

Research article

Pinning synchronization of neural networks under denial-of-service attacks

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Abstract: This work examined pinning synchronization in coupled networks with current-state coupling under denial-of-service (DoS) attacks, using a pinning impulsive control (PIC) scheme and the average-impulse-interval technique. Lyapunov-based sufficient conditions were established to guarantee stability for impulsive time-delay systems, and a comparison principle suitable for such systems was derived. To address the high dimensionality of stability criteria, inequality techniques and matrix decomposition were employed to obtain reduced-dimensional conditions. A numerical example illustrated the effectiveness and practicality of the proposed approach.

Keywords: synchronization; coupled networks; DoS Attacks; pinning impulsive control

1. Introduction

Research on neural networks has focused on stability, periodic, and chaotic attractors [1]. These issues, crucial for pattern recognition, associative memory, and dynamic optimization, have gained significant attention [2]. Neural networks are now widely applied in image recognition, speech processing, intelligent recommendation systems, autonomous driving, finance, and healthcare, enhancing intelligent capabilities across various industries [3].

Researchers have focused on selecting network technologies and communication protocols to enable complex networks to achieve desired functionalities. Synchronization among nodes is a key research direction in complex networks [4], aiming to align all nodes toward a unified state. Numerous synchronization phenomena have been identified.

Coupled neural networks have been extensively investigated, yielding substantial theoretical developments. With increasing attention to collective network dynamics, key research topics include synchronization under

unobservable states and impulsive perturbations [5], fixed-time stabilization and tracking control [6], and synchronization in uncertain interconnected networks with time-varying delays [7]. These studies reflect ongoing advances in understanding and regulating coordinated behaviors in complex networked systems.

Synchronization has been addressed using genetic-algorithm radial-basis-function (GA-RBF) neural-network controllers, adaptive and fuzzy control, global exponential synchronization with impulses [8], and optimal/robust designs [9]. Two scheduling paradigms—time-triggered and event-triggered—govern transmission and sampling for stability and performance. Event-triggered schemes are increasingly practical; notably, event-triggered impulsive control integrates impulses with event rules and has been applied to nonlinear systems [10], and Lyapunov stability of impulsive systems under event-triggered impulsive control was established in [11]; event-triggered synchronization using gain-scheduled controllers was explored in [12] for memristive neural networks, [13] presented an event-triggered prescribed-time synchronization strategy for

Takagi–Sugeno (T–S) fuzzy neural networks, and memory-aided quantized dynamic event-triggered control achieving finite-time synchronization under actuator faults [14]. Pinning impulsive control, which acts on a small subset of nodes, is effective for network synchronization [15] and has yielded advances via Lyapunov analysis, distributed-delay differential inequalities [16], distributed pinning for Boolean networks [17], and bio-inspired impulsive inputs such as tumour-necrosis-factor (TNF) stimulation [18].

In recent years, complex dynamical networks with delays have attracted considerable attention, as delays critically influence system stability [19]. Such delays commonly arise in sensor–controller–actuator communications and computations, motivating extensive research on delayed control systems. Notable contributions include observer design for discrete-time fractional-order neural systems [20], finite-time stabilization of switched hybrid models with impulse-driven delays [21], controllability of discrete-time hybrid systems with delayed inputs [22], and analyses of neural network dynamics under delays [23]. Delays further complicate synchronization, including in nabla fractional-order fuzzy neural networks [24], while exponential stability of time-delay systems with nonlinear impulses is studied in [25]. These results highlight that overlooking delay effects during controller design may lead to instability, underscoring the necessity of explicitly incorporating delays in control analysis.

Denial-of-service (DoS) attacks can disrupt data exchange among the plant, controller, and actuator, potentially causing instability. Recent studies have examined DoS effects in control systems, including finite-time consensus in variable-order fractional systems [26], compression strategies to mitigate congestion and bandwidth demands under DoS and quality-of-service (QoS) constraints [27], and robust distributed filtering in sensor networks using dynamic event-triggered mechanisms under DoS conditions [28].

This study addresses pinning synchronization under DoS attacks. An average impulse-interval framework is adopted, and a pinning impulsive control strategy incorporating a pinning ratio is proposed to ensure stability. Based on Lyapunov analysis, sufficient conditions for stability in impulsive time-delay systems subject to DoS interruptions are derived. Furthermore, a comparison

principle for delayed impulsive control systems under DoS attacks is developed, offering a theoretical basis for analyzing synchronization robustness in adversarial network environments.

Advanced inequality techniques and matrix decomposition are employed to obtain reduced-dimensional stability criteria suitable for high-order systems under intermittent attacks. A numerical simulation illustrates that the proposed impulsive pinning strategy maintains robustness and effectively drives delayed complex networks to synchronization under DoS conditions, demonstrating its applicability in cybersecurity-critical settings.

2. Preliminaries

Within a coupled neural network array, the evolution of the state variable associated with the i -th node is governed by:

$$\dot{x}_i(t) = -Dx_i(t) + Af_1(x_i(t)) + Bf_2(x_i(t - \tau)) + J, i = 1, \dots, N. \quad (2.1)$$

Consider agent i with state $x_i(t) = [x_{i1}(t), \dots, x_{in}(t)]^\top \in \mathbb{R}^n$, and let $D = \text{diag}\{d_1, \dots, d_n\} > 0$ denote a positive diagonal matrix, where each d_k represents the intrinsic decay rate of the corresponding neuron. The activation function at time t is given by $f_m(x(t)) = [f_m(x_1(t)), \dots, f_m(x_n(t))]^\top$, $m \in \{1, 2\}$, and τ denotes a fixed discrete delay.

We represent the coupling weights by the matrices: $A = (a_{ij})_{n \times n}$ (instantaneous), $B = (b_{ij})_{n \times n}$ (delayed), and define the external input as the fixed vector $J = (J_1, J_2, \dots, J_n)^\top$.

This study considers a neural network model in which the coupling depends solely on the current state.

$$\begin{aligned} \dot{x}_i(t) = & -Dx_i(t) + Af_1(x_i(t)) + Bf_2(x_i(t - \tau)) \\ & + c \sum_{j=1}^N l_{ij} \Gamma x_j(t) + J, \quad i = 1, \dots, N. \end{aligned} \quad (2.2)$$

The current-state inner coupling is given by the positive diagonal matrix $\Gamma = \text{diag}\{\gamma_1, \dots, \gamma_n\}$ with $\gamma_i > 0$. The inter-node coupling is described by $L = (l_{ij})_{N \times N}$, where $l_{ij}(t) = 1$ if a directed edge exists from node j to node i at time t , and $l_{ij}(t) = 0$ otherwise. The diagonal terms are set as $l_{ii} = -\sum_{j \neq i} l_{ij}$, ensuring each row of L sums to zero.

Assumption 2.1. For each nonlinear mapping $x_1 = (x_{11}, \dots, x_{1n})^\top$ and $x_2 = (x_{21}, \dots, x_{2n})^\top \in \mathbb{R}^n$, it holds that

$$\begin{aligned} & |f_m(x_{11}, x_{12}, \dots, x_{1n}) - f_m(x_{21}, x_{22}, \dots, x_{2n})| \\ & \leq \sum_{j=1}^n l_{f_m}^{ij} |x_{1j} - x_{2j}|. \end{aligned} \quad (2.3)$$

Let $s(t)$ be the desired trajectory described by

$$\dot{s}(t) = -Ds(t) + Af_1(s(t)) + Bf_2(s(t - \tau)).$$

Using $e_i(t) = x_i(t) - s(t)$ to denote the error, the impulse pinning controller can be written as

$$u_i(t) = \begin{cases} \sum_{k=1}^{+\infty} \mu e_i(t) \delta(t - t_k), & i \in D(t_k), \#D(t_k) = l_k, \\ 0, & i \notin D(t_k). \end{cases} \quad (2.4)$$

Consider the impulse instants $\{t_k\}$ forming a strictly increasing sequence

$$0 = t_0 < t_1 < \dots < t_k < \dots, \quad \lim_{k \rightarrow \infty} t_k = +\infty.$$

Denote the Dirac delta distribution by $\delta(\cdot)$, and for each t_k define the pinned-node set $D(t_k) = \{i_1, i_2, \dots, i_{l_k}\} \subseteq \{1, 2, \dots, N\}$. Let μ be the impulsive gain. Under the control law in (2.4), the error dynamics are given by

$$\begin{cases} \dot{e}_i(t) = -De_i(t) + Ag_1(e_i(t)) + Bg_2(e_i(t - \tau)) \\ \quad + c \sum_{j=1}^N l_{ij} \Gamma e_j(t), \quad t \neq t_k, \quad k \in \mathbb{N}, \\ e_i(t_k) = e_i(t_k^-) + \mu e_i(t_k^-), \quad i \in D(t_k), \\ e_i(t_k) = e_i(t_k^-), \quad i \notin D(t_k). \end{cases} \quad (2.5)$$

We assume the error $e_i(t)$ is right-continuous at each impulse instant t_k , so that $e_i(t_k) = e_i(t_k^+)$ and $e_i(t_k^-) = \lim_{h \rightarrow 0^-} e_i(t_k + h)$. The term $g(e_i(t), s(t))$ is given by $g(e_i(t), s(t)) = f(e_i(t) + s(t)) - f(s(t))$.

Assumption 2.2. During each DoS interval, control communication is unavailable. Let t_k° denote the start of the k -th attack, with $\mathcal{D}_k = [t_k^\circ, t_k^\circ + \Delta_k)$ and $\mathcal{W}_k = [t_k^\circ + \Delta_k, t_{k+1}^\circ)$. Over (T_1, t) , the attack duration is $\mathcal{D}(T_1, t) = \bigcup_k \mathcal{D}_k \cap (T_1, t)$ and the remaining interval is $\mathcal{W}(T_1, t) = (T_1, t) \setminus \mathcal{D}(T_1, t)$, with $|\mathcal{D}(T_1, t)| + |\mathcal{W}(T_1, t)| = t - T_1$. During $\mathcal{D}_k$, no control is applied, so $\dot{x}_i(t_k^+) = \dot{x}_i(t_k^-)$. Let L denote the total number of impulse instants within a finite observation window $[T_1, T]$,

with the portions inside and outside DoS periods denoted $L_{\mathcal{D}}$ and $L_{\mathcal{W}}$, respectively:

$$L_{\mathcal{D}} + L_{\mathcal{W}} = L, \quad \frac{L_{\mathcal{D}}}{L} = r,$$

then the error system (2.5) with (2.4) under DoS attack is shown as follows,

$$\begin{cases} \dot{e}_i(t) = -De_i(t) + Ag_1(e_i(t)) + Bg_2(e_i(t - \tau)) \\ \quad + c \sum_{j=1}^N l_{ij} \Gamma e_j(t), \quad t \neq t_k, \quad k \in \mathbb{N}, \\ \Delta e_i(t_k) = \mu \theta(t) e_i(t_k^-), \quad t = t_k, \quad k \in \mathbb{Z}_+, \end{cases}$$

where $\Delta e_i(t_k) = e_i(t_k^+) - e_i(t_k^-)$. The binary switching signal

$$\theta(t) = \begin{cases} 0, & t \in \mathcal{D}(T_1, t) \quad (\text{DoS interval}), \\ 1, & t \in \mathcal{W}(T_1, t) \quad (\text{normal interval}), \end{cases} \quad (2.6)$$

flags the availability of the control channel: $\theta(t) = 0$ while a DoS attack is active and $\theta(t) = 1$ whenever the network operates without interference. Then, based on the proposed impulsive control (2.6), equivalently, the error dynamics are given by

$$\begin{cases} \dot{e}_i(t) = -De_i(t) + Ag_1(e_i(t)) + Bg_2(e_i(t - \tau)) \\ \quad + c \sum_{j=1}^N l_{ij} \Gamma e_j(t), \quad t \neq t_k, \quad k \in \mathbb{N}, \\ e_i(t_k) = e_i(t_k^-) + \mu e_i(t_k^-), \quad i \in D(t_k), \quad t \in \mathcal{W}_k, \\ e_i(t_k) = e_i(t_k^-), \quad i \notin D(t_k) \vee (i \in D(t_k) \wedge t \in \mathcal{D}_k). \end{cases} \quad (2.7)$$

Definition 2.1. Assume there exist positive constants T_a and N_0 such that, for all $T \geq t \geq t_0$,

$$N(T, t) \geq \frac{T - t}{T_a} - N_0. \quad (2.8)$$

Let $N(T, t)$ be the number of impulse instants $\{t_k\}$ in $(t, T]$, where T_a is the average impulsive-interval constant and N_0 is the elasticity coefficient. When $N_0 = 1$, condition (2.8) guarantees at least one impulse within every interval of length T_a . The set $\epsilon^*(T_a, N_0)$ consists of all impulse sequences satisfying (2.8); for fixed T_a , larger N_0 permits higher impulse density.

Definition 2.2. Given an impulse sequence $\{t_k\}$, the equilibrium of system (2.7) is globally exponentially stable (GES) if there exist constants N and S such that

$$\|x(t, t_0, \varphi)\| \leq N \|\varphi\| e^{-S(t-t_0)}, \quad \forall t \geq t_0, \quad (2.9)$$

for every initial history $\varphi \in PC([-h, 0], \mathbb{R}^n)$.

Here, N and S are constants that depend on the average impulsive interval T_a and the elasticity coefficient N_0 . They are analytical quantities derived from the system's dynamics and stability analysis.

Remark 2.1. System (2.7) is termed globally uniformly exponentially stable (GUES) over $\epsilon^*(T_a, N_0)$ if there exist N and S such that (2.9) holds for all impulse sequences in this class.

Definition 2.3. A function $V : [t_0 - h, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}_+$ is said to belong to $\mathcal{V}_0$ if it is continuous on each $[t_{k-1}, t_k) \times \mathbb{R}^n$ with right limits at t_k , locally Lipschitz in x , satisfies $V(t, 0) = 0$, and there exist constants $\lambda_1, \lambda_2, \nu > 0$ such that $\lambda_1 \|x\|^\nu \leq V(t, x) \leq \lambda_2 \|x\|^\nu, \forall t, x$.

We term any $V \in \mathcal{V}_0$ that meets the prescribed rate conditions for system (2.7) an *exponential Lyapunov function* (ELF). $\beta_1, \beta_2 \in \mathbb{R}$, $w \in \mathbb{R}_+$, and $r \in (0, 1)$ if, for every history $\varphi \in PC((\tau - h, 0], \mathbb{R}^n)$,

$$D^+ V(t, \varphi(0)) \leq \beta_1 V(t, \varphi(0)) + \beta_2 V(t - h, \varphi(-h)), \quad t \neq t_k,$$

$$V(t, g_k(\varphi(0))) \leq w^{1-r} V(t^-, \varphi(0)), \quad t = t_k,$$

where $D^+ V(x(t))$ denotes the upper righthand Dini derivative of V evaluated along trajectories of system (2.7).

$$D^+ V(t, \varphi(0)) \triangleq \lim_{p \rightarrow 0^+} \sup \frac{1}{p} [V(t + p), \varphi(0) + pf(t, \varphi(0), \varphi(-h) - V(t, \varphi(0)))].$$

At time t_k , the pinning ratio is defined by

$$\frac{\sum_{i \in D_{t_k}} e_i^T(t_k^-) e_i(t_k^-)}{\sum_{k=1}^N e_i^T(t_k^-) e_i(t_k^-)} = \eta_k.$$

Since the pinning ratio η_k varies with the impulse instants, the following analysis establishes an explicit lower bound for η_k .

Remark 2.2. The constants N and S in Definition 2.2 are derived from the Lyapunov decay rate and are dependent on both the average impulsive interval T_a and the elasticity coefficient N_0 .

3. Main results

Lemma 3.1. Suppose system (2.7) admits an exponential Lyapunov function $V \in \mathcal{V}_0$ with associated rate coefficients $\beta_1 \in \mathbb{R}$ and $\beta_2 \geq 0$. $\beta_1 \in \mathbb{R}$, $\beta_2 \geq 0$ satisfying $\beta_1 + \beta_2 > 0$, parameters $w, r \in (0, 1)$, and a design constant $\tilde{\eta} > \beta_1 + \beta_2$, such that

$$\beta_1 + \frac{\beta_2}{w^{(1-r)}} e^{-\eta h} - \tilde{\eta} < 0. \quad (3.1)$$

Then, there holds

$$V(t, x(t)) \leq \Omega_k \exp[\tilde{\eta}(t - t_0)], \quad t \in [t_k, t_{k+1}), \quad (3.2)$$

where $\Omega_k = w^{k(1-r)} \tilde{V}(t_0)$, with $\tilde{V}(t_0) = \sup_{s \in [t_0 - \tau, t_0]} V(s)$, for all $k \in \mathbb{Z}_+^0$, η and $\tilde{\eta}$ represent different quantities, with η directly related to the design of the impulsive control strategy, and $\tilde{\eta}$ being an analytical quantity derived from the system's stability conditions.

Proof. Because $\beta_2 \geq 0$ and $w \in (0, 1)$, one can always pick a constant

$$\tilde{\eta} > \beta_1 + \frac{\beta_2}{w^{1-r}}$$

so that both $\tilde{\eta} > \beta_1 + \beta_2$ and condition (3.1) hold for every $h \geq 0$; hence the set of admissible η satisfying these requirements is nonempty.

For any prescribed $\theta > 0$, introduce the auxiliary function

$$\mathcal{L}_\theta(t) = \begin{cases} V(t) \exp[-(\tilde{\eta} + \theta)(t - t_k)], & t \in [t_k, t_{k+1}), k \in \mathbb{Z}_+^0, \\ V(t), & t \in [t_0 - h, t_0]. \end{cases} \quad (3.3)$$

We now demonstrate that the conclusion remains valid for any t^* satisfying either Case (I) or Case (II).

Case (I):

$$\begin{aligned} t^* &\in [t_0, t_1) \text{ such that } \mathcal{L}(s) \leq \mathcal{L}(t^*), \\ t_0 - h &\leq s \leq t^*. \end{aligned} \quad (3.4)$$

Case (II):

There exists $k \in \mathbb{Z}^+$ such that $t^* \in [t_k, t_{k+1})$,

$$\mathcal{L}(s) \leq \mathcal{L}(t^*) \quad \forall s \in [t_k, t^*], \quad (3.5)$$

$$\mathcal{L}(s) e^{\tilde{\eta}(t_k - t_{k-1})} \leq \frac{\mathcal{L}(t^*)}{w^{1-r}} \quad \forall s \in [t_{k-1}, t_k).$$

Assuming Case (I) holds and $t^* - h \geq t_0$, it follows from $\mathcal{L}(s) \leq \mathcal{L}(t^*)$ and (3.4) that:

$$\begin{aligned} & e^{\theta(t^*-t_0)} D^+ \mathcal{L}_\theta(t) \\ & \leq D^+ V(t) \Big|_{t=t^*} e^{-\bar{\eta}(t^*-t_0)} - (\bar{\eta} + \theta) V(t^*) e^{-\bar{\eta}(t^*-t_0)}. \end{aligned}$$

$$D^+ \mathcal{L}_\theta(t) \Big|_{t=t_k} \leq (\beta_1 - \bar{\eta} - \theta) \mathcal{L}(t^*) + \beta_2 \mathcal{L}(t^* - h) e^{-\bar{\eta}h}.$$

Due to $\bar{\eta} > \beta_1 + \beta_2$,

$$D^+ \mathcal{L}_\theta(t) \Big|_{t=t_k} < (\beta_1 - \bar{\eta} - \theta + \beta_2 e^{-\bar{\eta}h}) \mathcal{L}(t^*) < -\theta \mathcal{L}(t^*).$$

Similarly, if $t^* - h < t_0$ and $t_0 - \leq t^* - h < t_0$, then $D^+ \mathcal{L}_\theta(t) \Big|_{t=t_k} \leq (\beta_1 - \bar{\eta} - \theta + \beta_2) \mathcal{L}(t^*) < -\theta \mathcal{L}(t^*)$.

Then, suppose that Case (II), the criterion related to the presence of a local maximum is met. If $t^* - h \geq t_k$, then we deduce from (3.5) that

$$\begin{aligned} & e^{\theta(t^*-t_k)} D^+ \mathcal{L}_\theta(t) \Big|_{t=t^*} \\ & = D^+ V(t) \Big|_{t=t^*} e^{-\bar{\eta}(t^*-t_k)} - (\bar{\eta} + \theta) V(t^*) e^{-\bar{\eta}(t^*-t_k)} \\ & \leq (\beta_1 - \bar{\eta} - \theta + \beta_2 e^{-\bar{\eta}h}) \mathcal{L}(t^*) \\ & < \left(\beta_1 - \bar{\eta} - \theta + \frac{\beta_2}{w^{1-r}} e^{-\bar{\eta}h} \right) \mathcal{L}(t^*) < \theta \mathcal{L}(t^*). \end{aligned}$$

Analogously, if $t^* - h < t_k$, then invoking (3.5) together with the conditions $t_{k-1} \leq t^* - h < t_k$, $0 < w < 1$, and $\Omega > \beta_1 + \beta_2/w^{1-r}$ yields:

$$\begin{aligned} D^+ \mathcal{L}_\theta(t) \Big|_{t=t_k} & < (\beta_1 - \bar{\eta} - \theta + \frac{\beta_2}{w^{1-r}} e^{-\bar{\eta}h}) \mathcal{L}(t^*) \\ & < -\theta \mathcal{L}(t^*). \end{aligned}$$

Therefore, regardless of which scenario applies, we deduce that:

$$\begin{aligned} & e^{\theta(t^*-t_k)} D^+ \mathcal{L}_\theta(t) \Big|_{t=t^*} < -\theta \mathcal{L}(t^*). \\ D^+ \mathcal{L}(t) \Big|_{t=t^*} & = e^{\theta(t^*-t_k)} D^+ \mathcal{L}_\theta(t) \Big|_{t=t^*} + \theta \mathcal{L}_\theta(t^*) e^{\theta(t^*-t_k)} \\ & < -\theta \mathcal{L}(t^*) + \theta \mathcal{L}_\theta(t^*) e^{\theta(t^*-t_k)} = 0. \end{aligned}$$

Denote by $x(t; t_0, \phi)$ the solution of the system with initial condition $(t_0, \phi) \in \mathbb{R} \times PC([-h, 0], \mathbb{R}^n)$, and set $V(t) := V(t, x(t; t_0, \phi))$ with $\bar{V}(t_0) \neq 0$; we now proceed to show that:

$$V(t) \leq \Omega_k e^{\bar{\eta}(t-t_0)}, \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{Z}_{\geq 0},$$

which is equivalent to demonstrating that:

$$\mathcal{L}(t) \leq \Omega_k e^{\bar{\eta}(t-t_0)}, \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{Z}_+^0.$$

Base step ($k = 0$). Because $\mathcal{L}(t_0) = V(t_0) \leq \bar{V}(t_0) = \Omega_0$, We assert that

$$\mathcal{L}(t) \leq \Omega_0, \quad t \in [t_0, t_1).$$

Assume, to the contrary, that the claim does not hold. Then there exists $t^* \in [t_0, t_1]$ such that $\mathcal{L}(t^*) = \Omega_0$, $\mathcal{L}(t) < \Omega_0$ for all $t < t^*$, and $D^+ \mathcal{L}(t^*) \geq 0$. Since $\mathcal{L}(s) \leq \mathcal{L}(t^*)$ for all $s \leq t^*$, it follows that $D^+ \mathcal{L}(t^*) < 0$, a contradiction. Thus, the bound is valid for $k = 0$.

Induction hypothesis. Assume there exists $N \in \mathbb{Z}_+^0$ such that

$$\mathcal{L}(t) \leq \Omega_k, \quad t \in [t_k, t_{k+1}), \quad k = 0, 1, \dots, N,$$

where $\Omega_k = w^{k(1-r)} \bar{V}(t_0)$. We next show that the same estimate is valid for $k = N + 1$.

$$\mathcal{L}(t) \leq \Omega_k e^{\bar{\eta}(t-t_0)}, \quad t \in [t_k, t_{k+1}), \quad k \leq N.$$

Therefore, it can be concluded that:

$$\mathcal{L}(t) \leq \Omega_N e^{\bar{\eta}(t-t_0)}, \quad t \in [t_N, t_{N+1}).$$

We now proceed to prove that:

$$\begin{aligned} \mathcal{L}(t) & \leq \Omega_{N+1} e^{\bar{\eta}(t_{N+1}-t_0)}, \quad t \in [t_{N+1}, t_{N+2}], \\ \mathcal{L}(t_{N+1}) & = V(t_{N+1}) \\ & \leq w^{(1-r)} V(t_N) \\ & = w^{(1-r)} \mathcal{L}(t_N) e^{\bar{\eta}(t_{N+1}-t_N)} \\ & \leq w^{(1-r)} \Omega_N e^{\bar{\eta}(t_N-t_0)} e^{\bar{\eta}(t_{N+1}-t_N)} \\ & = \Omega_{N+1} e^{\bar{\eta}(t_{N+1}-t_0)}. \end{aligned}$$

Thus, the statement holds at $t = t_{N+1}$. Assume, for contradiction, that a violation occurs at some $t^* \in [t_{N+1}, t_{N+2}]$ such that:

$$\mathcal{L}(t^*) = \Omega_N e^{\bar{\eta}(t_N-t_0)}, \quad \mathcal{L}(t) \leq \mathcal{L}(t^*), \quad t \in [t_{N+1}, t^*], \quad D^+ \mathcal{L}(t^*) \geq 0.$$

In this situation, we have:

$$\mathcal{L}(s) \leq \Omega_N e^{\bar{\eta}(t_N-t_0)} = \frac{\Omega_{N+1}}{w^{1-r}} e^{\bar{\eta}(t_{N+1}-t_0)} = \frac{\mathcal{L}(t^*)}{w^{1-r}} e^{\bar{\eta}(t_{N+1}-t_N)}.$$

This contradiction—namely $D^+ \mathcal{L}(t^*) < 0$ versus $D^+ \mathcal{L}(t) \geq 0$ —proves the claim. Consequently, for all $t \in [t_k, t_{k+1})$ we have

$$\mathcal{L}(t) \leq \Omega_k e^{\bar{\eta}(t-t_0)}.$$

Hence, the proof is complete. $\square$

Based on the foregoing analysis, we arrive at the following theorem. In particular, Lemma 3.1 provides an exponential estimate for the Lyapunov function $V(t)$, which will be combined with the matrix inequalities in Theorem 3.1 to derive verifiable linear-matrix-inequality (LMI) conditions for exponential stability.

Theorem 3.1. Assume that the average impulsive interval satisfies $\bar{T} < T_a$, and that there exist a positive definite matrix $P \in \mathbb{R}^{n \times n}$, positive diagonal matrices $\Sigma_1, \Sigma_2 \in \mathbb{R}^{n \times n}$, which serve as weighting matrices in the inequality estimates of g_1 and g_2 , and constants $\beta_1 \in \mathbb{R}$, $\beta_2 > 0$ with $\beta_1 + \beta_2 > 0$, $w < 1$, and $r \in [0, 1]$, such that:

$$\Omega_1 = \begin{bmatrix} \Phi & I_N \otimes PA & I_N \otimes PB \\ * & -I_N \otimes \Sigma_1 & 0 \\ * & * & -I_N \otimes \Sigma_2 \end{bmatrix} < 0, \quad (3.6)$$

$$\Pi_2^T \Sigma_2 \Pi_2 \leq \beta_2 P, \quad (3.7)$$

$$\bar{\eta}(T_a) + (1-r) \ln w < 0, \quad (3.8)$$

$$[1 + (1+\mu)^{2(1-r)}] \eta_k \leq w^{(1-r)} \frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}, \quad (3.9)$$

$$\bar{\eta} > \beta_1 + \beta_2, \quad (3.10)$$

$$\beta_1 + \frac{\beta_2}{w^{(1-r)}} e^{-\bar{\eta}} - \bar{\eta} < 0, \quad (3.11)$$

where $\Phi_1 = I_N \otimes (-PD - D^T P + \Pi^T \Sigma_1 \Pi - \beta_1 P) + c(L \otimes P\Gamma)^T$, $\Pi = (l_{fm}^{ij})_{n \times n}$, and $\lambda > 0$.

Consequently, the error trajectory decays to zero exponentially.

Proof. We select a Lyapunov function given by

$$V(t) = \sum_{i=1}^N e_i^T(t) P e_i(t).$$

For $t \in [t_{k-1}, t_k)$, where $k \in \mathbb{N}$, differentiating $V(t)$ along the trajectory of (2.7) results in:

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t) P [-D e_i(t) + A g_1(e_i(t)) + B g_2(e_i(t - \tau_1))] \\ & + c \sum_{i=1}^N l_{ij} \Gamma_1 e_j(t). \end{aligned}$$

For positive definite diagonal matrices Σ_1, Σ_2 , we have:

$$\begin{aligned} 2e_i^T(t) P A g_1(e_i(t)) & \leq e_i^T(t) P A \Sigma_1^{-1} A^T P e_i(t) \\ & \quad + g_1^T(e_i(t)) \Sigma_1 g_1(e_i(t)) \\ & \leq e_i^T(t) P A \Sigma_1^{-1} A^T P e_i(t) \\ & \quad + e_i^T(t) \Pi_1^T \Sigma_1 \Pi_1 e_i(t). \end{aligned}$$

Similarly, we have:

$$2e_i^T P B g_2(e_i(t - \tau)) \leq e_i^T P B \Sigma_2^{-1} B^T P e_i + e_i^T(t - \tau) \Pi_2^T \Sigma_2 \Pi_2 e_i(t - \tau),$$

where $\Pi = (l_{fm}^{ij})_{n \times n}$ ($m = 1, 2$).

Let $e(t) = [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T$, and rewrite the coupling terms as follows:

$$2c \sum_{i=1}^N e_i^T(t) P \sum_{j=1}^N l_{ij} e_j(t) = 2c e^T(t) (L \otimes P\Gamma) e(t).$$

Then, we have:

$$\begin{aligned} V(t) = & \sum_{i=1}^N e_i^T(t) \Lambda e_i(t) + 2c e^T(t) (L \otimes P\Gamma) e(t) \\ & + \sum_{i=1}^N e_i^T(t - \tau) \Pi_2^T \Sigma_2 \Pi_2 e_i(t - \tau), \end{aligned}$$

$$\Lambda = -PD - DP + PA \Sigma_1^{-1} A^T P^T + \Pi_1^T \Sigma_1 \Pi_1 + PB \Sigma_2^{-1} B^T P.$$

Then, we have:

$$\begin{aligned} \dot{V}(t) & \leq \beta_1 \sum_{i=1}^N e_i^T(t) P e_i(t) + \beta_2 \sum_{i=1}^N e_i^T(t - \tau) P e_i(t - \tau) \\ & \leq \beta_1 V(t) + \beta_2 V(t - \tau). \end{aligned}$$

For the impulsive region attacked by DoS, when $t = t_k$, we have:

$$\begin{aligned} V(t_k^+) & = \sum_{i \in D(t_k)} (1 + \mu)^2 e_i^T(t_k^-) P e_i(t_k^-) + \sum_{i \notin D(t_k)} e_i^T(t_k^-) P e_i(t_k^-) \\ & \leq (1 + \mu)^2 \lambda_{\max}(P) \sum_{i \in D(t_k)} \|e_i(t_k^-)\|^2 + \lambda_{\max}(P) \sum_{i \notin D(t_k)} \|e_i(t_k^-)\|^2 \\ & \leq \lambda_{\max}(P) \left[(1 + \mu)^{2(1-r)} \eta_k + (1 - \eta_k) \right] \sum_{i=1}^N \|e_i(t_k^-)\|^2 \\ & \leq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \left[(1 + \mu)^{2(1-r)} \eta_k + (1 - \eta_k) \right] V(t_k^-) \\ & \leq w^{(1-r)} V(t_k^-). \end{aligned}$$

Consider an exponential Lyapunov function $V \in \mathcal{V}_0$ for the system, associated with rate coefficients $\beta_1, \beta_2 \in \mathbb{R}$ and a

parameter $w \in (0, 1)$. Based on (2.6), we can deduce the following comparison system:

$$\begin{cases} D^+ V(t) \leq \beta_1 V(t) + \beta_2 V(t-h), & t \geq t_0, t \neq t_k, \\ V(t) \leq w^{(1-r)} V(t^-), & t = t_k. \end{cases}$$

According to Lemma 3.1, we have:

$$V(t_k^+, x(t_{k+1})) \leq w^{(1-r)} V(t_k^-, x(t_k^-)).$$

Then we have:

$$L(t_k^+) = V(t_k^+) e^{-\bar{\eta}(t_k^+ - t_k^-)} \leq w^{(1-r)} V(t_k^-) e^{-\bar{\eta}(t_k^+ - t_k^-)} = w^{(1-r)} L(t_k^-).$$

For arbitrary $t \in [t_k, t_{k+1})$,

$$L(t) \leq w^{(1-r)k} L(t_0) = w^{(1-r)k} \bar{V}(t_0), \quad L(t) = V(t) e^{-\bar{\eta}(t-t_k)},$$

then $V(t) \leq W^{(1-r)k} \bar{V}(t_0) e^{\bar{\eta}(t-t_0)}$.

Therefore, for all $t \in [t_k, t_{k+1})$, we have

$$V(t, x(t)) \leq \Omega_k e^{\bar{\eta}(t-t_0)}.$$

Considering that the sequence $\{t_k\}$ satisfies $\{t_k\} \in \xi^*(T_a, N_0)$, it can therefore be deduced that

$$\begin{aligned} V(t, x(t)) &\leq w^{(1-r)N(t,t_0)} e^{\bar{\eta}(t-t_0)} \bar{V}(t_0) \\ &\leq w^{(1-r)\frac{t-t_0}{T_a} - N_0} e^{\bar{\eta}(t-t_0)} \bar{V}(t_0) \\ &= e^{-N_0} e^{\frac{(1-r)\ln w}{T_a} + \bar{\eta}(t-t_0)} \bar{V}(t_0). \end{aligned}$$

This bound is valid uniformly for every $t \geq t_0$, with $N(t, t_0)$ denoting the total impulse count on $[t_0, t]$. Actually,

$$\lambda_{\min}(P) \sum_{i=1}^N e_i^T(t) \Lambda e_i(t) \leq V(t, x) \leq \lambda_{\max}(P) \sum_{i=1}^N e_i^T(t) \Lambda e_i(t),$$

we can indicate that:

$$\|x(t, t_0, \phi)\| \leq N \|\phi\| e^{-k(t-t_0)}, \quad \forall t \geq t_0,$$

where $N = \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\right)^{\frac{1}{2}} e^{-N_0}$, $S = -\frac{\bar{\eta} + \frac{(1-r)\ln w}{T_a}}{2}$.

We can infer that the above system (2.7) is exponentially stable.

4. Numerical example

To verify the preceding theoretical conclusions, the leader system is characterized by the following cellular neural network.

$$\begin{aligned} \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} &= - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} + \begin{bmatrix} 1 + \frac{\pi}{4} & 20 \\ 0.1 & 1 + \frac{\pi}{4} \end{bmatrix} \begin{bmatrix} f_1(s_1(t)) \\ f_2(s_2(t)) \end{bmatrix} \\ &+ \begin{bmatrix} -\frac{\sqrt{2}\pi}{4} 1.3 & 0.1 \\ 0.1 & -\frac{\sqrt{2}\pi}{4} 1.3 \end{bmatrix} \begin{bmatrix} f_1(s_1(t-1)) \\ f_2(s_2(t-1)) \end{bmatrix} + J. \end{aligned} \quad (4.1)$$

The activation functions are defined as

$$f_i(x_i) = \frac{1}{2}(|x_i + 1| - |x_i - 1|), \quad i = 1, 2,$$

while the external input vector is $J = [0, 0]^T$. Accordingly, $\Pi = \text{diag}\{1, 1\}$. Figure 1 plots the leader's trajectory starting from the initial state $x_0 = [0.1, 0.2]^T$.

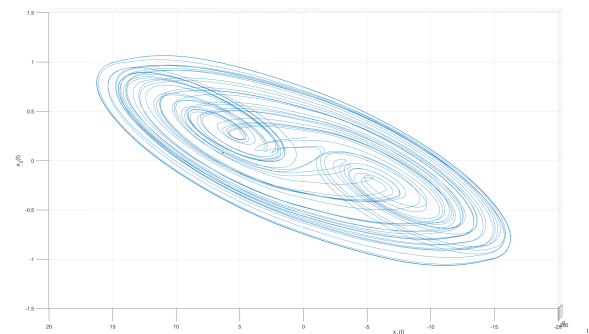


Figure 1. The figure depicts the trajectory of the leader system.

A network of sixteen cellular neural subsystems is considered, with topology in Figure 2 defining the coupling matrix L . Let $\Gamma = \text{diag}\{1, 1\}$ and $\tau = 1$. We set $\beta_1 = 25$, $\beta_2 = 37$, $c = 0.2$, and $N_0 = 2$. Solving (3.6)–(3.10) via the MATLAB LMI Toolbox (Theorem 3.1) yields

$$P = \begin{pmatrix} 0.0049 & 0.0002 \\ 0.0002 & 0.0141 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 0.0539 & 0.0018 \\ 0.0018 & 0.2719 \end{pmatrix},$$

$$\Sigma_2 = \begin{pmatrix} 0.1782 & 0.0004 \\ 0.0004 & 0.1776 \end{pmatrix}, \quad \frac{\lambda_{\min}(P)}{\lambda_{\max}(P)} = 0.3472.$$

□ With $\bar{\eta} = 63$ and $\mu = -1.1$, the pinning ratio requires $\eta_k \geq 0.25$. In simulation, we choose $T_a = 0.020$, $\eta_k = 0.625$, and $r = 0.2$. The impulse sequence adopted in the simulation, generated with $N_0 = 2$ and $T_a = 0.02$, is shown in Figure 3.

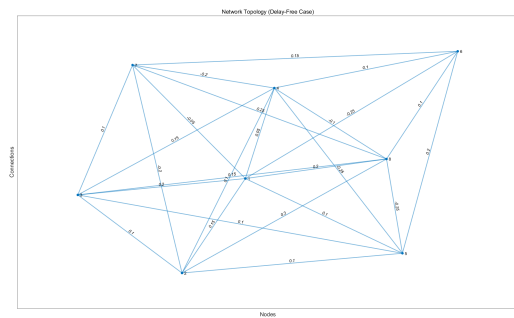


Figure 2. The network topologies: the case without delays.

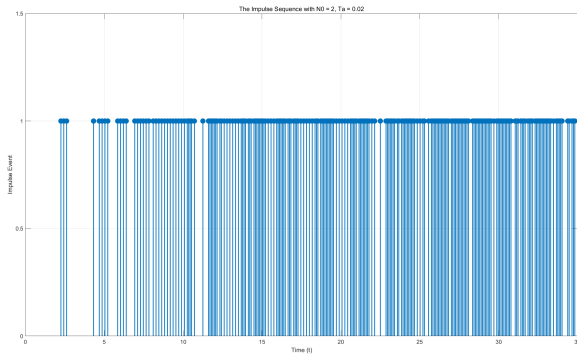


Figure 3. An impulse sequence is taken with parameters $N_0 = 2$ and $T_a = 0.02$.

At each impulse moment t_k , the nodes are arranged in descending order of their tracking errors: $\|e_{i_1}(t_k)\| \geq \|e_{i_2}(t_k)\| \geq \dots \geq \|e_{i_N}(t_k)\|$, and the largest $\lceil N\mu_k \rceil$ are selected for pinning, where μ_k is a fraction, not the impulse gain μ . With $\eta_k = 0.626$, this corresponds to pinning ten nodes. Figures 4 and 5 show the time evolution of the leader and sixteen follower states, confirming rapid synchronization.

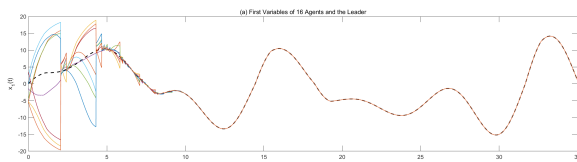


Figure 4. Simulation results with 10 pinned nodes, showing initial states of eight agents and the leader.

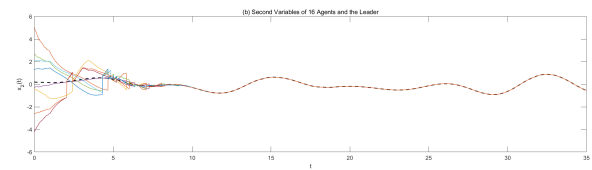
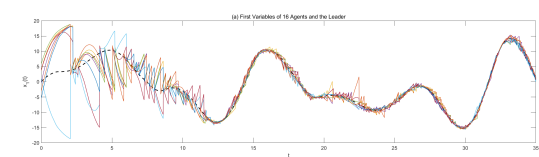


Figure 5. Synchronization results with 10 pinned nodes in the second layer.

As shown in Figure 6(a),(b), pinning only five nodes fails to synchronize with the leader, indicating that at least six nodes are required. This agrees with the theoretical analysis and supports the effectiveness of the proposed approach.

The simulations confirm successful synchronization under DoS attacks, where the DoS attack sequence with an attack frequency of 20% is depicted in Figure 7; the parameters T_a , μ , and η_k play key roles. Smaller T_a speeds up synchronization, but too frequent updates can destabilize the system. Increasing μ accelerates convergence, but higher values may cause instability. A larger η_k improves synchronization by reducing error faster, though excessive values can lead to instability. These results match theoretical predictions and highlight the need for careful parameter tuning.



(a) Synchronization results with 4 pinned nodes in the first layer.



(b) Synchronization results with 4 pinned nodes in the second layer.

Figure 6. The simulation outcomes under the scenario of four pinned nodes.

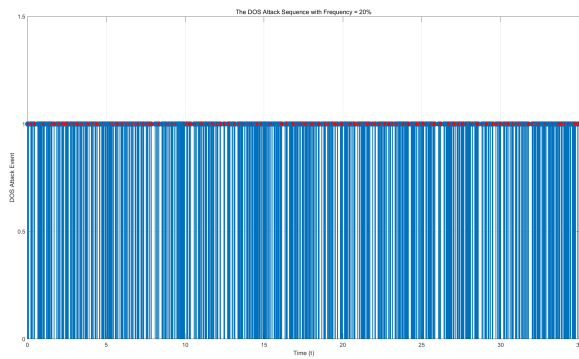


Figure 7. The DOS Attack Sequence with Frequency = 20%.

5. Conclusions

This work addressed pinning synchronization of coupled networks under DoS disruptions, proposing a pinning impulsive control (PIC) scheme with the pinning ratio. Using the average impulsive-interval framework and Lyapunov analysis, we derived conditions for exponential stability in impulsive time-delay systems. A novel comparison lemma for delayed impulsive systems was introduced to support the proofs. Numerical simulations confirmed reliable synchronization under DoS interruptions, highlighting the method's practical value. Future work will extend the framework to more complex topologies and incorporate additional delay types to enhance robustness and applicability.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest in this paper.

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