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Technical note

What happens if the opposite order from the recommended order is used for conducting integration by parts in applied calculus?

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Abstract: Integration by parts is an important technique in obtaining analytical solutions to many product-type integrands in integral calculus; hence, it is widely taught in undergraduate calculus courses. The most common approach for students to handle the processes of conducting integration by parts has been based on some rules of thumb, such as LIATE, which guides students to choose the order of the multiple functions forming a product-type integrand. This approach works well in most cases where integration by parts is applicable. However, some curious students, though not many, question what happens if they choose the opposite order from that guided by LIATE for product-type integrands involving a polynomial with an exponential function or a sinusoidal function. This short technical note intends to answer this long-overlooked question using some typical examples. It shows that the reverse approach for conducting integration by parts generally leads to a reduction formula for the specific product-type integrand, rather than providing the clear solution obtained by following the LIATE rule. Despite its weaknesses, the reverse approach demonstrated in this note may serve as a useful reference for students learning calculus, teachers who teach calculus, and other professionals who are interested in the subject.

Keywords: integration by parts, product-type integrands, opposite order, reverse approach, reduction formula

1. Introduction

Integration by parts is an important technique in obtaining analytical solutions to many

product-type integrands in integral calculus, differential equations, integral transforms, and other advanced mathematics [1–6]. Therefore, teaching students how to effectively use integration by parts to solve integrals involving product-type integrands is one of the key objectives in applied calculus courses in many undergraduate STEM degree studies worldwide [7–10]. The author recently shared some practices for effectively guiding engineering students and pre-service mathematics teachers to methodologically master integration by parts [3,6,9,11,12]. However, one question asked by several curious students regarding some special product-type integrands has been ignored because the teaching focus has been on guiding students to use the rule of thumb known as LIATE [1,11,12]. The question was: what happens if I choose the opposite order from that guided by LIATE for product-type integrands involving a polynomial with an exponential function or a sinusoidal function, i.e., $x^n e^x$ or $x^n \sin x$?

This short technical note intends to answer this long-overlooked question. Section 2 presents the common practices in using integration by parts guided by LIATE. Section 3 explains the background about the question asked by students and then presents an alternative procedure for solving integrals involving those product-type integrands through some typical examples. Brief discussions and conclusions are made in Section 4.

2. Integration by parts and the LIATE rule

2.1. Concept of integration by parts

Integration by parts is basically the inverse of the product rule of differentiation. If $u = f(x)$ and $v = g(x)$ are differentiable functions, by the product rule of differentiation,

$$\frac{d(uv)}{dx} = \frac{du}{dx}v + u\frac{dv}{dx},$$

multiply dx to both sides of the above equation to transfer it to its differential form [1]:

$$\cancel{dx} \times \frac{d(uv)}{\cancel{dx}} = \left(\frac{du}{\cancel{dx}}v + u\frac{dv}{\cancel{dx}}\right) \times \cancel{dx} \longrightarrow d(uv) = vdu + udv.$$

Apply the integral to both sides of the above differential equation,

$$\int d(uv) = \int vdu + \int udv \longrightarrow uv = \int vdu + \int udv$$

or

$$\int udv = uv - \int vdu \ . \tag{1}$$

Formula (1) is commonly known as *integration by parts*. It converts one combinative part of an integral to the other combinative part of the integral if the integrand can be treated as a product of two differentiable functions. Note that not all product-type integrands can be integrated analytically by integration by parts.

In practice, once u and dv on the left side of formula (1) are identified, v and du can be obtained through

$$\int udv = uv - \int vdu \longleftarrow\!\!\!

$$= -\frac{1}{2}x^2\cos 2x + \int (x)(\cos 2x dx)$$

$$= -\frac{1}{2}x^2\cos 2x + \left[(x)(\frac{1}{2}\sin 2x) - \int (\frac{1}{2}\sin 2x)(dx) \right] \longleftarrow \textit{2nd round of integration by parts}$$

$$= -\frac{1}{2}x^2\cos 2x + \left[\frac{1}{2}x\sin 2x - (-\frac{1}{4}\cos 2x) \right] + c$$

$$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c = \frac{1}{4}\left[2x\sin 2x + (1-2x^2)\cos 2x \right] + c.$$

Example 3: Evaluate $\int x^3 e^x dx$.

Solution: LIATE indicates $u = x^3$ and $dv = e^x dx$. By formula (2),

$$\int \underset{u}{x^3}\,\underset{dv}{e^x dx} = (\underset{u}{x^3})\left(\underset{v}{e^x}\right) - \int \left(\underset{v}{e^x}\right)\left(\underset{du}{3x^2 dx}\right) \longleftarrow v = \int e^x dx = e^x,\ \ du = (x^3)'dx = 3x^2 dx$$

$$= x^3e^x - 3\int x^2e^x dx = x^3e^x - 3\int \underset{u}{x^2}\,\underset{dv}{e^x dx} = x^3e^x - 3\left[x^2e^x - 2\int \underset{u}{x}\,\underset{dv}{e^x dx} \right]$$

$$= x^3e^x - 3x^2e^x + 6\left[(\underset{u}{x})\left(\underset{v}{e^x}\right) - \int \left(\underset{v}{e^x}\right)\left(\underset{du}{dx}\right) \right] = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + c$$

$$= (x^3 - 3x^2 + 6x - 6)e^x + c.$$

3. The question asked by curious students and the reverse approach

For product-type integrands involving a polynomial with an exponential function or a sinusoidal function, i.e., $x^n e^x$ or $x^n \sin x$, or the AE or AT combination by the LIATE rule [13], the first function u chosen for conducting integration by parts should be the polynomial (A). This choice ensures that the order of the polynomial is reduced by one after completing each round of integration by parts until the order is reduced to zero to obtain the final result, as demonstrated in Examples 2 and 3.

In the classroom, most students understood the logic of reducing the order of a polynomial to zero and hence used this rule in their attempts to deal with similar product-type integrands using integration by parts. However, a few students each semester asked whether the same result could be obtained if the order of choice were reversed. Typical answers to such a question were often indirect, such as “it would not produce an analytical solution as clear as the one obtained using the recommended way”, “it would make the integration more complicated than using the recommended way”, or “you would not be able to solve the problem if you could not further manipulate the obtained result properly by going in the opposite way”.

Are these the correct answers to the question? Or should we provide a more accurate procedure for answering the question in an alternative way (if possible) rather than taking the “recommended way” for granted? The author pondered whether answering this question was a worthwhile pedagogical point until he realized that providing those mathematically talented students with multiple strategies for handling the same mathematical problem in any suitable topic (not only in calculus) is a powerful

way to inspire them to love mathematics further when their curiosity is satisfied. This led the author to explore this question, and he eventually found an alternative way to deal with these types of integrands, as shared and demonstrated using some examples in this classroom note.

Example 4 derives a reverse reduction formula for an AE-type integrand; Example 5 applies the reduction formula obtained in Example 4 to the integral in Example 3 to obtain the same analytical solution achieved in Example 3 by LIATE. Example 6 derives a reverse reduction formula for an AT-type integrand, and Example 7 derives the same reduction formula by LIATE to mutually verify the correctness of the obtained reduction formula. Example 8 applies the reduction formula obtained in Examples 6 and 7 to the integral in Example 2 to obtain the same analytical solution achieved in Example 2 by LIATE. Following Example 8, Example 9 demonstrates the difference between the reverse reduction formula obtained in Example 6 and the reduction formula obtained by LIATE in Example 7 in solving an integral of a new AT-type integrand, from which the limitation of the reverse approach is exposed. Example 10 demonstrates the weaknesses of LIATE for students to understand the limitations of LIATE when using integration by parts.

Example 4: Evaluate $I(n)=\int x^n e^x dx \ \ (n\geq 1)$ by taking the order opposite to the order of LIATE.

Solution: This is the same question as Example 8 presented in [11]. Without loss of generality, let us use a positive integer *m* for *n* temporarily in the process as follows. Note that the order in this procedure is E before A, opposite to the order suggested by LIATE.

$$I(m)=\int \underbrace{e^x}_{u}\underbrace{x^m dx}_{dv}=\underbrace{(e^x)}_{u}\underbrace{(\frac{1}{m+1}x^{m+1})}_{v}-\int \underbrace{(\frac{1}{m+1}x^{m+1})}_{v}\underbrace{(e^x dx)}_{du}=\frac{1}{m+1}x^{m+1}e^x-\frac{1}{m+1}\int x^{m+1}e^x dx$$

$$=\frac{1}{m+1}x^{m+1}e^x-\frac{1}{m+1}I(m+1)\longleftarrow I(m+1)=\int x^{m+1}e^x dx.$$

$$\because I(m)=\frac{1}{m+1}x^{m+1}e^x-\frac{1}{m+1}I(m+1)\longrightarrow (m+1)I(m)=x^{m+1}e^x-I(m+1)$$

$$\therefore I(m+1)=x^{m+1}e^x-(m+1)I(m).$$

Let $n = m + 1$ ($m = n - 1$). The above formula becomes

$$I(n)=x^n e^x-nI(n-1). \tag{3}$$

This reverse formula is the same as the reduction formula obtained using the "recommended order" for conducting integration by parts in Example 8 in [11].

Example 5: Use the reduction formula (3) obtained in Example 4 to find the integral for $\int x^3 e^x dx$.

Solution: This question is the same as Example 3. The integral in Example 3 can be regarded as a specific case $I(3)=\int x^3 e^x dx$. By the reduction formula (3), solving *I*(3) requires knowing *I*(2), which in turn requires knowing *I*(1). It is easy to directly obtain *I*(0) as follows.

$$I(0) = \int x^0 e^x dx = \int e^x dx = e^x + (c).$$

Hence, by formula (3), $I(1)$, $I(2)$, and $I(3)$ can be obtained respectively as follows.

$$n = 1: \quad I(1) = x^1 e^x - I(1-1) = xe^x - I(0) = xe^x - e^x + (c) = (x-1)e^x + (c);$$
$$n = 2: \quad I(2) = x^2 e^x - 2I(1) = x^2 e^x - 2e^x(x-1) + (c) = (x^2 - 2x + 2)e^x + (c);$$
$$n = 3: \quad I(3) = x^3 e^x - 3I(2) = x^3 e^x - 3e^x(x^2 - 2x + 2) + (c) = (x^3 - 3x^2 + 6x - 6)e^x + c.$$

$I(3)$ is the same as the integral obtained in Example 3 using integration by parts by LIATE.

Example 6: Evaluate $I(n) = \int x^n \sin ax dx$ by taking the order opposite to the order of LIATE. Here, n (≥ 2) represents positive integers.

Solution: Without loss of generality, let us use a positive integer m for n temporarily in the process as follows. Note that the order in this procedure is T before A, opposite to the order suggested by LIATE. The constant is also deliberately omitted in the process for outlining the main procedure.

$$I(m) = \int x^m \sin ax dx = \int \underbrace{(\sin ax)}_{u}\underbrace{(x^m dx)}_{dv} = \underbrace{(\sin ax)}_{u}\underbrace{\left(\frac{1}{m+1}x^{m+1}\right)}_{v} - \int \underbrace{\left(\frac{1}{m+1}x^{m+1}\right)}_{v}\underbrace{(a\cos ax)dx}_{du}$$

$$= \frac{1}{m+1}x^{m+1}\sin ax - \frac{a}{m+1}\int x^{m+1}\cos ax dx = \frac{1}{m+1}x^{m+1}\sin ax - \frac{a}{m+1}\int \underbrace{(\cos ax)}_{u}\underbrace{(x^{m+1}dx)}_{dv}$$

$$= \frac{1}{m+1}x^{m+1}\sin ax - \frac{a}{m+1}\left(\underbrace{(\cos ax)}_{u}\underbrace{(\frac{1}{m+2}x^{m+2})}_{v} - \int \underbrace{(\frac{1}{m+2}x^{m+2})}_{v}\underbrace{(-a\sin ax)dx}_{du}\right)$$

$$= \frac{1}{m+1}x^{m+1}\sin ax - \frac{a}{m+1}\left(\frac{\cos ax}{m+2}x^{m+2} + \frac{a}{m+2}\int x^{m+2}\sin ax dx\right)$$

$$= \frac{1}{m+1}x^{m+1}\sin ax - \frac{a\cos ax}{(m+1)(m+2)}x^{m+2} - \frac{a^2}{(m+1)(m+2)}I(m+2)$$

$$\because I(m) = \frac{1}{m+1}x^{m+1}\sin ax - \frac{a\cos ax}{(m+1)(m+2)}x^{m+2} - \frac{a^2}{(m+1)(m+2)}I(m+2)$$

$$\therefore (m+1)(m+2)I(m) = (m+2)x^{m+1}\sin ax - ax^{m+2}\cos ax - a^2 I(m+2)$$

$$a^2 I(m+2) = (m+2)x^{m+1}\sin ax - ax^{m+2}\cos ax - (m+1)(m+2)I(m)$$

$$I(m+2) = \frac{1}{a^2}\left[(m+2)x^{m+1}\sin ax - ax^{m+2}\cos ax - (m+1)(m+2)I(m)\right]$$

Let $n = m + 2$ ($m = n - 2$ and $m + 1 = n - 1$). The above formula becomes

$$I(n) = \frac{1}{a^2}\left[nx^{n-1}\sin ax - ax^n\cos ax - n(n-1)I(n-2)\right]. \qquad (4)$$

This is a reverse reduction formula, the same as the one obtained using the "recommended order" guided by LIATE when applying integration by parts, demonstrated in Example 7 below.

Example 7: Evaluate $I(n) = \int x^n \sin axdx$ (n: positive integers) by the LIATE rule.

Solution: Following the LIATE rule, integration by parts goes as follows.

$$I(n) = \int x^n \sin axdx = \int \underbrace{(x^n)}_{u}\underbrace{(\sin axdx)}_{dv} = \underbrace{(x^n)}_{u}\underbrace{\left(-\frac{1}{a}\cos ax\right)}_{v} - \int \underbrace{\left(-\frac{1}{a}\cos ax\right)}_{v}\underbrace{(nx^{n-1}dx}_{du}$$

$$= -\frac{1}{a}x^n \cos ax + \frac{n}{a}\int x^{n-1}\cos axdx = -\frac{1}{a}x^n \cos ax + \frac{n}{a}\int \underbrace{(x^{n-1})}_{u}\underbrace{(\cos axdx)}_{dv}$$

$$= -\frac{1}{a}x^n \cos ax + \frac{n}{a}\left[\frac{1}{a}x^{n-1}\sin ax - \int \frac{n-1}{a}x^{n-2}\sin axdx\right]$$

$$= -\frac{1}{a}x^n \cos ax + \frac{n}{a^2}x^{n-1}\sin ax - \frac{n(n-1)}{a^2}I(n-2)$$

$$= \frac{1}{a^2}\left[nx^{n-1}\sin ax - ax^n \cos ax - n(n-1)I(n-2)\right].$$

This is indeed the same as the reduction formula (4), but its process is much shorter than the one obtained using the opposite order demonstrated in Example 6.

Example 8: Use the reduction formula (4) obtained in Example 6 or Example 7 to find the integral for $\int x^2 \sin 2xdx$ solved in Example 2.

Solution: The integral $\int x^2 \sin 2xdx$ can be regarded as a specific case of $I(n) = \int x^n \sin axdx$ for n = 2 and a = 2. It is easy to directly obtain $I(0)$ as follows.

$$I(0) = \int \sin 2xdx = -\frac{1}{2}\cos 2x + (c).$$

By the reduction formula (4),

$$I(2) = \frac{1}{2^2}\left[2x^{2-1}\sin 2x - 2x^2 \cos 2x - 2(2-1)I(2-2)\right] = \frac{1}{4}\left[2x\sin 2x - 2x^2 \cos 2x - 2I(0)\right]$$

$$= \frac{1}{4}\left[2x\sin 2x - 2x^2 \cos 2x - 2\left(-\frac{1}{2}\cos 2x\right)\right] = \frac{1}{4}\left[2x\sin 2x + (1-2x^2)\cos 2x\right] + c.$$

$I(2)$ is the same as the integral obtained in Example 2 using integration by parts by the LIATE rule.

Although the two procedures in Examples 6 and 7 result in the same reduction formula, there is a difference between them. This difference is demonstrated in the following example.

Example 9: Use the reduction formula (4) obtained in Example 6 or Example 7 to find the integral for $\int x^3 \sin xdx$.

Solution: The integral $\int x^3 \sin x dx$ can be regarded as a specific case of $I(n) = \int x^n \sin ax dx$ for $n = 3$ and $a = 1$. To use the reduction formula (4), $I(3\text{-}2) = I(1)$ must be known first. However, $I(1)$ cannot be derived from the reduction formula (4) because n must be a positive integer greater than or equal to 2. Fortunately, $I(1)$ can be determined by using integration by parts with the LIATE rule as follows.

$$I(1) = \int x \sin x dx = x(-\cos x) - \int (-\cos x) dx = -x \cos x + \sin x + (c).$$

By the reduction formula (4),

$$I(3) = \frac{1}{1^2}\left[3x^{3-1} \sin x - x^3 \cos x - 3(3-1)I(3-2)\right] = 3x^2 \sin x - x^3 \cos x - 6I(1)$$
$$= 3x^2 \sin x - x^3 \cos x - 6(-x \cos x + \sin x) = 3(x^2 - 2) \sin x + x(6 - x^2) \cos x + c.$$

Hence, conducting integration by parts by the LIATE rule is more generic than going the opposite way.

However, LIATE has some weaknesses that can lead students to apply the LIATE rule to some product-type integrands that do not have analytical solutions. The following examples demonstrate two of these weaknesses.

Example 10: Evaluate $\int \ln x \cos x dx$ and $\int e^x \arctan x dx$ by the LIATE rule.

Solution: For $\int \ln x \cos x dx$, by the LIATE rule, ln$x$ (L) is ahead of cos$x$ (T) in using integration by parts.

$$\int \ln x \cos x dx = \ln x (\sin x) - \int (\sin x)(\frac{dx}{x}) = \ln x \sin x - \int \frac{\sin x}{x} dx.$$

It is well-known in engineering mathematics that integral $\int \frac{\sin x}{x} dx$ has no analytical solution. Hence, integral $\int \ln x \cos x dx$ has no analytical solution even though LIATE seems to work in the first instance.

For $\int e^x \arctan x dx$, by the LIATE rule, arctan$x$ (I) is ahead of e^x (E) when using integration by parts.

$$\int e^x \arctan x dx = (\arctan x)(e^x) - \int (e^x)(\frac{dx}{1+x^2}) = e^x \arctan x - \int \frac{e^x}{1+x^2} dx.$$

Once again, integral $\int \frac{e^x}{1+x^2} dx$ has no analytical solution; hence, integral $\int e^x \arctan x dx$ has no analytical solution even though LIATE seemed to work in the first instance.

Recently, Guo [12] proposed a new rule, *Pet-LoPo-InPo*, to address key weaknesses in the LIATE rule. In *Pet-LoPo-InPo*, *P* in *Pet* is for polynomial functions $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$; *e* is for exponential functions e^{ax}, and *t* is for the primary trigonometric (or sinusoidal) functions $\sin ax$ or $\cos bx$. The small *e* and *t* indicate that the order of a product-type integrand combined by an exponential function with a sinusoidal function is exchangeable for using integration by parts. *Lo*

in *LoPo* is for logarithmic functions, and *Po* is for power functions x^r, where the index *r* can take any real number. *I* in *InPo* is for inverse trigonometric functions $\arctan x$, $\arcsin x$, or $\arccos x$; (*I*)*nPo* is for integer power functions x^n, where index *n* can only take integers. In most cases, *Pet-LoPo-InPo* provides not only the same guide as LIATE for integrals with product-type integrands but also a guide to students to avoid applying LIATE for integrals that do not have analytical solutions, such as the LT type and IE type demonstrated in Example 10. Details about *Pet-LoPo-InPo* are explained in [12].

4. Brief discussion and conclusion

Examples 4–9 showed that for product-type integrands involving a polynomial with an exponential function or a sinusoidal function, i.e., $x^n e^x$ or $x^n \sin x$, the recommended way using the LIATE rule indeed leads to a more straightforward process to directly obtain the analytical solution to the integral, compared with using the opposite order for integration by parts. The key step in using the opposite order is the substitution of a nominally higher-order integer (e.g., $m + 1$ or $m + 2$) by a nominally lower-order integer (e.g., $n = m + 1$ or $n = m + 2$) at the end of the most appropriate round of integration by parts. This key step also makes the whole process more complicated, echoing one of the answers given to the students: *It would make the integration more complicated than using the recommended way*. Missing this key step would lead to a situation somewhat described by another answer given to the students: *you would not be able to solve the problem if you could not further manipulate the obtained result properly by going in the opposite way*. Even if everything moves well by going the opposite way, the best solution to be obtained is a reduction formula that is not as clear as the analytical solution directly obtained with the recommended order, echoing the other answer given to the students: *it would not produce an analytical solution as clear as the one obtained using the recommended way*. Hence, the teacher's answers to the students regarding the question asked by curious students were generally correct.

However, these examples also reveal two untold truths regarding this question. Methodologically, although the whole process is more complicated when using the opposite order, the obtained reduction formula is a general expression for any appropriate product-type integrands involving a polynomial with an exponential function or a sinusoidal function, which can be called to solve other specific integrals of the same type. Pedagogically, in the classroom, it is useful to explain this point to students to support a growth-mindset approach to mathematics learning [14]. By directly comparing both ways of using integration by parts, students should be able to choose the most efficient or appropriate method in handling similar situations. This can not only boost students' procedural flexibility [15] but also create opportunities for students to evaluate approaches or appreciate mathematical wisdom in different approaches, leading to developed metacognition and strategic reasoning ability [16].

The reverse approach demonstrated in this note may serve as a useful reference for students learning calculus, teachers who teach calculus, and other professionals who are interested in the subject. However, this reverse approach for conducting integration by parts generally leads to a reduction formula for a specific product-type integrand, which is not as clear as the one obtained by following the LIATE rule for conducting integration by parts, despite LIATE's weaknesses.

Use of Generative-AI tools declaration

The author declares that Artificial Intelligence (AI) tools were not used in the creation of this article.

Conflict of interest

The author declares that there are no conflicts of interest.

Ethics declaration

Ethical approval was not required for this study, which does not directly involve human participants, animals, or sensitive personal data.

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