

Research article

A generalized nonlinear model for multi-engine single thrust vectored flyingwing

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Abstract: Superior maneuverability and stealth are two primary focal points in the sixth generation of aviation and aerospace concepts. A potential technology to deliver such enhanced maneuverability is thrust vectored control (TVC), which is currently limited only to vertical control during take-off and landing. The expansion of maneuvering to all three dimensions (including horizontal) enables TVC's ability to attain extreme maneuvers with the least effect on overall performance. However, developing controllers to achieve these maneuvers requires a detailed mathematical model to help understand the aircraft dynamics. Thus, this paper demonstrates the development of a nonlinear dynamic model in the first-order differential form for an aircraft employing a full flyingwing design with a tail-end single-thrust vectored engine with two wing-mounted forward thrusters. This design falls under the generic class of the multi-engine single-thrust vector system (MESTV) configuration. A feedback controller was incorporated to simulate and attain optimal performance in two realistic and critical flight scenarios: flight-envelop regulation and stall recovery. The developed model was tested with three phases of operation; climb and cruising, pitch, and yaw maneuvers. The simulation results show that the proposed configuration achieved horizontal (yaw) maneuvers together with vertical (pitch) maneuvers. Thus, the proposed engine configuration is well-suited to attain superior maneuverability.

Keywords: thrust vectored control; nonlinear dynamics; flying wing; turbulent wind; aircraft manoeuvring

1. Introduction

The aviation and aerospace industries widely depend on smooth mobility in various flight conditions. Conventional aircraft are developed around the traditional maneuvering method that uses control surfaces such as flaps and ailerons [1]. However, the operational capability of traditional (static) control surfaces is constrained to six degrees of freedom, where the system is developed against the well-known Euler angles in a three-dimensional coordinate system [2, 3]. Due to their extreme reliance on wind direction and speed, these systems have become obsolete for precision maneuvers and require additional control effort in current operations and extreme conditions. The dependency on wind brings about extreme unpredictability and a heightened necessity for user intervention. This inadvertently brings the safety of

operations into question, particularly in extreme conditions. Hence, novel control methods are needed that can either completely avoid the aerodynamic dependency or optimize the control effort requirements [4–8]. Thrust vectored controls (TVCs) and the gimbaled engine system have been a fast-growing and popular discovery as they support the avoidance of aerodynamic dependency of maneuvering. However, TVC relies on the functions correlating the controller parameters to the body in motion [9]. This correlation governs the attitude and angular velocity of an aircraft in a spherical infinite coordinate space that is independent of the wind speed and direction and thus is a more effective control method in modern air and space vehicle design [1]. This novelty allows the TVC to be dynamic and respond with its own self-generated forces. In extreme flight conditions, the TVC's ability to perform rapid and precise manoeuvres with minimal

mechanical effort reduces strain on the operator. However, current applications of TVC are mainly focused on single TVC engine architectures that can achieve only 4 degrees of freedom leading it to operate only in the vertical plane of motion. This allows TVC to be used in various fields, such as vectored trajectory predictions of interceptors in terminal guidance [5], orbital stability correction of satellites using multiple pressurized air vector nozzles [10], the use of vectored engines in booster rockets for stabilized retrieval [11], and pressurized air 10-thruster kill vehicles [3, 12]. To our best knowledge however, the only use of TVC in fixed-wing aircraft is in the vertical take-off and landings (VTOL) and pitch movement control of F35 fighter jets [13, 14]. The F35 does not explore the use of TVC for horizontal plane control [15]. Therefore, this presents opportunities for further development of TVC in fixed-wing aircraft around the multi-dimensional operability (spherical coordinate movement), for horizontal and vertical control [16, 17], making it capable of pitch, roll and yaw movements. Nonetheless, the type of aircraft used plays a crucial role in developing a dynamic model. The different types of aircraft (in the essence of wing-body shape) vary from control-canard, lifting-canard, tandem wing, aft-strake, flying wing, drop wing, etc [18]. However, generic blended wing-type aircraft are more prone to instability even in extreme conditions of minor magnitudes [19]. Thus, a blended wing design could significantly benefit from the TVC approach. Hence, in this paper, we develop a mathematical model of a blended-wing aircraft controlled by a TVC engine configuration. More specifically, this architecture brings forth the design to be a flying wing (blended wing type). The overall engine configuration consists of two stationary main engines placed symmetrically opposing the water line of the aircraft. The TVC engine to be used as the controller is strategically positioned to optimize manoeuvrability while adhering to the laws of physics. Hence, the configuration entails the TVC engine to be tail-mounted and capable of operating in both vertical and horizontal planes within a spherical infinite or quaternion space system. Using this configuration, we apply TVC for five degrees of motion (compared to the existing limitation of four degrees) without requiring additional major moving parts to operate [20, 21].

The developed mathematical model will provide a much deeper understanding of the operational capability of such a TVC architecture, enabling the development of controllers for efficient operation and robustness towards extreme condition navigation. The model integrates identifying control limits (saturation ranges), defining intermediate variables and non-dimensional aerodynamic force coefficients, deriving aerodynamic forces, defining non-dimensional aerodynamic moment coefficients, deriving aerodynamic moments, rotating of aerodynamic moments to CoG, deriving propulsion effects, and deriving rigid body dynamic model for the aircraft. While previous studies have contributed significantly to the understanding of TVC flying wing aircraft, they often rely on simplified assumptions, particularly regarding turbulence effects, transient dynamics, and control system integration. Table 1 provides a comparative overview of how this study extends beyond existing research.

The contributions of this paper (and advantages compared to previous works) can be summarized as follows:

- (1) We develop a comprehensive nonlinear dynamic model that accurately captures the effects of thrust vectoring, aerodynamic forces, and external disturbances on a full flying-wing aircraft. Unlike existing models that simplify actuator dynamics, this work explicitly accounts for thrust vectoring effects in all flight conditions, improving fidelity and real-world applicability.
- (2) We propose a multi-layered FTC framework that ensures robust stability even in the presence of actuator faults, control saturation, and aerodynamic uncertainties. The controller actively compensates for system deviations using adaptive feedback mechanisms, preventing performance degradation during yaw, pitch, and turbulence-induced disturbances.
- (3) A flight envelope controller is integrated into the model, ensuring that altitude, velocity, and maneuverability constraints are maintained even under aggressive thrust vectoring maneuvers. Unlike prior studies, which assume fixed envelope constraints, our approach enables real-time adaptation, improving

Table 1. Unique aspects of the current research.

Feature	Previous Researches	This Research
Turbulence Modeling	Gao and Fu (2019) focus on robust LPV modeling for wind disturbances, assuming a linearized disturbance model. Saha et al. (2020) [22] use multiple-timescale nonlinear control to handle turbulence but do not integrate full 6-DOF effects.	Explicitly integrates turbulence effects into TVC flight dynamics, using a full nonlinear 6-DOF model to evaluate disturbance propagation and control response.
Use of Flight Envelope Control	Gust response stabilization studies integrates multi-control-effectors for response control but does not include dynamic flight envelope adaptation [21]. Longitudinal static stability study focuses on pre-defined stability augmentation without real-time adaptation [23].	Implements real-time flight envelope adaptation, ensuring altitude and velocity limits are dynamically adjusted under disturbances.
Control System introduction	Fahmi and Woolsey (2021) develop a port-Hamiltonian control framework but do not address actuator nonlinearity. Mathematical phenomenology study done by Kim et al. (2021) quantifies agility but lacks adaptive control integration.	Implements two feedback control systems to optimize the TVC safe operation and flight envelope adherence.
Transient Aerodynamic Response Analysis	NASA's X-48B studies provide lateral-directional parameter estimation but lacks transient aerodynamic force analysis [9]. Khalili et al. (2021) Partial Diffusion Kalman Filter study focuses on real-time estimation of system states under transients but does not integrate actuator dynamics.	Captures transient oscillations in the body-fixed frame, revealing short-duration velocity deviations and their impact on stability.
Aircraft Dynamics Consideration	Gust loads studies evaluates open/closed loop response of a flying wing to disturbances but does not include fault recovery for example the study conducted by A Voß (2019). Hassan and Taha (2021) examine nonlinear roll control but do not incorporate thrust vectoring into stability analysis.	Uses a full nonlinear aircraft model, considering thrust vectoring interactions with aerodynamic forces.
Yaw Control and TVC Effectiveness	Yu and Shu (2017) developed a novel TVC control law for improved maneuverability but focus only on steady-state conditions. Lee et al. (2017) study nonlinear acceleration control for TVC but do not explore dynamic yaw response.	Demonstrates TVC effectiveness in counteracting yaw disturbances and turbulence effects.
Pitch and Roll Stability Considerations	Blended wing body pitch control studies explores novel pitch effectors for stability but does not integrate adaptive feedback [24]. Actuator fault control study for tailless aircraft focuses on reallocation but does not include multi-mode fault handling [19].	Analyzes pitch and roll stability under turbulence, demonstrating TVC's role in attitude correction.
Real-World Applicability	BWB aerodynamic design studies evaluate airframe performance but does not include real-world disturbance effects [25]. Shin and Li (2021) explore proportional navigation but do not integrate environmental factors into guidance.	Simulated under realistic turbulence and maneuvering conditions, demonstrating control effectiveness.
Application to UAVs and Future Flying Wings	NASA's X-48B in-flight stability study explores maneuver characteristics but does not focus on thrust vectoring integration Regan (2008) [26]. Swept UCAV control study by Bauersfeld et al. (2021) [12] validates aerodynamic properties but lacks control adaptation to flight phases.	Designed for UAVs and next-generation manned flying wing aircraft using thrust vectoring.
Computational/ Simulation Approach	Mathematical phenomenology study quantifies agility but lacks turbulence integration [27]. Experimental UCAV study validates aerodynamics but lacks fault-tolerant control testing [28].	Uses high-fidelity simulations to capture dynamic effects.

safety margins in complex flight scenarios.

- (4) We provide a detailed analysis of translational and rotational velocity components (u, v, w, p, q, r) , demonstrating how thrust vectoring and environmental disturbances influence body-frame dynamics. This study is the first to quantify transient aerodynamic effects on stability in a flying wing aircraft with TVC, providing deeper insights into control performance during rapid corrections and turbulence mitigation.
- (5) The findings of this study contribute directly to the design and stability control of UAVs and next-generation flying-wing aircraft, enhancing maneuverability through thrust vectoring rather than

conventional aerodynamic surfaces. The proposed nonlinear model and control framework offer a foundation for future autonomous and fault-tolerant flight systems, bridging the gap between theoretical research and real-world applications.

Through detailed mathematical modeling, control implementation, and simulation validation, this study demonstrates that thrust vectoring enables effective stabilization and maneuverability in a flying-wing aircraft. The results highlight the system's ability to maintain stability under real-world disturbances, providing a refined approach for nonlinear flight dynamics and control.

This paper is organized as follows: Section 2 gives a

mathematical model with assumptions and the operating philosophy. Section 3 contains the results obtained by introducing the model to real flight scenarios followed by a critical discussion on any discrepancy observed against the theoretical expectations. Section 4 concludes the paper.

2. Materials and methods

2.1. Operating assumptions

In developing the new architecture, the following assumptions (and their justifications) were made according to general aerodynamic theories in order to keep the computational complexity at a reasonable level while maintaining the applicability and practicality of the problem.

Assumption 2.1. Adverse yaw is not considered (no deflection dependency).

Justification: The roll moment of any aircraft is achieved using ailerons which inadvertently contributes to the pressure difference generated on either side of the aircraft. This differential pressure will cause the induced drag to increase, which results in premature yawing. Thus, eliminating the roll moment's contribution eliminates the need to address the adverse yaw [23].

Assumption 2.2. The tendency of adverse roll is not considered (no deflection dependency).

Justification: Blended wing aircraft inherently minimizes the coupling between yaw and roll moments as it achieves more balanced aerodynamic forces, preventing the tendency of yaw induced roll moment. (Due to the absence of a distinct tail section unlike conventional aircraft) [28].

Assumption 2.3. The Centre of Gravity (CoG) of the aircraft falls within its datum line irrespective of the mass distribution variation on TVC engine movement.

Justification: The aircraft is designed with a TVC, whose mass distribution is considered to ensure the movement of the engine has no significant effect to the balance of the aircraft. Compared to the total mass of the aircraft, the mass and inertia of the TVC engine is relatively smaller and has no influence on the CoG [25, 29].

Assumption 2.4. The fuel exit mass change is negligible.

Justification: The simulation is conducted in small time

intervals. Hence, the fuel consumption would be relatively smaller with respect to total aircraft mass [30, 31].

Assumption 2.5. Control effectiveness is assumed to be equivalent to conventional control; hence, the method of moment coefficient and change of actuator input is given by $\partial C_m / \partial u$.

Justification: The conventional method can be referred to as the products of inertia, which are considered to be operating within the range where the model still behaves as a single rigid body [24].

Assumption 2.6. The aircraft moments are assumed to be independent of the tuneable parameters, but considered as functions of states.

Justification: By considering moment dynamics as functions of states, all real-time attitude changes are taken into account without depending on tuneable parameters. Hence, making this assumption does not affect the overall aim of the paper [32, 33].

2.2. Derivation of mathematical model

In this section, we develop the mathematical model of the TVC aircraft, using a tri-engine configuration named 'tri-engine single thrust vector' (TESTV) as a subclass of MESTV, which comprises of two conventional engines on each wing at a distance R from the CoG and a tail-mounted TVC engine at a distance R_x from the CoG. The aircraft is modeled as a rigid body of mass m maneuvered in 3D space of non-rotating flat earth [34, 35]. The TVC system is a single-gimbaled engine that can rotate laterally and vertically, determining the aircraft's yaw and pitch movements. The vertical deflection ε (pitch) and horizontal deflection ϕ (yaw) of the TVC are tuneable parameters. Proper decoupling is required to assign corresponding eigenvectors to the TVC engine deflection effects, which will be detailed in the following sub-sections.

- TVC flyingwing design

In preliminary analysis, the model is derived for six degrees of freedom. The underlying forces acting on the aircraft are considered with respect to the body frame (fixed frame) of reference as shown in Figure 1.

- Basic aircraft parameters [36]

differentiating the linear momentum with respect to time.

$$F = m \frac{dV_c}{dt} \Big|_i. \quad (2.3)$$

However, the aircraft's linear momentum can be equivalent to the linear momentum of a point mass at the CoG. Thus, using Eq (2.3), a general force vector (F_B) can be defined under the rationalization between the inertial and body-fixed axes as follows:

$$F_B = m \frac{dV_c}{dt} \Big|_B + m(\omega_B \times V_c) \quad (2.4)$$

where the total velocity components (denoted by subscript 'C') are written with respect to the body reference frame (denoted by subscript 'B') and inertial reference frame (denoted by subscript 'i'). Then, we define the following state vector:

$$V_c = [u \quad v \quad w]^T \quad (2.5)$$

where u , v , and w are the velocity components in the x, y, and z directions respectively. Then, we also define,

$$\omega_B = [p \quad q \quad r]^T \quad (2.6)$$

where p , q , and r are the angular velocities about the x, y, and z axes, respectively. However, ω_B may be expressed as an angular velocity tensor using linear mapping. This is considered an infinitesimal rotation matrix acting as ' ω_B^R ' by direct measurement [25, 32] and can be written as follows:

$$\omega_B^R = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}. \quad (2.7)$$

By substituting for the definition of V_c and the matrix BR from (2.7) into Eq (2.4), the translational function can be written as follows:

$$F_B = m \begin{bmatrix} \dot{u} - rv + qw \\ \dot{v} + ru - pw \\ \dot{w} - qu + pv \end{bmatrix}. \quad (2.8)$$

However, as per Figures 2 and 3, forces pertaining to aircraft dynamics incorporates engine thrust (T_A), TVC thrust ($T_E(\cos \varepsilon + \cos \varphi)$, $T_E \sin \varphi$, $T_E \sin \varepsilon$, respectively, in the x, y, and z directions) gravitational forces (g_x, g_y, g_z) and wind exerted forces (Lift - L , Drag - D , Lateral - Y).

Thus, by introducing the above forces to F_B of Eq (2.8) the following equations are derived:

$$m(\dot{u} + wq - vr) = T_A + T_E(\cos \varepsilon + \cos \varphi) + g_x - D, \quad (2.9)$$

$$m(\dot{v} + ur - wp) = T_E \sin \varphi + g_y + Y, \quad (2.10)$$

$$m(\dot{w} - uq + vp) = -T_E \sin \varepsilon + g_z - L, \quad (2.11)$$

which then can be written as a (quadratic) state vector as follows:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} uq \\ ur \\ vp \\ vr \\ wp \\ wq \end{bmatrix} + \begin{bmatrix} T_A + T_E(\cos \varepsilon + \cos \varphi) \\ T_E \sin \varphi \\ -T_E \sin \varepsilon \end{bmatrix} + \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} + \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix}. \quad (2.12)$$

For brevity, by denoting $x_{1,1}$ as the linear state vector, $x_{1,2}$ as the quadratic product of translational velocity, u_1 as the thrust input matrix, F as the aerodynamic forces, and g as the gravitational disturbance vector, (2.12) can be re-written in the following form:

$$\frac{dx_{1,1}}{dt} = Cx_{1,2} + u_1 + g + F. \quad (2.13)$$

The forces u_1 in Eq (2.12) generated by the TVC engine are the only forces dependent on the tuneable parameters ε and φ , which are the angle of the TVC engine deflection about the y and z axes respectively. In reality, the aerodynamic forces ' D , Y , L ' in F are correlated with the coefficients of motion (C_D , C_Y , C_L) which are functions of the angle of attack, α (angle between aircraft waterline and vertical deflection to the z-direction, generally given by $(\tan^{-1} \frac{w}{u})$) and angle of side slip, β (angle between aircraft waterline and horizontal deflection to the y-direction, generally given by $(\sin^{-1} \frac{v}{V_c})$) (in accordance with the widely acknowledged aerodynamic formulas in scientific literature represented by Eqs (2.39)–(2.41)). However, in this paper the aerodynamic forces are assumed to be independent of the tuneable parameters, but functions of states. Thus, making this assumption does not affect the overall aim of the paper [28,29]. However, in achieving the expected maneuvers, the following constraints were imposed on the tuneable parameters to satisfy Assumption 6:

- To achieve a pitch moment, the TV engine can deflect about the y-axis in the angular range of $-\frac{\pi}{2} \leq \varepsilon \leq \frac{\pi}{2}$.
- To achieve a yaw moment, the TV engine can deflect about the z-axis in the angular range of $-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$.
- Both pitch and yaw cannot happen at the same time. Thus, when, $\varepsilon \neq 0$, $\varphi = 0$, and when $\varphi \neq 0$, $\varepsilon = 0$.
- The deflection angle of the TV engine changes according to a step function.

Derivation of rotational dynamics:

When the rotational motion of an element mass on an aircraft is considered, its angular momentum δh can be derived as follows:

$$\delta h = r_c \times \delta m(V_c + \omega_B \times r_c) \quad (2.14)$$

where,

$$h = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{zx} & 0 & I_{zz} \end{bmatrix}. \quad (2.15)$$

Then, summing the angular momentums for element masses over the whole body of the aircraft gives the total angular momentum of the aircraft:

$$\sum \delta h = \sum r_c \delta m \times V_c + \sum \delta m \{r_c \times (\omega_B \times r_c)\}. \quad (2.16)$$

In the same form of (2.15), considering the element mass to be very small $\delta m \rightarrow 0$, and measured from the centre of mass $\int r_c dm = 0$, the momentum around the CoG can be expressed as follows:

$$M = \frac{dh}{dt}. \quad (2.17)$$

Thus, considering the body reference frame, the expression of h from (2.17) can be extended by considering the moment inputs with respect to the body reference frame as follows:

$$M = \left. \frac{dh}{dt} \right|_B + (\omega_B \times h). \quad (2.18)$$

Then, by expanding the expression of h in Eq (2.18) in terms of the body axis frame and assimilating moment components (2.6) and products of inertia (2.15), the following moment function can be written:

$$M = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{zx} & 0 & I_{zz} \end{bmatrix} \left. \frac{d}{dt} \right|_B \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \omega_B \times \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{zx} & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (2.19)$$

Then, substituting (2.6) into (2.19) yields:

$$M = \begin{bmatrix} I_{xx}\dot{p} - I_{xz}\dot{r} \\ I_{yy}\dot{q} \\ -I_{zx}\dot{p} + I_{zz}\dot{r} \end{bmatrix} + \begin{bmatrix} -pqI_{zx} - qr(I_{yy} - I_{zz}) \\ (p^2 - r^2)I_{zx} - pr(I_{zz} - I_{xx}) \\ qrI_{xz} - pq(I_{xx} - I_{yy}) \end{bmatrix}. \quad (2.20)$$

However, moments pertaining to aircraft dynamics are caused by TVC thrust ($0, T_E \sin \varphi \cdot R_x, T_E \sin \varepsilon \cdot R_x$ respectively in the x, y , and z directions) and body axis components (L, M , and N respectively in the x, y , and z directions). Thus, by introducing those moments to M of Eq (2.20), the following equations can be derived:

$$I_{xx}\dot{p} - (I_{yy} - I_{zz})qr + I_{yz}(r^2 - q^2) - I_{zx}(pq + \dot{r}) + I_{xy}(rp - \dot{q}) = L, \quad (2.21)$$

$$I_{yy}\dot{q} - (I_{zz} - I_{xx})rp + I_{zx}(p^2 - r^2) - I_{xy}(qr + \dot{p}) + I_{yz}(pq - \dot{r}) = M \pm T_E \sin \varphi \cdot R_x, \quad (2.22)$$

$$I_{zz}\dot{r} - (I_{xx} - I_{yy})pq + I_{xy}(q^2 - p^2) - I_{yz}(rp + \dot{q}) + I_{zx}(qr - \dot{p}) = N \pm T_E \sin \varepsilon \cdot R_x + T_A R, \quad (2.23)$$

which then can be written in the following (quadratic) state-space form

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = (A')^{-1} \cdot B + \begin{bmatrix} p^2 \\ pq \\ pr \\ q^2 \\ qr \\ r^2 \end{bmatrix} + (A')^{-1} \begin{bmatrix} L_r \\ M \\ N \end{bmatrix} + (A')^{-1} \begin{bmatrix} 0 \\ T_E \sin \varphi \cdot R_x \\ T_E \sin \varepsilon \cdot R_x + T_A R \end{bmatrix} \quad (2.24)$$

where

$$A' = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{zx} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{yz} & I_{zz} \end{bmatrix} \quad (2.25)$$

and

$$B = \begin{bmatrix} 0 & I_{xx} & -I_{xy} & I_{yz} & (I_{yy} - I_{zz}) & -I_{yz} \\ -I_{zx} & -I_{yz} & (I_{zz} - I_{xx}) & 0 & I_{xy} & I_{zx} \\ I_{xy} & (I_{yy} - I_{xx}) & I_{yz} & -I_{xy} & -I_{zx} & 0 \end{bmatrix} \quad (2.26)$$

and

$$(A')^{-1} = \frac{1}{\Delta} \begin{bmatrix} (I_{yy}I_{zz} - I_{yz}^2) & (I_{zz}I_{xy} + I_{yz}I_{zx}) & (I_{yz}I_{xy} + I_{yy}I_{zx}) \\ (I_{zz}I_{xy} + I_{yz}I_{zx}) & (I_{xx}I_{zz} + I_{zx}^2) & (I_{xx}I_{yz} + I_{zx}I_{xy}) \\ (I_{yz}I_{xy} + I_{yy}I_{zx}) & (I_{xx}I_{yz} + I_{xy}I_{zx}) & (I_{xx}I_{yy} - I_{xy}^2) \end{bmatrix}. \quad (2.27)$$

Then, by using, $x_{2,1}$ to denote the linear state vector, $x_{2,2}$ to denote the quadratic product of angular velocity, u_2 to denote the resultant input matrix, and F_r to denote the body axis component vector, Eq (2.26) can be written in compact form as follows:

$$\frac{dx_{2,1}}{dx} = (A')^{-1}x_{2,2} + (A')^{-1}u_2 + (A')^{-1}F_r. \quad (2.28)$$

The inputs u_2 in Eq (2.28) generated by the TVC engine are the only moments dependent on the tuneable parameters for angular motions. In reality, the body moments L, M, N in F_r are generated due to coefficients of motion about the aerodynamic centre, $C_{l_{ac}}, C_{m_{ac}}, C_{n_{ac}}$ which are associated with the angle of attack (α) and angle of side slip β of the aircraft. The relationships are defined in Eqs (2.42)–(2.44), following the well-established aerodynamic formulas in scientific literature.

Considering (2.13) and (2.24), the following 6th-order nonlinear (quadratic) state equation for the TVC aircraft can be derived in the ordinary differential form $\dot{x} = f(x, u)$ as follows:

$$\frac{dx^{[1]}}{dx} = Kx^{[2]} + Gu + g \quad (2.29)$$

where

$$x^{[1]} = \begin{bmatrix} \dot{x}_{1,1} \\ \dot{x}_{2,1} \end{bmatrix}, \quad x^{[2]} = \begin{bmatrix} \dot{x}_{1,2} \\ \dot{x}_{2,2} \end{bmatrix}, \quad K = \begin{bmatrix} C & [0_{3 \times 6}] \\ [0_{3 \times 6}] & (A')^{-1}B \end{bmatrix},$$

$$G = \begin{bmatrix} [I_{3 \times 3}] & [0_{3 \times 3}] \\ [0_{3 \times 3}] & (A')^{-1} \end{bmatrix}, \quad u_i = \begin{bmatrix} u_1 + F \\ u_2 + F_r \end{bmatrix},$$

$$g = \begin{bmatrix} g_x & g_y & g_z & 0 & 0 & 0 \end{bmatrix}^T.$$

Note: For the vector $x_{(i,j)}$, i is used to indicate whether the variable is associated with the translational dynamics ($i = 1$) or moment dynamics $i = 2$, and j is used to indicate whether the state vector is linear $j = 1$ or quadratic $j = 2$.

Derivation of angular position using Euler kinematical equations:

The Euler's rotational matrices for each axis can be written as follows:

About the x-axis:

$$R^{(x)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \chi & -\sin \chi \\ 0 & \sin \chi & \cos \chi \end{bmatrix}.$$

About the y-axis:

$$R^{(y)} = \begin{bmatrix} \cos \Theta & 0 & \sin \Theta \\ 0 & 1 & 0 \\ -\sin \Theta & 0 & \cos \Theta \end{bmatrix}.$$

About the z-axis:

$$R^{(z)} = \begin{bmatrix} \cos \Phi & -\sin \Phi & 0 \\ \sin \Phi & \cos \Phi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

For any rigid body, the rate of change of Euler angles (θ) can be written as a function of the angular velocity vectors (ω_B) as $\dot{\theta} = f(\omega_B)$ where the reference frame is interpreted to be a fixed body (denoted by subscript B). Thus, considering the summation of angular velocity vectors in the x, y , and z axis, ω_B can be written w.r.t. the fixed reference frame.

$$\bar{\omega}_B = \bar{\omega}_x + \bar{\omega}_y + \bar{\omega}_z. \quad (2.30)$$

The Euler angular displacement is typically taken with reference to each axis and is denoted by ζ, θ , and ψ respectively about the x, y , and z axes. Moreover, the angular velocity can be further depicted by the rate of change of Euler angles:

$$\bar{\omega}_x = \begin{bmatrix} \dot{\zeta} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{\omega}_y = \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}, \quad \bar{\omega}_z = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}.$$

However, these angular velocities are represented in their respective axis reference frames. Thus, all vectors must be rotated to an individual reference frame to reflect a specific body motion. Hence, the following Euler rotations were done to satisfy the requirement for the summation of vectors (the body reference frame is denoted by superscript 'b' in this case).

$$\bar{\omega}_x^b = \bar{\omega}_x, \quad (2.31)$$

$$\bar{\omega}_y^b = R^{(x)} \bar{\omega}_y, \quad (2.32)$$

$$\bar{\omega}_z^b = R^{(x)} R^{(y)} \bar{\omega}_z. \quad (2.33)$$

Then, by substituting Eqs (2.31)–(2.33) into the sum of the angular velocity Eq (2.29), the following equation can be derived for $\bar{\omega}_B^b$:

$$\bar{\omega}_B^b = \bar{\omega}_x + R^{(x)}\bar{\omega}_y + R^{(x)}R^{(y)}\bar{\omega}_z. \quad (2.34)$$

Where, the rotation matrices $R^{(x)}$ and $R^{(y)}$ are written with the respective x -axis and y -axis displacements ζ and θ . By simplifying the Eq (2.34), the following can be derived:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \zeta & \sin \zeta \cos \theta \\ 0 & -\sin \zeta & \cos \zeta \cos \theta \end{bmatrix} \cdot \begin{bmatrix} \dot{\zeta} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}. \quad (2.35)$$

Re-arranging (2.35) to form the inverse rotational matrix, factoring out the common denominator in each element, and then simplifying the denominator using the trigonometric identity $\cos^2 x + \sin^2 x = 1$ yields,

$$\begin{bmatrix} \dot{\zeta} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\sin \zeta \sin \theta}{\cos \theta} & \frac{\cos \zeta \sin \theta}{\cos \theta} \\ 0 & \cos \zeta & -\sin \zeta \\ 0 & \frac{\sin \zeta}{\cos \theta} & \frac{\cos \zeta}{\cos \theta} \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (2.36)$$

Therefore, the simplified aircraft angular position equation from (2.36) can be written as,

$$\begin{bmatrix} \dot{\zeta} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \zeta \tan \theta & \cos \zeta \tan \theta \\ 0 & \cos \zeta & -\sin \zeta \\ 0 & \sin \zeta \sec \theta & \cos \zeta \sec \theta \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (2.37)$$

Equation (2.37) can be re-written as,

$$\dot{\Phi} = H(\zeta, \theta) \cdot \bar{\omega}_B. \quad (2.38)$$

Here, $H(\zeta, \theta)$ is the matrix function for the time rate of displacement (which is not a unitary rotation matrix).

These motions are influenced by well-established aerodynamic formulas, which describe their dependence on the aircraft's orientation and velocity relative to the airflow. These aerodynamic forces can be expressed as:

$$D = C_D \cdot \frac{1}{2} \rho V_c^2 \cdot S, \quad (2.39)$$

$$Y = C_Y \cdot \frac{1}{2} \rho V_c^2 \cdot S, \quad (2.40)$$

$$L = C_L \cdot \frac{1}{2} \rho V_c^2 \cdot S. \quad (2.41)$$

The corresponding aerodynamic coefficients are given by usual notations: $C_D = C_{D_0} + C_{D_\alpha} \cdot \alpha^2$, $C_Y = C_{Y_\beta} \cdot \beta$, and $C_L = C_{L_0} + C_{L_\alpha} \cdot \alpha$.

Beyond aerodynamic forces, rotational motion is governed by moments that result from the aircraft's inertia and angular velocities. These moments determine the aircraft's rotational response, and can be expressed as:

$$L_r = \frac{1}{2} \rho V_c^2 \cdot S b \cdot C_{l_{ac}}, \quad (2.42)$$

$$M = \frac{1}{2} \rho V_c^2 \cdot S c \cdot C_{m_{ac}}, \quad (2.43)$$

$$N = \frac{1}{2} \rho V_c^2 \cdot S b \cdot C_{n_{ac}}, \quad (2.44)$$

where ρ is the air density, V_c is the freestream velocity, S is the reference wing area, and c is the mean aerodynamic chord.

The corresponding aerodynamic coefficients are given by the following usual notations: $C_{l_{ac}} = C_{l_0} + C_{l_\beta} \cdot \beta$, $C_{m_{ac}} = C_{m_0} + C_{m_\alpha} \cdot \alpha$, and $C_{n_{ac}} = C_{n_0} + C_{n_\beta} \cdot \beta$.

The angles are defined as

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right) \text{ and } \beta = \sin^{-1}\left(\frac{v}{V_c}\right).$$

This derived model demonstrates the behavior of the aircraft in its translational velocities, rotational velocities, and angular position. Moreover, the outputs are presented w.r.t. the body reference frame.

In terms of model fidelity, even though we consider aerodynamic forces as functions of α and β , at this stage the framework can be extended to a higher-order coupling between thrust vector deflections and aerodynamic coefficients. We intentionally omitted the thrust-aerodynamic cross-interactions in-order to preserve clarity and computational tractability. However, this formulation remains a model that captures the dominant coupling between translational and rotational dynamics.

2.3. Model test operating conditions

The developed model of the TVC aircraft was tested across two distinct flight conditions, broadly categorized as 'ideal' and 'realistic' conditions.

Ideal condition: No external forces are acting on the aircraft.

Realistic condition: In actual flight environments,

meteorological disturbances are the most frequently occurring and challenging external disturbances, characterized by diverse atmospheric phenomena. For comprehension, it was reasonable for our model to consider a pre-built disturbance classifier to achieve realistically detailed and targeted outcomes [37, 38].

The models developed for the above phenomenon present a simplified base approach, while the realistic extreme wind conditions are more chaotic in nature and complex in mathematical frameworks [39, 40]. Two established turbulence mathematical models are the Dryden (DM) and the Von Kármán (VKM) wind turbulence models [41]. The DM is commonly applied to simulate turbulence at higher altitudes above the atmospheric boundary layer. It is often used to model the turbulence aircraft encounters during cruising or at higher altitudes in the atmosphere. The VKM is often used for modeling atmospheric turbulence at lower altitudes, typically near the Earth's surface. It is well-suited for simulating turbulence effects in the atmospheric boundary layer, the lowest part of the atmosphere influenced by the Earth's surface. To implement these models, operational altitudes for cruising have been defined as mid-level (above surface topology) to high-level (up to max service ceiling) [42, 43]. As per Federal Aviation Authority (FAA) definitions, both turbulence models are used simultaneously in the altitude range of 4000 *ft* to 45,000 *ft* [44]. For brevity, the turbulence intensity was kept at a moderate level.

2.4. Feedback controller (FC)

Given the aircraft's inherent vulnerability to open-loop instability, especially in the presence of turbulent wind conditions, a stabilizing feedback controller was needed. This controller ensures the aircraft's performance can be accurately evaluated, particularly amidst unstable flight dynamics and challenging environmental factors (such as turbulence). While the primary focus of this paper is the development of the aircraft model, the inclusion of the controller is for validating its behavior under real-world constraints. The two types of controllers designed are as follows.

2.4.1. Flight envelope control model

Controller for operational adherence to the flight envelope helps eliminate steady-state error and improve the system's transient response.

$$\Delta u_2(t) = K_A \cdot e_1(t) + K_{u_2} \cdot \int e_2(t)dt. \quad (2.45)$$

Here $\Delta u_2(t)$ is the TVS deflection input about the y -axis, $e_1(t)$ is the altitude error, $\int e_2(t)dt$ is the accumulated controller error over time, which works as an output correction memory function and K_A and K_{u_2} are controller gains with the ranges being $0.1 \leq K_A \leq 10$ and $0.01 \leq K_{u_2} \leq 1$, respectively. The error $e_1(t)$ for the flight envelope controller can be computed by,

$$e_1(t) = h_{max} - h_{current}. \quad (2.46)$$

Here, h_{max} is the maximum ceiling altitude of the aircraft, and $h_{current}$ is its current flight altitude. The error $e_2(t)$ of the controller is calculated by,

$$e_2(t) = \Delta u_2(desired) - \Delta u_2(current). \quad (2.47)$$

2.4.2. Stall recovery control model

The controller for prevention of stalling by adhering to AoA thresholds are given by,

$$\Delta u_2(t) = K_{a_1} \cdot e_3(t) + K_{a_2} \cdot \int e_3(t)dt. \quad (2.48)$$

Here, $\delta u_2(t)$ is the TVS deflection input about the y axis, $e_3(t)$ represents the angle of attack error, K_{a_1} and K_{a_2} are controller gains within the ranges $0.1 \leq K_{a_1} \leq 10$ and $0.01 \leq K_{a_2} \leq 100$, respectively. The error $e_3(t)$ for the above controller can be computed by,

$$e_3(t) = AoA_{critical} - AoA_{current}. \quad (2.49)$$

This feedback controller inherently balances competing objectives, such as altitude-tracking precision, thrust vector deflection effort, and actuator load. Hence, this can be interpreted as a Pareto-optimal control trade-off, where each operating point corresponds to a compromise between performance and efficiency. However, future optimizations of these gains will explicitly employ multi-objective formulations to map the trade-off surface.

2.4.3. Controller activation and switching

The activation and switching of the flight envelope controller (2.45) and stall controller (2.48) depend on the aircraft's altitude and angle of attack, respectively. This model triggers the flight envelope controller activation at altitudes below 5000 *ft* (hard deck) and above 45,000 *ft* (service ceiling). The stall recovery controller works as a local mechanism and activates when the $AoA > 0.35 \text{ rad}$. Table 2 presents the switching parameters of the two controllers,

Table 2. Controller switch conditions.

Scenario	Priority 1	Priority 2
Flight ceiling breached and aircraft stalling	Prevent stall, switch to stall controller	Manage altitude, switch to envelope controller
Flight ceiling breached and aircraft not stalling	Manage altitude, switch to envelope controller	Keep stall controller idle
Flight ceiling not breached and aircraft stalling	Prevent stall, switch to stall controller	Keep envelope controller idle
Hard deck breached and aircraft stalling	Prevent stall, switch to stall controller	Manage altitude, switch to envelope controller
Hard deck breached and aircraft not stalling	Manage altitude, switch to envelope controller	Keep stall controller idle
Ceiling hard deck not breached and not stalling	No controllers, maintain the idle setting	

In this research, we consider thrust vectoring as the sole effector to isolate and characterize its dynamic influence. In a practical application it would be worth unifying the same control architecture with conventional aerodynamic surfaces (e.g; elevons or split rudders) and thrust vectoring integrated towards control effort distribution for effective cross regime operations. Future work will investigate this cooperative TVC-surface strategy to enhance energy efficiency and redundancy.

3. Main results

This section shows the results obtained through the nonlinear quadratic state model (2.28) and the angular position model (2.36) during the following three operation conditions and two flight conditions mentioned earlier.

- Steady climb with a set angle of attack of 0.1 rad , transitioning into level flight with a cruising speed capped at 320 m/s .
- TVC engine positive pitch deflection of 0.17 rads for 5 seconds.

- TVC engine positive yaw deflection of 0.17 rads for 5 seconds.

The conditions mentioned above have been carefully selected to highlight the significant impact of TVC on the results. We opted the selection to reflect the standard operational limits reached by most large airliners [33].

The results are analyzed in the body-fixed frame, where velocity components u, v, w and rotational rates p, q, r describe motion relative to the aircraft's own axes rather than absolute movement in an inertial frame. This distinction is essential in correctly interpreting aerodynamic responses and control system behavior.

- Translational velocities (u, v, w)
 - u (longitudinal velocity): Influenced by thrust vectoring, drag forces, and flight envelope constraints that regulate cruise speed.
 - v (lateral velocity): Fluctuates due to yaw pulses, lateral disturbances, and corrective control inputs.
 - w (vertical velocity): A negative w does not indicate descent but rather downward airflow in the body-fixed frame, a direct result of angle of attack and aerodynamic effects.
- Rotational velocities (p, q, r)
 - p (roll rate): Dictated by thrust vectoring and lateral stability corrections.
 - q (pitch rate): Adjusted to maintain altitude and stability, particularly in transient conditions such as turbulence or maneuvering.
 - r (yaw rate): Driven by yaw pulses, asymmetric thrust, and corrective control efforts to maintain trajectory.
- Angular positions (θ, ζ, ψ)
 - θ (pitch angle): Reflects the aircraft's nose-up or nose-down orientation relative to the horizon, directly influenced by the angle of attack and pitch maneuvers.
 - ζ (roll angle): Represents the aircraft's banking or tilting about the longitudinal axis, affected by roll commands and lateral stability adjustments.
 - ψ (yaw angle): Indicates the heading direction relative to a fixed reference, primarily altered by yaw maneuvers and asymmetric thrust forces.

The momentary spikes observed in body-fixed velocity

and angular rates are due to rapid controller corrections during flight disturbances rather than actual excessive accelerations. Thus, the flight envelope controller plays a key role in dynamically maintaining altitude, speed, and maneuverability constraints under these conditions.

3.1. Climbing and level flight cruise

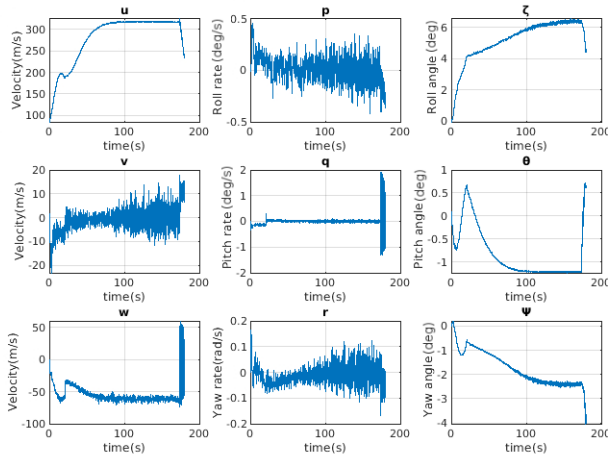


Figure 4. Aircraft dynamics during climb and level flight (under turbulence conditions with FC).

Figure 4 shows the model performance during turbulent conditions while the FC is in operation. The positive velocity gradient between $0 \leq t \leq 30$ shows the flight envelop controller attempting to elevate the aircraft beyond the minimum ceiling in Figure 4. Once the aircraft passes the 5000 ft at $t = 30$, the graph shows a change of gradient, which reflects the flight envelop controller switching off and the initial control input (operator input) switching back on. From $30 < t < 170$, the results show that the aircraft operates similarly to ideal conditions. The drop at $t = 170$ illustrates that the aircraft reaches maximum ceiling altitude, triggering the feedback controller for altitude governing. Thus, the aircraft is given a controlled dive to a lower altitude. The model achieved a stabilized state with the FC compared to the previous condition when the FC was absent in Appendix B subsection II. However, as the FC has no contribution to manipulating the lateral velocity v oscillations and roll p , the results show no change. Nevertheless, the system reflects significant improvements

in rotational velocity q and angular position ψ . The effect of chaotic wind conditions on angular position change has become reduced about the x-axis and more streamlined. The angular position about the y-axis is similar to ideal condition results; the flight envelop controller's effect can be seen $0 < t < 30$ with a positive deflection, which at $t = 30$ reverts back to its ideal path. The increase in the roll position is mainly due to a single-engine TVS's inability to counteract induced roll motions.

3.2. Results of induced pitch moment

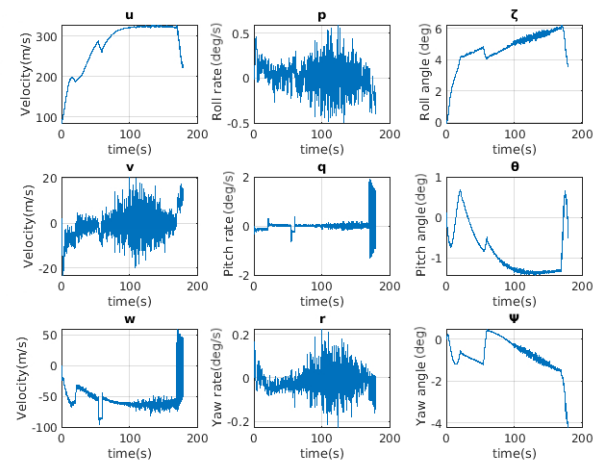


Figure 5. Aircraft dynamics during pitch maneuver (under turbulence conditions with FC).

Figure 5 presents the velocities and positions of the aircraft with the FC in operation. Similar to Appendix A subsection II, the pitch pulse is given at the same position for brevity. Movement in the x-axis: The first pulse point in u at $t = 10$ is due to the FC overcoming the minimum ceiling altitude, and the second pulse at $t = 50$ characterizes the pitch pulse. However, the drastic drop at $t = 180$ shows the system configuration to avoid the maximum ceiling. Movement in the y-axis: The y-axis velocity oscillates mainly due to the FC's lack of local lateral motion control. Movement in the z-axis: The velocity w can be seen as better regulated but is not smooth. A more sophisticated controller (compared to the current FC) is required to improve this performance. Rotational movements: Similar to the climb and cruise, a single-engine TV system does not control

these flight conditions. Hence, the dilation of oscillations in rotational velocity p cannot be reduced using any controller. Furthermore, due to the limitation of a local control system for lateral dynamics, the rotational velocity about the the z -axis ψ cannot be fine-tuned, which can aid in developing a more complex control algorithm. Similar to the results presented in Appendix A subsection III, the angular position is more controlled in the case of ζ and psi , with a deviation in ζ . Then, ψ shows a behavior similar to the expected physical phenomena. The position θ is nearly identical to the ideal condition.

3.3. Results of induced yaw moment

Figure 6 shows the results obtained under the same conditions described in Appendix B, subsection II with the feedback controller (FC) activated and the yaw pulse applied at the same time point.

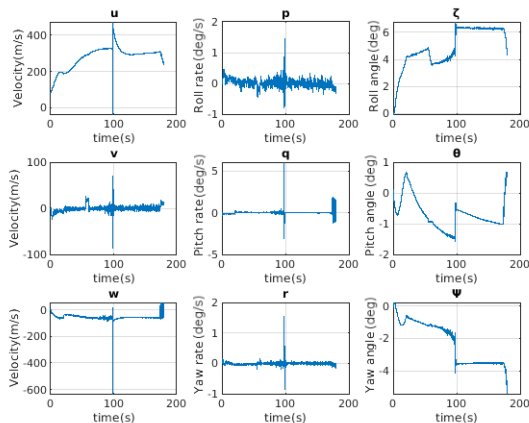


Figure 6. Aircraft dynamics during yaw maneuver (under turbulence conditions with FC).

Movement in the x -axis: The linear velocity was affected in three instances in this case: the minimum ceiling compensation $0 \leq t \leq 30$, the maximum ceiling correction $t = 100$, and the cruising speed regulation $t=180$. The effect between $0 \leq t \leq 30$ is observed due to the envelope controller trying to maintain the aircraft's trajectory within predefined safe altitude limits. The maximum ceiling is reached at two instances, $t = 100$ and $t = 180$, thus explaining the controlled decrease of velocity under the effect of the flight envelope controller upper boundary.

Movement in the y -axis: The system shows better control over velocity in the y -axis. The yaw pulse is recognizable in this case at $t = 50$. The ceiling altitude safety limit was represented by the spike at $t = 100$. As the model continuously goes through turbulence, the oscillation effect is controlled to a reasonable extent by the controllers, enough to range it closer to 0 ms^{-1} .

Movement in the z -axis: The changes in lift velocity w have become similar to the ideal condition but have not become equivalent. The effect of the flight envelope controller's lower bound can be seen from $0 \leq t \leq 30$ similar to the x -axis velocity. The controller's effort to dive the aircraft back to a safe altitude can be seen at $t = 100$ and $t = 180$.

Rotational movements: The angular velocities show a magnitude deviation at $t = 100$ due to the aircraft reaching maximum ceiling altitude. The amplitude of oscillations has reduced significantly and has come closer to 0 ms^{-1} .

The angular position of the aircraft has become much more stable and has avoided the effect of turbulence to a significant level. The system is deemed reliable with the use of FC in pitch deflection and cruising. However, it carries a significant positional deviation during the yaw motion in the global spatial context. These deviations can be seen between $30 < t < 50$ and at $t = 100$.

3.4. Results of local position

Apart from the velocity profiles presented above, the observed flight path behavior is presented in Table 3 below. Starting from the leftmost column, the figure illustrates the flight path in ideal conditions, turbulent conditions without FC, and while the FC is incorporated. Comparing the rightmost column to the middle column, a significant improvement was incurred in the system's stability and resilience towards operating in moderate turbulence conditions. The utilization of FC has enhanced the effect of β and α on yaw. This is implied by the corrected side slip of the aircraft position compared to the ideal motion. The FC does not compromise to the induced local yaw $[x - y]$ plane control sequence, which is vital in these results.

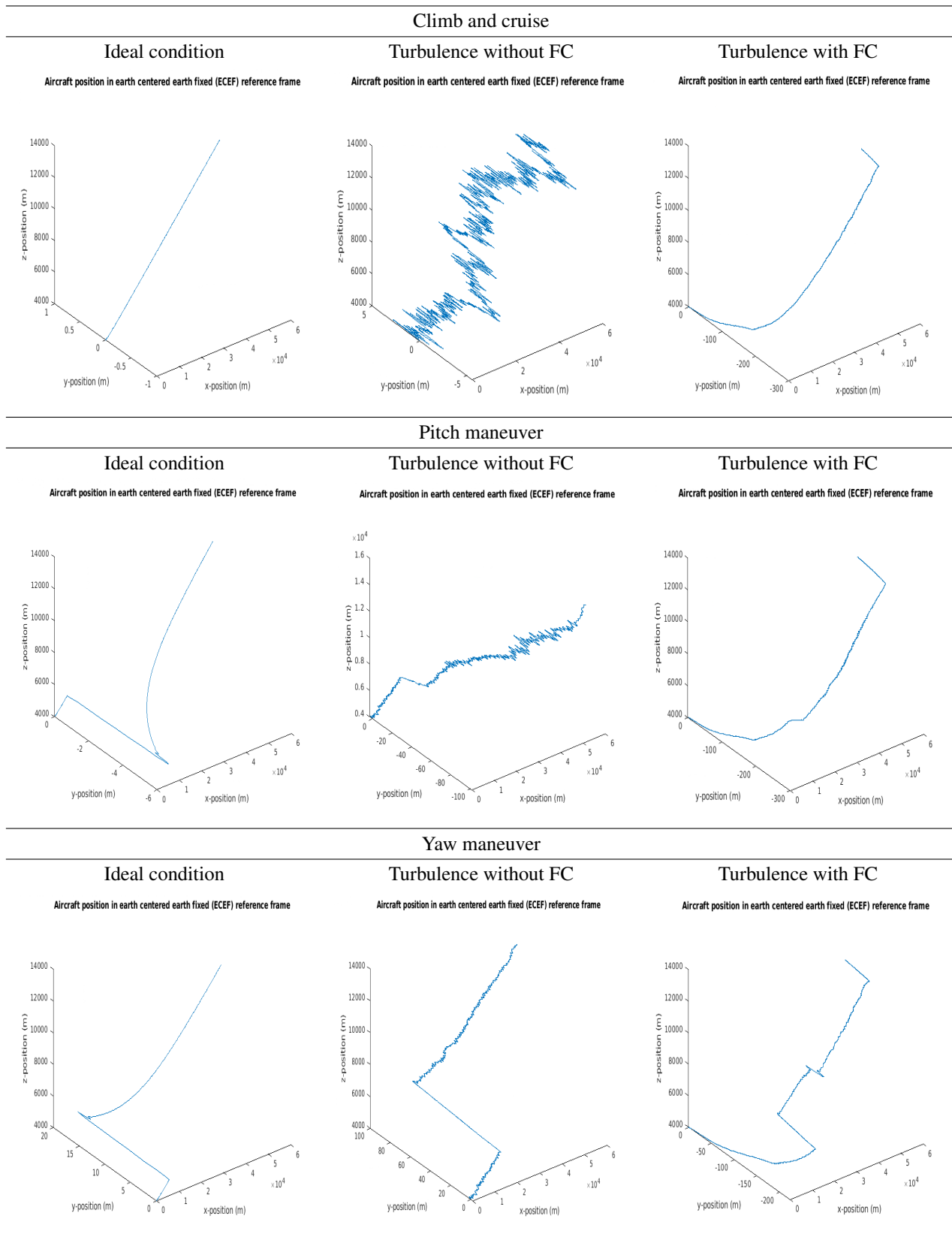
Table 3. All condition aircraft flight path behavior (3-dimensional view).

Table 4 above shows a breakdown of the discrepancies and deviations encountered (if any) in the test results presented above and FC's control effectiveness. Notice that the discrepancies encountered are, 'aerodynamic overshoot', 'actuator overshoot', and 'turbulent oscillations'. These discrepancies can be defined as follows:

- **Aerodynamic overshoot:** The state variables fluctuating beyond the expected region of operation due to the instantaneous motion.
- **Actuator overshoots:** The actuator's response surpassing the desired range of operation.
- **Turbulent oscillations:** The oscillations appear in state variables of the aircraft as an effect of wind turbulence.

Table 4. Discrepancy evasion (under turbulent conditions with FC in operation).

State	Climb and Cruise Discrep.	φ - Pitch		ε - Yaw		
		FC	Discrep.	FC	Discrep.	FC
u	N/A	✓	AcO	✓✓	N/A	✓
v	TO	xx	TO	xx	AeO	✓x
w	AcO	✓x	AcO	✓x	AcO	✓x
p	TO	xx	TO	xx	TO	✓x
r	TO	✓x	AeO & TO	xx	AeO & TO	✓x
ζ	TO	✓x	TO	✓x	AcO	✓x
Θ	N/A	✓	AcO	✓✓	N/A	✓
ψ	AcO	✓✓	AeO	✓✓	AcO	✓x

*Note: Discrepancy averted - ✓✓ No declared discrepancy - ✓ Discrepancy reduced - ✓x Discrepancy not averted- xx

Turbulent Oscillation - TO Actuator Overshoot - AcO Aerodynamic Overshoot - AeO

These overshoots can generally be elaborated as the system response exceeding the desired values during the transient phase. The occurrence of overshoots in this context are in accordance with established physical theories, mirroring the progressive outcomes depicted by the system dynamics model. Furthermore, the occurrence of overshoot serves to underscore the resilience of the dynamic model, even when operating under the assumption of instantaneous input deflections.

The integration of the FC has unveiled an optimal state of performance during moderate turbulence acting on the aircraft (Table 3). Hence, the above discrepancies can be neglected due to the fact that there is no apparent threat of resulting in critical dynamic motion failure. However, as we expanded the operational conditions to a more chaotic

nature, the observations posed an imminent risk of critical system failure. The observations made in such situations can be presented as turbulence oscillations, which can be defined as momentary fluctuations of the aircraft around its CoG in response to the continuous forces exerted by the wind. Hence, the model has the potential to further enhance the system's performance and resilience by incorporating a more complex control system.

4. Conclusions

In this paper, we developed a novel first-order dynamic mathematical model of a MESTV aircraft and identified an engine configuration that allows both vertical and horizontal maneuvers with minimal wind dependence. This study model was tested in turbulence-integrated evaluation for its stability under real-world attributes and aggressive maneuvers. Unlike prior research that assumes idealized or steady-state conditions, this study explicitly incorporated turbulence effects into the TVC framework, allowing for a more accurate representation of real-world aerodynamic behavior. The test results confirmed that operations such as climb and cruising and short-range pitch and yaw maneuvers are feasible with a single TV engine without compromising performance. The implementation of an advanced feedback control (FC) system demonstrated its effectiveness in stabilizing velocity and angular rate deviations induced by turbulence and yaw pulses. In addition, the study introduced a flight envelope management system that dynamically regulates altitude boundaries, ensuring operational safety under extreme conditions. The results provided detailed insights into the localized aerodynamic effects in the body-fixed frame, particularly in transient conditions where conventional TVC models fail to capture short-duration oscillatory responses. The findings of this research have direct applications in the development of next-generation UAVs and advanced manned aircraft that use thrust vectoring as a primary control mechanism. The integration of flight envelope controllers with the MESTV system presents enhanced maneuverability, robustness, and resilience to environmental disturbances in autonomous flight operations. Moreover, the transient response analysis presented in this study offers valuable insights to optimize

control strategies in TVC-based aerial systems, particularly in extreme flight conditions. Future work could focus on further refining the control mechanisms to address the issue of overshoot and enhance overall stability under various operational conditions.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that there is no conflict of interest in this paper.

Glossary

T_A, T_E	- Main, and Vector Engine Thrust
D, Y, L	- Drag, Lateral Force, Lift
L, M, N	- Total moment around x, y, z -axes
$q, \dot{q}, r$	- Angular velocity around x, y, z -axes
R	- Distance to Main engine from CoG
R_x	- Distance to Vector Engine from CoG
g_x, g_y, g_z	- Gravitational force in x, y, z direction
u, v, w	- Velocity in x, y, z direction
ϕ	- Vector Engine deviation for yaw
ε	- Vector Engine deviation for pitch
δm	- Variation of mass
r_c	- Distance to variable mass from CoG
I_{xx}, I_{yy}, I_{zz}	- Inertia in x, y, z directions
τ, θ, ψ	- Angular position about x, y, z axes

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Appendix

This developed model is validated using the real-life physical and aerodynamic data of the Boeing 747–400 [36] due to the readily available blueprint data through Boeing/NASA public records and the built-in tools of MATLAB 2024A. The simulations are conducted in MATLAB 2024A (Total Academic Headcount (TAH) license) software using 13th Gen Intel® Core™ i5-13450HX 2.40 GHz with 16.0GB RAM (15.7 GB usable).

This appendix provides baseline reference scenarios of aircraft dynamics under ideal conditions and turbulent conditions without active feedback control (FC). These results do not introduce contributions but serve as a comparative framework for the main text, where actual control interventions demonstrate stability improvements.

Appendix A. During ideal conditions

This section presents the expected flight behavior in ideal conditions, without turbulence, external disturbances, or actuator faults. These results are later contrasted with control-driven stabilization efforts in turbulent flight conditions.

A.1. Climbing and level flight cruising

This subsection analyzes the aircraft's steady climb and level cruise performance under ideal, disturbance-free conditions. The results provide a reference case for evaluating turbulence effects in Appendix B.

Figure A1 shows the reference results obtained in climbing and level flight cruising conditions.

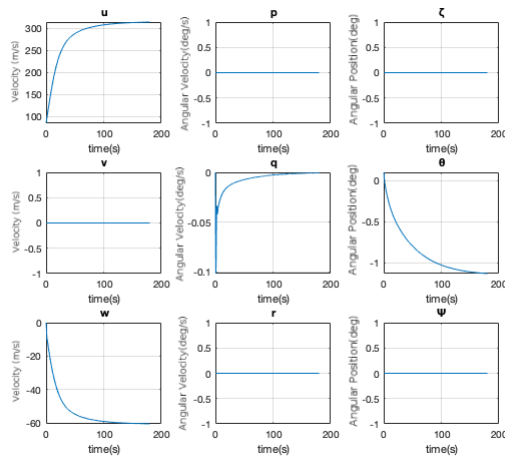


Figure A1. Reference flight behavior under Ideal conditions during climb and level flight cruise.

Translational motion

- Before climb initiation ($t < 10$): The forward velocity u increases as expected, driven by the combined thrust of the main and TVC engines. The lateral velocity v remains zero, as no external lateral disturbances are acting on the aircraft. The vertical velocity w increases steadily due to positive lift generation, indicating continuous altitude gain.
- During steady climb ($10 < t < 60$): The forward velocity u continues increasing, stabilizing at 320 m s^{-1} once the aircraft reaches cruise conditions. The vertical velocity w remains negative in the body-fixed frame, following the

aerodynamic convention where upward motion is along the negative z -axis. The lateral velocity v remains zero, confirming no unintended yaw or roll motions.

- During level flight cruise ($t > 60$): The aircraft reaches a steady-state condition, with velocity components maintaining equilibrium. The vertical velocity w stabilizes, ensuring the aircraft remains at constant altitude.

Rotational motion

- Before climb initiation ($t < 10$): The pitch rate q is near zero, as no moment is applied. The roll rate p and yaw rate r remain at zero, indicating no lateral disturbances.
- During climb and cruise ($10 < t < 60$): The pitch rate q momentarily increases as the aircraft adjusts its angle of attack but converges to zero in steady cruise. Roll (p) and yaw (r) rates remain zero, confirming pure forward flight stability.

Angular position

Pitch angle θ adjusts to -1.125 rad , aligning with expected equilibrium conditions. Roll ζ and yaw ψ angles remain at zero, ensuring no unintended deviations.

Key insights

- No turbulence allows stable climb and cruise behavior, free of disturbances or oscillations.
- Lift generation aligns with theoretical expectations, confirming model validity.
- Provides a benchmark for evaluating deviations in turbulence-affected cases.

A.2. TVC-Induced pitch maneuver

Figure A2 shows the aircraft dynamics during the TVC-induced pitch maneuver under ideal conditions.

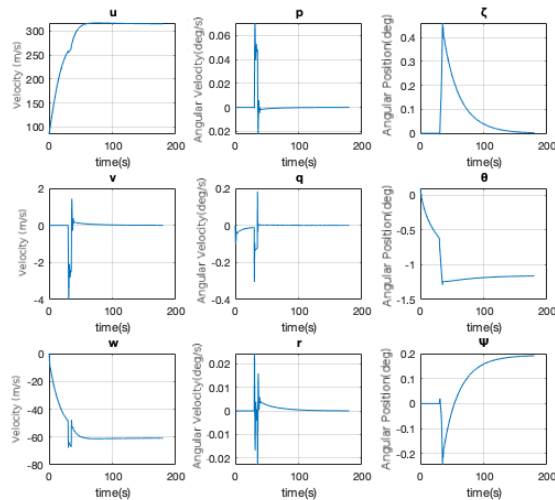


Figure A2. Aircraft dynamics during pitch maneuver.

Translational motion

- Before maneuver ($t < 30$): The forward velocity u steadily increases, driven by the combined thrust of the main and TVC engines. The lateral velocity v remains zero, as there are no external lateral disturbances. The vertical velocity w is positive (indicating upward movement due to lift), following the expected steady-state aerodynamic response.
- During maneuver ($30 < t < 35$): As the TVC engine deflects downward, the force component in the x -direction becomes $T_E \cos \epsilon$, temporarily reducing forward thrust. A sudden but brief increase in lateral velocity v occurs, due to the transient aerodynamic imbalance caused by the maneuver. Vertical velocity w increases sharply, driven by the increased lift and downward thrust component $T_E \sin \epsilon$.
- After maneuver ($t > 35$): The forward velocity u stabilizes and smoothly converges to 320 m s^{-1} , as the TVC engine returns to its neutral position. Lateral velocity v returns to zero, indicating recovery from any transient sideslip effects. Vertical velocity w stabilizes again, confirming a successful, stable pitch-up maneuver.

Rotational motion

- Before maneuver ($t < 30$): The roll rate p and yaw rate r remain zero, as expected in a pure forward flight scenario. The pitch rate q remains negligible, since no moment is applied.

- During maneuver ($30 < t < 35$): Pitch rate q increases sharply, as expected due to the applied pitch moment from TVC thrust. A brief roll rate p deviation is observed, caused by roll-yaw coupling effects from sideslip. Yaw rate r momentarily fluctuates, due to the coupling of angle of attack (α) and sideslip (β), but stabilizes once the maneuver is complete.

- After maneuver ($t > 35$): Pitch rate q returns to zero, indicating stable pitch control and maneuver completion. Roll rate p and yaw rate r also return to zero, confirming the absence of lingering coupling effects.

Angular position

- Before maneuver ($t < 30$): The pitch angle θ remains constant, as expected in steady flight. The roll angle ζ and yaw angle ψ remain at zero, indicating no lateral deviation.
- During maneuver ($30 < t < 35$): Pitch angle θ increases significantly, reflecting the aircraft's pitch-up motion. Yaw angle ψ shows a slight transient deviation, due to the coupling between β and α , but remains within expected limits.
- After maneuver ($t > 35$): The aircraft stabilizes at a new steady-state pitch angle, slightly below the peak deflection due to aerodynamic lift stabilization. Yaw angle ψ fully recovers, confirming that the maneuver remains controlled and does not introduce long-term instability.

Key insights

- No external disturbances allow a clean pitch response, without turbulence-induced deviations or undesired oscillations.
- Minimal roll and yaw coupling effects occur, confirming that the maneuver remains primarily a pure pitch maneuver under ideal conditions.
- The aircraft fully stabilizes after the maneuver, demonstrating predictable and well-behaved thrust vectoring dynamics.

A.3. TVC-Induced yaw maneuver

This subsection examines the yaw response of the aircraft when the TVC engine is deflected laterally by $\phi = 0.17 \text{ rad}$ at $t = 30 \text{ s}$, generating a pure yaw moment. Figure A3 shows the resulting aircraft dynamics for this maneuver.

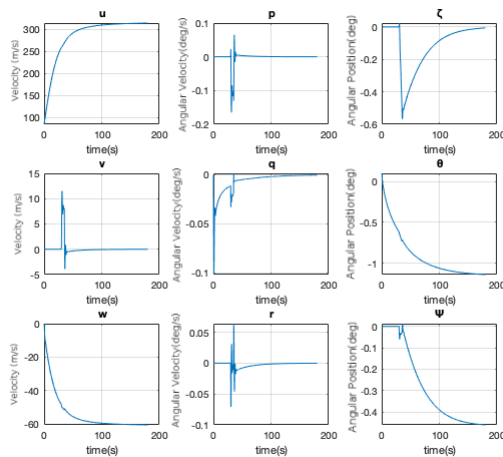


Figure A3. Aircraft dynamics during yaw maneuver.

Translational motion

- Before maneuver ($t < 30$): The forward velocity u steadily increases, driven by the combined thrust of the main and TVC engines. The lateral velocity v remains zero, as there are no external lateral disturbances. The vertical velocity w is positive (indicating upward movement due to lift), following the expected steady-state aerodynamic response.
- During maneuver ($30 < t < 35$): Forward velocity u remains nearly constant, as lateral thrust redirection has minimal impact on longitudinal motion. Lateral velocity v increases in the negative y -direction, confirming the expected side-slip effect caused by TVC yaw deflection. Vertical velocity w exhibits a brief fluctuation, reflecting minor aerodynamic interactions with lift forces.
- After maneuver ($t > 35$): Forward velocity u stabilizes, confirming minimal thrust loss due to yaw deflection. Lateral velocity v returns to near-zero, indicating successful recovery from the side-slip effect. Vertical velocity w remains stable, verifying that the yaw maneuver does not introduce altitude deviations.

Rotational motion

- Before maneuver ($t < 30$): The roll rate p , pitch rate q , and yaw rate r remain near zero, indicating stable level flight.
- During maneuver ($30 < t < 35$): Yaw rate r increases sharply, confirming the aircraft's intended yaw response. Roll rate p briefly deviates, due to roll-yaw aerodynamic coupling, where the advancing wing momentarily generates

asymmetric lift. Pitch rate q remains largely unaffected, confirming the dominance of yaw control.

- After maneuver ($t > 35$): Yaw rate r gradually decreases, stabilizing as the maneuver completes. Roll rate p returns to zero, confirming that roll-yaw coupling effects are transient. Pitch rate q remains unchanged, demonstrating that the maneuver does not significantly affect pitch dynamics.

Angular position

- Before maneuver ($t < 30$): The pitch angle θ , roll angle ζ , and yaw angle ψ remain at zero, confirming steady flight conditions.
- During maneuver ($30 < t < 35$): Yaw angle ψ increases, as expected from the applied yaw moment. Roll angle ζ experiences a minor transient deviation, due to yaw-induced rolling effects. Pitch angle θ remains largely unchanged, confirming that the maneuver does not introduce unintended pitch variations.
- After maneuver ($t > 35$): Yaw angle ψ stabilizes at a new steady-state value, aligning with the maneuver's intended trajectory change. Roll angle ζ returns to zero, verifying that roll-yaw coupling effects are temporary. Pitch angle θ remains stable, ensuring that altitude control is unaffected by yaw adjustments.

Key insights

- Yaw control remains predictable and stable, with minimal impact on other motion axes.
- The maneuver produces a clear, controlled yaw response, with only minor roll coupling effects.
- The aircraft fully stabilizes post-maneuver, confirming that the thrust vectoring system operates as expected in ideal conditions.

Appendix B. During turbulent flight conditions without FC

This section presents aircraft behavior under moderate turbulence, in the absence of active control compensation. These cases highlight natural instabilities, justifying the need for fault-tolerant control (FTC) and adaptive stabilization strategies, which are discussed in the main text.

B.1. Climbing and cruising

Figure B1 shows the aircraft dynamics under these turbulent conditions.

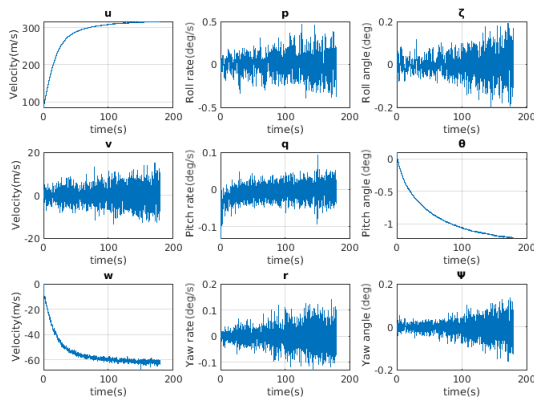


Figure B1. Aircraft dynamics during climb and level flight (under turbulence conditions without FC).

This subsection examines the aircraft's behavior under turbulent conditions during steady climb and level flight. The absence of feedback control allows turbulence-induced deviations to persist. conditions.

Translational motion

- Before reaching ceiling altitude: Forward velocity u exhibits steady acceleration, similar to the ideal case, but with minor fluctuations due to turbulent gusts. Lateral velocity v remains near zero on average, but exhibits oscillations, suggesting periodic side forces from turbulent disturbances. Vertical velocity w experiences continuous variations, closely following the ideal trend, but with added turbulence-induced oscillations.
- After reaching ceiling altitude: Forward velocity u stabilizes, but with persistent minor deviations, reflecting turbulence effects on cruise speed. Lateral velocity v continues to oscillate unpredictably, confirming that external forces disrupt directional stability in the absence of FC. Vertical velocity w remains generally negative, due to body-frame reference alignment, but exhibits frequent fluctuations, indicating that turbulence affects lift distribution.

Rotational motion

The roll rate p , pitch rate q , and yaw rate r are significantly more unstable than in the ideal scenario, oscillating without clear convergence. Yaw rate r fluctuates rapidly, suggesting that side gusts generate uncompensated lateral disturbances. Pitch rate q exhibits

transient deviations, correlating with variations in lift force due to turbulent airflow interactions. Roll rate p oscillates unpredictably, confirming that turbulence affects roll stability in the absence of feedback corrections.

Angular position

Pitch angle θ gradually converges, but remains in a state of continuous oscillation, confirming that turbulence disrupts smooth trajectory stabilization. Roll angle ζ and yaw angle ψ remain highly unstable, experiencing constant oscillations without a clear damping effect. The absence of FC prevents rapid stabilization, leading to prolonged positional deviations that persist throughout the flight.

Key insights

- Without FC, the aircraft exhibits significant oscillatory behavior, particularly in lateral and rotational motion.
- Turbulence affects not only vertical velocity but also introduces side-slip and roll disturbances.
- While forward velocity stabilizes, rotational instabilities remain unresolved, reinforcing the need for feedback control.
- This uncontrolled scenario serves as a baseline to evaluate the effectiveness of the FC system in mitigating turbulence-induced instabilities.

B.2. TVC-Induced pitch maneuver

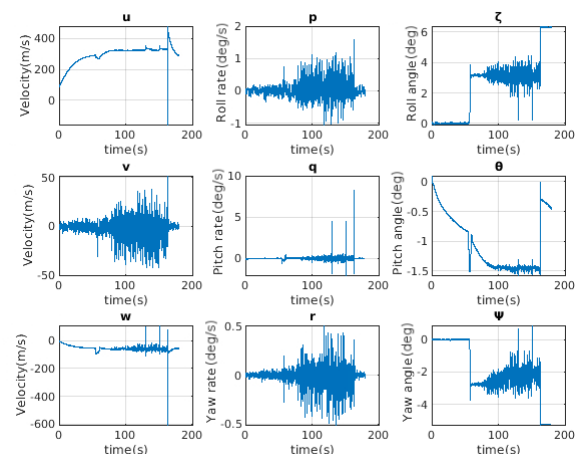


Figure B2. Aircraft dynamics during pitch maneuver (under turbulence conditions without FC).

This case evaluates the impact of turbulence on a pitch maneuver without active control compensation. The TVC engine deflects downward by $\epsilon = 0.17 \text{ rad}$ at $t = 30 \text{ s}$, but environmental disturbances cause deviations. Figure B2 shows the resulting aircraft dynamics.

Translational motion

- Before maneuver ($t < 50 \text{ s}$): The forward velocity u follows the expected acceleration trend, similar to ideal conditions, but exhibits minor distortions due to turbulence-induced drag variations. The lateral velocity v is unstable, fluctuating due to crosswinds that introduce side-slip effects before the maneuver begins. The vertical velocity w remains close to the ideal trajectory but oscillates significantly, reflecting uncompensated turbulence effects on lift.

- During maneuver ($50 < t < 55 \text{ s}$): A sharp drop in forward velocity u occurs, due to the thrust redirection. Side-slip effects increase, causing lateral velocity v to spike unpredictably. The vertical velocity w shows transient oscillations, amplified by turbulence.

- After maneuver ($t > 55 \text{ s}$): Forward velocity u recovers, reaching cruise speed faster than in the ideal case, as turbulent gusts contribute to momentum variations. Lateral velocity v remains unstable, confirming that turbulence affects directional stability, even after maneuver completion. Vertical velocity w continues fluctuating, reinforcing the necessity of active flight control for turbulence rejection.

Rotational motion

Pitch rate q initially follows the expected profile, but turbulence introduces irregular oscillations, affecting the smoothness of the maneuver. Roll rate p and yaw rate r exhibit unexpected deviations, confirming that turbulence amplifies roll-yaw coupling effects during pitch deflection. Yaw rate r fluctuates erratically, suggesting asymmetric turbulence contributions to the side force dynamics.

Angular position

Pitch angle θ increases as expected, but turbulence causes minor deviations from the ideal trajectory. Roll angle ϕ experiences transient disturbances, confirming lateral instability due to turbulence-exacerbated roll-yaw coupling. Yaw angle ψ exhibits unintended oscillations, reinforcing that without FC, turbulence significantly impacts maneuver precision.

Key insights

- Without FC, turbulence significantly affects all velocity and attitude variables, leading to increased oscillatory behavior.
- Uncompensated yaw deviations confirm that turbulence amplifies roll-yaw coupling, leading to increased positional instability.

B.3. TVC-Induced yaw maneuver

This case examines yaw deflection in turbulence without feedback control. The TVC engine deflects at $\phi = 0.17 \text{ rad}$ at $t = 30 \text{ s}$, initiating yaw rotation. Figure B3 shows the resulting aircraft dynamics.

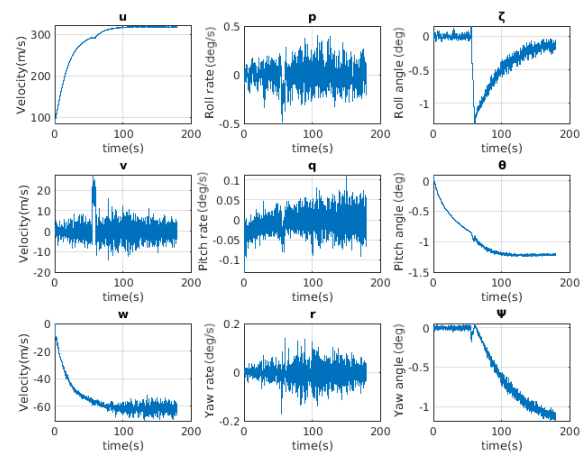


Figure B3. Aircraft dynamics during yaw maneuver (under turbulence conditions without FC).

Translational motion

- Before maneuver ($t < 50 \text{ s}$): Forward velocity u exhibits minor turbulence-induced fluctuations, but the overall acceleration trend remains comparable to the ideal case. Lateral velocity v is highly unstable, oscillating significantly due to crosswinds and turbulence effects. Vertical velocity w is disrupted by turbulence, preventing it from achieving the smooth convergence observed in ideal condition.

- During maneuver ($50 < t < 55 \text{ s}$): A yaw pulse is applied by deflecting the TVC engine, causing an expected velocity increment in u . Lateral velocity v increases sharply, but turbulence distorts the expected response, introducing unexpected side-slip variations. Vertical

velocity w oscillates erratically, indicating uncompensated aerodynamic disturbances due to turbulence-amplified side forces.

- After maneuver ($t > 55$ s): Forward velocity u stabilizes but shows turbulence-induced variations, unlike the ideal yaw response in Appendix A.3. Lateral velocity v remains unstable, emphasizing the impact of turbulence on directional control without FC. Vertical velocity w nears a converging trend but does not fully stabilize, reinforcing the necessity of active feedback control.

Rotational motion

Yaw rate r fluctuates unpredictably, due to turbulence amplifying roll-yaw coupling effects. Pitch rate q remains close to the expected profile, but turbulence causes minor deviations from ideal maneuver characteristics. Roll rate p experiences chaotic oscillations, confirming that yaw maneuvers in turbulence introduce significant cross-coupling effects.

Angular position

Yaw angle ψ exhibits a clear maneuver response, but remains affected by turbulence-driven oscillations. Roll angle ζ experiences unintended deviations, confirming lateral instability due to turbulence-exacerbated roll-yaw interactions. Pitch angle θ remains within expected bounds, but turbulence causes slight variations in the final convergence state.

Key insights

- Without FC, turbulence significantly disrupts yaw stability, increasing deviations from the expected trajectory.
- Lateral velocity fluctuations are amplified, leading to greater side-slip effects and roll-yaw coupling disturbances.
- Unlike the ideal maneuver, turbulence prevents complete stabilization post-maneuver, leaving residual oscillations in yaw and roll.
- Yaw rate deviations indicate that turbulence affects thrust vectoring efficiency, requiring active compensation to maintain control accuracy.

Conclusions on Appendix

- In ideal conditions, thrust vectoring enables precise control over pitch and yaw maneuvers without instability.
- Turbulence significantly degrades flight stability, particularly in yaw and roll axes.
- Without active control (FC), turbulence amplifies deviations, leading to persistent oscillations and instability.



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