Environmental Science

http://www.aimspress.com/journal/Environmental

AIMS Environmental Science, 9(3): 325–353.
DOI:10.3934/environsci.2022022
Received: 05 March 2022
Revised: 01 May 2022
Accepted: 31 May 2022
Published: 22 June 2022

Research article

Sustainable green production model considering variable demand, partial outsourcing, and rework

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Abstract: Social activities, economic benefits, and environmental friendly approach are very much essential for a sustainable production system. This is widely observed during the Covid-19 pandemic situation. The demand for essential goods in the business sector is always changing due to different unavoidable situations. The proposed study introduces a variable demand for controlling the fluctuating demand. However, a reworking of produced imperfect products makes the production model more profitable. Partial outsourcing of the good quality products has made the production system more popular and profitable. Separate holding cost for the reworked and produced products are very helpful idea for the proposed model. Moreover, consumption of energy during various purpose are considered. Separate green investment make the model more sustainable and eco-friendly. The main focus of the model is to find the maximum profit through considering optimum value of lot size quantity, average selling price, and green investment. The classical optimization technique is utilized here for optimizing the solution theoretically. The use of concave 3D graphs, different examples, and sensitivity analyses are considered here. Furthermore, managerial insights from this study can be used for industry improvement.

Keywords: imperfect production; remanufacturing; green investment; outsourcing; sustainability

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Abbreviation: SCM: supply chain management; EPQ: economic production quantity; DI: defective items; Outsur: Outsourcing; NA: not applied; SPDD: selling price dependent demand

1. Introduction

For the development of different business strategies in the modern competitive environment, sustainable production system plays an essential role. For the development of the society through economic benefit, social response, and environmental friendly approach, sustainable production model's importance is immeasurable [1]. The separate green investment for the production system makes the model more eco-friendly. However, energy consumption for the various cases of the production processes is very much essential for maximizing the profit through controlling of the energy [2]. A sustainable green production system [3] is an extremely important issue with respect to the present global situation. Carbon emissions, amount of energy consumptions, and wastes produced by the industry are major issues in the production system. Hence, the introduction of smart sustainable green production systems is the main concern in the management of modern industries [4].

The target of a smart sustainable green production system involves reducing the amount of traditional energy consumption, carbon emissions, and waste recycling to ensure a greener Earth [5–7]. To gain the maximum profit, a sustainable green production model is proposed in this model. The advancement of outsourcing technology in sustainable production systems has made it globalized, popular, and reliable. Furthermore, the introduction of outsourcing in sustainable production systems reduces the operational cost while maximizing the company's profit. In reality, no production system can produce perfect products forever. Defective production is a significant issue in the long run process. The proposed model considers defective production within modelling . Among all defective products, some products are eligible for the remanufacturing process.

This study introduces the concept of remanufacturing for such repairable items to reduce overall system costs. The fundamental concept is that only perfect-quality items should be outsourced after the remanufacturing to realize a high reputation and control of the market size by fulfilling customer demand and maximizing the company's profit. This model considers different cases of importance. Many researchers have derived the model on the basis of outsourcing, defective production, and reworking; they only considered a fixed demand with no greening cost. Based on the green concept, this model is formulated according to the lot size and greening cost. Additionally, unit production cost (UPC) includes labor cost, cost of development, and cost of tool/die. The proposed model presents a new direction in a sustainable green production system that considers variable demand and partial outsourcing.

1.1. Research gap

The following research gap can be drawn based in the existing literature.

- Several model on constant production rate and variable demand under sustainable smart production exist. However, the concept of defective items and remanufacturing of them under separate holding costs ,variable production costs, variable outsourcing costs have not been studied.
- Many production models are developed under partial outsourcing through production rate. However, some of them considered constant production under replenishment strategy. But, the effects of remanufacturing for the partially outsourced smart item with the investment of greening cost is yet not studied.

- A lot of research model on different inventory levels, and production uptime-downtime concepts through different proposed models already have been studied. But, how a sustainable smart production model can consist of the maximum profit through the defective items and their remanufacturing, partial outsourcing of good quality items, and separate holding cost through variable production rate, and selling price dependent demand has rarely been investigated.

1.2. Contribution

The model is based on the following two different cycle lengths: one is based on the replenishment process; and the other is based on a perfect-quality inventory. Different figures are included in this model to consider the different changes. This model considers two different examples with different parameters. In each example, smart production with a constant production rate and greening cost show the maximum profit as compared with other products. The focus involves determining the maximum profit of the total production system. The optimality of profit is verified analytically and numerically. The present study is a new direction of research based on the following points.

- The COVID-19 outbreak globally has led to unavoidable disruptions to major part of all the production companies. In order to overcome this difficulty, a smart production model is studied along with sustainability. This smart production model helps us to overcome to reduce wastes.
- To achieve the maximum profit and make the Earth more greener, a model based on sustainable smart production with the investment of greening cost is proposed in this study. Greening cost investment help us to reduce the environmental toxicity with eco-friendly sustainable living. Outsourcing is an crucial part of any production model to save money and time. Reduction of carbon emissions is one of the most important parts of the smart production system.
- Defective production is a common problem, whatever be the production system is smart or not. The proposed model considers the contribution of defective production in formulating the overall system. Generally, among all defective products, some products are repairable. The proposed model introduces the concept of remanufacturing for such repairable items.
- The present study considers outsource to control the market size by fulfilling the customer demand. Although many studies have been conducted on the basis of outsourcing, defective production, and reworking. But they considered variable production with a fixed demand. In this study, outsourcing, defective production, reworking with constant production rate and selling price dependent demand are considered together to achieve the sustainable goal.

1.3. Structure of this study

The rest of this paper is organized as follows. The purpose of the problem, related mathematical notations, and associated assumptions are provided in Section 3. Furthermore, Section 4 presents the mathematical modelling, and Section 5 presents the methodology used to determine the solution. Numerical applications are included in Section 6, and sensitivity analysis is described in Section 7. In Section 8, managerial insights of this study are provided. Finally, the conclusions of this study are presented in Section 9.

2. Literature review

Contributions of previous research with reference to the model proposed in this study and the gaps in the literature are discussed in this section. Furthermore, the novel contribution of this study is stated in the subsections. The contributions of previous researchers to imperfect production research are described in the first subsection. Further, remanufacturing, green investment, selling-price dependent demand, outsourcing, and sustainability are important keywords related to this model. However, for a better understanding of the research gap, a research gap determining Table 1 is provided.

2.1. Imperfect production

Almost all precautions and modern technology are introduced by the industry in the smart production system to consistently deliver perfect products. However, in real life, it is impossible to always produce perfect products. Owing to machine failure, human error, or any other cause, defective or imperfect production can occur in any production system. For customer satisfaction and goodwill, defective products must undergo a rework process to produce a perfect product. Sepehri et al. [5] derived an sustainable production-inventory model for imperfect production. They also considered preservation technology and investment for quality improvement but did not consider outsourcing, selling price-based demand. All of these factors are considered in the present model. Ahmed et al. [6] derived an imperfect production model in which they considered reworking, shortages, multi-period delay-in-payments, but did not consider outsourcing, greening costs, and selling price-based demand. All of these are considered in the present model.

Vandana et al. [7] developed an inventory model with the effect of energy and carbon emissions. They included trade-credit policy and inflation in their study. However, the model does not consider outsourcing, reworking, imperfect production and variable demand. Sepehri et al. [8] developed an inventory model in which they considered deterioration, trade credit, pricing, controllable carbon emissions but did not consider reworking, imperfect production, variable demand and outsourcing. Chen [9] developed a model based on optimizing pricing, rework decisions, and replenishment for defective and deteriorating products in a vendor-buyer channel but did not consider greening costs and outsourcing.

Kumar et al. [3] derived a production planning model with cost-effective analysis, uncertainty fuzzy number, and fuzzy linear programming problem but did not consider greening costs, reworking, imperfect production, variable demand and outsourcing. Through considering the multi-product production system, Bhuniya et al. [10] discussed an economic production model with imperfect products, reworked through failure rate strategy. But their model is lack of sustainability property. Dey and Giri [11] developed a new method for inspecting a combined vendor-buyer model with a defective production system. Malik and Sarkar [12] developed a model for disruption management of a multi-product defective production system without outsourcing.

It is clear from all these studies mentioned earlier that controllable production are fully dependent on how much imperfect products are produced. Every earlier research focussed on the imperfect production but no earlier research focus on the effect of imperfect production through partial outsourcing under variable demand. Thus, in the next section, the effects of remanufacturing with imperfect production are described.

2.2. Remanufacturing

All production companies continuously attempt to produce perfect items in their production system. However, in reality, it is impossible to produce all products perfectly. Owing to machine breakdown and other causes, imperfect production is obvious. It is profitable from the company's point of view to modify the imperfect product into a perfect product after remanufacturing. Hence, nowadays modern production companies used reworking processes in their production systems. Chiu et al. [13] developed a imperfect production model with reworking and partial outsourcing however, in this model, they did not consider selling price-dependent demand, and greening cost. Bhuniya et al. [2] established a smart production model with flexible production rate, maintenance policy, and backorder. However, they did not consider variable demand, imperfect production, reworking and outsourcing. Yadav et al. [14] derived a model with deterioration, sustainability, preservation technology, and waste management without any outsourcing and reworking.

Taleizadeh et al. [15] derived a deterministic production model with backorder, reworking, and disruption in the production system. In this model, they introduced a single-machine multi-item concept. Chiu et al. [16] developed a production model with scrap, reworking, and several deliveries. Chiu et al. [17] derived a production model by considering rework, multi-shipment policy, multi-item stock refilling, and incorporating an advanced rate.

From the past research it was shown that no research focussed on reworking of defective items under controllable production, and green investment. Hence there is a big research gap on the effect of reworking for the imperfect products. This research gap is discussed in the next section.

2.3. Green investment

The introduction of green cost investment in modern production systems is an extremely important issue with respect to the present global pandemic situation. Carbon emissions, energy consumption, and the huge waste of personal protection equipment (PPE), produced by the healthcare industry, involving Covid-19 treatment are major issues in the production system. Hence, the introduction of green production systems is the main concern in the management of modern industries across the globe. The main target of a smart sustainable green production system involves reducing the utilizing the traditional energy consumption, carbon emissions, and waste recycling to ensure a greener earth. Nia et al. [18] derived an inventory model under a shortage of multi-item multi-constraint. Manna et al. [19] derived a two-layer supply chain management model for defects and production. In this model, they also considered a bi-level credit period.

Raja et al. [20] derived a combined revenue management model for pricing, a firms greening and inventory decisions. Mishra et al. [21] derived a inventory model by introducing manageable environmental emission rates and deterioration. Dev et al. [22] derived a model based on the circulation of green products. They also considered reverse logistics in their model production planning system, but did not consider rework, outsourcing, and selling price-dependent demand. Mishra et al. [23] derived an inventory management model of optimum sustainability, deterioration, and backorder under manageable carbon emissions.

From the past research, every research just included green investment in their production system. No research studies still do not consider the effect of green investment on the sustainable production system under imperfect production and reworking. In this sense, how the reworking, imperfect production, and green investment connected with fluctuate demand for the sustainable production system are discussed in detail in the next section.

2.4. *Selling price dependent demand*

In the current competitive business environment, every industry continuously attempts to sell their products rapidly as compared to others. To realize this target, the management of modern companies follows various promotional efforts. Among all the existing promotional efforts, a discount on selling price is one of the most attractive efforts to attract customers. Alfares and Ghaithan [24] developed inventory model by considering quantity discounts, demand depends on price, and holding cost, which vary over time. Feng et al. [25] developed a model considering lot-sizing and pricing for unpreserved products when the demand depends on selling price, stocks, and termination date.

Maiti and Giri [26] developed an inventory model based on decisions and pricing policies with price-dependent demand. Li and Teng [27] developed a model with lot-sizing decisions, product freshness-dependent demand, reference price, selling price, and exhibited stocks. Mishra et al. [28] derived a model with the effects of a hybrid price, demand dependent on stocks, optimality of deteriorating inventory management, trade credit policy, and reworking. Khan et al. [29] derived an inventory model for unpreserved items with advanced payment, holding cost dependent on time, demand dependent on advertisement, and selling price.

From the past research, it was found that no research focussed on remanufacturing of defective items under controllable production and partial outsourcing. Hence, there is a big research gap on the effect of reworking for the variable demand. The proposed study overcomes this gap. Moreover, the outsourcing strategy with defective items and reworking needs further investigation. This research gap, which is discussed in the next section.

2.5. *Outsourcing*

To remain competitive in the market, to fulfill the customer's demand in time, and to increase the goodwill of a company, the most acceptable technique of the modern production industry involves partially outsourcing the service and product. Chu et al. [30]developed an inventory model for outsourcing, lot-sizing, limited inventory, and backlogging. Chen and Xiao [31] derived a supply chain model on production interruption with outsourcing policy, capacity distribution uncertainties, and demand. Li et al. [32] derived an inventory model for transportation, outsourcing, and production under the guidelines of carbon reduction.

Abriyantoro et al. [33] derived a production model for biomass outsourcing by considering stochastic optimization. They considered production planning as a constraint in the cement manufacturing industry. Heydari et al. [34] developed a supply model that coordinates quantity flexibility contracts with the effect of outsourcing decisions.

From the past research it may be concluded that every research just included outsourcing in their manufacturing system. No research still now consider the effect of outsourcing on a sustainable production model under variable demand and reworking. In this sense, how the sustainability is connected with partial outsourcing, reworking of defective products under variable demand, that is discussed in detail in the next section.

2.6. Sustainability

Modern industry management is continuously trying to realize a sustainable production system. This sustainable production system can significantly impact the economy, society, and environment throughout the world. Omair et al. [35] developed a model that includes sustainability, decision support framework, suppliers selection analytical hierarchical process, fuzzy inference system. Zhalechian et al. [36] derived a sustainable supply chain network model under mixed uncertainties. Furthermore, this model included a closed-loop location-routing-inventory. Tiwari et al. [37] derived an inventory model for imperfect and deteriorating items under carbon emissions. Lu et al. [38] derived a sustainable production-inventory model considering the Stackelberg game approach. In this model, they included cooperative investment in technology to reduce carbon emissions. Ullah et al. [39] derived a model considering ramification of remanufacturing, sustainablity for returnable products. Recently, Sarkar and Bhuniya [1] discussed a sustainable supply chain under manufacturing-remanufacturing and service strategy. But their model cannot considered partial outsourcing. The proposed model is developed compare to them.

The previous research details stated in this section mainly focussed on the variable demand. However, the effects of variable demand on the sustainable production system with controllable production, reworking, and outsourcing, which is a novel concept introduced in this research, is shown in the proposed model. All of these studies together have not yet been identified in past studies. Therefore, a strong, sustainable production system with partial outsourcing under reworking, where demand is connected with price, will be beneficial for industries.

Table 1. Contribution of the authors.

Author(s)	Greening Cost	Demand Rate	DI	Outsur	Rework	Model Type
Chen [9]	NA	SPDD	Yes	NA	Yes	SCM
Chiu et al. [13]	NA	Constant	Yes	Yes	Yes	Inventory
Taleizadeh et al. [15]	NA	Constant	Yes	NA	Yes	EPQ
Chiu et al. [16]	NA	Constant	Yes	NA	Yes	EPQ
Chiu et al. [17]	NA	NA	Yes	NA	Yes	SCM
Nia et al. [18]	Yes	Constant	NA	NA	NA	Inventory
Manna et al. [19]	Yes	Stock Dependent	Yes	NA	Yes	EPQ
Alfares and Ghaithan [24]	NA	Constant	NA	NA	NA	Inventory
Feng et al. [25]	NA	Price and Stock	NA	NA	NA	Inventory
Maiti and Giri [26]	NA	SPDD	NA	NA	NA	SCM
Li and Teng [27]	NA	SPDD and Stock	NA	NA	NA	Inventory
Mishra et al. [28]	NA	SPDD and Stock	NA	NA	Yes	Inventory
Khan et al. [29]	NA	SPDD	NA	NA	NA	Inventory
Chu et al. [30]	NA	Forecast	NA	Yes	NA	Production
Chen and Xiao [31]	NA	Uncertainties	NA	Yes	NA	SCM
Heydari et al. [34]	NA	Stochastic	NA	Yes	NA	SCM
This Paper	Yes	SPDD	Yes	Yes	Yes	Inventory

3. Problem definition, notation, and assumptions

3.1. Problem definition

In this section, the purpose of the problem along with notation and assumption are described in detail. The first problem purpose is described elaborately. Then, notation of the mathematical model are provided. Finally, the assumptions in the study are briefly described.

A sustainable production model with reworking, and partial outsourcing is proposed here. In this model, customer demand is considered based on market selling price and greening cost. Additionally, partial outsourcing is an important parameter from the customer's viewpoint because many customers are not interested in the delay-in-delivery of any product. Chiu et al. [13] considered a model by assuming partial outsourcing and defective production with rework but with a constant type of demand and without sustainability concept. Here, the proposed model is an improvement over the previously stated model. The main research question is to find the maximum profit through considering a sustainable production model with imperfect production, reworking, partial outsourcing, variable demand, and green investment. The main problem involves generating the highest profit and revenue by considering different costs in a production model in which partial outsourcing is highlighted with remanufacturing and greening costs. Furthermore, in this study, different cases are considered with different examples to derive the optimal profit. In this model, we consider the constant production rate with a greening cost, and in each case, optimality is numerically proved.

3.2. Notation

The following notation is considered to illustrate the model.

Decision variables

Q	lot size goods (units)
p	average selling price (\$/unit)
g_c	investment for greening product (\$/year)

Input parameters

P	production rate (unit/year)
K	setup cost (in-house) (\$/setup)
K^e	energy cost for in-house setup formation (\$/setup)
h	cost of holding products (\$/unit/unit time)
h^e	energy cost for holding the products per holding (\$/unit)
h_r	holding cost of each reworked good (\$/unit/unit time)
h_r^e	energy cost for holding reworked products per holding (\$/unit)
τ_1	scaling parameter of raw material cost for manufacturing system
τ_2	scaling parameter of the development cost for the product during manufacturing
τ_3	scaling parameter of tool/die cost
M_R	reworking cost (\$/unit)
M_R^e	energy cost for reworking the products (\$/unit)
A_π	constant type of outsourcing cost (\$/unit)
N_π	unit outsourcing cost (\$/unit)

R_1	reworking rate (units/year)
π	outsourcing portion of the lot size item ($0 < \pi < 1$)
α_1	connecting variable between K, A_π, where $A_\pi = [(1+\alpha_1)K]$ and $-1 \leq \alpha_1 \leq 0$
α_2	connection of C and N_π, $N_\pi = [(1+\alpha_2)C]$, and $\alpha_2 \geq 0$
R_π	the replenishment time horizon (time unit)
S_1	maximum inventory level for perfect product production ends.
S_2	inventory level of reworks of defective items becomes end
H	maximum inventory level of perfect products when outsourcing goods received
u_1	time of production ($\pi = 0$) (year)
u_2	time of reworking ($\pi = 0$) (year)
u_3	time of production down ($\pi = 0$) (year)
T	cycle length ($\pi = 0$) (year)
$I(t)$	good quality product inventory (units)
$Id(t)$	defective items inventory level (units)
TC	total operating cost per cycle ($/year)
s_{max}	maximum price ($/unit)
s_{min}	minimum price ($/unit)
σ_i	(i=1, 2, 3) scaling parameters
x	number of repairable defective products produced during fabrication
$E[x]$	expected value of x
d	production rate of the defective items

3.3. Assumptions

This study considered the following assumptions.

1. In this model, a fixed part, π, of the optimal lot size quantity Q ($0 < \pi < 1$) is outsourced, i.e., partial outsourcing is considered here. All the outsourced products are perfect. If $\pi = 0$, then the system becomes an in-house production system. If $\pi = 1$, then the system becomes a purchasing system [13].
2. In this model, a deterministic production model is considered with defective items. Remanufacturing is performed to increase the reputation of the company, to fulfill customer demand, and challenge the market size. Remanufacturing is possible only after investing in additional costs. The defective rate is also stochastic in terms of type. Among all defective items, only repairable items can undergo the remanufacturing process [12,40].
3. Here, a considerable unit production cost (UPC) depends on the production rate. Here, the UPC includes development costs, tool/die investments, and development investment. The development cost is inversely proportional to the production rate. Thus, the final expression of the function is as follows: $C(P) = (\tau_1 + \frac{\tau_2}{P} + \tau_3 P)$,
4. This model considers demand as the SPDD type. Typically, the demand for any item is considered as constant or variable. In this model, demand is considered as $D = \sigma_1 \frac{(s_{max} - p)}{(p - s_{min})} + \sigma_2 g_c$ [29].
5. The management of every production system is concerned with respect to the recycling of the waste produced by the production system to ensure a greener Earth. This model considers the investment costs in green products as :$IGP = \frac{\sigma_3 g_c^2}{2}$ [14].

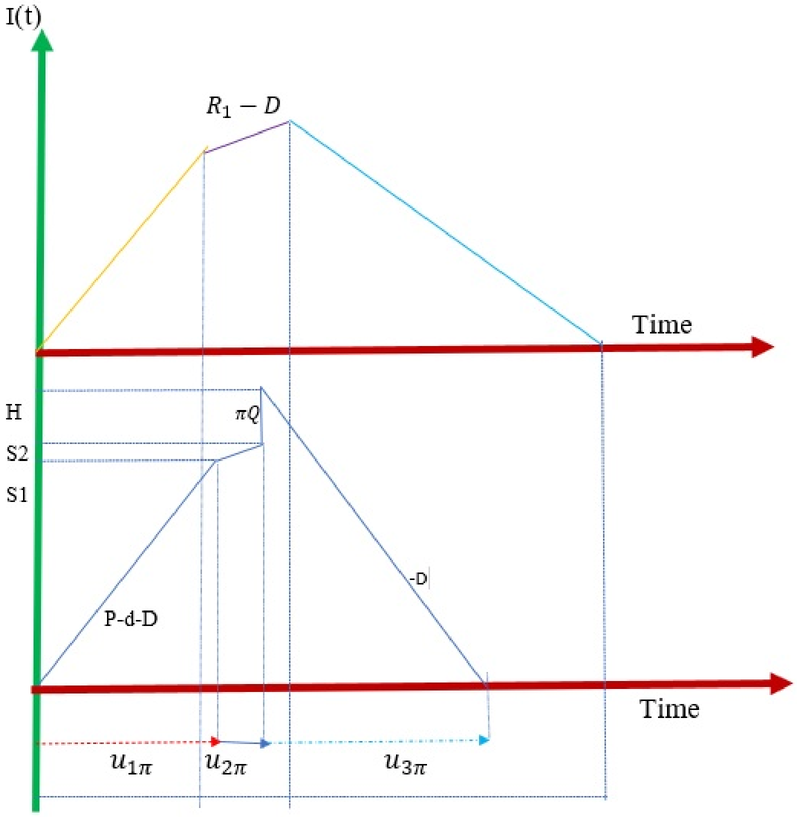

Figure 1. Perfect-quality inventory in the proposed system with respect to the system without an outsourcing plan.

4. Model formulation

In this section, different considerable costs are elaborated to formulate the proposed model. In the present socio-economic situation, outsourcing can play a vital role in the inventory system to supply daily customer requirements. In this model, the considerable production rate is considered as variable as opposed to constant to fulfill customer demand. In this model, demand is based on SPDD and green investment. Green investment is a new concept in inventory wherein the system is more profitable. Reworking for defective items with partial outsourcing leads to a more profitable model. When the production cycle ends, the reworking of faulty goods begins. A fixed π ($0 < \pi < 1$) portion is outsourced based on the optimal lot size quantity. The considerable outsourcing amount is assumed to be of perfect quality by the contractor. If $\pi = 1$, then this model considers the purchase system, and if $\pi = 0$, then an in-house production system is considered. A random portion x of faulty goods is produced during production rate d. Greening investments can move the inventory system through environmental benefits. From Figures 1 and 2, the following formulae can be directly obtained:

The level of perfect-quality on-hand inventory after the completion of in-house production is obtained by subtracting the defective products and demand of the products from the production rate using the following equations:

$$S_1 = (P - d - D)u_{1\pi} \tag{4.1}$$

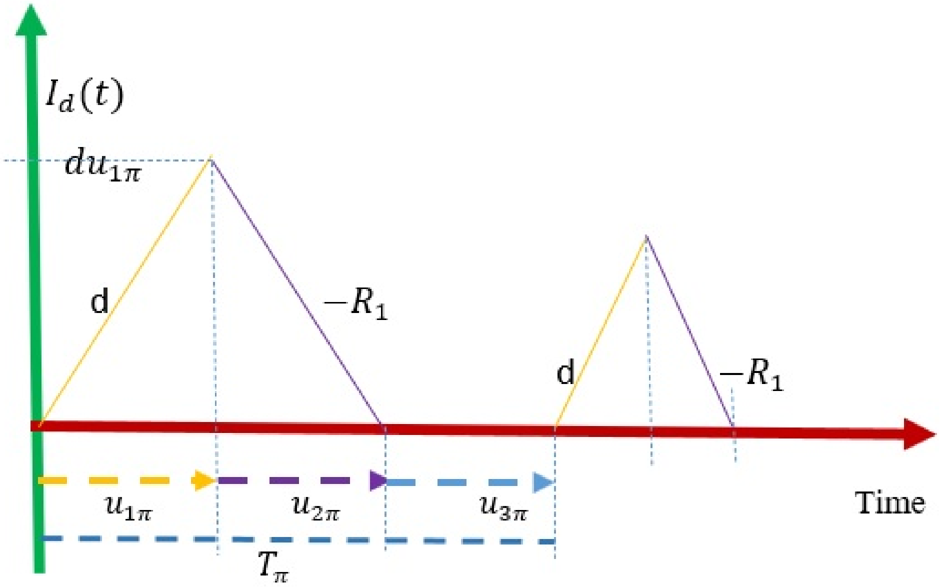

Figure 2. Level of on-hand defective item in the production and reworking times of the proposed system.

The level of the perfect-quality on-hand inventory when the reworking process ends is obtained by the sum of the perfect quality items on hand and the remaining remanufacturing items that cover the market demand in parallel, using the following formula:

$$S_2 = S_1 + (R_1 - D)u_{2\pi} \tag{4.2}$$

The maximum level of perfect-quality on-hand inventory when the outsourcing items are received is obtained by the sum of the after-rework process level of the per quality items with the outsourcing products, using the following formula:

$$H = S_2 + \pi Q = Du_{3\pi} \tag{4.3}$$

The following time indicates the production uptime, reworking time, and the production downtime when the outsourcing continues to fulfil the customer demand. In addition to the relationship with the perfect quality inventory, the reworking item inventory is presented here. Thus, the required formula is as follows:

$$u_{1\pi} = \frac{S_1}{(P-d-D)} = \frac{(1-\pi)Q}{P} \tag{4.4}$$

$$u_{2\pi} = \frac{x[(1-\pi)Q]}{R_1} \tag{4.5}$$

$$u_{3\pi} = \frac{H}{D} = \frac{S_2 + \pi Q}{D} \tag{4.6}$$

The cycle time is the sum of the perfect-quality item production time, which is known as the production uptime, reworking of defective products, and production downtime time. In general, the cycle time is calculated by dividing the number of lot sizes by market demand. Hence, the cycle time and repairable defective goods formula are considered as follows:

$$T = u_{1\pi} + u_{2\pi} + u_{3\pi} = \frac{Q}{D} \tag{4.7}$$

and

$$du_{1\pi} = xPu_{1\pi} = x[(1-\pi)Q] \tag{4.8}$$

Production setup cost (PSC)

By investing in the production setup, equipment can be prepared to process different batches of goods. By investing time, the output over the entire cycle time and next time can be obtained. This cost includes a fixed cost involved in the associated batch such that the cost is spread over the number of units that are produced. Some examples of production setup costs include the scrap cost of test units that are run on the machine and cost of the labor to configure the machine. In this model, the production setup cost is as follows:

$$PSC = K + K^e \tag{4.9}$$

Variable production cost (VPC)

Variable production cost is the type of expense that varies in proportion to the output of production. Variable production costs increase or decrease based on the output volume of the production system. Some examples of variable production costs include the costs of raw materials and packaging. In this study, the variable production costs are as follows:

$$VPC = (\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(1-\pi)Q \tag{4.10}$$

Fixed outsourcing cost (FOC)

The cost that is fixed and associated to outsource any product or service is termed as the fixed outsourcing cost. Some examples of fixed outsourcing cost include the cost of management and coordination of suppliers and cost of an outsourcing strategy. Then, the fixed outsourcing cost is as follows:

$$FOC = A_\pi \tag{4.11}$$

Variable outsourcing cost (VOC)

The cost that varies from time to tie and is associated with outsourcing any product or service is termed as variable outsourcing cost. Some examples of variable outsourcing costs include the cost of unplanned logistics activities and premium freight, cost of poor or substandard quality, cost of warranty, returns, and allowances. The variable outsourcing cost is as follows:

$$VOC = N_\pi(\pi Q) \tag{4.12}$$

Reworking cost (RC)

During the production process of a production system, imperfect products can be produced. There are two different approaches to mitigate this problem, namely by investing in development costs for improving the production process and by going through a reworking process. Defective products can be classified as reworkable and those that are not reworkable. The rework cost (RC) can be expressed as follows:

$$RC = (M_R + M_R^e)x[(1-\pi)Q] \tag{4.13}$$

Holding cost of reworked items (HCR)

Imperfect items underwent a rework process to ensure a perfect product. These reworked items were then ready for use. In this model, the holding cost for the reworked items is as follows:

$$HCR = (h_r + h_r^e)\frac{du_{1\pi}}{2}(u_{2\pi}) \tag{4.14}$$

Holding cost for perfect and defective items (HCPD)

Every production system produces a perfect product as well as defective products. All defective products underwent a reworking process to ensure a perfect product. In this model, this type of cost is represented as follows:

$$HCPD = (h + h^e)\left[\frac{S_1 + du_{1\pi}}{2}(u_{1\pi}) + \frac{S_1 + S_2}{2}(u_{2\pi}) + \frac{H}{2}(u_{3\pi})\right] \tag{4.15}$$

Investment in green products (IGP)

Management of every production system is concerned with recycling the waste produced by the production system to ensure a greener Earth. In this model, the investment in green products is expressed as follows:

$$IGP = \frac{\sigma_3 g_c^2}{2} \tag{4.16}$$

The total operating cost for this system, $TC(P, Q, p, g_c)$, includes the aforementioned costs in different situations in $u_{1\pi}, u_{2\pi}$, and $u_{3\pi}$ under a green environment. Hence, $TC(P, Q, p, g_c)$ is as follows:

Total cost (TC)

$$\begin{aligned} TC(Q,p,g_c) &= (PSC + VPC + FOC + VOC + RC + HCR + HCPD + IGP) \\ &= (K + K^e) + (\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(1-\pi)Q + A_\pi + N_\pi(\pi Q) \\ &+ (M_R + M_R^e)x[(1-\pi)Q] \\ &+ (h_r + h_r^e)\frac{du_{1\pi}}{2}(u_{2\pi}) + (h + h^e)\left[\frac{S_1 + du_{1\pi}}{2}(u_{1\pi}) + \frac{S_1 + S_2}{2}(u_{2\pi}) + \frac{H}{2}(u_{3\pi})\right] + \frac{\sigma_3 g_c^2}{2} \end{aligned}$$

Using the expressions of A_π and N_π in the aforementioned equation, the total cost $TC(P, Q, p, g_c)$ is as follows:

$$\begin{aligned}
TC(Q, p, g_c) &= (K + K^e) + (\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(1 - \pi)Q + (K + K^e)(1 + \alpha_1) \\
&+ (1 + \alpha_2)(\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(\pi Q) + (M_R + M_R^e)x[(1 - \pi)Q] \\
&+ (h_r + h_r^e)\frac{du_{1\pi}}{2}(u_{2\pi}) \\
&+ (h + h^e)\left[\frac{S_1 + du_{1\pi}}{2}(u_{1\pi}) + \frac{S_1 + S_2}{2}(u_{2\pi}) + \frac{H}{2}(u_{3\pi})\right] + \frac{\sigma_3 g_c^2}{2} \qquad (4.17)
\end{aligned}$$

The expected total cost per unit time $E[TCU(P, Q, p, g_c)]$ becomes

$$\begin{aligned}
E[TCU(Q, p, g_c)] &= \frac{1}{Q}\left(\sigma_1 \frac{(s_{max} - p)}{(p - s_{min})} + \sigma_2 g_c\right)\bigg[(K + K^e) + Q(\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(1 - \pi) \\
&+ (K + K^e)(1 + \alpha_1) + Q\pi(1 + \alpha_2)(\tau_1 + \frac{\tau_2}{P} + \tau_3 P) \\
&+ Q(1 - \pi)\zeta(M_R + M_R^e) \\
&+ \frac{Q^2((h_r + h_r^e) - (h + h^e))}{2}\left(\frac{\zeta^2(1 - \pi)^2}{R_1}\right) \\
&+ \frac{(h + h^e)Q^2}{2}\left(\frac{1}{D} - (\frac{1 - \pi^2}{P}) + \frac{\zeta(1 - \pi)}{R_1}(-2\pi)\right) + \frac{\sigma_3 g_c^2}{2}\bigg] \qquad (4.18)
\end{aligned}$$

where $E[x] = \zeta$,

Total expected profit (TEP)

The revenue is calculated as follows: Revenue= pD. Thus, the total expected profit is as follows:

$$\begin{aligned}
TEP(Q, p, g_c) &= p\left(\sigma_1 \frac{(s_{max} - p)}{(p - s_{min})} + \sigma_2 g_c\right) - \frac{1}{Q}\left(\sigma_1 \frac{(s_{max} - p)}{(p - s_{min})} + \sigma_2 g_c\right)\bigg[(K + K^e) + Q(\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(1 - \pi) \\
&+ (K + K^e)(1 + \alpha_1) + Q\pi(1 + \alpha_2)(\tau_1 + \frac{\tau_2}{P} + \tau_3 P) + Q(1 - \pi)\zeta(M_R + M_R^e) \\
&+ \frac{Q^2((h_r + h_r^e) - (h + h^e))}{2}\left(\frac{\zeta^2(1 - \pi)^2}{R_1}\right) \\
&+ \frac{(h + h^e)Q^2}{2}\left(\frac{1}{D} - (\frac{1 - \pi^2}{P}) + \frac{\zeta(1 - \pi)}{R_1}(-2\pi)\right) + \frac{\sigma_3 g_c^2}{2}\bigg] \\
&= p\left(\sigma_1 \frac{(s_{max} - p)}{(p - s_{min})} + \sigma_2 g_c\right) - \frac{1}{Q}\left(\sigma_1 \frac{(s_{max} - p)}{(p - s_{min})} + \sigma_2 g_c\right)\bigg[(K + K^e) \\
&+ Q(\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(1 - \pi) + (K + K^e)(1 + \alpha_1) \\
&+ Q\pi(1 + \alpha_2)(\tau_1 + \frac{\tau_2}{P} + \tau_3 P) + Q(1 - \pi)\zeta(M_R + M_R^e) \\
&+ \frac{Q^2((h_r + h_r^e) - (h + h^e))}{2}\left(\frac{\zeta^2(1 - \pi)^2}{R_1}\right)
\end{aligned}$$

$$+ \quad \frac{(h+h^e)Q^2}{2}\left(\frac{1}{D} - (\frac{1-\pi^2}{P}) + \frac{\zeta(1-\pi)}{R_1}(-2\pi)\right) + \frac{\sigma_3 g_c^2}{2}\Bigg]$$

5. Solution methodology

To solve the mathematical model, the classical optimization method is considered analytically. The decision variables Q, p, and g_c are optimized using a continuous optimization technique. Given that there are multiple decision variables, the Hessian matrix is used to test the globality of the solution. Thus, the decision variables can be used to obtain the optimum results Q^*, p^*, and g_c^* such that the optimal values of the decision variables are as follows:

$$Q^* = \frac{\Psi - \left(\sigma_1 \frac{(s_{max}-p)}{(p-s_{min})} + \sigma_2 g_c\right)\left[(\tau_1 + \frac{\tau_2}{P} + \tau_3 P)(1-\pi) + \pi(1+\alpha_2)(\tau_1 + \frac{\tau_2}{P} + \tau_3 P) + (1-\pi)\zeta M_R\right]}{\left(\sigma_1 \frac{(s_{max}-p)}{(p-s_{min})} + \sigma_2 g_c\right)\left[Q((h_r + h_r^e) - (h+h^e))\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right) + (h+h^e)Q\Theta_1\right]} \tag{5.1}$$

$$p^* = \frac{\sqrt{\Upsilon^2 - 4\zeta p^2(\sigma_1 + \sigma_2 g_c)\left[1 - \frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]} - \Upsilon}{2p^2(\sigma_1 + \sigma_2 g_c)\left[1 - \frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]} \tag{5.2}$$

$$g_c^* = \frac{\sqrt{\left[\sigma_1 \frac{(s_{max}-p)}{(p-s_{min})}\Theta_2\right]^2 - 8\sigma_2 g_c \Theta_2 (p-\Psi)\sigma_2} - \sigma_1 \frac{(s_{max}-p)}{(p-s_{min})}\Theta_2}{2\sigma_2 g_c \Theta_2} \tag{5.3}$$

Please refer to Appendix A for the calculations of first-order derivatives.

Proposition. The total expected profit function is convex at Q^*, p^*, g_c^* if
$\chi < 0$
$\chi\varphi > \vartheta^2$
$\chi(\varphi\tau - \Omega^2) + \theta(\vartheta\Omega - \varphi\vartheta) < \vartheta(\vartheta\tau - \Omega\theta)$

Proof. see Appendix B

6. Numerical examples

In this sections, some numerical examples are presented to validate the mathematical model numerically.

6.1. Example 1

The mathematical model is numerically tested to validate the theoretical solution. The following input parameter values are considered to illustrate the numerical example. In this example, K = 4950 (\$/setup); K^e = 50 (\$/setup); τ_1 = 320; τ_2 = 11910; τ_3 = 0.009; M_R = 47 (\$/unit); M_R^e = 3 (\$/unit);

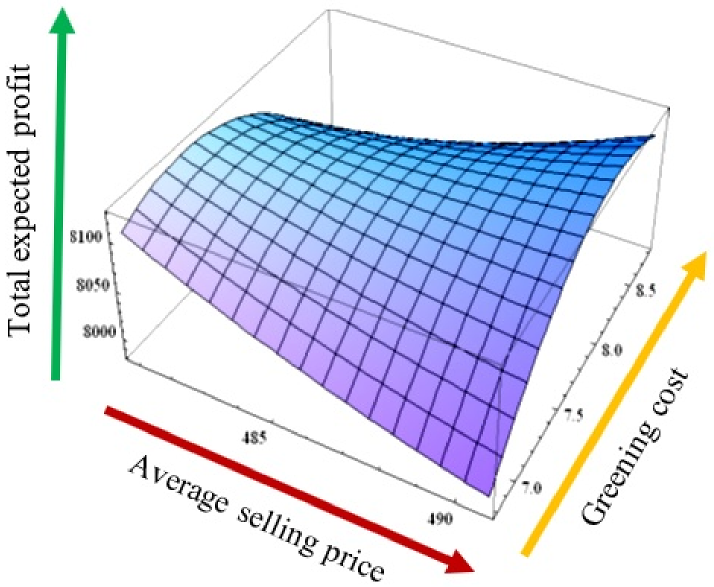

Figure 3. Total expected profit versus average selling price and greening cost.

α_1 = -0.3; α_2 = 0.3; s_{max} = 900 (\$/unit); s_{min} = 400 (\$/unit); σ_1 = 20; σ_2 = 6; σ_3 = 500; h_r = 23.01 (\$/unit/unit time);h_r^e = 2 (\$/unit/unit time); h = 14.9(\$/unit/unit time); h^e = 1 (\$/unit/unit time); E[x] = 0.2; π = 0.05; R_1 = 110 (units/year); and P = 315 (unit/year).

The optimal result of the decision variable is as follows: Q^* = 883.05 (units); p^* = 485.79 (\$/unit); g_c^* = 7.80 (\$/year); and at this optimal value, the total expected profit (TEP) is = 8105.92 (\$/year).

The optimality of the results is checked analytically as well as numerically. Here, H_{11} = -0.00987121 < 0; H_{22} = 0.0118433 > 0; and H_{33} = -0.424059 < 0. Figures 3 and 4 provide graphical representation. The concave 3D figures graphically support the optimality results of the total expected profit.

6.2. Example 2

The mathematical model is numerically tested to validate the theoretical solution. The following input parameter values are considered to illustrate numerical example.

Here K = 480 (\$/setup); K^e = 20 (\$/setup); τ_1 = 320 ; τ_2 = 900 ; τ_3 = 0.02 ; M_R = 95 (\$/unit); M_R^e = 5 (\$/unit); α_1 = -0.3; α_2 = 0.3; s_{max} = 900 (\$/unit); s_{min} = 400 (\$/unit); σ_1 = 10; σ_2 = 3; σ_3 = 300; h_r = 9 (\$/unit/unit time); h_r^e = 1 (\$/unit/unit time); h = 0.09 (\$/unit/unit time); h^e = 0.01 (\$/unit/unit time); E[x] = 0.62; π = 0.05; R_1 = 50 (units/year); P = 201 (unit/year).

The optimal result of the decision variable is as follows: Q^* = 192.85 (units); p^* = 461.11 (\$/unit); g_c^* = 1.45 (\$/year); and at these optimal values, the total expected profit (TEP) is = 4320.37 (\$/year). The optimality of the results is checked analytically and numerically.

Here, H_{11} = -0.0246569 < 0; H_{22} = 0.00458158 > 0; and H_{33} = -0.420143 < 0.

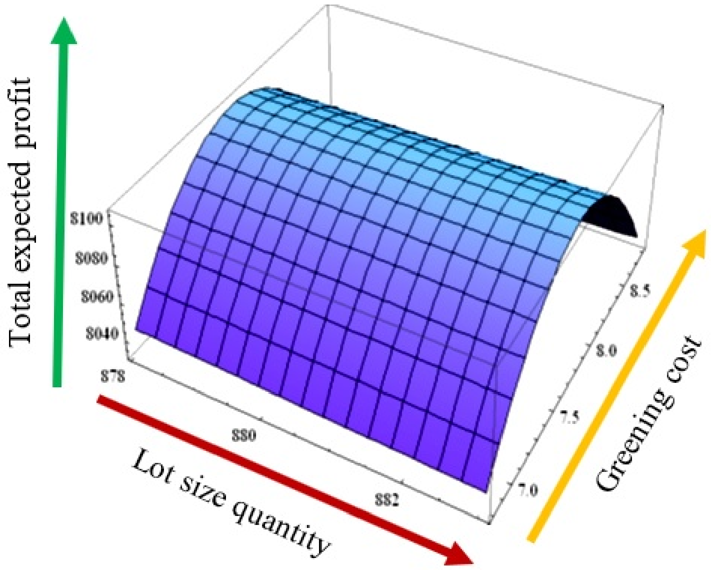

Figure 4. Total expected profit versus production lot size quantity and greening cost.

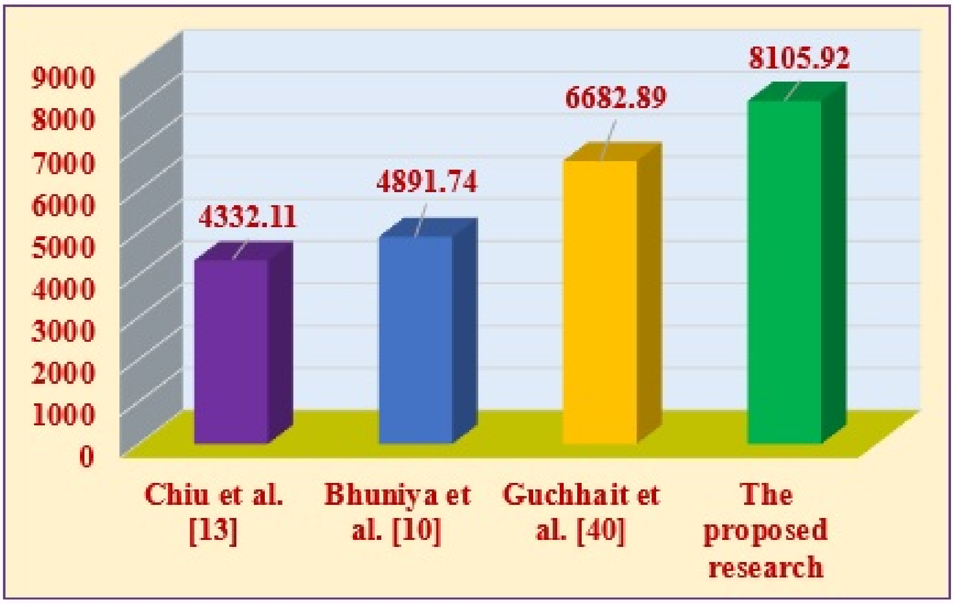

Figure 5. Comparison among the total expected profit of Example 1 and other studies in the literature review.

6.3. Discussions

From the above numerical experiment and their comparison among the previous research articles, it can be concluded that the TEP is maximum for the originally proposed model. All profit amount are numerically expressed using Mathematica 11.3.0 software. Figure 5 shows the comparison among the total expected profit of the Example 1 of the proposed research, Chiu et al. [13], Bhuniya et al. [10], and Guchhait et al. [40]. In the research article, Chiu et al. [13], there are only partial outsourcing with defective production, and reworking. But having constant demand and no eco-friendly approach. There research concept gives profit \$ 4332.11 per cycle. In addition of the previous stated research, variable demand concept of Bhuniya et al. [10] gives the profit \$ 4891.74 per cycle, and Guchhait et al. [40] gives the profit \$ 6682.89 per cycle. In comparison to these previous research the proposed model Example 1 gives the profit \$ 8105.92 per cycle.

For the scientific community, the proposed research gives more profit rather then the other previous research due to presence of the concept of variable demand with green investment, controllable production process, and partial outsourcing. However, energy cost in different section of the production process are considered here, which are very much helpful idea for the production management for analyzing their data analysis and maximum profit. The green investment with partial outsourcing makes the model more sustainable. Furthermore, selling price and greening cost dependent demand help to control the fluctuate market and for smooth running production process. Hence, the comparison among the previous research help in the validation of the original research idea.

7. Sensitivity analysis

Significant observations of cost parameters are numerically calculated, and the changes in this parameter effect are presented in Table 2 and Figure 6.

Table 2 shows the effects of cost parameters on total expected profit due to change such as (-50%, -25%, +25%, +50%). Based on the following sensitivity table, the following conclusions can be stated.

1. The most sensitive parameter is the cost of holding product. It has a significant effect on the expected profit. Decreasing the value of this parameter increases the TEP, and increasing its value has decreases TEP. However, holding cost maintains the quality and quantity of the products. Moreover, for the partial outsourcing it plays an important role.
2. The production setup cost of the producer has a significant impact on the TEP. Owing to small changes in the setup cost, TEP changes significantly. The TEP decreases when the setup cost increases and increases when the setup cost decreases. However, setup cost is the elementary cost of production started.
3. The sensitivity table clearly shows that the other cost such as reworking cost strongly affects the total expected profit. The decrease in value of cost function, increases the value of TEP. For maximizing the TEP, reworking of defective products can plays an important role in the sustainable production system.
4. Holding cost of each reworked good has a lower effect than the other cost parameters. An increase in value decreases the total expected profit, and a decrease in value increases the total expected profit.

Table 2. Sensitivity analysis table.

Parameters	change(%)	TEP (%)	Parameters	change (%)	TEP (%)
K	-50	+09.37	K^e	-50	+00.09
	-25	+04.43		-25	+00.04
	+25	-04.01		+25	-00.04
	+50	-07.63		+50	-00.09
M_R	-50	+08.15	M_R^e	-50	+00.51
	-25	+04.00		-25	+00.17
	+25	-03.91		+25	-00.25
	+50	-07.74		+50	-00.50
h	-50	+13.26	h^e	-50	+01.31
	-25	+11.09		-25	+00.65
	+25	-09.71		+25	-00.64
	+50	-07.54		+50	-01.28
h_r	-50	+02.91	h_r^e	-50	+00.26
	-25	+01.52		-25	+00.13
	+25	-01.38		+25	-00.13
	+50	-02.63		+50	-00.25

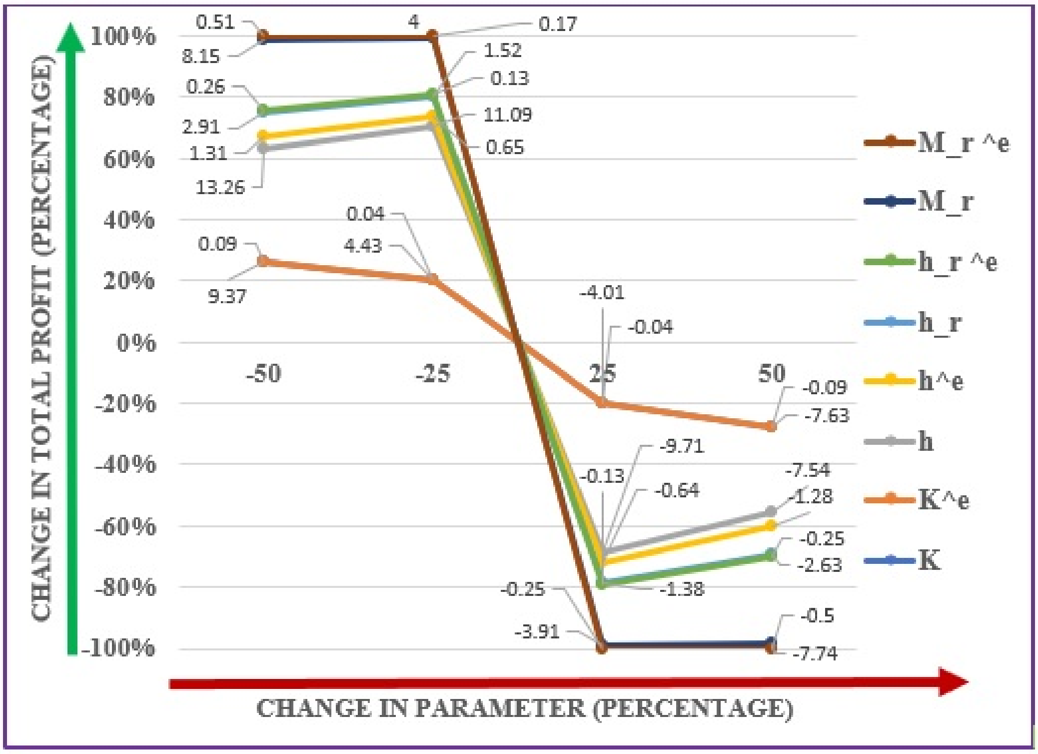

Figure 6. Percentage change in total profit vs change in percentage values of parameter.

5. Energy cost for holding reworked products, energy cost for reworking the products, holding cost of each reworked good sometimes has no effect or little effects on TEP.

8. Managerial insights

The following are the main recommendations for improving the overall profit, revenue, and goodwill of the industry:

1. The managements of large industries can effectively handle customers by using a constant production rate as opposed to a variable production rate. Furthermore, the research and development section of the company can increase market demand by considering the SPDD, introducing a trade credit period, advertisement, promotional, discount, and greening cost-dependent demand.
2. The management of a company can invest more capital in a smart production system, which can produce more smart products or outsource more products by using smart technology to control market demand as well as customer satisfaction in more effectively.
3. Management should concentrate on the reworking of defective products that are repairable and should carefully check the quality of the outsourced products. No defective products should be outsourced because it can damage the reputation of the companys production system.
4. The management should ensure that only repairable items undergo rework processes; otherwise, the production system can take more time to produce the scheduled items.
5. Management should invest more capital in greening costs to ensure that the company can fulfill corporate responsibility more effectively. This in turn will equally benefit the environment, economy, and society.

9. Conclusions

Currently, the market demand for any smart product is highly fluctuating and is dependent on many influential factors. Any changes in these factors can impact the revenue and profits of the modern industry. In the present study, the selling price and greening cost-dependent demand for smart products were in a sustainable production system. The total expected profit (TEP) for various cases were numerically and analytically optimized using decision variables. The numerical tool, Mathematica 11.3.0, was used to determine the numerical results and maximum TEP and to prove the global optimality. It was already demonstrated that smart products manufactured in a sustainable production system can easily provide a significant profit with the facility of global outsourcing. Additionally, given the variable customer demand, smart products could cover the entire competitive market with replacement, warranty, buyback, and reworking facilities.

For future extension of this model, there can be a demand that is dependent on stock, discount, advertisement, promotion, and trade credit. The cost effective subsidy policy, bio-fuels and animal fat-based biodiesel may be considered for future extension [41, 42]. In the future, this model can be expanded by considering measure of influences, associated network, centrality, power and relationship [43]. Further this model can be expanded to consider different maintenance policies, such as smart inspection systems as opposed to human inspection, preservative technology, and radio frequency identification [44], service level constraints and strategies under uncertainty [45].

Fuzzy random environment, interactive fuzzy programming approach [46], Tayyab and Sarkar [47] may be considered for future extension for this model. Given the current situation with respect to the pandemic, manufacturers are uncertain about the resumption of normal transportation facilities. Alternatively, the production of green products can be considered as it is closely related to sustainable outsourcing. In the present COVID-19 situation, the global business procedure easily fulfills and satisfies customer demand via online or online shopping systems or e-supply chain management. Another direction for development can involve incorporating the inspection costs and errors during the inspection and back ordering costs.

Appendix A

$TEP(.) = TEP(Q, p, g_c)$. The following expression indicates the second-order differentiation of TEP with respect to decision variables.

$$\begin{aligned}
\frac{\partial TEP(.)}{\partial Q} &= \frac{1}{Q^2}\Big(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\Big)\Big[(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)\\
&+ (K+K^e)(1+\alpha_1)\\
&+ Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e)\\
&+ \frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\Big(\frac{\zeta^2(1-\pi)^2}{R_1}\Big)+\frac{(h+h^e)Q^2}{2}\Theta_1+\frac{\sigma_3 g_c^2}{2}\Big]\\
&- \frac{1}{Q}\Big(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\Big)\Big[(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)\\
&+ (1-\pi)\zeta M_R+Q((h_r+h_r^e)-(h+h^e))\Big(\frac{\zeta^2(1-\pi)^2}{R_1}\Big)+(h+h^e)Q\Theta_1\Big]
\end{aligned}$$

Now equating zero, one can obtain

$$\begin{aligned}
0 &= \frac{1}{Q^2}\Big(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\Big)\Big[(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)\\
&+ (K+K^e)(1+\alpha_1)\\
&+ Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e)\\
&+ \frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\Big(\frac{\zeta^2(1-\pi)^2}{R_1}\Big)+\frac{(h+h^e)Q^2}{2}\Theta_1+\frac{\sigma_3 g_c^2}{2}\Big]\\
&- \frac{1}{Q}\Big(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\Big)\Big[(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)\\
&+ (1-\pi)\zeta M_R+Q((h_r+h_r^e)-(h+h^e))\Big(\frac{\zeta^2(1-\pi)^2}{R_1}\Big)+(h+h^e)Q\Theta_1\Big]
\end{aligned}$$

After simplifying the above equation, one can find

$$Q=\frac{\Psi-\Big(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\Big)\Big[(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+(1-\pi)\zeta M_R\Big]}{\Big(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\Big)\Big[Q((h_r+h_r^e)-(h+h^e))\Big(\frac{\zeta^2(1-\pi)^2}{R_1}\Big)+(h+h^e)Q\Theta_1\Big]}$$

where

$$\begin{aligned}
\Psi &= \frac{1}{Q}\left(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\right)\Bigg[(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+(K+K^e)(1+\alpha_1)\\
&+ Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e)\\
&+ \frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right)\\
&+ \frac{(h+h^e)Q^2}{2}\left(\frac{1}{D}-(\frac{1-\pi^2}{P})+\frac{\zeta(1-\pi)}{R_1}(-2\pi)\right)+\frac{\sigma_3 g_c^2}{2}\Bigg]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial TEP(.)}{\partial p} &= \sigma_1\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\Bigg[p-\frac{1}{Q}\Bigg\{(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)\\
&+ (K+K^e)(1+\alpha_1)\\
&+ Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e)\\
&+ \frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right)+\frac{(h+h^e)Q^2}{2}\Theta_1+\frac{\sigma_3 g_c^2}{2}\Bigg\}\Bigg]\\
&+ \left(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\right)\left[1-\frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]
\end{aligned}$$

Now equating zero, it is obtained

$$\begin{aligned}
0 &= \sigma_1\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\Bigg[p-\frac{1}{Q}\Bigg\{(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)\\
&+ (K+K^e)(1+\alpha_1)\\
&+ Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e)\\
&+ \frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right)+\frac{(h+h^e)Q^2}{2}\Theta_1+\frac{\sigma_3 g_c^2}{2}\Bigg\}\Bigg]\\
&+ \left(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\right)\left[1-\frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]
\end{aligned}$$

After simplifying the above equation, one can obtain

$$p=\frac{\sqrt{\Upsilon^2-4\zeta p^2(\sigma_1+\sigma_2 g_c)\left[1-\frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]}-\Upsilon}{2p^2(\sigma_1+\sigma_2 g_c)\left[1-\frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]}$$

where

$$\begin{aligned}
\Upsilon &= p\sigma_1(s_{min}-s_{max})+p\Big\{\sigma_1(s_{max}+s_{min})-2\sigma_2 g_c s_{min}\Big\}\left[1-\frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]\\
\zeta &= \sigma_1(s_{min}-s_{max})\Psi+s_{min}(\sigma_1 s_{max}+2g_c\sigma_2 s_{min})\left[1-\frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial TEP(.)}{\partial g_c} &= \frac{1}{Q}\left(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\right)\left(\frac{(h+h^e)\sigma_2 Q^2}{2D^2}-\sigma_3 g_c\right)+\sigma_2\left[p-\frac{1}{Q}\left\{(K+K^e)\right.\right.\\
&+ Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+(K+K^e)(1+\alpha_1)+Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)\\
&+ Q(1-\pi)\zeta(M_R+M_R^e)+\frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right)\\
&+ \left.\left.\frac{(h+h^e)Q^2}{2}\left(\frac{1}{D}-(\frac{1-\pi^2}{P})+\frac{\zeta(1-\pi)}{R_1}(-2\pi)\right)+\frac{\sigma_3 g_c^2}{2}\right\}\right]
\end{aligned}$$

Now equating zero, it is found

$$\begin{aligned}
0 &= \frac{1}{Q}\left(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\right)\left(\frac{(h+h^e)\sigma_2 Q^2}{2D^2}-\sigma_3 g_c\right)+\sigma_2\left[p-\frac{1}{Q}\left\{(K+K^e)\right.\right.\\
&+ Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+(K+K^e)(1+\alpha_1)+Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)\\
&+ Q(1-\pi)\zeta(M_R+M_R^e)+\frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right)\\
&+ \left.\left.\frac{(h+h^e)Q^2}{2}\left(\frac{1}{D}-(\frac{1-\pi^2}{P})+\frac{\zeta(1-\pi)}{R_1}(-2\pi)\right)+\frac{\sigma_3 g_c^2}{2}\right\}\right]
\end{aligned}$$

Now simplifying the above equation, it can be obtained

$$g_c=\frac{\sqrt{\left[\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}\Theta_2\right]^2-8\sigma_2 g_c\Theta_2(p-\Psi)\sigma_2}-\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}\Theta_2}{2\sigma_2 g_c\Theta_2}$$

where $\Theta_1=\left(\frac{1}{D}-(\frac{1-\pi^2}{P})+\frac{\zeta(1-\pi)}{R_1}(-2\pi)\right)$

$\Theta_2=\frac{1}{Q}\left(\frac{(h+h^e)\sigma_2 Q^2}{2D^2}-\sigma_3 g_c\right)$

Appendix B

The following expression indicates the second-order differentiation of TEP with respect to decision variables.

$$\begin{aligned}
\frac{\partial^2 TEP(.)}{\partial Q^2} &= -\frac{2}{Q^3}\left(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\right)\left[(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)\right.\\
&+ (K+K^e)(1+\alpha_1)+Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e)\\
&+ \left.\frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right)+\frac{(h+h^e)Q^2}{2}\Theta_1+\frac{\sigma_3 g_c^2}{2}\right]\\
&+ \frac{2\left(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\right)}{Q^2}\left[(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)\right.
\end{aligned}$$

$$
\begin{aligned}
&+ \quad (1-\pi)\zeta M_R + Q((h_r+h_r^e)-(h+h^e))\bigg(\frac{\zeta^2(1-\pi)^2}{R_1}\bigg) + (h+h^e)Q\Theta_1\bigg] \\
&- \quad \frac{\Big(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\Big)}{Q}\bigg[\frac{((h_r+h_r^e)-(h+h^e))\zeta^2(1-\pi)^2}{R_1} + (h+h^e)\Theta_1\bigg] \\
&= \quad \chi(say)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 TEP(.)}{\partial p^2} \quad &= \quad -2\sigma_1\frac{(s_{min}-s_{max})}{(p-s_{min})^3}\bigg[p-\frac{1}{Q}\bigg\{(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi) \\
&+ \quad (K+K^e)(1+\alpha_1)+Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e) \\
&+ \quad \frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\bigg(\frac{\zeta^2(1-\pi)^2}{R_1}\bigg)+\frac{(h+h^e)Q^2}{2}\Theta_1+\frac{\sigma_3 g_c^2}{2}\bigg\}\bigg] \\
&+ \quad 2\sigma_1\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\bigg[1-\frac{(h+h^e)\sigma_1 Q}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\bigg] \\
&+ \quad \bigg(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\bigg)\frac{2(h+h^e)\sigma_1 Q(s_{min}-s_{max})}{D^2(p-s_{min})^3} = \varphi(say)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 TEP(.)}{\partial g_c^2} \quad &= \quad -\frac{1}{Q}\bigg(\sigma_3+\frac{(h+h^e)Q^2\sigma_2^2}{D^3}\bigg)\bigg(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\bigg) \\
&- \quad \frac{\sigma_2}{Q}\bigg(\sigma_3 g_c-\frac{(h+h^e)\sigma_2 Q^2}{2D^2}\bigg) = \tau(say)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 TEP(.)}{\partial Q\partial p} \quad &= \quad \frac{\partial^2 TEP(.)}{\partial p\partial Q} = \sigma_1\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\frac{1}{Q^2}\bigg[(K+K^e)+Q(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi) \\
&+ \quad (K+K^e)(1+\alpha_1)+Q\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P)+Q(1-\pi)\zeta(M_R+M_R^e) \\
&+ \quad \frac{Q^2((h_r+h_r^e)-(h+h^e))}{2}\bigg(\frac{\zeta^2(1-\pi)^2}{R_1}\bigg)+\frac{(h+h^e)Q^2}{2}\Theta_1+\frac{\sigma_3\eta^2}{2}\bigg] \\
&- \quad \frac{\sigma_1}{Q}\frac{(s_{min}-s_{max})}{(p-s_{min})^2}\bigg[(\tau_1+\frac{\tau_2}{P}+\tau_3 P)(1-\pi)+\pi(1+\alpha_2)(\tau_1+\frac{\tau_2}{P}+\tau_3 P) \\
&+ \quad (1-\pi)\zeta(M_R+M_R^e)+\frac{((h_r+h_r^e)-(h+h^e))}{2}\bigg(\frac{\zeta^2(1-\pi)^2}{R_1}\bigg) \\
&+ \quad (h+h^e)Q\Theta_1+\frac{\sigma_3 g_c^2}{2}\bigg] \\
&- \quad \bigg(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\bigg)\frac{(h+h^e)\sigma_1}{D^2}\frac{(s_{min}-s_{max})}{(p-s_{min})^2} = \vartheta(say)
\end{aligned}
$$

$$
\frac{\partial^2 TEP(.)}{\partial Q\partial g_c} \quad = \quad \frac{\partial^2 TEP(.)}{\partial g_c\partial Q} = \bigg(\frac{\sigma_3 g_c}{Q^2}+\frac{(h+h^e)}{2D^2}\bigg)\bigg(\sigma_1\frac{(s_{max}-p)}{(p-s_{min})}+\sigma_2 g_c\bigg)+\frac{\sigma_2}{Q^2}\bigg\{(K+K^e)
$$

$$
\begin{aligned}
&+ \quad Q\Big(\tau_1 + \frac{\tau_2}{P} + \tau_3 P\Big)(1-\pi) + (K + K^e)(1+\alpha_1) + Q\pi(1+\alpha_2)\Big(\tau_1 + \frac{\tau_2}{P} + \tau_3 P\Big) \\
&+ \quad Q(1-\pi)\zeta(M_R + M_R^e) + \frac{Q^2((h_r + h_r^e) - (h + h^e))}{2}\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right) \\
&+ \quad \frac{(h + h^e)Q^2}{2}\Theta_1 \\
&+ \quad \frac{\sigma_3 g_c^2}{2}\Bigg\} - \frac{\sigma_2}{Q}\Bigg[\Big(\tau_1 + \frac{\tau_2}{P} + \tau_3 P\Big)(1-\pi) + \pi(1+\alpha_2)\Big(\tau_1 + \frac{\tau_2}{P} + \tau_3 P\Big) \\
&+ \quad (1-\pi)\zeta M_R + Q((h_r + h_r^e) - (h + h^e))\left(\frac{\zeta^2(1-\pi)^2}{R_1}\right) \\
&+ \quad (h + h^e)Q\Theta_1\Bigg] = \theta(say)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 TEP(.)}{\partial p \partial g_c} &= \frac{\partial^2 TEP(.)}{\partial g_c \partial p} = \frac{\sigma_1}{Q}\left(\frac{(h + h^e)\sigma_2 Q^2}{2D^2} - \sigma_3 g_c\right)\frac{(s_{min} - s_{max})}{(p - s_{min})^2} \\
&+ \sigma_2\left[1 - \frac{(h + h^e)\sigma_1 Q}{D^2}\frac{(s_{min} - s_{max})}{(p - s_{min})^2}\right] \\
&+ \left(\sigma_1\frac{(s_{max} - p)}{(p - s_{min})} + \sigma_2 g_c\right)\frac{2(h + h^e)\sigma_1\sigma_2 Q}{D^3}\frac{(s_{min} - s_{max})}{(p - s_{min})^2} = \Omega(say)
\end{aligned}
$$

$$
\mid H_{11} \mid = \left| \frac{\partial^2 TEP(.)}{\partial Q^2} \right| = \chi
$$

$$
\begin{aligned}
\mid H_{22} \mid = \begin{vmatrix} \frac{\partial^2 TEP(.)}{\partial Q^2} & \frac{\partial^2 TEP(.)}{\partial Q \partial p} \\ \frac{\partial^2 TEP(.)}{\partial p \partial Q} & \frac{\partial^2 TEP(.)}{\partial p^2} \end{vmatrix} &= \frac{\partial^2 TEP(.)}{\partial Q^2}\frac{\partial^2 TEP(.)}{\partial p^2} - \left(\frac{\partial^2 TEP(.)}{\partial Q \partial p}\right)^2 \\
&= \chi\varphi - \vartheta^2
\end{aligned}
$$

$$
\mid H_{33} \mid = \begin{vmatrix} \frac{\partial^2 TEP(.)}{\partial Q^2} & \frac{\partial^2 TEP(.)}{\partial Q \partial p} & \frac{\partial^2 TEP(.)}{\partial Q \partial g_c} \\ \frac{\partial^2 TEP(.)}{\partial p \partial Q} & \frac{\partial^2 TEP(.)}{\partial p^2} & \frac{\partial^2 TEP(.)}{\partial p \partial g_c} \\ \frac{\partial^2 TEP(.)}{\partial g_c \partial Q} & \frac{\partial^2 TEP(.)}{\partial g_c \partial p} & \frac{\partial^2 TEP(.)}{\partial g_c^2} \end{vmatrix} = \chi(\varphi\tau - \Omega^2) - \vartheta(\vartheta\tau - \Omega\theta) + \theta(\vartheta\Omega - \varphi\vartheta)
$$

Acknowledgments

The work is supported by the National Research Foundation of Korea (NRF) grant funded by the Korea Government(MSIT)(NRF2020R1F1A1064460).

Conflict of interest

The authors declare no conflict of interest.

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