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*Research article*

# Alternative methods for estimating the term structure of interest rates: Some evidence from the Spanish market

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**Abstract:** The development of sophisticated forecasting and modeling techniques based on statistical learning or Machine Learning has led to the application of these algorithms in various fields, including finance. In this paper, we described two methodologies that combined statistical learning techniques, i.e., Support Vector Regression (SVR) and Local Regression (LR), with the recursive or bootstrapping method to obtain the term structure of the interest rates in the Spanish public debt market in 2020. The analysis was replicated for 2023, enabling the robustness of the competing methodologies to be evaluated under a markedly different monetary-policy environment. These estimates were then compared with those obtained by the Nelson-Siegel model by calculating the theoretical price of the bonds in the sample. Although the Nelson-Siegel model remained one of the most popular methods for estimating the term structure of interest rates, the methods based on machine learning algorithms also performed well. Thus, our findings suggested that, in the Spanish government bond market during 2020, the Nelson-Siegel model outperforms the machine-learning alternatives considered, although the latter provides flexible and economically plausible estimates of the term structure of interest rates. Nevertheless, SVR generated the lowest average pricing errors in 2023. Consequently, the results indicate that the effectiveness of yield-curve estimation methodologies depends on prevailing market conditions.

**Keywords:** machine learning; Spanish public debt market; term structure of interest rates; nelson-siegel model; fixed income

**JEL Codes:** C38, C58, E43, G12

## 1. Introduction

The Term Structure of Interest Rates (TSIR) is one of the most important financial concepts with which to study and analyze fixed income assets, as it is a very useful tool for operators, analysts, and investors in financial markets, especially those in which public debt securities are issued and traded (Robles-Fernández & Abad-Romero, 2003).

TSIR is of great interest from a macroeconomic point of view because it is a key element in the transmission of monetary policy, as well as a possible indicator of inflation expectations. Monetary institutions manage and control shorter-term interest rates, while longer-term rates influence the level of aggregate demand and the level of saving and investment by economic agents (Goodfriend, 1998). Therefore, the TSIR can be considered as the heart of the transmission of monetary policy implemented by the different authorities concerned (Colletaz & Gourlaouen, 1990).

The main problem associated with the TSIR is that it is not directly observable due to the lack of enough zero-coupon bonds. Therefore, there are different techniques to estimate the TSIR. The literature divides this issue into two main approaches: the statistical approach and an equilibrium-oriented approach in the absence of arbitrage opportunities (Mineo et al., 2020).

The equilibrium-oriented approach is based on economic-financial theories that describe the state variables that characterize the economy of a given territory and their impact on short-term interest rates (Cox et al., 2005, Duffie & Kan, 1996, Vasicek, 1977). Contreras & Navarro (1993) proposed the use of exponential splines to estimate the intertemporal structure of interest rates in the Spanish market following the methodology of Vasicek and Fong (Vasicek & Fong, 1982). Some subsequent work showed that, in the case of the Spanish public debt book-entry market, the methods that provide the best results are the Nelson-Siegel method and the Svensson method since they produce flexible and smoothed time curves (Núñez, 1995). However, these methods have two drawbacks: On the one hand, they require data that is very laborious and costly to process and tabulate, and on the other hand, the zero-coupon curves may be artificially smoothed (Robles-Fernández & Abad-Romero, 2003).

On the other hand, the statistical approach is based on the construction of statistical models from data observed in financial markets (Heath et al., 1992). This approach can be divided into two broad groups: parametric methods and non-parametric methods. Parametric models adopt a specific functional form that enables the estimated coefficients or parameters to be interpreted from an economic point of view, notably the Nelson-Siegel model (Nelson & Siegel, 1987), whose initial approach was extended by Svensson (Svensson, 1994). This methodology is characterized by a high degree of economic interpretability since certain restrictions can be imposed on the parameters to adapt them to the economic-financial theory. However, the main drawback of this approach is the reduced ability of the model to conveniently fit the data, leading to a lower goodness-of-fit, so that its use may not be appropriate in some applications such as in the valuation of certain asset prices due to misspecification problems (Laurini & Moura, 2010). Non-parametric methods, on the other hand, do not assume an a priori functional form, so they are much more flexible than parametric methods and can even be very robust under the right conditions (Mineo et al., 2020).

There is no single method that consistently performs better than any other. Comparing competing machine-learning (ML) methodologies also raises important issues regarding model evaluation. Contributions have emphasized that performance assessment should not rely exclusively on a single accuracy measure but should also consider the model's intended practical application. In this regard, He et al. (2024) proposed a novel evaluation framework for statistical learning models in financial

applications. They highlighted the importance of selecting performance metrics that accurately reflect the economic objectives of the analysis. However, it is observed that in most applications and uses, the Nelson-Siegel model is used because it is a parsimonious and relatively simple method to implement in any software, and it has been shown to adapt quite well to simple shapes, providing very stable and smoothed results (Mineo et al., 2020).

The emergence of new techniques and algorithms related to statistical learning or machine learning has led to the application of these tools in many fields and domains, including finance. These techniques have demonstrated their strong predictive power in many financial applications. However, due to the complexity of the algorithms, it is quite difficult to validate and test the robustness and reliability of the results, leading to a loss of interpretability compared to traditional techniques. For this reason, this issue is often referred to as a "black box" problem, as it is very difficult to understand most of the operational processes that occur when implementing these tools (Bracke et al., 2019). Evidence also highlights the increasing importance of machine learning techniques in a variety of financial decision-making contexts. For instance, Mestiri (2024) demonstrated that machine learning and deep learning approaches can surpass traditional statistical methods in credit scoring applications. This reinforces the potential of data-driven techniques to model intricate and nonlinear relationships in financial datasets.

Thus, we focus on analyzing the performance of alternative methods for estimating the TSIR in the Spanish market. Specifically, we compare methods that combine statistical learning techniques, Support Vector Regression (SVR) and Local Regression (LR) with the recursive or bootstrapping method to obtain the TSIR in the Spanish public debt market in 2020. The basic hypothesis that we want to test is whether one of the developed methods performs significantly better due to the fact that the mean square errors of the theoretical prices estimated with respect to the dirty price are lower than the errors produced by the classical Nelson-Siegel method. The contributions of our research compared to other studies are significant and compelling. First, our research is tailored to the Spanish market, providing a localized perspective that may reveal unique characteristics not captured in broader studies. Second, we aim to test the performance of alternative methods for estimating the TSIR, which could potentially uncover more efficient or accurate methods. This is particularly important given the recognized limitations of the classical Nelson-Siegel method. In addition, our research is empirical in nature, focusing on the mean square errors of estimated theoretical prices relative to the dirty price. This practical approach could provide actionable insights that directly improve the estimation of the TSIR by potentially identifying superior methodologies for the Spanish market. On the other hand, this study contributes to the growing body of literature comparing machine-learning algorithms with traditional financial models. Research has demonstrated that machine-learning techniques often outperform traditional methods in forecasting and classification tasks involving complex nonlinear relationships. However, the evidence is less conclusive in applications where economic interpretability, model parsimony, and structural consistency are key requirements. In the context of term-structure estimation, the comparison between machine-learning approaches and benchmark financial models remains limited. This study contributes to this body of work by evaluating whether flexible statistical learning techniques can improve the estimation of the Spanish yield curve compared to one of the most widely used benchmark specifications: the Nelson-Siegel model.

Finally, for this analysis, we focus on the accuracy of cross-sectional estimation rather than modeling the dynamic evolution of yield-curve factors. Consequently, dynamic specifications, such as those proposed by Diebold and Li (2006), are reserved for future research. In Thus, this study

contributes to the growing body of literature comparing machine-learning methodologies with traditional financial models. Although machine learning techniques have demonstrated significant predictive potential in numerous financial applications, such as credit risk assessment, market forecasting, and firm classification (Bracke et al., 2019; Mestiri, 2024), there is conflicting evidence as to whether the added flexibility of these algorithms consistently results in improved economic or financial performance measures. Studies emphasize that model evaluation should consider not only predictive accuracy, but also interpretability, robustness, and suitability for the specific financial application under analysis (He et al., 2024). In the context of yield-curve estimation, however, comparisons between machine-learning techniques and benchmark term-structure models are relatively scarce. Therefore, we evaluate whether flexible non-parametric approaches, such as SVR and LR, improve upon the widely used Nelson-Siegel specification in terms of bond-pricing accuracy.

This paper contributes to the literature by replicating the complete estimation procedure using Spanish sovereign bond market data from two markedly different periods: 2020, characterized by the COVID-19 crisis and an exceptionally accommodative monetary policy stance, and 2023, characterized by elevated inflation and substantial monetary tightening by the European Central Bank. This enables us to evaluate whether the relative performance of Nelson-Siegel, SVR, and LR remains stable across market environments.

The remainder of this paper is therefore divided into three sections: In the second section, we describe the process used to obtain, clean, and tabulate the data required to carry out the empirical analysis, together with the methodology used to construct the proposed targets. In the third section, we present the results obtained providing a detailed comparison of the errors produced by each methodology. Finally, the main conclusions drawn from this study are presented in the last section, along with our future lines of research.

## 2. Materials and methods

### *2.1. Data*

The application proposed in this study for the pricing of government bonds is based on information from the Bank of Spain's public debt bulletins, in terms of simple buying and selling operations for the years 2020 and 2023. We select two bulletins containing the daily data for each month of the sample period; the first referring to the last day of the month in which the market is open, and the second referring to a day in the middle of this period, usually between the 14$^{th}$ and the 16$^{th}$. Discount bonds are excluded from the data-treatment process because they tend to be public debt strips. Bonds with a maturity of more than 30 years are also eliminated. The selection of bonds with coupons makes it necessary to know the exact dates on which they are paid. This information is obtained from SEND – Electronic Debt Negotiation System.

Knowing the cash flows generated by the bonds, together with the dates on which they were generated, makes it possible to calculate the gross price of the bond. This is the price actually paid on the market to acquire the bond in question. To obtain this price, it is first necessary to calculate the part proportional to the last coupon accrued. Once this is calculated, it is necessary to add the clean price of the bond to obtain the gross price. In this study, it is assumed that the clean price is the average price of the session relating to the date of quotation.

## 2.2. *Methodology*

Our main objective of this study is to compare the methods used to obtain the term structure of interest rates. It is therefore necessary to first estimate the zero-yield curve using the three methods that will later be compared to establish whether one of the performance samples is better than the others.

The first method used in this paper is the Nelson-Siegel model. This model, first proposed in 1987 (Nelson & Siegel, 1987), is one of the most widely used, making it a very popular method that has undergone significant extensions, improvements, and modifications. It is therefore the forerunner of the Svensson model (Svensson, 1994), the Nelson-Siegel dynamic model (Diebold & Li, 2006), the Nelson-Siegel Arbitrage-Free model (Christensen et al., 2011), and the DCOBS (Dynamic Constrained Smoothing B-Splines) method (He & Ng, 1999 and Laurini & Moura, 2010).

Although the Diebold and Li (2006) model has been widely accepted and applied in the literature (Badics et al., 2023 and Umar et al., 2022, among others), the original model is, to the best of our knowledge, the one most commonly used by central banks and in academic publications due to its parsimonious nature and the easy economic interpretation of the estimated parameters (Díaz et al., 2011). However, the Dynamic Nelson-Siegel (DNS) specification, which was developed by Diebold and Li (2006), enables the level, slope, and curvature factors to evolve over time. This provides a richer representation of yield curve dynamics. Implementing this approach usually requires a sufficiently long time series in order to model factor dynamics and changes in term premia adequately. As our aim of this study is to compare alternative estimation methodologies within a relatively short sample period, the focus is on the traditional Nelson-Siegel specification. Nevertheless, extending the analysis to a DNS framework would be a logical next step for this research. Moreover, developments in the term-structure literature have also introduced regression-based affine models capable of jointly pricing the cross-section and time-series evolution of bond yields. In particular, Adrian et al. (2013) proposed the ACM framework, which provides a computationally efficient methodology for estimating multi-factor arbitrage-free term structure models and extracting term premia. While these approaches are beyond the scope of this study, they represent promising alternatives for future comparisons with machine-learning-based estimation methods.

Thus, the Nelson-Siegel model is a parametric method that assumes a specific formal function that is expressed by the following expression:

$$f(t) = \beta_0 + \beta_1 * e^{-\frac{t}{\tau}} + \beta_2 * \frac{t}{\tau} * e^{-\frac{t}{\tau}} \quad (1)$$

This expression is an explicit function of time, since $t$ represents the time to maturity. This model is therefore made up of four parameters or coefficients to be estimated. Many researchers have interpreted the economic sense of the first three parameters, relating them to the specific factors that determine the performance curve. The first parameter, $\beta_0$, represents the level or intercept and captures the relationship between interest rates in the long run. The second parameter, $\beta_1$, measures the slope of the function, while the third parameter, $\beta_2$, controls the curvature of the term structure (Díaz et al., 2011).

Financial literature has given little attention to the interpretation of the last parameter ($\tau$) because it tends to be fixed as a pre-established value determined according to the criteria of coexistence, simplicity, and reliability (Diebold & Li, 2006). Nevertheless, this parameter plays a very important role because it determines the point of inflection and the position of the "hump", if there is one. Experience shows that this parameter is much more volatile and sensitive than the other parameters (Díaz et al., 2011).

Once the four parameters have been estimated using non-linear optimization routines, the spot rates that will form part of the term structure of the interest rates are obtained. This is done using the following discrete time expression:

$$R_{0,t} = EXP[-\beta_0 * t - (\beta_1 + \beta_2) * \tau * \left(1 - \exp\left(\frac{t}{\tau}\right)\right) + \beta_2 * \exp\left(\frac{t}{\tau}\right)]^{-1/t} - 1 \quad (2)$$

where $R_{0,t}$ is the spot rate at time $t$.

Machine learning has a number of tools and techniques that make it possible to perform non-parametric regressions that fit data very well. In this work, we use Support Vector Machines (SVR) with radial kernels and LR, using crossed validation procedures that optimize the control hyperparameters implied by this type of methodology. These techniques are implemented using the free R Studio software.

The selection of SVR and LR is motivated by the characteristics of the term-structure estimation problem and our objectives of the study. Unlike many machine learning applications that rely on highly complex, black-box architectures (such as deep learning frameworks employed in forecasting or classification problems), the estimation of the yield curve requires smoothness, stability, and economic plausibility across maturities. SVR provides a flexible non-linear framework capable of modeling complex relationships while maintaining good generalization properties in relatively small samples (Yang et al., 2002). LR, in turn, offers a transparent non-parametric alternative that adapts locally to the data, making it particularly suitable for modeling gradual changes along the maturity spectrum (James et al., 2013). Furthermore, unlike large-scale machine learning applications, which typically require extensive datasets, the Spanish sovereign bond market provides a relatively limited cross-sectional sample at each observation date. Consequently, parsimonious statistical learning approaches such as SVR and LR constitute a balanced compromise between flexibility, interpretability and computational simplicity. This is consistent with research emphasizing that the choice of methodology should align with the characteristics of the empirical problem rather than being based solely on forecasting performance (He et al., 2024).

The Support Vector Machine is one of the most successful machine learning algorithms for regression tasks, particularly in the area of time series forecasting and financial applications (Yang et al., 2002). This tool uses functions that quantify the similarity between pairs of observations called kernels, which can be linear and non-linear. In this work, we use a SVR with radial kernels, where the dependent variable to be explained is the performance of the different bonds and the independent variable is their duration to maturity. The radial kernel takes the following form, where $x_{ij}$ represents the $j$th observation of the training sample $i$, $x_{i'j}$ denotes the $j$th observation of the training sample $i'$, and $\gamma$ is a positive constant (James et al., 2013):

$$K(x_i, x_{i'}) = \exp\left(-\gamma * \sum_{j=1}^{p}(x_{ij} - x_{i'j})^2\right) \quad (3)$$

This algorithm depends on two parameters: gamma γ and $x_{ij}$-$x_{i'j}$. Both have been optimized using cross – validation procedures to find the values of the parameters that minimize the residuals or errors generated by the model. This procedure involves replicating the models many times, resulting in the model with the fewest errors (James et al., 2013). The use of SVR in this study follows the growing literature that applies machine-learning methods to financial prediction and valuation problems. However, unlike many applications that focus on classification tasks, stock-market prediction, or large-

scale financial datasets, we apply statistical-learning techniques to the specific problem of yield-curve estimation, where smoothness and pricing consistency are particularly important.

The second method developed in this study uses LR techniques to estimate the TSIR. The starting point of the LR is an approach that enables non-linear functions to be fitted in a very flexible way, using only the nearest observations to fit the point under study. The choice of the span parameter is very important in this algorithm, as it controls the flexibility of the non-linear fitting; the smaller it is, the more local and flexible the fitting of the model will therefore be, while as the span parameter increases, the fitting will tend to be more global, as it will tend to have a greater number of observations when running the regression, and the fitting will be much smoother (James et al., 2013). The optimal value of the span parameter is again obtained using crossed validation procedures that yield the model with the fewest errors.

A priori, this procedure could work very well for this type of estimation because the yield curve tends to have local behavior throughout its distribution. This phenomenon is motivated by the fact that short-term interest rates do not behave in the same way as long-term interest rates (Goodfriend, 1998). The Yield To Maturity (YTM) dependent variable is again estimated using the duration to maturity of the sampled bonds as the regression variable.

Regarding implementation details, to facilitate reproducibility, the SVR and LR models are estimated independently for each observation date using the bond yields to maturity as the dependent variable and the corresponding time-to-maturity values as the explanatory variable. For the SVR specification, a radial basis function kernel is employed. The tuning parameters are selected using K-fold cross-validation, where the optimal combination of the cost and gamma parameters is chosen as the one minimizing the cross-validation error. For the LR methodology, the span parameter is selected using the same cross-validation procedure. The final fitted curve is obtained using the optimal span value identified for each estimation date. All estimations are implemented in R Studio using standard machine-learning libraries. The resulting fitted yield curves are subsequently combined with the bootstrapping procedure described below to obtain the corresponding spot-rate curves displayed in Figures 1–4.

The process described so far makes it possible to determine the yield to maturity for each of the cash flows generated by each bond according to the accrual date. This information is then used to calculate the spot rates using a recursive procedure known as bootstrapping. This procedure is commonly used because of its simple and straightforward nature (Robles-Fernández & Abad-Romero, 2003). However, one of its main drawbacks is that it does not enable the estimation to be biased (Buse, 1970).

The objective is to find the implicit yield that the zero coupon bond will have, $K_{0,t}$. Estimating the different values of $K_{0,t}$ is done iteratively, starting with $K_{0,1}$. Given that there is no difference between a zero coupon bond and a coupon bond with one period to maturity, it can be concluded that:

$$K_{0,1} = i_1 \tag{4}$$

Once $K_{0,1}$ has been obtained, it is possible to calculate $K_{0,2}$:

$$V = \frac{V*i_2}{(1+K_{0,1})} + \frac{V*i_2+V}{(1+K_{0,2})} \Rightarrow K_{0,2} = \left(\frac{1+i_2}{1-i_2*\left(\frac{1}{1+K_{0,1}}\right)}\right)^{1/2} - 1 \tag{5}$$

The above expression can be generalized so that it is possible to obtain any $K_{0,n}$ as:

$$K_{0,n} = \left( \frac{1+i_n}{1-i_n * \sum_{j=1}^{n-1} \left( \frac{1}{1+K_{0,j}} \right)^j} \right)^{1/n} - 1 \quad (6)$$

This formula is used to calculate all spot rates.

Some authors point out that it is much more advisable to work with discount functions than with spot rates. The discount function can be defined as the current value of a single zero-coupon bond maturing in t (Nave-Pineda et al., 1996). The advantage of discount functions is that the price of a given bond depends linearly on this function (Vasicek & Fong, 1982).

The discount factors are calculated using the following expression, where *D(t)* is the discount function and $K_{0,n}$ is the spot rate:

$$D(t) = (1 + K_{0,n})^{-t} \quad (7)$$

Or equivalent:

$$D(t) = \frac{1}{(1+K_{0,n})^t} \quad (8)$$

The next step is to update each periodic coupon in the pricing date. The discount function makes this process very simple to implement, as it is only necessary to multiply each cash flow by its corresponding discount factor. All the updated cash flows are then added together so that the result is the theoretical price of the bond $\hat{P}_{i,t}$.

Finally, the error made is calculated using the mean square error metric, *MSE*, which is the difference between the square of the theoretical price of the bond and the purchase price. The *MSE* formula is as follows:

$$MSE = \frac{1}{n} * \sum_{i=1}^{n} (P_{i,t} - \hat{P}_{i,t})^2 \quad (9)$$

where *n* is the total number of bonds in the sample, $P_{i,t}$ is the bond purchase price, and $\hat{P}_{i,t}$ is the estimated theoretical price.

## 3. Results

In this section, we present the main results obtained by applying the proposed methodology to the selected data. First, the different TSIRs estimated using the proposed methodologies are described, followed by a discussion of which one best approximates the theoretical price to the gross price of the sampled bonds.

### *3.1. Estimation of the TSIRs*

The estimation of the yields to maturity of the bonds in the sample shows that they generally increase with maturity. However, there is a certain irregularity in short-term interest rates, since situations arise in which the interest rate is higher for shorter maturities and vice versa. This leads to the following requirements: 1) That the estimated term structure is sufficiently smooth, and 2) that the data are appropriately adjusted (Nave-Pineda et al., 1996).

We first estimate the term structure of interest rates using the parametric Nelson-Siegel model, which enables us to obtain the coefficients shown in Tables 1a and 1b for the period under review:

**Table 1a.** Estimated coefficients for the Nelson-Siegel model (2020).

| Date | $\boldsymbol{\beta_0}$ | $\boldsymbol{\beta_1}$ | $\boldsymbol{\beta_2}$ | $\boldsymbol{\tau}$ |
|---|---|---|---|---|
| 15/01/2020 | 0.0248745 | −0.0298006 | -0.0200430 | 5.6480313 |
| 31/01/2020 | 0.0233444 | −0.0279210 | -0.0227881 | 5.8693309 |
| 14/02/2020 | 0.0227555 | −0.0268688 | -0.0249561 | 5.4906812 |
| 28/02/2020 | 0.0196904 | −0.0245738 | -0.0198979 | 4.9915854 |
| 16/03/2020 | 0.0284156 | −0.0308105 | 0.0287803 | 20.0084041 |
| 31/03/2020 | 0.0248931 | −0.0237695 | -0.0233207 | 6.2406392 |
| 15/04/2020 | 0.0224609 | −0.0252775 | 0.0271052 | 13.2558309 |
| 30/04/2020 | 0.0170962 | −0.0196391 | 0.0291480 | 11.6729520 |
| 15/05/2020 | 0.0034779 | −0.0088623 | 0.0722481 | 10.7673486 |
| 29/05/2020 | −0.0109814 | 0.0081751 | 0.0987820 | 10.7039108 |
| 15/06/2020 | -0.0076882 | 0.0041602 | 0.0981274 | 10.9053925 |
| 30/06/2020 | −0.0068364 | 0.0014346 | 0.0885532 | 11.4368331 |
| 15/07/2020 | −0.0044964 | −0.0010392 | 0.0826325 | 12.0842964 |
| 31/07/2020 | −0.0207117 | 0.0187646 | 0.1089464 | 10.6107552 |
| 14/08/2020 | −0.0017604 | −0.0043437 | 0.0723564 | 12.9125278 |
| 31/08/2020 | −0.0027650 | −0.0040844 | 0.0788704 | 12.1457858 |
| 15/09/2020 | −0.0081672 | 0.0026659 | 0.0846762 | 12.3001398 |
| 30/09/2020 | −0.0074616 | 0.0012493 | 0.0819131 | 12.6327648 |
| 15/10/2020 | −0.0067714 | −0.0002202 | 0.0757134 | 13.3022629 |
| 30/10/2020 | −0.0139743 | 0.0076495 | 0.0877684 | 12.0395071 |
| 16/11/2020 | −0.0174410 | 0.0120500 | 0.0957406 | 11.9794511 |
| 30/11/2020 | −0.0197829 | 0.0145227 | 0.0990629 | 11.9599182 |
| 15/12/2020 | −0.0206475 | 0.0149126 | 0.0993349 | 12.0775709 |
| 31/12/2020 | −0.0199212 | 0.0144275 | 0.0988291 | 11.8075253 |
| Average | 0.0007334 | −0.0052999 | 0.0582326 | 10.9518102 |
| Standard Deviation | 0.0172713 | 0.0165050 | 0.0475682 | 3.3086211 |

Note: This table shows the estimated parameter values of the TSIR using the Nelson and Siegel (1987) model for the Spanish government bond market in 2020. The first parameter, $\beta_0$, represents the level or intercept and captures the relationship between interest rates in the long run. The second parameter, $\beta_1$, measures the slope of the function, while the third parameter, $\beta_2$, controls for the curvature of the term structure.

**Table 1b.** Estimated coefficients for the Nelson-Siegel model (2023).

| Date | $\boldsymbol{\beta_0}$ | $\boldsymbol{\beta_1}$ | $\boldsymbol{\beta_2}$ | $\boldsymbol{\tau}$ |
|---|---|---|---|---|
| 16/01/2023 | 0.0372478 | 0.1177957 | −0.1653783 | 0.8757848 |
| 31/01/2023 | 0.0333951 | −0.0154286 | 0.0000000 | 0.2517304 |
| 15/02/2023 | 0.0418165 | 0.1454189 | −0.2035754 | 1.2071238 |
| 28/02/2023 | 0.0376158 | −0.0151636 | −0.0000005 | 0.3284317 |
| 15/03/2023 | 0.0433272 | 0.0028545 | −0.0441000 | 2.4907380 |
| 31/03/2023 | 0.0410747 | −0.0041369 | −0.0284402 | 1.7223102 |
| 14/04/2023 | −0.0119737 | 0.0332180 | 0.1189406 | 17.6607179 |
| 28/04/2023 | 0.0416969 | −0.0075279 | −0.0282272 | 2.1708623 |
| 15/05/2023 | 0.0461506 | −0.0082380 | −0.0386630 | 2.8585851 |
| 31/05/2023 | 0.0449763 | −0.0000218 | −0.0470917 | 2.6001021 |
| 15/06/2023 | 0.0452325 | −0.0042616 | −0.0322356 | 3.0091088 |
| 30/06/2023 | 0.0394735 | 0.0182154 | −0.0434819 | 1.2813395 |
| 14/07/2023 | 0.0425447 | 0.0156091 | −0.0504904 | 2.5065882 |
| 31/07/2023 | 0.0449783 | −0.0098322 | 0.0000000 | 21.8245902 |
| 15/08/2023 | 0.0260780 | −0.0050392 | 0.0628056 | 11.1870926 |
| 31/08/2023 | 0.0426258 | −0.0071186 | −0.0131591 | 3.2429461 |
| 15/09/2023 | 0.0473502 | −0.0077023 | −0.0245740 | 3.6709883 |
| 29/09/2023 | 0.0496643 | 0.0108398 | −0.0660291 | 2.0639675 |
| 16/10/2023 | 0.0477840 | 0.0309841 | −0.0829405 | 1.7369484 |
| 31/10/2023 | 0.0484036 | 0.0797035 | −0.1522186 | 1.3481238 |
| 15/11/2023 | 0.0446700 | −0.0004447 | −0.0347384 | 2.2831401 |
| 30/11/2023 | 0.0441150 | 0.0314306 | −0.0842438 | 1.6292726 |
| 15/12/2023 | 0.0431577 | −0.0001580 | −0.0438155 | 4.0000000 |
| 29/12/2023 | −0.0273044 | 0.0484192 | 0.2989704 | 1.6292726 |
| Average | 0.0372541 | 0.0187256 | −0.0292785 | 3.8991568 |
| Standard Deviation | 0.0183860 | 0.0416593 | 0.0964243 | 5.3417198 |

Note: This table shows the estimated parameter values of the TSIR using the Nelson and Siegel (1987) model for the Spanish government bond market in 2023. The first parameter, $\beta_0$, represents the level or intercept and captures the relationship between interest rates in the long run. The second parameter, $\beta_1$, measures the slope of the function, while the third parameter, $\beta_2$, controls for the curvature of the term structure.

The TSIRs estimated for the days making up this period are increasing, indicating that the long-term interest rates are higher than short-term rates. From an economic perspective, the predominantly upward-sloping yield curve observed throughout most of 2020 is consistent with traditional term-structure theories. According to expectation-based interpretations, a positive slope indicates either anticipated higher short-term interest rates in the future, or a positive term premium required by investors for holding longer-term assets. In the Spanish market, this pattern is consistent with the highly accommodative monetary policy implemented by the European Central Bank during the pandemic, which helps keep short-term rates low while longer-term rates remain positive. This is in line with what has been observed for the YTM of coupon bonds, i.e., the longer the period to maturity,

the greater the profitability of this asset. Moreover, this curve is quite smooth, as no irregular behavior appears in the YTM, especially for bonds with short maturities.

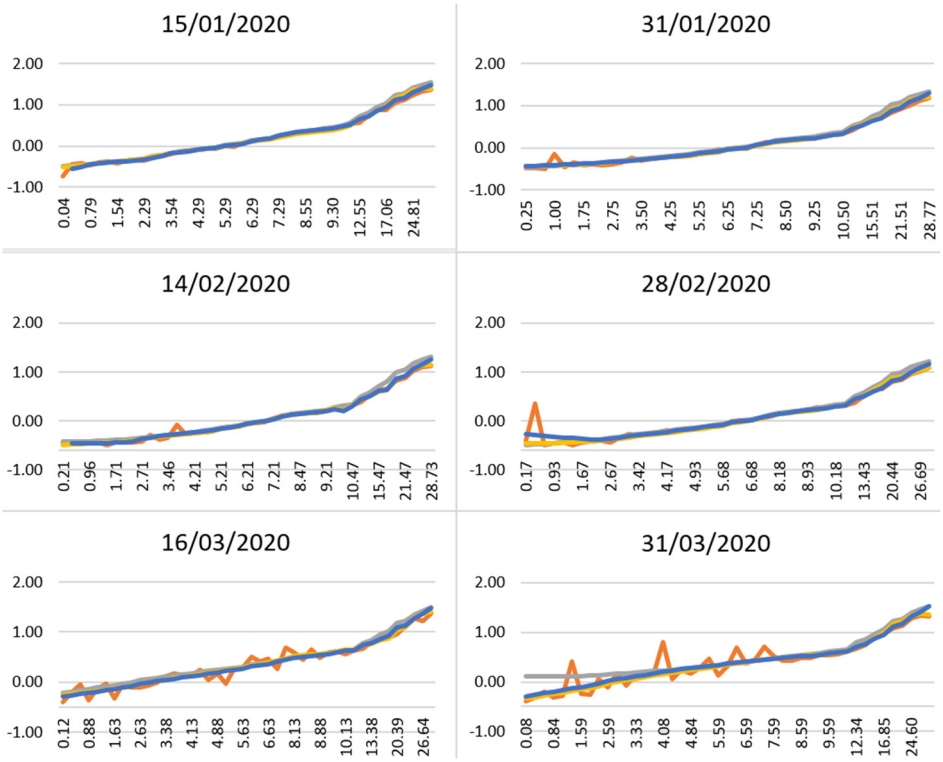
The average and typical deviations are also calculated to determine the volatility of the parameters (Table 1). The typical deviations of the first three parameters are much lower than those of the last coefficient ($\tau$), which means that it is much more volatile than the others. The greater variability observed in parameter $\tau$ is economically significant, since this coefficient determines the location of the yield curve's curvature. In 2020, financial markets were affected by periods of increased uncertainty related to the pandemic, resulting in temporary shifts in investor expectations and in the maturity segment where risk perceptions were concentrated. Consequently, stronger fluctuations in $\tau$ are consistent with changing market assessments of medium-term economic prospects. The extension of the analysis to 2023 provides an important robustness test. The year 2020 was characterized by extraordinarily low interest rates, unprecedented monetary intervention, and exceptional market conditions associated with the COVID-19 crisis. In contrast, 2023 reflected the effects of elevated inflation and a restrictive monetary policy stance. The replication of the complete estimation procedure under these distinct environments would help us demonstrate whether the relative ranking of methodologies could vary over time, indicating that the effectiveness of yield-curve estimation methodologies would depend on prevailing market conditions.

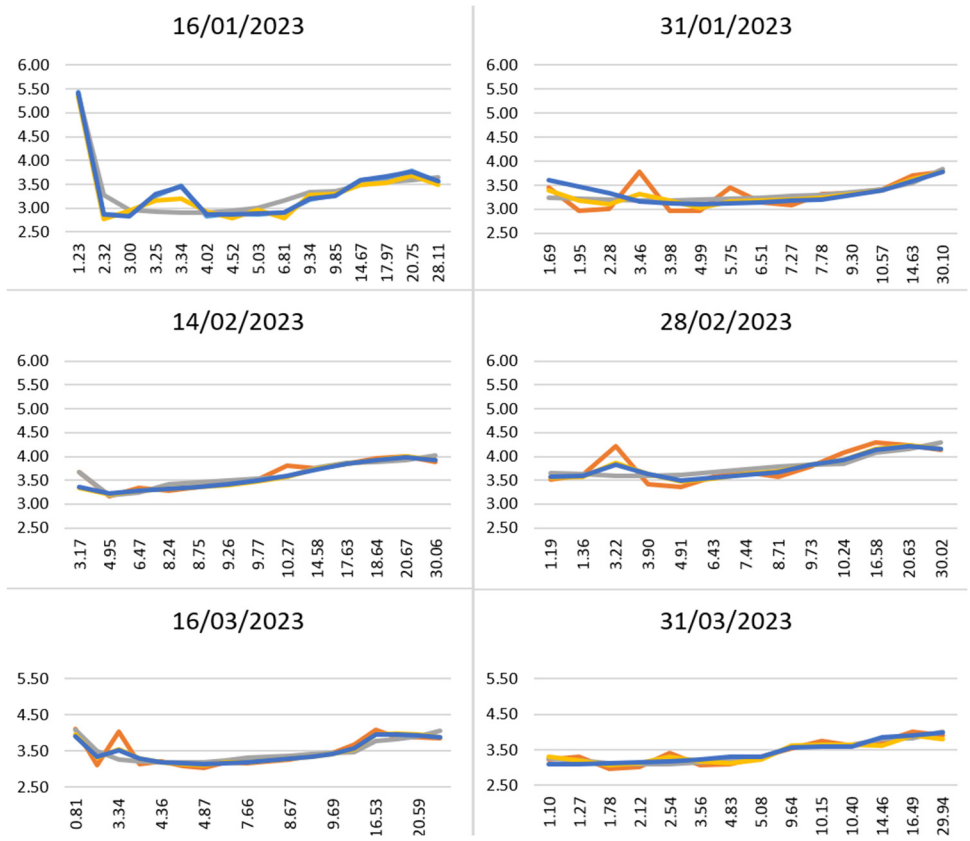
The two alternative methods proposed (SVR and LR), the results of which are shown below, can be classified as non-parametric, since no functional form has been imposed a priori into which a set of parameters can be integrated. Therefore, these models provide predictions directly for the given maturity vector. Graphs of the term structures of the interest rates estimated by the three methods, grouped by quarter, are shown below, together with the yield curve, to compare the spot rates obtained by the different methods.

The legend of Figure 1 can be interpreted as follows: YTM, yield to maturity; NS, an estimate of the term structure of interest rates using the Nelson-Siegel model; SVR, an estimate of the term structure of interest rates using the SVR algorithm; and LR, an estimate of the term structure of interest rates using LR. The same notation is used for the other graphs for the period under review.

Figures 1 and 2 show that the estimates of the zero yield curves using the three methods capture the long-term tendency of the yield to maturity of bonds in the Spanish public debt market to increase. It is also worth noting that these curves are fairly smooth, especially on the days in March when there are some irregularities in the short term. It should also be noted that forecasts for spot rates produced by the Nelson-Siegel model tend to be above the YTM, while the other two estimates tend to be more in line with it. For example, in the estimate for 31 March 2020, the curve of the Nelson-Siegel model is well above the YTM in the short term, which could indicate that the other two methods, which are related to machine learning algorithms, can adapt better to the performance of short-term coupon bonds.

The last two graphs (Figures 3 and 4) show the same results as the previous two, indicating that these methods are quite robust and produce very similar results when using data with characteristics very similar to those used previously. From all of the above, it can be concluded that the methods for estimating the TSIR proposed in the analysis can provide estimates of the term structure of interest rates that are sufficiently smooth for them to be adequately adapted to the data in the sample (Nave-Pineda et al., 1996), as was the case for the period examined in this work.

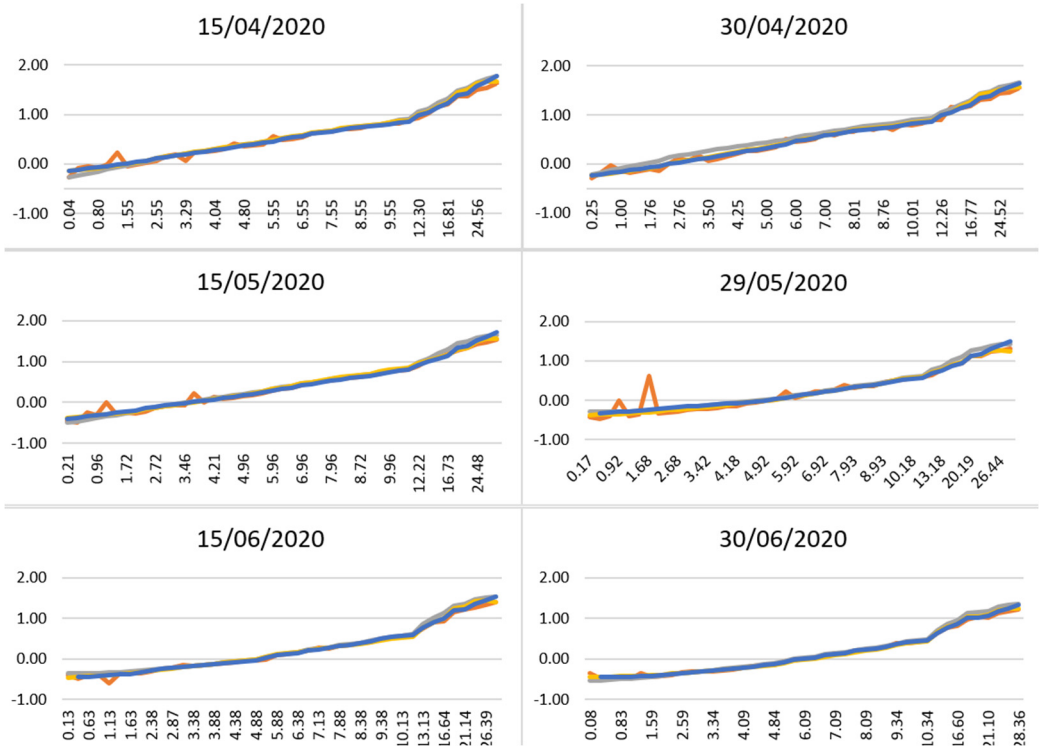



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**Figure 1.** TSIR estimates for the first quarter of 2020 and 2023. The horizontal axis shows time to maturity in years, and the vertical axis shows yields as a percentage. The same scale is used for both axes across all figures to facilitate comparison of the estimated yield curves over time. The horizontal axis shows time to maturity in years, and the vertical axis shows yields as a percentage. The same scale is used for both axes across all figures to facilitate comparison of the estimated yield curves over time. This figure shows estimates of yield curves for the case of the Spanish government bond market for the first quarter of 2020 and 2023, using three different methods to capture the long-term trend of bond yields to maturity. YTM, yield to maturity; NS, an estimate of the term structure of interest rates using the Nelson-Siegel model; SVR, an estimate of the term structure of interest rates using the Support Vector Regression algorithm; and LR, an estimate of the term structure of interest rates using Local Regression. The SVR and LR curves are obtained from cross-validated machine-learning models, estimated independently for each observation date using bond yield-to-maturity data and time-to-maturity as explanatory information.




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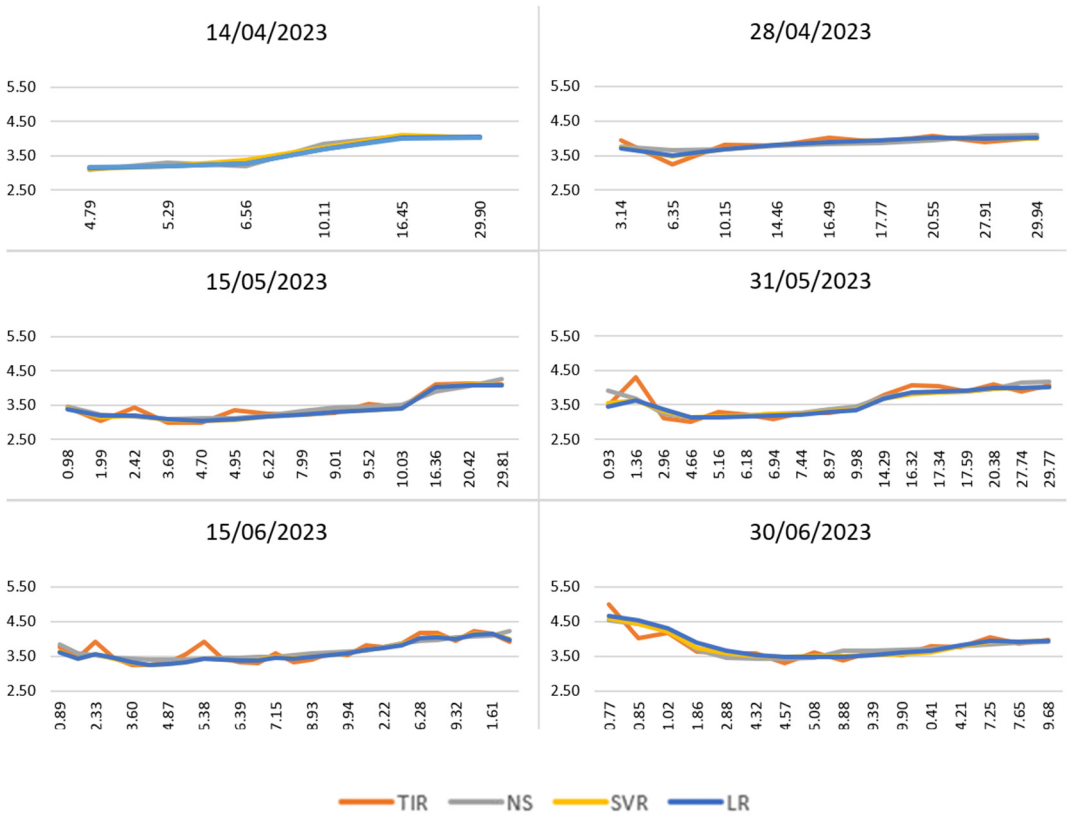



**Figure 2.** TSIR estimates for the second quarter of 2020 and 2023. The horizontal axis shows time to maturity in years, and the vertical axis shows yields as a percentage. The same scale is used for both axes across all figures to facilitate comparison of the estimated yield curves over time. The horizontal axis shows time to maturity in years, and the vertical axis shows yields as a percentage. The same scale is used for both axes across all figures to facilitate comparison of the estimated yield curves over time. This figure shows estimates of yield curves for the case of the Spanish government bond market for the second quarter of 2020 and 2023, using three different methods to capture the long-term trend of bond yields to maturity. YTM, yield to maturity; NS, an estimate of the term structure of interest rates using the Nelson-Siegel model; SVR, an estimate of the term structure of interest rates using the Support Vector Regression algorithm; and LR, an estimate of the term structure of interest rates using Local Regression. The SVR and LR curves are obtained from cross-validated machine-learning models, estimated independently for each observation date using bond yield-to-maturity data and time-to-maturity as explanatory information.

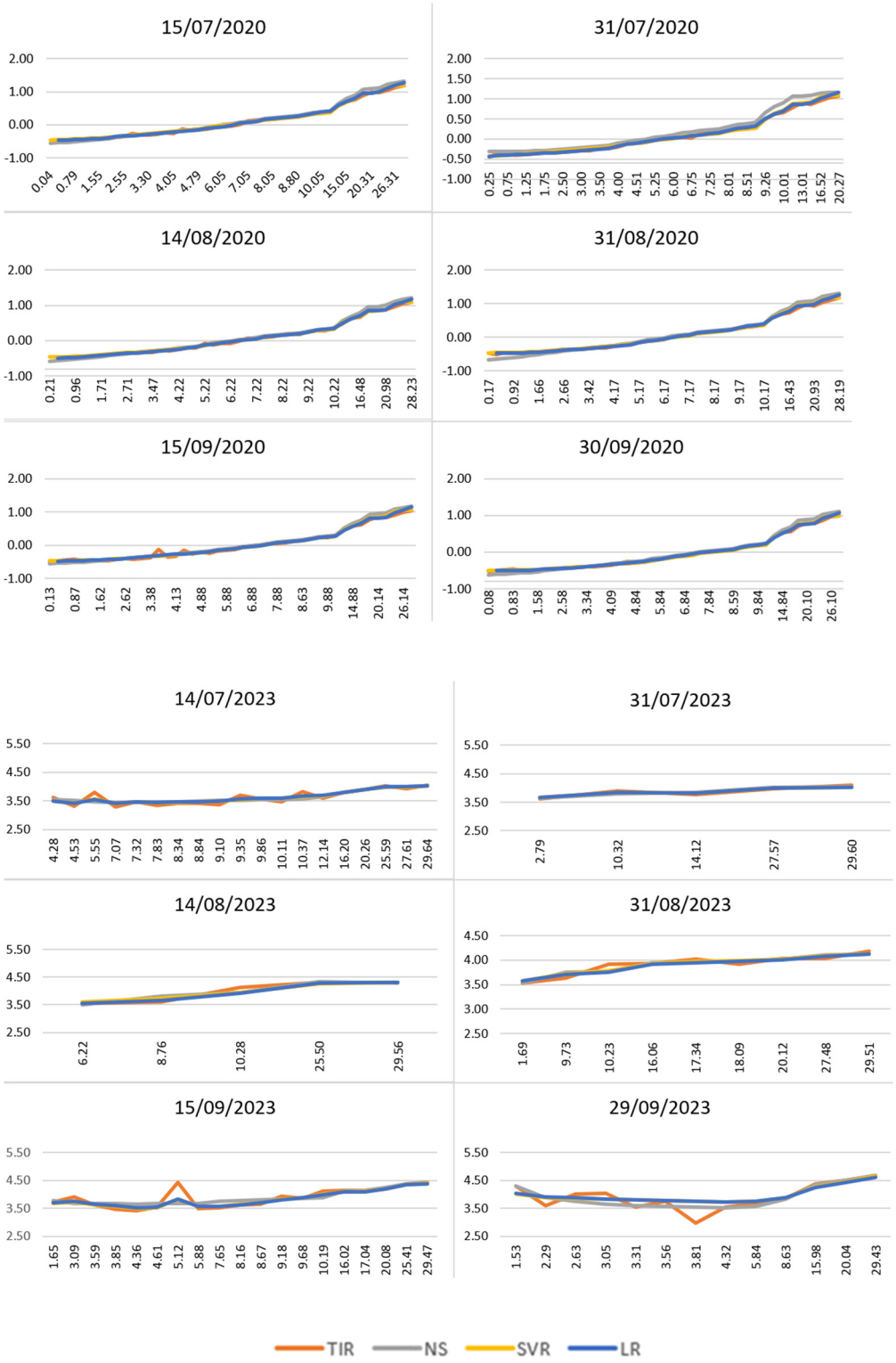



**Figure 3.** TSIR estimates for the third quarter of 2020 and 2023. The horizontal axis shows time to maturity in years, and the vertical axis shows yields as a percentage. The same scale is used for both axes across all figures to facilitate comparison of the estimated yield curves over time. This figure shows estimates of yield curves for the case of the Spanish government bond market for the third quarter of 2020 and 2023, using three different methods to capture the long-term trend of bond yields to maturity. YTM, yield to maturity;

NS, an estimate of the term structure of interest rates using the Nelson-Siegel model; SVR, an estimate of the term structure of interest rates using the Support Vector Regression algorithm; and LR, an estimate of the term structure of interest rates using Local Regression. The SVR and LR curves are obtained from cross-validated machine-learning models, estimated independently for each observation date using bond yield-to-maturity data and time-to-maturity as explanatory information.

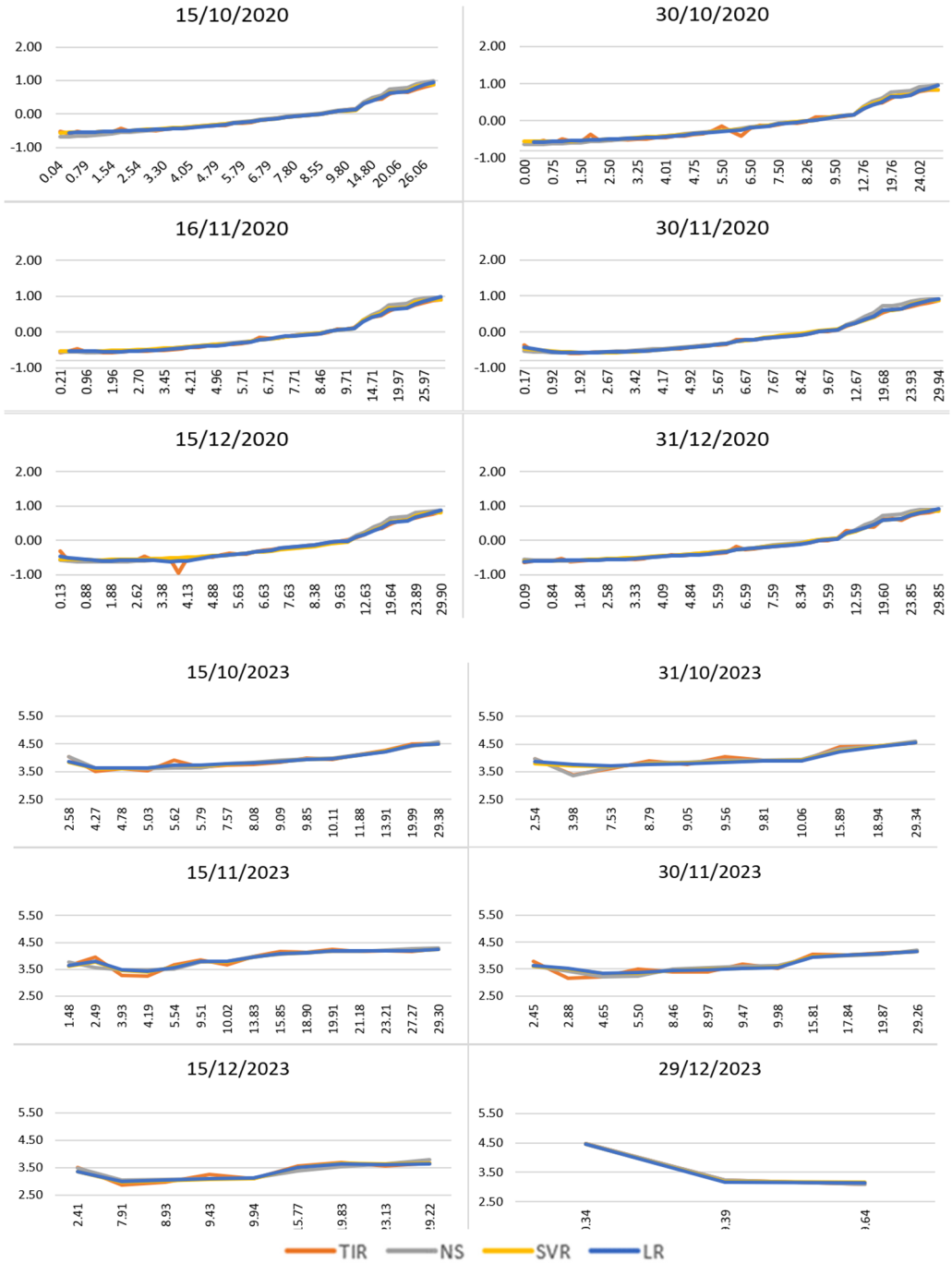



**Figure 4.** TSIR estimates for the fourth quarter of 2020 and 2023. The horizontal axis shows time to maturity in years, and the vertical axis shows yields as a percentage. The same scale is used for both axes across all figures to facilitate comparison of the estimated

yield curves over time. This figure shows estimates of yield curves for the case of the Spanish government bond market for the fourth quarter of 2020 and 2023, using three different methods to capture the long-term trend of bond yields to maturity. YTM, yield to maturity; NS, an estimate of the term structure of interest rates using the Nelson-Siegel model; SVR, an estimate of the term structure of interest rates using the Support Vector Regression algorithm; and LR, an estimate of the term structure of interest rates using Local Regression. The SVR and LR curves are obtained from cross-validated machine-learning models, estimated independently for each observation date using bond yield-to-maturity data and time-to-maturity as explanatory information.

These results are broadly consistent with other findings regarding the Spanish sovereign bond market. Núñez (1995) and Robles-Fernández and Abad-Romero (2003) reported that parsimonious parametric approaches, such as the Nelson-Siegel and Svensson models, tend to generate smooth, economically plausible yield curves, especially when the underlying term structure is relatively stable and monotonic. The curves estimated in this study display similar characteristics, suggesting that the information contained in Spanish government bond prices in 2020 can largely be represented by a limited number of latent factors associated with level, slope, and curvature. Moreover, the flexibility of SVR appears advantageous during periods characterized by steeper yield curves and greater dispersion in observed yields, as in 2023. LR generally provides intermediate performance, combining flexibility with a relatively smooth representation of the yield curve. Therefore, the results support the view that no single methodology dominates under all market circumstances.

The yield-curve estimates obtained for 2023 reveal several noteworthy differences relative to those observed in 2020. Most importantly, the overall level of interest rates is substantially higher across all maturities. This result is consistent with the sharp shift in monetary-policy conditions between the two periods. While 2020 was characterized by exceptionally low interest rates and unprecedented monetary support measures implemented by the European Central Bank during the COVID-19 crisis, the yield curves estimated for 2023 reflect an environment of elevated inflation and monetary tightening. The comparison between 2020 and 2023 therefore provides useful evidence regarding the temporal robustness of the methodologies. Moreover, while Nelson-Siegel displays very competitive performance during 2020, the estimates obtained for 2023 indicate that the greater flexibility of machine-learning methods becomes increasingly valuable when the yield curve exhibits stronger local nonlinearities and a steeper term structure. In particular, SVR frequently generates lower pricing errors than the alternative approaches during 2023, while LR also achieves competitive results across most observation dates.

Despite these differences, all three methodologies successfully capture the fundamental shape of the Spanish sovereign yield curve. The estimated curves remain smooth, monotonic over most maturity ranges, and consistent with the underlying economic environment. Consequently, the evidence suggests that the three approaches provide reliable estimates of the term structure of interest rates, although their relative performance varies according to prevailing market conditions. These findings reinforce the view that no single methodology uniformly dominates across all market environments. Instead, the advantages of parsimonious parametric specifications and flexible machine-learning techniques appear to depend on the characteristics of the yield curve being modeled and the broader monetary-policy regime in place.

*3.2. Comparison of the proposed methods for valuing fixed income assets*

The spot rates of the term structures could be used to calculate the theoretical price of the bonds in the sample, with the aim of finding out which method best approximates this price to the purchase price of the bond. The most appropriate method will be the one with the lowest mean square error (MSE) over all days in the period considered. Tables 2a and 2b show the MSE calculated from the residuals of each methodology.

**Table 2a.** Errors produced by each method measured by MSE and RMSE (2020).

| Date | NS | SVR | LR |
|---|---|---|---|
| 15/01/2020 | 0.084 | 0.829 | 1.150 |
| 31/01/2020 | 0.035 | 0.985 | 1.211 |
| 14/02/2020 | 0.154 | 1.690 | 1.464 |
| 28/02/2020 | 0.072 | 1.030 | 1.083 |
| 16/03/2020 | 0.772 | 1.586 | 1.327 |
| 31/03/2020 | 0.719 | 1.298 | 1.440 |
| 15/04/2020 | 0.104 | 0.372 | 0.680 |
| 30/04/2020 | 0.614 | 0.999 | 1.329 |
| 15/05/2020 | 0.125 | 1.037 | 0.809 |
| 29/05/2020 | 0.202 | 2.007 | 1.179 |
| 15/06/2020 | 0.133 | 0.819 | 1.002 |
| 30/06/2020 | 0.083 | 0.966 | 0.984 |
| 15/07/2020 | 0.089 | 1.194 | 0.928 |
| 31/07/2020 | 1.522 | 4.207 | 4.504 |
| 14/08/2020 | 0.116 | 1.235 | 1.007 |
| 31/08/2020 | 0.069 | 0.797 | 0.910 |
| 15/09/2020 | 0.080 | 0.813 | 1.011 |
| 30/09/2020 | 0.065 | 1.022 | 0.974 |
| 15/10/2020 | 0.062 | 0.935 | 0.981 |
| 30/10/2020 | 0.160 | 1.010 | 1.184 |
| 16/11/2020 | 0.119 | 0.637 | 1.172 |
| 30/11/2020 | 0.132 | 1.105 | 1.130 |
| 15/12/2020 | 0.243 | 1.110 | 1.455 |
| 31/12/2020 | 0.275 | 1.085 | 1.242 |
| MSE | 0.251 | 1.199 | 1.257 |
| RMSE | 0.501 | 1.095 | 1.121 |

Note: MSE is computed as the average squared difference between observed gross bond prices and the corresponding theoretical prices estimated by each methodology, that is, the residuals of each methodology in order to estimate the TSIR using the Nelson and Siegel (1987) model for the Spanish government bond market in 2020. RMSE is reported as the square root of the MSE and is expressed in the same units as bond prices, facilitating interpretation of pricing errors. Lower values indicate superior pricing performance. NS denotes Nelson-Siegel, SVR denotes Support Vector Regression, and LR denotes Local Regression (alternative methods to estimate the term structure of interest rates). YTM denotes the yield to maturity.

**Table 2b.** Errors produced by each method measured by MSE and RMSE (2023).

| Date | NS | SVR | LR |
|---|---|---|---|
| 16/01/2023 | 0.789 | 0.705 | 0.762 |
| 31/01/2023 | 1.100 | 0.426 | 0.761 |
| 15/02/2023 | 0.130 | 0.176 | 0.159 |
| 28/02/2023 | 0.863 | 0.252 | 0.286 |
| 15/03/2023 | 1.010 | 0.398 | 0.453 |
| 31/03/2023 | 0.216 | 0.113 | 0.264 |
| 14/04/2023 | 0.057 | 0.047 | 0.053 |
| 28/04/2023 | 0.081 | 0.162 | 0.187 |
| 15/05/2023 | 0.317 | 0.212 | 0.209 |
| 31/05/2023 | 0.850 | 0.694 | 0.688 |
| 15/06/2023 | 0.902 | 0.572 | 0.564 |
| 30/06/2023 | 0.675 | 0.502 | 0.568 |
| 14/07/2023 | 0.366 | 0.210 | 0.224 |
| 31/07/2023 | 0.028 | 0.017 | 0.015 |
| 15/08/2023 | 0.069 | 0.047 | 0.038 |
| 31/08/2023 | 0.054 | 0.047 | 0.051 |
| 15/09/2023 | 0.939 | 0.431 | 0.469 |
| 29/09/2023 | 0.668 | 0.948 | 0.956 |
| 16/10/2023 | 0.116 | 0.104 | 0.108 |
| 31/10/2023 | 0.052 | 0.225 | 0.255 |
| 15/11/2023 | 0.328 | 0.110 | 0.131 |
| 30/11/2023 | 0.248 | 0.239 | 0.252 |
| 15/12/2023 | 0.160 | 0.075 | 0.071 |
| 29/12/2023 | 0.004 | 0.008 | 0.008 |
| MSE | 0.418 | 0.280 | 0.314 |
| RMSE | 0.646 | 0.529 | 0.560 |

Note: MSE is computed as the average squared difference between observed gross bond prices and the corresponding theoretical prices estimated by each methodology, that is, the residuals of each methodology in order to estimate the TSIR using the Nelson and Siegel (1987) model for the Spanish government bond market in 2023. RMSE is reported as the square root of the MSE and is expressed in the same units as bond prices, facilitating interpretation of pricing errors. Lower values indicate superior pricing performance. NS denotes Nelson-Siegel, SVR denotes Support Vector Regression, and LR denotes Local Regression (alternative methods to estimate the term structure of interest rates). YTM denotes the yield to maturity.

The results presented in Table 2a show that the Nelson-Siegel model approximates the theoretical prices best to the gross prices in 2020, with a mean square error of 0.251, and SVR is the model that provides the most accurate TSIR estimates in 2023 with a mean square error of 0.280. Nevertheless, caution is required when interpreting this result. The findings indicate that the Nelson-Siegel specification is particularly effective in describing the Spanish sovereign yield curve during the financial environment observed in 2020 (Núñez, 1995; Díaz et al., 2011). The results suggest that the relatively smooth, policy-driven behavior of the Spanish yield curve in 2020 favors the Nelson-Siegel

specification. In these circumstances, the parsimonious representation of level, slope, and curvature offered by the Nelson-Siegel specification is sufficient to capture the main term-structure dynamics and generate lower pricing errors than machine-learning approaches. However, SVR yields the most accurate pricing estimates in 2023. This suggests that no single yield curve estimation method is universally superior, as their performance depends on prevailing market conditions

The root mean square error (RMSE) is also calculated, as this measure is on the same scale as the variable analyzed and can therefore be interpreted. For the Nelson-Siegel model in 2020, the RMSE increases to 0.501. However, the methods associated with the machine learning algorithms produce errors greater than one. On average, the SVR gives a lower RMSE than the LR algorithm. However, at the disaggregated level, there are some days where the LR technique produces a lower residual than the SVR. In 2023, the RMSE for the SVR Model is 0.529, very close to 0.560 for the LR Model and 0.646 of the Nelson Siegel Model. Although the Nelson-Siegel model produces the fewest overall pricing errors in 2020, machine-learning approaches offer valuable insights from a practical standpoint. In particular, SVR and LR impose fewer structural restrictions on the shape of the yield curve and are therefore potentially better suited to situations characterized by nonlinearities, abrupt market adjustments or irregular yield-curve configurations. Consequently, their value should be assessed not only through average pricing errors, but also through their ability to adapt to changing market conditions and capture local features of the term structure. This aligns with research emphasizing that model evaluation should be aligned with the intended financial application rather than relying solely on a single performance metric (He et al., 2024).

The results contribute to the broader debate regarding the relative merits of machine-learning and traditional financial models. Several studies report that machine-learning approaches can outperform classical specifications when complex nonlinear relationships dominate the data-generating process (Bracke et al., 2019; Mestiri, 2024). However, our findings suggest that greater flexibility does not automatically imply superior economic performance. In the case of the Spanish government bond market, the term structure in 2020 appears to be sufficiently captured by a parsimonious factor representation based on level, slope, and curvature components, which helps explain the strong performance of the Nelson-Siegel model. This result is also consistent with evidence showing the robustness of Nelson-Siegel-type specifications in the estimation of sovereign yield curves (Núñez, 1995; Díaz et al., 2011).

Even though the alternative methods for estimating the TSIR proposed in this paper are not quite as accurate when it comes to estimating the theoretical prices of bonds, the error made is quite small, as it is barely more than one unit on average. Moreover, most of the errors of the largest magnitude occur in the calculation of the theoretical prices of the bonds with the longest maturity, while very small errors are made for the bonds with the shortest maturities.

### *3.3. Statistical comparison of pricing errors*

To complement the comparison based on the magnitude of the pricing errors, we conduct further statistical analyses using the MSE series reported for each observation date. Specifically, non-parametric paired comparisons are conducted to assess the statistical significance of the differences observed across methodologies.[1]

[1] Due to the small sample size and the use of aggregated daily pricing losses, the reported statistical tests should be viewed

Table 3 first reports the descriptive statistics of the pricing errors generated by the three alternative approaches. The Nelson-Siegel model exhibits the lowest average error (0.251) and the most consistent errors throughout the sample period. In contrast, both machine-learning approaches produce larger average errors and greater variability, although their overall performance is very similar.

**Table 3a.** Descriptive statistics of pricing errors in 2020.

| Statistic | NS | SVR | LR |
|---|---|---|---|
| Mean | 0.251 | 1.199 | 1.257 |
| Median | 0.118 | 1.016 | 1.107 |
| Standard deviation | 0.356 | 0.720 | 0.810 |
| Minimum | 0.035 | 0.372 | 0.680 |
| Maximum | 1.522 | 4.207 | 4.504 |
| RMSE | 0.501 | 1.095 | 1.121 |

**Table 3b.** Descriptive statistics of pricing errors in 2023.

| Statistic | NS | SVR | LR |
|---|---|---|---|
| Mean | 0.418 | 0.280 | 0.314 |
| Median | 0.283 | 0.211 | 0.238 |
| Standard deviation | 0.376 | 0.253 | 0.274 |
| Minimum | 0.004 | 0.008 | 0.008 |
| Maximum | 1.100 | 0.948 | 0.956 |
| RMSE | 0.646 | 0.529 | 0.560 |

Note: NS denotes the Nelson–Siegel model, SVR denotes the support vector regression approach, and LR denotes the local regression approach.

**Table 4a.** Wilcoxon signed-rank tests for paired pricing errors in 2020.

| Comparison | Z statistic | p-value |
|---|---|---|
| NS vs SVR | −4.29 | <0.001 |
| NS vs LR | −4.29 | <0.001 |
| SVR vs LR | −0.54 | 0.589 |

**Table 4b.** Wilcoxon signed-rank tests for paired pricing errors in 2023.

| Comparison | Z statistic | p-value |
|---|---|---|
| NS vs SVR | −2.77 | 0.0056 |
| NS vs LR | −2.21 | 0.0268 |
| SVR vs LR | −2.80 | 0.0051 |

Note: The Wilcoxon signed-rank test is performed on paired MSE observations across the sample dates. The null hypothesis is that both methodologies generate pricing errors with the same median. A negative test statistic indicates superior performance of the first model reported in each comparison.

as robustness checks that support the comparative analysis, rather than as definitive inferential evidence.

To formally compare the pricing accuracy of the competing methodologies, Wilcoxon signed-rank tests for paired samples are conducted (Table 4). This non-parametric procedure is chosen because it does not require distributional assumptions and is suitable for the small sample size involved.

The results in 2020 indicate that the pricing errors generated by the Nelson-Siegel model are significantly lower than those obtained with either SVR or LR. However, no statistically significant differences are found between the two machine-learning methodologies. In 2023, the three comparisons are statistically significant. The negative sign in NS vs SVR and NS vs LR indicates that Nelson-Siegel tends to have greater errors than the machine learning methods; SVR and LR perform consistently better than NS throughout 2023. The comparison between SVR and LR is also significant, with SVR performing slightly better than LR on average.

As an additional robustness check, a Diebold-Mariano-type comparison based on the differential loss series is conducted. The results, shown in Table 5, are consistent with the previous findings and provide further evidence supporting the superior pricing performance of the Nelson-Siegel specification over the sample period analyzed.

**Table 5a.** Diebold-Mariano-type tests of predictive accuracy in 2020.

| Comparison | DM statistic | p-value |
|---|---|---|
| NS vs SVR | −6.10 | <0.001 |
| NS vs LR | −6.85 | <0.001 |
| SVR vs LR | −0.73 | 0.472 |

**Table 5b.** Diebold-Mariano-type tests of predictive accuracy in 2023.

| Comparison | DM statistic | p-value |
|---|---|---|
| NS vs SVR | 2.7370 | 0.0117 |
| NS vs LR | 2.3330 | 0.0288 |
| SVR vs LR | −2.2597 | 0.0336 |

Note: This table presents Diebold-Mariano-type tests, which are based on the differential loss series constructed from the mean squared error (MSE) values obtained on each observation date. The null hypothesis states that the two methodologies exhibit equal pricing accuracy. Negative statistics indicate lower pricing losses for the first model in each comparison. However, results should be interpreted with caution as the comparison is based on aggregated pricing losses rather than individual bond-level forecast errors.

Overall, the non-parametric and predictive-accuracy comparisons for 2020 (Table 5a) reach the same conclusion: The Nelson-Siegel model generates consistently lower pricing losses than machine-learning alternatives. However, the lack of significant differences between SVR and LR indicates that the two non-parametric approaches perform similarly within the considered market environment. The absence of statistically significant differences between the two machine-learning approaches is also noteworthy. This suggests that the superiority of one statistical learning architecture over another may depend more on the characteristics of the underlying yield curve than on the specific algorithm employed. Similar conclusions have been reported in several forecasting applications, where the benefits of increased model flexibility tend to diminish when the data-generating process is relatively stable and can be well approximated by low-dimensional factor structures. For 2023, Table 5b shows that SVR is the best method, LR falls somewhere in between, and Nelson-Siegel consistently provides

the worst fit for ETTI in 2023.

From a practical standpoint, the results suggest that traditional yield-curve models and machine-learning techniques should be viewed as complementary rather than competing approaches. While the Nelson-Siegel model provides the lowest pricing errors during the exceptional market conditions observed in 2020, the evidence from 2023 indicates that machine-learning techniques, particularly SVR, can achieve superior valuation performance in a markedly different interest-rate environment. This finding highlights the importance of adapting the choice of methodology to prevailing market conditions. For investors and portfolio managers, non-parametric methods may be particularly valuable when the yield curve exhibits local irregularities or nonlinear patterns that cannot be fully captured by parsimonious factor models. Similarly, financial institutions involved in fixed-income pricing, portfolio valuation, and risk management may benefit from combining traditional and machine-learning approaches, using each methodology in the market environments for which it appears best suited.

From a financial perspective, the evidence suggests that the relative performance of yield-curve estimation methods is not invariant across market regimes. The results for 2020 indicate that the Spanish sovereign yield curve could be effectively represented through a small number of economically interpretable factors, making the Nelson-Siegel specification particularly successful. This outcome is consistent with a market environment largely dominated by unconventional monetary policy, low interest rates, and strong central-bank intervention. In contrast, the results obtained for 2023 reveal that machine-learning techniques, especially SVR, outperform Nelson-Siegel in terms of pricing accuracy. This finding suggests that the post-pandemic monetary tightening cycle, together with higher inflation and increased market uncertainty, may have generated more complex yield-curve dynamics that are better captured by flexible non-parametric methods. Consequently, the empirical evidence indicates that no single methodology can be considered universally superior; rather, the effectiveness of a given approach appears to depend on the underlying economic and financial environment in which the yield curve is estimated.

## 4. Conclusions

Our findings of this study provide new evidence regarding the performance of alternative methodologies for estimating the term structure of interest rates in the Spanish sovereign bond market. The original analysis conducted for 2020 showed that the Nelson-Siegel specification produced the lowest pricing errors among the three approaches considered. However, the extension of the analysis to 2023 revealed a more nuanced picture. While Nelson-Siegel remained highly competitive, the machine-learning methodologies, particularly SVR, achieved lower average pricing errors during 2023. These results indicate that the relative performance of yield-curve estimation methodologies is not constant through time and may depend on the prevailing economic, financial, and monetary environment. The comparison between 2020 and 2023 is particularly informative because both years reflect markedly different market conditions. The year 2020 was characterized by the COVID-19 crisis, exceptionally low interest rates, and unprecedented intervention by the European Central Bank. In contrast, 2023 was characterized by elevated inflation, higher interest rates, and a restrictive monetary-policy stance. The fact that the ranking of methodologies changed across these two periods suggests that no single estimation approach can be considered universally superior. Instead, the effectiveness of each methodology depends on the characteristics of the yield curve being modeled and the broader

market environment in which the estimation is performed.

From an economic perspective, the 2020 results suggest that the relatively smooth and policy-driven behavior of the Spanish yield curve favors parsimonious factor-based specifications such as Nelson-Siegel. The level, slope, and curvature factors embedded in the model were sufficient to capture most of the information contained in the observed term structure. In contrast, the 2023 results indicate that the greater flexibility of machine-learning techniques become advantageous in a market environment characterized by higher rates and more heterogeneous yield dynamics. In particular, SVR demonstrated a superior ability to accommodate local nonlinearities while maintaining relatively low pricing errors.

The results also highlight the complementary nature of the methodologies analyzed. Nelson-Siegel retained important advantages in terms of interpretability, parsimony, and economic intuition. The estimated parameters can be directly associated with level, slope, and curvature components of the yield curve, making the model particularly attractive for policymakers and practitioners. By contrast, SVR and LR provided greater flexibility and required fewer assumptions regarding the functional form of the term structure. Consequently, machine-learning approaches may be especially useful during periods characterised by structural changes, market fragmentation, abrupt revisions of expectations, or stronger nonlinearities in interest-rate dynamics.

Several limitations should nevertheless be acknowledged. Although the inclusion of 2023 substantially broadens the empirical evidence, the analysis remains focused on a single sovereign bond market. Therefore, the conclusions should be interpreted as evidence obtained from the Spanish government debt market and should not necessarily be generalized to other countries or institutional settings. In addition, we focus on static yield-curve estimation techniques and do not consider dynamic specifications that explicitly model the time evolution of the term structure.

Future research could extend the analysis in several directions. First, longer sample periods covering multiple monetary-policy regimes and economic cycles would provide additional evidence regarding the temporal stability of the results. Second, cross-country comparisons could be used to determine whether the patterns identified in Spain are also observed in other sovereign debt markets. Finally, researchers could compare the methodologies considered here with dynamic and arbitrage-free term-structure frameworks, including the Nelson-Siegel model, affine term-structure models, and hybrid approaches combining economically interpretable factor structures with machine-learning algorithms. Such extensions would offer a broader assessment of the strengths and limitations of traditional parametric models and modern data-driven techniques in the estimation of interest-rate term structure.

## Author contributions

Marta Tolentino: Conceptualization, Methodology, Supervision, Data curation, Formal analysis, Investigation, Writing – review & editing.

Pedro Gento: Conceptualization, Methodology, Supervision, Writing – review & editing.

Francisco Jareño: Validation, Formal analysis, Writing – review & editing.

Juan Navas-Ureña: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Writing – original draft.

## Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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**Conflict of interest**

All authors declare no conflicts of interest in this paper.

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