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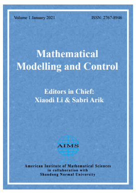

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Research article

A fractional mathematical model for COVID-19 outbreak transmission dynamics with the impact of isolation and social distancing

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Abstract: The Covid illness (COVID-19), which has emerged, is a highly infectious viral disease. This disease led to thousands of infected cases worldwide . Several mathematical compartmental models have been examined recently in order to better understand the Covid disease. The majority of these models rely on integer-order derivatives, which are incapable of capturing the fading memory and crossover behaviour observed in many biological phenomena. Similarly, the Covid disease is investigated in this paper by exploring the elements of COVID-19 pathogens using the non-integer Atangana-Baleanu-Caputo derivative. Using fixed point theory, we demonstrate the existence and uniqueness of the model's solution. All basic properties for the given model are investigated in addition to Ulam-Hyers stability analysis. The numerical scheme is based on Lagrange's interpolation polynomial developed to estimate the model's approximate solution. Using real-world data, we simulate the outcomes for different fractional orders in Matlab to illustrate the transmission patterns of the present Coronavirus-19 epidemic through graphs.

Keywords: fractional derivatives; fractional integral; Hyers Ulams stability; Lagrange's interpolation

1. Introduction

A novel coronavirus (SARS-CoV-2) emerged out of Wuhan city at the end of December, 2019 [1,2]. After spread to almost all countries and was declared a pandemic [3]. The outbreak has mostly been controlled in some countries since the end of April 2021, but still remains a series of public health and social-economic the problem in all over the world [4–6]. We know that most people infected with 2019-nCoV will experience mild to moderate respiratory illness, such as difficulty in breathing, sickness, cough, fever, and other symptoms [11, 13–15]. When an infected person sneezes, coughs, or comes in touch with a contaminated surface, the virus is mostly spread to healthy people via the mouth, nose, and eyes [7, 8]. The usual incubation time is 1 to 14 days, according to [9]. As the vaccine is not yet available everywhere, the control measures, for example, social distancing, wearing of masks, regular hand sanitation using sanitizer or soaps, and quarantine of the suspected individuals, are effective interventions that control the virus.

Mathematical models are useful tools for analysing outbreak dynamics. Mathematical models can be used to predict the virus's future spread, allowing the government to be prepared and take the necessary actions [18–20]. Several mathematical models were developed to better understand the behaviour of coronavirus disease. Researchers in [21] introduced a novel fractional model of COVID-19 while accounting for the effects of isolation and quarantine. The Mittag-Leffler, exponential kernels and power-law were used to simulate the proposed model. Mathematical modeling is an essential tool for formulating control strategies and forces guessing the spreading of COVID-19. To illustrate the dynamics of COVID-19, Chen and Zhao [7] established a susceptible, un-quarantine infected, quarantine infected, confirmed infected (SUQC) model. Similarly, Song et al. [17] formulated a mathematical model based on COVID-19 epidemiology, close contact, and the isolation of healthy persons. Acknowledge the means to control the spread of coronavirus in the people has been recorded in various mathematical modeling [12, 22].

Fractional calculus is the computation of integrals or derivatives of any positive real order. In recent years, there has been a lot of interest in the use of fractional differential equations. Many researchers use fractional differential equations in their research [24, 27, 46]. Therefore, many researchers have shown more interest in studying fractional derivative and fractional integrals. In the investigation of differential equation for optimization, numerical and qualitative importance was contributed by researchers [25, 26]. It is also notable that fractional differential equation has been characterized in various manners. It is well-known that regular kernels are not included in the definite integral. Therefore types of seeds have been included in different definitions. One crucial definition that has recently attracted the attention of researchers is the ABC derivative presented by Atangana, Baleanu, and Caputo [28] in 2016. This derivative exhibits the singular kernel by non-singular kernel, and So much has been studied [29, 30]. Because most of the non-linear problems are challenging to solve for their analytical or exact solution, different numerical methods have been introduced to solve these problems [10, 23, 31, 32]. The best numerical procedure has recently been developed to solve partial fractional differential equation under the ABC derivative; see for detail [33, 34].

The following model investigates the COVID-19 transmission dynamic. The model included seven compartments: susceptible population $\mathscr{S}$, expose population $\mathscr{E}$, infected papulation $\mathscr{I}$, Asymptomatic population $\mathscr{A}$, Quarantined population $\mathscr{Q}$, Hospitalized population $\mathscr{H}$ and the removed class $\mathscr{R}$ as:

$$\begin{cases} {}_0^{ABC}\mathscr{D}_t^{\alpha}[\mathscr{S}(t)] = \Lambda - (\delta+\mu)\,\mathscr{S}, \\ {}_0^{ABC}\mathscr{D}_t^{\alpha}[\mathscr{E}(t)] = \delta\mathscr{S} - (\pi+\mu)\,\mathscr{E}, \\ {}_0^{ABC}\mathscr{D}_t^{\alpha}[\mathscr{I}(t)] = \pi\sigma\,(1-c)\,\mathscr{E} + \phi\rho\mathscr{Q} + \gamma_1\mathscr{A} \\ \qquad\qquad\qquad - (\varphi_1+\omega_1+\mu_1+\mu)\,\mathscr{I}, \\ {}_0^{ABC}\mathscr{D}_t^{\alpha}[\mathscr{A}(t)] = \pi\,(1-\sigma)\,(1-c)\,\mathscr{E} + \gamma_A\mathscr{Q} \\ \qquad\qquad\qquad - (\gamma_1+\varphi_A+\mu)\,\mathscr{A}, \\ {}_0^{ABC}\mathscr{D}_t^{\alpha}[\mathscr{Q}(t)] = \pi c\mathscr{E} - (\gamma_A+\phi\rho+\mu)\,\mathscr{Q}, \\ {}_0^{ABC}\mathscr{D}_t^{\alpha}[\mathscr{H}(t)] = \omega_1\mathscr{I} - (\varphi_H+\mu_1+\mu)\,\mathscr{H}, \\ {}_0^{ABC}\mathscr{D}_t^{\alpha}[\mathscr{R}(t)] = \varphi_1\mathscr{I} + \varphi_A\mathscr{A} + \varphi_H\mathscr{H} - \mu\mathscr{R}, \end{cases} \tag{1.1}$$

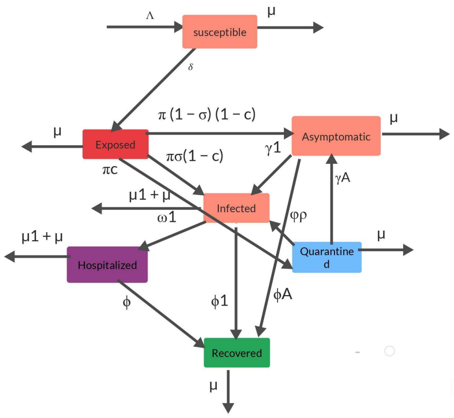

Figure 1. Flow diagram of the COVID-19 model (1.1).

with the following initial conditions $\mathscr{S}(0) > 0, \mathscr{E}(0) \geq 0, \mathscr{I}(0) \geq 0, \mathscr{A}(0) \geq 0, \mathscr{Q}(0) \geq, \mathscr{H}(t) \geq 0, \mathscr{R}(0) \geq 0$. The COVID-19 transmission dynamics are represented in the Figure 1 flow chart.

Some essential assumptions we put on the model are that all of the model (1.1) involved parameters be non-negative. The above model is examined from three perspectives. First, since the presented model (1.1) is recently defined, we use fixed point theory to demonstrate its existence. However, since stability is vital, we will examine Hyers-Ulam type stability for the proposed model. We refer to [43–45] for further information on the general models that use ordinary derivatives of fractional order. Furthermore, the model under consideration is nonlinear because it is sometimes impossible to discover an accurate solution to nonlinear situations. Therefore several numerical processes (methods) have been developed in the literature to deal with similar problems, see [39–42]. Therefore a Lagrange's interpolation polynomial approach is used to simulate the findings using Matlab-16.

We arranged the paper as Section 1 gives the introduction, section 2 describes preliminaries, section 3 shows the existence of the solution; section 4 offers uniqueness solution to the model, section 5 provides Hyers Ulams Stability analyses, and section 6 gives the numerical solution. Section 7 shows a graphical representation to support the analytical result. Finally, in section 8, we discuss the conclusion.

Table 1. Biological interpretations of parameters in model (1.1).

Parameters	Biological interpretations
Λ	Describes the birth rate
δ	The effective contact rate of social distancing
μ	Natural death rate
π	Shows the infected contact rate
σ	The rate of symptoms among infected people
μ_1	Showed the death rate due to the coronavirus
φ_1	Recovery rates of individuals in the infected class
φ_A	Recovery rates of individuals in the asymptomatic population
ω_1	The transmission rate from the infected class to the quarantined class
γ_1	Shows the transition rate from asymptomatic population to the infected class
γ_A	Represents the movement rate from the quarantined quarantined population to the asymptomatic population
c	Represented the rate of quarantine for exposed persons
φ_H	Recovery rate of individuals in the hospitalized population

2. Basic definitions and theorems

Definition 2.1. *Let $\mathcal{F} \in H^1(a,b), b > a$, for any $\alpha \in [0,1]$, the ABC fractional derivative is*

$$ {}_0^{ABC}D_t^{\alpha}\mathcal{F}(z) = \frac{\varkappa(\alpha)}{1-\alpha}\int_0^t \mathcal{F}'(\mathbb{S})\mathbb{E}_{\alpha}\left[\frac{-\alpha(t-\mathbb{S})}{1-\alpha}\right]d\mathbb{S}, $$

where $\mathbb{E}$ is a Mittag-Leffler function and $\varkappa(\alpha)$ satisfied $\varkappa(0) = \varkappa(1) = 1$.

Definition 2.2. *Let $\mathcal{F} \in H^1(a,b), b > a$, and $0 \le \alpha \le 1$, the ABC fractional integral is*

$$ {}_0^{ABC}\mathcal{I}_t^{\alpha}\mathcal{F}(z) = \frac{1-\alpha}{\varkappa(\alpha)}\mathcal{F}(z) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t \mathcal{F}(\mathbb{S})(t-\mathbb{S})^{\alpha-1}d\mathbb{S}. $$

Lemma 2.1. *The following Newton Leibniz formula is satisfied for any $\mathcal{F}(t) \in H^1(a,b)$*

$$ {}_0^{ABC}\mathcal{I}_t^{\alpha}\left({}_0^{ABC}\mathscr{D}_t^{\alpha}\mathcal{F}(t)\right) = \mathcal{F}(0) - \mathcal{F}(t). $$

3. Existence of the solution

Applying the Lemma (2.1) to (1.1), we get

$$ \begin{aligned} \mathscr{S}(t) - \mathscr{S}(0) =& \frac{1-\alpha}{\varkappa(\alpha)}\left(\Lambda - (\delta+\mu)\mathscr{S}\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \\ & \int_0^t (t-\mathbb{S})^{\alpha-1}\left(\Lambda - (\delta+\mu)\mathscr{S}\right)d\mathbb{S}. \\ \mathscr{E}(t) - \mathscr{E}(0) =& \frac{1-\alpha}{\varkappa(\alpha)}\left(\delta\mathscr{S} - (\pi+\mu)\mathscr{E}\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \\ & \int_0^t (t-\mathbb{S})^{\alpha-1}\left(\Lambda - (\delta+\mu)\mathscr{S}\right)d\mathbb{S}. \\ \mathscr{I}(t) - \mathscr{I}(0) =& \frac{1-\alpha}{\varkappa(\alpha)}\left(\pi\sigma(1-c)\mathscr{E} + \phi\rho\mathscr{Q} + \gamma_1\mathscr{A}\right. \\ & \left. -(\varphi_1+\omega_1+\mu_1+\mu)\mathscr{I}\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \\ & \int_a^t (t-\mathbb{S})^{\alpha-1}\left(\pi\sigma(1-c)\mathscr{E} + \phi\rho\mathscr{Q} + \gamma_1\mathscr{A}\right. \\ & \left. -(\varphi_1+\omega_1+\mu_1+\mu)\mathscr{I}\right)d\mathbb{S}. \\ \mathscr{A}(t) - \mathscr{A}(0) =& \frac{1-\alpha}{\varkappa(\alpha)}\left(\pi(1-\sigma)(1-c)\mathscr{E} + \gamma_A\mathscr{Q}\right. \\ & \left. -(\gamma_1+\varphi_A+\mu)\mathscr{A}\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-\mathbb{S})^{\alpha-1} \\ & \left(\pi(1-\sigma)(1-c)\mathscr{E} + \gamma_A\mathscr{Q} - (\gamma_1+\varphi_A+\mu)\right. \\ & \left.\mathscr{A}\right)d\mathbb{S}. \\ \mathscr{Q}(t) - \mathscr{Q}(0) =& \frac{1-\alpha}{\varkappa(\alpha)}\left(\pi c\mathscr{E} - (\gamma_A+\phi\rho+\mu)\mathscr{Q}\right) \\ & \int_0^t (t-\mathbb{S})^{\alpha-1}\left(\pi c\mathscr{E} - (\gamma_A+\phi\rho+\mu)\mathscr{Q}\right)d\mathbb{S}. \\ \mathscr{H}(t) - \mathscr{H}(0) =& \frac{1-\alpha}{\varkappa(\alpha)}\left(\omega_1\mathscr{I} - (\varphi_H+\mu_1+\mu)\mathscr{H}\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \\ & + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-\mathbb{S})^{\alpha-1}\left(\omega_1\mathscr{I} - \right. \\ & \left.(\varphi_H+\mu_1+\mu)\mathscr{H}\right)d\mathbb{S}. \\ \mathscr{R}(t) - \mathscr{R}(0) =& \frac{1-\alpha}{\varkappa(\alpha)}\left(\varphi_1\mathscr{I} + \varphi_A\mathscr{A} + \varphi_H\mathscr{H} - \mu\mathscr{R}\right) \\ & + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-\mathbb{S})^{\alpha-1}\left(\varphi_1\mathscr{I} + \varphi_A\mathscr{A}\right. \\ & \left.+\varphi_H\mathscr{H} - \mu\mathscr{R}\right)d\mathbb{S}. \end{aligned} $$

Suppose the function $\mathscr{G}_i$ for $i = 1, 2...7$ given below

$$ \mathscr{P}_1(t,\mathscr{S}) = \Lambda - (\delta+\mu)\mathscr{S} $$

$$\begin{aligned}
\mathcal{G}_2(t,\mathcal{E}) =& \delta\mathcal{S} - (\pi+\mu)\,\mathcal{E}\\
\mathcal{G}_3(t,\mathcal{I}) =& \pi\sigma\,(1-c)\,\mathcal{E} + \phi\rho\mathcal{Q} + \gamma_1\mathcal{A} - (\varphi_1+\omega_1\\
&+\mu_1+\mu)\,\mathcal{I}\\
\mathcal{G}_4(t,\mathcal{A}) =& \pi\,(1-\sigma)\,(1-c)\,\mathcal{E} + \gamma_A\mathcal{Q} - (\gamma_1+\varphi_A+\mu)\,\mathcal{A}\\
\mathcal{G}_5(t,\mathcal{Q}) =& \pi c\mathcal{E} - (\gamma_A+\phi\rho+\mu)\,\mathcal{Q}\\
\mathcal{G}_6(t,\mathcal{H}) =& \omega_1\mathcal{I} - (\varphi_H+\mu_1+\mu)\,\mathcal{H}\\
\mathcal{G}_7(t,\mathcal{R}) =& \varphi_1\mathcal{I} + \varphi_A\mathcal{A} + \varphi_H\mathcal{H} - \mu\mathcal{R}.
\end{aligned}$$

We defined

$$\begin{cases}
\Theta_1 = |\delta+\mu|\\
\Theta_2 = |\pi+\mu|\\
\Theta_3 = |\varphi_1+\omega_1+\mu_1+\mu|\\
\Theta_4 = |\gamma_1+\varphi_A+\mu|\\
\Theta_5 = |\gamma_A+\phi\rho+\mu|\\
\Theta_6 = |\varphi_H+\mu_1+\mu|\\
\Theta_7 = |\mu|\,.
\end{cases} \tag{3.1}$$

Let

- (p), let, for $\mathcal{S},\mathcal{S}^*,\mathcal{E},\mathcal{E}^*,\mathcal{I},\mathcal{I}^*,\mathcal{A},\mathcal{A}^*,\mathcal{Q},\mathcal{Q}^*,\mathcal{H},\mathcal{H}^*,\mathcal{R},\mathcal{R}^* \in L[0,1]$, there exits constants $W_i > 0$, for $i = 1,2,3,4,5,6,7$ such that $\|\mathcal{S}\| \le W_1, \|\mathcal{E}\| \le W_2, \|\mathcal{I}\| \le W_3, \|\mathcal{A}\| \le W_4, \|\mathcal{Q}\| \le W_5, \|\mathcal{H}\| \le W_6, \|\mathcal{R}\| \le W_7$ and $\eta_1,\eta_2 > 0$.

$$\|\mathcal{S}+\mathcal{E}+\mathcal{I}+\mathcal{Q}+\mathcal{H}\| \le \eta_1 \tag{3.2}$$

$$\|\mathcal{A}+\mathcal{R}\| \le \eta_2. \tag{3.3}$$

The $\mathcal{G}_i$ for $i \in N_1^7$ satisfy the Lipschitz condition provided that (p) is obeyed. First, we take for $\mathcal{G}_1(t,\mathcal{S})$, we have

$$\begin{aligned}
\mathcal{G}_1(\mathcal{S})\mathcal{S} - \mathcal{G}_1(\mathcal{S}^*) &= \|(\Lambda - (\delta+\mu)\,\mathcal{S}) - (\Lambda - (\delta+\mu)\,\mathcal{S}^*)\|\\
&\le |(\delta+\mu)|\,\|(\mathcal{S}-\mathcal{S}^*)\|\\
&\le \Theta_1\,\|(\mathcal{S}-\mathcal{S}^*)\|\,.
\end{aligned}$$

For the $\mathcal{G}_2(t,\mathcal{E})$, we have

$$\begin{aligned}
\mathcal{G}_2(\mathcal{E}) - \mathcal{G}_2(\mathcal{E}^*) &= \|(\delta\mathcal{S} - (\pi+\mu)\,\mathcal{E}) - (\delta\mathcal{S} - (\pi+\mu)\,\mathcal{E}^*)\|\\
&\le |(\pi+\mu)|\,\|(\mathcal{E}-\mathcal{E}^*)\|\\
&\le \Theta_2\,\|(\mathcal{E}-\mathcal{E}^*)\|\,.
\end{aligned}$$

For the $\mathcal{G}_3(t,\mathcal{I})$, we have

$$\begin{aligned}
\mathcal{G}_3(\mathcal{I}) - \mathcal{G}_3(\mathcal{I}^*) =& \|(\pi\sigma(1-c)\mathcal{E} + \phi\rho\mathcal{Q} + \gamma_1\mathcal{A}\\
&(\varphi_1+\omega_1+\mu_1+\mu)\mathcal{I}) - (\pi\sigma(1-c)\mathcal{E}\\
&+ \phi\rho\mathcal{Q} + \gamma_1\mathcal{A} - (\varphi_1+\omega_1+\mu_1+\mu)\mathcal{I}^*)\|\\
\le& |(\varphi_1+\omega_1+\mu_1+\mu)|\,\|(\mathcal{I}-\mathcal{I}^*)\|\\
\le& \Theta_3\,\|(\mathcal{I}-\mathcal{I}^*)\|\,.
\end{aligned}$$

For the $\mathcal{G}_4(t,\mathcal{A})$, we have

$$\begin{aligned}
\mathcal{G}_4(\mathcal{A}) - \mathcal{G}_4(\mathcal{A}^*) =& \|(\pi\,(1-\sigma)\,(1-c)\,\mathcal{E} + \gamma_A\mathcal{Q} - (\gamma_1+\varphi_A\\
&+\mu)\mathcal{A}) - (\pi(1-\sigma)(1-c)\mathcal{E} + \gamma_A\mathcal{Q}\\
&- (\gamma_1+\varphi_A+\mu)\mathcal{A}^*)\|\\
\le& |(\gamma_1+\varphi_A+\mu)|\,\|(\mathcal{A}-\mathcal{A}^*)\|\\
\le& \Theta_4\,\|(\mathcal{A}-\mathcal{A}^*)\|\,.
\end{aligned}$$

Now, for $\mathcal{G}_5(t,\mathcal{Q})$, we have

$$\begin{aligned}
\mathcal{G}_5(\mathcal{Q}) - \mathcal{G}_5(\mathcal{Q}^*) =& \|(\pi c\mathcal{E} - (\gamma_A+\phi\rho+\mu)\mathcal{Q}) - (\pi c\mathcal{E}\\
&- (\gamma_A+\phi\rho+\mu)\mathcal{Q}^*)\|\\
\le& |(\gamma_A+\phi\rho+\mu)|\,\|(\mathcal{Q}-\mathcal{Q}^*)\|\\
\le& \Theta_5\,\|(\mathcal{Q}-\mathcal{Q}^*)\|\,.
\end{aligned}$$

For the $\mathcal{G}_6(t,\mathcal{H})$, we have

$$\begin{aligned}
\mathcal{G}_6(\mathcal{H}) - \mathcal{G}_6(\mathcal{H}^*) =& \|(\omega_1\mathcal{I} - (\varphi_{\mathcal{H}}+\mu_1+\mu)\mathcal{H}) - (\omega_1\mathcal{I}\\
&- (\varphi_H+\mu_1+\mu)\mathcal{H}^*)\|\\
\le& |(\varphi_H+\mu_1+\mu)|\,\|(\mathcal{H}-\mathcal{H}^*)\|\\
\le& \Theta_6\,\|(\mathcal{H}-\mathcal{H}^*)\|\,.
\end{aligned}$$

Similarly, for the $\mathcal{G}_7(t,\mathcal{R})$, we have

$$\begin{aligned}
\mathcal{G}_7(\mathcal{R}) - \mathcal{G}_7(\mathcal{R}^*) =& \|(\varphi_1 I + \varphi_A\mathcal{A} + \varphi_H\mathcal{H} - \mu\mathcal{R}) - (\varphi_1\mathcal{I}\\
&+ \varphi_A\mathcal{A} + \varphi_H\mathcal{H} - \mu\mathcal{R}^*)\|\\
\le& |\mu|\,\|(\mathcal{R}-\mathcal{R}^*)\|\\
\le& \Theta_7\,\|(\mathcal{R}-\mathcal{R}^*)\|\,.
\end{aligned}$$

Hence, the $\mathcal{G}_i$, for $i = 1,2,...7.$ satisfies the lipschitz condition. We suppose that $\mathcal{S}(0) = \mathcal{E}(0) = \mathcal{A}(0) = \mathcal{I}(0) =$

$\mathscr{Q}(0) = \mathscr{H}(0) = \mathscr{R}(0) = 0$, we have

$$\mathscr{S}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_1(t, \mathscr{S}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_1(z, \mathscr{S}(z))dz$$
$$\mathscr{E}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_2(t, \mathscr{E}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_2(z, \mathscr{E}(z))dz$$
$$\mathscr{I}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_3(t, \mathscr{I}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_3(z, \mathscr{I}(z))\, dz$$
$$\mathscr{A}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_4(t, \mathscr{A}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_4(z, \mathscr{A}(z))\, dz$$
$$\mathscr{Q}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_5(t, \mathscr{Q}(t)) + \frac{\alpha}{\varkappa(\alpha_5^*)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \mathscr{G}_5(z, \mathscr{Q}(z))\, dz$$
$$\mathscr{H}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_6(t, \mathscr{H}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_6(z, \mathscr{H}(z))\, dz$$
$$\mathscr{R}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_7(t, \mathscr{R}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_7(z, \mathscr{R}(z))\, dz.$$

We defined the following iterative relation for (1.1) as

$$\mathscr{S}_n(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_1(t, \mathscr{S}_{n-1}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_1(z, \mathscr{S}_{n-1}(z))\, dz$$
$$\mathscr{E}_n(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_2(t, \mathscr{E}_{n-1}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_2(z, \mathscr{E}_{n-1}(z))\, dz$$
$$\mathscr{I}_n(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_3(t, \mathscr{I}_{n-1}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_3(z, \mathscr{I}_{n-1}(z))\, dz$$
$$\mathscr{A}_n(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_4(t, \mathscr{A}_{n-1}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_4(z, \mathscr{A}_{n-1}(z))\, dz$$
$$\mathscr{Q}_n(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_5(t, \mathscr{Q}_{n-1}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_5(z, \mathscr{Q}_{n-1}(z))\, dz$$
$$\mathscr{H}_n(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_6(t, \mathscr{H}_{n-1}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_6(z, \mathscr{H}_{n-1}(z))\, dz$$
$$\mathscr{R}_n(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_7(t, \mathscr{R}_{n-1}(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_7(z, \mathscr{R}_{n-1}(z))\, dz.$$

Using the supposition (P) and (3.1), the model (1.1) has a solution $\Omega = \max\{\Theta_1, \Theta_2, \Theta_3, \Theta_4, \Theta_5, \Theta_6, \Theta_7,\}$. The functions we defined as

$$\begin{aligned} \Pi 1_n(t) &= \mathscr{S}_{n+1} - \mathscr{S} \\ \Pi 2_n(t) &= \mathscr{E}_{n+1} - \mathscr{E}, \\ \Pi 3_n(t) &= \mathscr{I}_{n+1} - \mathscr{I}, \\ \Pi 4_n(t) &= \mathscr{A}_{n+1} - \mathscr{A}, \\ \Pi 5_n(t) &= \mathscr{Q}_{n+1} - \mathscr{Q}, \\ \Pi 6_n(t) &= \mathscr{H}_{n+1} - \mathscr{H}, \\ \Pi 7_n(t) &= \mathscr{R}_{n+1} - \mathscr{R}. \end{aligned} \tag{3.4}$$

Now, we using the definition (2.1) and (3.4), we get

$$\begin{aligned} \|\Pi 1_n\| &= \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_1(\mathscr{S}_n) - \mathscr{G}_1(\mathscr{S}_{n-1})\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \|\mathscr{G}_1(z, \mathscr{S}_n(z)) - \mathscr{G}_1(z, \mathscr{S}_{n-1}(z))\|\, dz \\ &\le \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_1 \|\mathscr{S}_n - \mathscr{S}\|. \end{aligned}$$

Similarly, using the same procedure for the other compartments of the model (1.1)

$$\begin{aligned} \|\Pi 2_n\| &= \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_2(\mathscr{E}_n) - x_2(\mathscr{E}_{n-1})\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \|\mathscr{G}_2(z, \mathscr{E}_n(z)) - \mathscr{G}_2(z, \mathscr{E}_{n-1}(z))\|\, dz \\ &\le \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha_2^*)\Gamma(\alpha)}\right] \Theta_2 \|\mathscr{E}_n - \mathscr{E}\|. \end{aligned}$$

$$\begin{aligned} \|\Pi 3_n\| &= \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_3(\mathscr{I}_n) - \mathscr{G}_3(\mathscr{I}_{n-1})\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \|\mathscr{G}_3(z, \mathscr{I}_n(z)) - \mathscr{G}_3(z, \mathscr{I}_{n-1}(z))\|\, dz \\ &\le \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_3 \|\mathscr{I}_n - \mathscr{I}\|. \end{aligned}$$

$$\begin{aligned} \|\Pi 4_n\| &= \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_4\mathscr{A}_n) - \mathscr{G}_4(\mathscr{A}_{n-1})\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \|\mathscr{G}_4(z, \mathscr{A}_n(z)) - \mathscr{G}_4(z, \mathscr{A}_{n-1}(z))\|\, dz \\ &\le \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_4 \|\mathscr{A}_n - \mathscr{A}\|. \end{aligned}$$

$$\|\Pi 5_n\| = \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_5(\mathscr{Q}_n) - \mathscr{G}_5(\mathscr{Q}_{n-1})\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \|\mathscr{G}_5(z, \mathscr{Q}_n(z)) - \mathscr{G}_5(z, \mathscr{Q}_{n-1}(z))\| \, dz \leq \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_5 \|\mathscr{Q}_n - \mathscr{Q}\|.$$

$$\|\Pi 6_n\| = \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_6(\mathscr{H}_n) - \mathscr{G}_1(\mathscr{H}_{n-1})\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \|\mathscr{G}_6(z, \mathscr{H}_n(z)) - \mathscr{G}_6(z, \mathscr{H}_{n-1}(z))\| \, dz \leq \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_6 \|\mathscr{H}_n - \mathscr{H}\|.$$

$$\|\Pi 7_n\| = \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_7(\mathscr{R}_n) - \mathscr{G}_7(\mathscr{R}_{n-1})\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \|\mathscr{G}_7(z, \mathscr{R}_n(z)) - \mathscr{G}_1(z, \mathscr{R}_{n-1}(z))\| \, dz \leq \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_7 \|\mathscr{R}_n - \mathscr{R}\|.$$

Hence, we have $\Pi_n(t) \to 0$ as $n \to \infty$.

4. Uniqueness of the solution

Theorem 4.1. With the supposition (P) the model (1.1) has unique solution if

$$\left[\frac{1-\phi_i}{\mathscr{B}(\phi_i)} + \frac{1}{\mathscr{B}(\phi_i)\Gamma(\phi_i)}\right] \Theta_i \leq 1, i \in \mathbb{N}_i^7. \tag{4.1}$$

Proof. We suppose that the solution of (1.1) is not unique , Let $\{\mathscr{S}^*(t), \mathscr{E}^*(t), \mathscr{I}^*(t), \mathscr{A}^*(t), \mathscr{Q}^*(t), \mathscr{H}^*(t), \mathscr{R}^*(t)\}$ be another solution of (1.1), such that

$$\mathscr{S}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_1(t, \mathscr{S}^*(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_1(z, \mathscr{S}^*(z)) \, dz$$

$$\mathscr{E}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_2(t, \mathscr{E}^*(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_2(z, \mathscr{E}^*(z)) \, dz$$

$$\mathscr{I}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_3(t, \mathscr{I}^*(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_3(z, \mathscr{I}^*(z)) \, dz$$

$$\mathscr{A}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_4(t, \mathscr{A}^*(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_4(z, \mathscr{G}^*(z)) \, dz$$

$$\mathscr{Q}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_5(t, \mathscr{Q}^*(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_5(z, \mathscr{Q}^*(z)) \, dz$$

$$\mathscr{H}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_6(t, \mathscr{H}^*(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_6(z, \mathscr{H}^*(z)) \, dz$$

$$\mathscr{R}(t) = \frac{1-\alpha}{\varkappa(\alpha)} \mathscr{G}_7(t, \mathscr{R}^*(t)) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \mathscr{G}_7(z, \mathscr{R}^*(z)) \, dz.$$

Then,

$$\|\mathscr{S} - \mathscr{S}^*\| = \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_1(t, \mathscr{S}(t)) - \mathscr{G}_1(t, \mathscr{S}^*(t))\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \|\mathscr{G}_1(z, \mathscr{S}(z)) - \mathscr{G}_1(z, \mathscr{S}^*(z))\| dz \leq \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_1 \|\mathscr{S} - \mathscr{S}^*\|.$$

Which implies that

$$\left[\frac{1-\alpha}{\varkappa(\alpha)} \Theta_1 + \frac{\Theta_1}{\varkappa(\alpha)\Gamma(\alpha)} - 1\right] \|\mathscr{S} - \mathscr{S}^*\| \geq 0. \tag{4.2}$$

By (4.1) , (4.2) is vaild for $\|\mathscr{S} - \mathscr{S}^*\| = 0$, which means that $\mathscr{S} = \mathscr{S}^*$. Similarly, we repeat the same fashion for the remaining compartment of the model (1.1), we have

$$\|\mathscr{E} - \mathscr{E}^*\| = \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_1(t, \mathscr{E}(t)) - \mathscr{G}_1(t, \mathscr{E}^*(t))\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \|\mathscr{G}_2(z, \mathscr{E}(z)) - \mathscr{G}_2(z, \mathscr{E}^*(z))\| dz \leq \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_2 \|\mathscr{E} - \mathscr{E}^*\|.$$

Which implies that

$$\left[\frac{1-\alpha}{\varkappa(\alpha)} \Theta_2 + \frac{\Theta_2}{\varkappa(\alpha)\Gamma(\alpha)} - 1\right] \|\mathscr{E} - \mathscr{E}^*\| \geq 0. \tag{4.3}$$

By using (4.1) which implies the (4.3) is valid for $\|\mathscr{E} - \mathscr{E}^*\| = 0$, which means that $\mathscr{E} = \mathscr{E}^*$. Similarly, for $\mathscr{I}$,we have

$$\|\mathscr{I} - \mathscr{I}^*\| = \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_3(t, \mathscr{I}(t)) - \mathscr{G}_3(t, \mathscr{I}^*(t))\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \|\mathscr{G}_3(z, \mathscr{I}(z)) - \mathscr{G}_3(z, \mathscr{I}^*(z))\| dz \leq \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right] \Theta_3 \|\mathscr{I} - \mathscr{I}^*\|.$$

Which implies that

$$\left[\frac{1-\alpha}{\varkappa(\alpha)}\Theta_3 + \frac{\Theta_3}{\varkappa(\alpha)\Gamma(\alpha)} - 1\right]\|\mathscr{I} - \mathscr{I}^*\| \geq 0. \quad (4.4)$$

From (4.1) , (4.4) result is valid for $\|\mathscr{I} - \mathscr{I}^*\| = 0$, which means that $\mathscr{I} = \mathscr{I}^*$. Now, for $\mathscr{A}$,we have

$$\begin{aligned}\|\mathscr{A} - \mathscr{A}^*\| =& \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_4(t, \mathscr{A}(t)) - \mathscr{G}_4(t, \mathscr{A}^*(t))\| \\ &+ \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \|\mathscr{G}_4(z, \mathscr{A}(z)) \\ &- \mathscr{G}_4(z, \mathscr{A}^*(z))\| dz \\ \leq& \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right]\Theta_4 \|\mathscr{A} - \mathscr{A}\mathscr{A}^*\|.\end{aligned}$$

Which implies that

$$\left[\frac{1-\alpha}{\varkappa(\alpha)}\Theta_4 + \frac{\Theta_4}{\varkappa(\alpha)\Gamma(\alpha)} - 1\right]\|\mathscr{A} - \mathscr{A}^*\| \geq 0. \quad (4.5)$$

By using (4.1) , the (4.5) is valid for $\|\mathscr{A} - \mathscr{A}^*\| = 0$, which means that $\mathscr{A} = \mathscr{A}^*$. Similarly, for $\mathscr{Q}$,

$$\begin{aligned}\|\mathscr{Q} - \mathscr{Q}^*\| =& \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_5(t, \mathscr{Q}(t)) - \mathscr{G}_5(t, \mathscr{Q}^*(t))\| \\ &+ \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \|\mathscr{G}_5(z, \mathscr{Q}(z)) \\ &- \mathscr{G}_5(z, \mathscr{Q}^*(z))\| dz \\ \leq& \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right]\Theta_5 \|\mathscr{Q} - \mathscr{Q}^*\|.\end{aligned}$$

Which implies that

$$\left[\frac{1-\alpha}{\varkappa(\alpha)}\Theta_5 + \frac{\Theta_5}{\varkappa(\alpha)\Gamma(\alpha)} - 1\right]\|\mathscr{Q} - \mathscr{Q}^*\| \geq 0. \quad (4.6)$$

By using (4.1) which implies the (4.6) is valid for $\|\mathscr{Q} - \mathscr{Q}^*\| = 0$, which means that $\mathscr{Q} = \mathscr{Q}^*$. Similarly, for $\mathscr{H}$,we have

$$\begin{aligned}\|\mathscr{H} - \mathscr{H}^*\| =& \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_6(t, \mathscr{H}(t)) - \mathscr{G}_6(t, \mathscr{H}^*(t))\| \\ &+ \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \|\mathscr{G}_6(z, \mathscr{H}(z)) \\ &- \mathscr{G}_6(z, \mathscr{H}^*(z))\| dz \\ \leq& \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right]\Theta_7 \|\mathscr{H} - \mathscr{H}^*\|.\end{aligned}$$

Which implies that

$$\left[\frac{1-\alpha}{\varkappa(\alpha)}\Theta_6 + \frac{\Theta_6}{\varkappa(\alpha)\Gamma(\alpha)} - 1\right]\|\mathscr{H} - \mathscr{H}^*\| \geq 0. \quad (4.7)$$

By using (4.1) , the (4.7) result is valid for $\|\mathscr{H} - \mathscr{H}^*\| = 0$, which means that $\mathscr{H} = \mathscr{H}^*$. Similarly, for $\mathscr{R}$,we have

$$\begin{aligned}\|R - R^*\| =& \frac{1-\alpha}{\varkappa(\alpha)} \|\mathscr{G}_7(t, \mathscr{R}(t)) - \mathscr{G}_7(t, \mathscr{R}^*(t))\| \\ &+ \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \|\mathscr{G}_7(z, \mathscr{R}(z)) \\ &- \mathscr{G}_7(z, \mathscr{R}^*(z))\| dz \\ \leq& \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right]\Theta_7 \|\mathscr{R} - \mathscr{R}^*\|.\end{aligned}$$

Which implies that

$$\left[\frac{1-\alpha}{\varkappa(\alpha)}\Theta_7 + \frac{\Theta_7}{\varkappa(\alpha)\Gamma(\alpha)} - 1\right]\|\mathscr{R} - \mathscr{R}^*\| \geq 0. \quad (4.8)$$

By using (4.1) which implies that (4.8) results is valid for $\|\mathscr{R} - \mathscr{R}^*\| = 0$, which shows that $\mathscr{R} = \mathscr{R}^*$. Thus the model (1.1) has a unique solution. □

5. Hyers Ulams Stability

The integral system Eqs. (3.4)-(3.4) is Hyers -ulam Stable if for $\eta_i > 0, \epsilon_i > 0$, for $i \in N_1^7$ [47] , with

$$\begin{aligned}&|\mathscr{S}(t) - \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_1(t, \mathscr{S}(t)) - \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \\ &\mathscr{G}_1(z, \mathscr{S}(z))\, dz| \leq \eta_1 \quad (5.1)\\ &|\mathscr{E}(t) - \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_2(t, \mathscr{E}(t)) - \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \\ &\mathscr{G}_2(z, \mathscr{E}(z))\, dz| \leq \eta_2 \quad (5.2)\\ &|\mathscr{I}(t) - \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_3(t, \mathscr{I}(t)) - \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \\ &\mathscr{G}_3(z, \mathscr{I}(z))\, dz| \leq \eta_3 \quad (5.3)\\ &|\mathscr{A}(t) - \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_4(t, \mathscr{A}(t)) - \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_a^t (t-z)^{\alpha-1} \\ &\mathscr{G}_4(z, \mathscr{A}(z))\, dz| \leq \eta_4 \quad (5.4)\\ &|\mathscr{Q}(t) - \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_5(t, \mathscr{Q}(t)) - \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \\ &\mathscr{G}_5(z, \mathscr{Q}(z))\, dz| \leq \eta_5 \quad (5.5)\\ &|\mathscr{H}(t) - \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_6(t, \mathscr{H}(t)) - \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \\ &\mathscr{G}_6(z, \mathscr{H}(z))\, dz| \leq \eta_6 \quad (5.6)\\ &|\mathscr{R}(t) - \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_7(t, \mathscr{R}(t)) - \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \int_0^t (t-z)^{\alpha-1} \\ &\mathscr{G}_7(z, \mathscr{R}(z))\, dz| \leq \eta_7 \quad (5.7)\end{aligned}$$

Now, we have $\dot{\mathscr{S}}, \dot{\mathscr{E}}, \dot{\mathscr{I}}, \dot{\mathscr{A}}, \dot{\mathscr{Q}}, \dot{\mathscr{H}}, \dot{\mathscr{R}}$, which implies that

$$\dot{\mathscr{S}}(t) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_1\left(t, \dot{\mathscr{S}}(t)\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_1 (z, \dot{\mathscr{S}}(z))dz \tag{5.8}$$

$$\dot{\mathscr{E}}(t) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_2\left(t, \dot{\mathscr{E}}(t)\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_2 \left(z, \dot{\mathscr{E}}(z)\right)dz \tag{5.9}$$

$$\dot{\mathscr{I}}(t) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_3\left(t, \dot{\mathscr{I}}(t)\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_a^t (t-z)^{\alpha-1}\mathscr{G}_3 \left(z, \dot{\mathscr{I}}(z)\right)dz \tag{5.10}$$

$$\dot{\mathscr{A}}(t) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_4\left(t, \dot{\mathscr{A}}(t)\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_a^t (t-z)^{\alpha-1}\mathscr{G}_4 \left(z, \dot{\mathscr{A}}(z)\right)dz \tag{5.11}$$

$$\dot{\mathscr{Q}}(t) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_5\left(t, \dot{\mathscr{Q}}(t)\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_5 \left(z, \dot{\mathscr{Q}}(z)\right)dz \tag{5.12}$$

$$\dot{\mathscr{H}}(t) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_6\left(t, \dot{\mathscr{H}}(t)\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_6 \left(z, \dot{\mathscr{H}}(t)\right)dz \tag{5.13}$$

$$\dot{\mathscr{R}}(t) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_7\left(t, \dot{\mathscr{R}}(t)\right) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_7 \left(z, \dot{\mathscr{R}}(t)\right)dz. \tag{5.14}$$

Such that

$$\left|\mathscr{S}-\dot{\mathscr{S}}\right| \le \epsilon_1\eta_1, \left|\mathscr{E}-\dot{\mathscr{E}}\right| \le \epsilon_2\eta_2, \left|\mathscr{I}-\dot{\mathscr{I}}\right| \le \epsilon_3\eta_3, \left|\mathscr{A}-\dot{\mathscr{A}}\right| \le \epsilon_4\eta_4, \left|\mathscr{Q}-\dot{\mathscr{Q}}\right| \le \epsilon_5\eta_5, \left|\mathscr{H}-\dot{\mathscr{H}}\right| \le \epsilon_6\eta_6, \left|\mathscr{R}-\dot{\mathscr{R}}\right| \le \epsilon_7\eta_7.$$

Theorem 5.1. We assume that (P), Satisfied then the FOM (1.1) is Hyers Ulam stable.

Proof. By using (4.1) , the FOM (1.1) has a unique solution, say $(\mathscr{S}, \mathscr{E}, \mathscr{I}, \mathscr{A}, \mathscr{Q}, \mathscr{H}, \mathscr{R})$. Let we suppose that $\left(\dot{\mathscr{S}}, \dot{\mathscr{E}}, \dot{\mathscr{I}}, \dot{\mathscr{A}}, \dot{\mathscr{Q}}, \dot{\mathscr{H}}, \dot{\mathscr{R}}\right)$ be an another solution for the considered model. We go ahead with first equation of the (1.1), satisfying Eqs. (3.4). Then

$$\begin{aligned}\left\|\mathscr{S}-\dot{\mathscr{S}}\right\| =& \frac{1-\alpha}{\varkappa(\alpha)}\left\|\mathscr{G}_1(t,\mathscr{S}(t)) - \mathscr{G}_1(t,\dot{\mathscr{S}}(t))\right\| + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} \\ & \int_0^t (t-z)^{\alpha-1}\left\|\mathscr{G}_1(z,\mathscr{S}(z)) - \mathscr{G}_1\left(z,\dot{\mathscr{S}}(z)\right)\right\|dz \\ \le & \left[\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\right]\Theta_1\left\|\mathscr{S}-\dot{\mathscr{S}}\right\|.\end{aligned}$$

Now, we taking $\eta_1 = \Theta_1$ and $\frac{1-\alpha}{\varkappa(\alpha)} + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)} = \Delta_1$, this implies $\left\|\mathscr{S}-\dot{\mathscr{S}}\right\| \le \eta_1\Delta_1$.

Similarly, for $\mathscr{E}(t), \dot{\mathscr{E}}(t), \mathscr{I}(t), \dot{\mathscr{I}}(t), \mathscr{A}(t), \dot{\mathscr{A}}(t), \mathscr{Q}(t), \dot{\mathscr{Q}}(t), \mathscr{H}(t), \dot{\mathscr{H}}(t), \mathscr{R}(t), \dot{\mathscr{R}}(t)$, we have

$$\begin{cases}\left\|\mathscr{E}-\dot{\mathscr{E}}\right\| \le \eta_2\Delta_2 \\ \left\|\mathscr{I}-\dot{\mathscr{I}}\right\| \le \eta_3\Delta_3 \\ \left\|\mathscr{A}-\dot{\mathscr{A}}\right\| \le \eta_4\Delta_4 \\ \left\|\mathscr{Q}-\dot{\mathscr{Q}}\right\| \le \eta_5\Delta_5 \\ \left\|\mathscr{H}-\dot{\mathscr{H}}\right\| \le \eta_6\Delta_6 \\ \left\|\mathscr{R}-\dot{\mathscr{R}}\right\| \le \eta_7\Delta_7.\end{cases} \tag{5.15}$$

Thus, the solution of the considered FOM is stable. □

6. Numerical algorithm

In this section,we find the numerical solution by using the numerical method of Lagrange's interpolation. For the solution of the system (1.1), we consider

$$\begin{cases}{}_0^{ABC}\mathscr{D}_t^\alpha[\mathscr{S}(t)] = \mathscr{G}_1(t,\mathscr{S}), \\ {}_0^{ABC}\mathscr{D}_t^\alpha[\mathscr{E}(t)] = \mathscr{G}_2(t,\mathscr{E}), \\ {}_0^{ABC}\mathscr{D}_t^\alpha[\mathscr{I}(t)] = \mathscr{G}_3(t,\mathscr{I}), \\ {}_0^{ABC}\mathscr{D}_t^\alpha[\mathscr{A}(t)] = \mathscr{G}_4(t,\mathscr{A}), \\ {}_0^{ABC}\mathscr{D}_t^\alpha[\mathscr{Q}(t)] = \mathscr{G}_5(t,\mathscr{Q}), \\ {}_0^{ABC}\mathscr{D}_t^\alpha[\mathscr{H}(t)] = \mathscr{G}_6(t,\mathscr{H}), \\ {}_0^{ABC}\mathscr{D}_t^\alpha[\mathscr{R}(t)] = \mathscr{G}_7(t,\mathscr{R}).\end{cases} \tag{6.1}$$

Using the Lemma (2.1) and (6.1), we get

$$\mathscr{S}(t) - \mathscr{S}(0) = \frac{1-\alpha}{\varkappa(\alpha)}x_1(t,\mathscr{S}) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_1(z,\mathscr{S})dz \tag{6.2}$$

$$\mathscr{E}(t) - \mathscr{E}(0) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_2(t,\mathscr{E}) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_2(z,\mathscr{E})dz \tag{6.3}$$

$$\mathscr{I}(t) - \mathscr{I}(0) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_3(t,\mathscr{I}) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_3(z,\mathscr{I})dz \tag{6.4}$$

$$\mathscr{A}(t) - \mathscr{A}(0) = \frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_4(t,\mathscr{A}) + \frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t (t-z)^{\alpha-1}\mathscr{G}_4(z,\mathscr{A})dz \tag{6.5}$$

$$\mathscr{Q}(t)-\mathscr{Q}(0)=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_5(t,\mathscr{Q})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t(t-z)^{\alpha-1}\mathscr{G}_5(z,\mathscr{Q})\,dz \tag{6.6}$$

$$\mathscr{H}(t)-\mathscr{H}(0)=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_6(t,\mathscr{H})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t(t-z)^{\alpha-1}\mathscr{G}_6(z,H)\,dz \tag{6.7}$$

$$\mathscr{R}(t)-\mathscr{R}(0)=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_7(t,\mathscr{R})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\int_0^t(t-z)^{\alpha-1}\mathscr{G}_7(z,\mathscr{R})\,dz. \tag{6.8}$$

Let [0,T] be the interval which we want to find the solution of the system (1.1). For this we divided the given interval as a set of points t_{m+1}, for m=0,1,2,..n, we have

$$\mathscr{S}(t_{m+1})=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_1(t_m,\mathscr{S})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}}(t_{m+1}-z)^{\alpha-1}\mathscr{G}_1(z,\mathscr{S})\,dz \tag{6.9}$$

$$\mathscr{E}(t_{m+1})=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_2(t_m,\mathscr{E})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}}(t_{m+1}-z)^{\alpha-1}\mathscr{G}_2(z,\mathscr{E})\,dz \tag{6.10}$$

$$\mathscr{I}(t_{m+1})=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_3(t_m,\mathscr{I})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}}(t_m-z)^{\alpha-1}\mathscr{G}_3(z,\mathscr{I})\,dz \tag{6.11}$$

$$\mathscr{A}(t_{m+1})=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_4(t_m,\mathscr{A})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}}(t_m-z)^{\alpha-1}\mathscr{G}_4(z,\mathscr{A})\,dz \tag{6.12}$$

$$\mathscr{Q}(t_{m+1})=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_5(t_m,\mathscr{Q})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}}(t_m-z)^{\alpha-1}\mathscr{G}_5(z,\mathscr{Q})\,dz \tag{6.13}$$

$$\mathscr{H}(t_{m+1})=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_6(t_m,\mathscr{H}(t))+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}}(t_m-z)^{\alpha-1}\mathscr{G}_6(z,\mathscr{H})\,dz \tag{6.14}$$

$$\mathscr{R}(t_{m+1})=\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_7(t_m,\mathscr{R})+\frac{\alpha}{\varkappa(\alpha)\Gamma(\alpha)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}}(t_m-z)^{\alpha-1}\mathscr{G}_7(z,\mathscr{R})\,dz. \tag{6.15}$$

Using lagrange's interpolation, we get

$$\begin{aligned}\mathscr{S}(t_{m+1})=&\mathscr{S}_0+\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_1(t_m,\mathscr{S})+\frac{\alpha}{\varkappa(\alpha)}\sum_{\zeta=1}^{m}[\frac{h^\alpha\mathscr{G}_1(t_\zeta,\mathscr{S})}{\Gamma(\alpha+2)}\\&((1+m-\zeta)^\alpha(2+m-\zeta+\alpha)-(m-\zeta)^\alpha(m-\zeta\\&+2+2\alpha))-\frac{h^\alpha\mathscr{G}_1(t_{\zeta-1},\mathscr{S})}{\Gamma(\alpha+2)}((1+m-\zeta)^\alpha\\&-(m-\zeta)^\alpha(1+m-\zeta+\alpha))].\end{aligned}$$

$$\begin{aligned}\mathscr{E}(t_{m+1})=&\mathscr{E}_0+\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_2(t_m,\mathscr{E})+\frac{\alpha}{\varkappa(\alpha)}\sum_{\zeta=0}^{m}[\frac{h^\alpha\mathscr{G}_2(t_\zeta,\mathscr{E})}{\Gamma(\alpha+2)}\\&((1+m-\zeta)^\alpha(2+m-\zeta+\alpha)-(m-\zeta)^\alpha(m-\zeta\\&+2+2\alpha))-\frac{h^\alpha\mathscr{G}_1(t_{\zeta-1},\mathscr{E})}{\Gamma(\alpha+2)}((m+1-\zeta)^\alpha\\&-(m-\zeta)^\alpha(m+1-\zeta+\alpha))].\end{aligned}$$

$$\begin{aligned}\mathscr{I}(t_{m+1})=&\mathscr{I}_0+\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_3(t_m,\mathscr{I})+\frac{\alpha}{\varkappa(\alpha)}\sum_{\zeta=0}^{m}[\frac{h^\alpha\mathscr{G}_3(t_\zeta,\mathscr{I})}{\Gamma(\alpha+2)}\\&((1+m-\zeta)^\alpha(2+m-\zeta+\alpha)-(m-\zeta)^\alpha(m-\zeta\\&+2+2\alpha))-\frac{h^\alpha\mathscr{G}_3(t_{\zeta-1},\mathscr{I})}{\Gamma(\alpha+2)}((m+1-\zeta)^\alpha\\&-(m-\zeta)^\alpha(m+1-\zeta+\alpha))].\end{aligned}$$

$$\begin{aligned}\mathscr{A}(t_{m+1})=&\mathscr{A}_0+\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_4(t_m,\mathscr{A})+\frac{\alpha}{\varkappa(\alpha)}\sum_{\zeta=0}^{m}[\frac{h^\alpha\mathscr{G}_4(t_\zeta,\mathscr{A})}{\Gamma(\alpha+2)}\\&((1+m-\zeta)^\alpha(2+m-\zeta+\alpha).-(m-\zeta)^\alpha(m-\zeta\\&+2+2\alpha))-\frac{h^\alpha\mathscr{G}_4(t_{\zeta-1},\mathscr{A})}{\Gamma(\alpha+2)}((m+1-\zeta)^\alpha\\&-(m-\zeta)^\alpha(m+1-\zeta+\alpha))].\end{aligned}$$

$$\begin{aligned}\mathscr{Q}(t_{m+1})=&\mathscr{Q}_0+\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_5(t_m,\mathscr{Q})+\frac{\alpha}{\varkappa(\alpha)}\sum_{\zeta=0}^{n}[\frac{h^\alpha\mathscr{G}_5(t_\zeta,\mathscr{Q})}{\Gamma(\alpha+2)}\\&((1+m-\zeta)^\alpha(2+m-\zeta+\alpha)-(m-\zeta)^\alpha(m-\zeta\\&+2+2\alpha))-\frac{h^\alpha\mathscr{G}_5(t_{\zeta-1},\mathscr{Q})}{\Gamma(\alpha+2)}((m+1-\zeta)^\alpha\\&-(m-\zeta)^\alpha(m+1-\zeta+\alpha))].\end{aligned}$$

$$\begin{aligned}\mathscr{H}(t_{m+1})=&\mathscr{H}_0+\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_6(t_m,\mathscr{H})+\frac{\alpha}{\varkappa(\alpha)}\sum_{\zeta=0}^{n}[\frac{h^\alpha\mathscr{G}_6(t_\zeta,\mathscr{H})}{\Gamma(\alpha+2)}\\&((1+m-\zeta)^\alpha(2+m-\zeta+\alpha)-(m-\zeta)^\alpha(m-\zeta\\&+2+2\alpha))-\frac{h^\alpha\mathscr{G}_6(t_{\zeta-1},\mathscr{H})}{\Gamma(\alpha+2)}((m+1-\zeta)^\alpha\\&-(m-\zeta)^\alpha(m+1-\zeta+\alpha))].\end{aligned}$$

$$\mathscr{R}(t_{m+1})=\mathscr{R}_0+\frac{1-\alpha}{\varkappa(\alpha)}\mathscr{G}_7(t_m,\mathscr{R})+\frac{\alpha}{\varkappa(\alpha)}\sum_{\zeta=0}^{n}[\frac{h^\alpha\mathscr{G}_7(t_\zeta,\mathscr{R})}{\Gamma(\alpha+2)}$$

$$((1+m-\zeta)^{\alpha}(2+m-\zeta+\alpha)-(m-\zeta)^{\alpha}(m-\zeta$$
$$+2+2\alpha))-\frac{h^{\alpha}\mathscr{G}_7(t_{\zeta-1},\mathscr{R})}{\Gamma(\alpha+2)}((1+m-\zeta)^{\alpha}$$
$$-(m-\zeta)^{\alpha}(m+1-\zeta+\alpha))].$$

This numerical scheme will be used in the next section.

7. Graphical representation of the solution

In this part of the paper, we apply the numerical method described in the previous section to demonstrate the graphical results of the model (1.1).

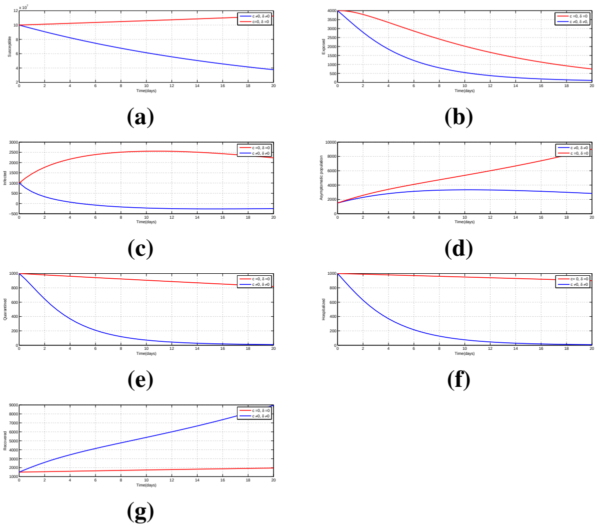

Figure 2. The effect of δ and c on the dynamical behavior of the system with $\alpha = 1$.

Table 2. Description of the parameters used in model (1.1).

Parameters	Parameters Values
δ	0.9497
μ	0.099
π	0.97
σ	0.3
ρ	0.09497
ϕ	0.5
μ_1	0.005
φ	0.762
φ_a	0.00922
γ_1	0.04
γ_A	0.01
c	2.54
φ_H	0.7

Individuals in the exposed, asymptomatic, infected, and hospitalized classes declined considerably with isolation and social distance parameters, but the suspectable and recovered population increased fast. These graphs demonstrate that these factors are useful in reducing Covid-19 infection in the population.

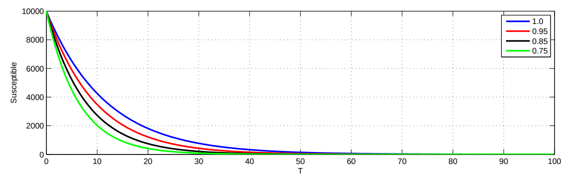

Figure 3. Dynamical behaviour of the susceptible population for various values of α.

The parameters used in simulations are given in Table.2, most of which are taken from previous published work [35–38]. Plotting in the Figure 2, the behavior of the model with two activate parameters, isolation rate ($c \neq 0$) and social distancing rate ($\delta \neq 0$), is shown by the bold blue line. In contrast, the bold red line shows the behavior of the model with parameters c and δ equal to zero.

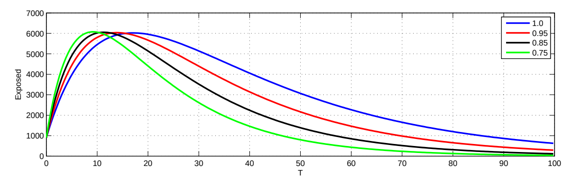

Figure 4. Dynamical behaviour of the exposed population for various values of α.

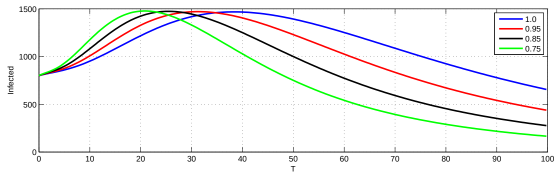

Figure 5. Dynamical behaviour of the infected population for various values of α.

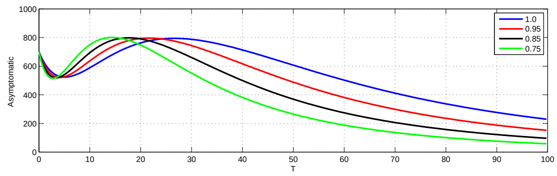

Figure 6. Dynamical behaviour of the asymptotic population for various values of α.

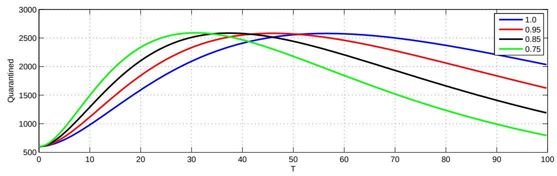

Figure 7. Dynamical behaviour of the quarantined population for various values of α.

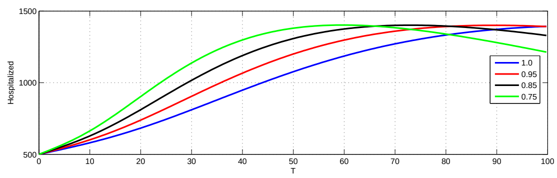

Figure 8. Dynamical behaviour of the hospitalized population for various values of α.

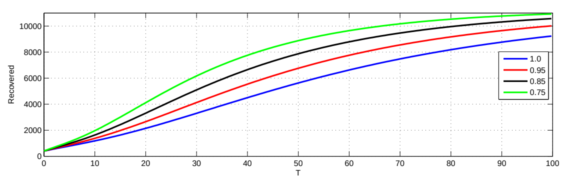

Figure 9. Dynamical behaviour of the recovered population for various values of α.

From Figures 3-9, we see that the susceptible, infection, asymptotic, quarantined, and hospitalized population will decrease, and consequently, the recovery will increase. We observed a rapid decrease in the population of exposed, asymptomatic, quarantined, hospitalized, and infected classes, which becomes more significant for fractional value compared to integer order $\alpha = 1$.

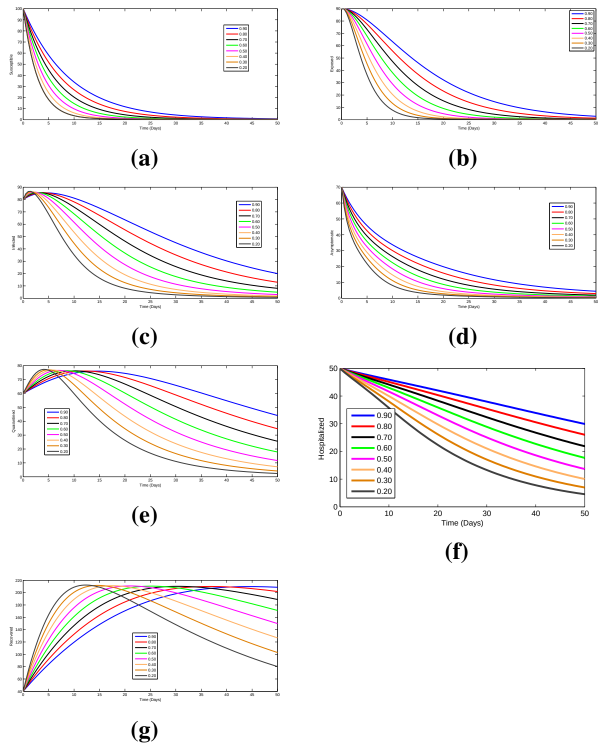

Figure 10. Dynamics behavior of the susceptible, exposed, infected, asymptomatic, quarantine, hospitalized,and recovered people when $\Lambda = 0.8$.

From Figure 10, we noted that the susceptible, exposed, and asymptomatic individuals are the smallest due to the smallest value of Λ. Otherwise, the largest value of the Λ could be high population density, close contact with the people, improper social distancing, and insufficient preventive measures. This will lead to a relatively high spread of the disease.

8. Conclusions

This work studied the transmission dynamics of the COVID-19 pandemic with asymptomatic, quarantine, and hospitalization individuals through an ABC fractional model. Initially, we formulated a mathematical model and analyzed the model using the fractional operator with the power-law kernel. We have used fixed point theory to conclude the existence of such a model in the real world. Then we have shown the unique solution of the model. Further, we investigated significant conditions for the Ulam Hyers type stability via non-linear functional analysis. The computational scheme is derived for the numerical simulation and is checked for available data. Further, We simulate the COVID-19 model for different transmission rate values to evaluate the dynamics of the separate compartments in the model. The dynamics of the COVID-19 pandemic and the impact of various control strategies by including additional classes into the current model will be addressed in future research. The vaccinated population and the number of pathogens in the environment are two compartments. The suggested model will include memory characteristics and non-locality and be an expanded version of the current model. This study's findings will help health care centers forecast how the coronavirus may affect the world in the future.

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Conflict of interest

We declare that we have no competing interests.

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