Data Science in Finance and Economics

http://www.aimspress.com/journal/dsfe

DSFE, 1(4): 393–407.
DOI: 10.3934/DSFE.2021021
Received: 29 November 2021
Accepted: 22 December 2021
Published: 30 December 2021

Research article

Dynamic tail dependence on China's carbon market and EU carbon market

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Abstract: This study explores the dynamic relationship between the European carbon emission price (EUA) and the Shenzhen carbon emission price (SZA) in the time and frequency domain. Since they represent major carbon emission rights prices in the markets, they show a close correlation and tail correlation between them. Given the current global implementation to reduce carbon economy and China's implementation of a dual-carbon policy, it is of great value to explore the dynamic relationship between the two major carbon markets. Firstly, this paper uses a wavelet method to decompose the returned sequence into different frequency components to certify the dependent construction under different time scales. Secondly, this paper uses a wide range of static and time-varying link functions to describe the tail-dependent. The empirical results show that under different time scales, the dependence construction between EUA and SZA has significant time variation. The results of this study have important policy implications for understanding the transmission of carbon prices between different markets, as well as for investors and policy makers.

Keywords: tail dependence; carbon market; wavelet; copula

JEL Codes: G15, F36, C40

1. Introduction

In order to cope with the challenges posed by global climate change and promote global greenhouse gas reduction, the "Kyoto Protocol" and United Nations Framework Convention on Climate Change (UNFCCC), which demonstrated general agreement on the need to mitigate climate change while allowing for continued economic growth and aims to limit greenhouse gas emissions in developed countries and curb global warming. Since the UNFCCC and Kyoto Protocol were signed, using market mechanisms was as a new path to solve the problem of greenhouse gas emission reduction represented by carbon dioxide. Thus, the global carbon trading market, the European Union Emissions Trading System (EU ETS), based on carbon dioxide emission rights has formed in 2005. It created the European Union Allowance (EUA) spot and its derivatives trading market occupies a dominant position in the global carbon market. Since then, there has been a lot of research on the European carbon emission market.

China are taking a more active part in exploring how to establish a domestic carbon market, since China, as a large developing country in the world, is facing tremendous pressure to reduce carbon emissions. For one thing, China establishes a domestic carbon market to achieve the efficiency of reducing emissions and reduce its disadvantages in the international carbon market. For other thing, carbon emissions trading market in China is the use of market mechanism to control and reduce greenhouse gas emissions and promote green low carbon transformation economic development way, which is an important system innovation, and strengthening the construction of ecological civilization, to carry out the important policy tool of international commitments. With the mutual influence of industries between countries, With China as the largest carbon emitter and supplier in the global carbon market in recent years, the fluctuation of carbon price of Chinese carbon market has been increasingly affected by the international market, especially with the European Union. More than that, the EU carbon price fluctuates violently since the development of the quota market. Meanwhile, the increasing of the interdependence between EU carbon market and Chinese carbon market, provide a more open channel for the transmission of risks between markets, since the EUA (EU carbon market) and SZA market (China's Shenzhen carbon market) are both carbon trading markets and have strong business similarities and they are with significant similarities of information transmission on macro fundamentals and risky contagion. Therefore, the interdependence and risk spillover effects of national carbon markets have become the core issues that investors must first solve to avoid carbon market risks and policymakers need to deal with to explore carbon market stability mechanisms.

Based on the reasons above, we firstly apply the asymmetric GJR-GARCH model to fit the marginal distribution of the carbon market, and then use the Wavelet-Copula method to study the EU carbon market and China's Shenzhen carbon market from the frequency domain. We further study the time-varying dynamic effects of the two carbon markets in the short, medium, and long term and the research conclusions would help China to establish a domestic carbon market.

2. Literature review

The literature referenced in this article is mainly composed of two parts. The first part is the relationship between European carbon market and Chinese carbon market. Given the importance of the two markets, some literature has explored the impact of the European carbon market on Chinese carbon market, and have provided meaningful materials and references for China's carbon emission market pricing by exploring the characteristics of European carbon emission prices (see Zhang et al., 2017; Hanif et al., 2021; Zeng et al., 2021; Boute and Zhang, 2019; Yang and Luo, 2020; Li and Duan, 2021; Ibrahim et al., 2021). The EUA market is currently the most complete carbon emission market in the world, so it has a spillover effect on other markets. Liu et al. (2021) suggest that volatility spillovers from the EUA spot market to future markets vary over time and are vulnerable to extreme events. Zhang and Zhang (2018) study the interdependence among the carbon market of China, Europe, and the USA. The results illustrate that, compared to the spillover effect from China to the EU and USA, the latter have a higher carbon emission spillover effect on the former. With the continuous advancement of the integration of the world economy, China's carbon emission market has become more closely linked with the international market. Xu (2021) finds the different risk spillover effect from the global crude oil market to the Shenzhen carbon market by combining the copula and CoVaR methods. Besides, some scholars find a close relationship between carbon and energy market. For example, Jiang et al. (2018) illustrate that carbon price is mainly influenced by its historical price.

The other part of literature is about the methodology used to analyze the correlation of those two important carbon markets. Many econometric methods, for example, Markov transformation VAR model (Chevallier, 2011), Dynamic model averaging (Koop and Tole, 2013), Empirical mode decomposition (EMD) (Zhu et al., 2015), Combinational Prediction Model Based on Variational Modal Decomposition (VMD), Peak Value Neural Network (SNN), Dynamic Conditional Correlation-generalized autoregressive conditional heteroskedasticity (DCC-GARCH) model (Güngör and Taştan, 2021) and complete Baba, Engle, Kraft and Kroner GARCH (BEKK-GARCH) model (Zhang and Sun, 2016) are applied in the study this issue. In recent years, many scholars have adopted a mixture of methods to study this problem. Since Patton (2006) proposes a variety of time-varying Copula to depict the dynamic interdependence structure of the market, the time-varying Copula model has been widely used in the analysis of the interdependence relationship and interdependence construction of different markets. Compared with other functions, the Copula function with multivariate joint distribution has the advantage of effectively describing the global and nonlinear correlation between variables, and can better describe the correlation in extreme cases for the characteristics of the tail structure. To our best knowledge, less literature using Copula method to study the dynamic correlation of the EUA and the Shenzhen Carbon price (SZA) in the time and frequency domain. However, considering the dynamic tail correlation between EU and China's carbon markets based on the time-varying Copula model, the effective fitting of edge distribution, the effective estimation of parameters in Copula modeling process and the selection of optimal Copula function are particularly important. Therefore, we adopt the Copula function to establish the ARMA-GJR-GARCH model and utilize the wavelet model and Copula model to analyze the dynamic correlation of EUA and the Shenzhen Carbon price (SZA) in the time domain and frequency domain.

The rest of the paper is outlined as follows. Section 2 is the review part. Section 3 introduces the methodology and data. Section 4 shows the empirical analysis. Section 5 concludes this paper.

3. Methodology and data

3.1. Methodology

3.1.1. ARMA-GJR-GARCH model and wavelet model

Before using the copula method to build a binary distribution model, we need to consider the marginal model. It is generally believed that financial time series returns always show important features, like fat tail and conditional heteroscedasticity. An autoregressive conditional heteroscedasticity (ARCH) model is first proposed to describe time series (Engle, 1982). Bollerslev et al. (1988) then extended the GARCH model, which has been applied extensively in analyzing financial time series. In the EGARCH model all constraints are non-negative. While the GJR-GARCH model is the latest application that takes into account asymmetric fluctuations and spillover effects after external shocks. In the GJR-GARCH model, past hysteresis can be used to measure asymmetry, considering earnings behavior, like long-tail distribution and leverage effects. The advantage of this model is that the parameter estimation method can reflect the time-varying characteristics between variables, so as to facilitate the study of the dynamic relationship between variables. This model assumes that the relationship between variables changes over time, so the size and trend of the correlation between variables can be measured by calculating conditional dynamic coefficients.

Inspired by existing research, we combined the ARMA (p, q) and GJR-GARCH models in this research. We describe the marginal density of carbon market price in the form of ARMA (p, q):

$$r_t = c + \sum_{i=1}^{p} \alpha_i r_{i-1} + \sum_{i=0}^{q} \beta_i \varepsilon_{i-1} \tag{1}$$

$$\varepsilon_t = \sigma^{1/2} v_t \tag{2}$$

where r_t represents the income of the carbon market, AR(p) and MA(q) are the processes of p and q lag, ε_t is the error, v_t is the standardized residual, and σ_t is the conditional variance term:

$$\sigma_t^2 = \psi + \phi \varepsilon_{t-1}^2 + \eta \varepsilon_{t-1}^2 d_{t-1} + \gamma \sigma_{t-1}^2 \tag{3}$$

where j is a constant and d_{t-1} is a dummy variable. When $\varepsilon_{t-1} < 0$, $d_{t-1} = 1$. Otherwise, $d_{t-1} = 0$. If $d_{t-1} = 0$, it will affect the conditional variances of "good news" (($\varepsilon_{t-1} > 0$) and "bad news" ($\varepsilon_{t-1} < 0$), and shows a leverage effect (Meng et al., 2020). "Good news" brings shocking ϕ, and "bad news" brings shocking $\phi + \eta$. If $\eta > 0$, the leverage effect leads to greater volatility in returns (Muteba and Mwambi, 2021; Jiang et al., 2019).

Different from Fourier transform, wavelet transform is about time and frequency (Mabrouk, 2020). Therefore, wavelet transform can obtain information by scaling operation functions.

The function $\psi(t)$ with space $L^2(R)$ can be expanded under wavelet base, which can be expressed in Equation (4).

$$WT_f(a, \tau) = \langle f(t), \psi_{\alpha,\tau}(t) \rangle = \frac{1}{\sqrt{a}} \int_R f(t) \psi^*(\tfrac{t-\tau}{a}) dt \tag{4}$$

where a is the increasing finer scale deviations and τ is the time position parameter.

3.1.2. Copula model

The Copula function connects the multivariate joint distribution and effectively describes the global and non-linear relevance between variables, and can better describe the relevance in extreme cases for the characteristics of the tail structure. The structural dependence between variables dominated by time series shows obvious time-varying characteristics. Patton (2006) proposed a variety of time-varying Copulas to describe the market dynamics dependence construction. The time-varying Copula model is applied extensively in the analysis of the interdependence and interdependence structure of different markets (See Jiang et al., 2020; Christoffersen et al., 2012; Jiang et al., 2017; Jiang et al., 2018; Abakah et al., 2021; Kumar et al., 2021; Naeem et al., 2021; Ma and Wang, 2021).

Copula models are always separated into two types. The static Copula functions (Nelsen, 1991) mainly include nine kinds of Copula.[1] While dynamic Copula models mainly include time-varying correlation Copula and variable structure Copula. Compared with the static Copula, the most important thing is that the parameters in the TVP-Copula model are time-varying. So, using the TVP-Copula model to analyze the dynamic relationship between variables, the influence of correlation and marginal distribution of modeling variables can be excluded, and the time-varying characteristics of parameters can be described. This paper mainly selects a class of time-varying Copula functions to describe the dynamic correlation structure between them under LL, AIC, and BIC criteria.

Taking the time-varying Normal copula model as an example, the PDF[2] of TVP-Normal copula function is:

$$C(u,v,\rho_t)=\int_{-\infty}^{\Phi^{-1}(u)}\int_{-\infty}^{\Phi^{-1}(v)}\frac{1}{2\pi\sqrt{1-\rho_t^2}}\exp\left[\frac{-(r^2+s^2-2\rho_t rs)}{2(1-\rho_t^2)}\right]drds \tag{5}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of the standard normal distribution function, and $\rho_t\in[-1,1]$ is the relevant parameter. Considering that the relevant parameter is time-varying, it is described by the following evolution equation:

$$\rho_t=\Lambda(w_\rho+\beta_\rho\rho_{t-1}+\alpha_\rho\times\frac{1}{q}\sum_{i=1}^{q}\Phi^{-1}(u_{t-i})\,\Phi^{-1}(v_{t-i})) \tag{6}$$

where the function $\Lambda(x)=\frac{1-e^{-x}}{1+e^{-x}}$ is to ensure that ρ_t is always in the interval [0, 1], and the lag order is generally less than 10.

3.2. Data

As the most important regional carbon emission allowance trading market in the world, the European Emissions Trading System is the main leader in international carbon trading prices. We select the Shenzhen carbon emission trading market price with the largest market transaction volume and the longest transaction time in China's carbon market. Besides, considering the time difference between the markets, we select weekly data[3] spanning from 2013/8/10 to 2021/8/28. Through matching, there are 408 transaction data in total, and we get the return series of the carbon prices by calculating the logarithmic difference of the price.

Figure 1 presents the return rates of EUA and SZA. The figure shows that the price trends in the two markets are quite different. The price of carbon emissions in Europe shows a continuous upward trend and a significant sharp increase after the 2020 epidemic. The carbon emission price in the Shenzhen market shows an opposite trend, with a downward trend since 2013. While in terms of volatility, The EUA market shows a small fluctuation range, while in SZA market there have been large fluctuations since 2019, especially during the epidemic period.

[1] The function each copula can be seen in Patton (2006).
[2] PDF is the probability distribution function.
[3] We use the closing prices of the weekly data of the two markets, and eliminate the no-data days in those two markets.

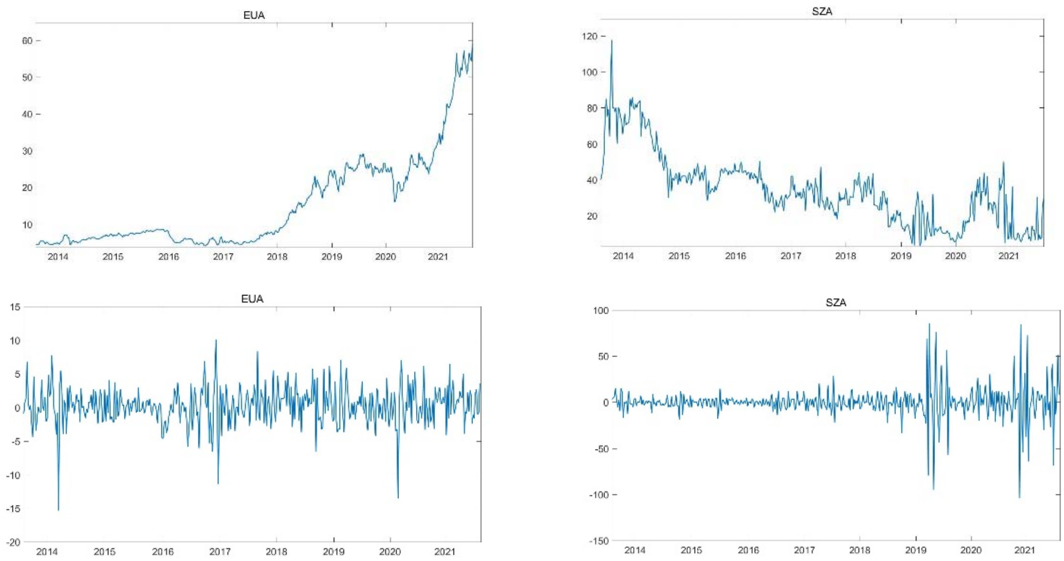

Figure 1. Time series plot of returns for two markets.

Table 1. Statistic description.

	EUA	SZA
Mean	0.2762	−0.0320
Median	0.2426	−0.1035
Max	10.0943	85.7881
Min	−15.3499	−104.1050
S. D.	2.7961	17.0905
Skew	−0.5702	−0.2592
Kurt	7.0968	15.4815
JB	306.6811	2646.463
Q(10)	6.0211	91.5870**
	[0.8134]	[0.0000]
Q^2(10)	19.2091**	207.5415***
	[0.0376]	[0.0000]
ARCH(10)	18.7024**	98.9740***
	[0.0442]	[0.0000]
ADF	−19.6805***	−18.0618***

Notes: EUA and SZA represent the European carbon emission price and the Shenzhen carbon emission price, separately. Weekly data for the period is from 2013/8/10 to 2021/8/28. ADF is to test the stationary of all series. Q (20) and Q2 (20) are the Ljung-box test (autocorrelation) with 20 lags. ARCH (10) is ARCH effects with 10 lags. *** represents the 1% level of significance.

Table 1 describes the statistics of the return series. It can be seen that within the sample range, the data of the two markets are quite different. The average value of EUA is positive, while that of SZA is negative. Besides, the standard deviation of SZA is relatively large, which means that the degree of dispersion of SZA is higher than that of EUA. In addition, the negative values of skewness indicate that both of the two markets exhibit left-tailing characteristics for positive shocks. The kurtosis of the return rates of the two markets are both greater than 3, which has significant peak characteristics. Skewness and kurtosis show the characteristics of "spikes and thick tails" in the yields of the two markets, which means that the two markets are easy to extreme risk events. The JB test shows that at the 1% significance level, all returns reject the null hypothesis of normal distribution. Q statistics show that SZA's return rate series are autocorrelated, but the Q-square statistics show that the return series of the two markets are autocorrelation. Lagrangian multiplier and ARCH test statistics show that all three series have significant fluctuation clustering characteristics. In addition, the return rates of the two markets are stable, which means that the selected sequence can be applied to further econometric analysis.

4. Empirical analysis

4.1. Marginal distribution results

Table 2 shows the parameter estimation results of ARMA-GJR-GARCH-partial t modeling that represents the marginal distribution of EUA and SZA return data. The results show that the parameters of the SZA mean equation are significant under most conditions, which shows that the past stock information in the SZA market is related to the current stock returns. However, the conditional wave equation of the Shanghai Stock Exchange Index is not significant. The lagging square residual parameter (ARCH) shows that the SZA is significant and the EUA is not significant. To a certain extent, it indicates that SZA is greatly affected by the previous period and its effectiveness is low. The volatility lag variance parameter (GARCH) of the two market returns is significant at the 1% confidence level, indicating that the previous price information of the two markets would affect the subsequent price information. In other words, it has a strong memory. The kurtosis parameter clearly shows that the time series of the carbon market is biased, with some characteristics such as sharp peaks and thick tails that are common in financial series. Q test and ARCH test are on autocorrelation and heteroscedasticity respectively on the standardized residual series. The results are not significant indicating that the financial series after fitting according to the marginal distribution no longer has autocorrelation and heteroscedasticity. To sum up, the ARMA-GJR-GARCH-partial t model constructed in this paper can effectively depict the marginal distribution of the two carbon market return series, and can effectively identify some sequence characteristics in prices and fluctuations. After fitting in the time-varying Copula model, the standardized residual can describe the dynamic tail structure relationship between EUA-SZA.

4.2. Wavelet decomposition

Next, we mainly use the wavelet multi-resolution analysis method to carry out the multi-scale decomposition of EUA and SZA return rates. Considering that our data set observations are far greater than 256 and less than 512, we use the wavelet method to decompose the time series into 7 quadrature components. They are represented as D1, D2, D3, D4, and D5, respectively, representing different time ranges of 2 to 4, 4 to 8, 8 to 16, 16 to 32, and 32 to 64 weeks. The breakdown structure is shown in Figures 2 and 3. The decomposition results of EUA show that, between 2014–2015 and 2017–2018, whether it is a low-frequency or high-frequency sequence, the fluctuations significantly exceed the fluctuations in other intervals. Besides, the results show that in SZA market with low-frequency D1–D2, the fluctuations in 2021 are significantly higher than those in the other bands of the frequency, mainly due to the outbreak in 2021.

Table 2. The estimates of the ARMA-GJR-GARCH model.

	EUA	SZA
Cst(M)	0.3189***	−0.1339
	(0.1180)	(0.1386)
AR(1)	0.2511	0.1277**
	(1.1603)	(0.0754)
MA(1)	−0.2830	−0.6182***
	(1.2685)	(0.0518)
Cst(V)	0.9250	1.8899
	(0.6796)	(1.4029)
ARCH(Alpha1)	0.1136	0.2932**
	(0.0882)	(0.1245)
GARCH(Beta1)	0.7390***	0.7410***
	(0.0928)	(0.0651)
GJR(Gamma1)	0.0778	0.1158
	(0.1529)	(0.1182)
Asymmetry	0.0407	-0.0617
	(0.0757)	(0.0652)
Tail	5.5565***	3.7082***
	(1.5477)	(0.6679)
Q(10)	3.7046	8.8228
	[0.8827]	[0.3574]
Q^2(10)	6,2118	11.3065
	[0.6235]	[0.1849]
ARCH(10)	0.6289	1.0171
	[0.7891]	[0.3281]

Notes: EUA and SZA represent the European carbon emission price and the Shenzhen carbon emission price, separately. Weekly data for the period is from 2013/8/10 to 2021/8/28. ADF is to test the stationary of all series. Q (20) and Q2 (20) are the Ljung-box test (autocorrelation) with 20 lags. ARCH (10) is ARCH effects with 10 lags. *** represents the 1% level of significance.

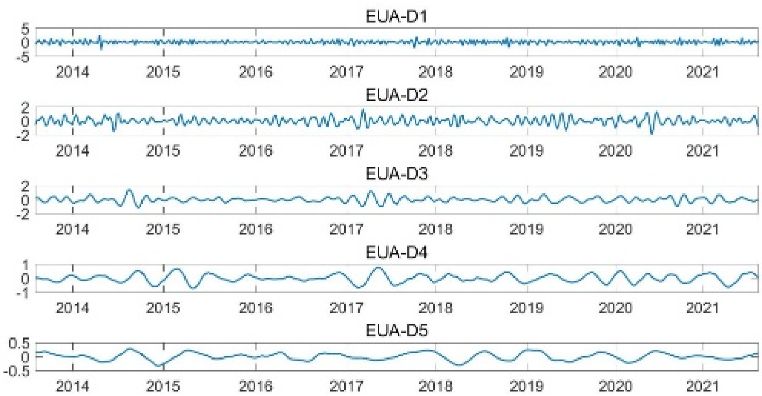

Figure 2. D1–D5 of EUA.

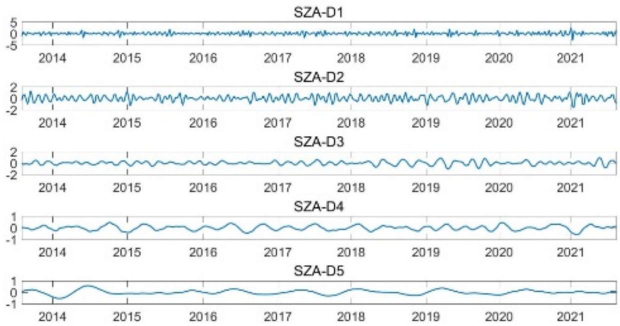

Figure 3. D1–D5 of SZA.

Table 3. Scale interpretation of the MRA scale levels.

Scale	Weekly scale
D1	2–4 weeks
D2	4–9 weeks
D3	8–16 weeks
D4	16–32 weeks
D5	32–64 weeks

Notes: D1–D2, D3 and D4–D5 represent low timescales, mediate timescales, and high timescales, respectively.

4.3. Copula estimations

In this part, we use many copulas to study the dependency structure of EUA and SZA to test tail dependency and determine the dependency parameters for the relationship at each frequency. The correlation results are shown in Table 4. And then we utilize the AIC criterion to select the best fitting Copula function. It shows that compared to the static Copula, the time-varying Copula function can better fit the tail relationship structure of EUA and SZA, which means that the optimal dependency structure varies over time. The Copula fitting values for each set of sequences are shown in the table. For the convenience of analysis, this section organizes the best fitting results in Table 5.

Table 4 shows that the dependence direction at different frequencies presents different results. The t of Normal and Student shows that the correlation between EUA and SZA is positive and relatively weak at the D1 frequency in the shortest time range. In the longest time range D5 frequency, the correlation is positive and weak. But in the medium and long-term frequency, the correlation transforms from a strong negative relationship to a positive relationship. Based on asymmetric connections such as TVP and SJC, the relationship between EUA and SZA also has a potential mid-to-long-term positive-tail dependency. The results of all link functions show a similar trend. In another word, the correlation becomes weaker as time extends.

From Table 5, we can find that for raw data, the best-fitting Copula is TVP Gumbel, which exhibits a special upper tail correlation, and its AIC value is −4.7827, indicating that EUA and SZA show a strong correlation under normal market circumstances. The TVP Gumbel coefficient between them is positive, showing a similar trend. While based on the results at different frequencies, the best-fitting Copula at the frequency is TVP Normal, showing no tail-related features.

Table 4. Bivariate copula estimates for EUA and SZA.

	Raw	D1	D2	D3	D4	D5
Normal	−0.0237	0.0094	−0.1342***	0.0862**	0.0286	−0.0027
	(0.0497)	(0.0497)	(0.0492)	(0.0495)	(0.0497)	(0.0497)
AIC	−0.2235	−0.0313	−7.3959	−3.0321	−0.3289	0.0019
Clayton	0.0001	0.0001	0.0001	0.0696	0.0001	0.0001
	(0.0624)	(0.0593)	(0.0659)	(0.0692)	(0.0790)	(0.0857)
AIC	0.0056	0.0056	0.0127	−1.6079	0.0075	0.0118
Rotated Clayton	0.0001	0.0258	0.0001	0.1019	0.1052	0.0798
	(0.0704)	(0.0583)	(0.0852)	(0.0740)	(0.0690)	(0.0649)
AIC	0.0083	−0.2399	0.0141	−3.3384	−3.9096	−2.2761
Plackett	0.9659**	0.9827**	0.6695***	1.1993**	1.0943**	0.9295**
	(0.4209)	(0.4369)	(0.2016)	(0.6418)	(0.6054)	(0.4071)
AIC	−0.0490	−0.0086	−7.0644	−1.4825	−0.3191	−0.2237
Frank	0.0001	0.0001	0.0001	0.3648	0.1679	0.0001
	(1.4949)	(8.3211)	(1.8976)	(1.7989)	(1.8850)	(1.9300)
AIC	0.0051	0.0050	0.0067	−1.4867	−0.2969	0.0052
Gumbel	1.1000***	1.1000***	1.1000***	1.1000***	1.1000***	1.1000***
	(0.0382)	(0.0320)	(0.0424)	(0.0291)	(0.0285)	(0.0302)
AIC	13.2447	6.6916	23.2318	−1.8376	0.0588	4.8576
Rotated Gumbel	1.1000***	1.1000***	1.1000***	1.1000***	1.1000***	1.1000***
	(0.0367)	(0.0346)	(0.0412)	(0.0307)	(0.0359)	(0.0435)
AIC	11.7082	9.2132	23.2780	1.0292	5.9479	17.3836
Student's t	−0.0234***	0.0088	−0.1377	0.0868	0.0331***	−0.0043
	(0.0036)	(0.0265)	(17.1445)	(0.9443)	(0.0000)	(0.0094)
	99.2046***	99.9927***	99.9883***	99.9896***	8.6421***	99.9998***
	(0.0029)	(0.0008)	(0.0087)	(0.0083)	(0.0028)	(0.0025)
AIC	−0.0439	0.1731	−7.2815	−2.8958	−3.6099	0.3974
SJC	0.0000	0.0009	0.0000	0.0149	0.0129	0.0105***
	(0.0673)	(0.0025)	(0.0022)	(0.1366)	(0.0126)	(0.0001)
	0.0005	0.0000	0.0001	0.0000	0.0000	0.0000
	(0.0025)	(0.0004)	(0.0001)	(0.0008)	(0.0179)	(0.0194)
AIC	2.5065	0.5404	6.3078	−3.6410	−2.7270	0.5791
Time-varying normal	−0.0614**	0.0644***	−0.4198***	0.0051	−0.1296	0.1229***
	(0.0253)	(0.0234)	(0.0648)	(0.0596)	(0.0830)	(0.0232)
	−0.1047	1.1758***	0.8506***	1.2788***	1.9513***	1.1014***
	(0.4300)	(0.2108)	(0.2436)	(0.1038)	(0.0893)	(0.0992)
	−0.3966	−1.9074***	−1.9788***	−1.9517***	−2.0359***	−1.9715***
	(0.8156)	(0.0666)	(0.0433)	(0.1544)	(0.1209)	(0.0675)
AIC	−0.2816	−12.1627	−14.0821	−33.9155	−84.0276	−32.9354

Continued on next page

	Raw	D1	D2	D3	D4	D5
Time-varying rotated Gumbel	0.3303	2.1574***	−1.1831	1.9934***	0.5771	0.3342***
	(0.7693)	(0.0724)	(0.8838)	(0.3389)	(0.7320)	(0.1127)
	0.3669	−1.1572***	1.4544**	−0.8291**	0.2925	0.3796***
	(0.4497)	(0.2116)	(0.6049)	(0.3266)	(0.2914)	(0.0383)
	−2.0931**	−2.1072***	−0.6723	−2.2464***	−1.6902**	−1.4184***
	(0.8833)	(0.3330)	(0.7069)	(0.2511)	(0.8878)	(0.1779)
AIC	−4.4360	−6.5791	−0.5041	−21.8799	−71.0901	−32.0794
Time-varying SJC	−16.3531***	1.6338***	−16.8954***	5.5656***	3.1622***	4.0148***
	(0.3671)	(0.6632)	(0.0095)	(1.1373)	(0.2143)	(0.0856)
	−2.0180***	−17.2009***	−2.1631***	−25.0000***	−22.8389***	−25.0000***
	(0.1061)	(2.4263)	(0.0017)	(3.4851)	(3.0786)	(1.4140)
	−0.0046***	1.5542	−0.0050***	−3.3630***	−1.6922**	−0.5161***
	(0.0012)	(1.4600)	(0.0000)	(1.0759)	(0.6902)	(0.0111)
	−22.3124***	−20.3901**	−19.0075***	−16.0509**	−17.6150***	−18.9901***
	(0.0040)	(10.1782)	(0.0097)	(7.5079)	(1.4847)	(0.2083)
	−5.3073***	−2.6585***	−4.1689***	−3.7669**	−0.0003	−3.8794***
	(0.2242)	(0.7348)	(0.0031)	(1.5521)	(0.0016)	(0.0441)
	−0.0208***	−0.0055***	−0.0109***	−0.0063**	0.0000	−0.0058***
	(0.0006)	(0.0011)	(0.0000)	(0.0030)	(0.0036)	(0.0004)
AIC	4.1431	−2.2552	10.3575	−27.0007	−42.7642	−111.4742
Time-varying Gumbel	0.1801**	2.0771***	2.7230***	1.1483***	0.6882***	0.4753
	(0.0827)	(0.0698)	(0.0033)	(0.0579)	(0.1113)	(0.3878)
	0.4115	−1.0345***	−2.7015***	−0.1066	0.1751**	0.3326**
	(0.7194)	(0.1243)	(0.0031)	(0.2013)	(0.0966)	(0.1291)
	−1.6474***	−2.1432***	−0.0449***	−2.2681***	−1.6521***	−1.5570
	(0.1927)	(0.0659)	(0.0010)	(0.0277)	(0.3404)	(1.3876)
AIC	−4.7827	−10.0386	−0.2082	−26.5840	−51.2170	−51.0338
Time-varying T-copula	−0.0614**	0.0580	−0.2381	0.0063	−0.1204*	0.1041
	(0.0252)	(0.1492)	(0.4254)	(0.4938)	(0.0728)	(0.2348)
	−0.1056	0.9802***	0.4034***	0.9733***	1.5381***	0.8809***
	(0.2574)	(0.0968)	(0.0187)	(0.0013)	(0.0764)	(0.0114)
	−0.4333***	−1.9028***	−0.2559***	−1.9375***	−2.0240***	−1.9595***
	(0.0730)	(0.0576)	(0.0105)	(0.0085)	(0.1467)	(0.0078)
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(2.1053)	(0.4936)	(0.0180)	(0.0088)	(0.0006)	(0.0261)
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.9089)	(0.0014)	(0.0001)	(0.0199)	(0.0004)	(0.0085)
	1.0000***	1.0000***	1.0000***	1.0000***	0.9998***	1.0000***
	(0.1866)	(0.4803)	(0.1722)	(0.0042)	(0.1083)	(0.0048)
AIC	−0.1248	−11.5335	−11.2433	−31.6107	−81.0220	−30.4326

Continued on next page

	Raw	D1	D2	D3	D4	D5
Time-varying Clayton	0.9454**	1.3161***	0.0000***	1.5140***	1.2096***	0.8273***
	(0.4757)	(0.3816)	(0.0000)	(0.2333)	(0.2643)	(0.2039)
	0.2125	−0.8905***	−0.2859***	−0.6548**	0.1940**	0.3303***
	(0.6436)	(0.3261)	(0.0000)	(0.2696)	(0.1216)	(0.0798)
	−2.9145**	−2.8353***	0.0000***	−2.8847***	−2.3196***	−1.6712***
	(1.4021)	(0.8882)	(0.0000)	(0.4447)	(0.5096)	(0.4869)
AIC	−2.8737	−4.7399	0.0225	−20.1380	−65.8131	−23.6189

Notes: Raw represents the full sample scale. D1–D2, D3 and D4–D5 represent low timescales, mediate timescales, and high timescales, respectively. The table displays the estimates for the different copula models for EUA and SZA. We estimate the parameters of different copula. Values in parenthesis are the standard error. We also show the AIC values for the small-sample bias. We set q as 10 in the TVP Gaussian and Student-t copulas. ***, ** and * means the 1%, 5% and 10% levels of significance, respectively.

Table 5. Best-fit copula and tail dependence of each group.

Scale	Best copula	Dependence structure
Raw	TVP-Gumbel	Lower tail independence and upper tail dependence
D1	TVP-Normal	Tail independence
D2	TVP-Normal	Tail independence
D3	TVP-Normal	Tail independence
D4	TVP-Normal	Tail independence
D5	TVP-Normal	Tail independence

Notes: Raw represents the full sample scale. D1–D2, D3 and D4–D5 represent low timescales, mediate timescales, and high timescales, respectively.

To facilitate a more direct judgment of the dynamic relationship between the tails of EUA and SZA at different frequencies, Figure 4 shows the time-varying path of the risk correlation coefficient. It illustrates that the relationship between EUA and SZA fluctuates significantly. That's the reason we adopt time-varying Copula for modeling. Specifically, EUA and SZA show a dynamic and special upper-tail correlation in the relationship between the original sequences. This relationship has shown a sharp decline in 2013 and the overall performance is relatively stable. However, the correlation has increased significantly since 2021. This should be attributed to the SZA market becoming more and more perfect in the price mechanism, the market price more fully reflects the supply and demand relationship of the Shenzhen carbon emission market, and the linkage with the EUA market is getting stronger.

In addition, the dependency relationships from D1–D5 all show TVP-Normal. Though the dynamic relationship between them does not show an obvious tail correlation, it maintains a consistent dependency relationship under all possible market conditions. Specifically, under the frequency D1, its dependence relationship fluctuates around 0, which shows a complex relationship between EUA and SZA in the short term. While under the frequency D2, the dependence relationship always appears to be negative. From the mid to long term, the dependency relationship will change significantly over time. Taking D3 and D4 as examples, the dependency relationship is mostly positive, but after the outbreak of 2020, the relationship changes from positive to negative. It illustrates that though the dependency relationship of EUA and SZA deviate in the short term, it has the same trend in the long term.

Figure 4. Time variations for each scale of bivariate copula of EUA-SZA.

5. Conclusion

In this paper, we adopt the weekly data, spanning from 2013/8/10 to 2021/8/28 of the return rate of the carbon trading spot price of the European union emissions market and Shenzhen carbon emissions trading market.

Firstly, we fit the two markets with marginal distributions by utilizing the ARMA-GJR-GARCH model. Secondly, the wavelet method is further used to decompose data into different frequencies. Finally, dynamic and static Copula are used to analyze the tail correlation effect between the two markets.

The results show that (i) Both markets have agglomeration effects and are biased, with common features such as spikes and thick tails in financial series. The standardized residuals after fitting are applied for time-varying Copula modeling and we study the dynamic tail structure relationship between SZA and EUA. (ii) The time-varying Copula function can better fit the tail relationship structure of EUA and SZA than the static Copula at different frequencies, meaning that the optimal dependence structure varies with time. Besides, in raw data, the best-fitting Copula of the two markets is TVP Gumbel. It shows a special upper-tail correlation, indicating that under normal circumstances, EUA and SZA show a similar trend and a strong market correlation in good market conditions. The best-fitting Copula at other frequencies is TVP Normal. But it does not show obvious tail-related features. (iii) The relationship between EUA and SZA fluctuates significantly. So, it shows strong time-varying characteristics, and the relationship between the two fluctuates sharply after the outbreak. It shows that, to a certain extent, the two carbon markets would be more vulnerable when impacted by external events. It is necessary to strengthen the perfect mechanism for the national carbon market, improve the price supervision mechanism, and avoid price fluctuations from affecting the domestic industrial structure adjustment and the development of low-carbon industries.

Acknowledgments

This work is supported by grants from the National Natural Science foundation for Young Scholars of China (72101063).

Conflict of interest

All authors declare no conflicts of interest in this paper.

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