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Research article

Adaptive control of a flexible satellite with an internal disturbance

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Abstract: This paper examines the well-posedness and adaptive stabilization of a satellite vibration caused by internal disturbances. The structure of the satellite is made up of two flexible, symmetrical wings that are connected to a central body. The wings are modeled as transmission Euler-Bernoulli beams with dynamic boundary conditions. We design high-gain adaptive regulators by measuring velocity feedback at the central body to quickly dampen vibrations. We demonstrate an exponential decay result for the system employing the multiplier method. The theoretical results are supported by a numerical simulation.

Keywords: flexible satellite; adaptive control; exponential decay; Euler-Bernoulli beams

1. Introduction

This study aims to investigate the adaptive boundary stabilization of a flexible satellite model subjected to internal disturbances. The satellite comprises a central body and two similar flexible wings, with its dynamics described as two Euler-Bernoulli beams attached to the central body (see [1]). Using Hamilton's principle, we get the equations of motion:

$$\begin{cases} \rho u_{tt}(x,t) + e u_{xxxx}(x,t) + \alpha u_t(x,t) = -f(u(x,t)), \\ x \in [0, l/2],\ t \geq 0, \\ \rho v_{tt}(x,t) + e v_{xxxx}(x,t) + \alpha v_t(x,t) = -f(v(x,t)), \\ x \in [l/2, l],\ t \geq 0, \end{cases} \tag{1.1}$$

subject to

$$\begin{cases} u(0,t) = u_x(0,t) = u_{xx}(l/2,t) = 0, & t \geq 0, \\ v(l,t) = v_x(l,t) = v_{xx}(l/2,t) = 0, & t \geq 0, \\ u(l/2,t) = v(l/2,t) = w(t), & t \geq 0, \\ m w_{tt}(t) = e u_{xxx}(l/2,t) - e v_{xxx}(l/2,t) + C(t), & t \geq 0, \end{cases} \tag{1.2}$$

and

$$\begin{cases} u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in [0, l/2], \\ v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), \quad x \in [l/2, l], \end{cases} \tag{1.3}$$

where $u(x,t)$ and $v(x,t)$ represent the left and right wings' transverse displacements at position x and time t. The wings have density ρ, bending stiffness e, and a viscous damping coefficient α. The term f represents an internal source that arises from disturbances affecting the satellite's dynamics, which is a locally Lipschitz continuous function in $\mathbb{R}$, with $F(u) = \int_0^u f(s)ds$ such that

$$f(u) = F'(u) \quad \text{and} \quad 0 \leq F(u) \leq u f(u), \qquad \forall u \in \mathbb{R}. \tag{1.4}$$

Here, m denotes the mass of the central body, and $C(t)$ refers to the control inputs applied to the central body, which will be determined later. The measured output of the center body is provided by

$$a(t) = w_t(t), \qquad t \geq 0. \tag{1.5}$$

The advancement of modern technology across various fields has driven a need for high-resolution imaging and precise control of satellite positioning. However, vibrations caused by internal or external forces can hinder these capabilities, motivating researchers to explore methods for vibration reduction. Control technology, in its various forms, has emerged as an effective solution. For instance, He and Ge [1] considered a satellite system (1.1)–(1.3) without disturbance (i.e., with $f \equiv 0$). They established an exponential stability result under the following control:

$$C(t) = -k_1 w(t) - k_2 w_t(t),$$

where k_i are control gains.

The exponential decay of the system (1.1)–(1.3) was established in [2] using a boundary control:

$$C(t) = -k_1 w(t) - k_2 w_t(t) - k_3 w_x(t) - k_4 w_{xt}(t), \qquad k_i \geq 0.$$

In [3], Je and Liu investigated system (1.1)–(1.3) with a bounded external disturbance $f(t)$ affecting the satellite's central body. Using the backstepping method, they designed a control law to regulate the satellite's vibrations. A similar result was achieved in [4] for the case of an internal disturbance $f(w)$ acting on the satellite's central body. In another study, Berkani [5] examined the asymptotic stability of system (1.1)–(1.3), incorporating viscoelastic damping in place of viscous damping and accounting for a distributed disturbance $f(x,t)$. For further research on the control of flexible satellites and spacecraft, see [6–8]. Numerous results have also been published concerning the control of flexible structures, including works such as [9–13].

In [14], Lasiecka and Toundykov studied a wave equation incorporating a source term and a localized damping. They successfully established the optimal decay rate for the system. In the case of the Petrovsky problem, extensive literature investigates the asymptotic behavior over time under internal damping effects (see [15–19]). Similarly, the von Kármán system has garnered significant attention in recent years; see [20–23] for further details. The notion of global stability, established via the Lyapunov method, plays a significant role in both theoretical research and practical scientific applications [24–27]. It is well known [28–30] that, in certain cases, stability can only be ensured through the application of appropriate control strategies.

We highlight here some notable contributions to adaptive control. Hassan and Tatar [31] investigated the boundary adaptive stabilization of a Timoshenko beam, designing high-gain adaptive regulators using velocity feedback measured at the right end. Kelleche and Tatar [32] investigated the stabilization of a nonlinear axially moving string using adaptive boundary control. In a related study, Kelleche and Tatar [33] addressed the adaptive stabilization of a Kirchhoff moving string employing high-gain adaptive output feedback. Furthermore, Kelleche and Saedpanah [34] examined the stabilization of an axially moving beam via adaptive boundary control, designing the control law based on a low-gain adaptive velocity feedback strategy.

To the best of our knowledge, the problem of satellite vibrations subjected to internal disturbance forces has not previously been addressed using adaptive control, as evidenced by the references above. This novelty constitutes a key contribution of the present work. In particular, we emphasize the distinction between the proposed high-gain adaptive control strategy and the conventional proportional-derivative (PD) control approach used in [35]. Furthermore, the existing literature appears to lack studies concerning the well-posedness and adaptive stability of flexible satellites in the presence of internal nonlinear disturbances. Motivated by this gap, we investigate the asymptotic stability of problem (1.1)–(1.3) under the impact of the following high-gain adaptive boundary controls:

$$\begin{cases} C(t) = -b(t)a(t), \\ b'(t) = da^2(t), \qquad b(0) = b_0, \ \ b_0, d > 0. \end{cases} \tag{1.6}$$

First, Section 2 introduces the necessary notations, assumptions, and technical lemmas. Section 3 examines the well-posedness through semigroup theory, Section 4 addresses the energy decay rate, and Section 5 focuses on numerical simulations.

2. Preliminary results

The energy of system (1.1)–(1.3) is

$$\begin{aligned} E(t) = &\frac{\rho}{2}\int_0^{l/2} u_t^2 dx + \frac{\rho}{2}\int_{l/2}^{l} v_t^2 dx + \frac{m}{2} w_t^2(t) + \frac{e}{2}\int_0^{l/2} u_{xx}^2 dx \\ &+ \frac{e}{2}\int_{l/2}^{l} v_{xx}^2 dx + \int_0^{l/2} F(u)dx + \int_{l/2}^{l} F(v)dx. \end{aligned} \tag{2.1}$$

This means the following.

Lemma 2.1. *The energy* (2.1) *satisfies*

$$E'(t) = -\alpha \int_0^{l/2} u_t^2 dx - \alpha \int_{l/2}^{l} v_t^2 dx - b(t)w_t^2(t) \le 0 \quad (2.2)$$

for $t \ge 0$.

Proof. Multiplying the first equation in (1.1) by u_t and integrating by parts over $(0, l/2)$ yields

$$\frac{d}{dt}\left[\int_0^{l/2} \frac{\rho}{2}u_t^2 + \frac{e}{2}u_{xx}^2 + F(u)dx\right] + eu_{xxx}w_t(t) = -\alpha \int_0^{l/2} u_t^2 dx. \quad (2.3)$$

Similarly, for the second equation in (1.1), we get

$$\frac{d}{dt}\left[\int_{l/2}^{l} \frac{\rho}{2}v_t^2 + \frac{e}{2}v_{xx}^2 + F(v)dx\right] - ev_{xxx}w_t(t) = -\alpha \int_{l/2}^{l} v_t^2 dx. \quad (2.4)$$

Combining (2.3), (2.4), and the last equation in (1.2), we obtain

$$E'(t) = -\alpha \int_0^{l/2} u_t^2 dx - \alpha \int_{l/2}^{l} v_t^2 dx + C(t)w_t(t),$$

using (1.5) and (1.6) to get (2.2). □

Proposition 2.1. *Let* $b(t)$ *be given in* (1.6)*, then*

$$b(t) \le b_1, \qquad \forall t \ge 0. \quad (2.5)$$

Here, $b_1 > 0$ *depends on the initial data and* b_0.

Proof. Let functional G be defined by

$$G(t) = E(t) + \frac{1}{2d}b^2(t).$$

Then, it follows from (1.5) and (1.6) that

$$\begin{aligned} G'(t) &= -\alpha \int_0^{l/2} u_t^2 dx - \alpha \int_{l/2}^{l} v_t^2 dx - b(t)w_t^2(t) + \frac{1}{d}b'(t)b(t) \\ &= -\alpha \int_0^{l/2} u_t^2 dx - \alpha \int_{l/2}^{l} v_t^2 dx \le 0. \end{aligned} \quad (2.6)$$

Consequently, we have

$$\frac{1}{2d}b^2(t) \le G(t) \le G(0).$$

By taking $b_1 = \sqrt{2dG(0)}$, we have (2.5). □

3. Well-posedness

Now, we state the well-posedness result for (1.1)–(1.3), which can be established using semigroup theory.

We also introduce the following spaces:

$$\mathcal{H} = H_0^2 \times L_0^2 \times H_l^2 \times L_l^2 \times \mathbb{R}^2,$$

where

$$L_a^2 = \left\{ u : [\frac{a}{2}, \frac{a+l}{2}] \longrightarrow \mathbb{R} | \int_{\frac{a}{2}}^{\frac{a+l}{2}} u dx < \infty, \right\}$$

and for $i, j \in \mathbb{N},\ j \le i,\ L_a^2 = H_a^{0,0}$,

$$H_a^{i,j} = \left\{ u : [\frac{a}{2}, \frac{a+l}{2}] \longrightarrow \mathbb{R} | u, u', \cdots, u^{(i)} \in L_a^2, \right. \\ \left. u(a) = u'(a) = \cdots = u^{(j)}(a) = 0 \right\}.$$

The inner product in $\mathcal{H}$ is

$$\begin{aligned}(y_1, y_2)_{\mathcal{H}} = {} & \frac{\rho}{2}\int_0^{l/2} \bar{u}_1\bar{u}_2 dx + \frac{\rho}{2}\int_{l/2}^{l} \bar{v}_1\bar{v}_2 dx + \frac{m}{2}\bar{w}_1\bar{w}_2 \\ & + \frac{e}{2}\int_0^{l/2} u_{1xx}u_{2xx}dx + \frac{e}{2}\int_{l/2}^{l} v_{1xx}v_{2xx}dx + w_1w_2,\end{aligned}$$

where $y_i = (u_i, \bar{u}_i, v_i, \bar{v}_i, w_i, \bar{w}_i)^T \in \mathcal{H},\ i = 1, 2.$

By introducing the state $y = (u, \bar{u}, v, \bar{v}, w, \bar{w})^T$, system (1.1)–(1.3) becomes

$$\begin{cases} \dfrac{dy}{dt} = Ay, \quad t > 0, \\ y(0) = y_0 = (u_0, \bar{u}_0, v_0, \bar{v}_0, w_0, \bar{w}_0)^T, \end{cases}$$

where $A : \mathcal{H} \longrightarrow \mathcal{H}$ is defined as

$$Ay = \begin{pmatrix} \bar{u} \\ -e/\rho u_{xxxx} - \alpha/\rho\bar{u} - 1/\rho f(u) \\ \bar{v} \\ -e/\rho v_{xxxx} - \alpha/\rho\bar{v} - 1/\rho f(v) \\ \bar{w} \\ e/m(u - v)_{xxx}|_{l/2} + 1/mC(t) \end{pmatrix}$$

and

$$D(A) = \begin{Bmatrix} (u, \bar{u}, v, \bar{v}, w, \bar{w}) | u, v \in H_0^{4,2},\ \bar{u}, \bar{v} \in H_l^{2,0}, \\ w, \bar{w} \in \mathbb{R} \text{ and } u_{xx}(l/2) = v_{xx}(l/2) = 0 \end{Bmatrix}.$$

Theorem 3.1. *Let $y_0 \in D(A)$, and assume that* (1.4) *holds. Then, the problem* (1.1)–(1.3) *admits a unique solution satisfying*

$$y \in C^0([0,T]; D(A)) \cap C^1([0,T]; \mathcal{H}),$$

for any $T > 0$.

Proof. To prove this theorem, we adopt the nonlinear semigroup approach; see [36, 37]. □

4. Asymptotic behavior

This section examines the exponential stability of the system (1.1)–(1.3) via the use of the Lyapunov functional and convex analysis. To do this, let

$$\mathcal{L}(t) = E(t) + \beta\Phi_1(t) + \beta\Phi_2(t),$$

where $\beta > 0$ and

$$\Phi_1(t) = \frac{\alpha}{2}\int_0^{l/2} u^2 dx + \frac{\alpha}{2}\int_{l/2}^{l} v^2 dx, \tag{4.1}$$

$$\Phi_2(t) = \rho\int_0^{l/2} uu_t dx + \rho\int_{l/2}^{l} vv_t dx + mww_t. \tag{4.2}$$

Proposition 4.1. *If β is small enough, we have*

$$\alpha_1 E(t) \le \mathcal{L}(t) \le \alpha_2 E(t), \tag{4.3}$$

where α_1 and α_2 are positive constants.

Proof. This proposition can be proven by combining Young's inequality, the Cauchy –Schwarz inequality, and Poincaré's inequality (see [38]). □

To demonstrate the main decay result of the system, it is frequently necessary to establish the following technical lemmas:

Lemma 4.1. *The function $\Phi_1(t)$ satisfies*

$$\Phi_1'(t) = \alpha\int_0^{l/2} uu_t dx + \alpha\int_{l/2}^{l} vv_t dx. \tag{4.4}$$

Proof. A direct computation gives (4.4). □

Lemma 4.2. *The function $\Phi_2(t)$ satisfies*

$$\begin{aligned}\Phi_2'(t) = {}& \rho\int_0^{l/2} u_t^2 dx - e\int_0^{l/2} u_{xx}^2 dx - \alpha\int_0^{l/2} uu_t dx \\ & +\rho\int_{l/2}^{l} v_t^2 dx - e\int_{l/2}^{l} v_{xx}^2 dx - \alpha\int_{l/2}^{l} vv_t dx - \int_0^{l/2} F(u)dx \\ & -\int_{l/2}^{l} F(v)dx + mw_t^2 + mwC(t).\end{aligned} \tag{4.5}$$

Proof. Putting (1.1) into $\Phi_2'(t)$, we obtain

$$\begin{aligned}\Phi_2'(t) = {}& \rho\int_0^{l/2} u_t^2 dx - \int_0^{l/2} u(eu_{xxxx} + \alpha u_t + f(u))dx \\ & +\rho\int_{l/2}^{l} v_t^2 dx - \int_{l/2}^{l} v(ev_{xxxx} + \alpha v_t + f(v))dx \\ & +mw_t^2 + mww_{tt}.\end{aligned} \tag{4.6}$$

We utilize integration by parts in accordance with condition (1.2) to simplify the terms in (4.6) as follows:

$$-e\int_0^{l/2} uu_{xxxx} dx = -euu_{xxx}(l/2,t) - e\int_0^{l/2} u_{xx}^2 dx \tag{4.7}$$

and

$$-e\int_{l/2}^{l} vv_{xxxx} dx = evv_{xxx}(l/2,t) - e\int_{l/2}^{l} v_{xx}^2 dx. \tag{4.8}$$

By using the assumption (1.4), we have

$$-\int_0^{l/2} f(u)u dx \le -\int_0^{l/2} F(u)dx \tag{4.9}$$

and

$$-\int_{l/2}^{l} f(v)v dx \le -\int_{l/2}^{l} F(v)dx. \tag{4.10}$$

Substituting Eqs (4.7)–(4.10) in (4.6), we obtain (4.5). This completes the proof. □

Theorem 4.1. *Under assumptions* (1.4) *and control* (1.6)*, there exist strictly positive constants A_0 and α such that*

$$E(t) \le A_0 e^{-\alpha t}, \qquad \forall t > 0.$$

Proof. By combining (2.2), (4.4), and (4.5), we obtain, for some $\theta > 0$,

$$
\begin{aligned}
\mathcal{L}_1'(t) =& -(\alpha-\rho\beta)\int_0^{l/2} u_t^2 dx - e\beta\int_0^{l/2} u_{xx}^2 dx \\
&-(\alpha-\rho\beta)\int_{l/2}^{l} v_t^2 dx - e\beta\int_{l/2}^{l} v_{xx}^2 dx \\
&-\beta\int_0^{l/2} F(u)dx - \beta\int_{l/2}^{l} F(v)dx \\
&-b(t)w_t^2(t) + m\beta w_t^2 - m\beta b(t) w w_t(t) \\
\le& -(\alpha-\rho\beta)\int_0^{l/2} u_t^2 dx - e\beta\int_0^{l/2} u_{xx}^2 dx \\
&-(\alpha-\rho\beta)\int_{l/2}^{l} v_t^2 dx - e\beta\int_{l/2}^{l} v_{xx}^2 dx \\
&-\beta\int_0^{l/2} F(u)dx - \beta\int_{l/2}^{l} F(v)dx \\
&-\left(b(t) - m\beta - m\beta\frac{b^2(t)}{4\theta}\right)w_t^2(t) + m\beta\theta w^2(t).
\end{aligned} \tag{4.11}
$$

We use Poincaré inequality as follows:

$$
\begin{aligned}
2w^2(t) &= u^2(l/2,t) + u^2(l/2,t) \\
&\le \frac{l}{2}\int_0^{l/2} u_x^2 dx + \frac{l}{2}\int_0^{l/2} u_x^2 dx \\
&\le \frac{l^3}{8}\int_0^{l/2} u_{xx}^2 dx + \frac{l^2}{8}\int_0^{l/2} u_{xx}^2 dx.
\end{aligned} \tag{4.12}
$$

With the help of Proposition 2.1, (4.11), and (4.12), we entail that

$$
\begin{aligned}
\mathcal{L}_1'(t) \le& -(\alpha-\rho\beta)\int_0^{l/2} u_t^2 dx - \left(e - \frac{ml^3}{16}\theta\right)\beta\int_0^{l/2} u_{xx}^2 dx \\
&-(\alpha-\rho\beta)\int_{l/2}^{l} v_t^2 dx - \left(e - \frac{ml^3}{16}\theta\right)\beta\int_{l/2}^{l} v_{xx}^2 dx \\
&-\beta\int_0^{l/2} F(u)dx - \beta\int_{l/2}^{l} F(v)dx \\
&-\left(b(t) - m\beta\frac{4\theta + b^2(t)}{4\theta}\right)w_t^2(t) \\
\le& -(\alpha-\rho\beta)\int_0^{l/2} u_t^2 dx - \left(e - \frac{ml^3}{16}\theta\right)\beta\int_0^{l/2} u_{xx}^2 dx \\
&-(\alpha-\rho\beta)\int_{l/2}^{l} v_t^2 dx - \left(e - \frac{ml^3}{16}\theta\right)\beta\int_{l/2}^{l} v_{xx}^2 dx \\
&-\beta\int_0^{l/2} F(u)dx - \beta\int_{l/2}^{l} F(v)dx \\
&-\left(b_0 - m\beta\frac{4\theta + b_1^2}{4\theta}\right)w_t^2(t).
\end{aligned}
$$

Now, we choose $\theta < \min\left\{\frac{16e}{ml^3}, \frac{\alpha}{\rho}\right\}$ and $\beta \le \frac{b_0}{mb_1^2}$ so that

$$
\mathcal{L}'(t) \le -A_1 E(t), \qquad \forall t > 0, \tag{4.13}
$$

where $A_1 = 2\min\left\{\frac{\alpha-\rho\beta}{\rho}, \frac{16e - m\theta l^3}{16e}, 2\beta, \frac{4\theta b_0 - m\beta 4\theta - m\beta b_1^2}{4\theta m}\right\} > 0$.

Combining (4.3) and (4.13) yields

$$
\mathcal{L}'(t) \le -\frac{A_1}{\alpha_2}\mathcal{L}(t) = -\alpha\mathcal{L}(t), \qquad \forall t > 0. \tag{4.14}
$$

Integrating over $(0,t)$, we have

$$
\mathcal{L}(t) \le \mathcal{L}(0)e^{-\alpha t}. \tag{4.15}
$$

Using (4.3) in (4.15), we get

$$
E(t) \le \frac{\mathcal{L}(0)}{\alpha_1}e^{-\alpha t} = A_0 e^{-\alpha t}.
$$

This concludes the proof. □

5. Numerical simulations

This section presents simulation results using the finite difference approach to demonstrate the system's performance. The parameters of the flexible satellite are defined as $(l,\ \rho,\ e,\ \alpha,\ m) = (20\,m,\ 3.24\times10^2\ kg/m,\ 1.195\times 10^4\ Nm^2,\ 0.1,\ 20\ kg)$. In practice, these parameters are typically unknown. In such cases, they can be estimated using appropriate estimation algorithms (see [35, 39, 40]).

The initial dates are specified as $(u_0(x), u_1(x)) = (v_0(x), v_1(x)) = (\sin(0.3\pi x), 0)$. The internal disturbance is modeled as $f(x) = x^3$.

To demonstrate the effectiveness of adaptive control (1.6), it is compared against PD control with constant gains. Satellite dynamics are simulated under three scenarios: (i) without control ($C(t) = 0$), (ii) with PD control ($C(t) = -200a(t)$), and (iii) with adaptive control (1.6) using $b_0 = 200$ and $d = 10$. The dynamics of the left wing $u(0,t)$, the right wing $v(l,t)$, and the center body $w(t)$ are shown in Figures 1, 2, and 3, respectively, throughout all three scenarios.

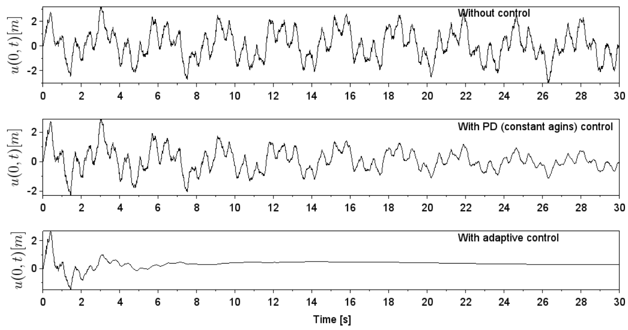

Figure 1. Left wings displacement at position $x = 0$.

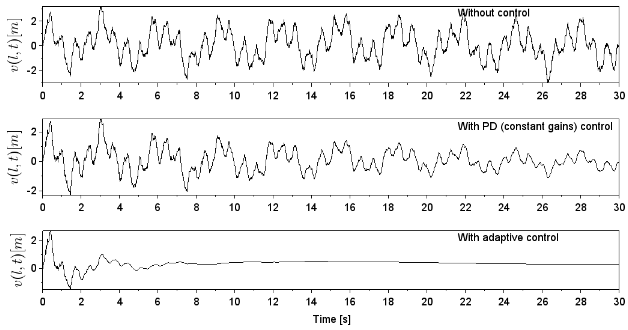

Figure 2. Right wings displacement at position $x = l$.

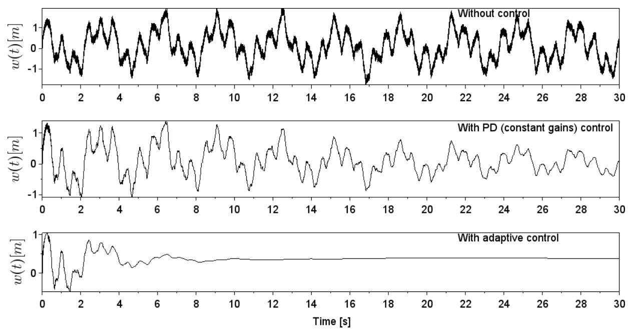

Figure 3. Central body displacement ($x = \frac{l}{2}$).

From the comparison of three cases in Figures 1–3, the proposed adaptive control (1.6) demonstrates superior performance, effectively suppressing the satellite vibrations within 10 seconds. In contrast, the PD control fails to achieve the desired level of vibration suppression within the 30 second simulation period. These results highlight the efficiency and robustness of the adaptive control in mitigating vibrations.

The temporal behavior of the adaptive control force and the PD control force is shown in the Figure 4.

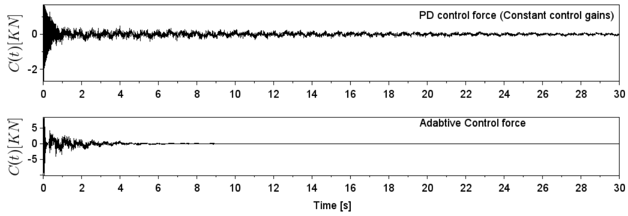

Figure 4. Control forces.

6. Conclusions

We have demonstrated that the proposed adaptive control based on low-gain output feedback ensures the exponential stabilization of the nonlinear satellite system under consideration. This result was established using a Lyapunov functional defined as the system's energy augmented by two suitable functionals. Furthermore, it was shown that the gain function remains bounded. The theoretical findings were corroborated through numerical simulations.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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