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Research article

Discovering AI tokens in the Fractal Markets Hypothesis and their time-frequency co-movements with the leading high-carbon cryptocurrency

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Abstract: In the AI era, we contribute to the literature by uncovering that the price dynamics of most AI tokens could be fully characterized by the processes driven by fractal Brownian motion, which robustly supports the principles of the fractal markets hypothesis. Using rescaled range (R/S, i.e., Fractal) analysis and the wavelet coherence technique, we analyzed daily log-returns from seven major AI tokens and Bitcoin over the period 2020–2024. Our empirical results rejected the weak form of the Efficient Market Hypothesis (EMH), supporting the Fractal Market Hypothesis (FMH) as a better explanation for the dynamics of AI crypto tokens. More importantly, the log-returns of all analyzed AI tokens, each exhibiting a Hurst exponent exceeding 0.58, provided evidence of persistent behavior and an inherent tendency toward positive price trajectories. These results implied that Fractal analysis can enhance investors' ability to model return dynamics and identify potential appreciation in AI tokens, particularly as short-term trading activity intensifies during episodes of elevated market turbulence. Finally, this work reveals that AI tokens exhibit strong coherence patterns with Bitcoin, varying across time and frequency domains, suggesting Bitcoin's limited role as a hedge against AI tokens. Crucially, this study highlights the significant role of AI tokens as potential safe-haven assets during market turmoil, offering valuable insights for portfolio diversification for crypto investors with intuitive and plausible results that carry strong policy implications.

Keywords: AI-related tokens; fractal market hypothesis; rescaled range analysis (Hurst exponent), efficient market hypothesis

JEL Codes: G11, G14, G15, G21, O33

1. Introduction

Since OpenAI's ChatGPT[1] was released in November 2022, artificial intelligence (AI) has rapidly transformed our world, driving innovation across sectors. This surge in AI interest has led to astronomical valuations of AI companies and a corresponding price surge in AI-related stocks (Cao et al., 2024). AI crypto tokens, also referred to as coins or tokens based on AI, belong to a category of cryptocurrencies that integrate AI technology into their operations. These digital assets combine AI technology with cryptocurrency markets. However, developing and deploying advanced AI systems requires significant computational resources and financial investment. Moreover, rooted in the foundational principles of decentralization, transparency, and security, blockchain technology offers a distinctive and promising approach to addressing these challenges (Ante, 2023). AI holds significant potential in advancing cryptocurrency applications and addressing the complex algorithms intrinsic to blockchain systems, particularly in the domains of data management and security (Yousaf et al., 2024a;2024b). One of the critical areas is data management and security. Moreover, AI systems require vast amounts of data to learn and improve. Through blockchains, training data can be shared across platforms and stakeholders, enabling more avenues for collaboration in AI research and development. Crypto can also be used to incentivize the sharing of AI data to foster a more inclusive ecosystem (Binance, 2023).

1.1. Overview of AI Tokens by market cap

According to the release of the Financial Times[2], AI-Crypto assets are emerging as one of the most potent and exciting investment themes in the crypto space for 2024. Currently, two prominent cryptocurrency data sources, including the CoinGecko and CoinMarketCap databases, are considered proper crypto data sources (Vidal-Tomás, 2022). AI crypto has been the fastest-growing blockchain sector since the end of 2023. In particular, considering the CoinGecko "AI coin" category, the market cap of AI tokens has been growing, reaching around $28.3 billion in February 2024. Lee (2024) studied the price returns of various AI crypto tokens between January 1 and February 29, 2024[3]. These tokens aim to leverage AI algorithms for trading decisions and other applications. According to the data source of CoinMarketCap's "AI & Big Data" category, the market capitalization of AI tokens stands at $30.1 billion, with a trading volume of $1.8 billion as of July 17, 2024 (Table 1). AI tokens are cryptocurrencies that fuel AI-driven initiatives such as portfolio management, image generation, and pathfinding. Native to blockchain platforms, they are designed to support AI development by incentivizing participation, optimizing resource allocation, and enabling decentralized AI marketplaces. In this paper, we explore the nature of AI tokens, examining their technological foundations, potential benefits, role in AI development, and implications for the future of fintech.

[1] ChatGPT, developed by OpenAI and launched on November 30, 2022, is a chatbot and virtual assistant. It utilizes large language models (LLMs) to allow users to shape conversations by adjusting parameters like length, format, style, detail, and language. Context from successive user prompts and replies informs its responses. https://inverodigital.com/insights/how-microsofts-copilot-and-chatgpt-will-change-business-as-we-know-it-forever/.

[2] See https://www.ft.com/content/69000488-0cdb-4aa0-9654-d8f2885454f0.

[3] See https://www.coingecko.com/en/categories/artificial-intelligence

Table 1. Summary statistics for top AI Tokens and Bitcoin by market capitalization.

Rank	Name	Ticker	Coinmark	Price	24h %	7d %	Volume(24h)	Market Cap
1	Bitcoin	BTC		$64,118.79	2.07%	16.18%	$32,525,071,311	$1,264,876,808,968
17	NEAR Protocol	NEAR		$6.12	2.26%	31.77%	$304,846,263	$6,747,097,149
26	Artificial Superintelligence Alliance	FET		$1.49	0.70%	25.3%	$195,448,929	$3,756,445,669
34	B Render	RNDR		$6.72	2.84%	5.44%	$184,189,801	$2,638,180,498
43	Injective	INJ		$25.99	2.46%	26.2%	$144,584,709	$2,427,122,944
45	Bittensor	TAO		$317.49	1.78%	25.6%	$65,757,184	$2,250,256,446
48	The Graph	GRT		$0.2115	2.42%	13.7%	$64,456,222	$2,019,247,377
80	Akash Network	AKT		$3.55	5.65%	0.42%	$16,477,080	$868,031,639

Note: 1. Source: Data collected from CoinMarketCap

2. Table 1 lists the most valuable AI cryptos and Bitcoin by market cap as of July 17, 2024. These cryptos are listed by market capitalization with the largest first and then descending in order.

1.2. Central objective of this study

Despite their growing market capitalization, exceeding $30 billion, and increasing integration into decentralized ecosystems, the financial behavior of these AI tokens remains largely underexplored[4]. Our central objective of this study is to determine whether AI token prices exhibit fractal behavior and whether their co-movements with Bitcoin, the most liquid and energy-intensive cryptocurrency, supports the FMH framework. To empirically investigate the applicability of FMH to AI token markets, we employ two key methodologies: The Hurst exponent, estimated via Rescaled Range (R/S) analysis, and wavelet analysis, including power spectra and coherence techniques. The Hurst exponent measures long-memory and persistence in time series data, helping to identify whether price dynamics deviate from randomness. Wavelet analysis, in turn, enables the decomposition of time series into time-frequency domains, revealing how volatility and dependencies evolve over time and across scales. These tools are particularly well-suited for testing FMH, as they detect static and dynamic fractal features (Ikeda, 2017; Celeste et al., 2020).

Another issue about the role of AI tokens is that researchers have documented that the energy-intensive cryptocurrency (Bitcoin) can act as a hedge or safe-haven in turbulent periods or against asset classes (Bouri et al., 2017; Selmi et al., 2018; Nkrumah-Boadu et al., 2022; Chen et al., 2024; Nguyen

[4] Most researchers have focused on mainstream cryptocurrencies such as Bitcoin and Ethereum, with limited empirical investigation into emerging tokens tied to AI innovation. We address that gap by examining the market dynamics of AI tokens, particularly in terms of persistence, volatility, and interdependence with Bitcoin during periods of financial stress.

et al., 2025). Additionally, Bitcoin is well-known for being the leading high-carbon cryptocurrency. To uncover the role of AI tokens in portfolio allocation against Bitcoin, we have made methodological advances in analyzing the time-frequency linkages between AI tokens and Bitcoin. By employing wavelet coherence analysis, we have thoroughly investigated the time-frequency dynamics between AI tokens and Bitcoin, the largest high-carbon cryptocurrency, and conducted additional tests on the Bitcoin-AI token data in this study. In recent years, wavelet coherence, a distinct method within wavelet analysis, has become increasingly prominent in academic discussions across financial markets. (e.g., Smolo et al., 2024; Bouri et al., 2023; Karamti and Belhassine, 2022; Das, 2021). Following the recent literature of Osman et al. (2024) and Smolo et al. (2022), we introduce the wavelet coherence analysis to capture the time-frequency linkages between Bitcoin and AI tokens. More importantly, the wavelet coherence approach is briefly implemented using the R package "biwavelet" for the wavelet coherence package by Gouhier et al. (2021) under this study[5].

Within this context, we also provide a comprehensive research review of the FMH and the growth of the cryptocurrency market to date. This highlights the gaps in research and sets the stage for our investigation. Besides, Ikeda (2017) found that 82% of world stock prices are consistent with the fractal market hypothesis rather than the efficient market hypothesis. In brief, the Fractal Market Hypothesis (FMH) developed by Peters (1994) focuses on addressing the limitations of the Efficient Market Hypothesis (EMH). Specifically, it seeks to understand why self-similarity occurs in financial market prices. FMH extends EMH by incorporating fractal properties and analyzing the interactions among various investor groups, shedding light on market behavior even during turbulent times (Niveditha, 2024). Moreover, the FMH interprets market dynamics by drawing on theories related to fractals and chaos and can provide insights into prevalent periods of market crashes and crises that impact market behavior (Kristoufek, 2013; Kristjanpoller et al., 2022) in contrast to the weak-form EMH, which posits that the market is efficient when prices follow a martingale (such as a random walk). Due to the unpredictability of price movements, the market is considered efficient. The EMH posits that the market is efficient when all available information is fully reflected. As a result, prices become unpredictable, and markets are considered efficient. Consequently, generating profits becomes an impossibility.

In contrast, the FMH explains market dynamics using theories of fractals and chaos. Most studies indicate robust evidence of the FMH shedding light on significant periods of crashes and crises, making it an appropriate approach to assess market efficiency during instability, such as the COVID-19 pandemic (Mensi et al., 2020; Okorie et al., 2021; Kristjanpoller et al., 2022; Niveditha, 2024, among others). However, researchers looking into major AI token price processes have yet to fully investigate their joint fractal characteristics and wavelet-based analysis. Such analyses can affirm the existence of fractal dynamics in their prices and either validate or refute the applicability of the FMH to AI tokens. This raises an important question: *Do AI token markets behave in accordance with FMH than EMH?* To address this, we explore three key research questions:

1. RQ1**:** Do AI token markets exhibit fractal properties, such as long-range dependence and persistence?
2. RQ2: Do wavelet power spectra reveal time-localized behavior in AI tokens during periods of market stress?
3. RQ3: Does the wavelet coherence between AI tokens and Bitcoin vary across time and scales, revealing dynamic interdependencies?

[5] The package is available at: http://www.glaciology.net/wavelet-coherence. For brevity, the R package code is not reported in the content. However, it is available from the author(s) upon request.

We empirically test two major hypotheses:

- H_1: AI tokens exhibit Hurst exponents greater than 0.5, indicating persistent price dynamics consistent with fractal Brownian motion.
- H_2: Time-frequency wavelet coherence between AI tokens and Bitcoin reveals non-stationary co-movement patterns, especially during crisis periods, validating FMH assumptions.

Our analysis [6] provides robust evidence favoring the FMH and contributes to a deeper understanding of how AI tokens behave under varying market conditions. Thus, this research contributes to the literature by examining the characteristics of AI-based tokens, offering valuable insights into investor and manager responses to AI's expanding influence in the global economy. Compared to the most recent literature, this study offers four key contributions. Unlike prior studies, we address several gaps in the literature across dimensions, as outlined below:

First, AI tokens are infantile products in the crypto ecosystem. A limited article focuses on the fractal analysis of some traditional cryptos, but the fractal behavior of the AI tokens market needs to be explored. Second, we quantitatively describe the stylized fact of long-term dependence and better estimate the Hurst exponent (H) for Brownian fractal signals with known H when H is greater than 0.5 in the AI token markets. Third, we suggest that AI tokens can be considered potential safe-haven assets through time-frequency wavelet analysis. Fourth, the evidence of strong coherence between most AI tokens and Bitcoin indicates that, while Bitcoin serves as a diversifier for AI tokens, it is ineffective as a hedge or safe-haven asset against them.

Last, the remainder of the paper is structured as follows: In Section 2, we review related literature. In Section 3, we introduce the theoretical framework for rescaled range analysis, wavelet transform, and power analysis. Section 4 shows our fractal and wavelet analysis findings, and we conclude in Section 5 with future directions.

2. Review of related literature

To date, research on the properties of new technological AI-assets is limited, primarily concentrating on the significance of the AI label and the diversification role of AI-themed assets in a portfolio. Wu and Chen (2022) demonstrated that US ETFs with the AI name possess a notable name premium on their underlying assets. Huynh et al. (2020) and Abakah et al. (2023) examined the diversification role of new technology assets against conventional assets, revealing a strong correlation between AI-themed assets and standard financial assets. This suggests a significant likelihood of large losses during distressed market periods, indicating a weak diversification ability of AI assets for undiversified portfolios. Taken together, Tiwari et al. (2021) demonstrated that AI stocks can effectively hedge against carbon prices. Moreover, Sharma et al. (2024) investigated the potential of technology-based assets, including fintech, robotics, and blockchain, to enhance the diversification of MSCI emerging market portfolios during and after the COVID-19 pandemic. In summary, we synthesized our findings with those in the literature and highlight the novel insights or distinct nuances in Table 2 below for details.

[6] To examine these hypotheses, we apply rescaled range (R/S) analysis to estimate the Hurst exponent of seven primary AI tokens and deploy wavelet transform techniques to evaluate their returns' dynamic power and coherence structure.

Table 2. Summary of relevant studies in the AI tokens.

Author(s)	Novel insights or distinct nuances	Sample	Approach	Publishing journal
Yousaf et al. (2024a)	AI coins, in particular, could provide cost-effective hedging opportunities for traditional assets such as gold, equities, real estate, bonds, and currencies, but not for the oil or cryptocurrency markets.	The artificial intelligence (AI) tokens, AI ETFs, and other asset classes	Quantile VAR approach	Journal of International Financial Markets, Institutions & Money
Yousaf et al. (2024b)	The short-term fluctuations predominantly influence the network's total shock transmission, while the long-term perspective has the potential to shift the role toward either a net transmitter or receiver of shocks.	1. AI stocks (MSFT-Microsoft, GOOG-Alphabet, AMZN-Amazon), 2.AI tokens (AGIX-SingularityNET, OCEAN-Ocean Protocol, FET-Fetch.ai), 3. Fossil fuel markets (WTI, BRENT, and GAS-natural gas)	QVAR-based quantile connectedness approach	Energy Economics
Almeida et al. (2024)	Most AI-crypto sectors tend to exhibit greater efficiency under extreme market conditions. The launch of ChatGPT 3 notably improved market efficiency, driving positive mean returns and enhanced liquidity in AI-associated sectors.	Several AI-Crypto index categories are used as proxies for Generative AI, AI Big Data, Distributed Computing, Cybersecurity, and for Top Crypto sectors.	Adjusted market inefficiency magnitude (AMIM)	Finance Research Letters
Saggu et al. (2024)	Identifying significant "ChatGPT effects," with AI-related crypto assets experiencing average returns ranging between 10.7% and 15.6% (35.5% to 41.3%) in the one-month (two-month) period after the ChatGPT launch.	The study consists of 16 crypto assets classified as 'AI'-related by coingecko.com (GAI) and a broader cohort of 86 crypto asset returns categorized as 'AI and Big Data'-related by coinmarketcap.com (CAI).	Synthetic difference-in-difference (SDID) model	Finance Research Letters
Ante and Demir (2024)	Following ChatGPT's launch, AI tokens recorded substantial abnormal returns, peaking at 41% within two weeks, with 90% of tokens achieving positive abnormal returns.	The study includes 15 AI-focused crypto assets.	The cumulative abnormal return (CAR) approach	Economics and Business Letters
Ali et al. (2024)	The study elaborates how speculative behavior influences the long-term trends and resilience of AI-based tokens amid economic fluctuations.	The researchers collect three AI-based tokens, AGIX-singularity (NET), cortex (CTXC), and NMR, for price prediction.	Using long short-term memory (LSTM) algorithm	Journal of Economic Studies
The current study	AI tokens follow fractal Brownian motion, which robustly supports the principles of the fractal markets hypothesis.	The study includes 7 major AI-crypto assets.	Rescaled Range (R/S) Fractal and Wavelet Power and Coherence Analysis	Working paper

Source: Authors' own elaboration.

The literature on digital currencies and AI is extensive, yet the intersection of these two fields is relatively new. Researchers have explored blockchain technology, the backbone of most cryptocurrencies, and the applications of AI in finance. The following content reviews key literature on blockchain, AI in finance, and the nascent concept of AI tokens. Almeida et al. (2024) note that the introduction of ChatGPT 3 greatly improved market efficiency, as evidenced by the positive performance and increased liquidity of AI-related cryptocurrencies. Saggu and Ante (2023) suggest that ChatGPT has revolutionized the crypto market since its launch in November 2022, especially in AI-related domains, drawing an unprecedented user base within a brief timeframe. Not surprisingly, after ChatGPT's introduction, AI-related crypto assets have demonstrated average returns ranging between −0.017% and 15.6% in the one-month period after its launch, outperforming other crypto assets (Chipolina, 2023). In addition, the cryptocurrency market was also affected by the launch of

ChatGPT, as reported by Ante and Demir (2023), who observed that 90% of AI-related tokens exhibited positive abnormal returns.

The fractal and multifractal characteristics of physical phenomena and empirical data have garnered significant interest due to their association with scale invariance, a property observed across scientific disciplines. Similarly, the vast array of financial instruments and the complexity of financial markets have presented growing challenges for academics and investors. In response to this, Peters (1994) introduced the concept of the FMH as an alternative to the EMH, emphasizing the scaling properties of return distributions rather than informational efficiency. Consequently, conventional asset pricing and risk management models may require reassessment. Multifractality has been extensively studied in financial time series, including cryptocurrency market (Mnif et al., 2020; Wątorek et al., 2021; Aslam et al., 2023), stock markets (Gaio et al., 2022; Bouoiyour et al., 2018), energy markets (Wang et al., 2022; Adekoya et al., 2023), and foreign exchange markets (Stošić et al., 2015).

3. Methodology

To explore the potential of AI tokens, we utilize fractal and wavelet analysis approaches, including a qualitative analysis of current AI token projects. The fractal and Wavelet Analysis approach is synthesized as follows.

3.1. Measuring fractals via Rescaled Range (R/S) analysis

The Rescaled Range (R/S) Fractal analysis is a statistical method used to assess the long-term memory of a time series. Given a time series $X = X_1, X_2, \ldots, X_n$, the logarithmic returns of AI tokens are given by:

$X_n = \log(P_t) - \log(P_{t-1})$

Next, the R/S analysis is performed step-by-step as follows:

Step 1: Compute the Mean

The mean of the subsample is determined as:

$$m = \frac{1}{n}\sum_{t=1}^{n} X_t \tag{1}$$

Step 2: Compute the Demeaned Series

The demeaned return series Y_t is calculated as:

$$Y_t = X_t - m, \quad t = 1, 2, \ldots, n. \tag{2}$$

Step 3: Calculate the Cumulative Sum of Demeaned Returns

The cumulative deviate series D_t is given by:

$$D_t = \sum_{i=1}^{t} Y_i, \qquad t = 1, 2, \ldots, n. \tag{3}$$

Alternatively, it can be rewritten as:

$$D_t = \sum_{i=1}^{t} X_i - tm$$

Step 4: Compute the Range R_t

The range is calculated as the difference between the maximum and minimum values of the cumulative deviation:

$$R_t = max(D_1, D_2, \ldots D_t) - min(D_1, D_2, \ldots D_t) \quad , \mathrm{t} = 1, 2, \ldots, n. \tag{4}$$

Step 5: Calculate the Standard Deviation S_t

The standard deviation of the series is given by:

$$S_t = \sqrt{\frac{\sum_{i=1}^{t}(X_i - m\ )^2}{t}} \quad , t = 1, 2, \ldots, n. \tag{5}$$

Step 6: Compute the Rescaled Range (R/S)

The Rescaled Range (R/S) is defined as:

$$(R/S)_t = \frac{R_t}{S_t} \ , \mathrm{t} = 1, 2, \ldots, n. \tag{6}$$

where $(R/S)_t$ represents the mean across different subsamples. The observations are divided into segments:

$[X_1, X_t]$, $[X_{t+1}, X_{2t}]$, $[X_{(m-1)t+1}, X_{mt}]$

where m = floor(n/t), assuming n observations are taken at equal time intervals.

Step 7: Estimate the Hurst Exponent H

The R/S scaling estimation follows a power-law relationship:

$$(R/S)_t = c\ t^H \tag{7}$$

where c is a constant, and H is the Hurst exponent. Taking the logarithm on both sides:

$$log\left(\frac{R}{S}\right) = logc + Hlog(t) \tag{8}$$

By plotting $ln(R/S)$ against *logt* in a scatter graph, the slope of the fitted line (estimated via least squares regression) provides an estimate of the Hurst exponent *H*.

In summary, R/S analysis helps us understand correlations and persistence in natural processes, making it relevant to fractal analysis.

3.2. Wavelet analysis for the FMH

Peters (2015) states that if FMH's claim is valid, heightened power at low scales (high frequencies) should be evident during critical times. Additionally, due to shifts in investor behavior during market crash periods, changes in the variance structure across frequencies might be observed before market turbulence occurs. The procedure includes wavelet power spectrum extraction from the given time series by applying the Continuous Wavelet Transform (CWT) method to examine variance evolution across timescales. Below is a brief description of the wavelet technique and wavelet power spectrum proposed by Torrence et al. (1998). The information is extracted by decomposing the scales into 4, 16, 64, and 256 days. A wavelet $\phi_{u,s}(t)$ denotes a real-valued square-integrable function that could be defined as

$$\phi_{u,s}(t) = \phi\,\frac{(t-u)}{\sqrt{s}} \tag{9}$$

Given a location (u), a scale (s), and a time (t), any given time series could be reconstructed from its wavelet transform, provided that the so-called admissibility condition is satisfied as follows:

$$C_{\phi} = \int_{0}^{\infty} \frac{|\phi(f)|^2}{f} df < +\infty \tag{10}$$

where $\phi(f)$ denotes the Fourier transform of the given wavelet. A wavelet must have zero means, i.e., $\int_{-\infty}^{\infty} \phi(\mathrm{t})\, dt = 0$, so that $\int_{0}^{\infty} \phi^2(\mathrm{t})\, dt = 1$, i.e., its square integrates to unity. To acquire the CWT (continuous wavelet transform) $W_x(u, s)$ of a given time series x_t, a wavelet $\phi(.)$ is generated by projecting $\phi^*(.)$ on the examined series x_t so that

$$W_x(u, s) = \int_{-\infty}^{\infty} \frac{\phi^* \frac{(t-u)}{s} dt}{\sqrt{s}} \ , \tag{11}$$

where ϕ^* (.) represents the complex conjugate of $\phi(.)$. The continuous wavelet transform decomposes the underlying series into frequencies or different timescales. Notably, the reconstructed signal retains all information without any loss while also maintaining the energy of the examined series, that is

$$x(t) = \frac{\int_{0}^{\infty} \int_{-\infty}^{\infty} W_x(u,s)\ \phi_{u,s}(t) du ds}{s^2 C_{\phi}}, \tag{12}$$

$$\left|\left|x\right|\right|^2 = \frac{\int_{0}^{\infty} \int_{-\infty}^{\infty} |\ W_x(u,s)|^2 du ds}{s^2 C_{\phi}} \tag{13}$$

where $|\ W_x(u, s)|^2$ denotes the wavelet power at scale s>0. Among the various wavelets available for analysis, we select the Morlet wavelet due to its common use in achieving a balance between time and frequency localization. The Morlet wavelet is specifically designed to address localization considerations in time as well as frequency regions given by

$$\phi(\mathrm{t}) = \pi^{-1/4}\, exp\left(i\omega_0 t - \frac{t^2}{2}\right) \tag{14}$$

where ω_0 is the central frequency.

3.3. A brief note on wavelet coherence

To validate the results of the Hurst exponent and wavelet analysis, we perform an alternative wavelet coherence to capture the time-frequency connections between Bitcoin and AI tokens. Using the wavelet coherence method from Grinsted et al. (2004), we can detect the volatility connectedness between Bitcoin and AI tokens across both time and frequency domains. Based on Torrence and Compo (1998), the cross-wavelet transform function | $W_{xy}(u, s)$) | of the two times series x(t) and y(t) is expressed utilizing the continuous wavelet transforms (CWT) of timeseries factors Wx (u, s) and Wy (u, s), and is given by:

$$W_{xy}(u, s) = W_x(u, s).W_y^*(\mathrm{u, s}) \tag{15}$$

where u and s denote the location and the scale, respectively. * is the complex conjugate. The CWT identifies the scales in the time-frequency domain where strong interactions occur within the time series. The cross-wavelet power highlights regions of significant shared power between two time series within the time-frequency space, with the wavelet squared coherence described specifically as follows:

$$R^2(u,s) = \frac{\left|S\left(s^{-1}W_{xy}(u,s)\right)\right|^2}{S(s^{-1}|W_x(u,s)|^2).S\left(s^{-1}\left|W_y(u,s)\right|^2\right)} \tag{16}$$

where R^2 and S (.) denote the wavelet squared coherency and the smoothing operator, respectively.

As a result of the aforementioned complexity of the employed wavelets and the reliance on squared coherence, the ability to determine the direction of the relationship is compromised, instead of coherence itself. To achieve this, a phase difference can be mathematically represented as follows:

$$\phi_{x,y}(u,s) = \tan^{-1}\left(\frac{\boldsymbol{lm}\{S(s^{-1}W_{xy}(u,s))\}}{\boldsymbol{Re}\{S(s^{-1}W_{xy}(u,s))\}}\right) \tag{17}$$

In Equation (17), ***lm*** represents the imaginary smoothed component, while ***Re*** denotes the real part of the smoothed cross-wavelet transform. Typically, a zero-phase difference signifies that the time series 'x' and 'y' are moving in inconsistent directions.

4. Data, results, and discussion

4.1. Data

The daily closing prices from seven primary AI tokens and Bitcoin are selected for the analyses. The period covers October 22, 2020, until July 17, 2024 (see Table 1). We utilize CoinMarketCap, a widely recognized and authoritative market data provider, as the source for our dataset. All data were collected from CoinMarketCap (https://coinmarketcap.com/view/ai-big-data/), comprising 1,357 daily observations per series and a total of 9,499 observations. This study uses daily log-returns on Bitcoin and AI tokens under study, selected based on market cap. The dataset includes the most liquid cryptocurrency, Bitcoin (BTC), along with seven of the most liquid AI tokens, namely NEAR Protocol (NEAR), Artificial Superintelligence Alliance (FET), Render (RNDR), Injective (INJ), Bittensor (TAO), The Graph (GRT), and Akash Network (AKT). More importantly, the considered cryptocurrencies were selected for the daily sample of the top seven AI tokens and the top cryptocurrency, Bitcoin, regarding market capitalization. Some factors associated with popular AI tokens include market cap and dominance, existing AI crypto initiatives, and blockchain platforms. These selected AI tokens are supported by their strong liquidity and inclusion in the top 7 market cap, which enriches the fractal market analysis. Our analysis focuses on the daily closing prices of seven primary AI tokens (i.e., NEAR, FET, RNDR, INJ, TAO, GRT, and AKT), constituting more than 68% of the market capitalization of all AI tokens. As depicted in Table 1, the market cap of each AI token relative to the total market cap of all AI tokens is 22.4% (NEAR), 12.8% (FET), 8.8% (RNDR), 8.1% (INJ), 7.5% (TAO[7]), 6.7% (GRT), and 2.9% (AKT).

Table 3 presents the descriptive statistics for AI tokens and Bitcoin analyzed in this study. While all average returns are positive, GRT market returns are an exception, and AI tokens demonstrate higher average returns than Bitcoin. Additionally, all analyzed series display excess kurtosis, indicating the presence of fat tails in the return distributions. The Jarque-Bera test results reject the hypothesis of normality distribution for all return series.

[7] Due to the limited sample period (March 2023 – July 2024) for the newer TAO token, the available data may be insufficient to accurately capture its long-term price dynamics and fractal properties. As a result, findings based on this timeframe could be highly volatile and unreliable for broader generalization. Therefore, the following chapter excludes analysis of the TAO token.

Table 3. Descriptive statistics and R/S analysis for daily log-returns.

	NEAR	FET	RNDR	INJ	GRT	AKT	BTC
Mean	0.00158	0.00253	0.00287	0.00256	−0.00017	0.00165	0.00118
Median	−0.00024	−0.00081	−0.00131	−0.00082	−0.00113	−0.00087	0.00019
Maximum	0.36103	0.36076	0.50183	0.40538	0.60952	0.34487	0.17182
Minimum	−0.44363	−0.43793	−0.43018	−0.41944	−0.48695	−0.27416	−0.17405
Std. Dev.	0.06806	0.07271	0.08217	0.07121	0.07243	0.06468	0.03306
Skewness	0.11797	0.26366	0.58804	0.17242	0.58587	0.71808	−0.17198
Kurtosis	7.37614	6.72853	7.66861	6.81134	15.4338	6.76443	6.32838
Jarque-Bera	1092.36	806.49	1318.31	832.95	8494.03	923.28	636.79
Probability	0.0000***	0.0000***	0.0000	0.0000	0.00000	0.0000	0.0000
Expected	value	under null	hypothesis	that H=0.5			
Hurst exponent	0.6048	0.6170	0.6327	0.6212	0.5845	0.6539	0.6053
t-stat	9.211	10.088	7.916	13.500	6.292	11.784	8.267
p-value	0.0001**	0.0001**	0.0002**	0.00001**	0.0022**	0.00002**	0.0002**
Observations Total:9,997							

Note: 1. For the normality test (Jarque Bera), ∗∗∗ means significant results at 0.1% level of significance.

2. Hurst exponent computed by *R/S*, roughness-length, and variogram techniques are employed for traditional hypothesis tests with a null hypothesis of H=0.50. Rejection of the null at the 0.05 and 0.01 two-tail significance level is reported by (∗) and (∗∗), respectively.

4.2. Results

Our analysis reveals that AI tokens exhibit several stylized facts about these emerging cryptocurrencies. These include some empirical results from our fractal and wavelet analysis as follows.

4.2.1. Fractal analysis evidence

All Hurst exponents of AI tokens give rise to Hurst coefficients (over 0.58) that are significantly larger than their expected values, suggesting that series exhibit a level of persistence similar to the work of Chen et al. (2024). Measuring fractal dimensions from the bottom of Table 3, it is observed that H(q) is 0.6048, indicating significant persistent characteristics when small fluctuations dominate the time series in this stage. Unsurprisingly, Hurst exponents of NEAR (H = 0.6048), FET (H = 0.6170), RNDR (H = 0.6327), INJ (H = 0.6212), GRT (H = 0.5845), and AKT (H = 0.6539) markets are greater than 0.5, suggesting that AI coin’s prices are characterized by long-range dependence and high persistence. These values also give rise to fractal Brownian signals with a known H value when $H \geq 0.5$, thereby providing empirical support for Hypothesis 1. It can be observed that the price series of AI tokens follows the fractional Brownian motion than random walks. Additionally, markets exhibit a prolonged response time to information. Accordingly, investors can achieve substantial profits by analyzing historical data for each AI coin’s price, so the seven AI crypto markets are inefficient.

We then compute a log (range) corresponding to log N and plot this graphically for the AI tokens and Bitcoin markets in Figure 1. A notable power-law scaling relationship (Mandelbrot, 2001) is observed. Subsequently, we obtain the estimate of the Hurst exponent (H) using Equation (8), which is given by $H = \log[F(n)]/\log(n)$. When plotting R/S against the number of groups on a log-log scale, the resulting graph should be linear, and the slope of this line is related to the Hurst parameter (H), which can be determined using linear regression.

Based on the fractal market hypothesis, Hurst exponents are estimated by R/S analysis, and all AI tokens are inefficient under daily returns. Overall, combining the empirical results with the reality of cryptocurrencies proves that the results of the fractal market hypothesis are more convincing.

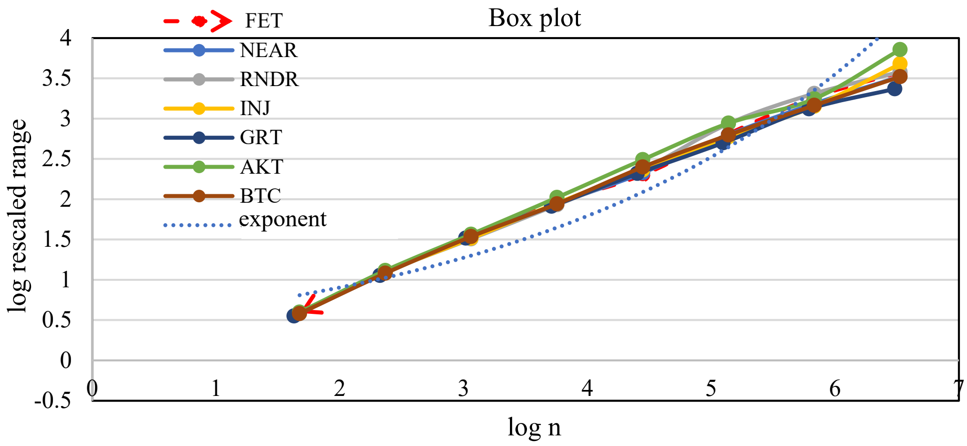


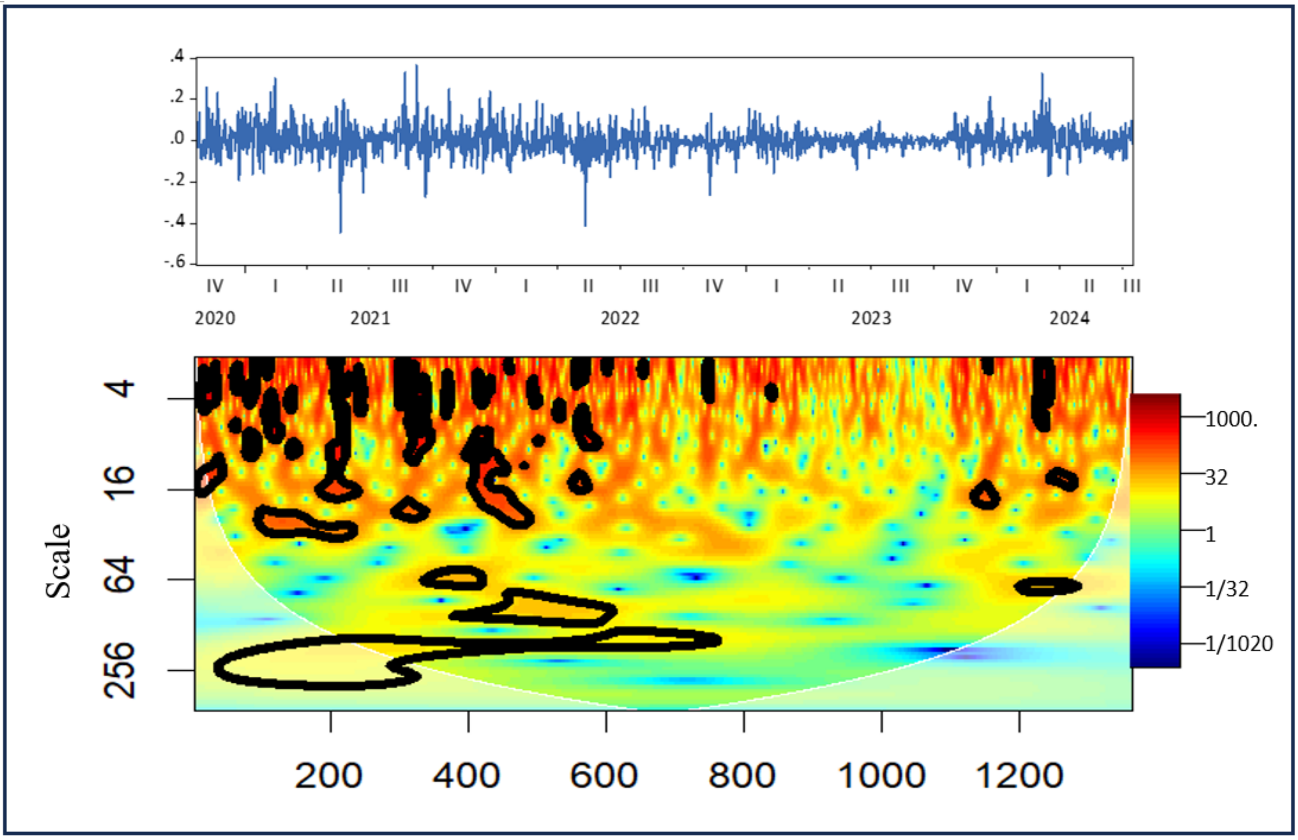
Figure 1. Graphical estimation of the Hurst exponent for the major AI tokens and BTC. Note: Plot the logarithm of the size (x-axis) of major AI tokens and Bitcoin versus the logarithm of the rescaled range (y- axis).

4.2.2. Wavelet analysis evidence

The upper chart in Figures 2–8 displays each AI crypto's daily volatility time-series trajectory. The findings show a significant volatility jump and note that the returns were significantly larger during the initial phase of the COVID-19 outbreak in March 2021. This can be attributed to several factors, including the significant impact of shocks from the first wave of the COVID-19 pandemic outbreak during this period, as well as favorable market conditions. It is clear that as soon as the spread of the COVID pandemic occurred globally, which corresponds to nearly the initial phase (2020-2021) of all charts, there were widespread market reactions across all those assets. Significant spikes in abnormal returns are apparent during the identified first wave of the COVID-19 pandemic outbreak period.

Wavelet power is depicted across the entire analysis period (bottom chart) and the evolution of the daily logarithmic returns (upper chart). Figures 2–7 (bottom chart) display the wavelet power spectra for the log-returns of the analyzed AI tokens. In each chart, the wavelet powers are tested for significance against the null hypothesis of a red noise (AR (1) process). The bold black curve delineates regions while the wavelet powers exhibit statistical significance for the crypto series, providing valuable insights into the behavior of financial markets during turbulent times. The hotter colors indicate higher power (variance) at specific scales (y-axis) and times (x-axis), with statistically significant areas outlined by a bold black curve. Given the daily data frequency, the scales are measured in day units. To interpret the powers shown in Figure 2, note that hotter colors correspond to a higher power (variance) for specific scales and times. The dominance of high frequencies at the D4-D32 scale is evident during the most turbulent times of the NEAR and other significant trend changes and corrections. Hence, the outcomes align with the predictions of the fractal markets hypothesis. As illustrated in Figure 3, hotter colors visually represent higher frequencies. We can observe that higher frequencies dominated during the COVID crisis period from 2020 to 2021. Figure

3 clearly shows a significant increase in FET power distribution at high frequencies during both the COVID-19 crisis period and the ongoing Russia-Ukraine war. During turbulent times, the shock remains persistent at smaller scales (i.e., high frequencies at 4,16, and 32 days). This suggests that the impact of shocks holds only for the short-term structure of FET. Thus, the FMH can explain these potential conditions. Kristoufek (2013) also shows this same analysis for several other non-US indices during the economic turbulence.



Note: 1. The coned line in the lower half of the graph indicates regions affected by 'edge effects'. These effects arise when the wavelet is centered near the start or end of the time series, potentially distorting the results for those periods.

2. Data beyond this line cannot be statistically inferred, as these regions fall within the cone of influence (Torrence and Compo, 1998). The color intensity, ranging from dark blue for low-power areas to bright orange for high-power spectra, depicts the power spectrum of the results.

Figure 2. NEAR log returns and wavelet power spectrum.

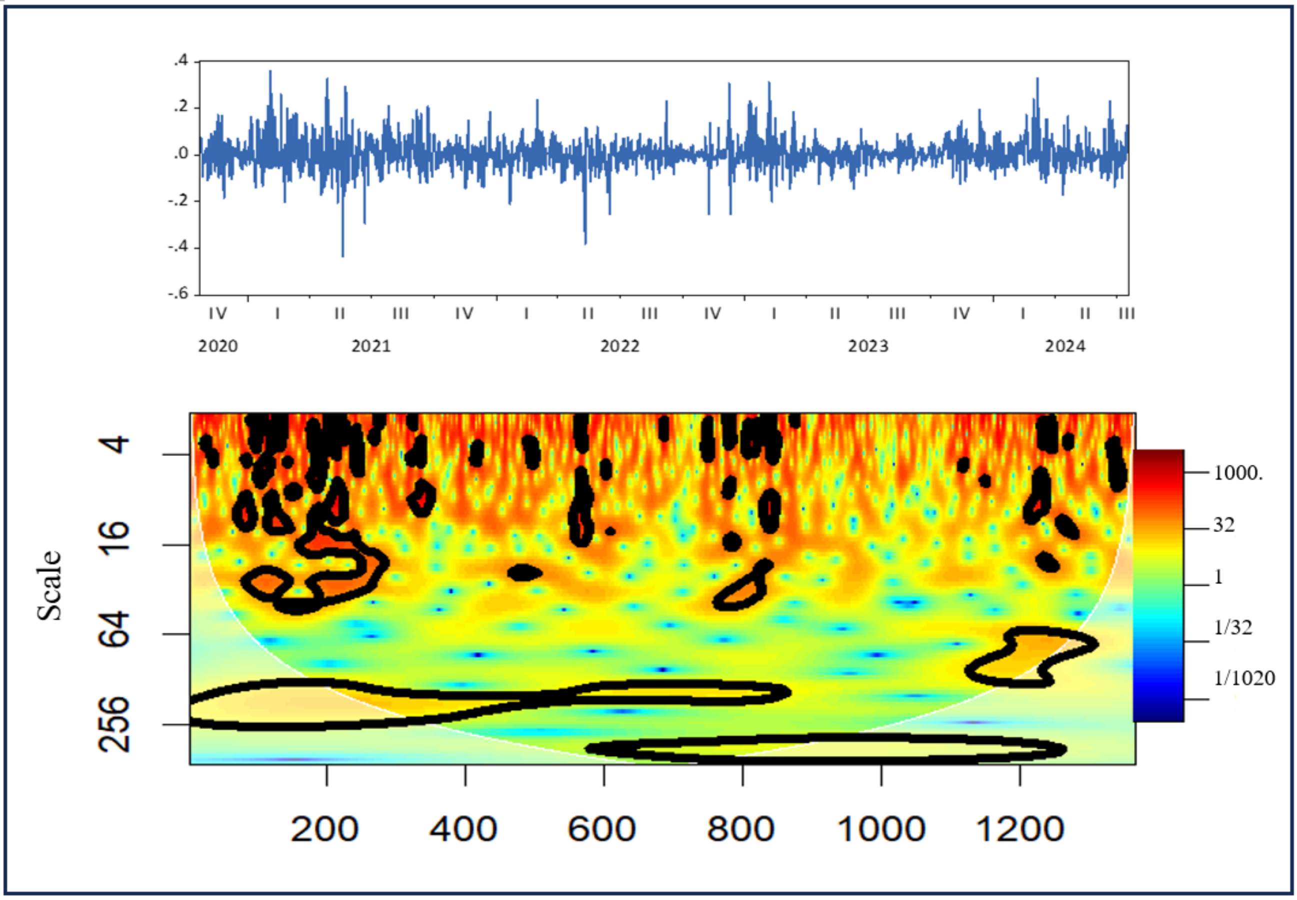


Figure 3. FET log returns and wavelet power spectrum.

During 2020–21[8], the wavelet power spectrum depicted in Figure 4 exhibits increased power at higher frequencies. This suggests that short-term investors dominate the market during turbulent periods, emphasizing the significance of those regions. Greater power (variance) at a specific scale (y-axis) and time coincides with a corresponding hotter color (x-axis), clearly showing that RNDR aligns with FMH and is particularly evident during the COVID-19 pandemic crisis. Not surprisingly, Figures 5, 6, 7, and 8 also show a high-power density localization at higher frequencies. This is consistent with FMH, indicating that clearing the market efficiency is impossible for AI coin markets and Bitcoin. GRT's story (Figure 6) is also similar to that of Bitcoin (Figure 8), and AKT (Figure 7) shares the same narrative.

Overall, the results obtained from the AI tokens and Bitcoin show a considerable response to the COVID-19 outbreak. Amid the health crisis caused by the global spread of the COVID pandemic, an overwhelming fear triggered what is now commonly called the 'Coronavirus Crash' in the global financial market. This abrupt downturn began in November 2020 in the study, leading to unprecedented volatility in the market.

[8] Regarding COVID-19 infection attacks, for example, in India (with a population of nearly 1.4 billion), Alpha and Delta of Covid-19 variants were detected between October and December 2020. The second wave of COVID-19 had a profound impact, with case numbers surging from mid-March 2021 and peaking at over 400,000 cases per day in May (according to Triambak et al. (2023) and WHO).

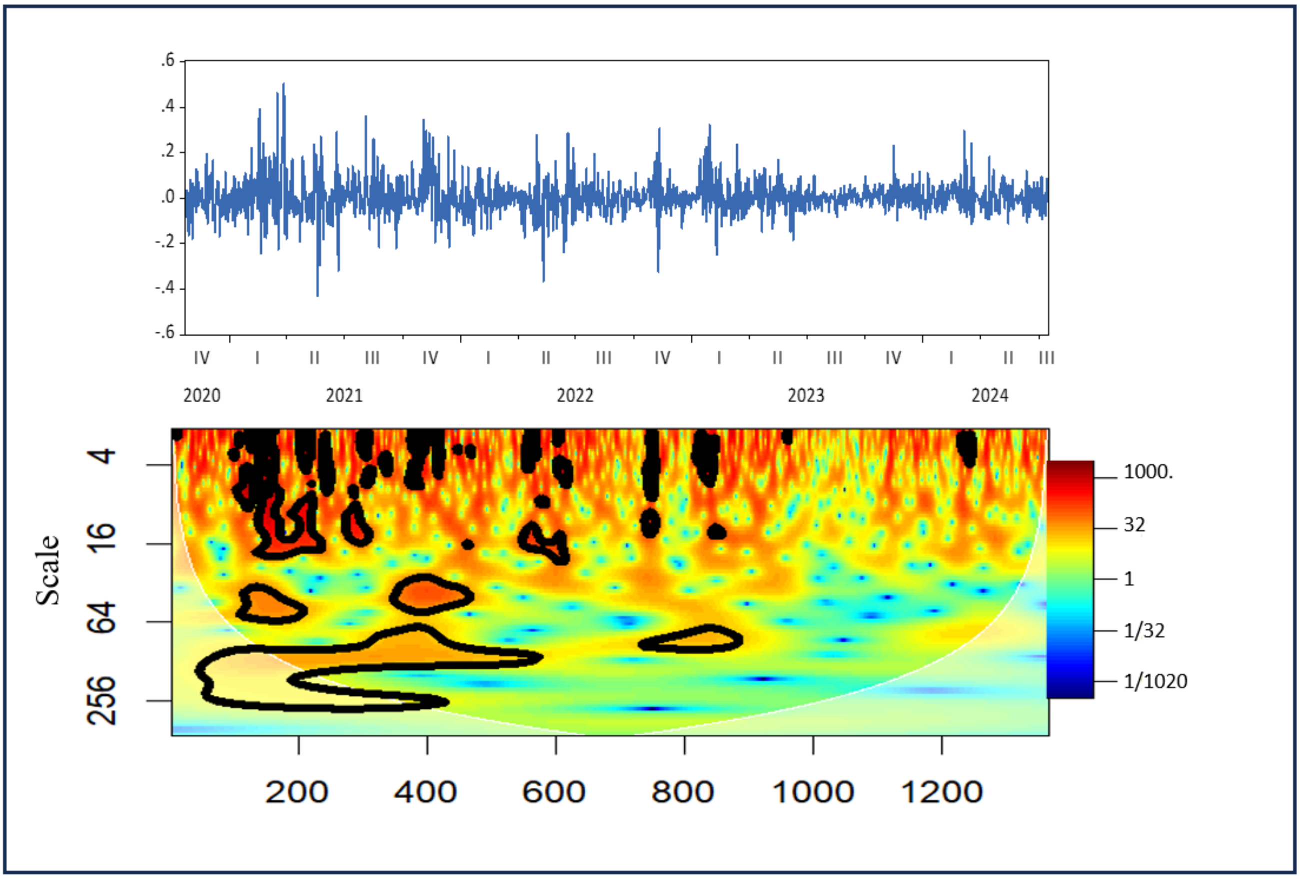


Figure 4. RNDR log returns and wavelet power spectrum.

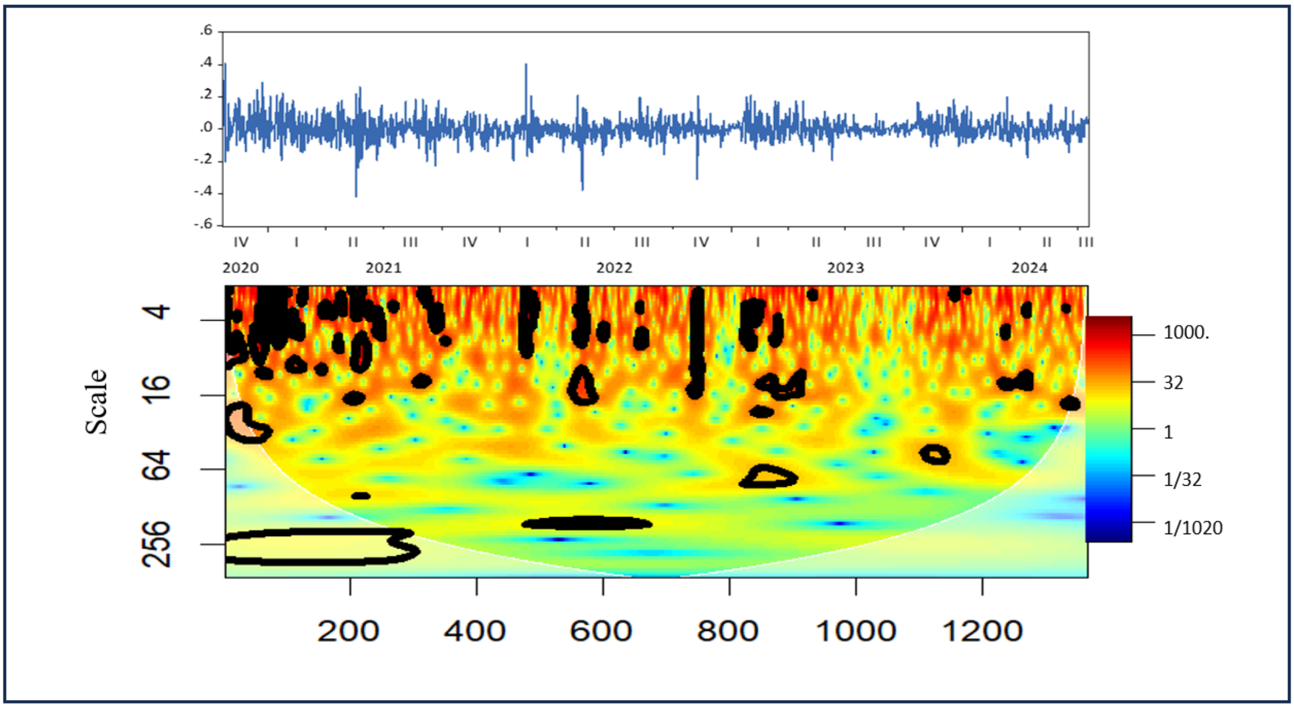


Figure 5. INJ log returns and wavelet power spectrum.

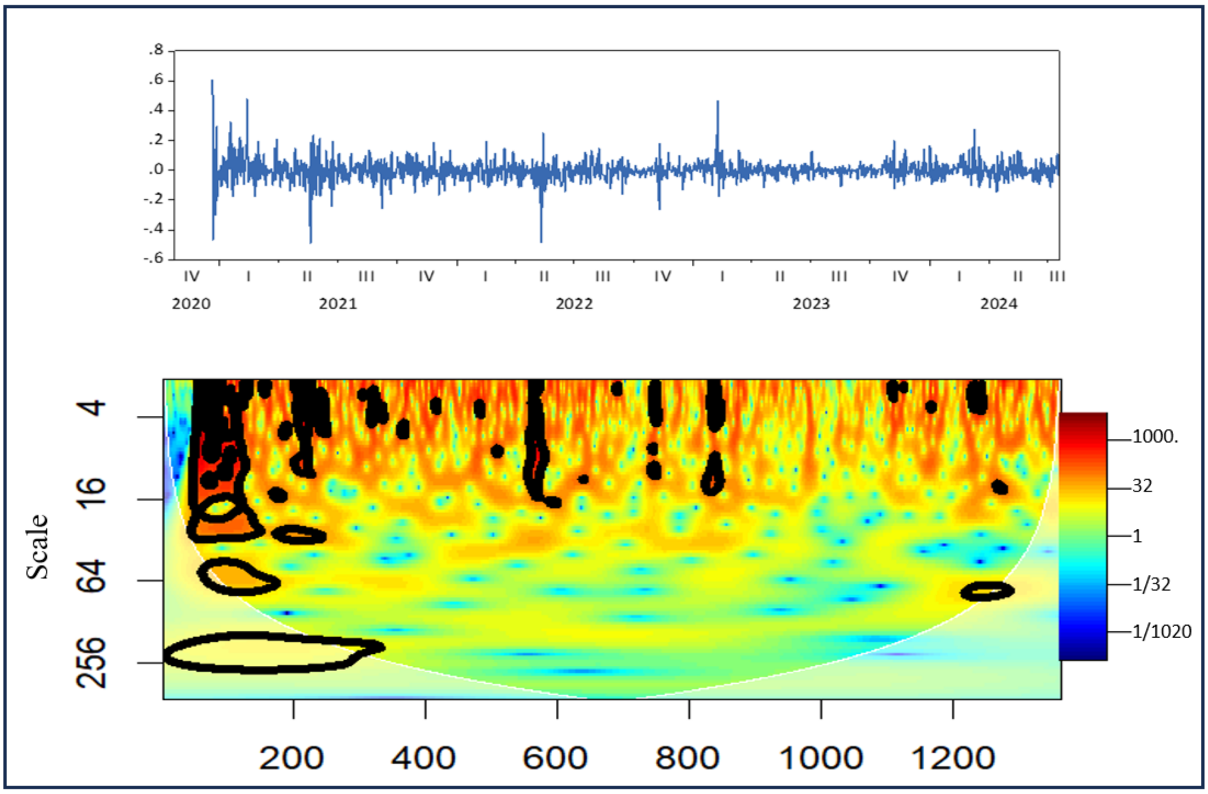


Figure 6. GRT log returns and wavelet power spectrum.

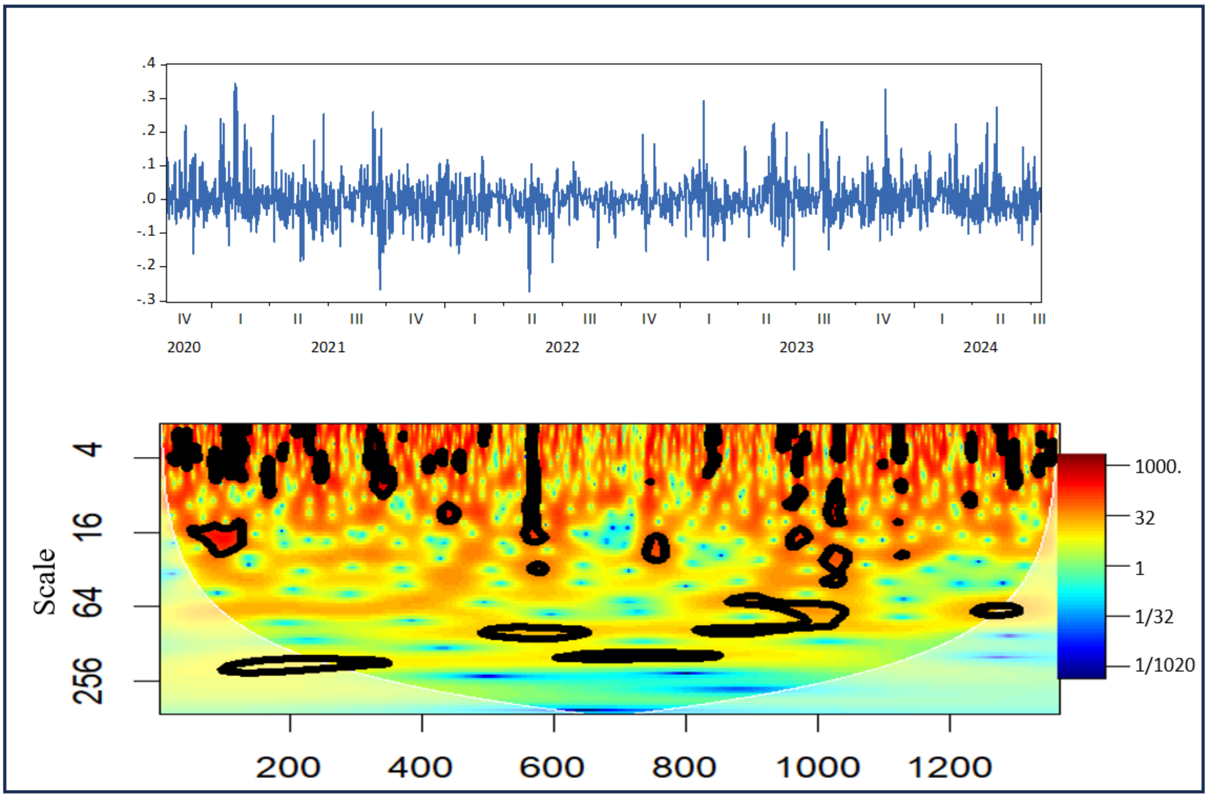


Figure 7. AKT log returns and wavelet power spectrum.

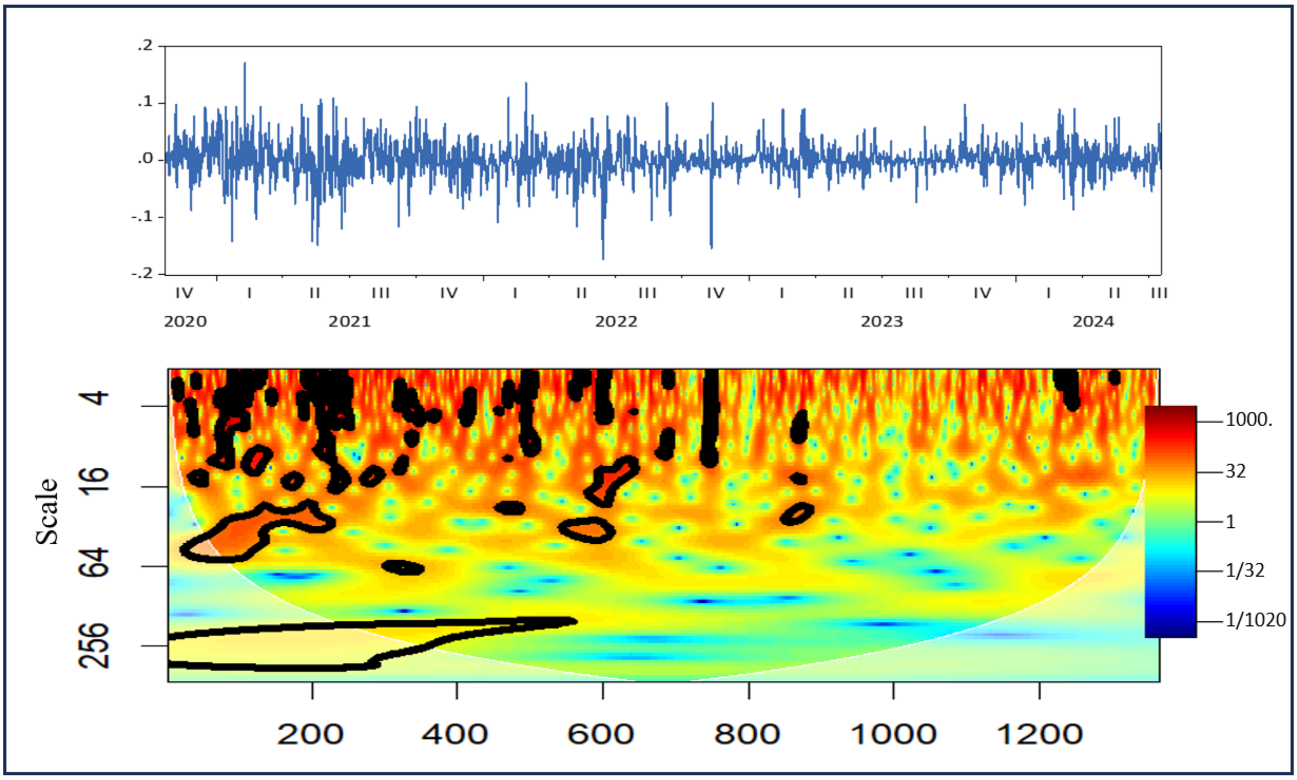


Figure 8. BTC log returns and the wavelet power spectrum.

4.2.3. Dependency testing: wavelet coherence for Bitcoin vs AI tokens

To capture the nonlinear dependencies and co-movement between Bitcoin and AI tokens, we conduct a dependency analysis using Wavelet coherence for Bitcoin–AI token pairs. Wavelet coherence analysis (Figure 9) examines the interconnectivity of AI tokens and their dependence on Bitcoin across different time horizons (on the *y*-axis), ranging from short-term (up to 16 days) to long-term (above 64 days), and utilizing daily data from October 22, 2020, to July 17, 2024 (on the *x*-axis).

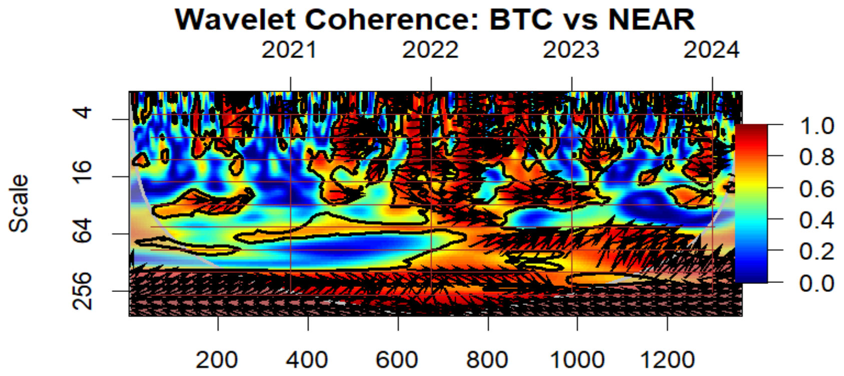
Not surprisingly, AI token markets signify significant interconnectedness, with regional co-movement spikes driven by external shocks such as economic turbulence like COVID-19, geopolitical risks like the Russia-Ukraine war, and Bitcoin price volatility.

To the best of our knowledge, this study expresses the first contribution using wavelet coherence and phase shift for potential implications on portfolio optimization, asset allocation, and hedging strategies between Bitcoin and AI tokens.

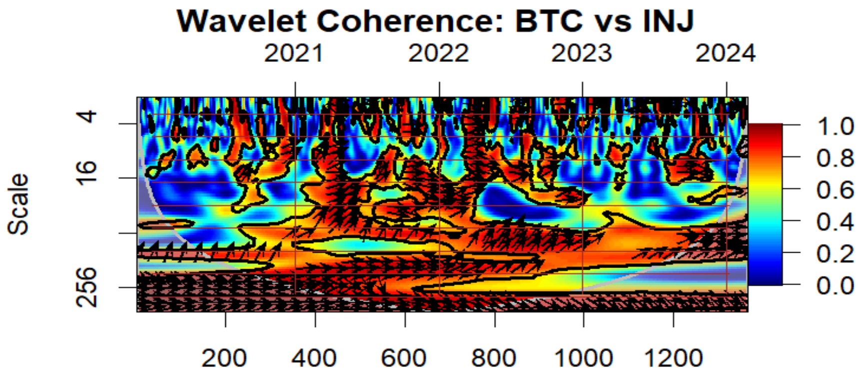
More specifically, the phase angles in Figure 9 (a–f) are tilted to the right arrows (→), indicating that the two times series are in phase, signifying positive co-movements and strong coherence between Bitcoin and AI tokens across most time horizons, where a few blue signals are observed. The findings are notable in several ways, revealing strong coherence between most AI tokens and Bitcoin. However, AI coin AKT and Bitcoin are weakly coherent following the release of ChatGPT, offering valuable insights for multi-horizon investors seeking hedging or safe-haven assets. According to Baur and Lucey (2010), a diversifier is defined as an asset that exhibits a positive correlation (but not a perfect one) with another asset or portfolio on average.

Overall, from the aforementioned results, it can be concluded that while Bitcoin is viewed as a diversifier for AI tokens, it is not effective as a hedge or safe-haven asset. Alternatively, as shown in Figure 9 (a–f), this result reveals that AI tokens exhibit a positive coherence with Bitcoin, the largest high-carbon cryptocurrency, across most time domains. Based on the study proposed by Baur and Lucey (2010), this suggests that Bitcoin is ineffective as a hedge or a safe-haven asset against these tokens. Crucially, this work underscores the potential of AI tokens to serve as safe-haven assets during periods of market turbulence, providing crypto investors with actionable insights for portfolio diversification, supported by intuitive and robust findings with significant policy implications.

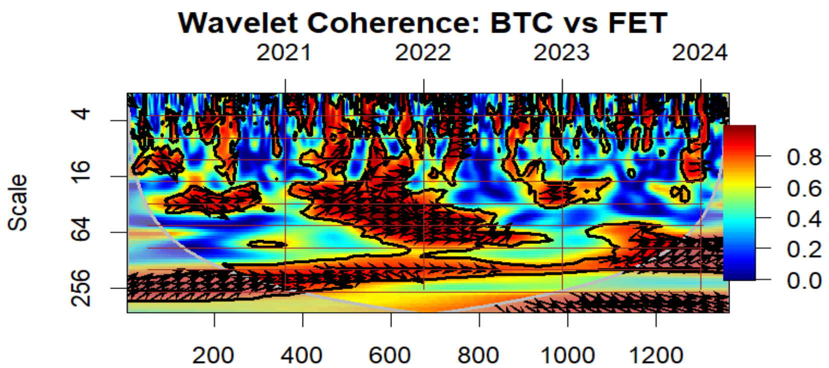
(a)



(b)



(c)



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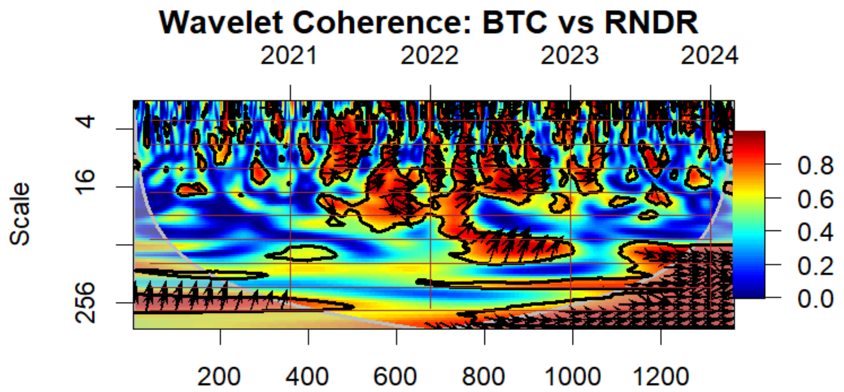
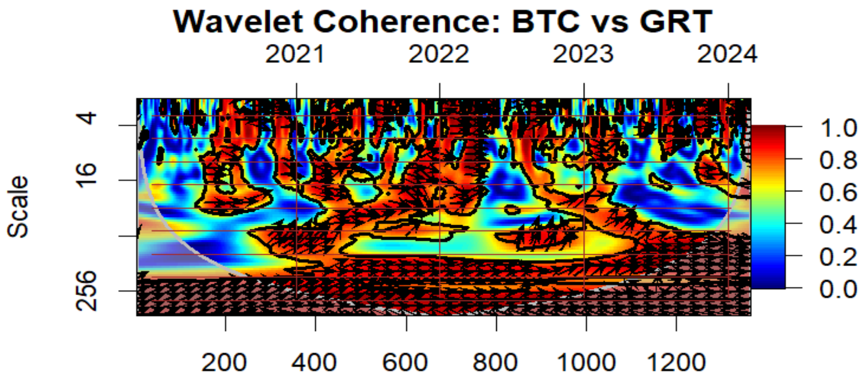
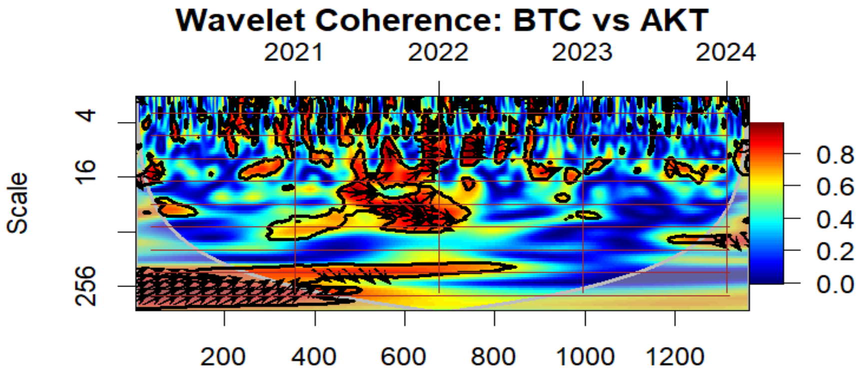
(d)



(e)



(f)



Note: 1. The color intensity indicates the strength of the correlation between the two assets at each timescale, with warmer colors (yellow to red) signifying stronger coherence (closer to 1), and cooler colors (blue) indicating weaker or no coherence. The black contour lines denote regions of statistically significant coherence at the 5% level. The cone of influence, depicted as a lighter-shaded border, marks the area where edge effects may distort the results and should be interpreted cautiously.

2. Directional arrows within the significant regions represent the phase relationship:

- Rightward arrows (→) indicate that Bitcoin and AI cryptos are positively correlated and move in the same direction.
- Leftward arrows (←) imply negative correlation (i.e., anti-phase movement).
- Upward or downward angles suggest lead-lag dynamics (e.g., AI cryptos leading BTC or vice versa).

Figure 9. Wavelet coherence between Bitcoin and AI token pairs.

4.3. Discussions

In line with FMH, which posits that AI token markets are dominated by heterogeneous agents operating at different investment horizons, our wavelet power spectra (Figures 2–8) reveal significant energy concentration at higher frequencies (shorter time scales), particularly during turbulent periods such as the COVID-19 pandemic and the Russia-Ukraine war. These findings indicate that short-term investors become more active during market crises, which is consistent with FMH's assertion that different investor groups dominate at different times depending on market conditions.

Moreover, the wavelet coherence analysis (Figure 9) reveals time-varying co-movements between Bitcoin and AI tokens, with significant coherence at various frequencies, particularly during periods of market stress. The presence of both in-phase and out-of-phase relationships across horizons reflects dynamic interdependencies. It supports the heterogeneity of investor behavior, as proposed by the FMH.

The economic intuition of the results shows that most AI tokens exhibit persistent behavior, with Hurst exponents exceeding 0.5, indicating a deviation from market efficiency (Naeem et al., 2021). Additionally, the wavelet power spectra provide evidence that short-term volatility becomes predominant during episodes of market turbulence, most notably during the COVID-19 crisis. In parallel, the wavelet coherence analysis reveals that AI tokens exhibit a consistently strong, albeit time-varying, co-movement with Bitcoin, thereby highlighting the limited efficacy of Bitcoin as a hedging vehicle for these assets. These findings provide empirical support for Hypothesis 2 and reinforce the applicability of the FMH within the context of AI cryptos. Specifically, the results suggest that AI tokens may act as short-term hedging instruments than reliable safe-haven assets during periods of market stress.

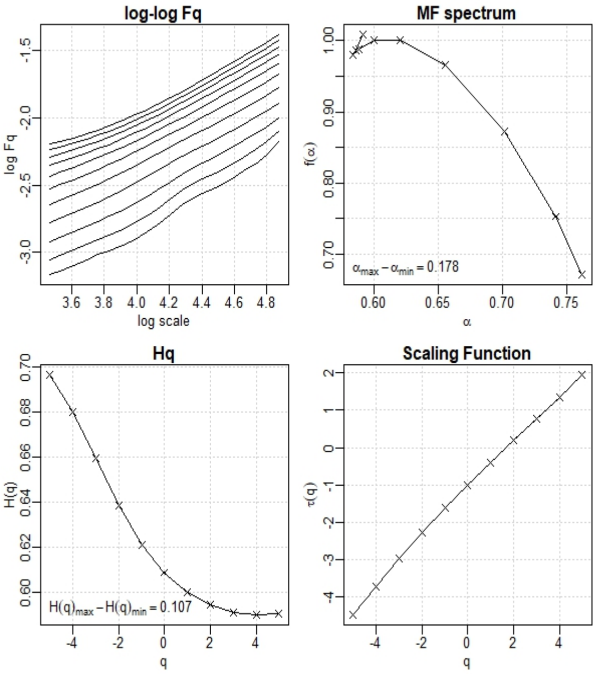
Importantly, the wavelet coherence analysis uncovers strong, time-varying co-movements between Bitcoin and AI tokens, especially during crises. This result corroborates earlier evidence from Shahzad et al. (2022) and Dutta et al. (2020), who demonstrate that Bitcoin's hedging effectiveness is regime-dependent and often diminishes during heightened market uncertainty. Our findings suggest that Bitcoin similarly fails to provide consistent diversification or safe-haven benefits for AI tokens during turbulent periods. This dynamic relationship aligns with the work of Ji et al. (2019), who report significant information spillovers between cryptocurrencies, energy, and commodity markets. Our time-frequency results further illustrate that co-movement strength varies across investment horizons, highlighting AI tokens' integration into broader macro-financial systems rather than isolation as a distinct asset class.

4.4. Robustness testing

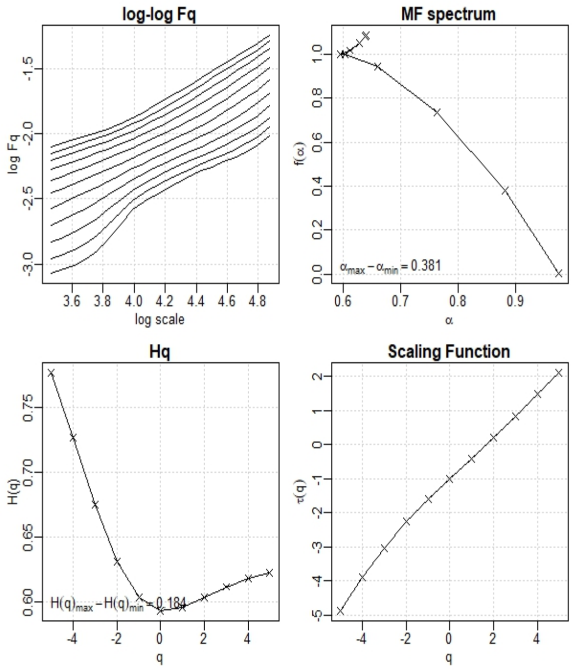
To assess the robustness of the multifractal phenomenon, we further explore the multifractal characteristics of AI tokens using MF-DFA, following Schadner (2021), as detailed in the Appendix. Over the past decade, fractal and multifractal scaling behaviors have been widely observed in natural time series generated by complex systems. However, to the best of our knowledge, no prior studies have entailed the multifractal properties or local persistence of AI token markets. This research provides novel perspectives on market dynamics[9] and contributes to the literature.

[9] Our approach is consistent with the latest developments in multifractal analysis, incorporating focus-based variant crossovers derived from the scaling-range adaptive methodology, and fluctuation functions calculated using maximally overlapping segments.

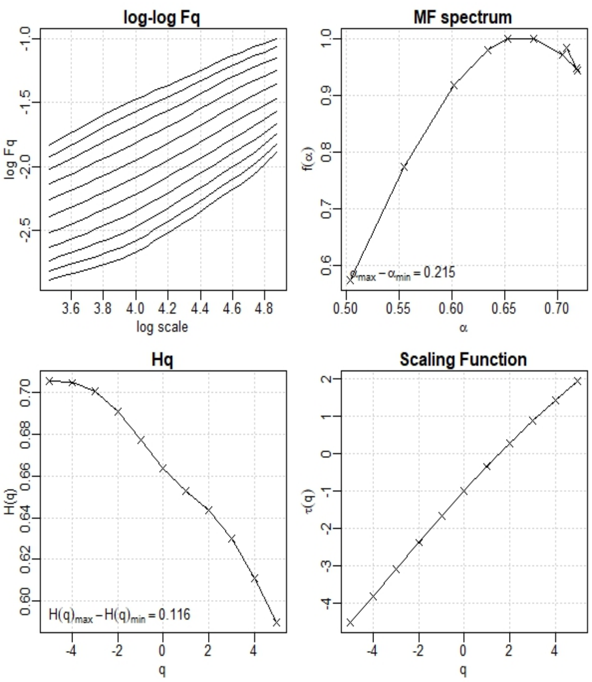
(a) NEAR



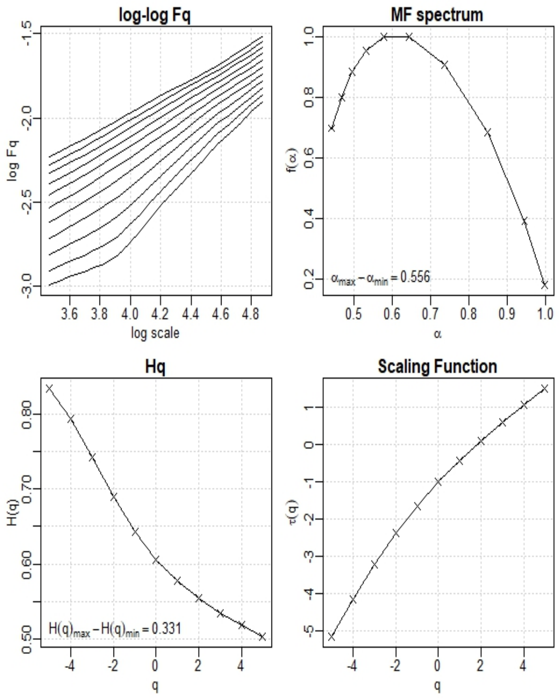
(b) FET



(c). RNDR



(d). INJ



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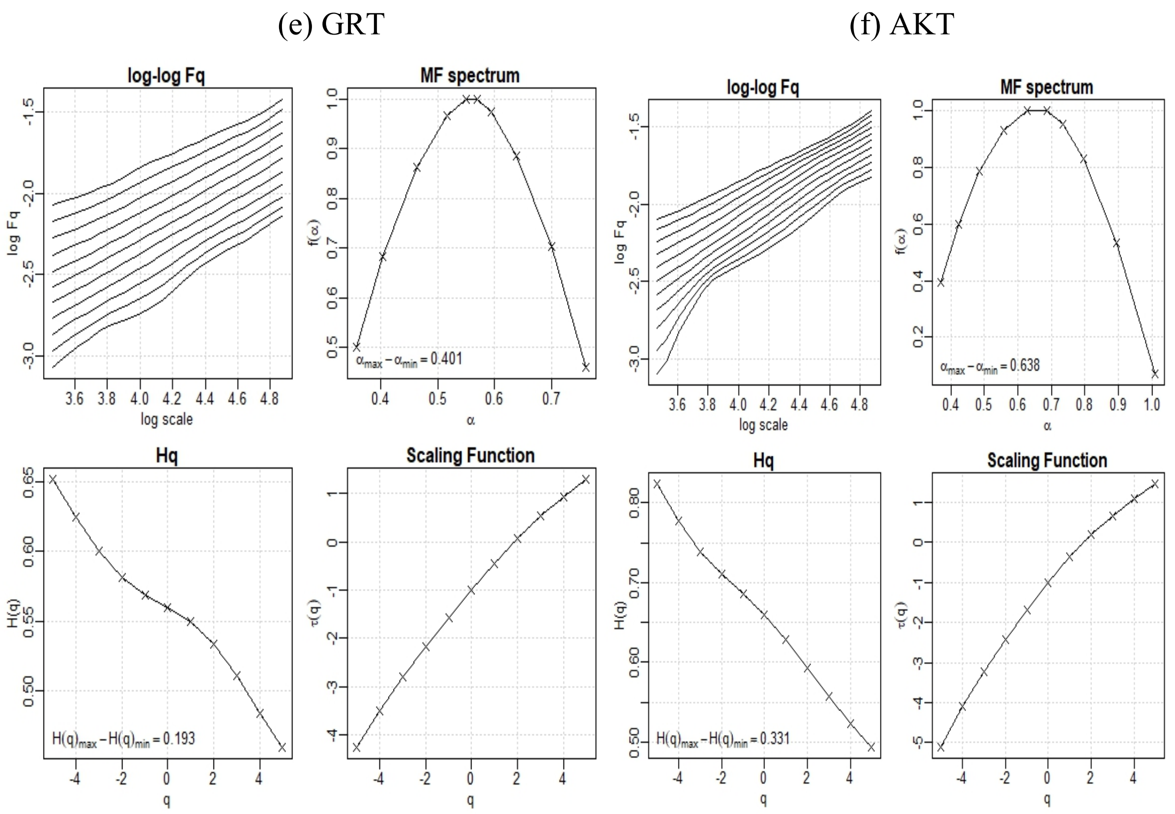

Note: MFDFA results for AI cryptocurrency returns. (a) Generalized Hurst exponent as a function of q (upper left panel); (b) Multifractal spectra f(α) versus α (upper right panel); (c) Fluctuation functions for q values ranging from -5 to 5 (lower left panel); and (d) Mass exponent τ(q) (lower right panel).

Figure 10. Multifractal fluctuation analysis for six major AI tokens.

For each panel (a)–(f) in Figure 10, the key findings are synthesized into the following four major takeaways.

1. The typical log–log plots in the upper left corner indicate that the series exhibit long-range power-law correlations. As a result, $F_q(s)$ follows a power-law relationship, $F_q(s) \infty s^{H(q)}$, for large values of s, where H(q) represents the generalized Hurst exponent.
2. The multifractal spectrum $f(\alpha)$, each curve shown in the upper right corner of each panel, has a significant width in all markets, indicating considerable variation in local fluctuation persistence.
3. The generalized Hurst exponent H(q), each curve depicted in the lower left corner, reveals that AI coin series exhibit different scaling behaviors for small and large fluctuations. Consequently, H(q) strongly depends on q, indicating significant multifractality.
4. The concavity of the scaling function τ(q), illustrated in the lower right corner, provides clear evidence of multifractality and the presence of multiple singularity exponents.

The robustness analysis, employing multifractal detrended fluctuation analysis (MF-DFA), confirms the presence of multifractal features and fluctuating market efficiency, consistent with Kristjanpoller et al. (2024) and Naeem et al. (2021). Finally, while we primarily address financial dynamics, we also raise environmental considerations. Echoing Ghosh and Bouri (2022) and Köhler and Pizzol (2019), we emphasize the need for further exploration into the sustainability profiles of AI tokens relative to Bitcoin's substantial carbon footprint. The primary characteristic of the Hurst exponent value is anti-persistency, which can likely be explained by the stylized fact that AI tokens are not an efficient market. The lack of regulatory oversight leads to significant price fluctuations and high volatility.

5. Conclusions, policy implications, and future directions

Here, we investigate AI tokens' fractal properties and time-frequency dynamics using a combination of rescaled range (R/S) analysis, wavelet power spectra, and wavelet coherence methods. Our empirical results offer evidence that AI token returns exhibit long memory and persistent behavior, which are consistent with the FMH. Additionally, wavelet power analysis reveals that short-term volatility dominates during crisis periods, while wavelet coherence shows that Bitcoin and AI tokens are highly interconnected across time and frequency domains, particularly during financial turbulence. These findings hold several practical implications. For crypto traders and investors, the observed persistence in AI token prices suggests opportunities to exploit trend-following strategies over short- to medium-term horizons. However, caution is warranted due to potential volatility clustering.

5.1. Policy implications

Our findings of this study carry several tentative implications for market participants, regulators, and researchers examining the emerging AI token market as follows:

First, the presence of long memory and persistent behavior across AI tokens indicates that these markets may deviate from traditional notions of informational efficiency. Such persistence could reflect underlying structural factors, including investor herding, speculative momentum, or the relatively illiquid and immature nature of these assets. For portfolio managers, such behavior warrants attention, particularly when constructing strategies around entry and exit timing or evaluating algorithmic trading models.

Second, the dominance of high-frequency volatility during crisis periods, as identified through the wavelet power spectra, suggests that the price dynamics of AI tokens are predominantly driven by short-term informational shocks rather than by longer-horizon fundamental trends. This outcome underscores the heterogeneity of market participants and implies that short-term liquidity provision and noise trading significantly impact price discovery during periods of elevated systemic risk.

Third, although AI tokens display time-varying coherence with Bitcoin, the strong positive co-movement undermines Bitcoin's effectiveness as a hedge or safe-haven for this asset class. This result contrasts with earlier studies that suggested Bitcoin's potential diversification role for commodities or equities. Hence, investors seeking true hedging instruments for AI tokens may need to consider alternative assets or risk management approaches.

Originality: This study on innovative themes is fresh, with many doubts and gaps in the literature due to its emergence as a new financial segment. Therefore, adopting a longitudinal approach to identify the evolution of efficiency in this AI token market is interesting and needs to be explored in the literature. Additionally, the study provides insights into the challenges and opportunities in this emerging field.

5.2. Future directions for AI tokens

This study is not without limitations. First, the AI token market is nascent, meaning that results based on current data may not generalize to more mature market phases. Second, although we discuss

Bitcoin's environmental footprint, we do not empirically test the relative sustainability of AI tokens, which limits the strength of our policy statements regarding environmental impact.

To address these gaps, researchers could explore several avenues. One line of direction is to apply machine learning or nonlinear forecasting models to assess the predictability of fractal dynamics over time. Additionally, follow-up studies could compare AI tokens across blockchain infrastructures or explore their interactions with traditional financial instruments. Importantly, conducting an empirical assessment of the energy consumption and carbon footprint of AI tokens would support the development of ESG-oriented digital asset frameworks.

Author contributions

The authors declare to have contributed equally to the conception, design, analysis, and drafting of the manuscript. All authors have read and approved the final manuscript.

Use of AI tools declaration

The authors declare they have not used AI (AI) tools in creating this article. However, we use Generative AI to polish the manuscript and improve its readability and language in the writing process.

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Conflict of interest

The authors declare there is no conflict of interest.

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