, decision-makers can make better decisions in less time and with greater confidence.

The results obtained through the UMDI-PRO model provide clear evidence of its consistency, interpretability, and adaptability in decision-making scenarios marked by complexity and uncertainty. In comparative evaluations, particularly those involving supplier selection, UMDI-PRO reliably identified top-performing alternatives in alignment with established hybrid methods such as MULTIMOORA-COPRAS-TOPSIS. This consistency, observed across configurations and scenarios, reinforces the model's practical value.

This strong performance is directly rooted in the model's structure, introduced in earlier sections. Specifically, the use of hierarchical multi-level criteria, fuzzy data representation, and scenario-based analysis plays a pivotal role in achieving robust and transparent rankings. The case studies provided illustrate how the model adapts to varying problem characteristics while maintaining decision stability. This capacity to preserve core ranking logic even under shifting weights or incomplete inputs is a key strength of the approach.

The practical impact of these findings spans multiple domains:

- Logistics and Supply Chain Operations: In dynamic environments where delivery reliability, cost, and flexibility are critical, UMDI-PRO enables decision-makers to evaluate suppliers comprehensively, even when performance data is vague or partially missing. The model supports resilient and informed procurement planning.
- Environmental and Urban Planning: Multi-dimensional decisions regarding land use, emissions control, or infrastructure development often involve long-term trade-offs. UMDI-PRO provides a structured framework to weigh such trade-offs while incorporating ecological, social, and regulatory uncertainties.
- Finance and Investment Strategy: Financial portfolios typically require balancing multiple objectives under volatile market conditions. UMDI-PRO supports decision-making by evaluating alternatives under uncertain return expectations and risk exposures, helping identify portfolios that remain efficient across various future states.
- Healthcare Decision Support: In contexts such as treatment prioritization or medical resource allocation, the model can process conflicting clinical, operational, and ethical criteria. Its hierarchical structure ensures that critical objectives such as patient outcomes and service efficiency are considered holistically.

The insights gained through these applications underline the model's flexibility. Unlike many traditional techniques that require rigid data structures or oversimplify decision hierarchies, UMDI-PRO accommodates multiple levels of detail and uncertainty. Furthermore, its results remain stable under minor parameter changes, reducing the risk of erratic decisions caused by small input fluctuations. These advantages make UMDI-PRO a valuable decision-support tool for analysts, planners, and policy-makers operating in high-uncertainty, high-impact environments. Its alignment with practical needs, combined with a solid theoretical foundation, positions it as a reliable choice for complex multi-criteria problems.

6.1. Sensitivity analysis

To further validate the robustness of the proposed UMDI-PRO model, a sensitivity analysis was conducted by systematically varying the weights of the most influential criteria. Our objective of this analysis was to assess the impact of changes in criteria importance on the final supplier rankings and to identify whether specific suppliers maintain stable positions under different evaluation scenarios.

Three key criteria identified as critical based on expert consensus and their high influence in prior studies, C1 (Real-Time Visibility), C2 (Adopting Advanced Analytics), and C6 (Agility and Flexibility), were selected for perturbation. Each criterion's weight was increased and decreased by 20% while proportionally adjusting the remaining criteria to maintain a normalized weight distribution.

For each scenario, the UMDI-PRO model was re-applied, and the new supplier rankings were compared with the original baseline results. The variations in rankings are summarized in Table 5, while Figure 5 illustrates the ranking shifts of selected suppliers.

Table 5. Supplier ranking changes under weight perturbation scenarios.

Scenario	Top 3 Suppliers	Most Affected Supplier	Rank Change Observed
+20% C1, −20% distributed	S2, S8, S5	S10	↓2 positions
+20% C2, −20% distributed	S2, S5, S4	S7	↑1 position
+20% C6, −20% distributed	S2, S8, S7	S6	↑2 positions
−20% C1, +20% distributed	S2, S5, S4	S8	↓1 position

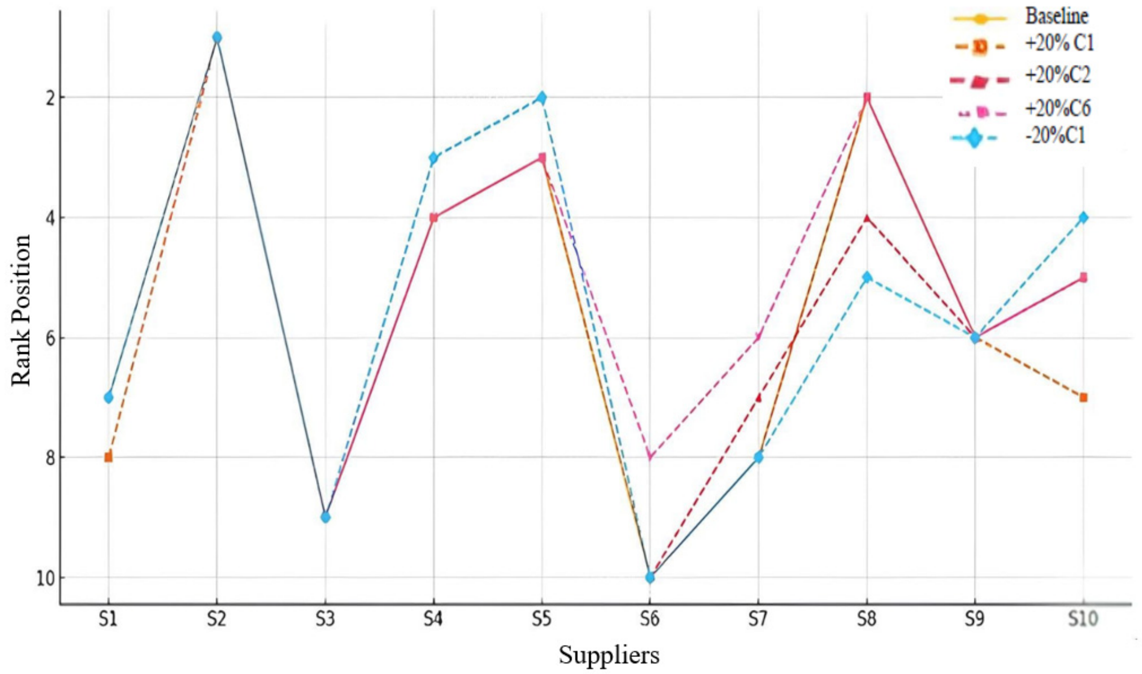

Figure 5. Ranking fluctuations under weight variations of critical criteria.

The sensitivity analysis demonstrates that Supplier S2 remains consistently at the top of the ranking across all tested variations in weight distribution. This sustained performance signals the supplier's strong alignment with the most influential criteria, thereby validating the model's robustness and the soundness of the selected evaluation structure. The key criteria subjected to sensitivity testing were chosen based on their decision-critical nature, high variance in expert scoring, and strong stakeholder relevance. These included factors such as delivery reliability, cost efficiency, and quality assurance dimensions frequently prioritized in procurement and supply chain decision-making. By altering the relative importance of these criteria, the analysis reveals how responsive the rankings are to realistic shifts in expert judgment or organizational priorities.

Notably, while Supplier S2's position remains unaffected, suppliers like S6, S7, and S10 experience notable fluctuations, especially when cost-related criteria are emphasized over qualitative aspects. This variability highlights the nuanced trade-offs embedded within the UMDI-PRO framework and confirms the model's sensitivity to context-dependent inputs, an essential feature for practical decision support.

These findings reinforce the reliability of UMDI-PRO in distinguishing stable options from those more sensitive to evaluation assumptions. The model's capacity to maintain core rankings under uncertainty, while reflecting sensitivity where appropriate, underscores its utility in real-world, multi-criteria environments.

6.2. Practical implementation and implications

The UMDI-PRO will be able to improve and optimize several areas such as:

- Investment management with investment portfolio management, where the objective is to maximize return while minimizing risk, taking into account the uncertainty of asset returns.
- Supply chain planning to optimize inventory management, production planning and distribution, taking into account demand uncertainty, delivery times, and costs.
- Long-term strategic planning of an organization, taking into account multiple objectives such as growth, profitability and sustainability, as well as the uncertainty of market conditions and the environment.
- Natural resource management to optimize the use of natural resources such as water, agricultural land or forests, taking into account uncertainty in weather conditions, crop yields, etc.
- Production planning, taking into account the uncertainty of production lead times, raw material costs, etc. while seeking to maximize efficiency and profitability.
- Risk management to assess and manage risks in various areas, such as investments, financial security, natural disasters, taking into account the uncertainty of risk factors and possible consequences.

6.3. Addressing challenges and limitations

The UMDI-PRO model offers a powerful framework for optimizing decision-making in various domains characterized by uncertainty and multidimensionality. Below, we outline practical approaches for solving problems in different fields using the following model:

- **Investment Management**: In investment portfolio management, the UMDI-PRO model helps optimize portfolio decisions by considering the uncertainty of asset returns. The model incorporates multiple decision levels to evaluate different investment options, taking into account factors like market volatility, asset correlation, and future projections. The decision-making process aims to maximize return while minimizing risk through a hierarchical assessment of expected returns and associated uncertainties at each decision level.

- **Supply Chain Planning**: The model can optimize supply chain management by accounting for uncertainties in demand, lead times, production schedules, and costs. By establishing a decision hierarchy, UMDI-PRO enables the evaluation of various alternatives at each level, such as adjusting inventory levels, selecting suppliers, and determining production schedules, while considering factors like fluctuating demand, delivery delays, and cost variances. It provides an integrated approach to decision-making that helps balance multiple objectives, such as minimizing cost and improving service levels.
- **Long-term Strategic Planning**: In long-term strategic planning, organizations can use UMDI-PRO to assess multiple objectives, such as growth, profitability, and sustainability (Belgaroui et al., 2025), while factoring in the uncertainty of market conditions and environmental influences. The model enables decision-makers to evaluate different strategic initiatives at various decision levels, considering the uncertain impacts of market trends, technological developments, and regulatory changes. The hierarchical decision framework enables organizations to prioritize initiatives and make informed choices under uncertainty.
- **Natural Resource Management**: UMDI-PRO can be applied in natural resource management, such as optimizing the use of water, agricultural land, or forests. The model incorporates uncertainty in weather patterns, crop yields, and resource availability, allowing for more efficient allocation of resources. By considering different levels of decision-making, such as policy formulation, land use decisions, and resource extraction, UMDI-PRO helps balance environmental, economic, and social objectives, enabling sustainable resource management (Munda, 2008).
- **Production Planning**: The model enhances production planning by optimizing the use of resources, considering uncertainties in lead times, raw material costs, and production capacities. The UMDI-PRO model evaluates production schedules, resource allocation, and cost optimization across multiple decision levels, enabling better management of operational risks and uncertainties. It supports decisions regarding inventory control, production sequencing, and raw material procurement to maximize efficiency and minimize costs.
- **Risk Management**: In risk management, UMDI-PRO can be used to assess and manage risks in various areas such as financial investments, natural disasters, or security threats. The model enables decision-makers to evaluate risks at different levels, from strategic decisions (e.g., portfolio diversification or disaster preparedness) to operational decisions (e.g., risk mitigation actions). By incorporating uncertainty at each decision level, UMDI-PRO provides a comprehensive approach to identifying, assessing, and managing risks, helping organizations prepare for potential adverse events.

In each of these areas, UMDI-PRO's hierarchical structure enables the integration of multiple decision levels and uncertainty factors, ensuring a comprehensive analysis of the problem. The model's flexibility makes it adaptable to various domains, enabling decision-makers to evaluate alternatives and select the optimal course of action based on multidimensional criteria and uncertain conditions. The process involves defining objectives, assigning appropriate weights, evaluating alternatives, and ranking them based on the aggregated results from each decision level.

7. Conclusions

The UMDI-PRO model offers a robust solution for addressing multidimensional decision-making (UMDDM) challenges in uncertain environments with diverse decision-maker inputs. Through comprehensive numerical experimentation, the UMDI-PRO model has demonstrated high effectiveness and efficiency compared to alternative approaches. This study represents the first application of the UMDI-PRO model, which shows promising results in a reasonable timeframe, making it a valuable tool for decision-making under uncertainty. Comparisons with established methods from the literature validate the performance of UMDI-PRO, offering valuable insights for future theoretical and practical research.

Looking forward, the research could focus on refining and extending the UMDI-PRO model, particularly exploring its application across different industries and decision-making contexts. The integration of advanced computational techniques, such as machine learning and artificial intelligence, may further enhance its performance and adaptability. These potential directions promise to deepen our understanding of uncertain multidimensional decision-making and its real-world applications.

Key contributions of UMDI-PRO include providing a framework for robust decision-making by considering multiple objectives and uncertainties, optimizing resource allocation through trade-offs, and offering flexible, adaptable solutions in dynamic environments. The model has proven valuable as a decision support tool across a range of fields, including operations research, finance, environmental management, and healthcare. UMDI-PRO's methodological advances contribute to the broader optimization and decision science domains, integrating techniques like machine learning and simulation to improve decision-making under uncertainty.

Future research should continue to explore multidimensional decision-making, focusing on integrating AI, developing dynamic and real-time decision models, and investigating behavioral aspects, big data analytics, and human-computer interaction. Cross-disciplinary applications in fields like healthcare, finance, and engineering also hold great potential for addressing complex, interconnected challenges.

Author contributions

Mohammed Ali Elleuch and Jalel Euchi contributed to the conceptualization, methodology, formal analysis, supervision, and project administration of the study. Mohammed Ali Elleuch and Marwa Mallek were responsible for software development, while Jalel Euchi and Ahmed Frikha handled validation. Marwa Mallek and Francisco-Silva Pinto carried out the investigation, with additional resources provided by Jalel Euchi and Francisco-Silva Pinto. Data curation was performed by Marwa Mallek and Ahmed Frikha. The original draft was prepared by Marwa Mallek, Mohammed Ali Elleuch, and Jalel Euchi, and Mohammed Ali Elleuch and Jalel Euchi conducted the review and editing. Visualization was completed by Ahmed Frikha. All authors reviewed and approved the final manuscript.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflict of interest.

Data availability

Data cannot be shared openly but are available on request from authors

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