

PREFACE TO “THE MATHEMATICS OF CONCRETE”

Although the concrete is a simple man-made material with initially-controlled composition (for instance, all ingredients are known beforehand, the involved chemical mechanisms are well studied, the mechanical strength of test samples is measured accurately), forecasting its behaviour for large times under variable external (boundary) conditions is not properly understood. The main reason is that the simplicity of the material is only apparent. The combination of the heterogeneity of the material together with the occurrence of a number of multiscale phase transitions either driven by aggressive chemicals (typically ions, like in corrosion situations), or by extreme heating, or by freezing/thawing of the ice lenses within the microstructure, and the inherent non-locality of the mechanical damage leads to mathematically challenging nonlinear coupled systems of partial differential equations (PDEs).

These systems essentially describe multi-physics in terms of balance laws, often by means of coupled fluxes interplaying at multiple length scales. For such complex PDEs scenarios, deep insight by mathematical modelling is needed to support the mathematical analysis of the underlying models, which then offers the theoretical basis for a proper numerical analysis of the discretisation schemes and efficient simulations.

We emphasize here a few worked out examples where both the modelling and mathematical analysis are inspired by specific case studies relevant in the context of civil engineering including (poro-)mechanics, chemical ingress, moisture transport, and high temperature impact. A careful reader will notice that none of the presented problems is completely solved and also that a couple of fundamental questions are open.

The collection of papers is as follows:

- G. Pijaudier-Cabot and D. Grégoire propose *A review of non-local continuum damage: modelling of failure?* The paper contains not only a review of known results, but also gives new insights in the matter, additionally pointing out new links with mathematical research.
- Extending results by A. Ainouz, M. Eden and M. Böhm deal with the *Homogenization of a pore-elasticity model coupled with diffusive transport and a first-order reaction for concrete*. This is a first attempt preparing the averaging of more complex phase transitions in materials with hierarchical substructures.
- Looking at basic mechanical scenarios translated in terms of coupled mass-spring systems, H. Notsu and M. Kimura prove the *Symmetry and positive definiteness of the tensor-valued spring constant derived from P₁-FEM for the equations of linear elasticity*.
- C. V. Nikolopoulos uses matched asymptotics expansions for the *Mathematical modelling of a mushy region formation during sulphatation of calcium carbonate*. The interest lies here on averaging microstructures with freely evolving boundaries described in the level-sets framework.

- A *One dimensional free boundary problem for adsorption phenomena* arising in the context of concrete carbonation is investigated by N. Sato, T. Aiki, Y. Murase, and K. Shirakawa. The authors obtain the existence and uniqueness of local-in-time solutions to their system.
- The classical problem of fast-reaction asymptotics is studied by D. Hilhorst and H. Murakawa within the framework of a *Singular limit analysis of a reaction-diffusion system with precipitation and dissolution in a porous medium*.
- T. Aiki and K. Kumazaki prove the *Uniqueness of solutions to a mathematical model describing moisture transport in concrete materials*. The particularity of the proposed system lies in the presence of differential inclusions.
- T. Fatima, E. Ijioma, T. Ogawa and A. Muntean consider the *Homogenization and dimension reduction of filtration combustion in heterogeneous thin layers*, pointing out the formation of combustion fingers in heterogeneous sheets.
- Having in mind the behaviour of cementitious materials exposed to heat gradients, O. Krehel, T. Aiki and A. Muntean treat *A thermo-diffusion system with Smoluchowski interactions : well-posedness and homogenization*. The attention is here very much focused on the coupling in fluxes induced by the presence of both Sorret and Dufour effects.

The employed mathematical techniques range from modern asymptotics methods in PDEs (periodic homogenization, large time behaviour, fast reaction limit, intermediate matched asymptotics) to weak convergence methods and fine integral estimates to handle free boundaries as well as to specific methods tuned to treat uniqueness of solutions to differential inclusions.

In this issue, the reader will find not only a wide spectrum of mathematical analysis results (like estimates in Bochner spaces or convergence proofs), but also practically important information on the structure of effective coefficients and up-scaled equations, error estimates for boundary layers for dimension reduction or fast reactions, as well as information on the numerical capturing of smoldering combustion fingers or on moving sharp reaction fronts in the sulphatation of concrete.

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