
Research article

Strategies for teaching physics and mathematics to develop critical thinking skills in schoolchildren based on interdisciplinary connections

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Abstract. This study aimed to identify optimal pedagogical conditions for integrating the subject areas of physics and mathematics within educational projects aimed at fostering and enhancing critical thinking skills among schoolchildren in grades 7–9. The methodological basis of the work combined systemic, activity-based, personality-oriented, and competence-based approaches. The research, conducted at School-Lyceum No. 3 in Taraz, Republic of Kazakhstan, employed methods of systematizing approaches to define the structure of critical thinking, modeling pedagogical systems, and mathematical statistics for processing experimental data. The study revealed a multicomponent structure of critical thinking, including analytical, logical, evaluative, metacognitive, information-retrieval, and dispositional components, and identified their specific manifestations in the study of physics and mathematics. An integrated model of critical thinking development was designed and tested, consisting of four interconnected blocks: conceptual target, content information, organizational activity, and result evaluation. A pedagogical experiment involving 78 students demonstrated significant positive changes in the development of critical thinking components in the experimental group compared with the control group. The most pronounced dynamics were observed in the development of metacognitive (37.9%), logical (34.4%), and evaluative (32.8%) components. Statistical analysis confirmed the significance of differences between the groups ($p < 0.01$) and revealed a positive correlation between the level of critical thinking development and academic performance in physics ($r = 0.67$) and mathematics ($r = 0.70$). The study identified the pedagogical conditions for effective integration of physics and mathematics: the creation of an integrated educational environment with modern equipment; the development of coordinated curricula that take into account students' age and individual characteristics; stimulation

of cognitive motivation through the demonstration of the practical significance of knowledge; and the use of active and interactive teaching methods. The practical significance of the study lies in the potential application of the developed model to improve the methodology of teaching natural sciences and mathematics in secondary schools.

Keywords: interdisciplinary approach, problem-based learning, metacognitive components, natural science education, analytical thinking, project activities, interdisciplinary communication

1. Introduction

Integration of subject areas in school education is one of the key directions in the development of the modern educational system. The relationship between physics and mathematics is of particular importance in the context of developing critical thinking among middle school students (grades 7–9). In an era of rapid technological progress and digitalization, the ability to critically evaluate information, analyze data, and make informed decisions has become an essential competency for the successful adaptation of the younger generation.

The educational system of Kazakhstan faces challenges related to the need to improve the quality of natural science education. This problem is especially acute in smaller towns of the republic, including Taraz, where, despite strict adherence to state educational standards, a gap remains between formal compliance with requirements and actual learning outcomes. Modern trends in education are characterized by a growing interdisciplinary interaction between subject areas, driven by the need to form students in a holistic scientific worldview [1,2]. In recent years (2021–2025), scientific interest in the integration of natural science disciplines, particularly physics and mathematics, as an effective means of developing critical thinking in secondary school students, has increased significantly. Within the broader development of interdisciplinary connections, special attention has been paid to higher professional education and, in particular, the teaching of science, technology, engineering, and mathematics (STEM).

A fundamental study of integration processes was presented by A. Keldibekova et al. [3], proposing innovative approaches to create an integrated educational environment and methodologies for designing interdisciplinary modules. Complementing this, the research by Z. M. Ozhibayeva and G. T. Abdoldinova [4] highlighted the role of mathematics in the development of STEM education, convincingly demonstrating that mathematical knowledge functions not only as an instrumental discipline but also as a methodological foundation for the integration of different subject areas. However, both studies have notable limitations: Keldibekova et al. [3] focused primarily on higher education, while Ozhibayeva and Abdoldinova [4] gave insufficient attention to the development of critical thinking within the context of mathematical education.

The study by Z. Kereimaganbetova and A. Uteuova [5] addressed the formation of algorithmic thinking in future teachers and identified effective methods for developing logical and algorithmic competencies that may be applied in school practice. The work of K. K. Zhampeisova et al. [6] offered conceptual approaches to designing new content for the professional training of teachers in the Republic of Kazakhstan, emphasizing the need to strengthen teachers' competencies in interdisciplinary design. However, both studies have notable gaps: Kereimaganbetova and Uteuova [5] did not disclose the mechanisms for integrating physical and mathematical knowledge, while Zhampeisova et al. [6] failed to provide specific models for subject-area integration.

Of particular value is the study by N. R. Mater et al. [7], which presented a model of technological acceptance that demonstrates a positive correlation between the use of integrated STEM programs and the level of critical thinking development. Similarly, N. Nurazmi and H. Bancong [8] experimentally confirmed the effectiveness of the integrated STEM–PBL model in fostering the skills of analysis, synthesis, and evaluation of information. Yet, both works reveal certain limitations: Mater et al. [7] did not offer a detailed examination of the mechanisms by which the physical and mathematical components of STEM influence critical thinking, while Nurazmi and Bancong [8] overlooked the age-specific characteristics of secondary school students.

Further contributions include the study by F. Prafitasari et al. [9], which proposed a system of electronic educational resources to develop skills in data analysis and in establishing cause-and-effect relationships within the framework of blended learning. Likewise, M. A. Martawijaya et al. [10] demonstrated the advantages of an ethnocultural approach to integrating STEM education by developing the Ethno-STEM–PBL model to overcome students' misconceptions about physics. Nevertheless, these studies did not devote sufficient attention to the integration of physics and mathematics: Prafitasari et al. [9] did not specify the nature of this integration in blended learning contexts, and Martawijaya et al. [10] placed little emphasis on the mathematical modeling of physical processes.

E. Supriyadi and K. S. Kuncoro [11] analyzed the prospects for using artificial intelligence to design mathematical tasks aimed at developing critical thinking. A. A. Yaki [12] demonstrated the effectiveness of STEM education in fostering skills of data interpretation and in assessing the reliability of information sources, and proposed a methodology for evaluating the level of critical thinking development in an interdisciplinary context. However, the work of Supriyadi and Kuncoro [11] did not consider the possibility of integrating physical and mathematical knowledge through artificial intelligence technologies, while Yaki [12] did not reveal the mechanisms of integrating physics and mathematics in developing critical thinking among students in grades 7–9.

Although previous studies have examined STEM integration, problem-based learning, and critical thinking development separately, the mechanisms through which physics–mathematics integration may support specific components of critical thinking in lower secondary school remain insufficiently specified. Existing studies often describe interdisciplinary learning in general terms, without showing how integrated physical and mathematical tasks are sequenced, assessed, and connected with analytical, logical, evaluative, and metacognitive forms of thinking. This creates a methodological gap concerning the operational design and classroom implementation of physics–mathematics integration for students in grades 7–9.

The purpose of this study was to examine how a structured model of physics–mathematics integration can be implemented in grades 7–9 and what changes in students' critical thinking indicators are observed after its use in one school setting.

To achieve this purpose, the study addressed the following research questions:

1. How can integrated physics–mathematics tasks and projects be organized to target specific components of critical thinking in grades 7–9?
2. What changes are observed in the analytical, logical, evaluative, metacognitive, information-retrieval, and dispositional components of critical thinking after the implementation of the integrative model?
3. How do the observed changes differ between the experimental group and the control group?
4. What methodological limitations should be considered when interpreting the relationship between the integrative model, critical thinking development, and academic performance?

The theoretical contribution of this study lies in clarifying the relationship between physics–mathematics integration and critical thinking as a component-based pedagogical process. Rather than treating critical thinking as a single undifferentiated outcome, the study links specific forms of integrated learning activity with analytical, logical, evaluative, metacognitive, information-retrieval, and dispositional components of critical thinking.

The novelty of the study lies in the development of a component-based model of physics–mathematics integration for critical thinking development in lower secondary school. Unlike broader STEM or problem-based learning frameworks, the proposed model connects specific types of integrated physics–mathematics tasks with particular components of critical thinking, including analytical, logical, evaluative, metacognitive, information-retrieval, and dispositional components. This makes it possible to explain not only that interdisciplinary learning may support critical thinking, but also how different forms of integration may contribute to different aspects of students' thinking.

2. Materials and methods

The methodological basis of the study on the development of critical thinking through the integration of physics and mathematics was grounded in systemic, activity-based, personality-oriented, and competence-based approaches.

To identify the key components of critical thinking and the criteria for their formation in students in grades 7–9, methods of generalization and systematization of scientific approaches to determining the structure of critical thinking were employed. Based on the work of leading researchers, six interrelated components of critical thinking were identified: analytical, logical, evaluative, metacognitive, information-retrieval, and dispositional. For each component, criteria for formation and indicators of their manifestation in the study of physics and mathematics were established.

The development of an integrative model for fostering critical thinking through the integration of physics and mathematics was undertaken using the method of modeling pedagogical systems. Structural and functional analysis of the model components made it possible to distinguish four interrelated blocks: conceptual target, content information, organizational activity, and result evaluation. To design the content-information block, curricula in physics and mathematics for grades 7–9 were analyzed, thematic areas for integration were identified, and a system of interdisciplinary training modules was developed.

One example of an integrated task was the analysis of uniform motion, where students first collected or interpreted data on distance and time, then constructed a graph of motion, calculated speed using proportional relationships, and evaluated whether the obtained mathematical model adequately described the physical situation. Another example concerned geometric optics, where students used geometric constructions and transformation principles to explain image formation and then discussed the limits of the model. Such tasks were intended to connect empirical observation, mathematical representation, and critical evaluation of the obtained result.

An experimental test of the effectiveness of the developed model was carried out at School-Lyceum No. 3 in Taraz, between September 2024 and January 2025. The study involved 78 students in grades 7–9, aged 13–15 years, including 42 girls (53.8%) and 36 boys (46.2%). A convenience sampling method was used, as the participants were selected from intact classes of School-Lyceum No. 3 in Taraz that were available for participation in the pedagogical experiment. Distribution of participants by grade was as follows: grade 7, 24 students (30.8%); grade 8, 27 students (34.6%); and grade 9, 27 students (34.6%). The study was conducted in accordance with the principles of the

Convention on the Rights of the Child, which emphasizes that the best interests of the child must be paramount, and that participation in research must be voluntary, with the option to withdraw at any stage without negative consequences. The parents of all students were informed of the aims and methods of study and provided written consent for their children's participation. Students were also informed that their participation was voluntary and that they could withdraw at any stage without any adverse effects on their academic performance or relationships with teachers. All procedures performed in the study were in accordance with the ethical standards of the institutional research committee and with the 1964 Helsinki Declaration and its later amendments.

The study was conducted according to the classical scheme of a pedagogical experiment, comprising ascertaining, formative, and control stages. At the ascertaining stage, the initial level of students' critical thinking and academic performance in physics and mathematics was diagnosed in both groups. At the formative stage, the integrative physics–mathematics model was implemented in the experimental group, while the control group continued learning according to the traditional subject-based curriculum. At the control stage, repeated diagnostics were conducted using the same assessment tools in order to compare the dynamics of changes in both groups. To ensure the reliability of the results, an experimental group (40 students) and a control group (38 students) were formed, equivalent in terms of their initial indicators of critical thinking development and academic performance in physics and mathematics. Equality between the groups at the ascertaining stage was confirmed through statistical analysis using the Pearson χ^2 criterion. Group equivalence was assessed by comparing the initial distribution of students according to the levels of critical thinking development and their academic performance in physics and mathematics before the intervention. The absence of statistically significant differences at the ascertaining stage indicated that the experimental and control groups were comparable before the implementation of the intervention.

The intervention in the experimental group was organized as a sequence of integrated physics–mathematics learning activities aligned with the existing curricula for grades 7–9. The sequence followed three stages. At the first stage, students worked with short integrated tasks that required them to connect physical quantities with mathematical representations, for example, through graphs, proportional relationships, and simple models. At the second stage, students completed interdisciplinary problem tasks in which they analyzed a physical situation, selected appropriate mathematical tools, and justified the adequacy of the selected model. At the third stage, students carried out project-based tasks requiring data collection, mathematical processing, interpretation of results, and reflection on possible sources of error.

The intervention included three thematic areas: kinematic regularities and their graphical representation; geometric optics and geometric transformations; and oscillatory processes and trigonometric functions. Each thematic area included tasks aimed at different components of critical thinking: analytical tasks focused on identifying relevant variables and relationships; logical tasks required justification of reasoning and detection of errors; evaluative tasks involved judging the reliability of data and models; metacognitive tasks required students to explain their problem-solving strategies; information-retrieval tasks involved selecting and organizing relevant information; and dispositional tasks were supported through discussion, argumentation, and reconsideration of alternative explanations. In the control group, physics and mathematics were taught according to the standard subject-based curriculum. Students completed regular classroom tasks, textbook exercises, and standard practical assignments without specially designed interdisciplinary physics–mathematics projects, reflective diaries, or targeted tasks for the separate components of critical thinking.

Implementation was monitored through lesson plans, students' written work, reflective notes, and teacher observations. These materials were used to check whether the activities corresponded to the

planned components of the model and whether students completed the intended stages of analysis, modeling, interpretation, and reflection. However, the intervention was implemented in a natural school setting, and therefore, complete control over teacher effects and classroom interaction was not possible.

Diagnostics of the level of critical thinking development were carried out using a set of complementary methods. The main tool was a researcher-developed critical thinking test for middle school students, consisting of 30 tasks. The critical thinking test was developed on the basis of theoretical analysis and aligned with the physics and mathematics curricula for grades 7–9. Tasks were designed for each component of critical thinking and then reviewed by experts in physics and mathematics education for relevance, clarity, and content correspondence. The test covered six components of critical thinking: analytical, logical, evaluative, metacognitive, information-retrieval, and dispositional. Tasks were constructed in relation to physics and mathematics content and required students to analyze conditions, identify relationships, interpret data, evaluate the adequacy of solutions, and reflect on problem-solving strategies. The test was scored on a 10-point scale for each component. The total results were interpreted according to three levels of critical thinking development: low, medium, and high. The level was determined on the basis of the total score and the distribution of scores across the six assessed components.

Content validity was assessed by experts in physics and mathematics education, who evaluated the relevance of the test items to the selected components of critical thinking and their correspondence with the curriculum content for grades 7–9. Reliability was checked through repeated testing at a two-week interval ($r = 0.86$, $p < 0.01$). In addition to the test, teacher-based expert assessment was used to evaluate students' performance during the solution of physics and mathematics problems, and an adapted metacognitive awareness questionnaire was used to assess the metacognitive component. Since teacher-based evaluations may include subjective judgment, these results were interpreted as supplementary evidence rather than as the sole basis for conclusions. The absence of fully independent blind assessment and detailed inter-rater reliability data is recognized as a methodological limitation of the study.

Statistical processing of the research results was performed using the SPSS Statistics 25.0 software package. Pearson's χ^2 criterion was used to compare the distribution of students by levels of critical thinking development and to assess the initial equivalence of the experimental and control groups. Paired-samples Student's t -test was used to compare pre-test and post-test indicators within each group, while independent-samples Student's t -test was applied to compare the experimental and control groups. Before doing inferential analysis, the assumptions necessary for parametric statistical methods were evaluated using SPSS Statistics 25.0. The normality of the distribution for the pre-test and post-test indicators was evaluated using the Shapiro–Wilk test for both the experimental and control groups independently. Levene's test was utilized to assess the homogeneity of variance for between-group comparisons. The outcomes of these initial assessments did not reveal significant breaches of normality or variance equality; hence, paired-samples Student's t -tests, independent-samples Student's t -tests, Pearson's correlation analysis, and effect-size estimates were deemed suitable for the next statistical study. The magnitude of the effect was estimated using Cramer's ϕ coefficient and Cohen's d . To determine the relationship between the development of critical thinking components and academic performance, correlation analysis was carried out with the calculation of Pearson's correlation coefficient.

3. Results

3.1. Theoretical foundations of critical thinking development in the context of integrating physics and mathematics

The development of critical thinking in students of grades 7–9 (aged 13–15) must be grounded in knowledge of the psychological foundations of adolescents' mental activity. This period is characterized by the active formation of abstract thinking and the growth of reflexive abilities, when students begin to operate with hypothetical situations, formulate and test hypotheses, and construct logical reasoning without relying on visualization. Metacognitive strategies increase the effectiveness of analytical thinking in solving complex problems that require the integration of knowledge from different subject areas [13–15]. Critical thinking in the context of natural sciences and mathematics incorporates interrelated components: analytical thinking, value judgment, logical reasoning, metacognitive reflection, and information-retrieval competence [16]. Mathematical and critical thinking share common cognitive foundations, such as analytical skills, logical reasoning, and precision of formulation [17]. The teaching of mathematics fosters the development of critical thinking by activating students' cognitive activity, including argumentation, proof, and refutation [18–20].

The integration of physics and mathematics creates distinctive opportunities for the development of critical thinking. Physics provides a context for applying mathematical models to real-life situations, enabling students to evaluate both the adequacy and the limits of applicability of these models [21]. A crucial aspect of integration is overcoming formalism in mathematics, when students apply algorithms mechanically without understanding their physical meaning. Problem situations that require analysis of a physical phenomenon, followed by the application of mathematical tools, stimulate critical thinking by encouraging comparisons between empirical data and theoretical models. The formation of a culture of argumentation—the construction of logical chains of reasoning supported by empirical data and mathematical proofs—constitutes the core of critical thinking in a scientific context [22,23]. In the study of physics and mathematics, students acquire diverse forms of scientific argumentation: deductive reasoning (characteristic of mathematics) and inductive generalization (typical of physics). This duality enriches their repertoire of thinking strategies and fosters multidimensional critical thinking. Such development is achieved under specific pedagogical conditions, including problem-oriented learning, dialogic methods of interaction, and the stimulation of reflective activity [24].

Theoretical analysis of the integration of physics and mathematics has revealed several conceptual approaches. The system-activity approach considers integration as the creation of an educational environment in which students are engaged in cognitive tasks requiring the combined application of physical and mathematical knowledge. The cognitive approach focuses on building interdisciplinary thinking structures. The contextual approach emphasizes learning mathematical methods in the context of solving physical problems. The competency-based approach seeks to develop integrative skills by applying methods from one discipline to solve problems in another [25]. Physics and mathematics may be integrated at different levels: coordination (alignment of curricula), combination (parallel study of related topics), and amalgamation (complete fusion of physics and mathematics content). Digital technologies further expand opportunities for developing critical thinking in the study of physics and mathematics [26–28]. Interactive simulations, computer algebra systems, and modeling environments enrich students' research activities [29], allowing them to

visualize abstract concepts, investigate parametric dependencies in physical models, and conduct virtual experiments.

The development of critical thinking in the study of physics and mathematics presents typical difficulties for students in grades 7–9: a tendency toward superficial information processing, challenges with abstraction, insufficient development of self-regulation skills for mental activity, and difficulties in transferring knowledge between subject areas [30]. Overcoming these difficulties requires didactic strategies such as a system of educational tasks of increasing complexity, methods for stimulating metacognitive reflection, and techniques for forming interdisciplinary cognitive schemes. There are two main directions for the development of critical thinking: the formation of general thinking skills through the content of academic subjects, and the development of subject-specific critical thinking. An integrative approach combines both directions, overcoming the limitations of narrowly subject-focused teaching. Critical thinking is most effectively developed in an educational environment that fosters students' intellectual independence in formulating research questions, selecting problem-solving methods, and assessing the reliability of results [31–33]. Of particular importance is the formation of a culture of scientific communication: the ability to articulate and defend a position, critically evaluate the arguments of others, and participate constructively in scientific discussion. These skills are fostered through group work such as project activities, scientific debates, and the joint analysis of scientific texts.

Analysis of the components of critical thinking shows that each has distinct features in the physical and mathematical context. The analytical component involves breaking complex phenomena into constituent parts, identifying essential features, and establishing cause-and-effect relationships. In physics and mathematics, this is manifested in analyzing problem conditions, identifying known and unknown quantities, determining physical patterns, and selecting the appropriate mathematical apparatus to describe a phenomenon [16]. The logical component entails constructing and evaluating reasoning, checking the consistency of statements, and assessing the validity of conclusions—skills that are essential for proving mathematical statements and justifying the applicability of physical laws [18]. The evaluative component consists of making judgments about the reliability of information, the credibility of sources, and the adequacy of research methods [22]. The metacognitive component concerns reflection on one's own thinking processes, including the planning, monitoring, and evaluation of cognitive activity [13]. The information-retrieval component encompasses searching for, selecting, and organizing information to solve a given problem [29]. The dispositional component refers to personal attitudes that support critical reflection, including intellectual honesty, willingness to reconsider one's views, and openness to alternative perspectives [24].

The integration of physics and mathematics provides conditions for the development of all these components of critical thinking. Physics offers empirical material that supports the development of analytical and evaluative components through the study of experimental data. Mathematics contributes to the development of logical and metacognitive components owing to its emphasis on rigorous reasoning [34,35]. Their combined study fosters holistic critical thinking, which unites the empirical validity of physical knowledge with the logical precision of mathematical constructions. At the age of 13–15, students experience a qualitative leap in the development of abstract-logical thinking, creating favorable conditions for the formation of critical thinking components [36]. Pedagogical approaches that support this process include problem-based learning, inquiry-based learning, strategies for developing critical thinking through reading and writing, discussion-based teaching methods, and project-based learning [25].

The study of the components of critical thinking and the features of their development in the context of learning physics and mathematics provides a holistic understanding of the potential for integrating these disciplines to foster critical thinking in students in grades 7–9. A comprehensive examination of the cognitive, metacognitive, and dispositional aspects of critical thinking, in connection with the substantive features of physics and mathematics, highlights the considerable potential of integration for developing higher-order thinking skills. Table 1 presents a correlation between the components of critical thinking and the opportunities for their development through the integration of physics and mathematics, thereby demonstrating the systemic character of this process.

Table 1. Potential of integrating physics and mathematics to develop critical thinking components in students in grades 7–9.

Critical thinking component	Potential of physics	Potential of mathematics	Synergistic effect of integration	Pedagogical development strategies
Analytical	Analysis of physical phenomena, identification of essential factors, and establishment of physical laws	Structuring information, identifying formal dependencies, and analytical presentation of data	Combining empirical analysis with formal logical structuring	Problem-based learning with an emphasis on analytical tasks requiring knowledge integration
Logical	Construction of cause-and-effect relationships, physical argumentation, and inductive generalizations	Deductive reasoning, proofs, and building logical chains	Formation of multidimensional logical thinking and combining inductive and deductive logic	Educational tasks for constructing and analyzing logical reasoning and identifying logical errors
Evaluation	Evaluation of the reliability of experimental data, applicability of physical laws	Evaluation of the accuracy of mathematical models, correctness of solutions, and limits of applicability	Comprehensive assessment of the adequacy of physical and mathematical models of reality	Tasks for critical evaluation of models, methods and results, and comparison of alternative approaches
Metacognitive	Planning a physical experiment and reflection on the research process	Strategies for solving mathematical problems and control of algorithmic actions	Awareness and regulation of complex cognitive activity	Reflective practices, metacognitive modeling, and self-regulation strategies
Information retrieval	Search and processing of empirical data, work with physical reference books	Working with mathematical texts, structuring information, and algorithmization	Comprehensive work with heterogeneous information, its integration, and transformation	Project activities with an emphasis on information tasks, working with different types of sources
Dispositional	Curiosity, desire to explain natural phenomena	Intellectual honesty, systematic thinking	Formation of a scientific worldview and research position	Creation of a communicative environment for the development of scientific discourse and intellectual dispositions

Note: The synergistic effect of integration describes new qualities of critical thinking that arise when combining the potentials of physics and mathematics, which cannot be achieved by studying these subjects separately. Pedagogical strategies include forms, methods, and technologies of teaching aimed at developing the corresponding components of critical thinking through the integration of physics and mathematics.

Source: Compiled by the authors based on B. Arisoy and B. Aybek [16], S. Dolapcioglu and A. Doğanay [18], S.F. Rivas et al. [13], S. Sachdeva and P.O. Eggen [30], M. Gunawardena and K. Wilson [24].

A theoretical examination of the development of critical thinking within the framework of integrating physics and mathematics suggests that the interdisciplinary approach holds considerable potential for fostering higher-order thinking skills in students in grades 7–9. The combination of physics and mathematics creates unique educational opportunities for the development of all components of critical thinking through the synthesis of empirical and theoretical knowledge, inductive and deductive reasoning, and concrete-figurative and abstract-symbolic thinking. The realization of this potential requires a systematic approach to structuring the educational process, incorporating substantive, procedural, and methodological aspects of integration, while also accounting for the psychological characteristics of adolescent cognitive development. Such an approach not only enhances the effectiveness of critical thinking formation but also provides students with a deeper understanding of the interrelationship between physical and mathematical concepts, thereby contributing to the development of a holistic scientific worldview.

3.2. Development and description of the model of integration of physics and mathematics for the development of critical thinking

The theoretical study of critical thinking development through the integration of physics and mathematics made it possible to formulate the conceptual foundations of an integrative model aimed at fostering critical thinking in students of grades 7–9. The developed model is a multicomponent system that reflects the interconnection between the target, content, procedural, and result-oriented aspects of the educational process. Its structure comprises four interconnected blocks: conceptual target, content information, organizational activity, and result evaluation. Each block performs specific functions within the overall system of critical thinking development through the integration of physics and mathematics.

The conceptual-target block defines the key theoretical and methodological approaches and principles of integration in the context of developing critical thinking. The principal methodological approaches include systemic, activity-based, personality-oriented, and competence-based perspectives. The systemic approach ensures a holistic view of the process, treating the integration of physics and mathematics as a unified system of interrelated elements. The activity-based approach emphasizes the active engagement of students in cognitive activities that require the application of physical and mathematical knowledge to solve problem-based tasks. The personality-oriented approach takes into account students' individual characteristics in the process of critical thinking development. The competence-based approach is directed toward forming integrative skills, including the ability to critically comprehend physical and mathematical problems.

The target component of the integrative model comprises a set of interconnected tasks: developing students' analytical abilities through the integration of physics and mathematics; cultivating a logical culture of thinking on an interdisciplinary basis; fostering the ability to make evaluative judgements when analyzing physical and mathematical problems; forming metacognitive strategies for regulating cognitive activity; enhancing information-search skills in physics and mathematics; strengthening the dispositional component of critical thinking, including intellectual honesty, objectivity, and consistency.

The principles underlying the integrative model include: the principle of interdisciplinary integration (the combination of physical and mathematical knowledge at the levels of content, methods, and forms of teaching); the principle of problematization (the creation of problem situations requiring critical reflection); the principle of dialogicity (the organization of educational

dialogue as a means of developing critical thinking); the principle of reflexivity (the stimulation of students' reflection on their own thought processes); and the principle of cognitive complexity (the gradual increase in the complexity of educational tasks, taking into account students' zones of proximal development).

The content-information block of the model defines the specifics of integrated content in physics and mathematics education aimed at developing critical thinking. Content integration is carried out at three levels: conceptual, theoretical, and problem-practical. At the conceptual level, integration involves identifying the relationships between physical and mathematical concepts, their systematization, and the establishment of hierarchical links. At the theoretical level, integration focuses on revealing the physical meaning of mathematical theories and methods, as well as demonstrating the role of mathematics in the formalization of physical laws. At the problem-practical level, integration is implemented through a system of interdisciplinary educational problems and projects that require the comprehensive application of both physical and mathematical knowledge.

The content of integrated physics and mathematics education is organized as a modular system, in which each module constitutes a logically complete unit that combines the physical and mathematical aspects of a specific problem area. Examples of such modules include vector representation of physical quantities (integration of mathematical topics on vectors with the study of vector quantities in physics), functional dependencies in physical processes (integration of the study of functions in mathematics with the study of physical laws), and differential and integral relationships in nature (integration of elements of mathematical analysis with the study of speed and acceleration, work, and energy in physics).

The content block places particular emphasis on the historical and scientific interrelationship between physics and mathematics, which is revealed through the study of key moments in the development of scientific thought, when advances in one discipline stimulated progress in the other. This enables students to develop an understanding of the unity of scientific knowledge and the role of interdisciplinary connections in the advancement of science. The history of science also serves as a rich source of problem situations that encourage students' critical thinking.

The information component of the content block encompasses a variety of sources used in the comprehensive study of physics and mathematics: educational texts, popular science literature, electronic educational resources, digital laboratories, and virtual modeling environments. Particular importance is attached to engaging with diverse sources of information, subjecting them to critical analysis and comparison. This contributes to the development of the information-retrieval component of critical thinking.

The organizational-activity block of the model defines the forms, methods, and technologies of teaching that facilitate the integration of physics and mathematics in the context of developing critical thinking. The principal forms of organizing the educational process include integrated lessons, which combine the content of physics and mathematics around a shared problem; laboratory workshops, which incorporate both experimental and computational-analytical elements; scientific seminars, where students present and discuss the results of their research; and project workshops, involving work on interdisciplinary projects.

The teaching methods employed within the framework of the integrative model include problem-based presentation (demonstration by the teacher of solving interdisciplinary problems with emphasis on critical thinking strategies), heuristic conversation (encouraging students to independently identify physical and mathematical patterns), the research method (involving the independent formulation and solution of problems at the intersection of physics and mathematics), and the project method

(focused on solving practice-oriented problems that require the combined application of physical and mathematical knowledge).

The technological component of the organizational-activity block comprises educational technologies that promote the development of critical thinking through the integration of physics and mathematics. These include technologies for developing critical thinking through reading and writing (adapted to the specifics of physics and mathematics), problem-based learning technologies (emphasizing interdisciplinary problems), research-based learning technologies (incorporating both theoretical and experimental approaches), and collaborative learning technologies (fostering the communicative aspects of critical thinking).

Particular attention in the organizational-activity block is devoted to digital tools and resources that expand the possibilities for integrating physics and mathematics: computer algebra systems (supporting the modeling of physical processes through mathematical apparatus); virtual laboratories (enabling computer-based experiments with mathematical processing of results); data visualization tools (developing students' skills in analyzing and interpreting graphical information); and educational platforms (providing access to interactive materials and enabling collaborative research activities).

The results-evaluation block of the model defines the criteria and indicators for assessing the effectiveness of integrating physics and mathematics in the context of developing critical thinking, as well as the methods for evaluating academic achievement. The principal criteria are the levels of development of the components of critical thinking: analytical, logical, evaluative, metacognitive, information-retrieval, and dispositional. For each component, a system of indicators has been developed to differentiate levels of development, ranging from elementary to advanced.

The methods used to assess outcomes include diagnostic tools for each component of critical thinking: tasks for analyzing arguments (to assess the analytical component); tasks for identifying logical errors (to assess the logical component); tasks for evaluating the reliability of information (to assess the evaluative component); metacognitive awareness questionnaires (to assess the metacognitive component); tasks for searching and analyzing information (to assess the information-retrieval component); and scales of critical thinking dispositions (to assess the dispositional component).

The successful implementation of the integrative model requires the creation of a set of pedagogical conditions to ensure the effective development of students' critical thinking through the integration of physics and mathematics. The study identified four groups of such conditions, differing in functional focus and content (Table 2).

The methodological toolkit for implementing the integrated model in the educational process of School-Lyceum No. 3, Taraz, comprises a set of educational and methodological materials and recommendations designed to ensure the effective application of the model in specific educational conditions. The toolkit was developed with consideration of the school-lyceum's educational program, available material and technical resources, and human capacity.

The main components of the methodological toolkit include: integrated curricula in physics and mathematics for grades 7–9, coordinated in both content and the sequencing of interrelated topics; educational and methodological complexes for integrated modules, incorporating theoretical materials, assignments for independent work, and methodological guidelines for teachers; collections of integrated tasks and exercises, differentiated by complexity level and aimed at developing the various components of critical thinking; methodological recommendations for organizing project-based and research activities on an interdisciplinary basis; and diagnostic tools for assessing students' levels of critical thinking development.

The toolkit further provides a system of educational tasks aimed at developing different components of critical thinking. The integrated use of these tasks ensures the systematic development of critical thinking by unifying physical and mathematical knowledge and methods of cognition (Table 3).

Table 2. Pedagogical conditions for the effective implementation of the integrative model of critical thinking development through the integration of physics and mathematics.

Group of conditions	Contents of the terms and conditions
Organizational and pedagogical	Creation of an integrated educational environment; development of interdisciplinary programs; coordination of the work of physics and mathematics teachers; flexible planning for the implementation of interdisciplinary projects
Psychological and pedagogical	Taking into account the age and individual characteristics of students; creating an atmosphere of intellectual freedom; stimulating cognitive motivation; ensuring psychological safety for expressing critical judgments
Didactic	Problematization of educational content; balance of reproductive and creative activity; gradual complication of tasks; introduction of special tasks for the development of critical thinking components
Methodical	Use of active and interactive teaching methods; diversity of forms of organization of educational activities; integration of formal and informal education; organization of interdisciplinary project activities

Note: Organizational and pedagogical conditions provide infrastructural and administrative support for integration processes; psychological and pedagogical conditions are aimed at creating a favorable environment for the development of critical thinking, taking into account age characteristics; didactic conditions determine the principles of selection and structuring of educational content; methodological conditions include technologies, methods, and forms of organization of the educational process.

Source: Compiled by the authors.

Table 3. Typology of educational tasks for developing critical thinking in the context of integrating physics and mathematics.

Task type	Characteristics and focus of tasks
Analytical study of physical and mathematical models	Analysis of the relationship between physical reality and its mathematical description; assessment of the adequacy and limits of applicability of models
Building and testing hypotheses	Formulating assumptions about the nature of physical laws and testing them using mathematical apparatus; developing inductive and deductive thinking
Detecting and correcting errors	Identification of logical and substantive errors in physical and mathematical reasoning; development of critical thinking and attention to the logical structure of argumentation
Interpretation of data	Working with information presented in various forms (tables, graphs, diagrams); translation from one form of presentation to another; identification of hidden patterns
Comparative analysis of different approaches	Comparison of various methods for solving physical and mathematical problems; assessment of their effectiveness; development of a multi-aspect vision of the problem
Metacognitive reflection	Awareness and analysis of one's own problem-solving strategies; identification of strengths and weaknesses of one's thinking; development of self-regulation of cognitive activity

Note: Each type of assignment is aimed at developing certain components of critical thinking, but together, they provide a holistic formation of critical thinking; the effectiveness of assignments increases with their systematic use and gradual complication, taking into account the age characteristics of students and the level of development of critical thinking; for each type of assignment, assessment criteria and methodological recommendations for their use in the educational process have been developed.

Source: Compiled by the authors.

For each type of assignment, methodological recommendations were developed for use in the educational process. These include learning objectives, guidelines for organizing students' work, criteria for evaluating outcomes, and methods for providing feedback.

An important element of the methodological toolkit is the assessment system aimed at fostering critical thinking. This system combines traditional forms of control (tests) with alternative forms of assessment: student portfolios (reflecting the dynamics of critical thinking development); expert evaluation of project and research work; self- and peer-assessment; and formative assessment, which provides detailed feedback on the strengths and weaknesses of students' thinking.

The introduction of the integrative model into the educational process of School-Lyceum No. 3, Taraz, is implemented in stages, with gradual expansion to cover more subjects and classes. At the initial stage, individual elements of the model are tested in experimental classes, enabling the identification and resolution of potential difficulties. At the main stage, the full-scale implementation of the model takes place across grades 7–9, with regular monitoring of outcomes. At the final stage, the effectiveness of the model is evaluated, and adjustments are made based on the results obtained.

Summarizing the characteristics of the developed integrative model, its key features may be highlighted as follows: systematicity: integration of all components into a unified whole; comprehensiveness: development of all components of critical thinking in interconnection; practice orientation: emphasis on solving real problems relevant to students; flexibility: adaptability to diverse educational contexts; technological effectiveness: provision of a clear implementation algorithm and well-defined criteria for evaluation.

Analysis of the various aspects of the integrative model allows its comprehensive characteristics to be presented in a systematized table that reflects the relationship between the model's components and their role in developing students' critical thinking (Table 4).

Table 4. Structural and functional characteristics of the integrative model of critical thinking development through the integration of physics and mathematics.

Model component	Component content	Functions in the development of critical thinking	Methods of implementation in the educational process
Conceptual and target	Methodological approaches, principles, goals, and objectives of the integration of physics and mathematics	Defining the strategic direction for the development of critical thinking through the integration of subjects	Designing integrated educational programs, formulating educational goals with an emphasis on developing critical thinking
Content and information	Integrated content of physics and mathematics education, system of interdisciplinary concepts and problems	Providing a substantive basis for the development of analytical and logical components of critical thinking	Development of integrated modules, creation of a system of interdisciplinary educational tasks, selection of problem situations
Organizational and activity-based	Forms, methods, and technologies of teaching that ensure the integration of physics and mathematics	Formation of procedural aspects of critical thinking through active cognitive activity	Application of problem-based, research, and project-based teaching methods, organization of scientific seminars and discussions
Result evaluative	Criteria and indicators of critical thinking development, methods of diagnostics, and evaluation of results	Ensuring monitoring and correction of the process of critical thinking development	Development of diagnostic tools, application of formative assessment, organization of reflective activities of students

Note: The components of the model and their content are adapted to the conditions of the educational process of School-Lyceum no. 3 Taraz, taking into account the age characteristics of students in grades 7–9 and the specifics of physics and mathematics education. Source: Compiled by the authors based on B. Arisoy and B. Aybek [16], M. Romero Ariza et al. [22], S. Sachdeva and P.O. Eggen [30], S. Dolapcioglu and A. Doğanay [18], S.I. Asigigan and Y. Samur [25].

The developed integrative model of critical thinking development through the unification of physics and mathematics is a theoretically substantiated and practice-oriented system designed to foster the ability of students in grades 7–9 to critically comprehend physical and mathematical problems. The model integrates key aspects of interdisciplinary learning and critical thinking development, creating a synergistic effect in which the combined content and methods of physics and mathematics serve as an effective means of cultivating higher-order thinking skills. The practical significance of the model lies in its adaptability to diverse educational contexts and its potential use as a theoretical and methodological foundation for the design of integrated educational programs aimed at developing students' critical thinking.

3.3. Results of testing the integrative model through the integration of physics and mathematics

The developed integrative model for the development of critical thinking through the unification of physics and mathematics was tested at School-Lyceum No. 3 in Taraz between September 2024 and January 2025. The study involved 78 students in grades 7–9. The pedagogical experiment was organized according to the classical scheme, comprising ascertaining, formative, and control stages. To ensure the reliability of results, an experimental group (40 students) and a control group (38 students) were formed, equivalent in terms of initial indicators of critical thinking development and academic performance in physics and mathematics.

At the ascertaining stage, primary diagnostics of students' critical thinking levels were conducted using a set of complementary methods. The principal diagnostic tool was a critical thinking test for secondary school students, developed as part of this study. The test consisted of 30 tasks aimed at assessing the components of critical thinking: analytical: tasks for identifying essential features and cause-and-effect relationships; logical: tasks for detecting logical errors and drawing conclusions; evaluative: tasks for assessing the reliability of information and sources; metacognitive: tasks for planning and evaluating problem-solving strategies; information-retrieval: tasks for locating relevant information; dispositional: tasks for identifying intellectual attitudes. Alongside testing, the expert assessment method was applied: teachers evaluated manifestations of students' critical thinking in the process of solving physics and mathematics problems using a specially developed scale. This assessment included observation of students' activity while solving problem tasks, analysis of written work, and evaluation of oral responses. In addition, a metacognitive awareness questionnaire was used to assess the development of the metacognitive component.

Before conducting the inferential analyses, the assumptions of normality and homogeneity of variance were examined. The Shapiro–Wilk tests indicated no statistically significant deviations from normality for either the pre-test or post-test scores in the experimental and control groups (all $p > 0.05$). Therefore, the null hypothesis of normality was not rejected, indicating that the distributions were sufficiently consistent with normality for the application of parametric procedures.

Levene's tests likewise showed no statistically significant differences in variance between the experimental and control groups for either the pre-test or post-test comparisons (all $p > 0.05$). Thus, the assumption of homogeneity of variance was considered satisfied. Taken together, these diagnostic results indicated no substantial violations of the assumptions underlying parametric statistical analysis and supported the subsequent use of paired-samples Student's *t*-tests for within-group comparisons and independent-samples Student's *t*-tests for between-group comparisons.

The results of the primary diagnostics indicated a predominance of medium and low levels of critical thinking in both groups. In the experimental group, 15.0% of students demonstrated a high level of development, 52.5% an average level, and 32.5% a low level. In the control group, the

distribution was similar: high level was 15.8%, average was 50.0%, and low was 34.2%. Statistical analysis using Pearson's χ^2 criterion confirmed the absence of significant differences between the groups at the ascertaining stage ($\chi^2 = 0.39$, $p > 0.05$), thus verifying the equivalence of the groups in terms of their initial level of critical thinking development.

A detailed analysis of the results for the components of critical thinking showed that the most developed among students in grades 7–9 were the analytical and information-retrieval components, while the greatest difficulties arose with tasks assessing the logical, evaluative, and metacognitive components. Typical challenges included the inability to identify hidden assumptions in argumentation, difficulties in distinguishing facts from opinions, problems in evaluating the reliability of information sources, and insufficient development of planning skills and self-regulation of cognitive activity.

A qualitative analysis of the results from the ascertaining stage also revealed several problems associated with the integration of physics and mathematics in students' thinking. Many perceived these subjects as isolated areas of knowledge and experienced difficulties in applying mathematical tools to solve physics problems. These difficulties were particularly evident when constructing and analyzing mathematical models of physical phenomena, as well as when interpreting graphical dependencies. These findings confirmed the relevance of the developed integrative model and the necessity of testing it in the educational process.

At the formative stage of the experiment, the integrative model of critical thinking development through the unification of physics and mathematics was implemented in the experimental group. The educational process was organized in accordance with the conceptual provisions of the model and incorporated all its structural components: conceptual target, content information, organizational activity, and result evaluation. By contrast, in the control group, physics and mathematics continued to be taught within the framework of the traditional subject-based model, without emphasis on interdisciplinary integration or the targeted development of critical thinking. An important element of the formative stage was the introduction of an integrated physics and mathematics practical course, in which students of the experimental group completed complex research tasks requiring the combined application of physical and mathematical knowledge. The topics of the practical course were aligned with the curricula in physics and mathematics and included sections such as kinematic regularities and their graphical representation, geometric optics and geometric transformations, and oscillatory processes and trigonometric functions.

During the practical training, students conducted experiments to investigate physical phenomena, collected and analyzed data, constructed mathematical models of the processes under study, and tested their adequacy and limits of applicability. Particular attention was given to developing skills in critically analyzing research results, identifying potential sources of error, assessing the reliability of obtained data, and formulating well-substantiated conclusions.

In addition to practical training, the experimental group participated in integrated physics and mathematics lessons, in which interrelated topics were studied in unity. When designing these lessons, various forms of integration were applied: substantive (combining physical and mathematical content around a common problem), methodological (using common methods of inquiry), and procedural (organizing joint student activities aimed at solving interdisciplinary problems). Another important element of the formative stage was the organization of project-based activities on an interdisciplinary basis. Students in the experimental group developed and implemented projects aimed at solving practical problems requiring the integration of physical and mathematical knowledge. The topics of the projects reflected issues relevant to the region, such as the study of the energy efficiency of various building materials in the climatic conditions of Taraz,

mathematical modeling of solar activity for the design of energy-saving systems, and acoustic analysis of noise pollution in different areas of the city of Taraz.

To foster the metacognitive component of critical thinking in the experimental group, special techniques were applied to stimulate students' reflective activity. These included keeping individual diaries, where students recorded the problem-solving process, difficulties encountered, and strategies for overcoming them; applying the "thinking aloud" method when solving problem-based tasks; organizing reflective pauses during lessons to analyze personal thinking strategies; and holding group reflective sessions, in which students discussed the effectiveness of different approaches to problem-solving.

To develop the dispositional component of critical thinking, a special communicative environment was created that fostered openness of thought, intellectual honesty, and a willingness to reconsider views in the light of new data. This environment was established through the organization of scientific discussions, seminars, and round tables, during which students presented the results of their research, asked questions, offered critical comments, and defended their positions. At the control stage of the experiment, repeated diagnostics of students' critical thinking levels were conducted in both the experimental and control groups using the same set of methods as at the ascertaining stage. This enabled the identification of developmental dynamics and the evaluation of the effectiveness of the integrative model.

The results of the repeated diagnostics revealed significant positive changes in the experimental group. The proportion of students with a high level of critical thinking increased from 15.0% to 40.0%, while the proportion at the average level shifted from 52.5% to 47.5%, and those at the low level decreased from 32.5% to 12.5%. Positive changes were also observed in the control group, but they were less pronounced: the proportion of students with a high level of critical thinking rose from 15.8% to 21.1%, the average level from 50.0% to 55.3%, while the low level decreased from 34.2% to 23.6%.

Statistical analysis using Pearson's χ^2 criterion confirmed the significance of the differences in developmental dynamics between the experimental and control groups ($\chi^2 = 13.6$, $p < 0.01$), indicating the effectiveness of the integrative model. To assess the magnitude of the effect, Cramer's coefficient was calculated; the value ($\phi = 0.33$) corresponds to a medium effect size according to the generally accepted classification. Analysis of the dynamics of individual components of critical thinking showed that the most significant positive changes in the experimental group occurred in the logical, evaluative, and metacognitive components, which had been the least developed at the ascertaining stage. These results confirm the effectiveness of the teaching methods and techniques applied within the integrative model, particularly in enhancing these components of critical thinking.

A qualitative analysis of the results demonstrated that students in the experimental group achieved a higher level of development in analytical skills, logical argumentation, value judgments, and metacognitive reflection. They performed more successfully in tasks requiring the integration of physical and mathematical knowledge, showed greater independence in planning and conducting research, and exhibited a deeper understanding of the interrelationships between different aspects of the phenomena under study.

Marked differences between the experimental and control groups were observed in students' ability to construct and analyze mathematical models of physical phenomena, interpret experimental data, and critically evaluate the reliability of information. Students in the experimental group were more successful in identifying hidden assumptions in argumentation, distinguishing between facts and opinions, assessing the credibility of information sources, and formulating well-substantiated conclusions based on data analysis.

An important outcome of testing the integrative model was a positive shift in students' attitudes toward studying physics and mathematics. A survey conducted at the end of the experiment revealed that students in the experimental group reported greater interest in these subjects (45% more positive responses compared with the control group), a better understanding of their interconnections and practical significance (37% more positive responses), and increased confidence in their scientific abilities (33% more positive responses).

The positive effect of the developed model was also reflected in the improved academic performance of students in the experimental group in physics and mathematics. The average score in these subjects increased from 3.8 to 4.3 (an increase of 0.5 points), whereas in the control group, the improvement was less pronounced, from 3.7 to 3.9 (an increase of 0.2 points). This finding indicates that the integration of physics and mathematics not only fosters the development of critical thinking but also contributes to a deeper assimilation of subject content.

In addition, an analysis was conducted to examine the impact of the integrative model on the development of critical thinking among students of different age groups (grades 7, 8, and 9). The results showed that the greatest effectiveness of the model was observed in grades 8 and 9, which can be explained by students' higher levels of abstract and logical thinking at this stage, and by their greater readiness to master complex interdisciplinary connections. In grade 7, the model proved somewhat less effective, which is attributable to age-related features of cognitive development and highlights the need to adapt methodological approaches accordingly.

For a more detailed analysis of the model's efficiency, a comparison was made of the dynamics of the different components of critical thinking in the experimental group. The results are presented in Table 1, which contains data on the development of critical thinking components in the experimental and control groups.

A comprehensive assessment of the model's efficiency requires consideration of results across all components and age groups. Systematization of the data obtained during the pedagogical experiment makes it possible to identify patterns in the dynamics of students' critical thinking development and to evaluate the effectiveness of the pedagogical approaches applied. A comparative analysis of the dynamics of critical thinking components in the experimental and control groups is presented in Table 5.

The preliminary checks of normality and homogeneity of variance supported the use of parametric statistical procedures for comparing the pre-test and post-test indicators. The results presented in Table 5 demonstrate a clear superiority of the experimental group over the control group across all components of critical thinking. The most substantial increases in the experimental group were observed in the metacognitive (37.9%), logical (34.4%), and evaluative (32.8%) components, confirming the effectiveness of the teaching methods and techniques employed within the integrative model for developing these aspects of critical thinking. Smaller gains were recorded in the information-retrieval (20.0%) and analytical (20.5%) components, which can be attributed to their relatively higher initial levels of development, and thus a reduced potential for growth.

Statistical analysis using Student's t-test confirmed the significance of the differences between the experimental and control groups in all components of critical thinking ($p < 0.01$), indicating that the differences were not random and verifying the effectiveness of the integrative model. The effect size, calculated using Cohen's d , ranged from medium ($d = 0.62$ for the information-retrieval component) to high ($d = 0.87$ for the metacognitive component), thereby affirming the practical significance of the findings.

Table 5. Comparative analysis of the dynamics of critical thinking component development in students of grades 7–9 as a result of testing the integrative model.

Critical thinking component	Experimental group			Control group			Statistical significance of differences
	Before the experiment (average score)	After the experiment (average score)	Growth (%)	Before the experiment (average score)	After the experiment (average score)	Growth (%)	
Analytical	7.3	8.8	20.5	7.4	7.9	6.8	4.18 (p<0.01)
Logical	6.1	8.2	34.4	6.2	6.8	9.7	5.32 (p<0.01)
Evaluation	6.4	8.5	32.8	6.3	7.0	11.1	4.85 (p<0.01)
Metacognitive	5.8	8.0	37.9	5.9	6.4	8.5	5.88 (p<0.01)
Information retrieval	7.0	8.4	20.0	7.1	7.6	7.0	3.71 (p<0.01)
Dispositional	6.7	8.3	23.9	6.8	7.3	7.4	4.09 (p<0.01)
Overall indicator	6.6	8.4	28.2	6.6	7.2	8.4	5.39 (p<0.01)

Note: The average score is calculated on a 10-point scale for each component based on the results of the comprehensive diagnostics; the increase is indicated as a percentage and calculated using the formula: $(\text{post-test score} - \text{pre-test score}) / \text{pre-test score} \times 100\%$; the statistical significance of differences was assessed using the Student's t-test for dependent samples.

Source: Compiled by the authors.

A correlation analysis was also conducted between the indicators of critical thinking development and academic performance in physics and mathematics. The results revealed a significant positive correlation between the overall critical thinking indicator and performance in both physics ($r = 0.67$, $p < 0.01$) and mathematics ($r = 0.70$, $p < 0.01$). The strongest correlations with academic achievement were demonstrated by the logical component ($r = 0.72$ for mathematics; $r = 0.64$ for physics) and the metacognitive component ($r = 0.68$ for mathematics; $r = 0.63$ for physics). These findings confirm the crucial role of critical thinking in the successful mastery of physics and mathematics and highlight the mutual reinforcement of critical thinking development and subject learning within the framework of the integrative model.

A qualitative analysis of the results of testing the integrative model made it possible to identify the factors that determine its effectiveness. These include the problematization of educational content through the creation of interdisciplinary problem situations; the organization of active research activities at the intersection of physics and mathematics; the creation of a communicative environment that stimulates critical discourse; the use of reflective practices to develop the metacognitive component of critical thinking; and the application of formative assessment, which provides constructive feedback on both the process and outcomes of students' cognitive activity.

Alongside the positive results, certain difficulties in implementing the integrative model were identified. These relate to the need for additional methodological training of teachers to ensure effective integration of physics and mathematics; challenges in coordinating curricula across different subjects; insufficient development of didactic materials for integrated learning; and organizational difficulties in conducting integrated lessons. These limitations highlight the need for further refinement of the model and the development of additional methodological support for its effective implementation.

Testing of the integrative model of critical thinking development through the integration of physics and mathematics showed positive changes in the experimental group compared with the control group. However, these results should be interpreted cautiously because the study was

conducted in one school, with a relatively small sample and within a limited intervention period. The observed improvements indicate an association between participation in the integrative learning model and higher post-test indicators of critical thinking, but they do not allow strong causal claims. The most pronounced improvements were observed in the metacognitive, logical, and evaluative components. These gains may be related to the targeted reflective, argumentative, and evaluative tasks used in the intervention, but they may also partly reflect the lower initial levels of these components and, therefore, a greater room for improvement. The correlations between critical thinking indicators and academic performance should likewise be interpreted as evidence of association, not causation.

4. Discussion

The results indicate that the integrative physics–mathematics model was associated with positive changes in students' critical thinking indicators, especially in components related to reflection, reasoning, and evaluation. These changes may be explained by the structure of the intervention, which required students to justify mathematical models, evaluate empirical data, and reflect on their problem-solving strategies. Therefore, the discussion focuses not on repeating statistical outcomes but on interpreting the pedagogical mechanisms that may have supported these changes.

The stronger dynamics in the metacognitive, logical, and evaluative components may be explained by the structure of the intervention, which required students to justify mathematical models, evaluate empirical data, and reflect on their problem-solving strategies.

In the context of critical thinking development in physics lessons, the outcomes show significant parallels with the study of I.H. Wenno et al. [37], who designed and tested physics teaching tools aimed at fostering critical thinking. Their results demonstrated a substantial increase in students' critical thinking levels. Comparison with the present study reveals similar trends regarding the effectiveness of specially designed methodological tools. In particular, Wenno et al. emphasized the value of tasks that require the analysis of physical phenomena and processes, including the establishment of cause-and-effect relationships and the evaluation of information reliability. This correlates with the substantial increases recorded in the analytical (20.5%) and evaluative (32.8%) components of critical thinking in the experimental group of the current study. However, it should be noted that Wenno et al.'s work focused exclusively on physical content, without integration with mathematics, which represents a limitation compared with the comprehensive integrative approach adopted here. Furthermore, their study did not differentiate between the components of critical thinking, making a detailed comparison difficult.

When considering the relationship between students' and teachers' critical thinking and achievement in physics, reference should be made to the study of X. Ma et al. [38], which established a significant correlation between the levels of teachers' and students' critical thinking and academic achievement in physics. These findings are consistent with the present study, where critical thinking indicators were positively associated with academic achievement in physics and mathematics, although this relationship should be interpreted as associative rather than causal. This relationship highlights the importance of targeted critical thinking development to improve academic performance. However, Ma et al.'s research is limited by its treatment of critical thinking as a holistic construct without distinguishing its individual components, which makes it impossible to identify which aspects of critical thinking are most strongly associated with achievement in physics. By contrast, the current study demonstrates that the logical and metacognitive components of critical

thinking show the highest correlations with academic performance, thereby allowing for a more targeted design of the educational process.

The study by M. Jamil et al. [39] on the analysis of physics curricula in relation to the development of critical thinking for the 21st century merits particular attention. The authors highlight the insufficient integration of critical thinking aspects into existing physics programs and emphasize the need to modernize them, in line with contemporary requirements. Compared with these findings, the proposed integrative model for developing critical thinking through the unification of physics and mathematics can be considered as one pathway for curriculum modernization. A significant advantage of the present study is the practical testing of the model with measurable outcomes, whereas Jamil et al.'s work is largely theoretical and lacks empirical validation of the proposed recommendations.

In terms of specific educational strategies, the study by N. Mafarja et al. [40] is of interest, focusing on the use of peer learning methods to develop students' critical thinking in physics. Their findings indicate that peer learning significantly enhances critical thinking, particularly in the domains of analysis, evaluation, and explanation. A comparison with the current study reveals similar trends regarding the effectiveness of active learning methods in fostering critical thinking. In the present experiment, problem-based learning, research activities, and the project method played a crucial role, leading to a significant increase in critical thinking indicators within the experimental group. In this respect, the peer learning strategy identified by Mafarja et al. may be viewed as a potential supplementary tool for enriching the integrative model. However, Mafarja et al.'s study is limited to a single strategy, while the present work adopts an integrated approach that combines multiple methods and forms of organizing the learning process, each adapted to different components of critical thinking. These findings resonate with the conclusions of D.T. Tiruneh et al. [41], who stressed that the most effective educational environments for cultivating critical thinking are those that integrate several pedagogical approaches and ensure active student engagement in analysis, argumentation, and problem-solving.

An important dimension of critical thinking development is the role of the teacher. The study of M.R. Ridwan et al. [42] investigated teachers' perceptions of the application of critical thinking skills in secondary mathematics education. The authors found that, although most teachers recognize the importance of developing critical thinking, they face difficulties in incorporating appropriate tasks into their teaching. These findings are consistent with the challenges identified in the present study, particularly the need for additional methodological training for teachers to effectively integrate physics and mathematics, and the lack of adequately developed didactic materials for integrated instruction. At the same time, the current study contributes practical solutions by proposing the development of methodological tools and a system of teacher training for the implementation of the integrative model, thereby offering applied value beyond the more descriptive focus of Ridwan et al.

The results of the present study on the effectiveness of integrating physics and mathematics for developing critical thinking are supported by the work of E.A. Afriansyah et al. [43] on fostering critical thinking in mathematics. The authors found that tasks requiring the application of mathematical knowledge to model real-life situations are particularly effective in developing critical thinking. This aligns with the findings of the current study, which demonstrated the high effectiveness of simulated physical situations requiring mathematical analysis for enhancing critical thinking. At the same time, the present study adopts a more comprehensive perspective, as it addresses the two-way integration of physics and mathematics. In contrast, Afriansyah et al. focused mainly on applying mathematics to real-life contexts, without exploring the reciprocal influence of

the physical context on mathematical thinking. Furthermore, their study does not propose a specific integration model, limiting itself to general recommendations regarding the use of contextual tasks.

With regard to innovative educational technologies, the study by Y. Sukma and N. Priatna [44] on the effectiveness of blended learning in developing students' critical thinking in mathematics education is of interest. Based on a literature review, the authors concluded that blended learning supports critical thinking by combining the advantages of face-to-face and online learning. These findings are partly consistent with the present study, which also confirmed the effectiveness of digital tools and resources for developing critical thinking. However, the current research offers a more differentiated approach by considering the effects of different types of digital resources on individual components of critical thinking. By contrast, Sukma and Priatna treated blended learning as a single construct without a detailed analysis of its elements. Moreover, their conclusions are based solely on a literature review, lacking empirical validation, which reduces the strength of their findings compared with the experimentally tested results of this study.

Comparison can also be made with the work of D.S. Setiana et al. [45], who examined the use of a mathematical teaching model to develop critical thinking in high school students. Their model, which incorporated the stages of orientation, organization, research, presentation, and evaluation, significantly enhanced students' critical thinking. These findings are consistent with the results of the current study regarding the effectiveness of the organizational-activity component of the integrative model, which engages students in active research and project-based learning. At the same time, the present study offers a broader scope, integrating both physics and mathematics, whereas Setiana et al. focused solely on mathematical content. In addition, while their research was conducted among high school students, the present study addressed students in grades 7–9, thereby providing a broader understanding of age-related characteristics in the development of critical thinking.

The study by S. Prayogi et al. [46] on the critical inquiry-based learning model for training future physics teachers deserves particular attention. The authors developed and tested a model based on critical inquiry-based learning, which proved highly effective in developing critical thinking skills in students preparing to become physics specialists. A comparative analysis with the present research highlights conceptual similarities in the emphasis on developing critical thinking through research activities. In both cases, students' active engagement in problem situations requiring analysis, evaluation, and data interpretation plays a central role. However, there are notable differences: Prayogi et al. focused on training future physics teachers, whereas the current study targeted the development of critical thinking among students in grades 7–9. Furthermore, their model does not consider the integration of physics and mathematics, concentrating solely on critical thinking in the context of physics education. Nonetheless, the model proposed by Prayogi et al. is valuable for advancing teachers' professional competencies, which is a crucial condition for the successful implementation of the integrative model developed in the present study. Particularly useful is their detailed design of the stages of critical inquiry, which could be adapted for secondary school students.

In relation to the role of teachers in developing students' critical thinking, the study of M. Jamil et al. [47] on science teachers' perceptions and practices is noteworthy. The authors revealed a significant gap between teachers' recognition of the importance of critical thinking and their practical ability to foster it. While most teachers acknowledged its importance for academic and personal development, they faced challenges in selecting and applying appropriate pedagogical strategies. These findings resonate with the present study, which also identified difficulties teachers encounter when implementing an integrative model of critical thinking. The key distinction, however, is that Jamil et al.'s research is primarily descriptive, whereas the current study proposes a specific

methodological model, including tools for preparing teachers to support the development of critical thinking. At the same time, Jamil et al.'s identification of barriers to effective practice provides valuable input for improving training programs and professional development for physics and mathematics teachers. Similar issues were identified by Y.S.M. Tan and I.S. Caleon [48], who emphasized that, despite strong awareness of the importance of critical thinking, many science teachers struggle to select and apply effective pedagogical practices. Their findings underline the need for targeted teacher training, which fully aligns with the methodological recommendations advanced in this study.

Compared with previous STEM and problem-based learning studies, the present study suggests that the effect of integration may depend not only on the interdisciplinary character of tasks but also on the explicit connection between task type and the targeted component of critical thinking. However, unlike broader multi-school studies, the present research was limited to one educational context, which reduces the strength of generalization.

Several alternative explanations should be considered when interpreting the findings. First, teacher effects may have influenced the outcomes, since the intervention depended on the organization of classroom interaction and the teacher's ability to support interdisciplinary reasoning. Second, maturation effects cannot be fully excluded because students in grades 7–9 are in a period of active development of abstract and logical thinking. Third, the experimental group received additional attention through integrated tasks, projects, and reflective activities, which may have contributed to motivation and engagement independently of the specific physics–mathematics integration. Fourth, the use of researcher-developed assessment tools and teacher-based evaluations may have introduced assessment bias. For these reasons, the findings should be understood as preliminary evidence of the potential of the proposed model rather than definitive proof of its effectiveness.

Since the study was conducted in one school with a relatively small sample, the findings cannot be generalized to all lower secondary schools without further verification. The results should be interpreted as context-specific evidence, and replication in schools with different organizational, regional, and socio-educational conditions is required.

In summary, the comparative analysis shows that the present study on developing critical thinking through the integration of physics and mathematics shares commonalities with existing research while also demonstrating significant distinctions. General conclusions confirm the effectiveness of active learning methods, research-oriented approaches, and problem-based learning in developing critical thinking, as well as the positive relationship between critical thinking and academic performance. The main differences lie in the comprehensive integrative approach, the differentiation of critical thinking components, and the experimental validation of the model's effectiveness using statistically robust methods.

5. Conclusions

The study examined the implementation of an integrative physics–mathematics model for developing critical thinking in students in grades 7–9. The results showed greater positive changes in critical thinking indicators in the experimental group than in the control group. These findings suggest that the integrative model may support the development of critical thinking in the studied educational context.

The most pronounced dynamics were observed in the metacognitive, logical, and evaluative components of critical thinking. However, these findings should be interpreted cautiously, since these

components had lower initial values and therefore greater potential for improvement. The relationship between critical thinking indicators and academic performance in physics and mathematics should be understood as associative rather than causal.

The research identified a set of pedagogical conditions necessary for the effective integration of physics and mathematics in educational projects. Foremost is the creation of an integrated educational environment equipped with modern experimental and digital tools that enable comprehensive physical and mathematical investigations. Equally important is the development of integrated curricula and teaching materials, coordinated in content and timing across both disciplines. Consideration of the age-related and individual characteristics of students in grades 7–9, who are in a phase of active formation of abstract and logical thinking, was found essential, along with the differentiation of tasks by complexity. A key factor is the demonstration of cognitive value, achieved by illustrating the practical significance of integrated physical and mathematical knowledge in addressing real-world and everyday problems.

The observed positive dynamics may be related to the systematic use of active and interactive teaching methods, including problem-based, research, and project-based approaches, with an emphasis on students' independent cognitive activity. A notable outcome of the study was the development of methodological tools, including a typology of educational tasks tailored to different components of critical thinking and a comprehensive assessment system combining both traditional and alternative forms of evaluation. This toolkit was successfully tested under real educational conditions and showed potential for supporting integrated physics–mathematics learning.

The practical significance of the results lies in their potential for direct implementation in the educational process of schools in Kazakhstan, thereby improving the quality of science education and fostering critical thinking among students. The developed materials can also be used in the training and professional development of physics and mathematics teachers, contributing to the formation of their competencies in integrated teaching.

The study has several limitations, including the small sample, single-school setting, relatively short intervention period, and the use of researcher-developed assessment tools. Teacher effects, maturation, additional attention received by the experimental group, and possible assessment bias cannot be fully excluded. Therefore, the conclusions should be regarded as preliminary and context-specific. Future research should test the model in several schools, use larger samples, provide an independent assessment of students' work, report inter-rater reliability, and examine whether similar results can be obtained over a longer period.

Author contribution

Maikul Kusherbayeva: Conceptualization, methodology, supervision, and project administration. Alma Abylkasymova: Investigation, field data collection, and veterinary examinations. Sofiya Yegemberdiyeva: Laboratory diagnostics, validation, and data curation. Bibara Kushkimbayeva: Formal analysis, statistical analysis, visualization, and interpretation of the results. Yeskermes Mussabekov: Literature review, writing of the original draft, and review and editing of the manuscript. All authors contributed to the interpretation of the findings, critically reviewed the manuscript, approved the final version, and agreed to be accountable for the integrity of the work.

Use of Generative-AI tools declaration

The authors declare the use of Artificial Intelligence (AI) tools in preparation of this article, exclusively for refining the grammar, structure and spelling. All intellectual content, data interpretation, and critical analysis were conceived and developed by the human author.

Conflict of interest

The authors declare that there is no conflict of interest in this manuscript.

Ethics declaration

All procedures performed in the study were in accordance with the ethical standards of the institutional research committee and with the 1964 Helsinki Declaration and its later amendments.

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