

Research article

Stabilization of 2-D discrete-time switched positive linear systems via weak switched linear copositive Lyapunov function

Mengying Sun¹, Dadong Tian^{2,*} and Yongping Zhang^{1,*}

¹ School of Mathematics and Statistics, Shandong University, Weihai 264209, China

² College of Information Science and Engineering, Shandong Agricultural University, Taian 271018, China

* **Correspondence:** Email: dadongtian0520@163.com, ypzhang@sdu.edu.cn.

Abstract: In this paper, we study the stabilization problem of 2-D discrete-time switched positive linear systems. First, by using the common linear copositive Lyapunov function, it is showed that the 2-D switched positive linear system is asymptotically stable under arbitrary switching. Then, a weak common linear copositive Lyapunov function is introduced. By designing new time-dependent switching signals, the asymptotic stability of the 2-D system with disturbance is demonstrated. Finally, a function with a wider range of adaptation - weak switched linear copositive Lyapunov function is considered, and the asymptotic stability of the corresponding system under a given switching signal is proved. A numerical example demonstrates the effectiveness of the obtained results.

Keywords: 2-D discrete-time switched positive linear systems; stabilization; switching signal; weak switched linear copositive Lyapunov function

1. Introduction

Two-dimensional (2-D) discrete-time systems are dynamical systems whose information propagation occurs in two independent directions [1]. Such a system is used to describe the dynamics of various practical and physical systems in various fields, such as integrated circuit design, image processing, and communication [2, 3]. These systems can be described in many forms, such as in the Rosser model and the Fornasini-Marchesini model [4, 5]. These models are based on specific mathematical structures and constraints to accurately capture the behavior of systems on a two-dimensional plane. If the state and output of the 2-D system remain nonnegative at all times and at any spatial position, the system is called a 2-D positive system [3, 6]. This non-negative requirement has important physical significance and practical needs in many practical applications. For example, in the modeling of heat transfer processes, temperature cannot be negative; in the analysis

of biological systems, the population number must also be non-negative [7–9]. Despite its long history, the stability and performance analysis of 2-D systems have remained one of the most active topics in the control field over the past few decades [10, 11].

Switched 2-D systems are a special kind of two-dimensional dynamical system that consist of multiple 2-D subsystems and a switching rule [12, 13]. In a switched 2-D system, the system switches between 2-D subsystems at different times according to the switching rules. Each 2-D subsystem has its own characteristics and dynamic behaviors. 2-D switched linear systems, as the name implies, have the characteristics of both switched systems and linear systems. The stability problem of 2-D linear switched systems is mainly divided into two aspects: the stability problem of switched systems under arbitrary switching signals and the stability problem of switched systems under specific switching signals. The stability analysis of two-dimensional discrete systems has

attracted considerable interest, and some significant results were obtained in [14–16]. It is well known that even if each subsystem in the switched 2-D system is stable, the whole system may still be unstable under arbitrary switching conditions. In [17], a sufficient condition for the asymptotic stability of 2-D switched systems is given. The preliminary results are based on the commonly used Lyapunov function, and the linear matrix inequality (LMI) conditions for stability are given. In order to reduce the conservatism of the initial method, a multi-Lyapunov function was proposed, which also gives LMI conditions. [1] studied the sufficient conditions for the asymptotic stability of 2-D discrete-time switched systems represented by the Rosser model of state feedback control, using the common and multiple Lyapunov function method, and the results are given in the form of LMI.

Stability under constrained rules. In general, although the 2-D linear system does not satisfy the stability condition under any switching rule, we can design appropriate switching rules, that is, the switching system can still be stable under certain constraints. In this case, the common Lyapunov function, and multi-Lyapunov function method are too conservative. The concept of average dwell time switching is proposed in [18]. Under the condition that each subsystem is stable, as long as the switching is ‘average slow’, that is, if the dwell time of the switching rule is not less than some constant, the system is stable. A switching law was designed in [19], which requires that the average dwell time is large enough, and the total activation time ratio between the Hurwitz stable subsystem and the unstable subsystem is not less than a specified constant, so that the whole switching system is exponentially stable. [20, 21] applied the average dwell time to the study of exponential stability and stabilization of 2-D switched systems. [22] continued by considering a more general 2-D switched system with time delay.

For the stability problem of 2-D switched linear systems, an important problem is the asymptotic stability of the system under arbitrary switching [23]. This problem usually needs to find conditions to ensure the existence of the Lyapunov function. However, many obvious examples show that the systems involved cannot meet this requirement. This motivates us to study the Lyapunov function under fewer

constraints by designing appropriate switching signals to make the system asymptotically stable.

Based on the above analysis, in this paper, we mainly study the stabilization problem of 2-D switched linear systems. In particular, we have made the following contributions:

First, by using the common linear copositive Lyapunov function method, we give a sufficient condition for the asymptotic stability of the 2-D system with disturbance under given conditions.

Second, the weak common linear copositive Lyapunov function method is introduced. By designing a new time-dependent switching signal, the asymptotic stability of the 2-D system under a given switching is proved. The main results are also extended to general switched linear time-varying systems with disturbance.

Finally, we consider the more applicable weak switched linear copositive Lyapunov function method, and also prove the asymptotic stability of the 2-D system under a given switching. Therefore, Theorem 3.2 can be regarded as a special case of Theorem 3.4. The result of Theorem 3.4 can also be extended to general 2-D switched linear systems, which is not necessarily positive.

The rest of this article is as follows. Some necessary notations, definitions, and problem formulations are presented in Section 2. Our main results and the proofs are provided in Section 3. Section 4 gives an example to justify the efficiency of the obtained results, and the conclusions are stated in Section 5.

2. Preliminaries

$\mathbb{R}$ and $\mathbb{N}$ represent the sets of real and natural numbers, respectively. $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. $\mathbb{R}^n$ is the set of n -dimensional real vectors and $\mathbb{R}_+^n := \{\mathbf{x} \in \mathbb{R}^n, x_j \geq 0, 1 \leq j \leq n\}$. For $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, we write $\mathbf{x} \geq \mathbf{y}$ ($\mathbf{x} > \mathbf{y}$, $\mathbf{x} < \mathbf{y}$) if $x_j \geq y_j$ ($x_j > y_j$, $x_j < y_j$) for $1 \leq j \leq n$. $\mathbb{R}_{n \times n}$ represents the space of $n \times n$ -dimensional real matrices. For a matrix or vector A , we write $A > 0$ (≥ 0 , ≤ 0 , < 0) if all entries of A are positive (nonnegative, nonpositive, or negative). I_n denotes the n -dimensional identity matrix.

Consider the following 2-D discrete-time switched

positive linear system:

$$\begin{aligned} \begin{bmatrix} \mathbf{x}^h(k+1, l) \\ \mathbf{x}^v(k, l+1) \end{bmatrix} &= A_{\sigma(k, l)} \begin{bmatrix} \mathbf{x}^h(k, l) \\ \mathbf{x}^v(k, l) \end{bmatrix} + \omega(k, l) \\ &= \begin{bmatrix} A_{\sigma(k, l)}^{11} & 0 \\ 0 & A_{\sigma(k, l)}^{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}^h(k, l) \\ \mathbf{x}^v(k, l) \end{bmatrix} + \begin{bmatrix} \omega^h(k, l) \\ \omega^v(k, l) \end{bmatrix} \end{aligned} \quad (2.1)$$

where $\mathbf{x}^h(k, l)$ is the horizontal state in $\mathbb{R}^{n_1}$, and $\mathbf{x}^v(k, l)$ is the vertical state in $\mathbb{R}^{n_2}$. The piecewise continuous function $\sigma(k, l) : \mathbb{N}_0 \times \mathbb{N}_0 \rightarrow \{1, 2, 3, \dots, m\}$ is the switching rule, where m is the number of subsystems (modes), k and l are two positive integers, and $n = n_1 + n_2$. $A_{\sigma(k, l)} = (a_{\sigma(k, l)}^{ij})$ is an $n \times n$ diagonal matrix for each $\sigma(k, l) \in \{1, 2, 3, \dots, m\}$, and $\omega^h(k, l) : \mathbb{N}_0 \times \mathbb{N}_0 \rightarrow \mathbb{R}^{n_1}$ and $\omega^v(k, l) : \mathbb{N}_0 \times \mathbb{N}_0 \rightarrow \mathbb{R}^{n_2}$ are the disturbances.

When $\omega(k, l) = \mathbf{0}$, system (2.1) reduces to

$$\begin{bmatrix} \mathbf{x}^h(k+1, l) \\ \mathbf{x}^v(k, l+1) \end{bmatrix} = A_{\sigma(k, l)} \begin{bmatrix} \mathbf{x}^h(k, l) \\ \mathbf{x}^v(k, l) \end{bmatrix}. \quad (2.2)$$

System (2.1) or (2.2) is said to be positive if, for nonnegative initial conditions and arbitrary switching, the corresponding trajectory $\mathbf{x}(k, l) \geq 0$, $k, l = 0, 1, 2, \dots$. It is obvious that system (2.2) is positive if and only if A_i is a nonnegative matrix for each $i \in \{1, 2, 3, \dots, m\}$. System (2.1) is positive if A_i is a nonnegative matrix for each $i \in \{1, 2, 3, \dots, m\}$ and $\omega(k, l) \geq 0$.

For the stability of the system, a common method used in this note to analyze asymptotic stability under arbitrary switching is the common linear copositive Lyapunov function (CLCLF). That is to say, $V(\mathbf{x}(k, l)) = \xi^T \mathbf{x}(k, l)$ is said to be a CLCLF of system (2.1) or (2.2) if there exists an n -dimensional vector $\xi > 0$ such that

$$\xi^T (I_n - A_{\sigma(k, l)}) > 0. \quad (2.3)$$

In many cases, system (2.1) or (2.2) may not have a CLCLF, i.e., condition (2.3) is only a sufficient condition for asymptotic stability under arbitrary switching. Thus, we introduce a class of weak common linear copositive Lyapunov functions (WCLCLF) which relax the restriction on the existence of CLCLF. $V(\mathbf{x}(k, l)) = \xi^T \mathbf{x}(k, l)$ is said to be a weak common linear copositive Lyapunov function, if there exists an n -dimensional vector $\xi > 0$ satisfying

$$\xi^T (I_n - A_{\sigma(k, l)}) \geq 0. \quad (2.4)$$

For asymptotic stability of system (2.1) or (2.2) under condition (2.4), we design an appropriate switching rule such that system (2.1) or (2.2) generated by it is asymptotically stable. Then, we research weak switched linear copositive Lyapunov functions (WSLCLF) which relax the restriction on the existence of WCLCLF.

3. Main results

Theorem 3.1. *If there exists an n -dimensional vector $\xi > 0$ such that*

$$\xi^T (I_n - A_{\sigma(k, l)}) > 0, \quad \sigma(k, l) \in \langle m \rangle \quad (3.1)$$

and

$$\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \xi^T \omega(k, l) < \infty \quad (3.2)$$

hold, then SPLS (2.1) is asymptotically stable under arbitrary switching.

Proof. Let

$$\begin{aligned} V(\mathbf{x}(k, l)) &= V^h(\mathbf{x}(k, l)) + V^v(\mathbf{x}(k, l)) \\ &= \xi_h^T \mathbf{x}^h(k, l) + \xi_v^T \mathbf{x}^v(k, l) \\ &= \xi^T \mathbf{x}(k, l), \\ V^+(\mathbf{x}(k, l)) &= V^h(\mathbf{x}(k+1, l)) + V^v(\mathbf{x}(k, l+1)) \\ &= \xi_h^T \mathbf{x}^h(k+1, l) + \xi_v^T \mathbf{x}^v(k, l+1). \end{aligned}$$

So, we get

$$\begin{aligned} &V^h(\mathbf{x}(k+1, l)) - V^h(\mathbf{x}(k, l)) \\ &= \xi_h^T (\mathbf{x}^h(k+1, l) - \mathbf{x}^h(k, l)) \\ &= \xi_h^T (A_{\sigma(k, l)}^{11} \mathbf{x}^h(k, l) + \omega^h(k, l) - \mathbf{x}^h(k, l)) \\ &= \xi_h^T (A_{\sigma(k, l)}^{11} - I_{11}) \mathbf{x}^h(k, l) + \xi_h^T \omega^h(k, l), \quad k \geq 0. \end{aligned}$$

Note that

$$\begin{aligned} &V^h(\mathbf{x}(k, l)) \\ &= V^h(\mathbf{x}(0, l)) + \sum_{s=0}^{k-1} \xi_h^T (A_{\sigma(s, l)}^{11} - I_{11}) \mathbf{x}^h(s, l) + \sum_{s=0}^{k-1} \xi_h^T \omega^h(s, l), \end{aligned}$$

where $k \geq 1$. From (3.1) and (3.2), we obtain

$$V^h(\mathbf{x}(k, l)) \leq V^h(\mathbf{x}(0, l)) + \sum_{s=0}^{k-1} \xi_h^T \omega^h(s, l),$$

and

$$0 < \sum_{k=0}^{\infty} \xi_h^T (I_{11} - A_{\sigma(k,l)}^{11}) x^h(k, l) < \infty. \quad (3.3)$$

On the other hand, we consider

$$\begin{aligned} & V^v(x(k, l+1)) - V^v(x(k, l)) \\ &= \xi_v^T(x^v(k, l+1) - x^v(k, l)) \\ &= \xi_v^T(A_{\sigma(k,l)}^{22} x^v(k, l) + \omega^v(k, l) - x^v(k, l)) \\ &= \xi_v^T(A_{\sigma(k,l)}^{22} - I_{22}) x^v(k, l) + \xi_v^T \omega^v(k, l), \quad l \geq 0. \end{aligned}$$

This indicates that

$$\begin{aligned} & V^v(x(k, l)) \\ &= V^v(x(k, 0)) + \sum_{s=0}^{l-1} \xi_v^T (A_{\sigma(k,s)}^{22} - I_{22}) x^v(k, s) + \sum_{s=0}^{l-1} \xi_v^T \omega^v(k, s), \end{aligned}$$

where $l \geq 1$. From (3.1) and (3.2), we obtain

$$V^v(x(k, l)) \leq V^v(x(k, 0)) + \sum_{s=0}^{l-1} \xi_v^T \omega^v(k, s),$$

and

$$0 < \sum_{l=0}^{\infty} \xi_v^T (I_{22} - A_{\sigma(k,l)}^{22}) x^v(k, l) < \infty. \quad (3.4)$$

Set $\psi_1(k, l) = \xi_h^T (I_{11} - A_{\sigma(k,l)}^{11}) x^h(k, l)$ and $\psi_2(k, l) = \xi_v^T (I_{22} - A_{\sigma(k,l)}^{22}) x^v(k, l)$. It follows from (3.3) and (3.4) that $\lim_{k \rightarrow \infty} \psi_1(k, l) = 0$ and $\lim_{l \rightarrow \infty} \psi_2(k, l) = 0$. Therefore, we can derive $\lim_{k \rightarrow \infty} x^h(k, l) = 0$ and $\lim_{l \rightarrow \infty} x^v(k, l) = 0$. Eventually, we prove $\lim_{k, l \rightarrow \infty} x(k, l) = 0$. $\square$

Remark 3.1. Consider a 2-D switched positive linear system described by (2.1) with $m=2, n=4$,

$$A_1 = \begin{pmatrix} 0.2 & 0.1 & 0 & 0 \\ 0.1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0.4 & 0.6 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0.6 & 0.3 & 0 & 0 \\ 0.2 & 0.85 & 0 & 0 \\ 0 & 0 & 0.5 & 0.3 \\ 0 & 0 & 0.1 & 0.3 \end{pmatrix},$$

and $\xi^T = (1, 2, 1, 0.5)$. We get

$$I_4 - A_1 = \begin{pmatrix} 0.8 & -0.1 & 0 & 0 \\ -0.1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.2 & -0.2 \\ 0 & 0 & -0.4 & 0.4 \end{pmatrix},$$

$$I_4 - A_2 = \begin{pmatrix} 0.4 & -0.3 & 0 & 0 \\ -0.2 & 0.15 & 0 & 0 \\ 0 & 0 & 0.5 & -0.3 \\ 0 & 0 & -0.1 & 0.7 \end{pmatrix},$$

$\xi^T (I_4 - A_1) = (0.6, 0.9, 0, 0) \geq 0$, and $\xi^T (I_4 - A_2) = (0, 0, 0.45, 0.05) \geq 0$. However, this contradicts condition (3.1) in **Theorem 3.1**. The system is not asymptotically stable under arbitrary switching.

Next, we consider the stabilization of the switched positive linear system (2.1). System (2.1) is said to be stabilizable if there exists a switching rule under which each solution of system (2.1) asymptotically converges to zero.

Set

$$\xi_h^T (I_{11} - A_{\sigma(k,l)}^{11}) = \theta_{\sigma(k,l)}^1 = (\theta_{\sigma(k,l)}^{11}, \theta_{\sigma(k,l)}^{12}, \dots, \theta_{\sigma(k,l)}^{1n_1}),$$

$$\xi_v^T (I_{22} - A_{\sigma(k,l)}^{22}) = \theta_{\sigma(k,l)}^2 = (\theta_{\sigma(k,l)}^{21}, \theta_{\sigma(k,l)}^{22}, \dots, \theta_{\sigma(k,l)}^{2n_2}),$$

where I_{11} and I_{22} are identity matrices. Define the index sets $\mathcal{J}_{\sigma(k,l)}^{11} = \{i \in \langle n_1 \rangle : \theta_{\sigma(k,l)}^{1i} > 0\}$ and $\mathcal{J}_{\sigma(k,l)}^{22} = \{i \in \langle n_2 \rangle : \theta_{\sigma(k,l)}^{2i} > 0\}$ for each $\sigma(k, l) \in \langle m \rangle$, which are subsets of the sets $\langle n_1 \rangle$ and $\langle n_2 \rangle$. Characterize the following time-dependent switching rule $\sigma(k, l)$ satisfying

(S1) There exists a sequence of time instants k_{1j}, k_{2j}, l_{1j} and l_{2j} such that

$$\bigcup_{(k,l) \in \{(k_{1j}, l_{1j}), (k_{1j+1}, l_{1j}), \dots, (k_{2j}, l_{2j})\}} \mathcal{J}_{\sigma(k,l)}^{11} = \langle n_1 \rangle, \quad j = 1, 2, \dots$$

$$\bigcup_{(k,l) \in \{(k_{1j}, l_{1j}), (k_{1j+1}, l_{1j}), \dots, (k_{2j}, l_{2j})\}} \mathcal{J}_{\sigma(k,l)}^{22} = \langle n_2 \rangle, \quad j = 1, 2, \dots$$

where $\{1_j\}_{j=1}^{\infty}$ and $\{2_j\}_{j=1}^{\infty}$ are strictly increasing sequences of positive integers, $1_j \leq 2_j < 1_{j+1} \leq 2_{j+1}$, $0 \leq k_{1j} \leq k_{2j} < k_{1j+1} \leq k_{2j+1}$, $\lim_{j \rightarrow \infty} k_{1j} = \infty$, $k_{2j} - k_{1j} \leq N_1$, and $N_1 > 0$ is an integer. $0 \leq l_{1j} \leq l_{2j} < l_{1j+1} \leq l_{2j+1}$, $\lim_{j \rightarrow \infty} l_{1j} = \infty$, $l_{2j} - l_{1j} \leq N_2$, and $N_2 > 0$ is an integer.

Remark 3.2. In the time-dependent switching rule designed above, $\sigma(k, l)$ is only constrained on part of the time instants k_{1j}, k_{2j}, l_{1j} , and l_{2j} (which are not necessarily switching instants). Under condition (3.1), switching rule (S1) is equivalent to $\sum_{i=1}^{2j} \xi_h^T (I_{11} - A_{\sigma(k_i, l_i)}^{11}) > 0$ and $\sum_{i=1}^{2j} \xi_v^T (I_{22} - A_{\sigma(k_i, l_i)}^{22}) > 0$.

Assumption 3.1. All the diagonal entries of the subsystem matrix A_i are positive for $i \in \langle m \rangle$.

Remark 3.3. Assumption 3.1 seems to have some limitations. It is difficult to estimate the i th component of the state when some diagonal entries of the subsystem matrices are zero. In this case, the stability anyalysis of the discrete time SPLS (2.1) becomes complex. We will continue to research this in the future.

Theorem 3.2. Suppose that Assumption 3.1 is satisfied. If there exists an n -dimensional vector $\xi \geq 0$ such that

$$\xi^T (I_n - A_{\sigma(k,l)}) \geq 0, \quad \sigma(k,l) \in \langle m \rangle \quad (3.5)$$

and

$$\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \xi^T \omega(k,l) < \infty \quad (3.6)$$

hold, then SPLS (2.1) is stabilizable under the switching signal satisfying (S1).

Proof. Similar to the proof of Theorem 3.1, we can get

$$V^h(x(k,l)) \leq V^h(x(0,l)) + \sum_{s=0}^{k-1} \xi_h^T \omega^h(s,l), \quad (3.7)$$

$$0 < \sum_{k=0}^{\infty} \xi_h^T (I_{11} - A_{\sigma(k,l)}^{11}) x^h(k,l) < \infty, \quad (3.8)$$

$$V^v(x(k,l)) \leq V^v(x(k,0)) + \sum_{s=0}^{l-1} \xi_v^T \omega^v(k,s), \quad (3.9)$$

and

$$0 < \sum_{l=0}^{\infty} \xi_v^T (I_{22} - A_{\sigma(k,l)}^{22}) x^v(k,l) < \infty. \quad (3.10)$$

Set

$$\psi_1(k,l) = \xi_h^T (I_{11} - A_{\sigma(k,l)}^{11}) x^h(k,l),$$

$$\psi_2(k,l) = \xi_v^T (I_{22} - A_{\sigma(k,l)}^{22}) x^v(k,l).$$

According to (3.8) and (3.10), we have

$$\lim_{k \rightarrow \infty} \psi_1(k,l) = 0, \quad \lim_{l \rightarrow \infty} \psi_2(k,l) = 0.$$

Therefore, for any constant $\epsilon > 0$, there exists a common integer $N_\epsilon > 0$ such that $\psi_1(k,l) < \epsilon$, $\psi_2(k,l) < \epsilon$ for $k, l \geq N_\epsilon$, and

$$\sum_{p=N_1}^{N_2} \sum_{s=N_1}^{N_2} \xi^T \omega(p,s) < \epsilon, \quad \forall N_2 \geq N_1 \geq N_\epsilon. \quad (3.11)$$

Next, we divide the proof into the following two parts to show that $\lim_{k,l \rightarrow \infty} x(k,l) = 0$.

Case 1:

If $k_{1_j} = k_{2_j}$, $l_{1_j} = l_{2_j}$, there are sufficiently large k_{1_j} , l_{1_j} such that k_{1_j} , $l_{1_j} \geq N_\epsilon$. From the switching rules (S1), we can get

$$\theta_{\sigma(k_{1_j},l)}^{1i} > 0, i \in \langle n_1 \rangle, \quad \theta_{\sigma(k,l_{1_j})}^{2i} > 0, i \in \langle n_2 \rangle.$$

Hence,

$$0 \leq \theta_{\sigma(k_{1_j},l)}^{1i} x_i^h(k_{1_j},l) \leq \psi_1(k_{1_j},l) < \epsilon, i \in \langle n_1 \rangle,$$

and

$$0 \leq \theta_{\sigma(k,l_{1_j})}^{2i} x_i^v(k,l_{1_j}) \leq \psi_2(k,l_{1_j}) < \epsilon, i \in \langle n_2 \rangle.$$

It shows that

$$\begin{aligned} 0 \leq x_i^h(k_{1_j},l) &< \frac{\epsilon}{\theta_{\sigma(k_{1_j},l)}^{1i}} < \frac{\epsilon}{\theta}, \\ 0 \leq x_i^v(k,l_{1_j}) &< \frac{\epsilon}{\theta_{\sigma(k,l_{1_j})}^{2i}} < \frac{\epsilon}{\theta}, \quad i \in \langle n \rangle \end{aligned} \quad (3.12)$$

where $\theta = \min\{\min\{\theta_p^{1i} > 0 : p \in \langle m \rangle, i \in \langle n_1 \rangle\}, \min\{\theta_p^{2i} > 0 : p \in \langle m \rangle, i \in \langle n_2 \rangle\}\}$.

Add inequality (3.7) to inequality (3.9). Then, combining (3.11) and (3.12), we obtain

$$\begin{aligned} &V^h(x(k,l)) + V^v(x(k,l)) \\ &\leq V^h(x(k_{1_j},l)) + V^v(x(k,l_{1_j})) + \sum_{p=k_{1_j}}^k \sum_{s=l_{1_j}}^l \xi^T \omega(p,s) \\ &= \xi_h^T x^h(k_{1_j},l) + \xi_v^T x^v(k,l_{1_j}) + \sum_{p=k_{1_j}}^k \sum_{s=l_{1_j}}^l \xi^T \omega(p,s) \\ &< \sum_{i=1}^n \frac{\xi_i \epsilon}{\theta} + \epsilon \quad (k \geq k_{1_j} + 1, l \geq l_{1_j} + 1), \end{aligned}$$

that is, $\lim_{k,l \rightarrow \infty} V(x(k,l)) = 0$. So, we can immediately get that $\lim_{k,l \rightarrow \infty} x(k,l) = 0$.

Case 2:

Assume that $k_{1_j} \neq k_{2_j}$ and $l_{1_j} \neq l_{2_j}$. Because $\bigcup_{(k,l) \in \{(k_{1_j},l), (k_{1_j+1},l), \dots, (k_{2_j},l)\}} \mathcal{J}_{\sigma(k,l)}^{11} = \langle n_1 \rangle$, there exists at least one $(k^*,l) \in \{(k_{1_j},l), (k_{1_j+1},l), \dots, (k_{2_j},l)\}$ for each $i \in \langle n_1 \rangle$, such that $i \in \mathcal{J}_{\sigma(k^*,l)}^{11}$. This implies that $\theta_{\sigma(k^*,l)}^{1i} > 0$.

Thus,

$$\theta_{\sigma(k^*,l)}^{1i} x_i(k^*,l) \leq \psi_1(k^*,l) < \epsilon, \quad i \in \langle n_1 \rangle. \quad (3.13)$$

On the other hand, since $x^h(k, l) \geq 0$ and $\omega(k, l) \geq 0$, we have that

$$x_i^h(k+1, l) \geq \gamma_i x_i^h(k, l), \quad i \in \langle n_1 \rangle, \quad (3.14)$$

where $\gamma_i = \min\{a_{ii}^{(p)} : p \in \langle m \rangle\}$. Since $A_{\sigma(k, l)} \geq 0$, from (3.5), $\theta_{\sigma(k^*, l)}^{li} > 0$, and Assumption 3.1, we get $0 < a_{\sigma(k^*, l)}^{ii} < 1$. Therefore, $0 < \gamma_i < 1$. From (3.14), we obtain

$$x_i^h(k^*, l) \geq \gamma_i^{k^* - k_{1j}} x_i^h(k_{1j}, l), \quad i \in \langle n_1 \rangle.$$

This together with (3.13) yields

$$x_i^h(k_{1j}, l) \leq \gamma_i^{-N_1} \frac{\epsilon}{\theta_{\sigma(k^*, l)}^{li}} < \frac{(\gamma')^{-N_1}}{\theta} \epsilon, \quad i \in \langle n_1 \rangle \quad (3.15)$$

where $\gamma' = \min\{\gamma_i : i \in \langle n_1 \rangle\}$. Similar to the above process, we have

$$x_i^v(k, l_{1j}) \leq \gamma_i^{-N_2} \frac{\epsilon}{\theta_{\sigma(k, l^*)}^{li}} < \frac{(\gamma'')^{-N_2}}{\theta} \epsilon, \quad i \in \langle n_2 \rangle \quad (3.16)$$

where $\gamma'' = \min\{\gamma_i : i \in \langle n_2 \rangle\}$.

Adding inequality (3.7) to inequality (3.9) and combining (3.15) and (3.16), we obtain

$$\begin{aligned} & V^h(x(k, l)) + V^v(x(k, l)) \\ & \leq V^h(x(k_{1j}, l)) + V^v(x(k, l_{1j})) + \sum_{p=k_{1j}}^k \sum_{s=l_{1j}}^l \xi^T \omega(p, s) \\ & = \xi_h^T x^h(k_{1j}, l) + \xi_v^T x^v(k, l_{1j}) + \sum_{p=k_{1j}}^k \sum_{s=l_{1j}}^l \xi^T \omega(p, s) \\ & < \left(\sum_{i=1}^{n_1} \xi_i \frac{(\gamma')^{-N_1}}{\theta} + \sum_{i=1}^{n_2} \xi_i \frac{(\gamma'')^{-N_2}}{\theta} \right) \epsilon + \epsilon \\ & = \left(1 + \frac{\gamma^{-N}}{\theta} \sum_{i=1}^n \xi_i \right) \epsilon, \quad k \geq k_{1j} + 1, l \geq l_{1j} + 1, \end{aligned}$$

where $\gamma = \min\{\gamma', \gamma''\}$, and $N = \max\{N_1, N_2\}$; that is, $\lim_{k, l \rightarrow \infty} V(x(k, l)) = 0$. So, we immediately get that $\lim_{k, l \rightarrow \infty} x(k, l) = 0$. Theorem 3.2 is complete. $\square$

Note that

$$\begin{bmatrix} x^h(k+1, l) \\ x^v(k, l+1) \end{bmatrix} = |A_{\sigma(k, l)}| \begin{bmatrix} x^h(k, l) \\ x^v(k, l) \end{bmatrix} + |\omega(k, l)|, \quad k, l = 0, 1, 2, \dots$$

where $|A_{\sigma(k, l)}| = (|a_{ij}^{\sigma(k, l)}|)$ is a matrix, and $\sigma(k, l) \in \langle m \rangle$. Therefore, each solution $x(k, l)$ of system satisfies $|x(k, l)| \leq$

$y(k, l)$, for $k, l = 0, 1, 2, \dots$, where $y(k, l)$ is the solution of the following SPLS:

$$\begin{bmatrix} y^h(k+1, l) \\ y^v(k, l+1) \end{bmatrix} = |A_{\sigma(k, l)}| \begin{bmatrix} y^h(k, l) \\ y^v(k, l) \end{bmatrix} + |\omega(k, l)|, \quad k, l = 0, 1, 2, \dots$$

As a result, Theorem 3.2 can be easily extended to the general discrete-time switched linear system (not necessarily positive).

The above theorem addresses the case of the existence of a common vector ξ . However, sometimes finding a common vector is very difficult or even impossible. Therefore, we consider the case of finding a set of vectors $\xi_j, j \in \langle m \rangle$.

Consider the following 2-D discrete-time switched positive linear system:

$$\begin{aligned} \begin{bmatrix} x^h(k+1, l) \\ x^v(k, l+1) \end{bmatrix} &= A_{\sigma(k, l)} \begin{bmatrix} x^h(k, l) \\ x^v(k, l) \end{bmatrix} \\ &= \begin{bmatrix} A_{\sigma(k, l)}^{11} & 0 \\ 0 & A_{\sigma(k, l)}^{22} \end{bmatrix} \begin{bmatrix} x^h(k, l) \\ x^v(k, l) \end{bmatrix}. \end{aligned} \quad (3.17)$$

Take the Lyapunov function:

$$V(x(k, l)) = \xi_{\sigma(k, l)}^T x(k, l).$$

Let

$$\begin{aligned} \xi_j^{T_h} - \xi_i^{T_h} A_j^{11} &= T_{ij}^1 = (\gamma_{1ij}^1, \dots, \gamma_{n_1 ij}^1), \\ G^1 &= \{(i, j) : \xi_i^{T_h} A_j^{11} < \xi_j^{T_h}\}. \end{aligned}$$

Let

$$\begin{aligned} \xi_j^{T_v} - \xi_i^{T_v} A_j^{22} &= T_{ij}^2 = (\gamma_{1ij}^2, \dots, \gamma_{n_2 ij}^2), \\ G^2 &= \{(i, j) : \xi_i^{T_v} A_j^{22} < \xi_j^{T_v}\}, \end{aligned}$$

$$I_{ij}^1 = \{s : \gamma_{sij}^1 > 0, s \in \langle n_1 \rangle, i, j \in \langle m \rangle\},$$

$$I_{ij}^2 = \{s : \gamma_{sij}^2 > 0, s \in \langle n_2 \rangle, i, j \in \langle m \rangle\}.$$

Assumption 3.2. For any given positive integer N , there exist $k^*, l^* > N$, such that $\langle \sigma(k^* + 1, l), \sigma(k^*, l) \rangle \in G^1$ and $\langle \sigma(k, l^* + 1), \sigma(k, l^*) \rangle \in G^2$.

Theorem 3.3. Assuming there exists a set of vectors ξ_j , such that $\xi_i^T A_j \leq \xi_j^T, i, j \in \{1, 2, \dots, m\}$, then system (3.17) is asymptotically stable under the switching signal that satisfies Assumption 3.2.

Proof. It is straightforward to obtain

$$\begin{aligned}
 & V^h(\mathbf{x}(k+1, l)) - V^h(\mathbf{x}(k, l)) \\
 &= \xi_{\sigma(k+1, l)}^{T_h} \mathbf{x}^h(k+1, l) - \xi_{\sigma(k, l)}^{T_h} \mathbf{x}^h(k, l) \\
 &= \xi_{\sigma(k+1, l)}^{T_h} A_{\sigma(k, l)}^{11} \mathbf{x}^h(k, l) - \xi_{\sigma(k, l)}^{T_h} \mathbf{x}^h(k, l) \\
 &= (\xi_{\sigma(k+1, l)}^{T_h} A_{\sigma(k, l)}^{11} - \xi_{\sigma(k, l)}^{T_h}) \mathbf{x}^h(k, l) \\
 &\leq 0.
 \end{aligned}$$

According to the condition of Theorem 3.3, it can be inferred that $V^h(\mathbf{x}(k, l))$ is monotonically decreasing with respect to k if l is fixed. Likewise, $V^v(\mathbf{x}(k, l))$ is monotonically decreasing with respect to l if k is fixed. In the following, we will prove that $\lim_{k \rightarrow \infty} V^h(\mathbf{x}(k, l)) = 0$, $\lim_{l \rightarrow \infty} V^v(\mathbf{x}(k, l)) = 0$.

From the definition of T_{ij}^1 , it can be known that $\lim_{k \rightarrow \infty} T_{\sigma(k+1, l)\sigma(k, l)}^1 \mathbf{x}^h(k, l) = 0$, so for any constant $\epsilon > 0$, there exists an integer $N'_\epsilon > 0$ such that

$$T_{\sigma(k+1, l)\sigma(k, l)}^1 \mathbf{x}^h(k, l) < \epsilon \quad (3.18)$$

when $k > N'_\epsilon$. On the other hand, based on Assumption 3.2, there exists an integer $k^* > N'_\epsilon$, such that $\langle \sigma(k^* + 1, l), \sigma(k^*, l) \rangle \in G^1$. Without loss of generality, let $\sigma(k^* + 1, l) = i$, $\sigma(k^*, l) = j$. Then, $T_{ij}^1 > 0$. Since $V^h(\mathbf{x}(k, l))$ is monotonically decreasing with respect to k , we get from (3.18) that $V^h(\mathbf{x}(k, l)) \leq V^h(\mathbf{x}(k^*, l)) \leq \alpha T_{ij}^1 \mathbf{x}^h(k^*, l) < \alpha \epsilon$, $k > k^*$, where $\alpha = \max_{s \in (n_1), (i, j) \in G^1} \frac{\xi_{sj}}{\gamma_{sij}}$, and ξ_{sj} is the s th entry of the vector ξ_j , thus $\lim_{k \rightarrow \infty} V^h(\mathbf{x}(k, l)) = 0$.

Likewise, from the definition of T_{ij}^2 , it can be known that $\lim_{l \rightarrow \infty} T_{\sigma(k, l+1)\sigma(k, l)}^2 \mathbf{x}^v(k, l) = 0$, so for any constant $\epsilon > 0$, there exists an integer $N''_\epsilon > 0$ such that

$$T_{\sigma(k, l+1)\sigma(k, l)}^2 \mathbf{x}^v(k, l) < \epsilon \quad (3.19)$$

when $l > N''_\epsilon$. Based on Assumption 3.2, there exists an integer $l^* > N''_\epsilon$ such that $\langle \sigma(k, l^* + 1), \sigma(k, l^*) \rangle \in G^2$. Let us assume $\sigma(k, l^* + 1) = i$, $\sigma(k, l^*) = j$. Then, $T_{ij}^2 > 0$. Since $V^v(\mathbf{x}(k, l))$ is monotonically decreasing with respect to l , we get from (3.19) that $V^v(\mathbf{x}(k, l)) \leq V^v(\mathbf{x}(k, l^*)) \leq \beta T_{ij}^2 \mathbf{x}^v(k, l^*) < \beta \epsilon$, $l > l^*$, $\beta = \max_{s \in (n_2), (i, j) \in G^2} \frac{\xi_{sj}}{\gamma_{sij}}$, thus $\lim_{l \rightarrow \infty} V^v(\mathbf{x}(k, l)) = 0$.

In summary, $\lim_{k, l \rightarrow \infty} V(\mathbf{x}(k, l)) = 0$, thus $\lim_{k, l \rightarrow \infty} \mathbf{x}(k, l) = 0$. The proof of Theorem 3.3 is complete. $\square$

For the case when the set S and G^2 are empty, we can design appropriate switching signals as follows.

(S2) There exists a sequence of time instants k_{1j}, k_{2j}, l_{1j} , and l_{2j} such that

$$\begin{aligned}
 & \bigcup_{(k, l) \in \{(k_{1j}, l), (k_{1j+1}, l), \dots, (k_{2j}, l)\}} I_{\sigma(k+1, l)\sigma(k, l)}^1 = \langle n_1 \rangle, \quad j = 1, 2, \dots \\
 & \bigcup_{(k, l) \in \{(k, l_{1j}), (k, l_{1j+1}), \dots, (k, l_{2j})\}} I_{\sigma(k, l+1)\sigma(k, l)}^2 = \langle n_2 \rangle, \quad j = 1, 2, \dots
 \end{aligned}$$

where $\{1_j\}_{j=1}^\infty$ and $\{2_j\}_{j=1}^\infty$ are strictly increasing sequences of positive integers. $1_j \leq 2_j < 1_{j+1} \leq 2_{j+1}$, $0 \leq k_{1j} \leq k_{2j} < k_{1j+1} \leq k_{2j+1}$, $\lim_{j \rightarrow \infty} k_{1j} = \infty$, $k_{2j} - k_{1j} \leq N_1$, and $N_1 > 0$ is an integer, $0 \leq l_{1j} \leq l_{2j} < l_{1j+1} \leq l_{2j+1}$, $\lim_{j \rightarrow \infty} l_{1j} = \infty$, $l_{2j} - l_{1j} \leq N_2$, and $N_2 > 0$ is an integer.

Remark 3.4. In fact, switching rule (S2) only requires information on the switching signal at a subset of specific time instants (which are not necessarily switching instants). It is obvious that switching rule (S2) is equivalent to $\sum_{i=1}^{2j} (\xi_{\sigma(k_i, l)}^{T_h} - \xi_{\sigma(k_i+1, l)}^{T_h} A_{\sigma(k_i, l)}^{11}) > 0$ and $\sum_{i=1}^{2j} (\xi_{\sigma(k, l_i)}^{T_v} - \xi_{\sigma(k, l_i+1)}^{T_v} A_{\sigma(k, l_i)}^{22}) > 0$.

Theorem 3.4. Assume that Assumption 3.1 is satisfied. If there is an existing a set of vectors ξ_j , such that $\xi_i^T A_j \leq \xi_j^T$, $i, j \in \{1, 2, \dots, m\}$, then system (3.17) is asymptotically stable under the switching signal that satisfies (S2).

Proof. First, we will prove $\lim_{k \rightarrow \infty} V^h(\mathbf{x}(k, l)) = 0$. Based on the above proof, we have $\lim_{k \rightarrow \infty} T_{\sigma(k+1, l)\sigma(k, l)}^1 \mathbf{x}^h(k, l) = 0$, so for any constant $\epsilon > 0$, there exists an integer $N'_\epsilon > 0$ such that we have $T_{\sigma(k+1, l)\sigma(k, l)}^1 \mathbf{x}^h(k, l) < \epsilon$ when $k > N'_\epsilon$. Choosing a sufficiently large k_{1j}, k_{2j} , we have

$$T_{\sigma(k+1, l)\sigma(k, l)}^1 \mathbf{x}^h(k, l) < \epsilon, \quad k_{1j} \leq k \leq k_{2j}. \quad (3.20)$$

Next, we discuss the two cases separately.

Case 1:

$k_{1j} = k_{2j}$. Assume $\sigma(k_{1j} + 1, l) = i$, $\sigma(k_{1j}, l) = j$. Because of the definition of T_{ij}^1 and the switching signal (S2), we have $T_{ij}^1 > 0$, so $V^h(\mathbf{x}(k, l)) \leq V^h(\mathbf{x}(k_{1j}, l)) \leq \alpha T_{ij}^1 \mathbf{x}^h(k_{1j}, l) < \alpha \epsilon$, $k > k_{1j}$. Thus, $\lim_{k \rightarrow \infty} V^h(\mathbf{x}(k, l)) = 0$.

Case 2:

$k_{1j} \neq k_{2j}$. According to the switching signal (S2), there exists $(\tilde{k}, l) \in \{(k_{1j}, l), (k_{1j+1}, l), \dots, (k_{2j}, l)\}$ such that $s \in I_{\sigma(\tilde{k}+1, l)\sigma(\tilde{k}, l)}^1$ and $\gamma_{s\sigma(\tilde{k}+1, l)\sigma(\tilde{k}, l)}^1 > 0$. Let $\sigma(\tilde{k} + 1, l) = i$, $\sigma(\tilde{k}, l) = j$.

From the above analysis, we have

$$\gamma_{sij}^1 x_s^h(\tilde{k}, l) \leq T_{ij}^1 x^h(\tilde{k}, l) < \epsilon, \quad s \in \langle n_1 \rangle. \quad (3.21)$$

On the other hand, because $A_i \geq 0$ for all $i \in \langle m \rangle$ and $x(k, l) \geq 0$, we have

$$x_s^h(k+1, l) \geq \eta_s x_s^h(k, l), \quad k_{1j} \leq k \leq \tilde{k}, \quad s \in \langle n_1 \rangle. \quad (3.22)$$

Here, $\eta_s = \min \{a_p^{ss} : p \in \{1, 2, \dots, m\}\}$, and a_p^{ss} is the s -th diagonal entry of matrix A_p . Due to the condition of Theorem 4 and $\gamma_{sij}^1 > 0$, we have $0 < \eta_s < 1$. From (3.22), we obtain $x_s^h(\tilde{k}, l) \geq \eta_s^{\tilde{k}-k_{1j}} x_s^h(k_{1j}, l) \geq \eta_s^N x_s^h(k_{1j}, l)$, $s \in \{1, 2, \dots, n_1\}$. Taking this together with (3.21), we have $x_s^h(k_{1j}, l) \leq \frac{\epsilon}{\eta_s^N \gamma_{sij}^1} \leq \frac{\epsilon}{\gamma \eta^N}$, $s \in \{1, 2, \dots, n_1\}$, where $\eta = \min\{\eta_s : s \in \{1, 2, \dots, n_1\}\}$ and $\gamma = \min\{\gamma_{sij}^1 : \gamma_{sij}^1 > 0, s \in \{1, 2, \dots, n_1\}, i, j \in \{1, 2, \dots, m\}\}$. Thus, $V^h(x(k, l)) \leq V^h(x(k_{1j}, l)) < \frac{n_1 \beta \epsilon}{\gamma \eta^N}$, $k \geq k_{1j} + 1$, $\beta = \max\{\beta_{si} : s \in \{1, 2, \dots, n_1\}, i \in \{1, 2, \dots, m\}\}$. Then, $\lim_{k \rightarrow \infty} V^h(x(k, l)) = 0$. Likewise, we have $\lim_{l \rightarrow \infty} V^v(x(k, l)) = 0$. In conclusion, we get $\lim_{k, l \rightarrow \infty} V(x(k, l)) = 0$. $\square$

Remark 3.5. For the special circumstance where $\xi_i = \xi$ for $i \in \langle m \rangle$, Theorem 3.4 degenerates into Theorem 3.2, which means Theorem 3.2 is a special case of it.

4. Numerical example

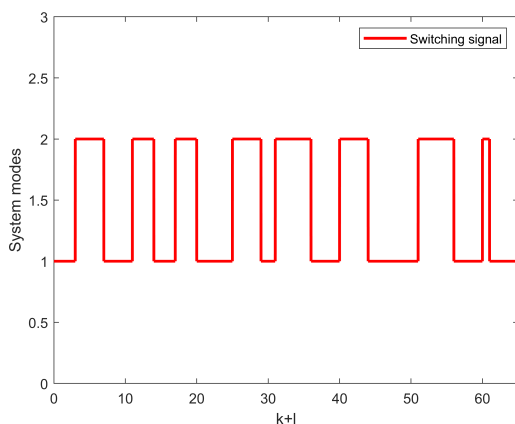


Figure 1. The designed signal of system (2.1).

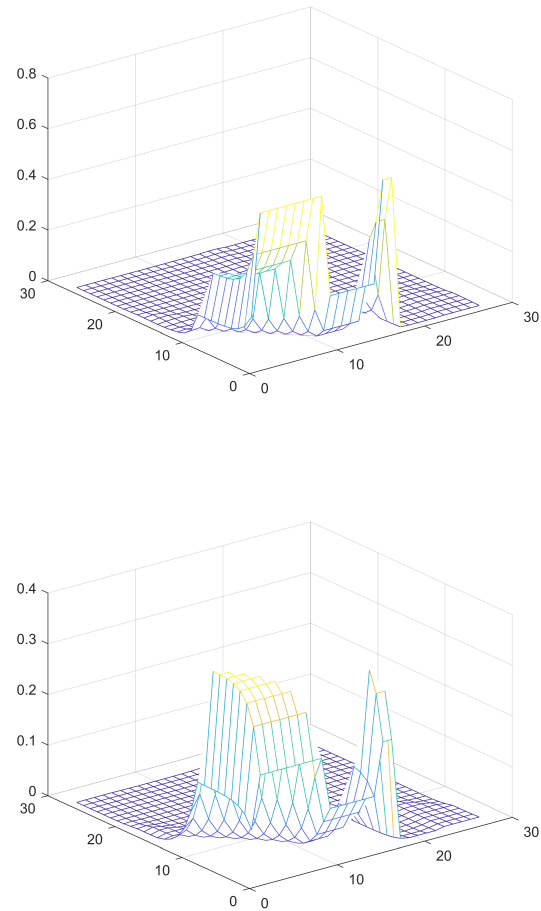


Figure 2. The estimate for $x^h(i, j)$ under the switching signal in Figure 1.

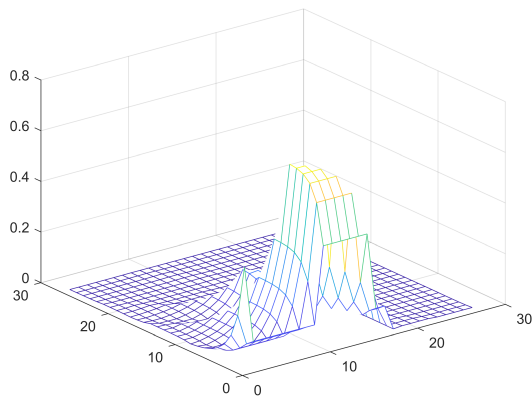
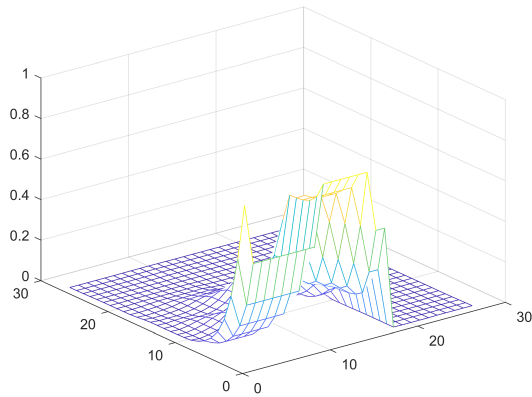


Figure 3. The estimate for $x^v(i, j)$ under the switching signal in Figure 1.

In the following, two numerical examples are provided to demonstrate the main results.

Example 4.1. Consider the switched positive system with disturbance in the form of (2.1), which is composed of two positive subsystems. The two positive subsystem matrices are given by

$$A_1 = \begin{pmatrix} 0.2 & 0.1 & 0 & 0 \\ 0.1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0.4 & 0.6 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0.6 & 0.3 & 0 & 0 \\ 0.2 & 0.85 & 0 & 0 \\ 0 & 0 & 0.5 & 0.3 \\ 0 & 0 & 0.1 & 0.3 \end{pmatrix}.$$

For the disturbance $\omega(k, l) = \frac{1}{1+(k+l)^2}(|\sin(k+l)|, |\cos(k+l)|, |\sin(2(k+l))|, |\cos(2(k+l))|)$, we cannot find a vector ξ such that $\xi^T(I_4 - A_1) > 0$, and $\xi^T(I_4 - A_2) > 0$. However, we have $\xi^T(I_4 - A_1) = (0.6, 0.9, 0, 0) \geq 0$ and $\xi^T(I_4 - A_2) = (0, 0, 0.45, 0.05) \geq 0$ when $\xi = (1, 2, 1, 0.5)$. Using Theorem 3.2, the system above is asymptotically stable under the switching signal as shown in Figure 1. The trajectories $x^h(i, j)$ and $x^v(i, j)$ are shown in Figures 2 and 3.

Example 4.2. Consider the switched positive system in the form of (3.17) which is composed of two positive subsystems. The two positive subsystem matrices are given by

$$A_1 = \begin{pmatrix} 0.7 & 0.4 & 0 & 0 \\ 0.6 & 0.2 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0.6 & 0.8 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0.3 & 0.4 & 0 & 0 \\ 0.2 & 0.1 & 0 & 0 \\ 0 & 0 & 0.75 & 0.5 \\ 0 & 0 & 0.3 & 0.4 \end{pmatrix}.$$

It is obvious that there is not a common vector ξ such that $\xi^T A_j \leq \xi^T, i, j = 1, 2$. However, if we choose $\xi_1 = (1, 0.5, 1, 1)^T$, $\xi_2 = (0.5, 0.5, 1.08, 0.9)^T$, it yields

$$A_1^T \xi_1 - \xi_1 = (0, 0, 0, 0),$$

$$A_1^T \xi_2 - \xi_1 = (-0.35, -0.2, -0.028, -0.064),$$

$$A_2^T \xi_2 - \xi_2 = (-0.25, -0.25, 0, 0),$$

$$A_2^T \xi_1 - \xi_2 = (-0.1, -0.05, -0.03, 0).$$

So, we have $\xi_i^T A_j \leq \xi_j^T, i, j = 1, 2$. When the switching signal is as in Figure 4 and the initial condition is $(1, 0.5, 0.5, 1)^T$, the corresponding solutions of system (3.17) are asymptotically stable. The trajectories $x^h(i, j)$ and $x^v(i, j)$ are shown in Figures 5 and 6.

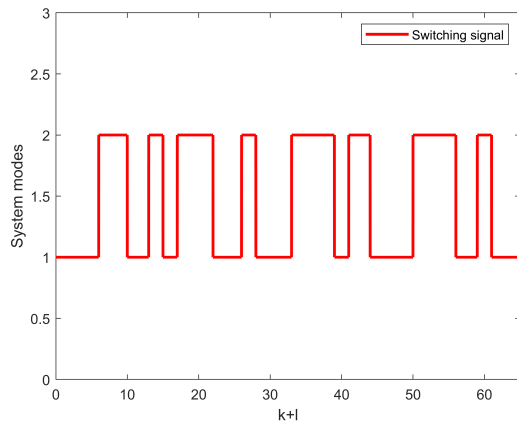


Figure 4. The designed signal of the system (3.17).

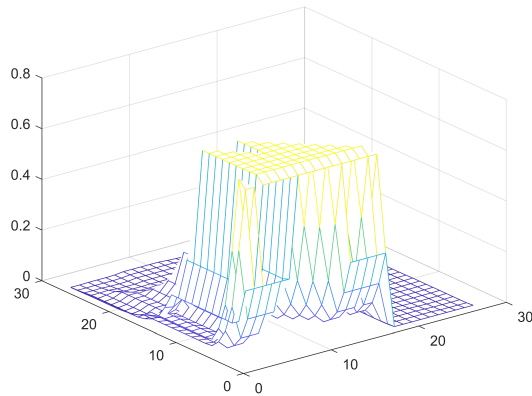
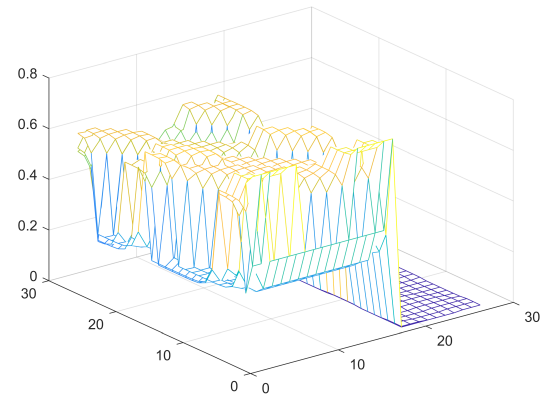


Figure 6. The estimate for $x^v(i, j)$ under the switching signal in Figure 4.

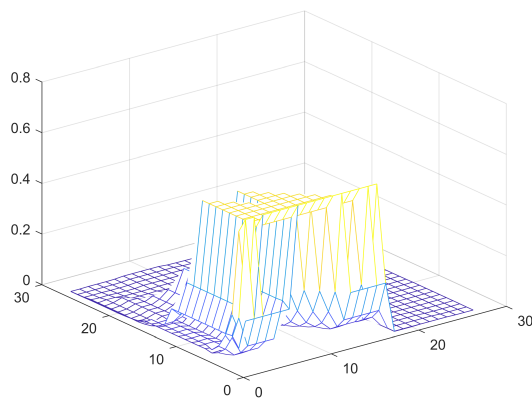


Figure 5. The estimate for $x^h(i, j)$ under the switching signal in Figure 4.

5. Conclusions

The stabilization problem of 2-D switched positive linear systems with disturbance is studied in this paper. Analogous to the 1-D system, first, it is proved that the 2-D switched positive linear system with disturbance is asymptotically stable under arbitrary switching by using the common linear copositive Lyapunov function. Then, the weak common linear copositive Lyapunov function and the weak switched linear copositive Lyapunov function are introduced, and the asymptotic stability results of the corresponding system under the given switching rules are obtained. Finally, a numerical example shows the effectiveness of our results. In the future, the stabilization problem of continuous 2-D switched positive systems will be considered.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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