



Research article

Optimal harvesting policy for a single-species θ -logistic model with seasonal succession

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Abstract: In this paper, a seasonally switching single-species θ -logistic harvesting model with alternating closed and open seasons is developed, and the effects of nonlinear density dependence on fish population persistence and sustainable harvesting are investigated. Theoretical analysis identifies a critical closed-season duration threshold that depends strictly on harvesting intensity. When fishing pressure is excessive, the population persists in a stable positive periodic state if the closed season exceeds this threshold; otherwise, it inevitably tends toward extinction. Notably, while the closed-season threshold is independent of the density-dependence parameter, the population biomass levels and harvesting outcomes are profoundly influenced by this parameter. We analyze the maximum annual sustainable yield (MASY) within the persistence region and demonstrate that it is attained only in the absence of a closed season under idealized continuous harvesting. Numerical simulations confirm that a larger density-dependence parameter enhances population recovery, raises the stable periodic population level, and increases the MASY. Furthermore, a resilience-constrained yield-efficiency index (RCYEI) is proposed by integrating annual sustainable yield, catch per unit effort (CPUE), and ecological safety constraints to circumvent the drawbacks of yield-only optimization. The results indicate that placing greater emphasis on yield raises the optimal harvesting intensity, whereas stricter ecological safety requirements limit the allowable harvesting pressure. A larger density-dependence parameter also expands the ecologically feasible region, enabling the population to withstand higher fishing pressure under identical safety requirements. These findings provide rigorous theoretical insights and practical guidance for designing sustainable seasonal harvesting strategies under ecological safety constraints.

Keywords: seasonal harvesting; θ -logistic model; maximum annual sustainable yield; single species; periodic solutions; global stability

1. Introduction

Fishery resources make substantial contributions to global food security, employment, and economic growth. However, steady increases in global fishing effort and technological advancements have intensified marine fishing pressure, driving heavy exploitation of fish stocks and precipitous declines in numerous populations [1, 2]. The Food and Agriculture Organization (FAO) of the United Nations reports that from 1974 to 2021, the proportion of marine fish stocks harvested at biologically sustainable levels dropped from 90% to 62.3%, while stocks fished at biologically unsustainable levels increased from 10% to 37.7% [3]. In China, the collapse of Chinese shrimp stocks in the 1980s and the continuous decline of many commercially vital fishery resources reflect the long-term ecological pressure exerted on marine environments [4–6]. Furthermore, the International Union for Conservation of Nature (IUCN) formally declared the Chinese Paddlefish extinct and the Yangtze Sturgeon extinct in the wild in 2022 [7]. These alarming collapses underscore that marine resource conservation remains an urgent and complex challenge. Consequently, balancing fishery harvesting with long-term population persistence is a primary concern in contemporary fisheries management.

Mathematical modeling provides an analytical framework for understanding fish population dynamics and evaluating harvesting strategies. Because fish reproduction, migration, and growth are fundamentally governed by seasonal environmental changes, the intrinsic growth rate, carrying capacity, and harvesting rate are frequently modeled as periodic functions in population dynamics models [8–10]. This concept has also been extended to impulsive control and time-delayed systems to characterize biological and epidemiological population dynamics in periodic environments [11–16]. In practice, however, many fisheries enforce seasonal fishing moratoriums, which represent periodic but discontinuous management policies [17–22]. Under such regulations, harvesting is completely banned during the closed season to protect spawning adults and juveniles and resumes during the open season. To mathematically model this switching mechanism, researchers often employ square waves, sawtooth waves, or piecewise-constant functions [23–25]. For instance, Feng et al. [26] and Liu et al. [27] established a single-species seasonal Michaelis–Menten-type harvesting model to describe population dynamics under natural recovery during closed seasons and nonlinear harvesting during open seasons:

$$\begin{cases} \frac{dx}{dt} = rx \left(1 - \frac{x}{G}\right), & t \in (n\omega, n\omega + \bar{\omega}], \\ \frac{dx}{dt} = rx \left(1 - \frac{x}{G}\right) - \frac{qEx}{cE + lx}, & t \in (n\omega + \bar{\omega}, (n+1)\omega], \end{cases} \quad (1.1)$$

where $x(t)$ is the population density and $\frac{qEx}{cE + lx}$ represents a nonlinear Michaelis–Menten-type harvesting rate. The parameter ω denotes the environmental period, while $\bar{\omega}$ and $\omega - \bar{\omega}$ correspond to the lengths of the closed and open seasons, respectively ($n \in \mathbb{N}_0$). This model provides a robust foundation for analyzing the effects of seasonal regulation on population persistence and periodic dynamics.

While existing models predominantly utilize classical logistic growth, natural populations generally exhibit nonlinear density dependence. There are distinct biological variations in compensatory capacity across different species during low-density recovery, environmental saturation, and harvesting disturbances. Assuming linear density regulation ($\theta = 1$) for all populations may lead to distorted evaluations of stock dynamics and result in biased sustainable harvesting assessments. To capture more flexible density regulation, the θ -logistic model has been widely implemented in population ecology [28–31],

taking the form

$$\frac{dx}{dt} = rx \left[1 - \left(\frac{x}{K} \right)^\theta \right], \quad (1.2)$$

where $\theta > 0$ is the shape parameter determining density dependence. When $\theta = 1$, the system reduces to the classical logistic equation. For $0 < \theta < 1$, strong density regulation operates at low densities, resulting in relatively slow population recovery. Conversely, for $\theta > 1$, density regulation acts primarily near the carrying capacity K , enabling rapid recovery for depleted populations [32]. Empirical studies on birds, mammals, and aquatic invertebrates have utilized the θ -logistic framework to confirm that density dependence is rarely linear and directly affects extinction risks [33–37]. Furthermore, Sibly et al. [38] and Hatton et al. [39] estimated θ distributions from extensive fish time-series data, demonstrating frequent deviations from the linear baseline ($\theta = 1$). Thus, it is imperative to investigate how the density-dependence parameter θ influences the dynamics of seasonally harvested fisheries.

In this paper, we formulate a seasonally switching θ -logistic harvesting model with alternating closed and open seasons by synthesizing systems (1.1) and (1.2). Maximizing yield under seasonal harvesting while preserving long-term biological persistence is a fundamental challenge. Previous studies on annual sustainable yield and maximum annual sustainable yield (MASY) offer valuable guidance for optimal harvesting policies [15, 40, 41]. However, strategies that strictly maximize yield often achieve MASY at the cost of high harvesting intensity and severe stock depletion, which diminishes harvesting efficiency and weakens biological resilience. To quantify harvesting efficiency, catch per unit effort (CPUE) is widely adopted as an index of relative stock abundance and economic return in commercial fisheries [42–44]. We incorporate CPUE into our seasonal harvesting model to measure economic output per unit fishing effort. Nonetheless, optimizing yield and CPUE alone cannot fully safeguard fish populations against sudden collapse. Threshold-based management principles emphasize that resource exploitation must operate within clearly defined ecological boundaries [45–48]. Motivated by these principles, we impose a resilience constraint requiring that the long-term periodic population biomass be maintained above a prescribed fraction $\alpha \in (0, 1)$ of its unharvested carrying capacity. Integrating this safety boundary, we propose a resilience-constrained yield-efficiency index (RCYEI) equipped with a preference weight parameter to balance annual sustainable yield against CPUE. By unifying economic yield, harvesting efficiency, and ecological safety within a single framework, this index provides a quantitative tool for determining optimal seasonal harvesting strategies that harmonize economic utility with biological resilience.

To our knowledge, seasonal harvesting models that simultaneously integrate nonlinear density dependence and ecological safety constraints remain largely unexplored. To bridge this gap, we develop and analyze a seasonally switching single-species θ -logistic harvesting model. The primary contributions and novelties of this study are threefold:

- 1) We incorporate nonlinear density dependence into a seasonally switching harvesting framework, providing a rigorous global dynamics analysis of periodic solutions and deriving explicit closed-season thresholds to prevent population collapse.
- 2) We formulate the RCYEI, which jointly integrates MASY, CPUE, and biological resilience constraints, providing a balanced management tool to optimize fishing efficiency while maintaining stock abundance above prescribed safety thresholds.
- 3) We conduct a systematic evaluation of the density-dependence parameter θ , revealing its critical role in determining stock recovery speed, expanding ecologically feasible harvesting regions, and

guiding sustainable management policies under safety constraints.

The remainder of this paper is organized as follows. In Section 2, we investigate the global dynamics of the seasonally switching single-species θ -logistic harvesting model, establishing the existence, uniqueness, and global asymptotic stability of positive T -periodic solutions and deriving the critical closed-season threshold for persistence. In Section 3, we derive the explicit piecewise formulation of the positive T -periodic solution, quantify annual sustainable yield, and prove the existence and uniqueness of the MASY. In Section 4, we propose the RCYEI by incorporating CPUE and resilience safety constraints, systematically evaluating optimal seasonal harvesting policies under varying management preferences and density-dependence parameters. Finally, Section 5 summarizes our main conclusions and highlights directions for future research.

2. Global dynamics of a single-species model with seasonal harvesting

This section investigates the dynamics of a single-species population subject to seasonal harvesting. We formulate a seasonally switching θ -logistic model and compare its qualitative behavior with the corresponding continuous-harvesting system. Sufficiently strong harvesting inevitably causes extinction under year-round exploitation, whereas introducing a sufficiently long closed season can guarantee population persistence. By introducing the coordinate transformation $y = x^\theta$, we reduce system (2.2) to the general periodic switching framework of Ding et al. [49]. Utilizing their threshold theory, we derive the critical closed-season threshold and completely characterize the global dynamics of system (2.2). Numerical simulations validate our analytical results and illustrate how the density-dependence parameter shapes the stable periodic biomass level.

2.1. Model formulation

We consider the following θ -logistic population model subject to constant-effort harvesting during the open season:

$$\begin{cases} \frac{dN}{dt} = rN \left[1 - \left(\frac{N}{K} \right)^\theta \right], & t \in (n\omega, n\omega + \bar{\omega}], \\ \frac{dN}{dt} = rN \left[1 - \left(\frac{N}{K} \right)^\theta \right] - qEN, & t \in (n\omega + \bar{\omega}, (n+1)\omega], \end{cases} \quad (2.1)$$

where $n \in \mathbb{N}_0$. Here, $N(t)$ denotes the population biomass at time t , $r > 0$ is the intrinsic growth rate, $K > 0$ is the carrying capacity, and $\theta > 0$ is the density-dependence parameter. When $\theta = 1$, model (2.1) reduces to the classical logistic harvesting model. The parameter $q > 0$ represents the catchability coefficient, $E > 0$ is the harvesting effort, and qEN denotes the constant-effort harvesting rate. The parameter $\omega > 0$ represents the period of the seasonal environmental cycle, while $\bar{\omega}$ and $\omega - \bar{\omega}$ denote the durations of the closed and open seasons, respectively.

Introducing the dimensionless variables and parameters

$$x = \frac{N}{K}, \quad \bar{t} = rt, \quad T = r\omega, \quad \bar{T} = r\bar{\omega}, \quad \delta = \frac{qE}{r}$$

and dropping the bar over the time variable $\bar{t}$ for convenience, system (2.1) is rescaled to the dimension-

less form:

$$\begin{cases} \frac{dx}{dt} = x(1 - x^\theta), & t \in (nT, nT + \bar{T}], \\ \frac{dx}{dt} = x(1 - x^\theta) - \delta x, & t \in (nT + \bar{T}, (n+1)T], \end{cases} \quad (2.2)$$

where $0 \leq \bar{T} < T$, $\delta > 0$, and $\theta > 0$.

When $\bar{T} = 0$, system (2.2) reduces to the continuous harvesting model:

$$\frac{dx}{dt} = x(1 - x^\theta) - \delta x. \quad (2.3)$$

The trivial equilibrium of model (2.3) is $x_0 = 0$. Furthermore, when $0 < \delta < 1$, model (2.3) admits a unique positive equilibrium:

$$x^* = (1 - \delta)^{\frac{1}{\theta}}.$$

Before analyzing the seasonal switching system, we first establish the global stability of equilibria for the continuous harvesting model (2.3).

Proposition 2.1. *For any initial value $x(0) > 0$, the following statements hold for system (2.3):*

- (i) *If $\delta > 1$, then $x_0 = 0$ is the unique non-negative equilibrium and is globally asymptotically stable.*
- (ii) *If $\delta = 1$, then $x_0 = 0$ is the unique non-negative equilibrium and is globally asymptotically stable.*
- (iii) *If $0 < \delta < 1$, then the positive equilibrium $x^* = (1 - \delta)^{1/\theta}$ exists and is globally asymptotically stable, whereas the trivial equilibrium $x_0 = 0$ is unstable.*

Proof. For the case $\delta > 1$, since $1 - \delta < 0$, the positive equilibrium $x^* = (1 - \delta)^{1/\theta}$ does not exist in $\mathbb{R}^+$, leaving $x_0 = 0$ as the unique non-negative equilibrium. Furthermore, $\frac{dx}{dt} = x(1 - \delta - x^\theta) < 0$ for all $x > 0$. Therefore, the trivial equilibrium $x_0 = 0$ is globally asymptotically stable in model (2.3).

For the case $\delta = 1$, we have $1 - \delta = 0$, so the only non-negative equilibrium is $x_0 = 0$. Furthermore, $\frac{dx}{dt} = -x^{\theta+1} < 0$ for all $x > 0$. Therefore, $x_0 = 0$ is globally asymptotically stable.

For the case $0 < \delta < 1$, both the trivial equilibrium $x_0 = 0$ and the positive equilibrium $x^* = (1 - \delta)^{1/\theta}$ exist. Moreover, $\frac{dx}{dt} < 0$ for $x > x^*$ and $\frac{dx}{dt} > 0$ for $0 < x < x^*$. Hence, the positive equilibrium x^* is globally asymptotically stable, while the trivial equilibrium $x_0 = 0$ is unstable. $\square$

Proposition 2.1 demonstrates that under continuous year-round harvesting, the population faces inevitable extinction once fishing intensity exceeds the intrinsic growth threshold ($\delta \geq 1$). Preventing stock collapse requires implementing a recovery period during which fishing is suspended. Thus, the fundamental management question is determining the exact duration of the closed season required to maintain stock persistence.

2.2. Threshold dynamics

To establish the global dynamics and extinction threshold of system (2.2), we employ the periodic switching dynamical systems theory developed by Ding et al. [49]. We first restate their core analytical framework and fundamental theorems.

Lemma 2.2 (Ding et al. [49]). Consider a general periodic switching dynamical system of the form

$$\begin{cases} \frac{dx}{dt} = xf(t, x, \alpha), & t \in (nT, nT + \bar{T}], \\ \frac{dx}{dt} = xg(t, x, \alpha), & t \in (nT + \bar{T}, (n+1)T], \end{cases} \quad (2.4)$$

where $f(t, x, \alpha)$ and $g(t, x, \alpha)$ are continuous and T -periodic in t . Assume that f and g satisfy the following standard structural hypotheses:

- (H1) $f(t, 0, \alpha) \geq f(t, x, \alpha)$ and $g(t, 0, \alpha) \geq g(t, x, \alpha)$ for all $x > 0$;
- (H2) There exist positive constants M_1 and M_2 such that $f(t, x, \alpha) \leq 0$ for all $x \geq M_1$ and $g(t, x, \alpha) \leq 0$ for all $x \geq M_2$;
- (H3) $f_x(t, x, \alpha) < 0$ and $g_x(t, x, \alpha) < 0$ for all $x > 0$.

Let $\mathcal{F}(T, \bar{T}, \alpha) = \int_0^{\bar{T}} f(t, 0, \alpha) dt + \int_{\bar{T}}^T g(t, 0, \alpha) dt$ denote the associated threshold function, and let ω^* be the unique non-negative root of $\mathcal{F} = 0$ with respect to a critical parameter ω . The generic dynamical behavior of system (2.4) is characterized as follows:

- (i) **(Theorem 2.1 in [49])** Suppose that (H2) holds. If $\omega > \omega^*$, then system (2.4) admits at least one positive T -periodic solution. Furthermore, if (H3) also holds, then this positive T -periodic solution is unique and globally attractive.
- (ii) **(Theorem 2.2 in [49])** Suppose that (H1) holds. If $\omega < \omega^*$, then the trivial equilibrium 0 of system (2.4) is globally asymptotically stable.
- (iii) **(Theorem 2.3 in [49])** Suppose that (H1) holds with strict inequalities for $x > 0$. If $\omega = \omega^*$, then the trivial equilibrium 0 of system (2.4) is globally asymptotically stable.

For $x > 0$, applying the bijective coordinate transformation $y = x^\theta$ reduces system (2.2) to the simplified switching system:

$$\begin{cases} \frac{dy}{dt} = yf(t, y, \delta), & t \in (nT, nT + \bar{T}], \\ \frac{dy}{dt} = yg(t, y, \delta), & t \in (nT + \bar{T}, (n+1)T], \end{cases} \quad (2.5)$$

where the nonlinear switching rates are defined as

$$f(t, y, \delta) = \theta(1 - y), \quad g(t, y, \delta) = \theta(1 - \delta - y).$$

System (2.5) matches the general switching framework (2.4). We now verify that system (2.5) satisfies hypotheses (H1)–(H3).

Lemma 2.3. For any $\theta > 0$ and $\delta > 0$, system (2.5) satisfies hypotheses (H1)–(H3):

- (i) Hypothesis (H1) holds with strict inequalities for all $y > 0$;
- (ii) Hypothesis (H2) holds with $M_1 = M_2 = 1$;
- (iii) Hypothesis (H3) holds for all $y > 0$.

Proof. For any $y > 0$, direct computation yields

$$f(t, 0, \delta) - f(t, y, \delta) = \theta y > 0, \quad g(t, 0, \delta) - g(t, y, \delta) = \theta y > 0,$$

confirming (H1) with strict inequalities. Next, for any $y \geq 1$, we observe that

$$f(t, y, \delta) = \theta(1 - y) \leq 0, \quad g(t, y, \delta) = \theta(1 - \delta - y) \leq 0,$$

which verifies (H2) with $M_1 = M_2 = 1$. Finally, differentiating f and g with respect to y gives

$$f_y(t, y, \delta) = g_y(t, y, \delta) = -\theta < 0, \quad \text{for all } y > 0,$$

which confirms (H3). $\square$

Having verified hypotheses (H1)–(H3), we compute the explicit threshold function $\Phi(\delta, \bar{T})$ for system (2.5). Substituting $f(t, 0, \delta) = \theta$ and $g(t, 0, \delta) = \theta(1 - \delta)$ into the threshold integral yields

$$\Phi(\delta, \bar{T}) = \int_0^{\bar{T}} f(t, 0, \delta) dt + \int_{\bar{T}}^T g(t, 0, \delta) dt = \theta [\delta \bar{T} + (1 - \delta)T].$$

Consequently, the sign of $\Phi(\delta, \bar{T})$ is completely categorized by the harvesting intensity δ :

- (i) If $\delta > 1$, setting $\Phi(\delta, \bar{T}^*) = 0$ defines a unique critical closed-season duration threshold:

$$\bar{T}^* = \frac{\delta - 1}{\delta} T \in (0, T).$$

In this regime, $\Phi(\delta, \bar{T}) > 0$ for $\bar{T} > \bar{T}^*$, $\Phi(\delta, \bar{T}) = 0$ for $\bar{T} = \bar{T}^*$, and $\Phi(\delta, \bar{T}) < 0$ for $\bar{T} < \bar{T}^*$.

- (ii) If $\delta = 1$, we obtain $\Phi(1, \bar{T}) = \theta \bar{T}$. Thus, $\Phi(1, \bar{T}) > 0$ for $\bar{T} > 0$, and $\Phi(1, \bar{T}) = 0$ when $\bar{T} = 0$.
- (iii) If $0 < \delta < 1$, then $\Phi(\delta, \bar{T}) > 0$ strictly holds for all $0 \leq \bar{T} < T$.

Because the coordinate transformation $y = x^\theta$ is a strictly increasing diffeomorphism on $(0, +\infty)$, all qualitative dynamical properties—including the existence, uniqueness, and stability of periodic solutions—are preserved under this transformation. Combining Lemma 2.2, Lemma 2.3, and the threshold analysis above, we establish the global persistence criteria for system (2.2).

Proposition 2.4. *System (2.2) admits a unique globally asymptotically stable positive T -periodic solution $x^p(t)$ if one of the following conditions holds:*

- (i) $\delta > 1$ and $\bar{T} > \bar{T}^*$;
- (ii) $\delta = 1$ and $\bar{T} > 0$;
- (iii) $0 < \delta < 1$ and $0 \leq \bar{T} < T$.

Proof. Under any of the conditions (i)–(iii), our threshold analysis guarantees that $\Phi(\delta, \bar{T}) > 0$, matching the threshold condition in Lemma 2.2(i). Because hypotheses (H2) and (H3) hold by Lemma 2.3, Lemma 2.2(i) directly implies that system (2.5) admits a unique globally attractive positive T -periodic solution $y^p(t)$. Thus, system (2.2) possesses a unique globally attractive positive T -periodic solution given by $x^p(t) = (y^p(t))^{1/\theta}$.

To rigorously establish local asymptotic stability, we compute the Floquet multiplier ρ associated with the linearized variational equation around $x^p(t)$:

$$\rho = \exp \left\{ \int_0^{\bar{T}} [1 - (\theta + 1)(x^p(t))^\theta] dt + \int_{\bar{T}}^T [1 - \delta - (\theta + 1)(x^p(t))^\theta] dt \right\}.$$

Integrating the governing ordinary differential equations (ODEs) $\frac{(x^p)'}{x^p} = 1 - (x^p)^\theta$ on $(0, \bar{T}]$ and $\frac{(x^p)'}{x^p} = 1 - \delta - (x^p)^\theta$ on $(\bar{T}, T]$ over one period and applying the periodicity condition $\ln x^p(T) - \ln x^p(0) = 0$ yields the integral identity:

$$\int_0^T (x^p(t))^\theta dt = \bar{T} + (1 - \delta)(T - \bar{T}).$$

Substituting this identity into the expression for ρ , we obtain

$$\rho = \exp \left\{ -\theta [\bar{T} + (1 - \delta)(T - \bar{T})] \right\} = e^{-\Phi(\delta, \bar{T})} < 1.$$

Because $\rho < 1$, the periodic solution $x^p(t)$ is locally asymptotically stable. Combined with global attractivity, this establishes the global asymptotic stability of $x^p(t)$ for system (2.2). $\square$

Complementing Proposition 2.4, the following proposition specifies the conditions leading to complete population extinction.

Proposition 2.5. *The trivial equilibrium $x_0 = 0$ of system (2.2) is globally asymptotically stable if one of the following conditions holds:*

- (i) $\delta > 1$ and $\bar{T} < \bar{T}^*$;
- (ii) $\delta > 1$ and $\bar{T} = \bar{T}^*$;
- (iii) $\delta = 1$ and $\bar{T} = 0$.

Proof. Under condition (i), we have $\Phi(\delta, \bar{T}) < 0$. Because hypothesis (H1) is satisfied by Lemma 2.3, Lemma 2.2(ii) directly proves the global asymptotic stability of the trivial equilibrium $x_0 = 0$. Under conditions (ii) and (iii), we have $\Phi(\delta, \bar{T}) = 0$. Since (H1) holds with strict inequalities for $x > 0$ by Lemma 2.3, applying Lemma 2.2(iii) confirms that $x_0 = 0$ is globally asymptotically stable. $\square$

Propositions 2.4 and 2.5 establish a sharp threshold classification for seasonal harvesting in system (2.2). When harvesting pressure exceeds the intrinsic recovery rate ($\delta > 1$), the closed-season threshold $\bar{T}^*$ represents the exact biological boundary separating persistence from extinction. From a fisheries management perspective, higher harvesting effort strictly mandates a longer closed season to preserve population sustainability.

2.3. Numerical examples

In this subsection, we conduct numerical simulations to illustrate and validate our theoretical theorems for system (2.2). For all simulations in Figures 1 and 2, we set $\theta = 1$ and $T = 1$.

Example 2.6. This example verifies the global asymptotic stability of the positive T -periodic solution under the persistence regimes specified in Proposition 2.4. Numerical trajectories from distinct initial values are shown in Figure 1.

- (a) Let $\delta = 1.5$. Here, $\delta > 1$, and the critical threshold is $\bar{T}^* \approx 0.33$. For $\bar{T} = 0.6 > \bar{T}^*$, Proposition 2.4(i) implies that the positive T -periodic solution is globally asymptotically stable, as confirmed in Figure 1(a).
- (b) Let $\delta = 1$. Here, $\bar{T}^* = 0$. For $\bar{T} = 0.6 > 0$, Proposition 2.4(ii) guarantees global asymptotic stability of the positive T -periodic solution, as illustrated in Figure 1(b).
- (c) Let $\delta = 0.5$ and $\bar{T} = 0$. Proposition 2.4(iii) implies that the positive equilibrium of the continuous harvesting model is globally asymptotically stable, as shown in Figure 1(c).
- (d) Let $\delta = 0.5$ and $\bar{T} = 0.3 > 0$. Proposition 2.4(iii) implies global asymptotic stability of the positive T -periodic solution, as depicted in Figure 1(d).

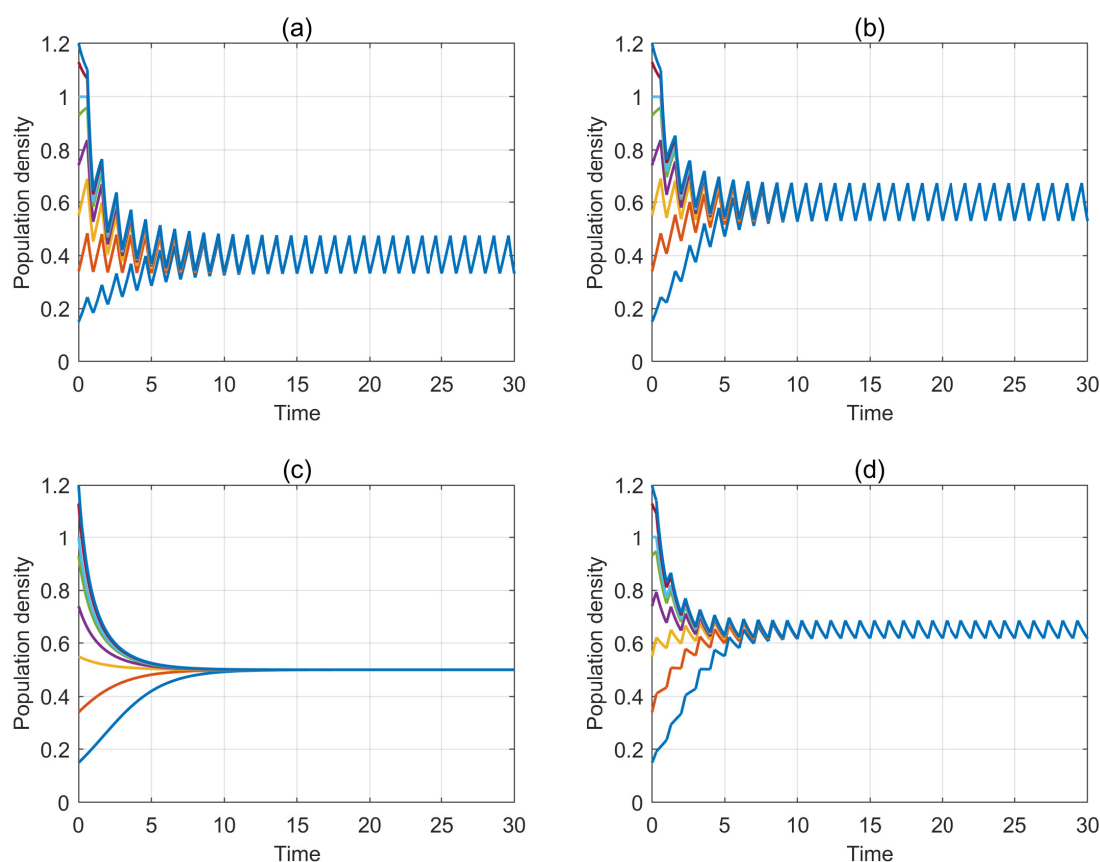


Figure 1. Dynamics of population density $x(t)$ illustrating the global asymptotic stability of the positive T -periodic solution under the conditions of Proposition 2.4. Curves of various colors represent trajectories originating from different initial values.

Example 2.7. This example validates the extinction scenarios predicted by Proposition 2.5. Numerical trajectories confirming population collapse are shown in Figure 2.

- (a) Let $\delta = 1.5$. The critical threshold is $\bar{T}^* \approx 0.33$. For $\bar{T} = 0.3 < \bar{T}^*$, Proposition 2.5(i) implies that the trivial equilibrium 0 is globally asymptotically stable, as illustrated in Figure 2(a).
- (b) Let $\delta = 2.6$. The critical threshold is $\bar{T}^* \approx 0.62$. For $\bar{T} = 0.6 < \bar{T}^*$, Proposition 2.5(i) confirms that

the population tends toward extinction, as shown in Figure 2(b).

- (c) Let $\delta = 2.6$ and $\bar{T} = \bar{T}^* \approx 0.62$. Proposition 2.5(ii) proves that the critical threshold boundary itself leads to global extinction, as verified in Figure 2(c).
- (d) Let $\delta = 1$ and $\bar{T} = 0$. Proposition 2.5(iii) confirms global asymptotic stability of the origin 0, as shown in Figure 2(d).

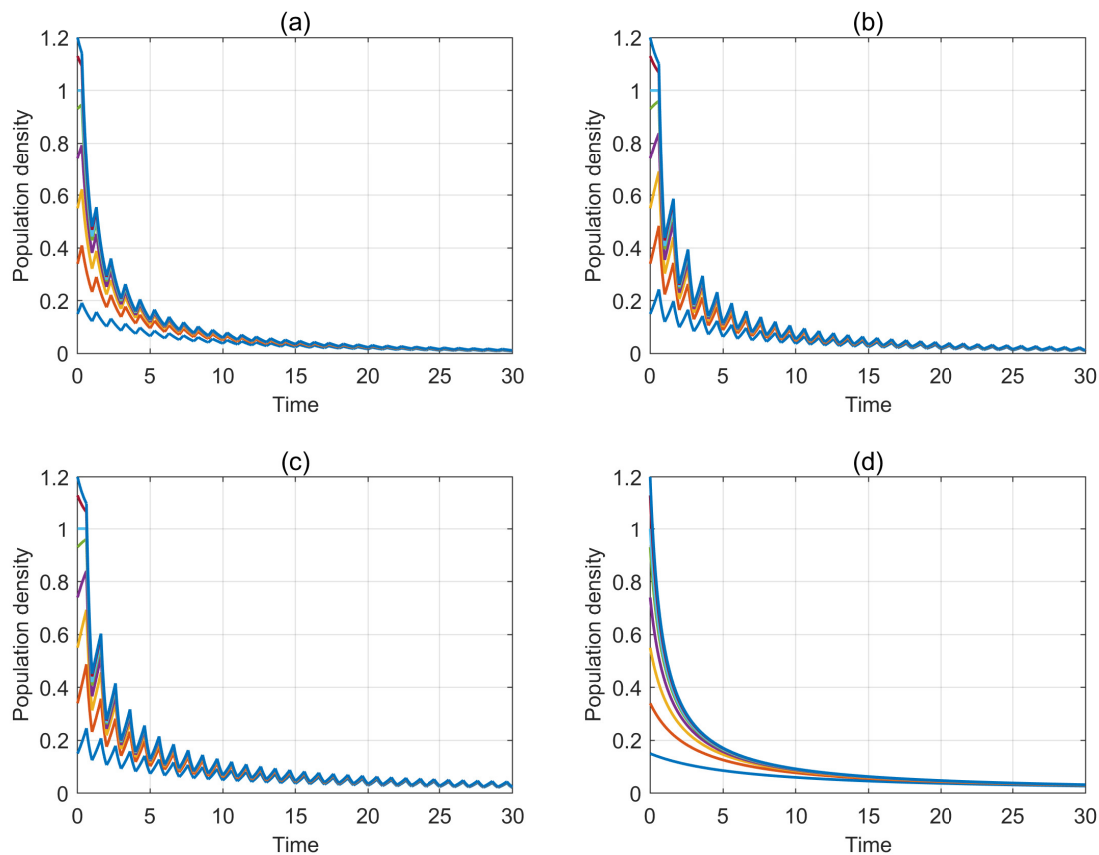


Figure 2. Dynamics of population density $x(t)$ illustrating population extinction under the conditions of Proposition 2.5, where the trivial equilibrium 0 is globally asymptotically stable.

The numerical simulations above validate our theoretical threshold classification. In physical variables, when harvesting intensity exceeds intrinsic growth ($qE > r$), the critical closed-season duration threshold

$$\bar{\omega}^* = \frac{qE - r}{qE} \omega$$

determines whether the fish stock persists or collapses. Notably, because the parameter θ does not appear in this expression, the critical closed-season threshold is completely independent of nonlinear density dependence.

Nevertheless, once biological persistence is secured ($\bar{T} > \bar{T}^*$), θ strongly influences the actual periodic population abundance. We examine this quantitative effect through numerical simulations

across three distinct regimes: $0 < \theta < 1$, $\theta = 1$ (classical logistic), and $\theta > 1$. The comparative asymptotic periodic dynamics are illustrated in Figure 3.

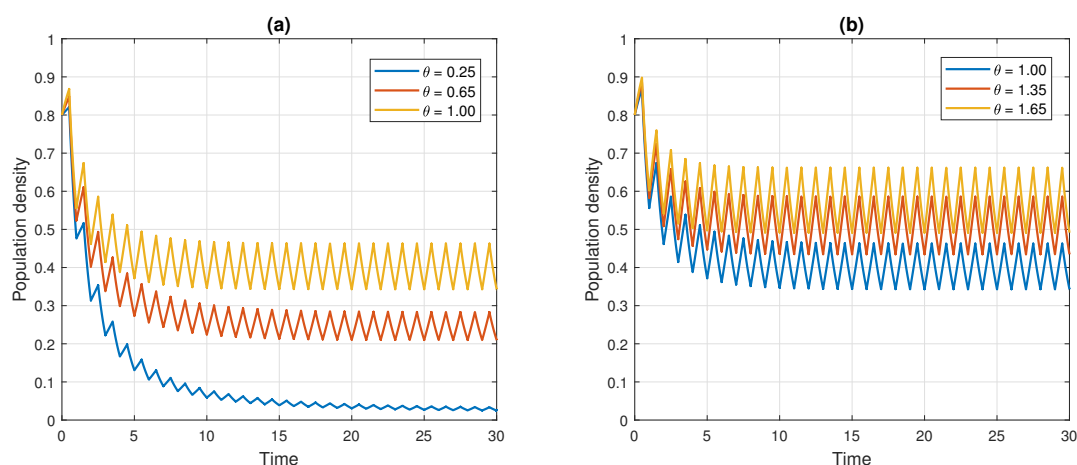


Figure 3. Effect of the density-dependence parameter θ on the asymptotic periodic dynamics of population density $x(t)$. Parameters are fixed at $\delta = 1.2$, $T = 1$, $\bar{T} = 0.5$, and $x(0) = 0.8$. Panel (a) compares $\theta \in \{0.25, 0.65, 1.00\}$, while Panel (b) compares $\theta \in \{1.00, 1.35, 1.65\}$. The values $\theta = 0.25$ and 0.65 are representative of slowly recovering species such as certain elasmobranchs or long-lived demersal fish, while $\theta = 1.35$ and 1.65 correspond to rapidly recovering species like small pelagics (e.g., anchovies, sardines) with strong compensatory dynamics.

As shown in Figure 3, θ exerts a profound influence on periodic biomass oscillations. In Figure 3(a) ($0 < \theta \leq 1$), the system is highly sensitive to changes in θ : Both minimum periodic density and oscillation amplitude increase substantially as θ increases toward 1. This occurs because smaller θ values impose severe density regulation at low biomass levels, significantly retarding population recovery during closed seasons. In Figure 3(b) ($\theta \geq 1$), larger θ values further elevate the overall population density above the linear logistic baseline ($\theta = 1$), although sensitivity diminishes as θ increases.

These insights emphasize that the closed-season threshold $\bar{T}^*$ represents merely a bare minimum survival boundary rather than an optimal management policy. Species with identical extinction thresholds may exhibit vastly different recovery rates and standing biomasses due to species-specific density dependence (θ). Consequently, uniform closed-season regulations provide inadequate protection for slowly recovering populations ($0 < \theta < 1$). For such species, precautionary management requires longer closed seasons, reduced harvesting intensity, or explicit biomass safety thresholds. Motivated by these requirements, the next section derives the explicit periodic solution to quantify annual sustainable yield and investigate optimal harvesting policies.

3. Maximum annual sustainable yield

In this section, we investigate the MASY within the admissible parameter region where system (2.2) admits a globally asymptotically stable positive T -periodic solution. To align with practical management cycles, we normalize the environmental period to $T = 1$ (one year). Unlike the classical maximum sustainable yield (MSY) [50], which assumes continuous harvesting at an invariant steady state, our

seasonal framework allows biomass to fluctuate periodically, requiring only annual periodic recovery. We thus use the MASY to specifically denote the cumulative harvest aggregated over the full annual cycle while ensuring long-term population persistence.

We first construct the explicit piecewise formulation of the positive T -periodic solution $x^p(t)$. Let $x_1^p(t)$ and $x_2^p(t)$ denote the trajectories during the closed and open seasons, respectively:

$$x^p(t) = \begin{cases} x_1^p(t), & t \in (0, \bar{T}], \\ x_2^p(t), & t \in (\bar{T}, 1]. \end{cases}$$

By continuity and periodicity, the solution satisfies the boundary matching conditions:

$$x_1^p(0) = u_0, \quad x_1^p(\bar{T}) = x_2^p(\bar{T}), \quad x_2^p(1) = u_0,$$

where $u_0 = x^p(0)$ is the unique periodic initial value. Solving the Bernoulli differential equations (detailed algebraic steps are given in Appendix A) yields the explicit periodic initial value:

$$u_0(\delta, \bar{T}) = \begin{cases} \left[\frac{(1-\delta)(e^{\theta[\bar{T}+(1-\delta)(1-\bar{T})]} - 1)}{e^{\theta[\bar{T}+(1-\delta)(1-\bar{T})]} - \delta e^{\theta\bar{T}} + \delta - 1} \right]^{1/\theta}, & \delta \neq 1, \\ \left[\frac{e^{\theta\bar{T}} - 1}{e^{\theta\bar{T}}(1 + \theta(1 - \bar{T})) - 1} \right]^{1/\theta}, & \delta = 1. \end{cases} \quad (3.1)$$

During the closed season $t \in (0, \bar{T}]$, the explicit trajectory is

$$x_1^p(t) = \left(\frac{u_0^\theta e^{\theta t}}{1 - u_0^\theta + u_0^\theta e^{\theta t}} \right)^{1/\theta}, \quad (3.2)$$

and during the open season $t \in (\bar{T}, 1]$, the explicit trajectory is

$$x_2^p(t) = \begin{cases} \left[\frac{(x_1^p(\bar{T}))^\theta (1-\delta) e^{\theta(1-\delta)(t-\bar{T})}}{1 - \delta + (x_1^p(\bar{T}))^\theta (e^{\theta(1-\delta)(t-\bar{T})} - 1)} \right]^{1/\theta}, & \delta \neq 1, \\ \left[\frac{(x_1^p(\bar{T}))^\theta}{1 + \theta(x_1^p(\bar{T}))^\theta(t - \bar{T})} \right]^{1/\theta}, & \delta = 1. \end{cases} \quad (3.3)$$

Based on this explicit periodic solution, we define the annual sustainable yield $H_\theta(\delta, \bar{T})$ as the cumulative harvest over one calendar year:

$$H_\theta(\delta, \bar{T}) = \delta \int_{\bar{T}}^1 x_2^p(t; u_0(\delta, \bar{T})) dt. \quad (3.4)$$

When $\theta = 1$, system (2.2) reduces to the classical logistic seasonal harvesting model studied by Yu et al. [41], where the MASY is attained at the boundary $\bar{T} = 0$. We now generalize this analysis to the θ -logistic framework and prove that this optimal boundary structure holds universally for all $\theta > 0$.

Theorem 3.1. Assume $\theta > 0$ and $0 \leq \bar{T} < T$. Among all admissible parameter pairs $(\delta, \bar{T})$ for which system (2.2) admits a positive T -periodic solution, the annual sustainable yield $H_\theta(\delta, \bar{T})$ attains its unique global maximum at

$$(\delta^*, \bar{T}^*) = \left(\frac{\theta}{\theta + 1}, 0 \right),$$

yielding the MASY:

$$H_{\theta, \max} = \frac{\theta}{(\theta + 1)^{1 + \frac{1}{\theta}}}.$$

At this maximizing point, the corresponding population biomass remains constant at the optimal equilibrium level:

$$x^p(t) \equiv \left(\frac{1}{\theta + 1} \right)^{\frac{1}{\theta}}, \quad t \in [0, T].$$

Proof. Let $x^p(t)$ be the positive T -periodic solution corresponding to an admissible pair $(\delta, \bar{T})$. Integrating the governing differential equations over one period $[0, T]$ and invoking the periodicity condition $x^p(T) = x^p(0)$, we obtain

$$\begin{aligned} 0 &= \int_0^{\bar{T}} (x_1^p)'(t) dt + \int_{\bar{T}}^T (x_2^p)'(t) dt \\ &= \int_0^{\bar{T}} x_1^p(t) [1 - (x_1^p(t))^\theta] dt + \int_{\bar{T}}^T x_2^p(t) [1 - (x_2^p(t))^\theta] dt - \delta \int_{\bar{T}}^T x_2^p(t) dt. \end{aligned}$$

By definition, the annual sustainable yield over period T is $H_\theta(\delta, \bar{T}) = \frac{\delta}{T} \int_{\bar{T}}^T x_2^p(t) dt$. Rearranging the integral identity above yields

$$H_\theta(\delta, \bar{T}) = \frac{1}{T} \int_0^T x^p(t) [1 - (x^p(t))^\theta] dt.$$

Define the biological production function $F(x) = x(1 - x^\theta)$ for $x > 0$. Differentiating with respect to x gives

$$F'(x) = 1 - (\theta + 1)x^\theta, \quad F''(x) = -\theta(\theta + 1)x^{\theta-1} < 0.$$

Thus, $F(x)$ is strictly concave and attains its unique global maximum at $x_* = (\frac{1}{\theta+1})^{1/\theta}$, with maximum value $F(x_*) = \frac{\theta}{(\theta+1)^{1+1/\theta}}$. Applying integral inequality bounds, we have

$$H_\theta(\delta, \bar{T}) = \frac{1}{T} \int_0^T F(x^p(t)) dt \leq F(x_*) = \frac{\theta}{(\theta + 1)^{1 + \frac{1}{\theta}}}.$$

The upper bound is attained if and only if the periodic solution remains constant at the optimal biomass level throughout the year: $x^p(t) \equiv x_*$.

If $\bar{T} > 0$, then during the closed season $(0, \bar{T})$, the constant solution must satisfy $(x_*)' = 0$, which implies $x_*(1 - x_*^\theta) = 0$, forcing $x_*^\theta = 1$. This directly contradicts $x_*^\theta = \frac{1}{\theta+1} < 1$. Consequently, achieving the MASY strictly requires eliminating the closed season: $\bar{T} = 0$.

When $\bar{T} = 0$, system (2.2) operates as continuous harvesting. For $0 < \delta < 1$, the unique stable positive equilibrium is $x^* = (1 - \delta)^{1/\theta}$, yielding the steady-state annual harvest $G(\delta) = \delta(1 - \delta)^{1/\theta}$. Differentiating $G(\delta)$ gives

$$G'(\delta) = (1 - \delta)^{\frac{1}{\theta}-1} \left(1 - \frac{\theta + 1}{\theta} \delta \right).$$

Setting $G'(\delta) = 0$ confirms that $G(\delta)$ attains its unique maximum at $\delta = \frac{\theta}{\theta+1}$. Substituting this optimal harvesting intensity into $G(\delta)$ yields the exact MASY value $H_{\theta, \max}$. $\square$

Theorem 3.1 establishes the theoretical upper bound for annual yield across the entire biological persistence region. The global maximum is achieved under continuous year-round harvesting ($\bar{T} = 0$) at moderate fishing intensity ($\delta = \frac{\theta}{\theta+1} < 1$). In practical fisheries management, however, zero-closure policies are often unfeasible because seasonal moratoriums are ecologically indispensable for protecting spawning aggregations and juvenile growth.

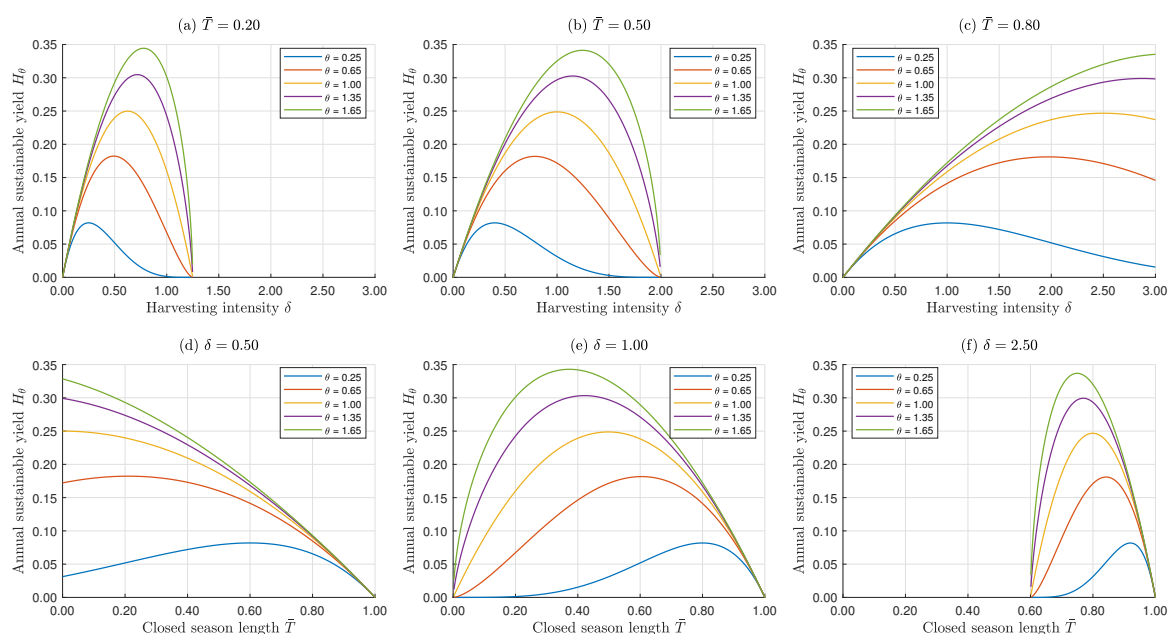


Figure 4. Numerical evaluations of annual sustainable yield $H_{\theta}(\delta, \bar{T})$ for $T = 1$. Panels (a)–(c) illustrate yield across harvesting intensity δ under fixed closed seasons $\bar{T} \in \{0.20, 0.50, 0.80\}$. Panels (d)–(f) illustrate yield across closed-season duration $\bar{T}$ under fixed harvesting intensities $\delta \in \{0.50, 1.00, 2.50\}$. Different θ values correspond to distinct ecological types: $\theta = 0.65$ (slow recovery, e.g., long-lived demersal fish), $\theta = 1.00$ (classical logistic), and $\theta = 1.35$ (fast recovery, e.g., small pelagics).

To illustrate this quantitative trade-off, consider a standard logistic fishery ($\theta = 1, T = 1$). While unconstrained year-round harvesting ($\bar{T} = 0$) yields the theoretical maximum $H_{1, \max} = 0.25$ at $\delta^* = 0.5$, introducing a mandatory seasonal closure of 20% of the year ($\bar{T} = 0.2$) yields $H_1 \approx 0.2384$ at an adjusted intensity $\delta \approx 0.62$. Thus, enforcing a closure of slightly over two months incurs only a 4.64% sacrifice in annual yield while significantly enhancing stock protection, highlighting the practical value

of seasonal closures as a cost-effective conservation tool. To investigate this trade-off systematically across a wider spectrum of operating conditions, we evaluate how annual sustainable yield behaves under fixed management constraints through numerical simulations in Figure 4.

As demonstrated in Figure 4(a)–(c), when the closed-season duration $\bar{T}$ is fixed, annual yield displays a unimodal response to harvesting intensity δ . Extending the closed season shifts the yield-maximizing harvesting intensity to higher levels, as longer recovery windows allow populations to withstand more intense open-season fishing without collapsing. Figure 4(d)–(f) reveal that when fishing effort is weak ($\delta = 0.5 < 1$), yield is maximized at $\bar{T} = 0$. However, under severe fishing pressure ($\delta = 2.5 > 1$, Figure 4(f)), sustainable yield is entirely zero unless the closed season exceeds the biological persistence threshold $\bar{T}^*$. Moreover, across all regimes, species with stronger density dependence ($\theta > 1$) consistently generate higher sustainable yields and require shorter recovery periods than linear or sublinear species.

4. Resilience-constrained yield-efficiency index

While maximizing annual sustainable yield $H_\theta(\delta, \bar{T})$ identifies productive fishing regimes, optimizing yield in isolation is a risky management strategy. Achieving the MASY typically requires high fishing effort that depresses standing stock biomass, driving down CPUE and increasing vulnerability to environmental collapse. This realization aligns with modern reference-point frameworks, which emphasize balancing socio-economic utility against strict biological limits [51].

To measure economic efficiency, CPUE is formally defined as catch per unit harvesting effort:

$$C_\theta(\delta, \bar{T}) = \frac{\delta \int_{\bar{T}}^T x_2^p(t; \delta, \bar{T}) dt}{\delta(T - \bar{T})} = \frac{1}{T - \bar{T}} \int_{\bar{T}}^T x_2^p(t; \delta, \bar{T}) dt. \quad (4.1)$$

Thus, CPUE equals the mean open-season population biomass, reflecting harvesting efficiency. However, even a combined yield-CPUE metric cannot prevent populations from dropping to dangerously low densities during fishing seasons. To guarantee ecological safety, we introduce a resilience constraint requiring that periodic stock biomass never fall below a prescribed safety fraction $\alpha \in (0, 1)$ of carrying capacity:

$$\min_{t \in [0, T]} x^p(t; \delta, \bar{T}) \geq \alpha.$$

We formally define the resilience-constrained admissible parameter set as

$$\mathcal{S}_{RC}(\alpha) = \left\{ (\delta, \bar{T}) \in (0, \infty) \times [0, T] : x^p(t; \delta, \bar{T}) \text{ exists and } \min_{t \in [0, T]} x^p(t; \delta, \bar{T}) \geq \alpha \right\}. \quad (4.2)$$

Any management policy outside $\mathcal{S}_{RC}(\alpha)$ is deemed ecologically illegal and excluded from optimization. Synthesizing multi-objective utility theory [52], we propose a unified management index.

Definition 4.1 (Resilience-constrained yield-efficiency index). For a safety threshold $\alpha \in (0, 1)$, let $H_{\theta, \max} = \max_{\mathcal{S}_{RC}(\alpha)} H_\theta(\delta, \bar{T})$ and $C_{\theta, \max} = \max_{\mathcal{S}_{RC}(\alpha)} C_\theta(\delta, \bar{T})$, both of which are strictly positive on the feasible set $\mathcal{S}_{RC}(\alpha)$. For a management preference weight $w \in [0, 1]$, the RCYEI is defined as

$$J(\delta, \bar{T}) = w \frac{H_\theta(\delta, \bar{T})}{H_{\theta, \max}} + (1 - w) \frac{C_\theta(\delta, \bar{T})}{C_{\theta, \max}}, \quad (\delta, \bar{T}) \in \mathcal{S}_{RC}(\alpha). \quad (4.3)$$

Remark 4.2. For any $(\delta, \bar{T}) \in \mathcal{S}_{RC}(\alpha)$, both $\delta > 0$ and $T - \bar{T} > 0$ hold by construction, while the resilience constraint ensures $x_2^p(t; \delta, \bar{T}) \geq \alpha > 0$ for all $t \in [\bar{T}, T]$. Hence,

$$C_\theta(\delta, \bar{T}) = \frac{1}{T - \bar{T}} \int_{\bar{T}}^T x_2^p(t; \delta, \bar{T}) dt \geq \alpha > 0, \quad H_\theta(\delta, \bar{T}) = \delta \int_{\bar{T}}^T x_2^p(t; \delta, \bar{T}) dt \geq \delta(T - \bar{T})\alpha > 0.$$

Taking the maximum over $\mathcal{S}_{RC}(\alpha)$ yields $H_{\theta, \max} > 0$ and $C_{\theta, \max} > 0$. Consequently, the index $J(\delta, \bar{T})$ in (4.3) is well defined.

Remark 4.3. Multi-objective fisheries management can also be approached through Pareto frontier analysis or lexicographic ordering [53, 54]. Pareto analysis effectively characterizes feasible trade-offs among competing objectives but does not by itself specify a unique operational policy. Lexicographic ordering provides clear priority rules but imposes strict rankings among objectives and may limit flexible compromises when multiple management goals are considered simultaneously. In contrast, our resilience-constrained index provides an intuitive decision framework: By adjusting the weight w , managers can flexibly balance catch volume and harvesting efficiency, while identifying solutions where both yield and CPUE remain favorable. Furthermore, restricting the optimization within $\mathcal{S}_{RC}(\alpha)$ ensures that ecological safety remains a non-negotiable constraint, integrating operational flexibility with stock conservation.

Here, w regulates the trade-off between gross catch volume ($w \rightarrow 1$) and economic harvesting efficiency/stock abundance ($w \rightarrow 0$). In fisheries management, biological limit reference points to prevent recruitment overfishing are standardly benchmarked at 20% of unexploited stock biomass [55], directly motivating our choice of $\alpha = 0.20$ as the baseline precautionary threshold. To investigate how management preferences and species recovery rates jointly determine optimal policies under safety constraints ($\alpha = 0.20$), we evaluate $J(\delta, \bar{T})$ across parameter combinations in Figure 5.

As shown in Figure 5, across all scenarios, optimal harvesting is consistently achieved under continuous fishing ($\bar{T}^* = 0$), reaffirming that moderated year-round exploitation maintains higher stability and efficiency than intense seasonal pulse fishing. Crucially, when management values harvesting efficiency ($w = 0.50$ or 0.75), the optimal harvesting intensity δ^* is restrained well below the unconstrained MASY level ($w = 1.00$). For example, for $\theta = 1.00$, shifting from pure yield maximization ($w = 1.00$, $\delta^* = 0.50$) to balanced efficiency ($w = 0.75$, $\delta^* = 0.46$) sacrifices less than 1% of gross yield while substantially increasing CPUE and standing biomass. Furthermore, comparing rows reveals that species with strong density dependence ($\theta = 1.35$) expand the ecologically feasible region $\mathcal{S}_{RC}(\alpha)$, sustaining higher fishing effort without breaching safety thresholds.

Finally, we examine how tightening the safety threshold α restricts allowable fishing policies under a fixed preference weight $w = 0.75$. While $\alpha = 0.20$ acts as the minimum biological boundary, conventional target biomass levels (such as B_{MSY} in classical production models) are commonly set near 0.50 of carrying capacity [56]. Furthermore, the presence of varying recovery capacities (θ) implies that sensitive species require an even safer operational buffer. We thus introduce $\alpha = 0.60$ alongside the baseline $\alpha = 0.20$ and the unconstrained case as a stricter conservation scenario, observing whether and how an elevated safety threshold constrains the achievable maximum RCYEI and alters optimal policies. The comparative distributions are illustrated in Figure 6.

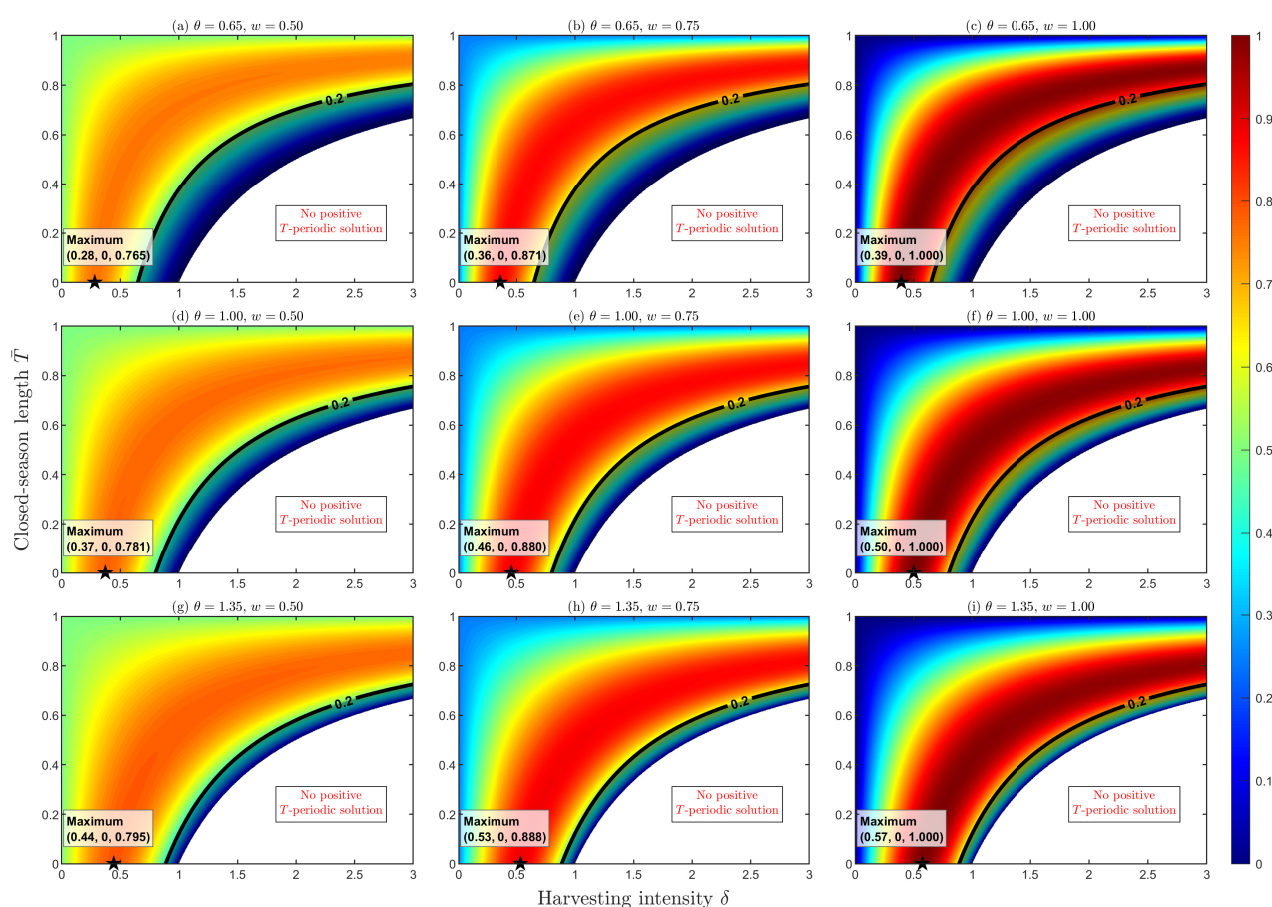


Figure 5. Contour evaluations of RCYEI $J(\delta, \bar{T})$ across density-dependence parameters $\theta \in \{0.65, 1.00, 1.35\}$ (rows) and yield preference weights $w \in \{0.50, 0.75, 1.00\}$ (columns) under safety threshold $\alpha = 0.2$. The solid black curve denotes the resilience boundary $\min x^p(t) = 0.2$. Black pentagrams indicate optimal management policies $(\delta^*, \bar{T}^*, J^*)$. The θ values again represent different fish life-history types as described earlier.

Figure 6 demonstrates that moderate conservation requirements ($\alpha = 0.20$, middle column) do not alter optimal harvesting policies, as balanced efficiency optimization inherently maintains standing biomass above 20% of carrying capacity. However, enforcing stringent conservation standards ($\alpha = 0.60$, right column) renders previously optimal aggressive policies illegal. To comply with strict safety mandates, optimal harvesting intensity must be significantly reduced (e.g., for $\theta = 1.00$, δ^* drops from 0.46 to 0.40), resulting in a necessary reduction in overall index performance. Notably, species with superior recovery capacity ($\theta = 1.35$) suffer considerably smaller performance penalties under strict conservation constraints than sensitive species ($\theta = 0.65$).

Figures 5 and 6 highlight the management risks of misidentifying θ under data limitations. While the optimal closure remains $\bar{T}^* = 0$, the recommended harvesting intensity δ^* depends strongly on θ . Overestimating θ (e.g., assuming $\theta = 1.00$ instead of 0.65) increases δ^* from 0.36 to 0.46, potentially causing stock biomass to decline below α . Conversely, underestimating θ for resilient stocks ($\theta = 1.35$ vs. $\theta = 1.00$) reduces the estimated sustainable yield potential by 17.7% ($H_{\theta, \max}$ decreases from 0.304 to 0.250), leading to substantial unnecessary yield losses. These results are consistent with

empirical evidence that insufficient information on stock status increases the risk of inappropriate exploitation decisions in unassessed fisheries [57]. Therefore, our results suggest that under data scarcity, conservative parameter assumptions or stricter biomass safety thresholds can provide a more precautionary management strategy.

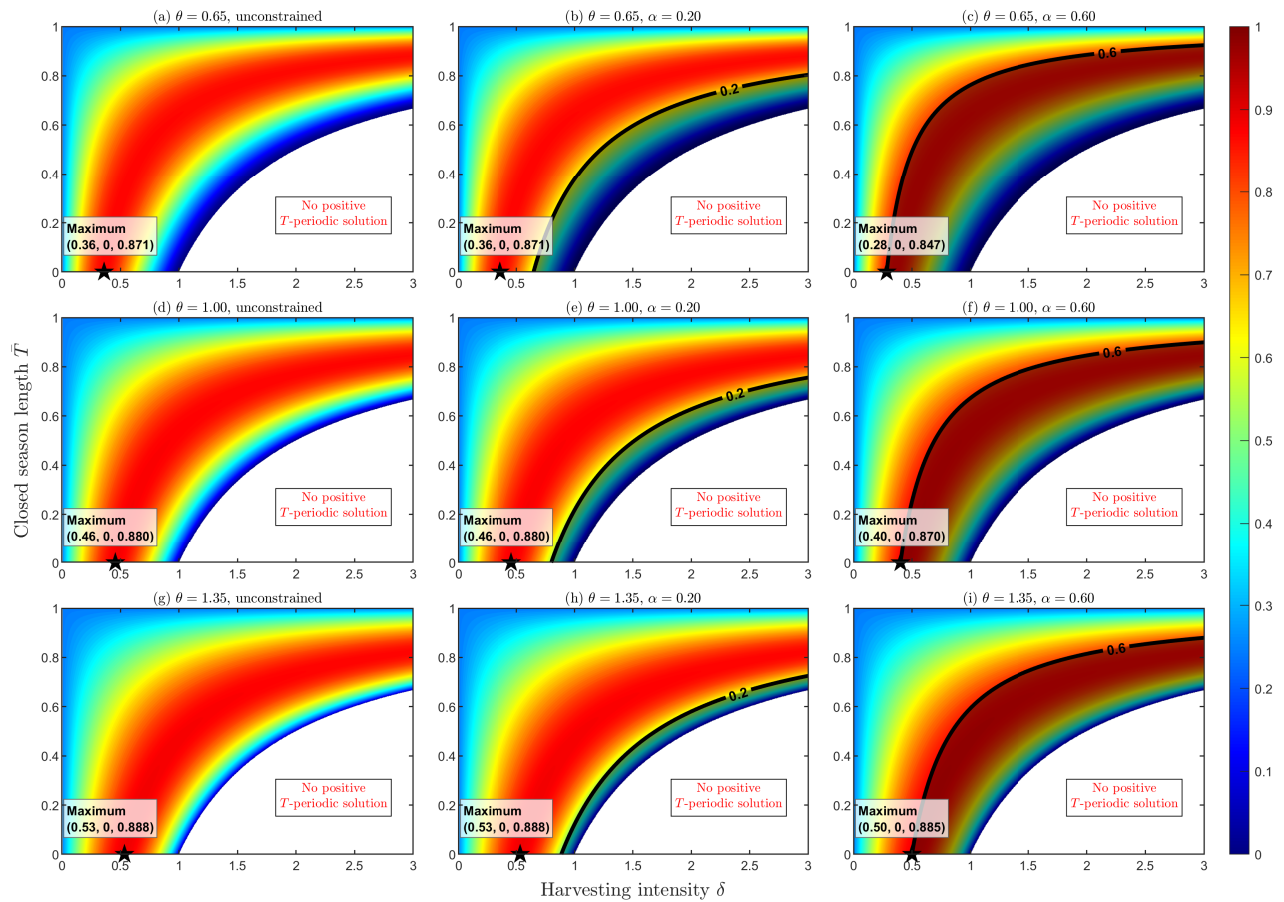


Figure 6. Evaluations of RCYEI $J(\delta, \bar{T})$ across different safety constraints (unconstrained, $\alpha = 0.2$, and $\alpha = 0.6$; columns) and $\theta \in \{0.65, 1.00, 1.35\}$ (rows) for $w = 0.75$. White zones indicate extinction regions; dark gray zones indicate persistent populations violating the safety constraint; and colored regions denote legally admissible sets $\mathcal{S}_{RC}(\alpha)$.

5. Discussion and conclusions

Balancing resource exploitation with ecological conservation is the central pillar of sustainable fisheries management. While seasonal fishing moratoriums are widely implemented to facilitate stock recovery, their biological efficacy depends intricately on harvesting intensity, closure duration, and species-specific density dependence. In this study, we developed and analyzed a seasonally switching single-species θ -logistic harvesting model with alternating closed and open seasons, providing an analytical framework for evaluating seasonal fishery regulations.

Our mathematical analysis established a sharp threshold mechanism governing stock persistence. When fishing intensity exceeds the intrinsic growth rate ($qE > r$), there exists a critical closed-season

duration threshold $\bar{\omega}^* = \frac{qE-r}{qE}\omega$. If the closed season exceeds this threshold, the population persists and converges to a unique globally asymptotically stable periodic cycle; otherwise, extinction is inevitable. Conversely, when fishing pressure is moderate ($qE < r$), the population persists regardless of closure duration. While this critical persistence boundary is independent of θ , the standing biomass, recovery speed, and sustainable yield are profoundly shaped by nonlinear density regulation. Assuming linear logistic growth ($\theta = 1$) for sublinear species ($\theta < 1$) severely overestimates recovery capacity and the MASY, creating a false sense of security that risks unexpected stock collapse. Conversely, applying linear models to superlinear species ($\theta > 1$) underestimates sustainable yield potential by up to 17.7% (as exemplified by $\theta = 1.35$), unnecessarily forfeiting harvestable yield.

To overcome the severe ecological drawbacks of unconstrained yield maximization, we formulated the RCYEI, which integrates annual sustainable yield, CPUE, and explicit biomass safety constraints. Our results demonstrate that incorporating harvesting efficiency ($w < 1$) restrains optimal fishing effort below aggressive MASY levels, substantially increasing stock abundance and economic return per unit effort at minimal yield cost. Furthermore, while stringent conservation standards ($\alpha = 0.60$) necessarily constrain allowable fishing intensity, species with robust density dependence ($\theta > 1$) exhibit superior resilience, sustaining higher performance under strict conservation mandates.

Robustness to parameter uncertainty. In real-world fisheries management, the catchability coefficient q and intrinsic growth rate r are often estimated with non-negligible errors. To assess the practical reliability of our optimal policies, we conducted a sensitivity analysis by perturbing these parameters within $\pm 10\%$ of their nominal values. For the RCYEI-optimal strategy (e.g., $w = 0.75$, $\alpha = 0.20$, $\theta = 1.00$), the corresponding optimal harvesting intensity δ^* and closed-season length $\bar{T}^*$ remained within the feasible safety region, and the resulting RCYEI value dropped by less than 10% under these perturbations. This indicates that the recommended management policies are reasonably robust to typical estimation errors in q and r , thereby enhancing the practical applicability of the RCYEI framework.

Theoretical optimum versus real-world seasonal closures. Although Theorem 3.1 shows that the global maximum of annual sustainable yield is achieved at $\bar{T} = 0$ (continuous year-round harvesting at moderate intensity), this result should not be misinterpreted as a call to abolish seasonal fishing moratoriums. In practice, closed seasons are indispensable for protecting spawning aggregations, juvenile recruitment, and age structure diversity, which are not captured by the single-species biomass model. Moreover, seasonal closures reduce bycatch, minimize habitat damage, and align with socio-cultural traditions in many coastal communities. Thus, while the mathematical optimum provides a valuable benchmark, actual management must balance it with ecological, biological, and socioeconomic constraints that favor periodic closures. Our RCYEI framework, by incorporating safety thresholds and efficiency, offers a pragmatic compromise that respects both theoretical insights and real-world necessities.

Several extensions of this research warrant future investigation. First, our model assumes constant parameters within seasons and focuses on a single spatial patch. Expanding the framework to incorporate stochastic environmental fluctuations, age-structured populations, and marine protected area (MPA) spatial networks with larval dispersal would enhance its realism for complex marine ecosystems. Second, fitting species-specific θ parameters from empirical catch-effort time series will be crucial for translating

these mathematical insights into operational, species-specific fisheries management advice.

Use of AI tools declaration

The authors utilized AI-assisted language tools to enhance the linguistic clarity and readability of the manuscript. The authors reviewed and edited the resulting text and take full responsibility for the final content.

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Conflicts of interest

Shengqiang Liu is an editorial board member for *Mathematical Biosciences and Engineering* and was not involved in the editorial review or the decision to publish this article. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Appendix

A. Derivation of the explicit periodic solution

Here, we outline the key steps leading to the explicit formulas for $u_0(\delta, \bar{T})$, $x_1^p(t)$, and $x_2^p(t)$ given in Section 3. The system is solved on each subinterval using the standard Bernoulli transformation $z = x^{-\theta}$, which linearizes the equations. On the closed season $(0, \bar{T}]$, the equation $x' = x(1 - x^\theta)$ becomes $z' = -\theta z + \theta$, yielding

$$x_1^p(t) = \left(\frac{u_0^\theta e^{\theta t}}{1 - u_0^\theta + u_0^\theta e^{\theta t}} \right)^{1/\theta},$$

which is equivalent to (3.2). At $t = \bar{T}$, we obtain $x_1^p(\bar{T})$ in terms of u_0 . On the open season $(\bar{T}, T]$, the equation $x' = x(1 - \delta - x^\theta)$ leads to $z' = -\theta(1 - \delta)z + \theta$ for $\delta \neq 1$ (and $z' = \theta$ for $\delta = 1$), yielding

$$x_2^p(t) = \begin{cases} \left[\frac{(x_1^p(\bar{T}))^\theta (1 - \delta) e^{\theta(1-\delta)(t-\bar{T})}}{1 - \delta + (x_1^p(\bar{T}))^\theta (e^{\theta(1-\delta)(t-\bar{T})} - 1)} \right]^{1/\theta}, & \delta \neq 1, \\ \left[\frac{(x_1^p(\bar{T}))^\theta}{1 + \theta(x_1^p(\bar{T}))^\theta(t - \bar{T})} \right]^{1/\theta}, & \delta = 1, \end{cases}$$

which corresponds to (3.3). Imposing the periodicity condition $x_2^p(T) = u_0$ then gives a linear algebraic equation for $u_0^{-\theta}$, which is solved explicitly to produce (3.1). The algebraic simplifications involve standard exponential manipulations and are omitted here for brevity.



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