AIMS Bioengineering

http://www.aimspress.com/journal/Bioengineering

AIMS Bioengineering, 7(4): 179–193.
DOI: 10.3934/bioeng.2020016
Received: 09 May 2020
Accepted: 04 June 2020
Published: 24 June 2020

Research article

Analysis of meningitis model: A case study of northern Nigeria

Isa Abdullahi Baba[1,*], **Lawal Ibrahim Olamilekan**[1], **Abdullahi Yusuf**[2,3] **and Dumitru Baleanu**[4,5,6]

[1] Department of Mathematical Sciences, Bayero University, Kano, Nigeria
[2] Department of Computer Engineering, Biruni University, Istanbul Turkey
[3] Department of Mathematical Sciences, Federal University Dutse, Nigeria
[4] Department of Mathematics, Cankaya University, Ankara, 06530, Turkey
[5] Institute of Space Sciences, Bucharest, Romania
[6] Department of Medical Research, China Medical University Hospital, Taichung, Taiwan

* **Correspondence:** Email: iababa.mth@buk.edu.ng; Tel: +2348133624929.

Abstract: A new strain of meningitis emerges in northern Nigeria, which brought a lot of confusion. This is because vaccine and treatment for the old strain was adopted but to no avail. It was later discovered that it was a new strain that emerged. In this paper we consider the two strains of meningitis (I 1 and I 2). Our aim is to analyse the effect of one strain on the dynamics of the other strain mathematically. Equilibrium solutions were obtained and their global stability was analysed using Lyaponuv function. It was shown that the stability depends on magnitude of the basic reproduction ratio. The coexistence of the two strains was numerically shown.

Keywords: basic reproduction ratio; equilibrium solution; global stability; meningitis; two strain

1. Introduction

The inflammation of meninges that surround membranes of the spinal cord and brain is called meningitis [1]. It is a bacterial and protozoa caused disease. It infects both children, young and older adult. Meningitis is popularly known to be a disease which spreads quickly in an isolated geographical settlement like students hostel, military quarters, school and prison yard [2]. There are pathogenic micro-organisms that are responsible for the spread of meningitis among individual in a society. These include listeria monocytogenes,streptococcus pneumonia, Group B streptococcus, Neisseria meningitides and Haemophilias, it is a transmissible disease [3]. This disease infects individuals based on their age group. Some of the pathogen are found in new born babies, they

include streptococcus pneumonia, Group streptococcus, listeria monocytogenes and Escherichia while streptococcus pneumonia, Neisseria meningitides, influenza type B and Group B streptococcus are found in children. Meningitis infects teeth in adult, the pathogen responsible for this infection include, streptococcus and Neisseria meningitides [3]. Meningitis is a deadly disease. It kills if the symptoms is not identified early enough. No amount of treatment and control can prevent the death by meningitis if it is discovered at late time. The major symptoms of the disease are headache, vomiting and sensitivity to light [3].

SIR model was used as a basis tool for modelling meningitis,this include incorporating seasonality [4,5] as spatial temporal model [6] to show how it spreads among individuals.

Mathematical model helps in studying meningitis virus and bacteria through past meningitis epidemic. Some model were used to study the spread and control of infectious disease Martinez et al. [2] used discrete mathematical model to study spread and control meningococcal meningitis, they considered a model with five compartments viz; susceptible, asymptotic infected, infected with symptoms, carries, recovered and dead class. Broutin et al. [7] used some mathematical tool to conduct time series analysis and wavelength method to study meningococcal meningitis in nine (9) Africa countries, according to their result, it was stated that international co-operation in public health and cross discipline studies are highly recommended to help in controlling this infectious disease. Miller and Shahab [8] recommended effectiveness of immunization with respect to cost constrain to control epidemic meningococcal meningitis. Irving et al. [9] adopt deterministic compartment model to investigate the effectiveness of simple structure model in controlling epidemic of meningococcal meningitis.

Therefore mathematical modelling plays a vital role in investigating the spread and control of meningitis disease, it makes it easy to identify what an individual should avoid in order to be free from infection of the disease.

Zamfara is a state in north-western Nigeria and has a population of about 4.1 million of which about 800,000 of children are under five [10]. It was at the centre of the largest meningitis outbreak in 21st century with 7,140 suspect meningitis cases and 553 death reported between December 2016 to 2017 [10]. The over attack was 155 per 100,000 population and children 5–14 years accounted for 47.

Most of the previous researches show that there are interactions among the multiple strains of disease such as tuberculosis, dengue fever, meningitis, HIV, influenza, malaria fever and other sexually transmitted related disease [11–15]. And it shows that any strain with largest basic reproduction ratio eliminates other strains. It is also investigated and showed the coexistence of multiple strain using exponential growth, co-infection, super-infection method and application of various methods to control coexistence of the strain [15]. Since new strain are still evolving, there is need for more studies on the coexistence of multiple strains.

Unlike other diseases as mentioned above, most meningitis models in literature only considered a single strain, hence there is need to study multiple strain of the disease and understand its qualitative properties. Here we are motivated by what happened in Zamfara State, Northern part of Nigeria in 2018 [10]. A new strain of meningitis surfaced and government and medical practitioners thought it was the old strain. So, the vaccine and treatment of the old strain were given for the new strain. This leads to the death of many people. Our main objective is to investigate this phenomena mathematically. This paper consists of five (5) sections and is arranged as follows: section 1 is the introduction, section 2 is the formation of model, section 3 is the study of existence of equilibrium and computation of

reproduction ratio, section 4 is the stability analysis of equilibrium while section 5 is the discussion of result and numerical simulation.

2. Formulation of the model

This model of meningitis consists of system of six differential equations. The compartments are $S(t)$, $C_1(t)$, $C_2(t)$, $I_1(t)$, $I_2(t)$, $R(t)$ which represent the population of susceptible, carrier of infection with respect to strain 1, carrier of infection with respect to strain 2, ill individual with respect to strain 1, ill individual with respect to strain 2 and recovered individual at time t, respectively.

Due to birth, immigration and other population growth factors, we assume constant recruitment in to susceptible population and there is no double infection. The variable as well as parameters as used in the model are all positive. Meaning of variables and parameters are given in Table 1 and Figure 1 gives the schematic diagram of the model. With the above assumptions, the model is given by the system of ODE as follows:

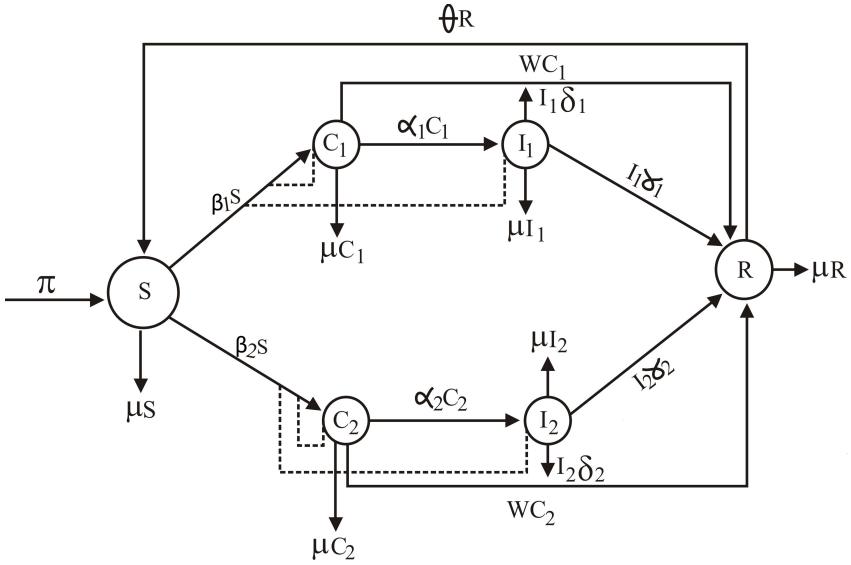

Figure 1. Schematic diagram of the model.

$$
\begin{aligned}
\frac{dS}{dt} &= \Pi + \theta R + \beta_1 S(C_1 + I_1) + \beta_2 S(C_2 + I_2) - \mu S,\\
\frac{dC_1}{dt} &= \beta_1 S(C_1 + I_1) - (\alpha_1 + \mu + \omega)C_1,\\
\frac{dI_1}{dt} &= \alpha_1 C_1 - (\mu + \delta_1 + \gamma_1)I_1,\\
\frac{dC_2}{dt} &= \beta_2 S(C_2 + I_2) - (\alpha_2 + \mu + \omega)C_2,\\
\frac{dI_2}{dt} &= \alpha_2 C_2 - (\mu + \delta_2 + \gamma_2)I_2,\\
\frac{dR}{dt} &= I_1\gamma_1 + I_2\gamma_2 + \omega C_1 + \omega C_2 - (\theta + \mu)R.
\end{aligned}
\tag{2.1}
$$

Table 1. Description of the parameters.

Parameter	Description
Π	Recruitment rate
θ	Loss of immunity
β_1	Effectiveness contact rate due to strain 1
β_2	Effectiveness contact rate due to strain 2
μ	Natural death rate
α_1	Progression rate from C_1 to I_1
α_1	Progression rate from C_2 to I_2
δ_1	Disease - induced mortality due to strain 1
δ_2	Disease - induced mortality due to strain 2
ω	Natural recovery rate
γ_1	Recovery rate from disease due to strain 1
γ_2	Recovery rate from disease due to strain 2

3. Analysis of the model

In this section, some important properties of the proposed model such as boundedness, existence of equilibrium and basic reproduction number will be analyzed.

3.1. Boundedness and positivity

The system trajectories are confined within a compact set. Then, the total population $N(t) = S(t) + C_1(t) + C_2(t) + I_1(t) + I_2(t) + R(t)$. Thus taking the derivative leads to

$$\begin{aligned}\frac{dN(t)}{dt} &= \frac{dS(t)}{dt} + \frac{dC_1(t)}{dt} + \frac{dC_2(t)}{dt} + \frac{dI_1(t)}{dt} + \frac{dI_2(t)}{dt} + \frac{dR(t)}{dt} \\ &= \Pi - N\mu - (\delta_1 I_1 + \delta_2 I_2).\end{aligned} \tag{3.1}$$

Therefore

$$\frac{dN(t)}{dt} \leqslant \Pi - N\mu \;\; which\ implies \;\; \frac{dN(t)}{dt} + N\mu \leqslant \Pi. \tag{3.2}$$

Consequently,

$$N(t) \leqslant \frac{\Pi}{\mu} + Ce^{-\mu t}, \tag{3.3}$$

where C is constant. The initial value condition at $t = 0$ gives

$$N(0) \leq \frac{\Pi}{\mu} + C. \tag{3.4}$$

This implies that

$$C = N(0) + \frac{\Pi}{\mu}. \tag{3.5}$$

We get

$$\lim_{t\to\infty} N(t) \leqslant \lim_{t\to\infty}(\frac{\Pi}{\mu} + (N(0) - \frac{\Pi}{\mu})e^{-\mu t}) = \frac{\Pi}{\mu}, \tag{3.6}$$

and this gives

$$\lim_{t\to\infty} N(t) \leqslant \frac{\Pi}{\mu}. \tag{3.7}$$

Hence the population is bounded above.

For the Positivity Let $t_0 > 0$. In the model, if the initial conditions

$$S(0) > 0, C_1(0) > 0, I_1(0) > 0, C_2(0) > 0, I_2(0) > 0, R(0) > 0, \tag{3.8}$$

then for all

$$t \in [0, t], S(t), C_1(t), I_1(t), C_2(t), I_2(t), R(t) \tag{3.9}$$

will remain positive in $\mathbb{R}^6_+$.

Since all the parameters used are positive, we can place lower bounds on each of the equations given in the model. Thus,

$$\begin{aligned}
\frac{dS}{dt} &= \Pi + \theta R + \beta_1 S(C_1 + I_1) + \beta_2 S(C_2 + I_2) - \mu S \geq -\mu S,\\
\frac{dC_1}{dt} &= \beta_1 S(C_1 + I_1) - (\alpha_1 + \mu + \omega)C_1 \geq -(\alpha_1 + \mu + \omega)C_1,\\
\frac{dI_1}{dt} &= \alpha_1 C_1 - (\mu + \delta_1 + \gamma_1)I_1 \geq -(\mu + \delta_1 + \gamma_1)I_1,\\
\frac{dC_2}{dt} &= \beta_2 S(C_2 + I_2) - (\alpha_2 + \mu + \omega)C_2 \geq -(\alpha_2 + \mu + \omega)C_2,\\
\frac{dI_2}{dt} &= \alpha_2 C_2 - (\mu + \delta_2 + \gamma_2)I_2 \geq -(\mu + \delta_2 + \gamma_2)I_2,\\
\frac{dR}{dt} &= I_1\gamma_1 + I_2\gamma_2 + \omega C_1 + \omega C_2 - (\theta + \mu)R \geq -(\theta + \mu)R.
\end{aligned} \tag{3.10}$$

Solving the differential inequality, we get

$$\begin{aligned}
S(t) &\geq e^{-\mu t} \geq 0,\\
C_1(t) &\geq e^{-(\alpha_1+\mu+\omega)t} \geq 0,\\
I_1(t) &\geq e^{-(\mu+\delta_1+\gamma_1)t} \geq 0,\\
C_2(t) &\geq e^{-(\alpha_2+\mu+\omega)t} \geq 0,\\
I_2(t) &\geq e^{-(\mu+\delta_2+\gamma_2)t} \geq 0,\\
R(t) &\geq e^{-(\theta+\mu)t} \geq 0.
\end{aligned} \tag{3.11}$$

Hence the proof.

3.2. Existence of equilibrium

In order to obtain the equilibrium solution , we equate the system of differential equations to zero and solve them simultaneously as follow:

$$\begin{aligned}
&\Pi + \theta R - \beta_1 S(C_1 + I_1) - \beta_2 S(C_2 + I_2) - \mu S = 0,\\
&\beta_1 S(C_1 + I_1) - (\alpha_1 + \mu + \omega)C_1 = 0,\\
&\beta_2 S(C_2 + I_2) - (\alpha_2 + \mu + \omega)C_2 = 0,\\
&\alpha_1 C_1 - (\mu + \delta_1 + \gamma_1)I_1 = 0,\\
&\alpha_2 C_2 - (\mu + \delta_2 + \gamma_2)I_2 = 0,\\
&\gamma_1 I_1 + \gamma_2 I_2 + \omega C_1 + \omega C_2 + \delta_1 I_1 + \delta_2 I_2 - (\mu + \theta)R = 0
\end{aligned} \tag{3.12}$$

Then, At DFE: $S_0, I_1 = I_2 = 0$ implies that $C_1 = C_2 = 0$
From Eq (3.12),

$$\Pi + \theta R - \mu S = 0. \tag{3.13}$$

Putting $R = 0$ into Eq (3.13), it implies that $S = \dfrac{\Pi}{\mu}$, $\therefore E_0 = [\dfrac{\Pi}{\mu}, 0, 0, 0, 0, 0]$. When $I_2 = 0 \Rightarrow C_2 = 0$ and $I_1 \neq 0 \Rightarrow C_2 \neq 0$
Then,

$$\begin{aligned}
S_1 &= \frac{\Omega_1\Omega_3}{\beta_1(\Omega_3 + \alpha_1)},\\
R_1 &= -\frac{(\Pi\Omega_3\beta_1 + \Pi\alpha_1\beta_1 - \mu\Omega_1\Omega_3)(\omega\Omega_3 + \alpha_1\gamma_1)}{\beta_1(\omega\theta\Omega_3^2 + \omega\theta\Omega_3\alpha_1 + \theta\Omega_3\alpha_1\gamma_1 + \theta\alpha_1^2\gamma_1 - \Omega_1\Omega_2^2\Omega_3 - \Omega_1\Omega_3\Omega_5\alpha_1)}\\
C_1 &= -\frac{\Omega_3\Omega_5(\Pi\Omega_3\beta_1 + \Pi\alpha_1\beta_1 - \mu\Omega_1\Omega_3)}{\beta_1(\omega\theta\Omega_3^2 + \omega\theta\Omega_3\alpha_1 + \theta\Omega_3\alpha_1\gamma_1 + \theta\alpha_1^2\gamma_1 - \Omega_1\Omega_2^2\Omega_3 - \Omega_1\Omega_3\Omega_5\alpha_1)}\\
I_1 &= -\frac{\alpha_1\Omega_5(\Pi\Omega_3\beta_1 + \Pi\alpha_1\beta_1 - \mu\Omega_1\Omega_3)}{\beta_1(\omega\theta\Omega_3^2 + \omega\theta\Omega_3\alpha_1 + \theta\Omega_3\alpha_1\gamma_1 + \theta\alpha_1^2\gamma_1 - \Omega_1\Omega_2^2\Omega_3 - \Omega_1\Omega_3\Omega_5\alpha_1)}
\end{aligned} \tag{3.14}$$

This equilibrium solution exists only when $I_1 \geqslant 0, C_1 \geqslant 0, R_1 \geqslant 0$ if $(\frac{\Omega_1+\alpha_1}{\mu\Omega_1\Omega_3}) \geqslant 1$. When $I_1 = 0 \Rightarrow C_1 = 0$ and $I_2 \neq 0 \Rightarrow C_2 \neq 0$, Then,

$$\begin{aligned}
S_2 &= \frac{\Omega_2\Omega_4}{\beta_2(\Omega_4 + \Omega_2)},\\
R_2 &= \frac{(\Pi\Omega_4\beta_2 + \Pi\alpha_2\beta_2 - \mu\Omega_2\Omega_4)(\omega\Omega_4 + \alpha_2\gamma_2)}{\beta_2(\omega\theta\Omega_4^2 + \omega\theta\Omega_4\alpha_2 + \theta\Omega_4\alpha_2\gamma_2 + \theta\alpha_2^2\gamma_2 - \Omega_2\Omega_4^2\Omega_5 - \Omega_2\Omega_4\Omega_5\alpha_2)}\\
C_2 &= -\frac{\Omega_4\Omega_5(\Pi\Omega_4\beta_2 + \Pi\alpha_2\beta_2 - \mu\Omega_2\Omega_4)}{\beta_2(\omega\theta\Omega_4^2 + \omega\theta\Omega_4\alpha_2 + \theta\Omega_4\alpha_2\gamma_2 + \theta\alpha_2^2\gamma_2 - \Omega_2\Omega_4^2\Omega_5 - \Omega_2\Omega_4\Omega_5\alpha_2)}\\
I_2 &= \frac{\Omega_5(\Pi\Omega_4\beta_2 + \Pi\alpha_2\beta_2 - \mu\Omega_2\Omega_4)}{\beta_2(\omega\theta\Omega_4^2 + \omega\theta\Omega_4\alpha_2 + \theta\Omega_4\alpha_2\gamma_2 + \theta\alpha_2^2\gamma_2 - \Omega_2\Omega_4^2\Omega_5 - \Omega_2\Omega_4\Omega_5\alpha_2)}.
\end{aligned} \tag{3.15}$$

This equilibrium solution exists only when $I_2, C_2, R_2 \geqslant 0$ if $\frac{(\Omega_4+\Pi)}{\mu\Omega_2\Omega_4}$, where $\Omega_1 = \alpha_1 + \mu + \omega$, $\Omega_2 = \alpha_2 + \mu + \omega$, $\Omega_3 = \mu + \delta_1 + \gamma_1$, $\Omega_4 = \mu + \delta_2 + \gamma_2$ and $\Omega_5 = \theta + \mu$.

3.3. Basic reproduction number

Basic reproduction number is the number of secondary infection caused by one infected individual in a wholly susceptible population. Here, it is obtained using next generation matrix as in [16];

$$\mathbf{F} = \begin{pmatrix} \beta_1 S(C_1 + I_1) \\ \beta_2 S(C_3 + I_2) \end{pmatrix} \tag{3.16}$$

$$\mathbf{V} = \begin{pmatrix} (\alpha_1 + \mu + \omega)C_1 \\ (\alpha_2 + \mu + \omega)C_2 \end{pmatrix} \tag{3.17}$$

Now,

$$\partial\mathbf{F}(\mathbf{E_0}) = \begin{pmatrix} \beta_1 S_0 & 0 \\ 0 & \beta_2 S_0 \end{pmatrix} \tag{3.18}$$

$$(\partial\mathbf{V})^{-\mathbf{1}} = \begin{pmatrix} \dfrac{1}{\alpha_1 + \mu + \omega} & 0 \\ 0 & \dfrac{1}{\alpha_2 + \mu + \omega} \end{pmatrix} \tag{3.19}$$

Then,

$$\partial\mathbf{F}(\mathbf{E_0})(\partial\mathbf{V})^{-\mathbf{1}} = \begin{pmatrix} \frac{\beta_1 S_0}{\alpha_1+\mu+\omega} & 0 \\ 0 & \frac{\beta_2 S_0}{\alpha_2+\mu+\omega} \end{pmatrix} \tag{3.20}$$

The matrix F is non-negative and it is called transition matrix which is responsible for the infection while the matrix V is known as a transmission matrix for the model.
From $\partial F(E_0)(\partial V)^{-1}$ in above,

$$R_1 = \frac{\beta_1 S_0}{\alpha_1 + \mu + \omega} = \frac{\beta_1 \Pi}{\alpha_1 + \mu + \omega},\ R_2 = \frac{\beta_2 S_0}{\alpha_2 + \mu + \omega} = \frac{\beta_2 \Pi}{\alpha_2 + \mu + \omega}, \tag{3.21}$$

where $S_0 = \frac{\Pi}{\mu}$. So that R_0 is the spectral radius of the matrix $\partial F(E_0)(\partial V)^{-1}$. Therefore $R_0 = max.(R_1, R_2)$, Hence

$$R_0 = max.(\frac{\beta_1 \Pi}{\alpha_1 + \mu + \omega}, \frac{\beta_{12} \Pi}{\alpha_2 + \mu + \omega}). \tag{3.22}$$

Profile of the basic reproduction number is given in Figure 2 below.

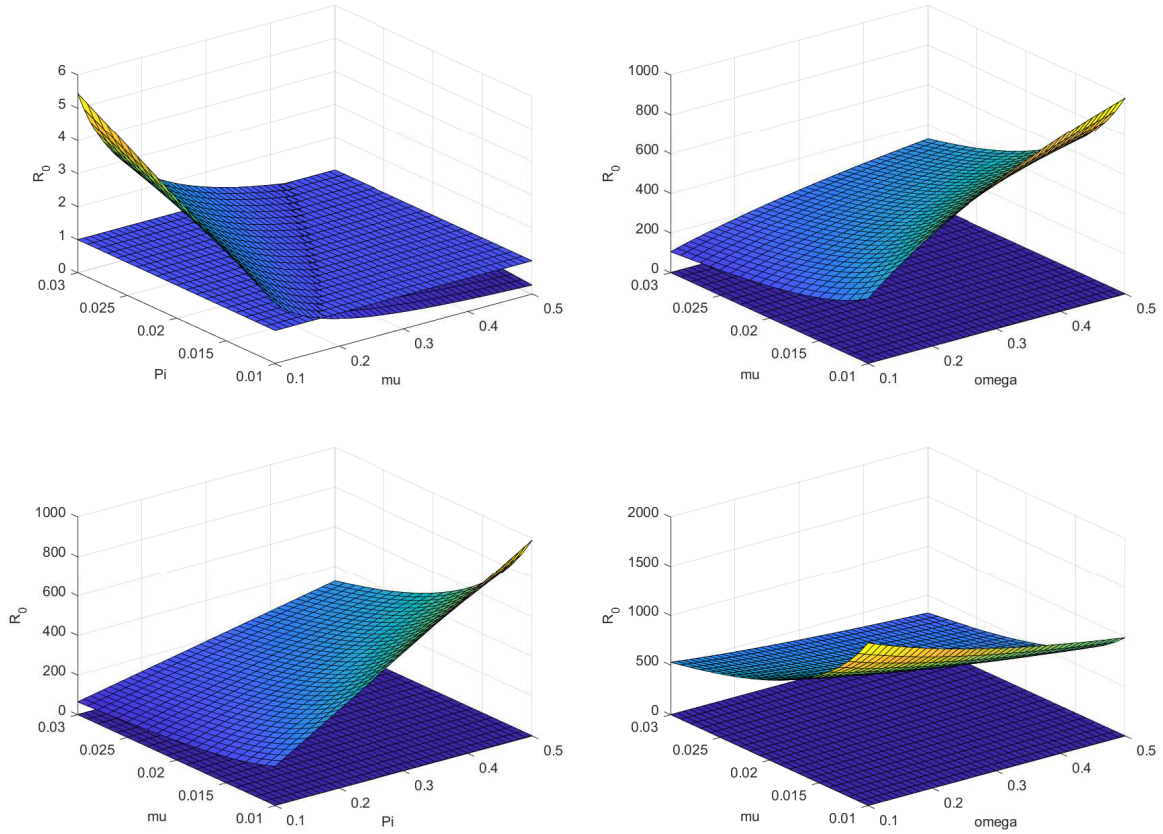

Figure 2. Profile of the basic reproduction number.

4. Stability analysis of the equilibrium solution

Here, we study the global stability of the equilibrium solutions using Lyaponuv function as in the following [17–19].

Theorem 4.1. *The disease free equilibrium, E_0 is globally asymptotically stable if $R_1 < 1$ and $R_2 < 1$.*

Proof. Consider the Lyaponuv function

$$V(S_0, C_{1.0}, C_{2.0}, I_{1.0}, I_{2.0}, R_0) = g(\frac{S}{S_0}) + I_{1.0} + I_{2.0} + C_1 + C_{2.0} + g(\frac{R}{R_0}). \tag{4.1}$$

where $g(x) = x - 1 - \ln x$, since $I_1, I_2 > 0$ then,

$$V(S, C_1, C_2, I_1, I_2, R) \geqslant 0. \tag{4.2}$$

Now we need to show that $\dot{V} < 0$.

$$\begin{aligned}
&\dot{V}(S, C_1, C_2, I_1, I_2, R) = (1 - \frac{S_0}{S})\dot{S} + \dot{I}_1 + \dot{I}_2 + \dot{C}_1 + \dot{C}_2 + (1 - \frac{R_0}{R})\dot{R} \\
&= (1 - \frac{S_0}{S})[\Pi + \theta R - \beta_1 S(C_1 + I_1) - \beta_2 S(C_2 + I_2) - \mu S] + \beta_1 S(C_1 + I_1) - (\alpha_1 + \mu + \omega)C_1 \\
&+ \beta_2 S(C_2 + I_2) - (\alpha_2 + \mu + \omega)C_2 + \alpha_1 C_1 - (\mu + \delta_1 + \gamma_1)I_1 + \alpha_2 C_2 - (\mu + \delta_2 + \gamma_2)I_2 \\
&+ (1 - \frac{R_0}{R})[\gamma_1 I_1 + \gamma_2 I_2 + \omega C_1 + \omega C_2 - \mu R - \theta R] \\
&= 2\mu S_0 - \frac{\mu S_0^2}{S} - \mu S + (\mu + \omega + \alpha_1)C_1[\frac{\beta_1 S_0}{(\mu + \omega + \alpha_1)} - 1] + (\mu + \omega + \alpha_2)C_2[\frac{\beta_2 S_0}{(\mu + \omega + \alpha_2)} - 1] \\
&+ (\beta_1 S_0 - \mu - \delta_1 - \frac{R_0\gamma_1}{R})I_1 \\
&+ (\beta_2 S_0 - \mu - \delta_2 - \frac{R_0\gamma_2}{R})I_2 + (\alpha_1 + \omega - \frac{R_0\omega}{R})C_1 + (\alpha_2 + \omega - \frac{R_0\omega}{R})C_2 - (\frac{RS_0}{S} + R_0)\theta - (R - R_0)\mu \\
&= 2\mu S_0 - \frac{\mu S_0^2}{S} - \mu S - (\mu + \omega + \alpha_1)C_1[1 - R_1] - (\mu + \omega + \alpha_2)C_2[1 - R_2] - (\frac{R_0\gamma_1}{R} + \mu + \delta - \beta_1 S_0)I_1 \\
&- (\frac{R_0\gamma_2}{R} + \mu + \delta - \beta_2 S_0)I_2 - (\frac{R_0\omega}{R} - \alpha_1 - \omega)C_1 - (\frac{R_0\omega}{R} - \alpha_2 - \omega)C_2 - (\frac{RS_0}{S} + R_0)\theta - (R - R_0)\mu \\
&= \mu S_0(2 - \frac{S_0}{S} - \frac{S}{S_0}) - (\mu + \omega + \alpha_1)C_1[1 - R_1] - (\mu + \omega + \alpha_2)C_2[1 - R_2] - (\frac{R_0\gamma_1}{R} + \mu + \delta - \beta_1 S_0)I_1 \\
&- (\frac{R_0\gamma_2}{R} + \mu + \delta - \beta_2 S_0)I_2 - (\frac{R_0\omega}{R} - \alpha_1 - \omega)C_1 - (\frac{R_0\omega}{R} - \alpha_2 - \omega)C_2 - (\frac{RS_0}{S} + R_0)\theta - (R - R_0)\mu.
\end{aligned} \tag{4.3}$$

But $2 - \frac{S_0}{S} - \frac{S}{S_0} < 0$ by the relationship between arithmetic and geometric mean, $\therefore \dot{V} \leqslant 0$.

Theorem 4.2. *E_1 is globally asymptotically stable if $R_1 < 1$.*

Proof. Consider the Lyaponuv function:

$$V(S, C_{1.1}, C_{2.1}, I_{1.1}, I_{2.1}, R_1) = g(\frac{S}{S_1}) + g(\frac{I_1}{I_{1.1}}) + I_2 + g(\frac{C_{1.1}}{C_1}) + C_2 + g(\frac{R}{R_1}), \tag{4.4}$$

where $g(x) = x - 1 - \ln x$, since $I_1 > 0$, then $V(S, C_1, C_2, I_1, I_2, R) \geqslant 0$.

Now we need to show that $\dot{V} < 0$.

$$\dot{V}(S, C_1, C_2, I_1, I_2, R) = (1 - \frac{S_1}{S})\dot{S_1} + (1 - \frac{I_{1.1}}{I_1})\dot{I_1} + \dot{I_2} + (1 - \frac{C_{1.1}}{C_1})\dot{C_1} + \dot{C_2} + (1 - \frac{R_1}{R})\dot{R}$$
$$= (1 - \frac{S_1}{S})[\Pi + \theta R - \beta_1 S(C_1 + I_1) - \beta_2 S(C_2 + I_2) - \mu S]$$
$$+ (1 - \frac{C_{1.1}}{C_1})[\beta_1 S(C_1 + I_1) - (\alpha_1 + \mu + \omega)C_1]$$
$$+ (1 - \frac{I_{1.1}}{I_1})[\alpha_1 C_1 - (\mu + \delta_1 + \gamma_1)I_1] + \beta_2 S(C_2 + I_2) - (\alpha_2 + \mu + \omega)C_2 + \alpha_2 C_2 - (\mu + \delta_2 + \gamma_2)I_2$$
$$+ (1 - \frac{R_1}{R})[\gamma_1 I_1 + \gamma_2 I_2 + \omega C_1 + \omega C_2 - \mu R - \theta R]$$
$$= 2\mu S_1 - \frac{\mu S_1^2}{S} - \mu S + (\mu + \omega + \alpha_1)C_1[\frac{\beta_1 S_1}{(\mu + \omega + \alpha_1)} - 1]$$
$$+ (\beta_1 S_1 - \mu - \delta_1 - \frac{R_1\gamma_1}{R})I_1 + (\alpha_1 + \omega - \frac{R_1\omega}{R})I_1$$
$$- [\beta_1 S(C_1 + I_1) - (\alpha_1 + \mu + \omega)C_1]\frac{C_{1.1}}{C_1} - \frac{\theta R S_1}{S} - \theta R_1 - (R - R_1)\mu - [\alpha_1 C_1 - (\mu + \delta_1 + \gamma_1)I_1]\frac{I_{1.1}}{I_1}$$
$$+ [\beta_2 S_1 - \mu - \delta_2 - \frac{R\gamma_2}{R}]$$
$$= \mu S_1(2 - \frac{S_1}{S} - \frac{S}{S_1}) - (\mu + \omega + \alpha_1)C_1[1 - R_1] - [\gamma_1 R + \mu R + \delta_1 R - \beta_1 S_1]I_1$$
$$- (R - \frac{RS_1}{S})\theta - (R - R_1)\mu - (\frac{\omega R_1}{R} - \alpha_1 - \omega)C_1 \tag{4.5}$$

But $2 - \frac{S_1}{S} - \frac{S}{S_1} < 0$ by the relationship between arithmetic and geometric mean, $\therefore \dot{V} < 0$

Theorem 4.3. *E_2 is globally asymptotically stable if $R_2 < 1$.*

Proof. Consider the Lyaponuv function:

$V(S, C_{1.2}, C_{2.2}, I_{1.2}, I_{2.2}, R) = g(\frac{S}{S_2}) + C_1 + I_1 + g(\frac{C_2}{C_{2.2}}) + \frac{I_2}{I_{2.2}} + g(\frac{R}{R_2})$, where $g(x) = x - 1 - \ln x$, since $I_1 > 0$, then $V(S, C_1, C_2, I_1, I_2, R) \geqslant 0$.

Now we need to show that $\dot{V} < 0$.

$$
\begin{aligned}
\dot{V}(S, C_1, C_2, I_1, I_2, R) &= (1 - \frac{S_2}{S})\dot{S} + \dot{C_1} + \dot{I_1} + (1 - \frac{C_{2.2}}{C_2})\dot{C_2} + (1 - \frac{I_{2.2}}{I_2})\dot{I_2} + (1 - \frac{R_2}{R})\dot{R} \\
&= (1 - \frac{S_1}{S})[\Pi + \theta R - \beta_1 S(C_1 + I_1) - \beta_2 S(C_2 + I_2) - \mu S] + \beta_1 S(C_1 + I_1) - (\alpha_1 + \mu + \omega)C_1 \\
&+ \alpha_1 C_1 - (\mu + \delta_1 + \gamma_1)I_1 \\
&+ (1 - \frac{C_{2.2}}{C_2})[\beta_2 S(C_2 + I_2) - (\alpha_2 + \mu + \omega)C_2] + (1 - \frac{I_{2.2}}{I_2})[\alpha_2 C_2 - (\mu + \delta_2 + \gamma_2)I_2] \\
&+ (1 - \frac{R_2}{R})[\gamma_1 I_1 + \gamma_2 I_2 + \omega C_1 + \omega C_2 - \mu R - \theta R] \\
&= 2\mu S_2 - \frac{\mu S_2^2}{S} - \mu S + (\mu + \omega + \alpha_2)C_2[\frac{\beta_2 S_2}{(\mu + \omega + \alpha_2)} - 1] + (\beta_2 S_2 - \mu - \delta_2 - \frac{R_2\gamma_2}{R})I_2 \\
&+ (\alpha_2 + \omega - \frac{R_2\omega}{R})C_2 - [\beta_2 S(C_2 + I_2) - (\alpha_2 + \mu + \omega)C_2]\frac{C_{2.2}}{C_2} - \frac{\theta R S_2}{S} - \theta R_2 - (R - R_2)\mu \\
&- [\alpha_2 C_2 - (\mu + \delta_2 + \gamma_2)I_2]\frac{I_{2.2}}{I_2} \\
&= \mu S_2(2 - \frac{S_2}{S} - \frac{S}{S_2}) - (\mu + \omega + \alpha_2)C_2[1 - R_2] - [\gamma_2 R + \mu R + \delta_2 R - \beta_2 S_2]I_2 \\
&- (R - \frac{RS_2}{S})\theta - (R - R_2)\mu - (\frac{\omega R_2}{R} - \alpha_2 - \omega)C_2.
\end{aligned} \tag{4.6}
$$

But $2 - \frac{S_2}{S} - \frac{S}{S_2} < 0$ by the relationship between arithmetic and geometric mean, $\therefore \dot{V} < 0$.

5. Numerical simulations

In this chapter numerical examples are given out using the parameter values in Table 2. Figure 3 shows how the disease from both strains die out when $\max(R_1, R_2) < 1$. Figure 4 and 5 show how strain 1 and 2 persist when $R_1 > 1$ and when $R_2 > 1$ respectively. Finally Figure 6 shows how both strain 1 and 2 persist when $\min(R_1, R_2) > 1$.

Table 2. Description of parameter values used in the model.

Parameter	E_0	E_1	E_2	E_3
Π	0.0381	0.0381	0.0381	0.381
θ	0.9	0.9	0.9	0.9
β_1	0.00174	10	0.00174	10
β_2	0.00174	0.00174	10	10
μ	0.1177	0.1177	0.1177	0.1177
α_1	0.104	0.104	0.104	0.104
α_2	0.104	0.104	0.104	0.104
δ_1	0.747	0.747	0.747	0.747
δ_2	0.747	0.747	0.747	0.747
ω	0.896	0.896	0.896	0.896
γ_1	0.253	0.253	0.253	0.253
γ_2	0.253	0.253	0.253	0.253

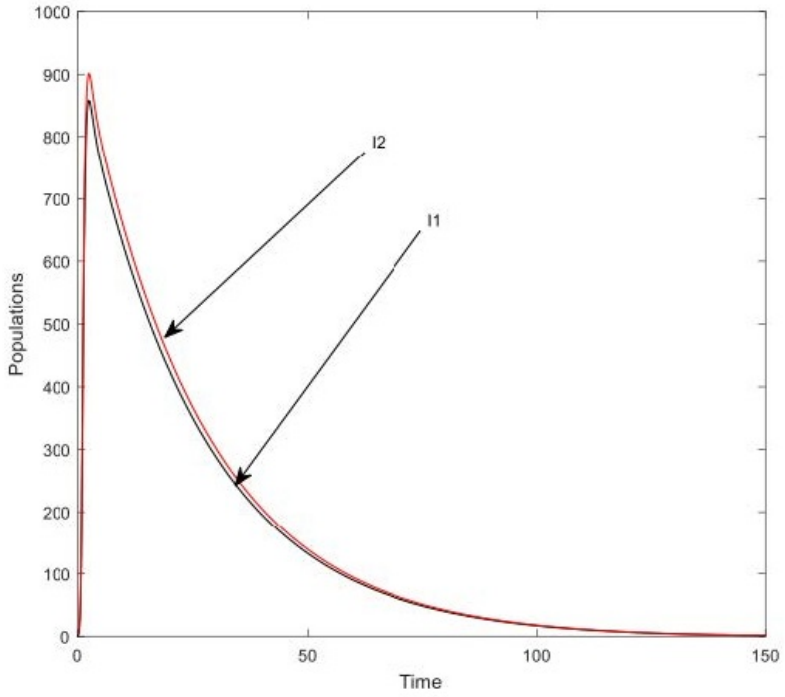

Figure 3. Disease free equilibrium: $\max\{R_1, R_2 < 1\}$.

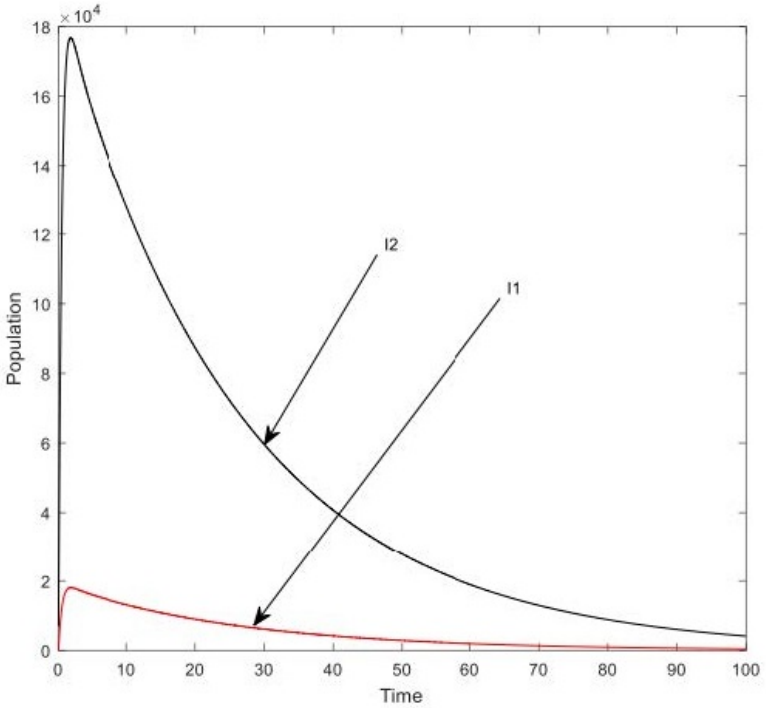

Figure 4. Endemic with respect to strain 2: $R_1 > 1$.

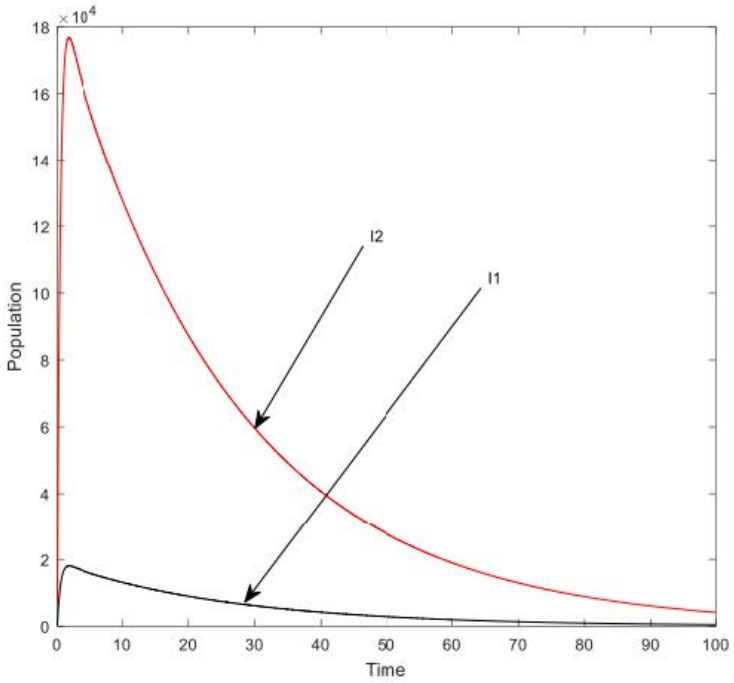

Figure 5. Endemic with respect to strain 2: $R_2 > 1$.

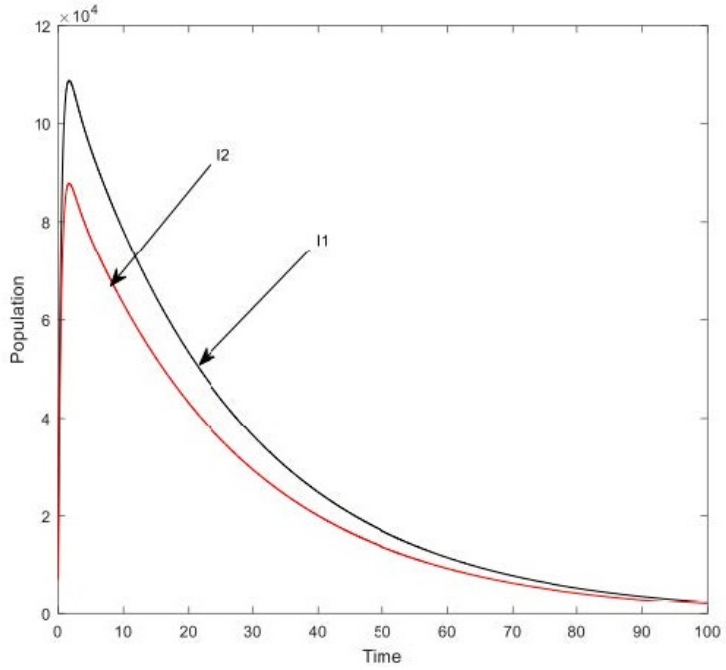

Figure 6. Endemic with respect to both strains 2: $\min\{R_1, R_2 > 1\}$.

6. Conclusion

In this paper, a model consisting of two strains of meningitis is studied. Three equilibrium points were obtained:

E_0 : disease free equilibrium, I_1 and I_2 are both zero.

E_1 : Endemic equilibrium for I_1 only and I_2 is zero.

E_2 : Endemic equilibrium for I_2 only and I_1 is zero.

But the endemic equilibrium for I_1 and I_2 is difficult to find due to the non-linear nature of the model, hence we show its stability numerically.

The method of next generation matrix was used to obtain two basic reproduction ratios for strain 1 and 2, and it was proved that the stability of these equilibrium points depend on the nature of basic reproduction ratios. Lyaponuv function was used to show the global stability of the equilibrium solutions. When $\min.(R_1, R_2) < 1$, the disease free equilibrium is globally stable and the disease dies out. And when the basic reproduction ratio is greater than 1 for each endemic equilibrium, then such equilibrium is globally stable and the disease at such equilibrium dies out.

Acknowledgments

We would like to thank the reviewers for their useful suggestions.

Conflict of interest

Authors declare that they have no conflict of interest.

References

1. Howlett WP (2015) *Neurology in Africa: Clinical Skills and Neurological Disorders*, UK: Cambridge University Press.
2. Martínez MJF, Merino EG, Sánchez EG, et al. (2013) A mathematical model to study meningococcal meningitis. *Procedia Comput Sci* 18: 2492–2495.
3. CDC Centers for Disease control, Bacterial Meningitis. USA, 2017. Available from: `http//www.who.int/gho/epidemic_disease/meningitis/suspected_cases_death_text/en/.`
4. Dushoff J, Plotkin JB, Levin SA, et al. (2004) Dynamical resonance can account for seasonal of influenza epidemic. *Proc Natl Acd Sci USA* 101: 16915–16916.
5. Stone L, Olinky R, Huppert A (2007) Seasonal dynamic of recurrent epidemics. *Nature* 446: 533–536.
6. Rvanchev LA (1968) Modeling experiment of a large epidemics by a means of computer. *Trans USSR Acad Sci Ser Math Phy* 180: 294–296.
7. Broutin H, Philippon S, De Magny GC, et al. (2007) Comparative study of meningitis dynamics across nine African countries: a global perceptive. *Int J Health Geogr* 6: 29.

8. Miller MA, Shahab CK (2005) Review of the cost effectiveness of immunization strategies for the control of epidemic meningococcal meningitis. *Pharmacoeconomics* 23: 333–343.
9. Irving TJ, Blyuss KB, Colijn C, et al. (2012) Modeling meningococcal meningitis in the African meningitis belt. *Epidemiol Infect* 140: 897–905.
10. Kwambana-Adams BA, Amaza RC, Okoi C, et al. (2018) Meningococcus serogroup C clonal complex ST-10217 outbreak in Zamfara State, Northern Nigeria. *Sci Rep* 8: 14194.
11. Chowell G, Miller MA, Viboud C (2008) Seasonal influenza in the United States, France and Australia: transmission and prospects for control. *Epidemiol Infect* 136: 852–864.
12. Bootsma MCJ, Ferguson NM (2007) The effect of public health measure on the 1918 influenza pandemic in US cities. *Proc Natl Acad Sci USA* 104: 7588–7593.
13. Chowell G, Ammon CE, Hengartner NW, et al. (2006) Transmission dynamics of the great influenza pandemic of 1918 in Geneva, Switzerland: assessing the effect of hypothetical interventions. *J Theor Biol* 241: 193–204.
14. Mills CE, Robins JM, Lipstich M (2004) Transmissibility of 1918 pandemic influenza. *Nature* 432: 904–906.
15. Chauchemez S, Valleron AJ, Boelle PY, et al. (2008) Estimating the impact of school closure on influenza transmission from Sentinel data. *Nature* 452: 750–754.
16. Diekmann O, Heesterbeek JAP, Roberts MG (2010) The Construction of next-generation matrices for compartmental epidemic models. *J R Soc Interface* 7: 873–885.
17. Sene N (2020) SIR epidemic model with Mittag-Leffler fractional derivative. *Chaos Soliton Fract* 137: 109833.
18. Sene N (2019) Stability analysis of the generalized fractional differential equations with and without exogeneous inputs. *J Nonlinear Sci Appl* 12: 562–572.
19. Sene N (2020) Global asymptotic stability of the fractional differential equations. *J Nonlinear Sci Appl* 13: 171–175.

AIMS Press