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Research article

Wavelet-based systematic risk estimation for GCC stock markets and impact of the embargo on the Qatar case

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Abstract: Systematic risk is one of the well-known indices involved in the market situation study. One of the disadvantages of scientific studies of market indices is the lack of involving extreme changes such as embargos and other crises in the model. The present paper attempts to study the impact of the embargo on systematic risk using wavelets as a mathematical-statistical tool. The proposed mathematical model was applied to the case of the Golf Council Countries (GCC) market, with the Qatar case as an example of an embargoed country. The time series applied corresponds to the Qatar stock exchange index active trade over the period January 01, 2017, to December 31, 2021, which was characterized by the main GCC embargo period against Qatar. The findings in the present work permit understanding the impact of such a crisis on the market and allow a good description of the behavior of the market during the embargo, which makes a good basis for managers, policymakers, and investors.

Keywords: wavelets CAPM; systematic (market) risk; scaling; GCC stock markets; Qatar exchange All Shares (QEAS); Embargo

JEL Codes: G11, C02, C22

1. Introduction and motivations

In this work, the wavelet theory is applied to estimate the systematic risk, known also as the capital asset pricing model (CAPM), for one of the leading markets in the GCC region, the Qatar market, based on its stock exchange, abbreviated as QEAS. Our principal aim is to study the impact of embargoes in general on systematic risk. Recall that the GCC region is one of the most important areas in the world as it constitutes the largest energy supplier to the other parts of the world by owning the largest natural reserves of petroleum and gas. Due to crises such as embargoes, financial indicators represented by their time series are subject to volatility and high fluctuations. Sophisticated statistical and mathematical tools are needed to describe these markets well. Wavelet theory has been proven to be a powerful tool in the economic/financial field by its ability in localizing and analyzing fluctuations and volatility. Qatar is one of the important representative markets in the GCC region and also in the world. It is an important exporter of gas, and it also has a strategic geographical position which makes it an important station for the exchange between oriental and occidental continents.

In the present paper, we propose not to develop in detail a literature review on the systematic risk or the capital asset pricing model, as this was done in a huge number of studies. However, we refer to many basic studies for young researchers to be familiar with the field of the CAPM (Sharpe (1964); Lintner (1965b); Mossin (1966); Black (1972b); Black et al. (1972a); Breeden (1979); Roll (1977)). Advanced researchers may refer to other recent and updated studies using wavelets and other tools for advanced researchers (Fama and French (1996); Kothari and Shanken (1998); Fama and French (1993); Aydogan (1989); Banz (1981); Basu (1977); Chae and Yang (2008); Chan and Lakonishok (1993); Fama and French (2004, 2006); Galagedera (2007); Gursoy and Rejepova (2007); Handa et al. (1989, 1993); Ho et al. (2000); Karan and Karadagli (2001); Merton (1973); Perold (2004)).

Some empirical extensions of the CAPM taking into account the time variations of beta and time variations of the risk premium or both of them have also been developed: Marfatia (2017a,b); Ben Mabrouk (2020); Ben Mabrouk et al. (2008, 2010).

The principal aim of the present paper is to investigate the role of the time factor in the estimation of systematic risk in critical periods such as embargoes. We thus aim to show that the involvement of the time scale in the mathematical model may explain better the economic situation in the severe moment of the embargo. This is very important in economic and financial studies as it permits the different agents related to the market to get suitable decisions. Agents may be managers, policymakers, investors, and also consumers. The main mathematical idea is the use of non-uniform wavelets adapted to non-uniform time scales and non-uniform sub-periods in the whole period of study to understand or to express accurately the impact of the time scale in the model. We, therefore, try, in this work, to provide a better comprehension of the Qatar GCC financial markets in the face of a modern financial theory such as the CAPM.

Qatar is a very particular place as it takes a strategic geographical position as a point of linkage between the Far East and the West. It is also one of the main exporters of gas and also a main consumer country of many products from both east markets such as China and the West such as the USA. Also, especially after the last embargo by other GCC countries and Egypt, Qatar has implemented many new industries and plans to overcome the embargo and to transform from a consumer to a producer country. In particular, due to the effects of the recent Russia/Ukraine war, Qatar has become an essential destination for many European countries to fill the needs and the shortages in liquefied gas. Many contracts have been signed in this context. Also, in a competitive framework, GCC countries have each planned a vision such as the KSA 2030-vision and Qatar 2030-vision in order to create advanced countries capable of sustaining their own development and providing a high standard of living for all their people. Particularly, Qatar has invested heavily in creating a world-class education sector and high-tech logistics hubs. This vision will have a direct impact on the national market as well as the rest of the Arab Gulf and international markets. This makes it of interest to study such a market, understand its complexities and test its capability to resist crashes and crises such as the last embargo.

The proposed procedure acted on samples composed of stocks in the Qatar market traded actively over a critical period strongly and directly related to the recent embargo. The period of study is also characterized by the recent pandemic COVID-19, which had strong effects on nearly all the worldwide markets and economies. Qatar as part of the GCC area gathers the largest workers' community. The sample of the study was based on the Qatar stock exchange index QEAS actively traded over the period January 01, 2017, to December 31, 2021, which is characterized by the main GCC embargo period against Qatar.

As Qatar is a principal element of the GCC region, the study of its case may be compared to those based on other elements from the same region such as Saudi Arabia. Indeed, in Ben Mabrouk (2020), the author considered a wavelet variant of the CAPM designated for the exploitation of the Saudi market. The main novelty in Ben Mabrouk (2020) may be resumed in the use of non-uniform (non-fixed or dynamic) intervals of time and non-uniform wavelets, differently from existing literature. So, in the present work, we use non-uniform wavelets in addition to non-uniform time intervals. Also, in the case of the Saudi Tadawul market, a somehow and quite common point may be in the period of study as the end of the period due to Ben Mabrouk (2020) intersects the present one as the outbreak of the Qatar embargo.

Ben Mabrouk (2020) proposed to estimate the CAPM for the Saudi TADAWUL as a GCC representative market by using wavelets to explore the time scale impact. In Ben Mabrouk (2020), a second point of difference consisted of a wavelet method to overcome the problem of missing data by providing a prediction procedure able to predict a short time interval series on an arbitrarily small set of backward and/or forwards (past and/or future, prior and/or post) values. In the present work, we improve Ben Mabrouk (2020) by applying non-uniform scales in both the time intervals and the wavelets.

In this article, we essentially explored the complexities of the systematic risk beta during times of crisis, especially the embargo in the framework of the CAPM. We analyze time series data due to Qatar stock market QEAS to understand the impacts of the embargo on the CAPM estimation of the systematic risk beta. The examination of the effect(s) of the embargo on the risk beta will affect in the role of investor sentiment during such a period of uncertainty. The main idea relies on the use of non-uniform time scales and non-uniform wavelets to estimate the risk beta. The contribution of the study, as well as the theoretical support, may be described in a few points:

- The use of non-uniform time scales for the time subdivision of the period of study.
- The use of non-uniform wavelets for the exploration of the mathematical model of CAPM.
- These points induce together a non-uniform systematic beta contrary to existing works where the wavelets or the time intervals do not apply the non-uniform principle (or both of them as in the classical CAPM).
- The fit or the impact of the crises, especially non-prevented shocks, are more adequately modeled by using the non-uniform principle in both the time supports and in the wavelet functional/statistical tool.

The rest of the present work is organized as follows. In section 2, a literature review on the CAPM is developed. Variants including the time scale and wavelets are reviewed. Section 3 is concerned with the development of our main contribution based on the non-uniform wavelet CAPM. Section 4 is devoted to the empirical results and their discussions. Section 5 is a conclusion. Section 6 is an appendix concerned with a brief review of the wavelet tool.

2. Literature review

The CAPM was introduced as a model for estimating systematic risk. It was initially formulated by Sharpe (1964) and next revised or improved in Lintner (1965a) and Lintner (1965b). Next, even though the CAPM has been dominant as an estimator of the capital cost for companies, and the performance of managed portfolios, it has suffered from many drawbacks from both theoretical failures due to its assumptions, and in the empirical evaluation due to the problems of data availability, accuracy, and exactness.

Markowitz (1952) proposed to include the expected return on investments and the portfolio risk to get an optimal portfolio by including many stocks based on profitability. This profitability factor was rejected by Vasichek et al. (1972), where the authors claimed that the dispersion of returns may reflect an uncertainty in the risk. This is why the variance of the profitability was proposed as a new measure for the total risk.

Many variants of the CAPM have been proposed in the literature of economic/financial studies according to the limitations confronted. Brennan (1973) and Litzenberger and Ramaswamy (1979) proposed the CAPM with taxes to overcome the assumption of market perfection aspect without taxes. Lévy (1978) proposed the CAPM with transaction costs to answer the drawback of the market perfection aspect without transaction costs. The CAPM in continuous time was developed in Merton (1973) to deal with the non-necessity of mono-periodicity. In Sharpe (1970a,b,c), other variants were proposed to deal with the CAPM with non-homogeneous anticipations. These studies show among other causes the necessity of more investigations of the systematic risk, and that the problem is still open for investigations.

In Sharpe (1964) and Lintner (1965b), the authors claimed that the CAPM may relate the supply/demand activities to the prices of equilibrium securities via a linear dependence of profitability and total risk. The authors therefore proposed the estimation of the prices of transferable securities which permits balanced supply/demand, and yields an eventual market equilibrium.

Mathematically or quantitatively speaking, the CAPM is based on the estimation of the linearity coefficient usually known as the systematic risk beta of the portfolio components to the market risk. For a portfolio composed by many actions, the systematic risk β_i relative to an action (asset) i is evaluated as the ratio of the covariance between the rate of return of the action (asset) i and the rate of return of the portfolio by the square of market risk (see Sharpe (1964), Sharpe (1970b), Vasichek et al. (1972)).

In the literature, there have been many critics of the CAPM. Roll (1977) related criticisms of the representative portfolio of the market which groups assets due to investments, stocks, bonds, real estate and human capital. Desmoulins-Lebeault (2003) explored the eventual relationship between the ineffectiveness of the CAPM and the non-normality of the returns distribution. Magni (2007a) concluded that a project is profitable whenever its internal rate of return is greater than the risk-adjusted cost of capital.

Since approximately the two last decades, there has been an interest of including the time scale into the CAPM to get a systematic risk beta relative to many horizons. Many studies such as Fernandez

(2006) on the Chile's Santiago stock market, Rhaiem, Ben Ammou and Ben Mabrouk (2007a) and Rhaiem, Ben Ammou and Ben Mabrouk (2007b) on the French CAC40 showed that the time scale or factor has a great impact on the estimate of beta. Aktan et al. (2009), Levhari and Lévy (1977), Marfatia (2014, 2015, 2017a,b, 2020) and Handa et al. (1993) concluded that the time factor in the interval of returns may induce different estimators. This rejected the idea that beta depends solely on the entire period and thus relatively stable to the scale.

In Maghyereh and Abdoh (2021) the authors investigated the eventual impact of oil price shocks on bank risk in the GCC countries over the period from January 2006 to September 2020 by using both conditional value-at-risk (CoVAR) and marginal expected shortfall to capture market-based systemic risk. The authors concluded that oil supply shocks may be considered as major drivers behind increases in the GCC members' bank risk rather than oil demand shocks. Also, related to our idea on the importance of the time scale or the time factor, the authors claimed that the change in bank risk in response to shocks is strongly related to time periods. In particular, factors such as those considered here and also in Ben Mabrouk (2020) and dealing with financial crises and the COVID-19 pandemic strongly affect the bank risk. See also Maghyereh et al. (2022).

Compared to our present work, Masih et al. (2010) applied also wavelet techniques for the estimation of the systematic risk beta in the GCC region. The authors concluded as usual that a multiscale tendency is observed in the beta according to time scale. The main difference with our work here and to the one due to Ben Mabrouk (2020) is that both wavelets and time periods are uniform. Also, the authors in Masih et al. (2010) analyzed the impact of time scales on the Value at Risk.

Trabelsi and Naifar (2017) assessed empirically the exposure of Islamic stock indexes to systemic tail events by applying the Conditional Value-at-Risk and the Delta Conditional Value-at-Risk. They concluded that the systemic risk has a moderate adverse effect on Islamic indexes, with a lower level in GCC countries.

In Yousaf and Yarovaya (2022), the bivariate VAR-asymmetric-BEKK-GARCH model was applied to study the returns, volatility spillovers and time-varying correlations among GCC stock markets based on some special factors such as Islamic stocks, oil, gold, bonds and real estate. The empirical study was conducted over a specific period including both the global financial crisis and the recent COVID-19 pandemic. The basic idea is to split the time period into special sub-periods depending on pre- and post-COVID-19, pre- and post-Global financial crisis and during the COVID-19 crisis. The empirical results concluded that effectively the global financial crisis and COVID-19 are strong causes of instability. However, gold served as a hedge and safe haven against most of the GCC stock markets in all sample periods.

The systemic risk was investigated in Al-Yahyaee et al. (2021) in the framework of Islamic law known as Sharia for the GCC countries by using copula and conditional value-at-risk. The authors declared that an evidence of time-varying tail exists between Islamic stock markets and many GCC stock markets. Also, no evidence of systemic risk for Islamic Sukuk and GCC markets was found.

Mensi et al. (2022) examined the dynamic return spillovers between gold and oil commodity futures and many other European equity sectors. The authors applied the Diebold and Yilmaz (2012) approach. The results showed that the time-varying symmetric and asymmetric return spillover are strongly dependent on crises such as oil crisis, Brexit referendum and COVID-19. A quite similar study has been conducted in Yousaf, Beljid, Chaibi, et al. (2022), where the authors focused on the return and volatility transmission between NFTs, Defi assets and other sectors such as oil, gold, Bitcoin, and S&P 500 using the TVP-VAR framework.

Naeem et al. (2022) examined the interdependence between financial markets and potential risk by using the Tail Event Network. The study confirmed the impact of time scale and the effect of crises such as the COVID-19 pandemic.

The authors in Torun et al. (2020) applied a non parametric wavelet method to examine the causal information flow between daily spot returns and their corresponding futures to show the impact of time scales. The authors concluded that each market may show distinct causality patterns compared to other markets. This may support our present study when compared to the recent works on GCC markets such as Ben Mabrouk (2020).

Knif et al. (1995) analyzed the lead and lag structures of the Finnish and Swedish stock market based on univariate spectral and cross-spectral analysis to investigate the impact of the time factor. In Haven et al. (2012), Monte Carlo simulations combined with wavelet methods were applied in the denoising of option price data.

Gradojevic et al. (2020) examined arbitrage dynamics by using a wavelet-based regression approach. The authors considered market risk indicators to predict arbitrage. The finding concluded that arbitrage returns are strongly affected by the time scale. In particular, they are increased (decreased) in periods when the volatility risk and correlations are relatively low (high).

In Sun and Meinl (2012) and Sun et al. (2015), the authors developed wavelet-based denoising and decomposing algorithms to investigate high-frequency financial data mining by adopting a linear scaling approximation algorithm, a new nonlinear filtering algorithm based on the linear maximal overlap discrete wavelet transform. See also Sun and Xu (2018). Also, Firouzi and Wang (2019) conducted a comparative study based on wavelets for exchange rates and order flow.

Katusiime (2022) considered time-Frequency connectedness for East Africa countries in the framework of the COVID-19 pandemic by applying Diebold and Yilmaz (2009). The authors examined precisely the return and volatility connectedness dynamics. Also, Liow et al. (2021) examined the volatility connectedness and spillover transmissions across markets for stock, public real estate, bonds, commodity futures and foreign exchange within the Chinese economy.

Ben Mabrouk (2020) proposed to estimate the CAPM for the Saudi TADAWUL as a GCC representative market by using wavelets to explore the time scale impact. The main difference with the present work is the use of non-uniform intervals of time in Ben Mabrouk (2020). Also, in Ben Mabrouk (2020), a second point of difference consisted of a wavelet method to overcome the problem of missing data by providing a prediction procedure able to predict a short time interval series on an arbitrarily small set of backward and/or forwards (past and/or future, prior and/or post) values.

3. The non-uniform wavelet CAPM

The original model of the CAPM is mathematically formulated by the equation

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + u_{i,t}, \tag{1}$$

where $R_{i,t}$ is the return rate of an action i at the period time t, $R_{m,t}$ is the return of the market measured by means of a general index at the same period of time t. The parameter β_i is a specific factor to each action i, indicating the relation between the fluctuations of the action i return rate and the fluctuations of the general index of the market, called often the beta coefficient or the systematic risk. The factor $u_{i,t}$

is a random factor representing the hidden fluctuations of $R_{i,t}$ that are not explained by the market, or generally an error term. Finally, the parameter α_i is added to guarantee a null expectation of $u_{i,t}$.

This simple model may be easily explained. Indeed, it designates that the CAPM split the total variability of an action into a first part due to the influence of the market (corresponding to the systematic risk), and a second part known as the diversified or the specific risk due to the specific characteristics of the action (corresponding to the variations of the specific prices of such action).

Many variants of the mathematical CAPM formulation have been developed in the literature based on different concepts such as expected returns (Sharpe (1964)), risk-free asset (Black (1972b)), CAPM with transaction costs (Black et al. (1972a)), etc.

In the present paper, we assume that the CAPM defines the required return on investment according to the equation

$$E(R_i) = r_f + \beta_i(E(R_m) - r_f), \tag{2}$$

where $E(R_i)$ is the asset's expected return, r_f is the risk-free rate, $E(R_m)$ is the expected return of the market portfolio, and β_i is the measure of risk for asset i evaluated as

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} = \frac{\sigma_{im}}{\sigma_m^2}. \tag{3}$$

In empirical finance, β_i is estimated usually via the ordinary least square estimate from the linear regression

$$R_{it} - r_{ft} = \alpha_{it} + \beta_i(R_{mt} - r_{ft}) + \varepsilon_{it}, \tag{4}$$

where ε_{it} is the error term, while α_{it} is a constant. Consequently, the risk beta allows decomposing the variance of an asset i as

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_{\varepsilon_i}^2. \tag{5}$$

Hence, the variance σ_i^2 can be decomposed into a first component $\beta_i^2\sigma_m^2$ corresponding to the firm's systematic risk, and a second one $\sigma_{\varepsilon_i}^2$ corresponding to the firm's unsystematic risk.

Now, the use of wavelets in the CAPM consists in decomposing the variance into sub-variances relative to scales or the levels j, which will be called the variance of the series at the scale j (Percival and Walden (2000)). Let υ_X^2 be the variance of a time series $X(t)$ and $\upsilon_X^2(j)$ be the variance of the j-level approximation of $X(t)$, and we have

$$\upsilon_X^2 = \sum_{j=1}^{\infty} \upsilon_x^2(j). \tag{6}$$

Define L as the length of the wavelet filter, $N_j = [N/2^j]$ as the number of wavelet coefficients at the level j and $L_j = [2^{-j}(L-2)(2^j - 1)]$ as the number of boundary wavelet coefficients at the level j. We get an estimator of the variance at the level j as

$$\widehat{\upsilon}_X^2(j) = \frac{1}{2^j(N_j - L_j)} \sum_{k=L_j-1}^{N_j-1} d_{j,k}^2. \tag{7}$$

Similarly, the covariance at the level j for a couple of series (X, Y) is estimated by

$$\widehat{\upsilon}_{XY}^2(j) = \frac{1}{2^j(N_j - L_j)} \sum_{k=L_j-1}^{N_j-1} d_{j,k}^X d_{j,k}^Y. \tag{8}$$

The application of wavelets to the CAPM starts by decomposing the returns into wavelet decomposition (Gençay et al. (2003)) to get next an estimator of the risk beta at the level j as

$$\widehat{\beta}_i(j) = \frac{\widehat{\upsilon}_{R_iR_m}(j)}{\widehat{\upsilon}^2_{R_m}(j)}, \tag{9}$$

where $\widehat{\upsilon}_{R_iR_m}(j)$ is the wavelet covariance of the component i of the portfolio with the market at the scale j. $\widehat{\upsilon}^2_{R_m}(j)$ is the wavelet variance of the market at the scale j.

The final step is next conducted by a determination coefficient $R_i^2(j)$ at the level j which illustrates the explanatory power of market returns on the determination of portfolio returns. Let

$$R_i^2(j) = \beta_i(j)^2 \frac{\sigma^2_{R_m}(j)}{\sigma^2_{R_i}(j)}. \tag{10}$$

Our improvement idea in the present work consists in adapting non-uniform wavelets instead of regular ones to describe the systematic risk. The aim and the motivation are based on our thought that perturbations and instabilities, even those due to causes that seem independent or outside the market, economy, and financial institutions and components, may induce instabilities and irregularities to the market components. The best way in our opinion is to involve the non-regular or non-uniform behavior of these instabilities in the model.

One of the possibilities is to involve it in the time factor or the time scale by using non-uniform wavelets. Indeed, in relation to the CAPM, the authors in Brennan and Zhang (2020) noticed the non-uniform aspect for both the mean returns and the betas according to the choice of the time interval over which returns are measured, yielding thus to the idea of using stochastic (thus, non-uniform) time intervals to describe better the market. This motivates our idea here which aims to apply non-uniform wavelet sampling for the study of the CAPM.

Also, in Sarraj and Ben Mabrouk (2021), the authors applied non-uniform wavelets to study the systematic risk in one of the very emergent markets due to Tunisia, where the economic, financial, and political situations are still very unstable. The use of non-uniform wavelets showed performance, robustness, and efficiency in describing the market. Such a market is a good example of cases of knowing instabilities due to revolutions and political movements.

Non-uniform wavelets are the last variants of wavelet. They were initially developed in Pelissier and Studer (2018); Dubeau et al. (2004) as an alternative to the existing variant. They involve some modifications of the original way which was based on perfect representations of data by means of a set of uniformly-spaced samples by considering instead non-uniformly-spaced samples. The new idea yielded an extension of the concept of multiresolution to a framework of the non-uniform multiresolution, non-uniform wavelet transform, and inverse transform.

The existing case of the non-uniform wavelets (which will be applied here) is based on the Haar wavelet system and multiresolution. Let Haar scaling function $\varphi = \chi_{[0,1[}$ be the Haar scaling function, and $\psi = \chi_{[0,1/2[} - \chi_{[1/2,1[}$ be the mother wavelet. Consider a time interval $I = [0,T]$ and a partition $\{\Delta_m\}_{m=0}^{M}$ of I, with $M \in \mathbb{N}$ satisfying $\Delta_m = \{x_k^{(m)}\}_{k=0}^{2^m}$, where

$$x_0^{(m)} = 0 < \cdots < x_k^{(m)} < \cdots < x_{2^m}^{(m)} = T, \quad \text{and} \quad x_k^{(m-1)} = x_{2k}^{(m)},$$

for all $m \in \{1, 2, \ldots, M-1, M\}$, and $k \in \{0, \ldots, 2^{m-1}\}$. For each m, define

$$\varphi^{(m)}(x) = \varphi_k^{(m)}(x) = \varphi\Big(\frac{x - x_k^{(m)}}{x_{k+1}^{(m)} - x_k^{(m)}}\Big) = \chi_{[x_k^{(m)}, x_{k+1}^{(m)}[}(x), \ \ x \in [x_k^{(m)}, x_{k+1}^{(m)}[$$

the non-uniform scaling function, and

$$\psi^{(m)}(x) = \psi_k^{(m)}(x) = \alpha_k^{(m)}\chi_{[x_{2k+1}^{(m+1)},x_{2k+2}^{(m+1)}[}(x) - (1-\alpha_k^{(m)})\chi_{[x_{2k}^{(m+1)},x_{2k+1}^{(m+1)}[}(x)$$

the non-uniform mother wavelet, where

$$\alpha_k^{(m)} = \frac{x_{2k+1}^{(m+1)} - x_k^{(m)}}{x_{k+1}^{(m)} - x_k^{(m)}}.$$

The approximation spaces of the non-uniform multiresolution analysis are

$$V_m = spann\{\varphi_k^{(m)};\ k = 0,\dots,2^m-1\}, \tag{11}$$

while the detail spaces are

$$W_m = spann\{\psi_k^{(m)};\ k = 0,\dots,2^m-1\}. \tag{12}$$

Using the non-uniform wavelets, and the non-uniform sub-periods, we obtain what we call non-uniform systematic risk ${}^{NU}\beta$ estimated analogously to (9) at the level j as

$${}^{NU}\beta_i(j) = \frac{{}^{NU}\upsilon_{R_iR_m}(j)}{{}^{NU}\upsilon_{R_m}^2(j)}, \tag{13}$$

where ${}^{NU}\upsilon_{R_iR_m}(j)$ is the wavelet covariance of the component i of the portfolio with the market at the scale j and ${}^{NU}\upsilon_{R_m}^2(j)$ is the wavelet variance of the market at the scale j, relatively to the non-uniform multiresolution (11). Next, as previously, we will apply a non-uniform determination coefficient ${}^{NU}R_i^2(j)$ at the level j as

$${}^{NU}R_i^2(j) = {}^{NU}\beta_i(j)^2\frac{{}^{NU}\sigma_{R_m}^2(j)}{{}^{NU}\sigma_{R_i}^2(j)}. \tag{14}$$

One way to get a non-uniform subdivision is to consider for the correspondence scale and dynamic days a random variable $(T_j,\ j\in\mathbb{N})$ for which the level j corresponds to a number $N_j = T_{j+1} - T_j$ of dynamic days. The process stops at a maximum level J_{max} corresponding to approximately one year. One suitable choice is to generate a random sequence $(T_j)_j$ satisfying $\lim_{j\to\infty}\frac{T_{j+1}}{T_j} = \infty$ or equivalently $\lim_{j\to\infty}(T_{j+1} - T_j) = \infty$.

Our idea is motivated by several past works. The authors in Brennan (1986); Brennan and Durrett (1987) and Sewell (2021) constructed an interval subdivision scheme generalizing the well-known Kakutani's method of interval subdivision into nonuniform sub-intervals, Kakutani (1976). The process consists in splitting at each stage, all intervals of maximal length into pieces whose lengths have certain fixed ratios $r_1 < r_2 < \dots$ summing to 1. AS compact intervals are all homomorphic to the unit interval, and the process may be applied easily to any other interval. In the present work, we split our time period according to this scheme. More precisely, an explicit way is provided in Sewell (2021), Chapter 2, Example 3. We recall it in brief here. The unit interval splits according to the partition $[0,\frac{1}{2}[$, $[\frac{1}{2},\frac{2}{3}[$, $[\frac{2}{3},1]$. Notice that these intervals are obtained by applying the similarities $S_1(x) = \frac{x}{2}$, $S_2(x) = \frac{x+3}{6}$ and $S_3(x) = \frac{x+2}{3}$ to the unit interval, respectively. We apply this process at level 6 to get a subdivision of our time period into a partition of sub-periods $\mathbb{T}_i = S_{i_1}\circ S_{i_2}\circ\cdots\circ S_{i_6}([0,T])$, where $i = (i_1, i_2, \dots, i_6) \in \{1,2,3\}^6$. The same process will be applied to the nonuniform wavelets reviewed in the previous section.

Concerning the wavelet type, of course, there are many types of wavelets, such as Haar, and Daubechies, which are in fact widely used in statistical applications. Higher-order Daubechies wavelets suffer from a major drawback related to the size of the time series or the statistical series, which has to be a power of 2. If not, a problem of boundary coefficients computation is always occurring. In practice, we even cut off the series to some 2-power size, or we extend it to reach such a size type. Both methods yield in general biased measures. Daubechies wavelets are not explicit, and thus we always use their values at integer points to compute coefficients via a discrete convolution.

Moreover, Haar wavelets permit us to compute the coefficients in a simple way as average and difference operators. We guarantee the same variance between the vector of the wavelet coefficients and the data. Thus, we guarantee consequently preservation of the information. By applying Haar wavelet filters, we get a weighted difference between consecutive returns or local averages of the original returns. These facts simplify widely the practical applications.

From a theoretical point of view, it is known in wavelet theory that any time series (statistical series) with finite variance is estimated by means of its wavelet series as in Equations (20) and (22), and this decomposition is independent of the wavelet basis. More precisely, we have $\|X - X_J\|_2 = O(2^{-J})$ independently of the mother wavelet ψ. (See for example Härdle et al. (2012).

It is simple to calculate and can easily be implemented, as it does not depend on a particular model selection criterion and model specific parameter choices. The proposed filtering method has the translation invariance property, has the ability to decompose an arbitrary length series without boundary adjustments. It permits also to preserve the entire sample, unlike many two-sided filters where data loss occurs from the beginning and the end of the studied sample.

Studies like Levhari and Levy confirmed that if a time horizon shorter than the true one is used, we may obtain a biased beta. Other studies such as Cohen et al. (1986) showed that the beta estimates were sensitive to return intervals. In our opinion, the major problem in the time sub-periods or timescale laws in market studies is that the market itself is uncertain and full of randomness. So, applying fixed sub-divisions may not follow the uncertainty in the market movement.

A recent study due to Sarraj and Ben Mabrouk (2021) confirmed the robustness of non-uniform wavelets in estimating beta against strong movements in the market. In our paper, we improve this study by adopting in addition non-uniform intervals. Moreover, we conducted in the empirical section a comparison with a recent technique due to Diebold and Yilmaz (2012) to further show the performance of the present idea.

4. Empirical results and discussions

4.1. The wavelet CAPM processing

In this section, we empirically test a non-uniform wavelet methodology to estimate the systematic risk beta on daily data for the Qatar QEAS index as one of the main representatives of the whole GCC market collected during the period from January 01, 2017, to December 31, 2021. We focused on a portfolio composed of 45 actions as listed in Table 1 with corresponding sectors.

The choice of this market is motivated by the fact that QEAS is the best representative index of the Qatar market directly supervised by the Capital Market Authority. It is also considered as one of the GCC-influencing markets in the Middle East. It lists more than 40 publicly traded companies. We considered in the present work a sample of 43 companies due to the lack of sufficient data for some components of the market.

Table 1. QAES components and sectors.

Sector	Company Name	Abbreviation
Financials	Commercial Bank of Qatar	COMB
	Dlala	DBIS
	Doha Insurance	DICO
	Doha Bank	DOBK
	Inma Holding Company	IHGS
	Al Kaleej Takaful	KINS
	Masraf al rayan	MARK
	Lesha Bank LLC	QFBQ
	General Insurance & Reinsurance Company	QGIR
	Qatar International Islamic Bank	QIIB
	Qatar Islamic Insurance Company	QIIC
	Qatar Insurance Company	QINS
	Qatar Islamic Bank	QISB
	Qatar National Bank	QNBK
	Qatar Oman Investment Company	QOIS
Consumer	Amal	AHCS
	Industries Qatar	IQCD
	Al Meera	MERS
	Qatar Industrial Manufacturing Company	QIMC
	Salam International	SALM
	Widam Food	WDAM
	Zad Holding Company	ZHCD
Real Estate	Barwa Company	BRES
	Ezdan	ERES
	Mazaya	MRDS
	Alijarah Holding Company	NLCS
	United Development Company	UDCD
Materials	Mesaieed Petrochemical Holding	MPHC
	Qatar Aluminum Manufacturing Company	QAMC
	Qatar National Cement Company	QANC
	Qatari Investors Group	QIGD
Energy	Gulf International	GISS
	Qatar Fuel	QFLS
	Qatar Gas Transport Company	QGTS
Technology	Mannai Corporation	MCCS
	Ooredoo	ORDS
	Vodafone Qatar	VFQS
Industrials	Gulf Warehousing Company	GWCS
	Estithmar Holding	IGRD
	Qatar Navigation	QNNC
Healthcare	Medicare	MCGS
	Qatar German Company for Medical Devices	QGMS
Utilities	Qatar Electricity and Water	QEWC

We propose to study the relationship between excess return on each individual stock and the time scales of the market portfolio using the usual OLS estimator for β_i issued from the regression (4). The daily return of each stock is calculated as the log-price difference

$$R_{it} = \log P_{i,t} - \log P_{i,t-1}, \tag{15}$$

where $P_{i,t}$ is the price of asset i at day t. The market return R_{mt} is taken as the log-difference of the index due to the action i as

$$R_{mt} = \log C_t - \log C_{t-1}, \tag{16}$$

where C_t is the index value at day t.

Table 2 shows the descriptive statistics of excess return on the stocks in the sample and on the proxy for the QEAS portfolio. The statistics correspond precisely to the excess return for each company relative to the risk-free. The risk-free rate of return is measured by the return on government securities and is fixed at 4.03%.

The first computations yielded an approximately zero median for all assets in the present sample. Moreover, the Jarque-Bera test leads to $JB = 1$, which rejected the null hypothesis at the 5% significance level.

Notice firstly from Table 2 that an approximately zero mean is obtained for all the stocks. The flatness and distortion features of all stocks' returns are different from each other. The Jarque-Bera test leads to $JB = 1$, which supports our previous conclusion about the null hypothesis at the 5% significance level.

Notice also from Table 2 a non-vanishing skewness (while being low) for all the components of the market, which rejects the symmetry in the distributions, while the kurtosis values are different from 3 for all the financial series used, which implies that the distributions of the data are not normal.

Table 2. Descriptive statistics of excess returns.

Stocks	Mean	Minimum	Maximum	SD.	Skewness	Kurtosis
QEAS	2.978	2.116	3.81	0.3451	0.004615	2.927
COMB	$2.249\ 10^{-6}$	−0.6482	0.09531	0.02535	−13.4	345.9
DBIS	−0.000364	−0.1055	0.09539	0.02442	0.5072	7.615
DICO	$6.889\ 10^{-5}$	−2.41	2.373	0.1661	−0.1884	198.9
DOBK	$-8.815\ 10^{-5}$	−0.1055	0.09531	0.01696	−0.5379	11.62
IHGS	−0.000352	−0.1054	0.09531	0.02663	0.5358	6.092
KINS	$9.023\ 10^{-5}$	−0.7521	0.09531	0.03394	−8.368	196.6
MARK	0.000203	−0.1042	0.06788	0.01258	−0.8129	13.97
QFBQ	−0.0005217	−0.1331	0.09546	0.0214	0.371	10.01
QGIR	−0.0009149	−2.435	2.362	0.2155	−0.1732	113.6
QIIB	0.0003182	−0.1041	0.0545	0.01301	−0.9795	12.54
QIIC	0.0004595	−2.312	2.357	0.09582	0.6705	567.9
QINS	−0.0007106	−0.1054	0.09527	0.0195	−0.1045	7.459
QISB	0.0004462	−0.105	0.0953	0.0133	0.07665	12.8
QNBK	0.0002488	−0.1054	0.09531	0.01432	0.01629	9.617
QOIS	−0.0001144	−2.344	2.346	0.09688	0.04341	552.7
AHCS	−0.000201	−0.1054	0.09531	0.01932	0.2477	9.326
IQCD	0.0002106	−0.1053	0.09531	0.01569	−0.1871	8.165
MERS	$7.921\ 10^{-5}$	−0.08965	0.0601	0.01153	-0.1469	11.26
QIMC	−0.0002749	−0.09933	0.09528	0.0194	0.02775	6.652
SALM	−0.0002349	−0.1054	0.09531	0.02309	0.1502	8.808
WDAM	−0.0004806	−0.1054	0.08536	0.01693	−0.8069	11.08
ZHCD	0.0005682	−0.1707	0.1707	0.0251	0.03893	15.9
BRES	$-6.555\ 10^{-5}$	−0.1053	0.08437	0.01453	-0.9508	14.49
ERES	−0.0001246	−0.1054	0.09531	0.02757	0.1924	6.389
MRDS	−0.000344	−0.1054	0.09509	0.02046	0.3823	9.034
NLCS	−0.0004132	−0.1052	0.09531	0.01984	0.1039	8.553
UDCD	−0.0002239	−0.1054	0.09531	0.01803	−0.07352	11.49
MPHC	0.0002431	−0.1052	0.09525	0.01946	0.1978	10.47
QAMC	0.0003196	−0.1054	0.3902	0.02567	3.401	50.26
QANC	−0.0002827	−0.1053	0.09522	0.01751	−0.2567	9.367
QIGD	−0.0007775	−0.1054	0.09531	0.0227	0.6582	8.112
GISS	−0.0004754	−0.1054	0.09531	0.02127	0.1556	8.254
QFLS	0.0002605	−0.1054	0.09531	0.01429	−0.6656	15.14
QGTS	0.0002916	−0.1054	0.09531	0.01613	−0.09852	12.66
MCCS	$2.07\ 10^{-5}$	−2.358	2.334	0.1633	−0.1573	198.8
ORDS	−0.0003126	−0.1053	0.0953	0.01727	−0.2193	8.73
VFQS	$-9.844\ 10^{-5}$	−0.1052	0.09457	0.01658	0.3381	10.93
GWCS	−0.0001496	−0.1054	0.09531	0.01719	−0.2074	9.884
IGRD	0.0008226	−0.1051	0.09531	0.02447	0.5105	7.842
QNNC	−0.0001685	−0.1054	0.07671	0.01646	−0.4203	10.49
MCGS	0.0002232	−0.1048	0.09531	0.02209	0.4055	7.847
QGMS	0.0009412	−0.1054	0.09531	0.03014	0.4482	6.292
QEWC	−0.0002564	−0.1052	0.09518	0.01336	−0.2952	12.72

Moreover, heavy tails are present for nearly all the components, except for the QEAS index, which looks to be normal. COMB, KINS, and QAMC companies present solely high absolute skewness values, which means data points favor one side of the distribution. Recall that financial time series present always stylized facts, where the distribution tails have to be described via the kurtosis.

We may also observe small standard deviations for all the stocks, which reflects the small deviation of the data around the mean. Moreover, it is noticeable that the market has a slow movement.

In this paper, we propose a wavelet multi-scaling approach that decomposes the data into low- and high-frequency components through the application of a non-uniform discrete wavelet transform and non-uniform time intervals. The methodology of the paper allows us to estimate the volatility or the fluctuation behavior of the data being entirely disentangled from the data seasonality. In some past literature, to eliminate intraday seasonality, for example, we needed to apply daily and/or weekly aggregate data.

However, daily volatility is always related to high-frequency data. In wavelet approaches, it is not necessary to eliminate as in many past works features like weekends, fixed holidays, moving holidays, and days with 15 longest zero returns. Eliminating such features may fall into fractal and multifractal data. These circumstances are strongly related to the so-called stylized facts. The wavelet theory is a promised tool that is widely used to detect the facts in financial series.

The first step in our analysis consists in projecting Equation (4) relative to time scales to test the effect of time on the systematic risk beta. This will be conducted by splitting the market returns into crystals or horizons relative to different time scales relative to the specific laws.

In order to compare with existing studies, we will consider the 2-scale law as the classical basis for comparison. Therefore, the correspondence scale and dynamic days applied here will be based on a random variable (T_j, $j \in \mathbb{N}$) for which $T_j \simeq 2^j$. The process stops at a maximum level $J_{max} = 6$.

The coefficients of the linear regressions will be estimated by the usual OLS of the returns $(R_{it} - r_f)^j$ on the one of the market $(R_{mt} - r_f)^j$ for each level j. This leads to a j-level mathematical formulation as

$$(R_i - r_f)^j = \alpha_i^j + \beta_i^j (R_m - r_f)^j + \varepsilon_i^j = \alpha_i^j + \beta_i^j D_m^j + \varepsilon_i^j. \tag{17}$$

Since we employ daily data in our analysis, wavelet scales are such that scale 1 is associated with 2–4 day dynamics, scale 2 with 4–8 day dynamics, scale 3 with 8–16 day dynamics, scale 4 with 16–32 day dynamics, scale 5 with 32–64 day dynamics and scale 6 with 64–128 day dynamics. Since the portfolio updating is carried out every year, scale 6 is the highest scale at which we can calculate the beta of each stock because scale 7 corresponds to 128–256 day dynamics (approximately one year). The CAPM model is estimated up to scale 6 of the market portfolio because scale 7 includes not only the detail D_7 but also the approximation A_7. Therefore, when recomposing the market excess returns at scale 7, we cannot disentangle the detail component from the approximation. In this framework, scales 1–2 are assumed to be short term horizons, scales 3–4 are associated to medium terms, and scales 5–6 are the long term levels.

We propose now in this section to discuss the validity of the main hypotheses about the time-scale stability of the risk beta, and the linear dependence between actions' returns and their systematic risks. Table 3 represents the estimations of stocks' betas at the scales $j = 1, 2, 3, 4, 5, 6$.

Table 3. Estimations of the excess return of actions on the market for 2-scale law.

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
COMB	1.0395	0.7897	0.9915	0.3597	0.9753	1.1810
DBIS	−0.0103	−0.0443	−0.4461	0.2567	0.0636	0.8929
DICO	−0.1304	−0.5796	0.6933	−0.1304	0.4189	1.9900
DOBK	−0.1551	0.1674	0.313	0.6173	0.1485	0.2639
IHGS	−0.1361	0.0209	0.1094	0.1912	0.0545	1.4870
KINS	1.3138	0.952	0.9751	0.4793	0.9635	1.3830
MARK	−0.2454	0.0358	0.3095	0.3558	0.4222	0.5987
QFBQ	−0.1140	−0.1584	0.3055	0.6515	0.5634	1.0530
QGIR	−0.0245	0.2423	0.3008	0.1599	0.8155	0.3306
QIIB	−0.0964	0.1358	0.3181	0.5401	0.6006	0.5350
QIIC	0.2817	−0.0886	0.1028	0.1269	0.0299	0.1182
QINS	−0.1025	−0.0182	0.0279	0.1017	−0.3009	−0.2604
QISB	−0.2702	0.1763	0.4997	0.7178	0.6347	0.3641
QNBK	0.1283	−0.5917	0.3261	0.5080	0.5825	0.7007
QOIS	−0.3780	−0.1213	−0.3282	−0.1112	−0.0961	0.6496
AHCS	0.1733	0.4107	0.1719	0.6036	0.0745	1.0980
IQCD	−0.3145	0.0691	0.5733	0.9356	0.7342	0.6613
MERS	0.0948	−0.2304	0.1154	0.1594	0.2479	0.1831
QIMC	0.0420	0.025	0.0467	0.2515	0.0096	0.1984
SALM	0.0461	−0.0915	−0.2258	0.2975	0.3898	1.5670
WDAM	0.0395	0.1495	−0.1736	0.2686	0.1480	1.0140
ZHCD	0.2135	0.1966	0.3298	0.175	0.2667	−0.1280
BRES	0.5103	0.6647	0.555	0.9232	0.4259	0.3103
ERES	0.0656	−0.3626	−0.0685	0.2460	0.7772	1.9670
MRDS	0.1475	−0.0283	−0.0907	0.6074	0.0962	1.2240
NLCS	−0.0338	−0.2681	0.4015	0.4664	0.1196	0.8081
UDCD	−0.0777	0.0533	0.2724	0.6247	0.0288	1.0350
MPHC	−0.2739	0.1327	0.6116	0.7728	0.2094	0.4171
QAMC	0.0286	0.0312	0.4245	−0.5494	0.3807	0.6642
QANC	−0.1016	0.0488	-0.0845	0.2014	0.1669	0.2559
QIGD	0.4437	0.4252	0.3884	0.8267	0.6975	1.9800
GISS	0.4773	0.6548	0.7185	0.651	0.6716	1.3540
QFLS	0.3506	0.3542	0.4853	0.7545	0.1823	0.4046
QGTS	0.4350	0.5304	0.6096	0.7519	0.4142	0.5377
MCCS	−0.4273	0.0529	0.0494	0.1479	0.2905	0.2137
ORDS	0.0195	−0.1938	0.2328	0.4751	0.8450	0.7455
VFQS	0.0335	−0.0285	0.2599	0.3270	0.4763	1.0290
GWCS	−0.1039	0.0366	0.0015	0.1443	0.4561	0.4921
IGRD	−0.0947	−0.0373	0.184	0.2383	0.0868	0.0691
QNNC	0.0107	−0.0043	−0.2961	0.0388	0.4308	0.1105
MCGS	−0.2080	−0.2196	0.443	0.469	0.2254	0.9952
QGMS	0.1664	−0.1126	−0.1738	0.1175	0.4411	1.1910
QEWC	0.1017	−0.3848	0.2116	0.1262	0.1817	0.7034
Mean	0.0666	0.0649	0.2435	0.3692	0.3184	0.7346

Table 3 reflects a quite positive relationship between multi-scale return and systematic risk coefficients, especially at higher horizons for a major part of actions. A time-changing behavior may also be detected for the beta. This means that the QEAS index is more efficient at scales 5 and 6. Therefore, the CAPM is a multi-scale phenomenon, and quite longer periods are more relevant in explaining the relationship between stock return and its beta. However, this relevance is perturbed for short to medium periods, which may be explained effectively by the impact and the influence of the strong phenomenon of the embargo on the market.

As it is not predicted in advance, the embargo affected negatively and perturbed the market movement. However, the market has rapidly absorbed this negative effect and returned to find and reach a positive behavior from medium (period of correction and absorption of the shock) to higher scales (period of recovery). However, even though the analysis reflects maybe the real behavior of the market, it does not advise on how individuals such as investors, managers, and policymakers may behave at the moment of the shock. These facts may be resolved by the involvement of other factors such as policy actions, and local economic policy in the model.

Table 3 shows that the linear dependence is always justified even though being negative and very weak in many cases and on all 6 scales. This may be explained effectively by the embargo phenomenon where the weakness and perturbations appear clearly at short to sometimes medium levels. The table shows also that no law may be expected simultaneously for all the contributions of the D_m^i of all actions relative to the increasing time scale.

In addition, the negative coefficients are not very significant (in absolute values less than 0.4) which explains the weak influence of these distressed companies on the whole market. We notice in Table 3 two negative values at level 6, three values at level 5, and two values at level 4. The mean of the coefficients is positive for all levels. This indicates, in other words, the resistance of the market against the crash.

To further explain such contributions, and consequently the behavior of the market according to the betas, we computed in Table 4 the determination coefficient R^2 relative to the estimations provided in Table 3.

We notice from Table 4 that the coefficient R^2 is increasing according to the time scale at the high scales, while for low and medium levels some perturbations are clearly present for many actions. This means that the major influencing parts of the market portfolio on the actions returns, are localized in high frequencies. This indicated that at low horizons the market is going down, although the linearity is strong at high levels. Economically, these findings may be due to the influence of the embargo reigning at the beginning of the period.

The results make it possible to quantify the correlation between the return of the stock and its beta at different time scales. Table 4 shows some positivity and significance for the relationship between the endogenous and exogenous variables at all scales although it is very weak in some cases. We notice null values and small ones in many rows such as DBIS, QGIR, QIMC, MCCS, QWCS, QNNC, etc.

However, almost all these zero or small values are located in low horizons. This confirms our conclusion about the negative shock due to the embargo at the beginning of the period, and the covering efforts leading to the equilibrium of the market and its positive developments at higher horizons.

Table 4. The determination coefficient R^2 relative to Table 3.

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
COMB	0.2274	0.1376	0.1933	0.0404	0.2015	0.278
DBIS	0.0000	0.0005	0.0006	0.0230	0.0275	0.234
DICO	0.0001	0.0003	0.0018	0.0022	0.0065	0.0751
DOBK	0.0105	0.0163	0.0086	0.0346	0.0462	0.2556
IHGS	0.0001	0.0024	0.0003	0.0085	0.0036	0.4096
KINS	0.0426	0.0929	0.1219	0.1494	0.1942	0.3106
MARK	0.0459	0.0014	0.0712	0.1509	0.2418	0.3742
QFBQ	0.0040	0.0099	0.0219	0.0527	0.1256	0.2775
QGIR	0.0000	0.0040	0.0041	0.0086	0.0148	0.0583
QIIB	0.0082	0.0167	0.0597	0.2010	0.2344	0.2511
QIIC	0.0006	0.0004	0.0033	0.0047	0.0138	0.0245
QINS	0.0034	0.0002	0.0003	0.0059	0.0148	0.0728
QISB	0.0521	0.0319	0.1636	0.1861	0.2989	0.4148
QNBK	0.0116	0.0772	0.1966	0.2035	0.2148	0.4207
QOIS	0.0010	0.0034	0.0049	0.0053	0.0253	0.1112
AHCS	0.0115	0.0022	0.0095	0.0643	0.1577	0.3331
IQCD	0.0559	0.0035	0.1423	0.2282	0.2674	0.4513
MERS	0.0081	0.0657	0.0157	0.0381	0.0588	0.0535
QIMC	0.0005	0.0000	0.0003	0.0009	0.0357	0.0698
SALM	0.0006	0.0025	0.0105	0.0376	0.0365	0.2912
WDAM	0.0008	0.0094	0.0110	0.0154	0.0390	0.4422
ZHCD	0.0076	0.0092	0.0126	0.0214	0.0268	0.0453
BRES	0.0829	0.1696	0.1665	0.1999	0.2988	0.5268
ERES	0.0010	0.0006	0.0155	0.0207	0.0708	0.5083
MRDS	0.0069	0.0003	0.0017	0.0025	0.1580	0.4299
NLCS	0.0004	0.0059	0.0261	0.0464	0.0932	0.4692
UDCD	0.0026	0.0017	0.0004	0.0248	0.1491	0.1590
MPHC	0.0080	0.0300	0.0093	0.0901	0.0905	0.1877
QAMC	0.0002	0.0002	0.0236	0.0308	0.0852	0.0910
QANC	0.0012	0.0045	0.0028	0.0085	0.0231	0.2800
QIGD	0.0494	0.0628	0.0346	0.1718	0.2034	0.7518
GISS	0.0763	0.1170	0.1303	0.1406	0.1502	0.3520
QFLS	0.0773	0.0326	0.0988	0.1322	0.1504	0.4489
QGTS	0.0933	0.1269	0.1565	0.1744	0.1751	0.3721
MCCS	0.0005	0.0000	0.0009	0.0097	0.0160	0.0308
ORDS	0.0002	0.0209	0.0208	0.1504	0.2869	0.3179
VFQS	0.0006	0.0004	0.0269	0.0698	0.0780	0.3541
GWCS	0.0046	0.0006	0.0000	0.0162	0.1159	0.2067
IGRD	0.0023	0.0003	0.0014	0.0051	0.0186	0.0900
QNNC	0.0001	0.0000	0.0010	0.0051	0.0328	0.0905
MCGS	0.0128	0.0149	0.0138	0.0414	0.0698	0.2791
QGMS	0.0052	0.0021	0.0023	0.0031	0.0228	0.2135
QEWC	0.0077	0.0173	0.1097	0.0344	0.0422	0.5120

Next, as the interpretation or the comprehension of tables is always not easy, we propose to test the link between the market return and the actions graphically by plotting for the 2-scale time law the recomposed crystal of the excess return on the stock versus the corresponding crystal on the market portfolio in Figure 1 (daily stock returns versus corresponding stock beta at different time scales).

The inspection of the figure confirms the relationship between the average betas of stocks and average returns at every scale. Moreover, it is clearly observed on these plots that the linear dependence between the beta and the return becomes stronger as the scale increases. This evidence supports the resistance of the market against the embargo effects and the covering at higher horizons.

Besides, some instability appears at low levels due to the embargo. This perturbation may influence negatively investments at short horizons. This is one motivation to think about stronger mathematical tools that permit us to go in both the microscopic and macroscopic scales of the market to get more accurate conclusions and accurate forecasting about the future of the market. One proposition is the use of non-uniform wavelets as well as time scales. We thus propose to estimate a non-uniform beta as stated in Equations (13) and (14).

We propose next to adopt the wavelet method to discover more aspects of the QEAS market and to show the behavior and dynamics of the market. We apply a wavelet decomposition of the QEAS index at level 6. The result is illustrated by Figure 2.

The next step consists in estimating the non-uniform wavelet beta. Empirical results are gathered in Table 5. It represents the wavelet estimations of the betas of each stock component to the levels $J = 6$.

Notice from Table 5 that more dependence between the market and actions is detected especially at higher levels. Also, this dependence is getting up according to the time scale. In many cases, the classical methods yielded zero (or approximately zero) betas (Table 3). These cases are now more represented by means of wavelet betas. We cite for example DBIS, QGIR, QIMC, MCCS, QWCS, QNNC where the beta takes zero or approximately zero values for many scales as shown in Table 3. We get here more significant betas (non-zero) even being negative for some cases. Furthermore, we notice clearly a positive increasing mean for the betas in Table 5.

Next, to confirm the linear dependence and the efficiency of the wavelet tool, we established as previously a computation of the R^2 coefficient of determination in Table 6 relative to the betas of each stock component estimated in Table 5 at the wavelet levels $J = 1, 2, 3, 4, 5, 6$.

Table 6 shows a coherence with results on wavelet estimation of beta. We notice that the market is going to be efficient at high scales, and the dependence increases as the level increases.

The findings in Tables 5 and 6 are confirmed in Figure 3, which illustrates the average of wavelet excess return of the actions against the wavelet excess return of the market at the different wavelet levels $j = 1, 2, 3, 4, 5, 6$.

Notice from Tables 3 and 5 that by looking at the mean values at the bottom of the tables, the mean values are positive and increasing. This leads to thinking about a regrouping of actions according to the sectors. Therefore, the final step in the investigation of the QEAS market will consist of a panel study based on the classification of the market components into sectors, which allows a global view of the market. Classification into the sector is already provided in Table 1. Table 7 shows the results of such a classification.

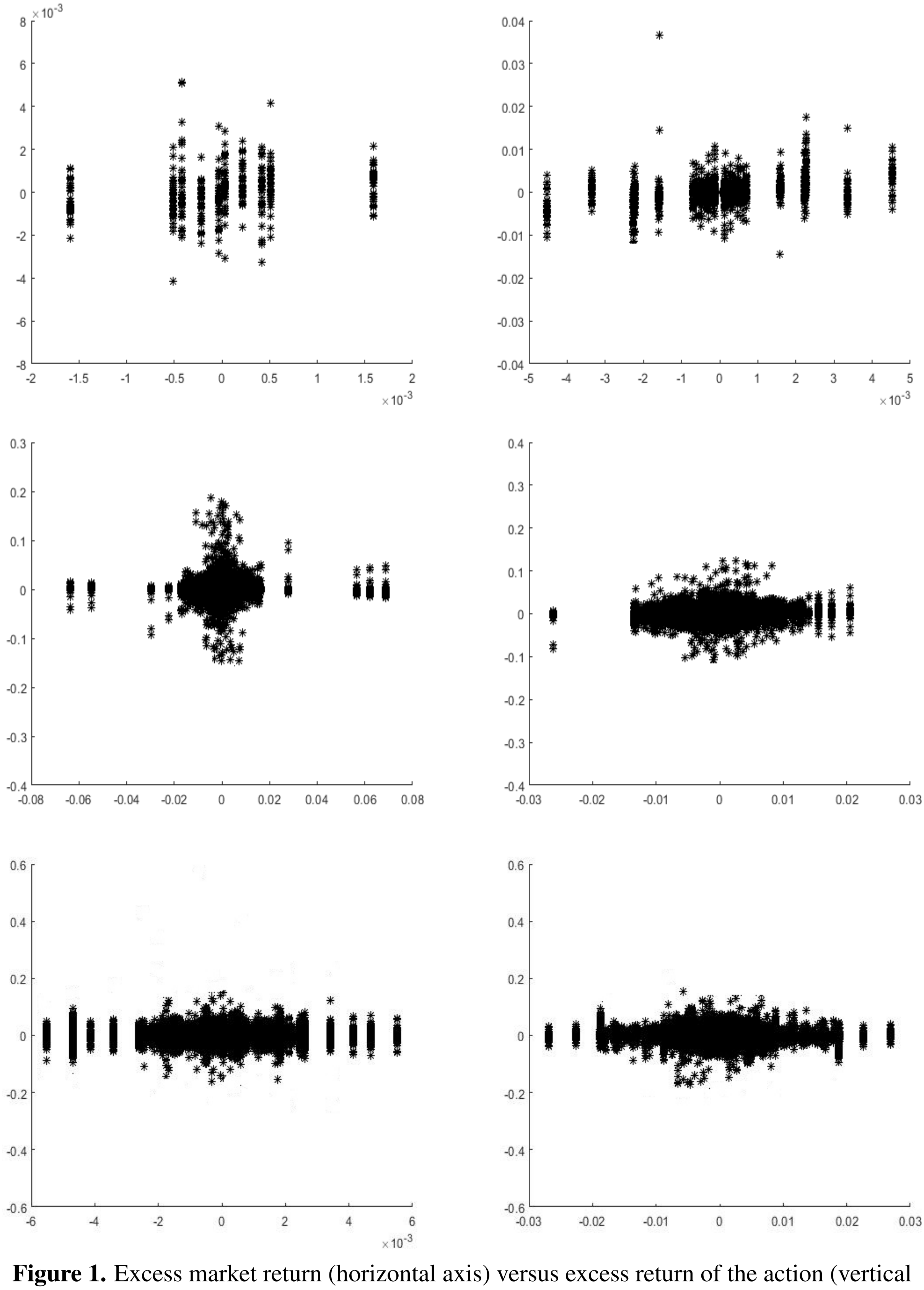

Figure 1. Excess market return (horizontal axis) versus excess return of the action (vertical axis) for different time scales with the 2-scale law.

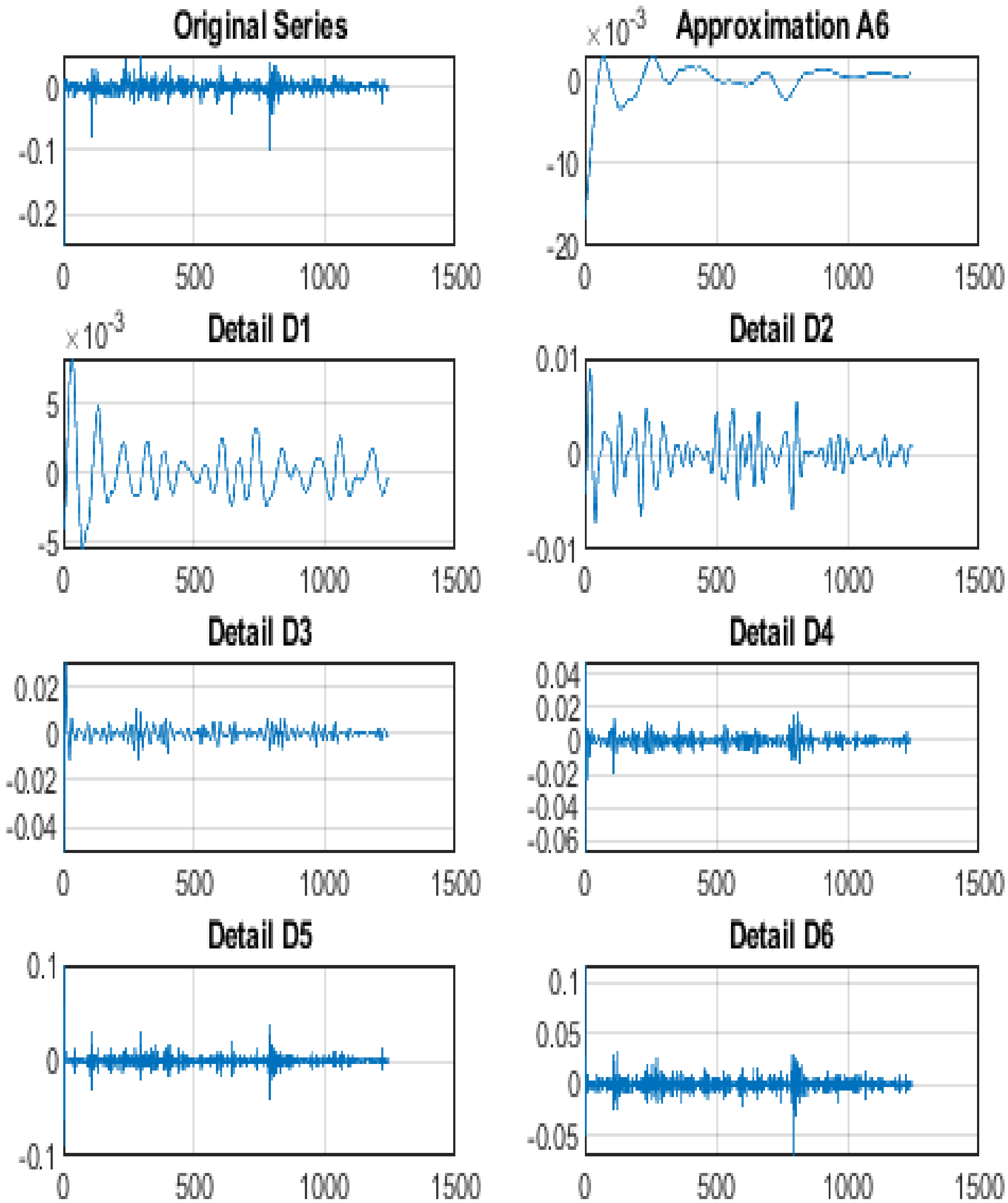

Figure 2. Wavelet decomposition of QEAS return series at level 6.

Table 5. Nonuniform wavelet beta for levels $J = 1$ to $J = 6$.

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
COMB	0.2609	0.4012	0.5333	0.5820	0.7797	0.7988
DBIS	0.1131	−0.3305	0.0515	0.0833	0.0936	0.7465
DICO	0.1453	−0.7293	0.0223	0.2009	0.2903	0.4933
DOBK	−0.2217	0.1534	0.4486	0.4735	0.4822	0.5815
IHGS	−0.1401	0.0842	0.0930	0.1670	0.4446	1.6950
KINS	0.5009	0.4539	0.6470	0.7249	0.7467	1.0780
MARK	−0.3782	0.0279	0.4291	0.5352	0.5848	0.5882
QFBQ	0.0985	−0.2313	0.2076	0.4429	0.4804	0.9456
QGIR	0.1049	0.1496	0.3119	0.4255	0.4852	0.6246
QIIB	−0.1546	0.0603	0.5815	0.5902	0.6155	0.6922
QIIC	0.2412	−0.0120	0.1615	0.2537	0.3144	0.7476
QINS	−0.0784	0.0177	0.1560	0.2393	0.7153	0.8020
QISB	0.1613	0.3281	0.4129	0.6184	0.6789	0.7807
QNBK	0.0522	0.2970	0.5344	0.6630	0.7509	0.8305
QOIS	−0.3225	-0.5229	0.3418	0.4095	0.4610	0.6139
AHCS	0.0276	0.3493	0.4694	0.4428	0.6632	0.9792
IQCD	−0.4886	0.2300	0.6832	0.7021	0.8094	0.9693
MERS	0.0022	−0.1265	0.1307	0.1604	0.2933	0.4053
QIMC	−0.0779	0.0693	0.0887	0.0888	0.1105	0.2875
SALM	−0.1410	−0.0173	0.0558	0.1308	0.9672	1.5230
WDAM	0.1299	−0.3355	0.0753	0.1611	0.4598	0.8070
ZHCD	−0.0604	0.0749	0.2411	0.2482	0.3821	0.5400
BRES	0.8139	0.3314	0.5415	0.7733	0.8359	0.9816
ERES	0.2257	0.9525	0.0120	0.5732	0.8025	1.6240
MRDS	0.0737	0.2708	0.2845	0.3953	0.4440	1.2220
NLCS	−0.3088	0.0093	0.3138	0.4351	0.4810	1.0760
UDCD	0.0820	0.2030	0.4193	0.5313	0.6782	0.7180
MPHC	−0.3723	0.1067	0.2783	0.4711	0.6803	0.7804
QAMC	0.0281	0.0405	0.0692	0.2772	0.3317	0.4353
QANC	−0.1512	0.0093	0.0445	0.0621	0.3091	0.4671
QIGD	0.5655	0.6003	0.6243	0.8972	0.3112	1.8700
GISS	0.6862	0.6901	0.7853	0.8165	0.9292	0.8629
QFLS	0.5634	0.4042	0.5103	0.5223	0.5360	0.5749
QGTS	0.7129	0.5064	0.7281	0.7362	0.7369	0.7659
MCCS	−0.0868	-0.5120	0.1940	0.0709	0.4619	0.5531
ORDS	0.1005	-0.3783	0.1560	0.4751	0.9770	0.9194
VFQS	0.1418	-0.2051	0.1926	0.3533	0.4193	1.0110
GWCS	−0.0109	0.0158	0.0383	0.1645	0.4272	0.4759
IGRD	0.0145	-0.0870	0.0312	0.1694	0.2503	0.6798
QNNC	−0.0760	0.0164	0.0607	0.0303	0.2925	0.4282
MCGS	0.3688	0.0608	0.1210	0.5603	0.6274	1.0980
QGMS	−0.0923	0.1924	0.2534	0.2724	0.3800	1.3100
QEWC	0.0342	0.0812	0.3265	0.3246	0.3687	0.5346
Mean	0.0718	0.0861	0.2945	0.4013	0.5323	0.8353

Table 6. Wavelet R^2 relative to Table 5 for each level.

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
COMB	0.0170	0.0766	0.1459	0.0499	0.1124	0.1800
DBIS	0.0105	0.0127	0.0199	0.0115	0.0816	0.2020
DICO	0.0033	0.0038	0.0036	0.0122	0.0188	0.1309
DOBK	0.0189	0.0171	0.0990	0.1872	0.1473	0.3869
IHGS	0.0030	0.0012	0.0251	0.0315	0.0443	0.4361
KINS	0.0344	0.0463	0.0215	0.0888	0.1628	0.1029
MARK	0.0922	0.0405	0.1980	0.2250	0.3904	0.3990
QFBQ	0.0027	0.0104	0.0110	0.0615	0.1527	0.1063
QGIR	0.0016	0.0026	0.0034	0.0112	0.0149	0.0575
QIIB	0.0173	0.0121	0.2166	0.2557	0.2187	0.5569
QIIC	0.0104	0.0102	0.0233	0.0244	0.0398	0.3912
QINS	0.0016	0.0101	0.0185	0.0103	0.1481	0.1859
QISB	0.0617	0.0157	0.2621	0.3634	0.4451	0.5424
QNBK	0.0013	0.1574	0.0563	0.3660	0.5373	0.5518
QOIS	0.0107	0.0041	0.0144	0.0492	0.0499	0.5436
AHCS	0.0340	0.0639	0.0502	0.2069	0.2102	0.5367
IQCD	0.1059	0.0226	0.2418	0.4596	0.4618	0.5404
MERS	0.0111	0.0138	0.0230	0.0364	0.2487	0.5247
QIMC	0.0013	0.0024	0.0023	0.0112	0.0261	0.0372
SALM	0.0242	0.0334	0.0524	0.2039	0.2160	0.6156
WDAM	0.0059	0.0105	0.0453	0.0300	0.1171	0.3578
ZHCD	0.0005	0.0110	0.0211	0.0933	0.0518	0.2892
BRES	0.3097	0.1773	0.3585	0.5422	0.5579	0.6507
ERES	0.0084	0.1167	0.2132	0.3426	0.4004	0.6235
MRDS	0.0175	0.0222	0.1016	0.2752	0.3551	0.5849
NLCS	0.0256	0.0156	0.0563	0.1245	0.3381	0.6491
UDCD	0.0142	0.0025	0.1945	0.2592	0.3716	0.5328
MPHC	0.0401	0.0032	0.1652	0.1672	0.2261	0.4411
QAMC	0.0001	0.0007	0.0114	0.2004	0.3314	0.5391
QANC	0.0068	0.0016	0.0210	0.2315	0.5345	0.5571
QIGD	0.0654	0.0736	0.0932	0.2008	0.3417	0.6476
GISS	0.1134	0.1160	0.1610	0.2370	0.1919	0.2689
QFLS	0.1557	0.0862	0.1583	0.2232	0.2642	0.3667
QGTS	0.1827	0.1168	0.2546	0.2449	0.2830	0.4834
MCCS	0.0021	0.0012	0.0021	0.2149	0.2519	0.6643
ORDS	0.0035	0.0458	0.0113	0.3265	0.4798	0.5089
VFQS	0.0086	0.0145	0.0450	0.0230	0.1004	0.3617
GWCS	0.0012	0.0006	0.1201	0.1271	0.2075	0.5199
IGRD	0.0251	0.0011	0.2069	0.2502	0.3029	0.5112
QNNC	0.0021	0.0101	0.0119	0.2005	0.3924	0.5076
MCGS	0.0315	0.0007	0.0444	0.1931	0.2186	0.4336
QGMS	0.0013	0.0147	0.1117	0.1385	0.3117	0.5258
QEWC	0.0106	0.0619	0.1071	0.2434	0.3735	0.5460

Table 7. The mean wavelet beta for each sector.

Sectors	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Financials	0.0255	0.0098	0.3288	0.4273	0.5282	0.8012
Consumer	−0.0869	0.0349	0.2492	0.2763	0.5265	0.7873
Real Estate	0.1773	0.3534	0.3142	0.5416	0.6483	1.1240
Materials	0.0175	0.1892	0.2541	0.4269	0.4081	0.8882
Energy	0.6542	0.5336	0.6746	0.6917	0.7340	0.7346
Technology	0.0518	−0.3651	0.1809	0.2998	0.6194	0.8278
Industrials	−0.0241	−0.0183	0.0434	0.1214	0.3233	0.5280
Healthcare	0.1382	0.1266	0.1872	0.4164	0.5037	1.2040
Utilities	0.0342	0.0812	0.3265	0.3246	0.3687	0.5346

In the present case, it appears from Table 7 that a majority of the sectors (Financials, Real Estate, Materials, Energy, Healthcare, Utilities) have maintained their cohesion during the crisis, which seems evident through the betas, which in turns remained positive and increasing, especially at the medium and long term levels. This confirms the fact that the whole market is resistant to the embargo, and that these sectors are somehow independent of the other GCC countries.

The imported needs in these sectors are not related strongly to the GCC. Sectors such as energy are encouraging for investors and maybe reliable partners and sources of energy. This fact is confirmed clearly during the Russian/Ukrainian war which leads European countries as the largest consumers of energy, especially gas, to change their direction towards Qatar and conclude many contracts in this field to supply Europe with gas.

The sector of consumers has been perturbed at the short horizons due to the dependence on GCC countries in many consumer products such as foods which are somewhat natural as Qatar and generally the GCC region is an importing country more than a self-producing in this sector. Technology and industrial sectors are also perturbed at short horizons due to the partnership with other countries in the GCC region such as telecommunications, spare parts, etc.

Globally, most sectors show a positive relationship in the medium to high levels. By applying the wavelet model, which generally allows going further into the microscopic state of the market, we notice that the positive relationship between the market and sectors becomes more and more important for large scales. These conclusions join previous ones about the ability of this market to remain stable during the blockade and thus make it a good center and destination for investors. These findings are reinforced by Figures 4 and 5 where the sectors' portfolio returns are regressed versus the market return.

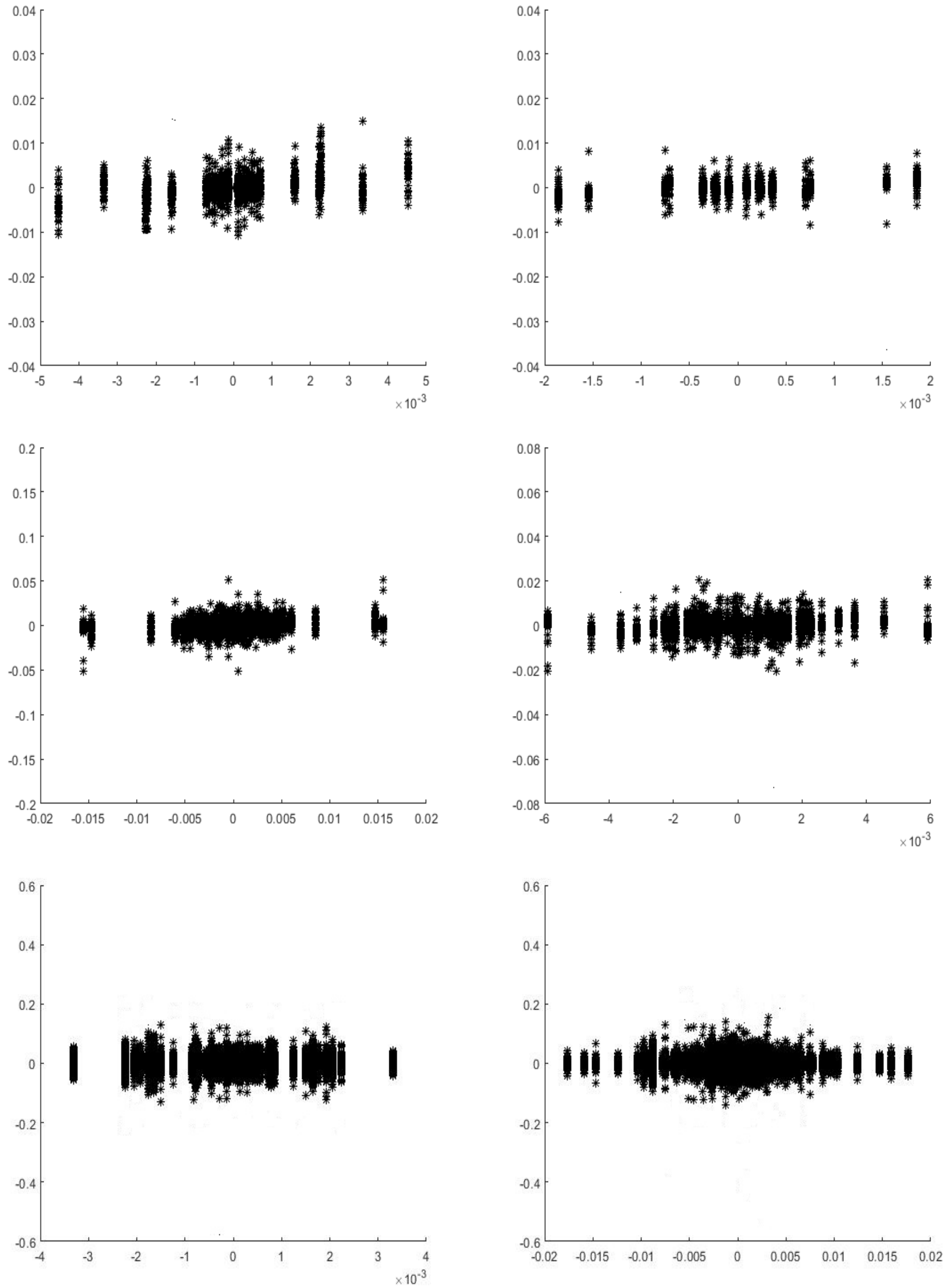

Figure 3. Wavelet excess market return (horizontal axis) versus wavelet excess return of the action (vertical axis) at levels j=1,2,3,4,5,6.

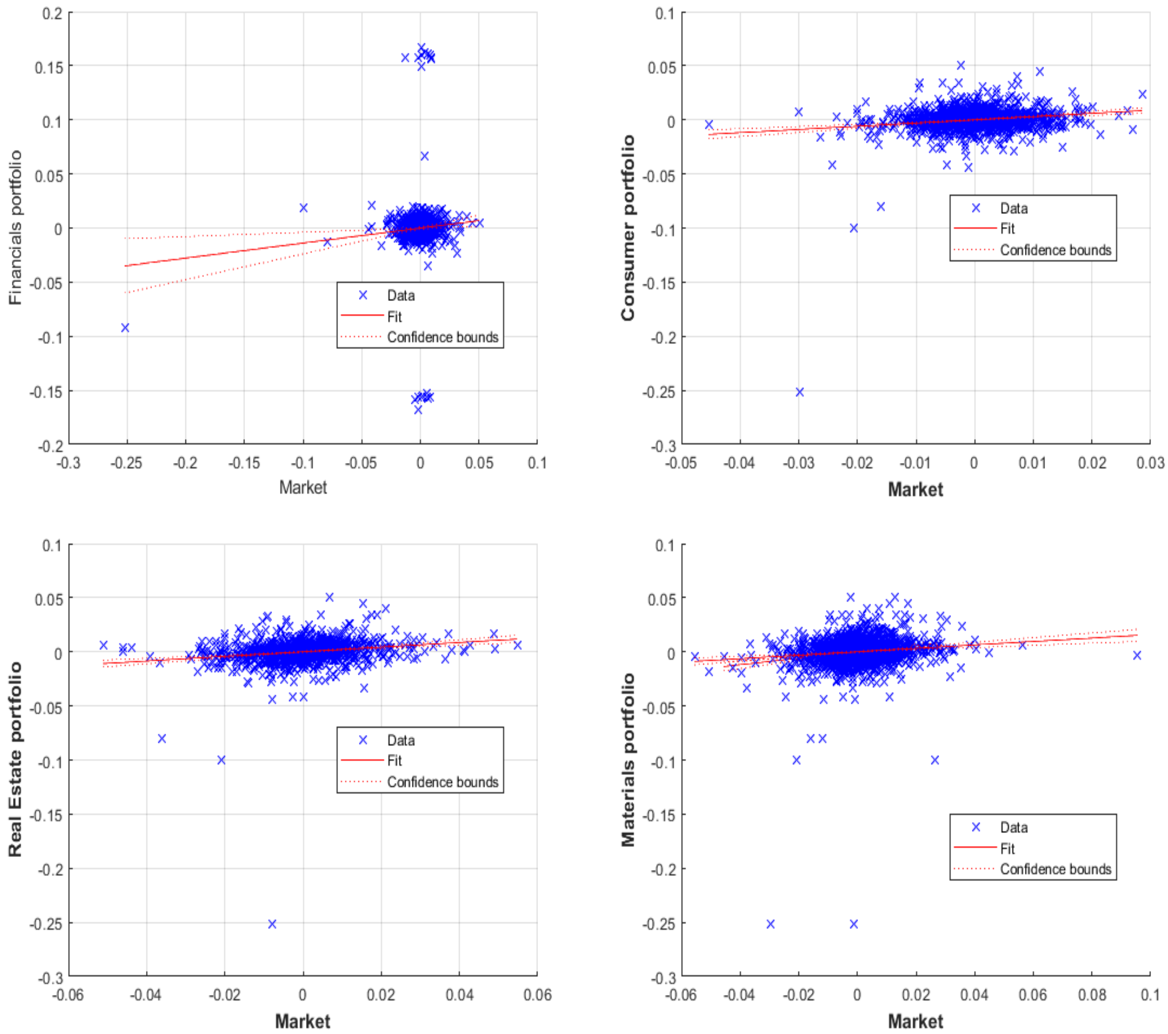

Figure 4. Regression of sectors' portfolio return (vertical axis) versus the market return (horizontal axis).

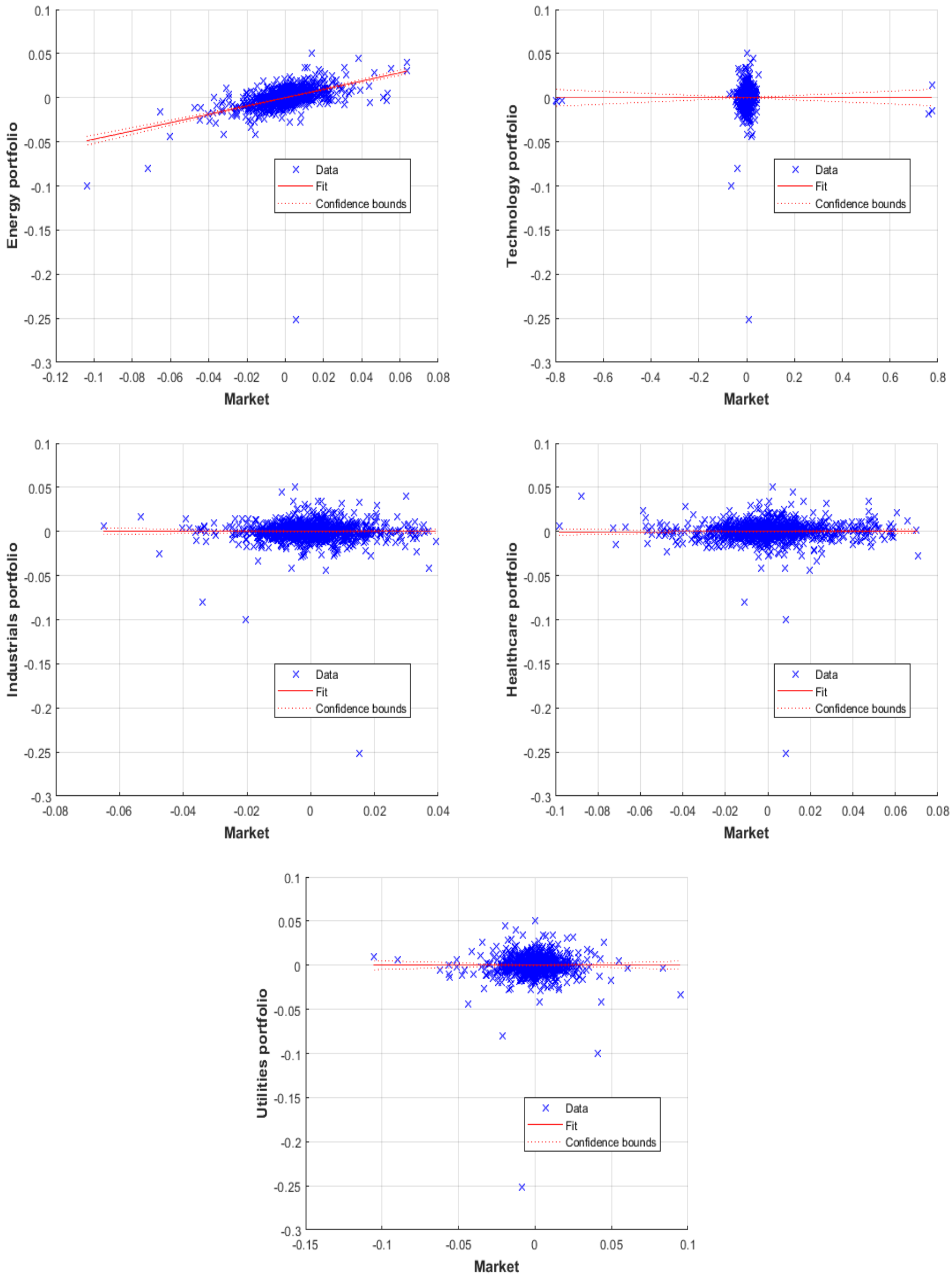

Figure 5. Regression of sectors' portfolio return (vertical axis) versus the market return (horizontal axis).

Although over the past few years, Qatar has tried similarly to other members of the GCC region, to diversify the national incomes away from oil and gas, we notice that this sector is still the most important one, and majorly, it was the main sector that keeps the market somewhat stable against the embargo.

4.2. Comparison with DY approach, Diebold and Yilmaz (2012), Diebold and Yilmaz (2009)

For robustness, efficiency and performance of our method ahead of past research, a subsection is added due to the application of another model due to Diebold and Yilmaz (2012, 2009) as suggested in the review comments of the editor(s) and the reviewers.

The essential aim of this subsection is to conduct a comparison between our main approach of risk transmission based on wavelets with other approaches of risk transmission such as Diebold and Yilmaz (2012, 2009). The authors may also refer to Klöβner and Wagner (2013), Naeem et al. (2022), Mensi et al. (2022) and Yousaf and Yarovaya (2022) for more investigations on the DY approach. However, as our main object of the present work is to investigate the systematic risk beta, and not to compare it with other indexes, we will not go into developing all the details of the DY approach and will restrict its application to the QEAS sectors instead of all the stock market indices. We however recall that some attempts on all the indices did not yield satisfactory results.

Figure 6 illustrates the matrix correlation due to the different sectors. Table 8 presents the static return spillover matrix across the sectors of the Qatar stock exchange index QEAS. The non-principal diagonal elements estimate the directional risk spillover effect of the two interactions. The column 'FROM' is due to each sector's exposure to aggregate risk spillovers from the other sectors, while the line 'TO' reflects the total risk spillover from each sector to the remaining sectors. The higher values in the column 'From' indicate that the sector faces more exposure to volatility, and the higher values in the line 'TO' indicate that the sector is facing more exposure to volatility in the remaining sectors.

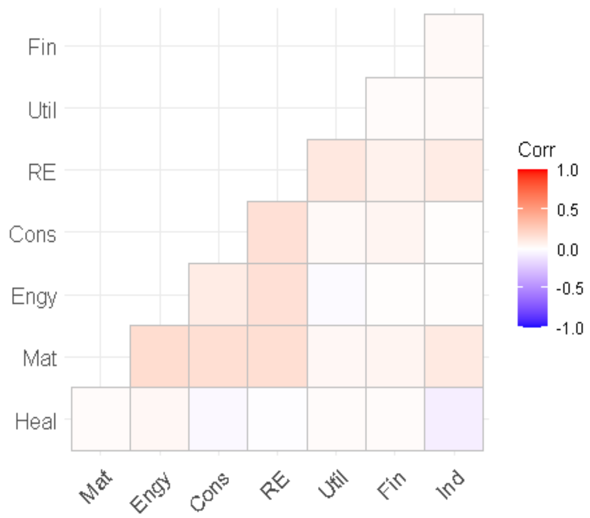

Figure 6. The matrix correlation due to DY approach.

Next, to illustrate the dynamic spillover returns according to DY approach, we plotted in Figure 7 the total returns spillover of the listed sectors on the Qatar stock market, over the entire period studied without removing the outliers (at left) and after removing the outliers (at right).

Table 8. Static return spillover matrix.

	Financials	Consumer	Real Estate	Materials	Energy	Industrial	Health	Utility	FROM
Financials	93.47	0.72	0.63	0.68	0.24	0.27	3.39	0.59	6.53
Consumer	0.96	89.13	3.36	0.94	1.18	1.41	0.49	2.54	10.87
Real Estate	0.98	11.36	80.48	0.85	2.52	0.68	0.50	2.63	19.52
Materials	0.59	3.44	3.30	87.24	1.14	1.78	0.31	2.20	12.76
Energy	0.65	4.48	10.18	2.52	69.38	2.46	0.48	9.85	30.62
Industrial	0.71	1.37	0.52	1.11	0.60	92.81	1.30	1.58	7.19
Health	0.13	0.69	0.65	0.10	0.77	0.45	96.82	0.40	3.18
Utility	1.02	1.45	3.33	1.02	0.76	0.93	0.25	91.23	8.77
TO	5.04	23.51	21.97	7.23	7.20	7.98	6.73	19.79	99.45
Inc.Own	98.51	112.63	102.45	94.47	76.58	100.79	103.54	111.02	cTCI/TCI
NET	-1.49	12.63	2.45	-5.53	-23.42	0.79	3.54	11.02	14.21/12.43
NPT	5.00	4.00	4.00	2.00	1.00	4.00	3.00	5.00	

Figure 7. Total connectedness.

The following graphics present the directional return spillovers from the others to each of the eight asset classes. More precisely, Figure 8 illustrates the directional returns spillovers from eight asset classes. Figure 9 represents the To spillover graphs. Figure 10 is designated for the Net spillover graphs. Figure 11 represents the Net Pairwise returns Spillovers. Finally, Figure 12 illustrates the Network representation. We just recall the special abbreviations, Financials (Fin), Consumer (Con), Real Estate (RE), Materials (Mat), Energy (Engy), Industrial (Ind), Health (Heal), and Utility (Util).

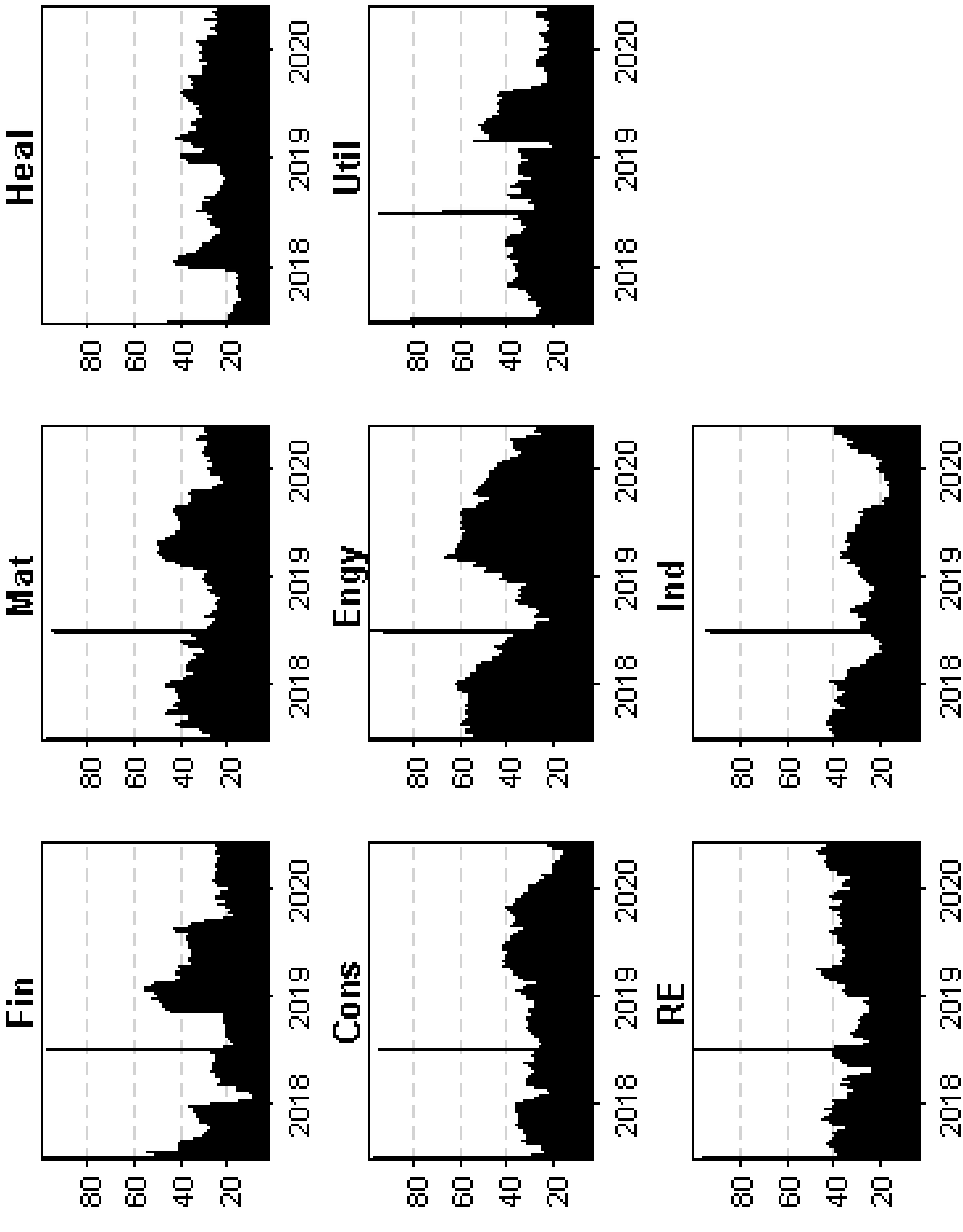

Figure 8. Directional returns spillovers, FROM eight asset classes.

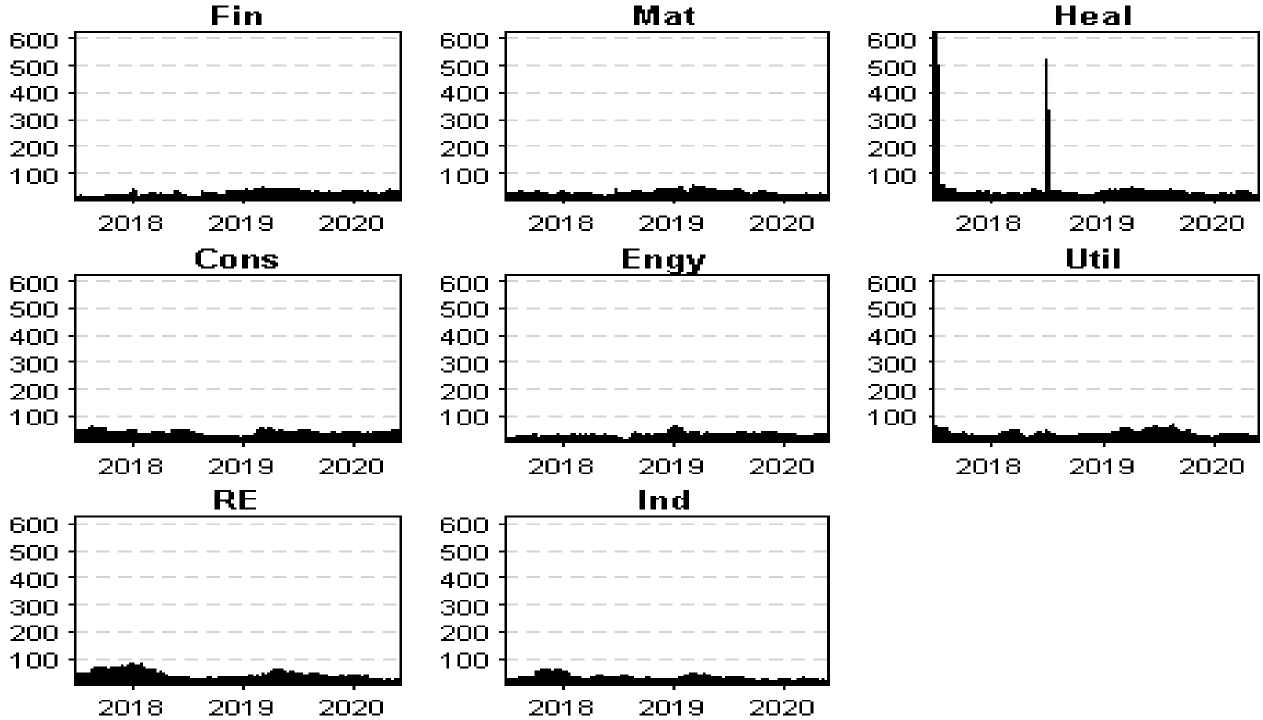

Figure 9. Directional returns spillovers, TO eight asset classes.

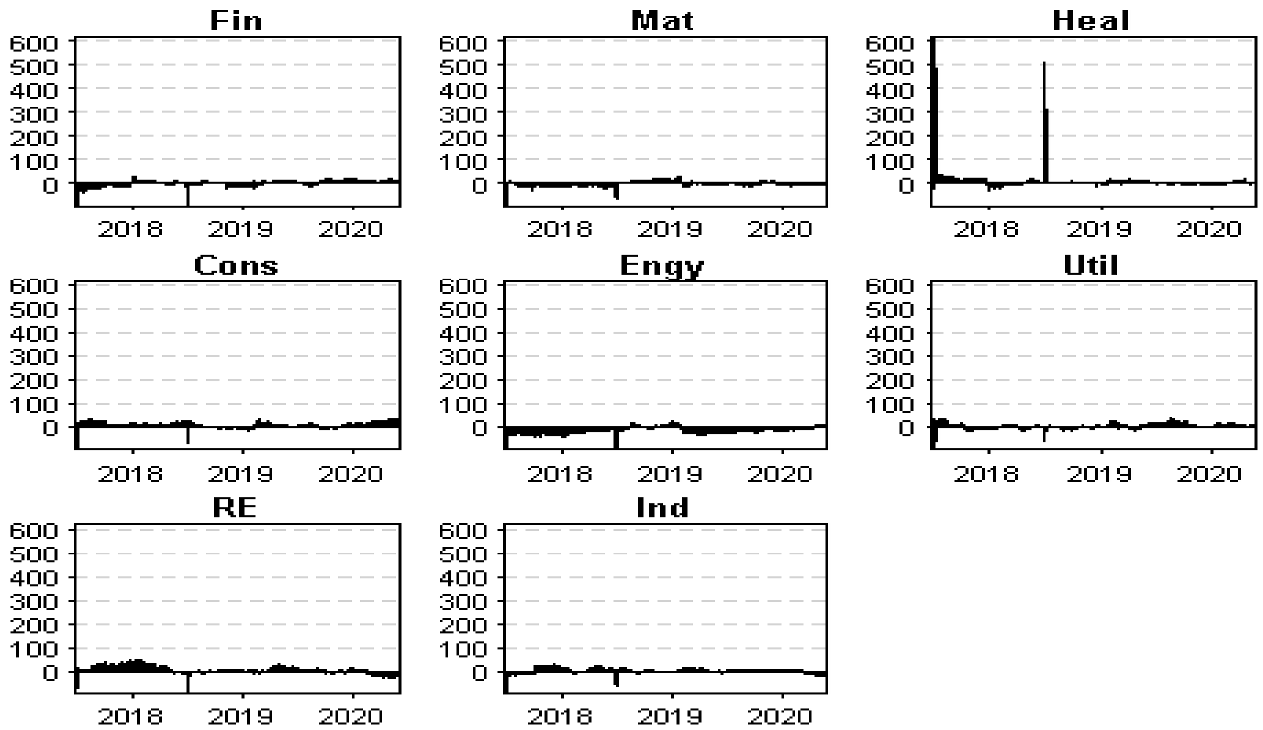

Figure 10. Directional returns spillovers, NET four asset classes.

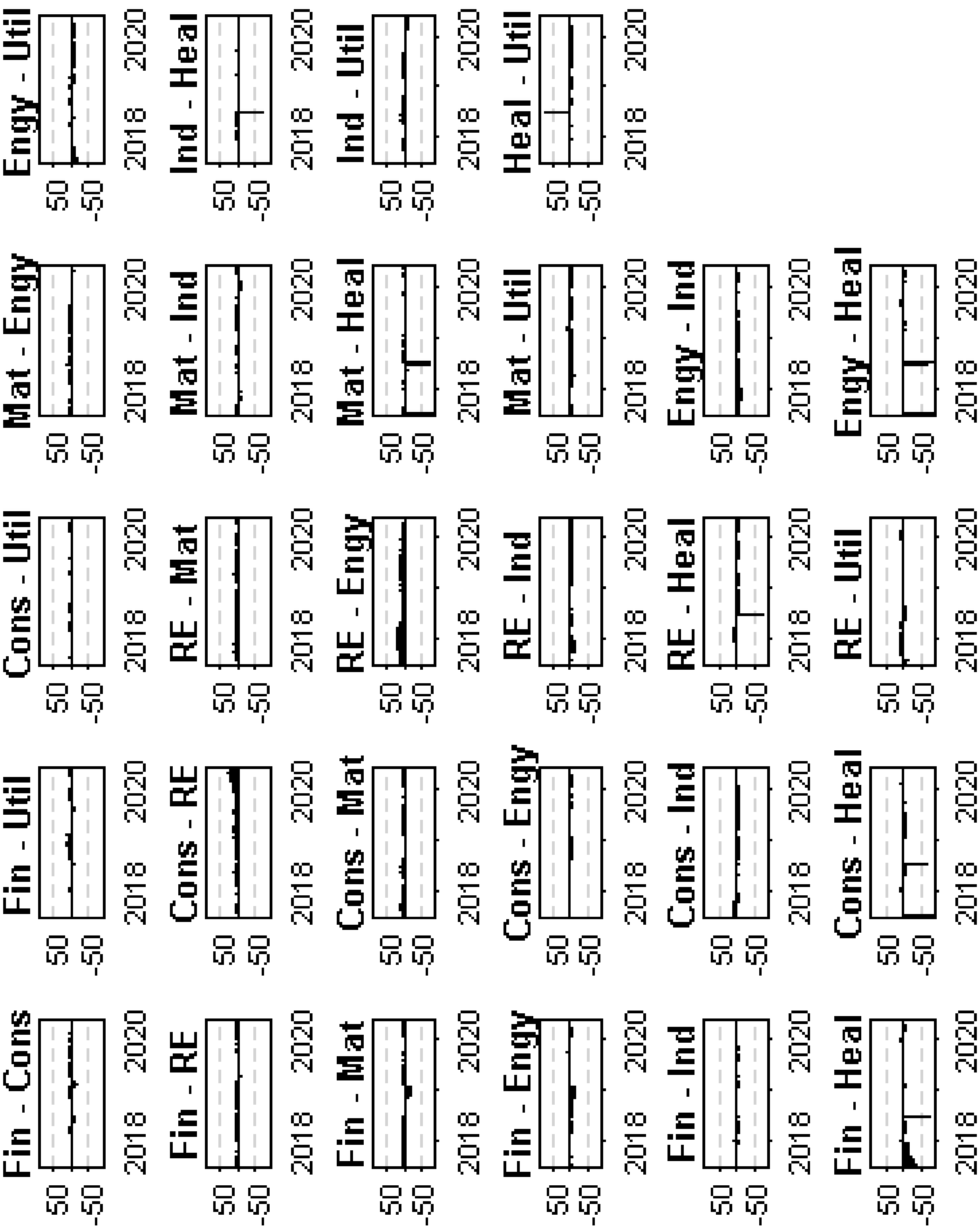

Figure 11. Net pairwise returns spillovers.

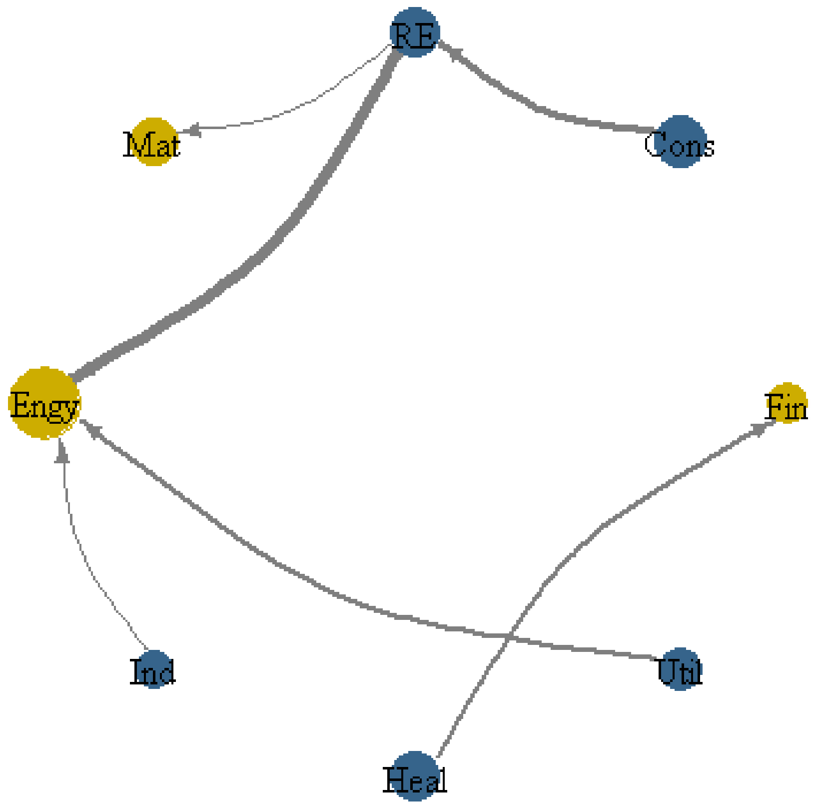

Figure 12. Network representation.

Notice from the spillover analysis that the total return spillover index is 99.45%, indicating higher and more significative connectedness among stock exchange sectors. The row "TO" in Table 8 indicating the contribution of each variable to the system reveals that the return spillovers are highest from the consumer discretionary (23.51%), real estate (21.97%), and utilities (19.79%).

The analysis of Figures 6, 7, 8, 9, 10, 11, 12, and Table 8 shows an asymmetry in the risk spillover relationships, with different sectors. In terms of the direction of risk absorption, the sector of energy has the highest degree of risk absorption, reaching 30.62, followed by the sector of real estate, which has a degree of 19.52. The third direction is due to the materials sector which reaches 12.76, followed by the sector of consumers, reaching 10.87. Next, from the point of view of the most significant risk spillover effects, the sector of real estate is the highest reaching approximately 25, followed by the consumer's sector with 23.51, and next by the utility sector reaching approximately 19.80.

These empirical findings yield that the sectors of energy, real estate, and materials absorb risk spillovers, while the sectors of real estate, consumer, and utility export risk spillovers in the financial system. For investors, these findings mean that investing in the energy, real estate, and materials sectors may provide a hedge against risk. On the other hand, investing in real estate, consumer, and utility sectors may expose investors to increased risk. In theoretical finance, this yields the need from regulators for increased oversight and regulation of the last sectors to reduce their risk-exporting behavior and prevent potential risk. Also, we may need to develop financial stability policies to reduce risk and promote more stability in the market against crises.

4.3. *Examination of some stylized facts*

Returning now again to the major affecting facts in financial series investigation, stylized facts are linked to high-frequency noises, and may therefore decrease the efficiencies and accuracy of the model. Wavelets are used as promising filtering tools to localize and filter these extreme values. Theoretically speaking, there is no prior choice for fixing the wavelet applied, as it needs generally to carry out reasonable data processing to the tested signal in order to enhance the accuracy of the numerical simulation. The wavelet transform possesses the principal ability to act simultaneously on time and frequency domains, or space and frequency domains. The main difference between wavelets relies only on the number of moments; however, it did not differ widely in the processing. It needs just to extend sometimes the level of decomposition.

In Figure 13 below, we plotted the result of the comparison between the actual probability distributions of the major sectors due to those absorbing the risk spillovers, and those exporting it, and the theoretical normal distribution for the returns due to these sectors.

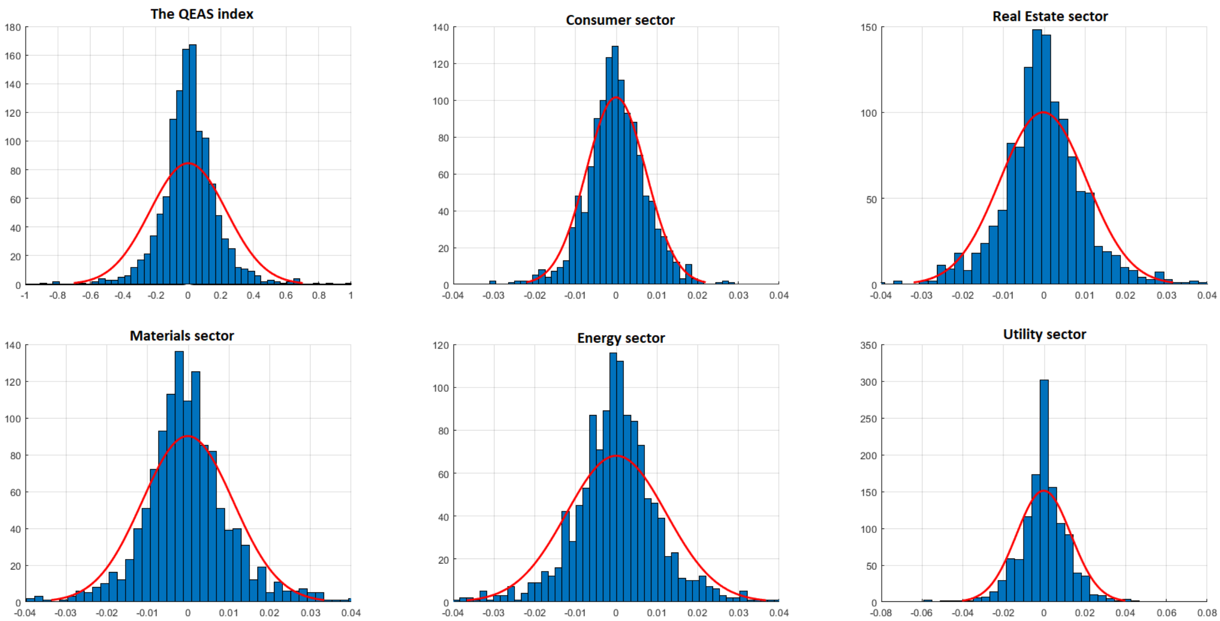

Figure 13. Comparison between a theoretical normal distribution and the actual probability distributions of sectors absorbing/exporting spillovers.

Besides, the Figures 14, 15, 16, 17, 18, and 19 illustrate further inspection based on the Q-Q plots of the approximation of different sectors' indexes at different levels $J = 1$ to $J = 6$. The straight lines line represent the Standard Normal distribution. Whenever this line is parallel to the observed distribution, we estimate that the returns follow the Gaussian distribution.

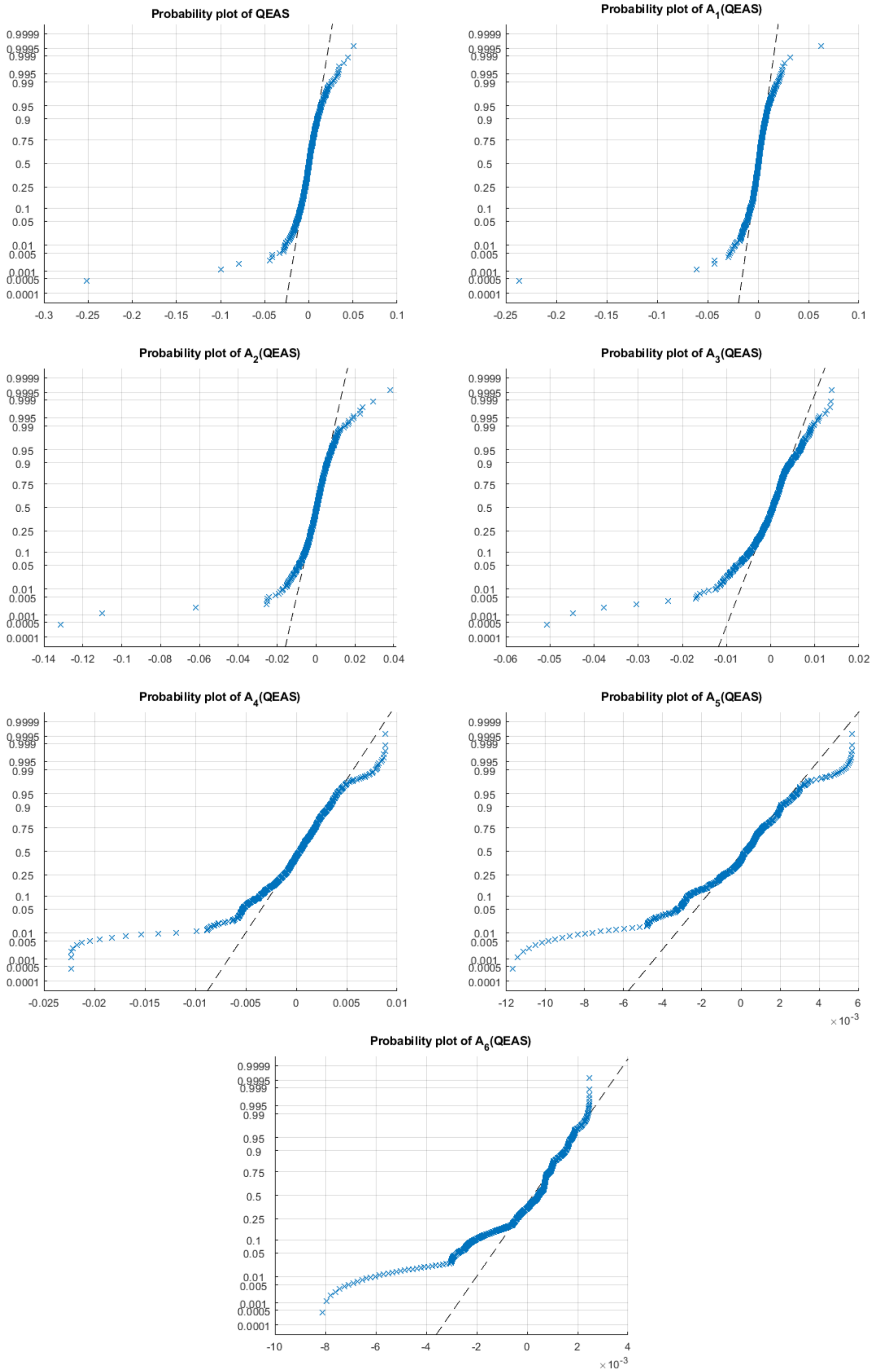

Figure 14. Q-Q plots for QEAS index at the levels $J = 1, 2, 3, 4, 5, 6$.

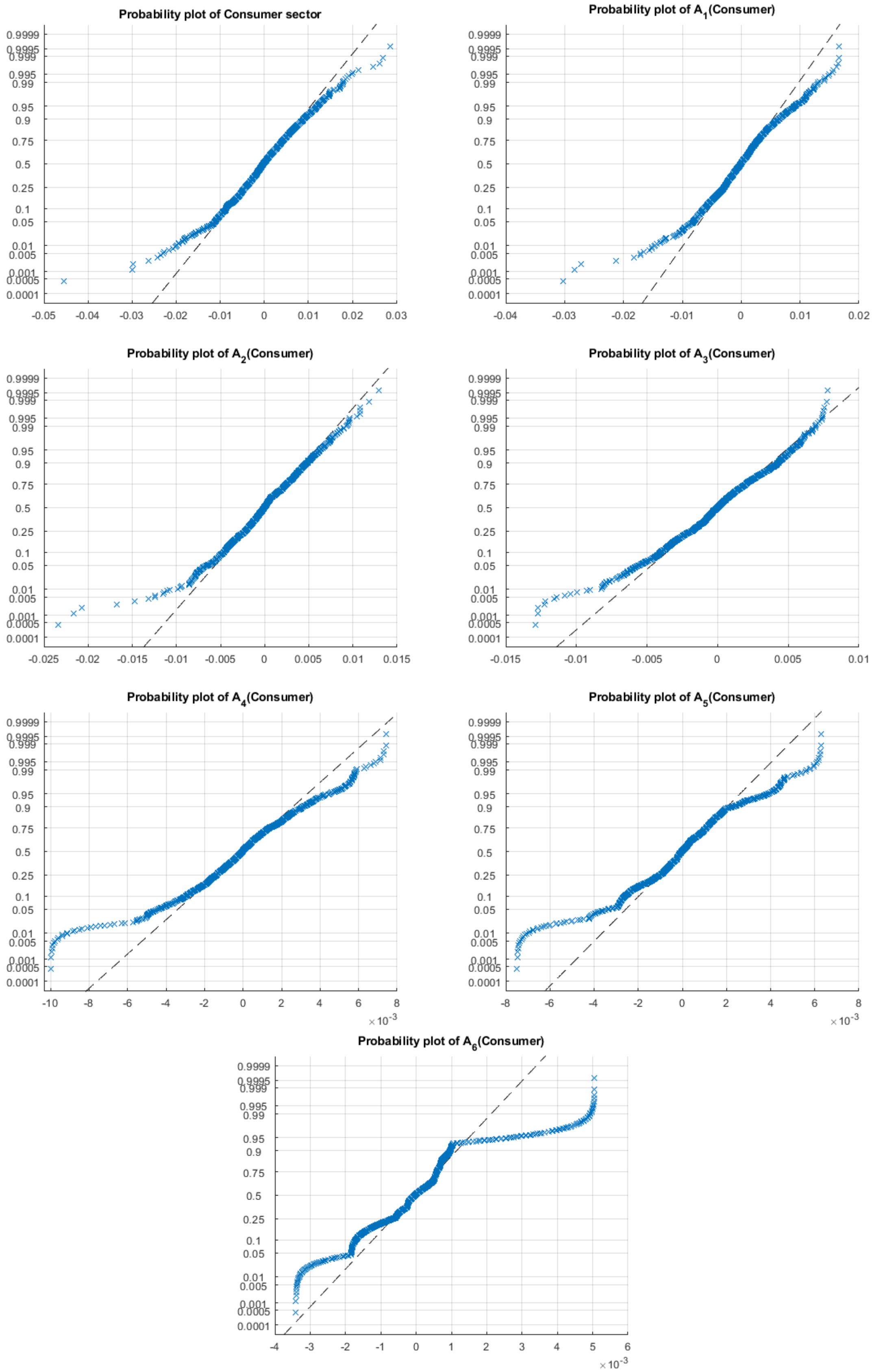

Figure 15. Q-Q plots for Consumer sector at the levels $J = 1, 2, 3, 4, 5, 6$.

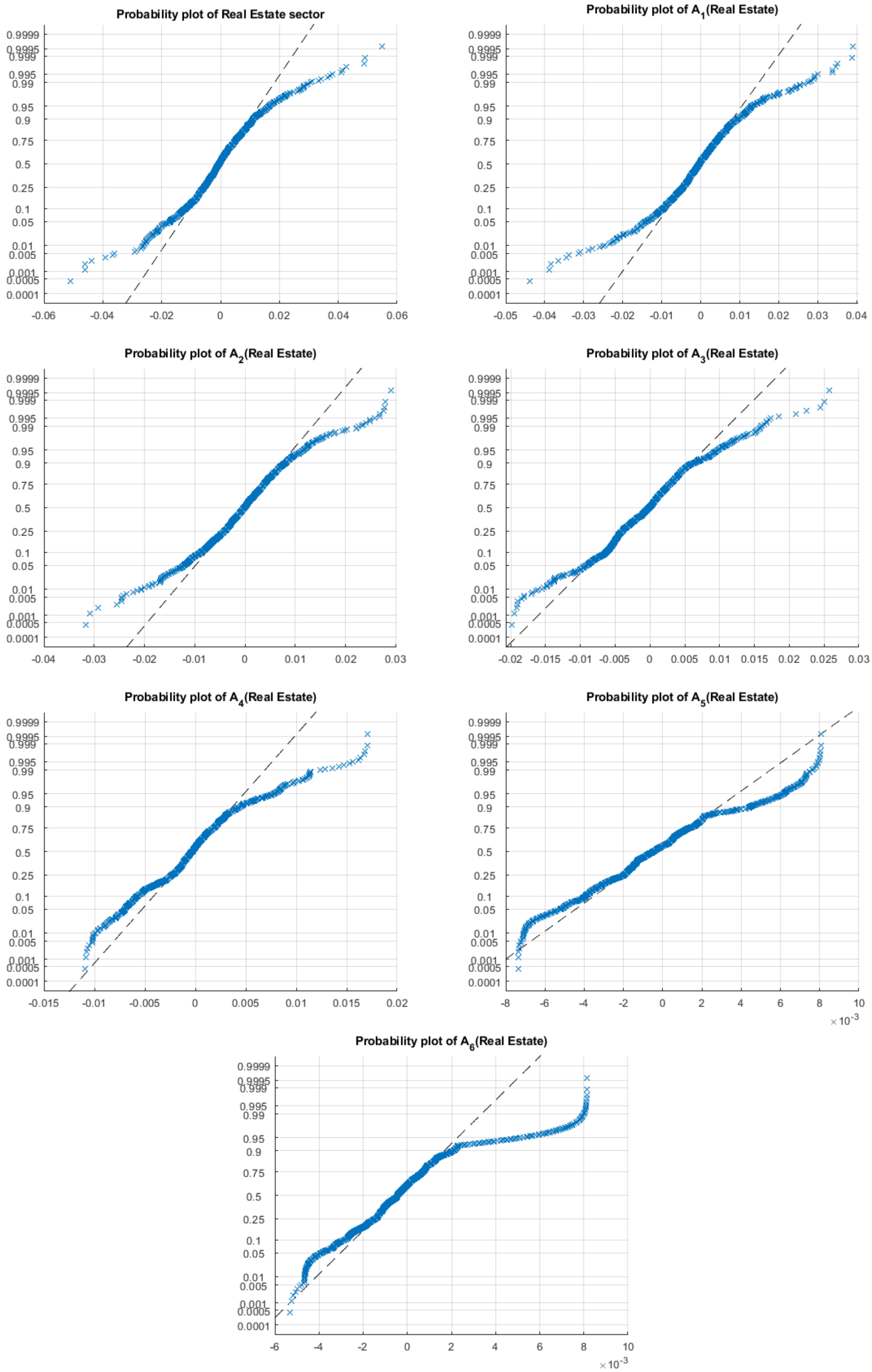

Figure 16. Q-Q plots for Real Estate sector at the levels $J = 1, 2, 3, 4, 5, 6$.

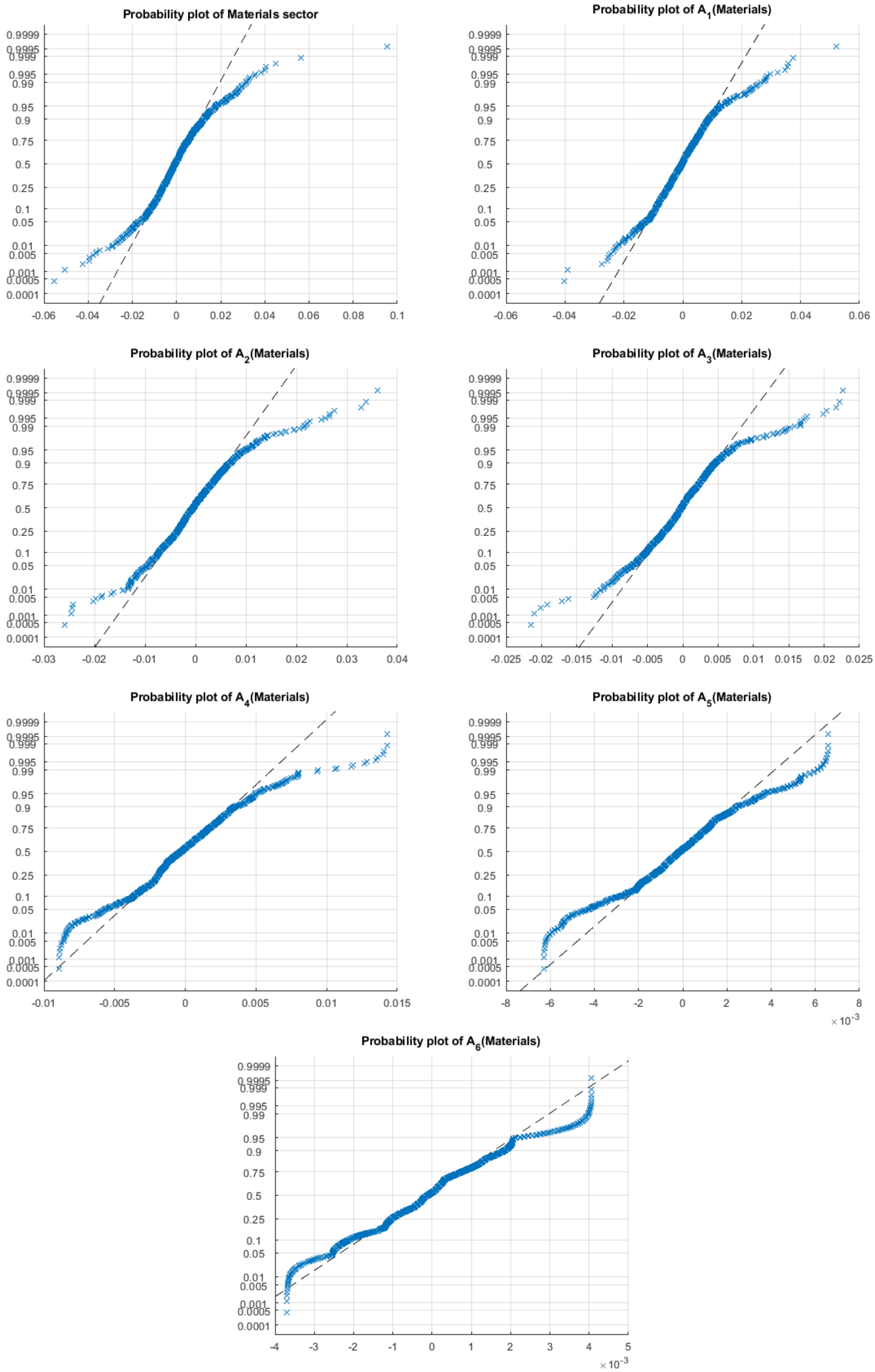

Figure 17. Q-Q plots for Materials sector at the levels $J = 1, 2, 3, 4, 5, 6$.

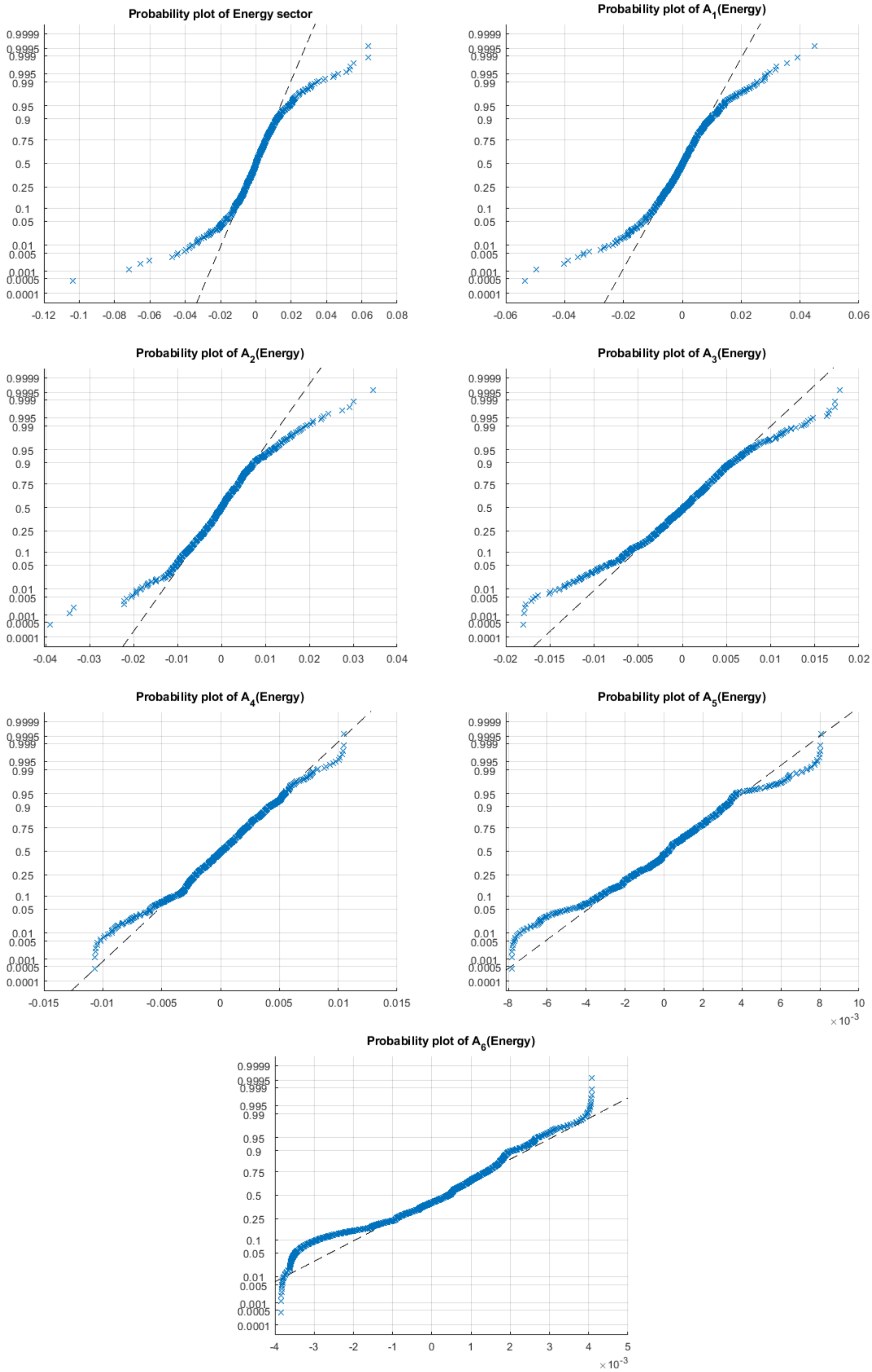

Figure 18. Q-Q plots for Energy at the levels $J = 1, 2, 3, 4, 5, 6$.

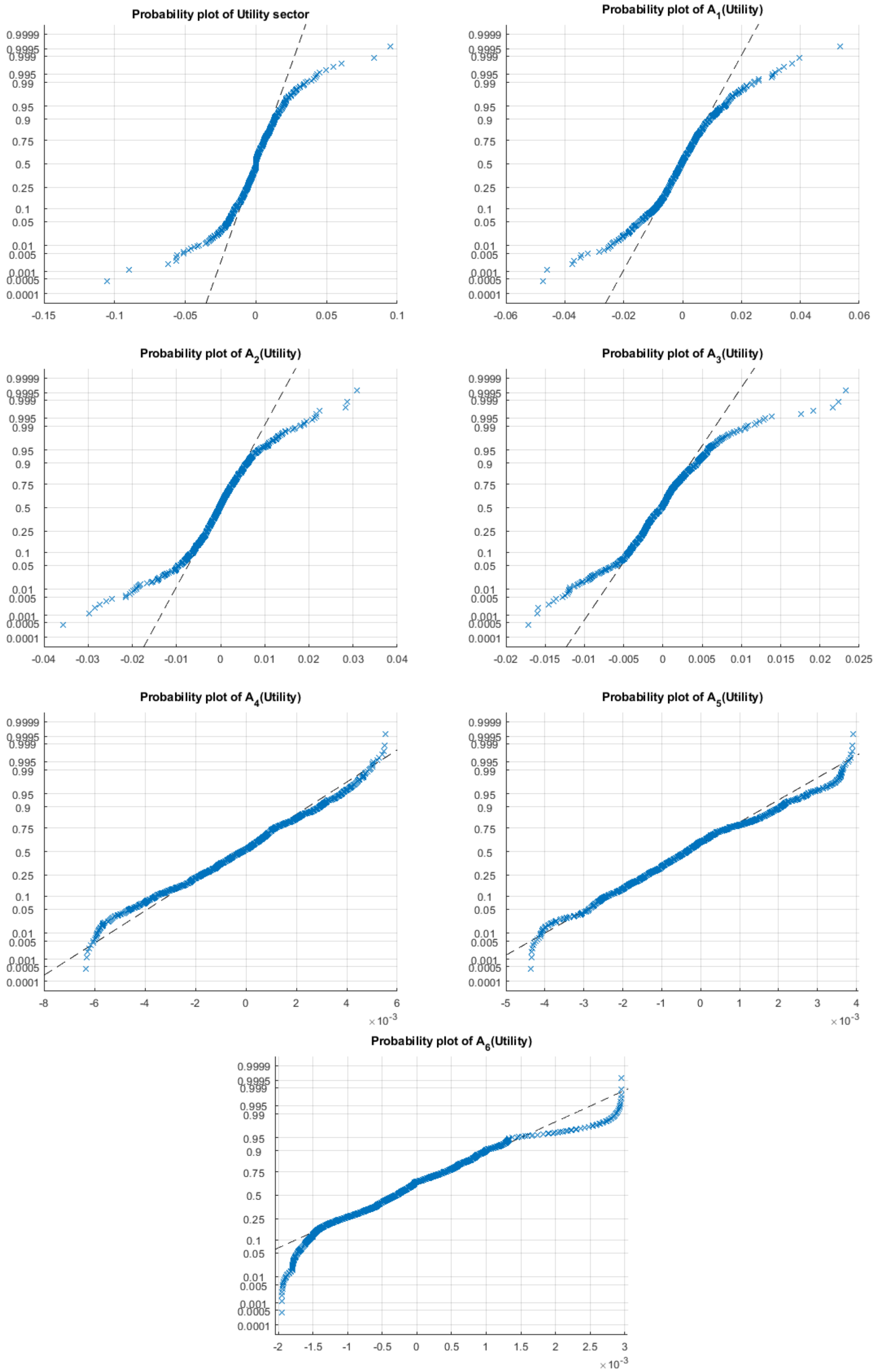

Figure 19. Q-Q plots for Utility sector at the levels $J = 1, 2, 3, 4, 5, 6$.

By investigating the plots in Figure 13 we notice easily that the distributions of returns differ from the normal distribution for all the indices or sectors. By investigating Figures 14, 15, 16, 17, 18, and 19, we see that there are differences, mostly in the tails. This confirms the findings previously mentioned.

The figures confirm the fat tails stylized fact presence. Moreover, we notice that the use of the wavelet levels of decomposition explains more clearly the time factor effect on the model. Recall that the period considered includes the main embargo crisis. The plots show the instability due to such a crisis according to time scales, and the propagation of the risk. For the whole market, for example, the plot of the QEAS index (Figure 14) at low levels (short horizons) shows an early perturbation, which becomes more clear at high horizons, yielding the strong presence of stylized factors. The same observations may be concluded for the Consumer sector (Figure 15). The Real Estate sector (Figure 16), although it presents also the presence of stylized facts, it possesses some minor instability at medium levels. The energy and Materials sectors seem to be the most stable ones with already stylized facts shown easily (Figure 17, and Figure 18). Finally, Figure 19 shows the presence of stylized facts due to the embargo getting more stable at medium to high levels.

The whole set of sectors shows an accelerated recovery phase taken in the middle of the period, explaining the role of the government, and policymakers in recovering the stability of the market. As an extension to the present findings, we may propose to study the evolution of standard deviation, beta coefficients, unconditional coefficients, and the range of returns for different stages by using a combination between wavelet analysis and change point analysis as suggested in the conclusion. The findings confirm also the dependence to the time scale of the statistical measures of the evolution of financial assets' returns.

5. Conclusions

In the present work, we developed a mathematical model for the systematic risk beta known also CAPM by involving a non-uniform wavelet basis to emphasize the dependence on the time scale of the market behavior. The empirical study is conducted using the Qatar stock exchange QEAS as a representative index of one of the most important cases in the so-called GCC region. Our aim was to focus on the impact of the embargoes on the systematic risk evaluation and thus on the veracity of the CAPM and its resistance against crises.

The findings confirm our idea of using non-uniform (dynamic) subdivisions of a whole period for accurately estimating the beta and for understanding the situation of the market. The present study permitted us to conclude that even in severe moments such as embargoes, the Qatar market succeeded to overcome the crises and to recover from the failure that appeared at the beginning of the period. It also confirmed that the use of classical fixed periods is not adequate for the comprehension of the market, especially in the perturbation periods. The stability of the systematic risk beta depends strongly on the time scales and the way of varying such factors. It seems that the uncertainty in the market is the major factor behind the failure of the use of a prior fixed subdivision.

The application of wavelets in the present case showed that the market is stable at medium to high scales, which explains that the Qatar stock market is resistant to shock and crises. The predictions of the CAPM are more relevant at the higher horizons in a multi-scale framework as compared to other horizons. This means that such a market may be encouraging for investments in long time horizons.

The findings indicate that this choice of time scale and wavelet functional basis is adequate for the estimation of the systematic risk beta transmission. Also, the results indicate that the crises such as the embargo and geopolitical tensions in general have a significant impact on the market. At the same time, the results showed that investors' behaviors may be crucial in maintaining the status of the market, and thus increase or decrease the risk of investment.

The study also shows important insights into the dynamics of markets during crises, highlighting the need for sophisticated modeling techniques to effectively capture the complexities of these markets. The study indicates finally the need to develop and innovate important methods and insights to understand the movement of markets and their dynamics during crises.

Finally, it is worth noticing that while it is certainly important to study systemic risk during the embargo, it would be interesting to also study risk propagation in the period prior to the paper's sample and compare the results for the two periods. A study in this direction is proposed to compare the risk propagation by using a combination of wavelets with change point analysis. For instance, the reader may refer to Balalaa and Ben Mabrouk (2023), where a step forward is conducted in this subject.

Use of AI tools declaration

The authors declare that Artificial Intelligence (AI) tools haven't been used in the creation of this article.

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Conflict of interest

The authors declare that no conflicts of interest in this paper.

Appendix: Wavelet time series processing

Wavelet analysis starts from one source function ψ known as the mother wavelet, and next composes dilation-translation copies to get a complete system for finite energy time series. Each wavelet basis element is defined for $j, k \in \mathbb{Z}$ as a copy of ψ at the scale j and the position k by $\psi_{j,k}(t) = 2^{-j/2}\psi(2^j t - k)$. The quantity 2^j corresponds to the frequency of the series, while the index k localizes volatility or fluctuations. For $j \in \mathbb{Z}$ fixed, $W_j = span(\psi_{j,k},\ k)$, known as the j-level detail space. A time series $X(t)$ is projected onto W_j yielding a component $DX_j(t)$ given by

$$DX_j(t) = \sum_k d_{j,k}\psi_{j,k}(t). \tag{18}$$

The $d_{j,k}$ are the detail coefficients of the series $X(t)$, expressed by means of the ordinary inner product in the functional space $L^2(\mathbb{R})$ as

$$d_{j,k} =< X, \psi_{j,k} >= \int_{\mathbb{R}} X(t)\psi_{j,k}(t)dt. \tag{19}$$

The spaces W_j's form an orthogonal decomposition covering the space of finite energy series $L^2(\mathbb{R})$. This means that the series $X(t)$ can be completely reconstructed as a sum of its projections on the detail spaces and that these projections are mutually uncorrelated. In wavelet theory, the mother wavelet yields a second function called the father wavelet or scaling function denoted here by φ. Similarly to ψ, the function φ yields dilation-translation copies $\varphi_{j,k}(t) = 2^{-j/2}\varphi(2^j t - k)$ generating subspaces V_j. The sequence $(V_j)_j$ is called a multi-resolution analysis (multi-scale analysis) on $\mathbb{R}$ and V_j is called the j-level approximation space. It is well known in wavelet theory that $V_j \subset V_{j+1}$, $j \in \mathbb{Z}$, which means that the approximation of the time series at the level j and $j+1$ can be viewed from each other and so from any horizon $p \geq j+1$. In physics-mathematics, this is called the zooming rule. It holds also that for all $j \in \mathbb{Z}$, $f(t) \in V_j$ iff $f(2t) \in V_{j+1}$, which reflects the fact that not only the signal f from horizon j can be seen in the horizon $j+1$, but also his contracted or dilated copies. As for the detail subspaces, the approximation subspaces V_j's satisfy also a completeness relation, meaning that no information is lost when considering all approximations, and a second property meaning that all the information is lost at finer scales. Finally, the Vj's satisfy a shift-invariance property in the sense that $f(t) \in V_j$ iff $f(t-k) \in V_j$, $j,k \in \mathbb{Z}$, which means that the multi-resolution analysis permits to detect the properties of the signal along the whole time support. Combining all the properties above, we deduce that the approximation space is decomposed into a low-level approximation part supplemented with a detail part. Under these properties, the following decomposition is proved for $j \in \mathbb{Z}$,

$$X(t) = \sum_j DX_j(t) = \sum_{j \leq J} DX_j(t) + \sum_{j \geq J+1} DX_j(t). \tag{20}$$

The component $AX_J(t) = \sum_{j \leq J} DX_j(t)$ is called the approximation of $X(t)$ at the level J and it reflects the trend or the global shape of $X(t)$. It also belongs to the space V_J. Thus, using the definition of the V_J's, the component $AX_J(t)$ may be expressed using the basis $(\varphi_{J,k})_k$ as

$$AX_J(t) = \sum_k a_{J,k}\varphi_{J,k}(t), \tag{21}$$

where the $a_{J,k}$ are the approximation coefficients of the series $X(t)$ expressed by $a_{J,k} =< X, \varphi_{J,k} >$. As a result, we obtain the following relation known as the wavelet decomposition of $X(t)$,

$$X(t) = AX_J(t) + \sum_{j \geq J+1} DX_j(t). \tag{22}$$

It is composed of one part reflecting the global behavior of the series, and a second part reflecting the higher frequency oscillations or the fine-scale deviations of the series near its trend. In practice, we cannot obviously compute the complete set of coefficients. We thus fix a maximal level of decomposition J, and consider the decomposition for any $J_0 < J$,

$$X_J(t) = AX_{J_0}(t) + \sum_{J_0 < j \leq J} DX_j(t). \tag{23}$$

There is no theoretical method for the exact choice of the parameters J_0 and J. However, the minimal parameter J_0 does not have an important effect on the total decomposition and is usually chosen to be 0. But, the choice of J is always critical. One selects J related to the error estimates.

In finance, economics, management, and generally actuarial sciences, compared to classical theories, wavelet analysis is still less used, although it proved good results, and it needs to be more developed. Recently, the literature has started growing rapidly. See Aktan et al. (2009), Arfaoui et al. (2021), Ben Mabrouk et al. (2008), Ben Mabrouk et al. (2010), Conlon et al. (2008), Cifter and Ozun (2007), Cifter and Ozun (2008), Fernandez (2006), Gençay et al. (2002), Gençay et al. (2003), Gençay et al. (2005), Hubbard (1998), In and Kim (2006), In and Kim (2007), In et al. (2008), Magni (2007b), Percival and Walden (2000), Sharkasi et al. (2006), Xiong et al. (2005), Yamada (2005).

To estimate the CAPM with wavelets, the first step is to pass by the variance and the covariance of the statistics (time) series and introduce the analogs for the components due to the wavelet decomposition.

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