



Research article

Vaccination and combined optimal control measures for malaria prevention and spread mitigation

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Abstract: Malaria is a life-threatening mosquito-borne infectious disease prevalent in tropical regions, primarily transmitted to humans by the bites of infected *Anopheles* mosquitoes. This study presents a mathematical model analysis aimed at understanding the dynamics of malaria transmission and the effectiveness of various prevention strategies. Despite being preventable and curable, malaria continues to pose significant public health challenges, notably due to the risk of recurrent infections if improperly treated. The proposed deterministic model establishes the positivity and boundedness of solutions alongside the local stability of equilibria. A sensitivity analysis is conducted to identify key parameters impacting the basic reproduction number (R_0), which is crucial for evaluating intervention strategies. The findings indicate that although the current vaccines are not 100% effective, vaccination could significantly contribute to malaria control alongside existing preventive measures, such as mosquito nets and insecticide spraying. The study underscores the need for a comprehensive approach combining multiple strategies to effectively reduce malaria transmission and improve health outcomes in endemic regions. Overall, this research highlights the importance of mathematical modeling in formulating effective disease control policies.

Keywords: epidemiology; vector–host model; malaria; vaccination; optimal control

1. Introduction

Malaria is a fatal disease caused by parasites that enter the human body through the bites of infected mosquitoes. It remains a major global health concern, especially in warm climates with high

mosquito populations. Malaria affects millions annually, significantly impacting public health and socioeconomic development. The disease poses the greatest risk to vulnerable populations, including young children and pregnant women, and is predominantly found in tropical countries. Although preventable and curable, malaria does not spread directly from person to person [1]. It is primarily transmitted through the bites of infected female *Anopheles* mosquitoes, though blood transfusions and contaminated needles can also spread the infection.

Being one of the life-threatening parasitic diseases existing in the world, malaria continues to show its detrimental impacts on the health and livelihood of people, particularly in the tropics and subtropics. Moreover, the history and pattern of urbanization are greatly affected by the recurrence of this virulent epidemic [2, 3]. Symptoms of malaria can range from mild to life-threatening. Mild symptoms include fever, chills, and headache, while severe symptoms may involve fatigue, confusion, seizures, and difficulty breathing. Symptoms typically begin within 10–15 days of being bitten by an infected mosquito. If left untreated, *P. falciparum* malaria can progress to severe illness and death within 24 hours. Five *Plasmodium* parasite species cause malaria in humans, with *P. falciparum* and *P. vivax* posing the greatest threats. *P. falciparum* is the deadliest malaria parasite and is most prevalent in Africa, while *P. vivax* is the dominant species in most countries outside sub-Saharan Africa. Other species that can infect humans include *P. malariae*, *P. ovale*, and *P. knowlesi*.

The effects of malaria are extensive and multifaceted. Due to global warming, the prominence of malaria has increased significantly in recent times, thereby affecting the life cycles of both the vector and the parasite [4]. The immediate impact includes symptoms such as fever, chills, and severe flu-like symptoms. Malaria can quickly lead to complications such as organ failure, anemia, and cerebral malaria, which can be fatal without prompt treatment. Malaria also has profound socioeconomic consequences. It places a significant burden on healthcare systems, diverting resources and straining already limited infrastructure in endemic regions. The costs associated with the prevention, diagnosis, and treatment of malaria can be a heavy financial burden for individuals, families, and communities. Additionally, malaria can hinder economic development by reducing productivity, decreasing school attendance, and limiting workforce participation. Controlling malaria effectively requires a multifaceted approach, incorporating mosquito population management, medical treatments, immunization, and community-based interventions. Strategies such as distributing insecticide-treated bed nets, indoor residual spraying with insecticides, eliminating mosquito breeding sites, prompt diagnosis, and effective treatment with antimalarial drugs help reduce transmission. Treatment options include artemisinin-based combination therapies (ACTs) and preventive medications like intermittent preventive therapy (IPT). The introduction of the RTS,S malaria vaccine provides some protection, though research continues on its long-term efficacy. Additionally, disease surveillance, rapid diagnostic methods, and health education initiatives enhance malaria prevention and treatment efforts. Mathematical modeling plays a key role in analyzing transmission patterns and evaluating the effectiveness of various interventions, assisting policymakers in implementing efficient disease control strategies. Despite the World Health Organization's (WHO) goal of eliminating malaria by 2017, persistent malaria transmission, especially in border areas, has become a critical concern in the Southeast Asia region [5]. Vaccines and research into new interventions are also crucial for long-term control and elimination [6–10].

Numerous research studies have been conducted to investigate the impact of malaria on human populations [11–18]. For instance, Onifade et al. [19] developed a mathematical model incorporating resistance testing as a potential control strategy. This model enabled quantifiable predictions about

the effects of utilizing testing on the prevalence of malaria's multidrug-resistant strains. Similarly, Chiyaka et al. [20] proposed a deterministic model to conceptually evaluate the impacts of personal protective measures, treatment, and immunization techniques on malaria's transmission dynamics. In recent advancements, Ayalew et al. [21] introduced a nonlinear deterministic mathematical model to study malaria's transmission dynamics. This model integrates a class of aware individuals and diverse control interventions, including treated bed nets, indoor residual spraying, awareness campaigns targeting susceptible individuals, and treatment for infected persons. Furthermore, age-structure considerations have been highlighted as a crucial factor in modeling malaria, the world's deadliest vector-borne disease. Dinsa et al. [22] suggested that incorporating fractional operators into such models could provide new insights into malaria's transmission dynamics and the effectiveness of control interventions. Blayneh et al. [23] utilized a problem to evaluate the impacts of treatment on malaria transmission. Okosun [24] extended this line of research by examining the effects of treatment and vaccination strategies, including waning immunity, through a model incorporating these factors. Zhuolin Qu et al. [53] used numerical simulations to reproduce the heterogeneity in the age distributions of immunity profiles and infection status created by frequent exposure. They also studied the impact of vaccination. This research underscored the importance of hypothetical vaccinations and their combination with treatment techniques in mitigating malaria transmission. The importance of modeling studies in understanding the influence of various control strategies on malaria transmission patterns is underscored by the WHO's position statement on insecticide-treated nets (ITNs) [25]. Inspired by this, our study employs an SVIRSI (susceptible–vaccinated–infected–recovered–susceptible–infective) framework to incorporate vaccination as a key compartment. Additional control measures, such as treatment of infected individuals, personal protection, and insecticide spraying, are also included.

Time-dependent control interventions have proven effective in the modeling of other infectious diseases, including HIV/AIDS, tuberculosis, influenza, and SARS [26–33]. In the context of vector-borne diseases, optimal control theory has been widely applied to investigate control strategies [23, 34–39]. A nonlinear dynamics model named SEIMQR (susceptible–exposed–infected–mutant–quarantined–recovered) was proposed to study the transmission mechanism of COVID-19, predict its development tendency and control strategies by Zhenhua Yu et al. [40]. Later, to investigate the rumor dissemination process and prevent its spread, a unique IRLTS (ignorant–rumor–spreader–lurker–truth–spreader–stifler) rumor propagation model was presented, which takes into account the influence of lurking psychology as well as the rumor's truth [41]. Subsequently, analyzing the impact of the malicious as well as truth spreaders, a IS2TR model was proposed to predict the rumor propagation process [42].

In this study, our primary objective was to formulate a model that captures host–vector interactions and incorporates control measures such as personal protection, treatment, and insecticide spraying. By leveraging optimal control theory, we aimed to identify and evaluate the most effective combinations of these interventions to minimize malaria transmission; develop an SVIRSI mathematical model to explore malaria's transmission dynamics, incorporating the effects of vaccination and immunity; analyze the model's stability to identify equilibrium points and determine the conditions necessary to curb malaria spread; formulate optimal control strategies that account for resource constraints in vector control, treatment, and vaccination to minimize malaria transmission; perform numerical simulations to evaluate the effectiveness of various control approaches under different scenarios; and conduct a sensitivity analysis to identify the key variables influencing malaria's dynamics and inform targeted intervention strategies.

The findings of the study are:

- The SVIRSI model accurately depicts malaria's transmission dynamics, illustrating the important role of vaccination and immunity in disease propagation.
- The stability analysis identified important intervention levels, suggesting conditions for effectively controlling or eliminating malaria transmission.
- The most effective control techniques identified here dramatically lower malaria transmission rates, emphasizing the significance of allocating resources among vaccination, treatment, and vector control.
- Numerical simulations indicated that integrated control methods reduce malaria incidence more effectively than separate ones.
- The sensitivity analysis found crucial parameters, such as vaccination rate, that have a significant impact on the model's behavior.
- On the basis of our findings, we propose that public health officials emphasize integrated control methods that include vaccination and treatment efforts for the greatest impact.

The structure of the paper is as follows:

In Section 2, we present the development of the mathematical model used in this study. The section provides a detailed description of the model's formulation, focusing on the interactions between human and mosquito populations and incorporating various control strategies. The positivity and boundedness of the solutions, as well as the characterization of the disease-free equilibrium (DFE) and endemic equilibrium (EE) points, are established. Additionally, the basic reproduction number (R_0) is derived, and a local stability analysis of the DFE is explored in the supplementary file for a deeper theoretical understanding. Section 3.1 explores the impact of varying parameter values on both human and mosquito populations. This section uses numerical simulations to demonstrate how different scenarios influence malaria's transmission dynamics and control outcomes. In Section 3.2, we conduct a sensitivity analysis to identify the most influential parameters affecting the system's dynamics. This analysis employs the Latin hypercube sampling (LHS) and partial rank correlation coefficient (PRCC) methods to ensure robust and comprehensive insights. Section 4 delves into the application of optimal control theory. This section discusses the implementation of time-dependent control strategies and evaluates the effectiveness of various control measures, such as vaccination, insecticide spraying, and personal protection, in minimizing malaria transmission. Finally, Sections 5 and 6 summarize the key findings of the study, highlighting the implications of the results and providing insights for future research directions.

2. Mathematical model

The classical susceptible–vaccinated–infected–removal–susceptible (SVIRS) model is a widely used framework for predicting infectious disease's dynamics. In this study, we adapt and extend this approach to develop a mathematical model specifically tailored for malaria. The human population is modeled using the SVIRS framework to account for vaccination and immunity loss, while the mosquito population is described using the susceptible–infected (SI) model, reflecting the fact that mosquitoes do not recover from the infection. The model integrates these two compartments to capture the interactions between humans and mosquitoes, forming a comprehensive framework to study malaria

dynamics. Based on the flow diagram (Figure 1) the model reads:

$$\begin{cases} S'_h(t) = \tau_1 - \beta \frac{S_h I_m}{N} + \rho R_h - (\omega + \mu) S_h, \\ V'_h(t) = \omega S_h - 0.7 \beta \frac{V_h I_m}{N} - (\mu + \gamma_1) V_h, \\ I'_h(t) = \beta \frac{S_h I_m}{N} + 0.7 \beta \frac{V_h I_m}{N} - (\gamma_2 + \delta + \mu) I_h, \\ R'_h(t) = \gamma_1 V_h + \gamma_2 I_h - (\rho + \mu) R_h, \\ S'_m(t) = \tau_2 - \alpha \frac{S_m I_h}{N} - \eta S_m, \\ I'_m(t) = \alpha \frac{S_m I_h}{N} - \eta I_m \end{cases} \quad (2.1)$$

for $t \in [0, \infty)$ with the initial conditions

$$S_h(0) = S_{h0}, \quad V_h(0) = V_{h0}, \quad I_h(0) = I_{h0}, \quad R_h(0) = R_{h0}, \quad S_m(0) = S_{m0}, \quad I_m(0) = I_{m0}. \quad (2.2)$$

2.1. Model formulation

The model considers two interacting populations: humans and mosquitoes. The human population is divided into four compartments: $S_h(t)$, $V_h(t)$, $I_h(t)$, and $R_h(t)$. The mosquito population is divided into two compartments: $S_m(t)$, and $I_m(t)$.

Table 1. State variables and their descriptions.

Notation	Definition
$S_h(t)$	Susceptible humans at time t
$V_h(t)$	Vaccinated humans at time t
$I_h(t)$	Infected humans at time t
$R_h(t)$	Recovered humans at time t
$S_m(t)$	Susceptible mosquitoes at time t
$I_m(t)$	Infected mosquitoes at time t

The total population sizes are defined as;

$$\begin{aligned} N(t) &\equiv S_h(t) + V_h(t) + I_h(t) + R_h(t) \quad (\text{total human population}); \\ M(t) &\equiv S_m(t) + I_m(t) \quad (\text{total mosquito population}). \end{aligned}$$

2.2. Parameters

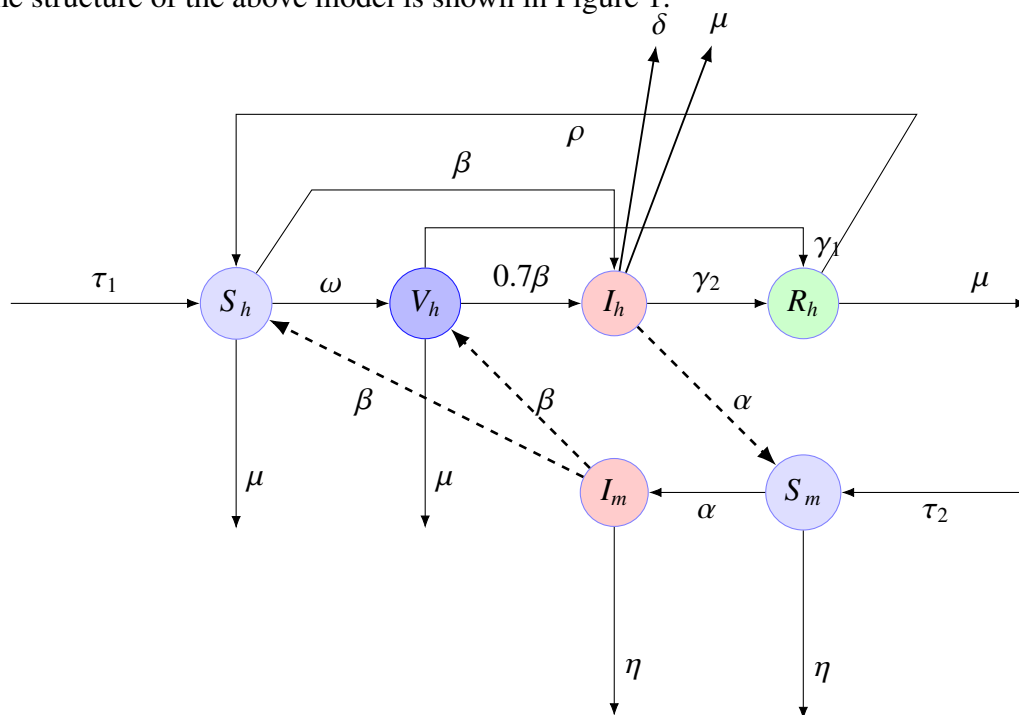
The model incorporates the following parameters:

Table 2. Model parameters, descriptions, and values.

Notation	Definition	Value	Unit	Source
τ_1	Recruitment/birth rate of humans	0.0000548	population/day	Estimated
τ_2	Recruitment/birth rate of mosquitoes	$\frac{1}{15}$	population/day	[43]
μ	Natural death rate of humans	0.0000548	1/day	[43]
η	Natural death rate of mosquitoes	$\frac{1}{15}$	1/day	[43]
β	Transmission rate (mosquito to human)	1.52	1/day	[43], [44], [45], estimated
α	Transmission rate (human to mosquito)	0.09	1/day	[43]
ω	Vaccination rate of susceptible humans	0.6	1/day	Assumed
γ_1	Immunity gain rate (vaccinated humans)	$\frac{1}{15}$	1/day	Assumed
γ_2	Recovery rate of infected humans	$\frac{1}{21}$	1/day	[43]
ρ	Immunity loss rate (recovered humans)	$\frac{1}{183}$	1/day	Assumed
δ	Disease-induced death rate (infected humans)	0.001	1/day	[43]

2.3. Disease transmission dynamics

The structure of the above model is shown in Figure 1.

**Figure 1.** Diagram of the SVIRS model for malaria.

The model captures the following transmission dynamics.

Susceptible humans (S_h) can become vaccinated (V_h) at a rate ω . Susceptible and vaccinated humans can become infected (I_h) through bites from infected mosquitoes (I_m) at the rates β and 0.7β , respectively. Infected humans can either recover (R_h) at a rate γ_2 or die from the disease at a rate

δ . Vaccinated individuals can recover (R_h) at a rate γ_1 . Recovered individuals lose immunity and return to the susceptible class at a rate ρ . Susceptible mosquitoes (S_m) become infected (I_m) by biting infected humans (I_h) at a rate α .

2.4. Related theorems to the model

Theorem 1. *The closed region $\Omega = \{(S_h, V_h, I_h, R_h, S_m, I_m) \in \mathbb{R}_+^6 : 0 < N \leq \frac{\tau_1}{\mu}, 0 < M \leq \frac{\tau_2}{\eta}\}$ is positively invariant set for the system in Eq (2.1).*

Theorem 2. *For the system (2.1), the DFE P_0 is locally asymptotically stable if $R_0 < 1$.*

The proofs of these theorems are given in the supplementary file for a deeper theoretical understanding.

2.5. Basic reproduction number

The basic reproduction number is an important threshold for studying infectious illness models. The basic reproduction number of the system (2.1) is obtained by the next-generation matrix method. The detailed calculation for obtaining the basic reproduction number (R_0) is given in the supplementary file. The basic reproduction number R_0 is

$$R_0 = \rho(FV^{-1}) = \frac{\sqrt{\alpha\beta\tau_2k_4k(0.7\omega + k_2)}}{N\eta\sqrt{k_3}}.$$

where $k_1 = (\omega + \mu)$, $k_2 = (\mu + \gamma_1)$, $k_3 = (\gamma_2 + \delta + \mu)$, $k_4 = (\rho + \mu)$, and $k = \frac{\tau_1}{k_1k_2k_4 - \gamma_1\rho\omega}$.

3. Results and discussion

3.1. Analysis and computations

The proposed model is a vaccination model for malaria, an infectious disease. The introduction of malaria vaccines marks a major breakthrough in combating one of the most lethal diseases, which primarily impacts low- and middle-income nations, especially in sub-Saharan Africa. The RTS,S/AS01 vaccine, also known as Mosquirix, has demonstrated its ability to reduce severe cases of malaria caused by *Plasmodium falciparum*, the most dangerous malaria parasite [46]. After the RTS,S/AS01 vaccine was recommended by the WHO, the R21/Matrix-M vaccine was the second malaria vaccine to be recommended by the WHO. The WHO updated their recommendation for these vaccines in October 2023. With a focus on regions of moderate and high transmission, the WHO advises children living in malaria-endemic areas to receive malaria vaccinations to avoid *P. falciparum* malaria. Children should begin receiving the four doses of the malaria vaccine at the age of five months. In regions with highly seasonal transmission or where the risk of malaria is still high during the third year of life or later, a fifth dose administered a year after the fourth dose may be considered. Endorsed by the WHO, it is primarily recommended for young children in high-risk regions, who face the greatest threat from malaria-related illness and death. These vaccines function by activating the immune system to target the parasite in its initial stages, thereby inhibiting its development and further spread. Although its efficacy is moderate, it complements other malaria control strategies, including insecticide-treated

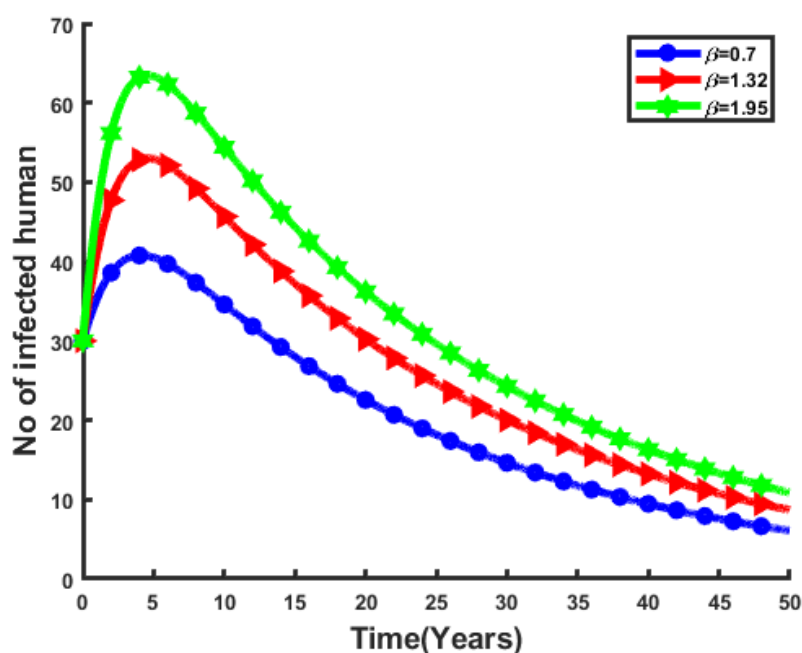
nets, insecticide spraying, and access to antimalarial treatments. Current research aims to enhance the vaccine's effectiveness, prolong its protective benefits, and create advanced vaccines to address other malaria parasites, such as *Plasmodium vivax*. Incorporating vaccination programs into comprehensive public health initiatives could significantly lower malaria's prevalence and save numerous lives.

Despite malaria being a major national concern in many countries, malaria vaccination has not been implemented on a large scale globally due to a variety of challenges. One key issue is the limited effectiveness of current vaccines like RTS,S/AS01, which provide only partial protection and require multiple doses and booster shots to maintain immunity. This creates logistical difficulties, particularly in regions with underdeveloped healthcare systems. Additionally, the high costs of the vaccine's production, transportation, and administration pose significant financial barriers for widespread distribution in resource-limited settings. Current vaccines also target only *Plasmodium falciparum*, leaving other species like *Plasmodium vivax* unaddressed. Socioeconomic factors, such as low levels of public awareness, vaccine hesitancy, and unequal access to health services, further restrict their adoption. Global funding often focuses on more established measures, such as insecticide-treated nets and antimalarial drugs, which have shown greater immediate impact. Lastly, the complex life cycle of the malaria parasite and its capacity to develop resistance continue to present scientific and operational hurdles, delaying the development of more comprehensive and effective vaccines. These issues collectively hinder the large-scale rollout of malaria vaccination programs. However, the importance of an efficient vaccination program cannot be overstated. Among all prevention strategies for infectious diseases, vaccination is often the first approach considered for disease prevention.

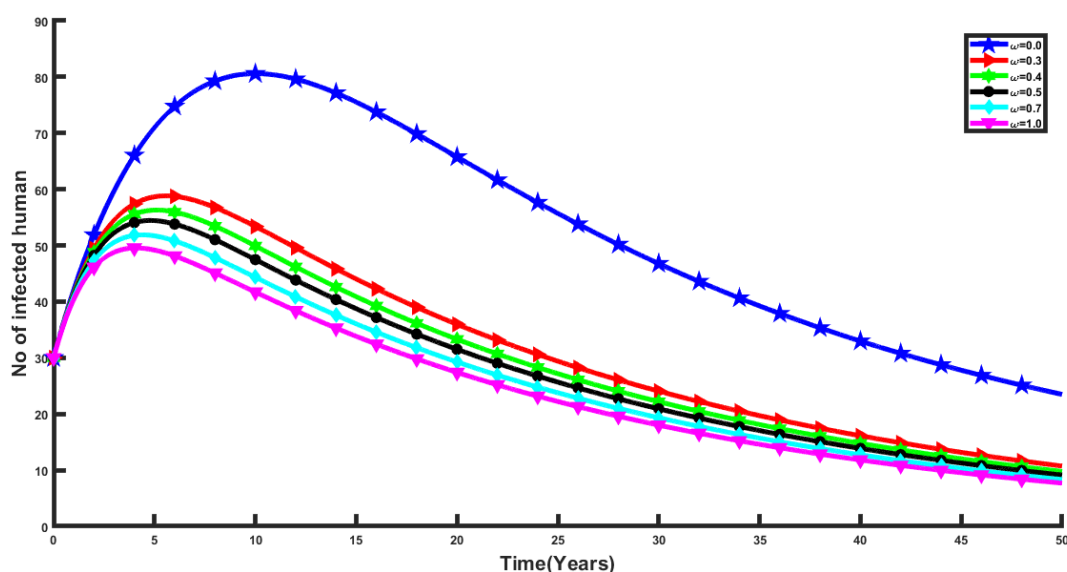
With this in mind and to draw a conclusion in support of a widespread vaccination program, we conduct a mathematical analysis using the proposed model. To consider the impact of vaccination, we now assume that a portion of susceptible humans receive the vaccine. Keeping this in mind, we proceed with our further analysis in this section. We use specific parameter values in the proposed model for this purpose; some values are taken from published articles, while others are assumed. Those parameter values which we use are presented in Table 2.

When discussing an infectious disease, the primary focus is on the infection rate. The infection rate, or incidence rate, is the risk of infection within a population, measuring the frequency of new infection cases over a specific period. Thus, the infection rate is a critical concern in the study of infectious diseases. In the proposed model, the infection rate comprises two components: The human infection rate and the mosquito infection rate. First, we consider the human infection rate, represented by the parameter β in the model. Figure 2(a) illustrates how a small change in the parameter β affects the number of infected humans. As the value of β increases, the number of infected individuals also rises.

Vaccination is one of the most effective means to save lives and improve public health. However, the impact of immunization against the most threatening infectious agents on life expectancy remains a subject of debate. Key issues include the relative influence of nutrition and infectious diseases on demographic patterns. As more people get vaccinated, it is expected that fewer individuals will come into contact with the virus. A successful and efficient vaccination program can significantly reduce infection rates. Therefore, the vaccination rate is a crucial concern. As the vaccination rate increases, the likelihood of infection decreases, consequently lowering the overall infection rate. This relationship is illustrated in Figure 2(b).



(a) Effect for different rates of infection on humans.



(b) Effect for different rates of vaccination on humans.

Figure 2. Effect for varying rates of infection and vaccination on humans.

Vaccines offer strong protection, but this protection takes time to build. To achieve full immunity, people must receive all the required doses of a vaccine. For two-dose vaccines, the first dose provides partial protection, while the second dose enhances that protection. It takes a few weeks after the second dose for protection to reach its maximum level. Even then, there is still a chance of being infected, as vaccines do not provide complete immunity. However, vaccinated individuals who become sick are

likely to experience milder symptoms. In Figure 2(b), it is shown that the changes in the number of infected humans for a varying vaccination rate $\omega > 0.3$ is small. Herd immunity occurs when a critical proportion of the population becomes immune, lowering disease transmission enough to prevent large outbreaks. In the graph, infection levels drop substantially as ω increases from 0.0 to 0.5, but beyond $\omega = 0.5$, the additional benefit of increasing ω further is minimal. This suggests that herd immunity is likely achieved around $\omega = 0.5$. However, it indicates that the threshold lies between 30% and 40% vaccination (that is, between $\omega = 0.3$ and $\omega = 0.4$), where the infection curves are still distinguishable but converging. Understanding the precise herd immunity threshold would provide valuable insights for policymakers, helping them optimize vaccination efforts while conserving resources.

Free-riding in vaccination programs, including for malaria and other infectious diseases, occurs when individuals choose not to get vaccinated, relying instead on the immunity of others to protect them. This can undermine herd immunity and lead to outbreaks. Free-riding can occur in vaccination programs that contain: public awareness and education, dispelling misinformation, incentives for vaccination, social and community influence, policy and regulation, and reducing barriers to vaccination. But free-riding in vaccination happens if herd immunity can be achieved with a vaccination rate of 50% or less. People therefore decide against getting vaccinated, depending instead on other people's immunity to keep them safe. Assuming that they will stay safe even if they do not receive the vaccination. Public health initiatives may be weakened by this practice, which may also keep the population from achieving the required immunity threshold. Mandatory vaccination laws can be put in place to combat free-riding. Studies like those by Kawata and Nakabayashi demonstrate how mandates can promote compliance and lessen the perceived burden of immunization [47]. Enhancing vaccination safety is yet another crucial tactic. According to Fukai et al. [48], people were deterred from taking additional doses by even mild side effects. Improving vaccine formulas to reduce adverse effects may contribute to higher voluntary immunization rates and less reliance on enforcement actions. By incorporating these tactics into public health regulations, vaccination campaigns can be reinforced and long-term disease control can be guaranteed.

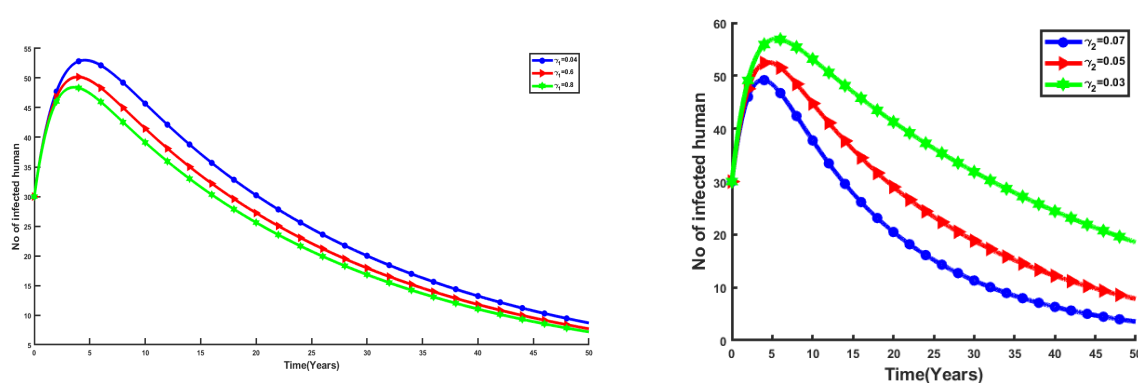


Figure 3. (a) Effect of different rates of recovery with vaccine on humans, (b) Effect of different rates of natural recovery on humans.

It is generally rare for vaccinated people to suffer severe illness or die. The effect of vaccines usually results in faster recovery for vaccinated individuals, depending on the vaccine's effectiveness. If the

effectiveness of the vaccine is increased, the recovery rate of vaccinated people will also increase, leading to a decreasing number of infected individuals. This relationship is depicted in Figure 3(a).

While vaccines help the human body recover faster from infections, people can also develop immunity through their own immune systems. The immune system in humans is a highly intricate defense mechanism that safeguards the body from harmful invaders, including bacteria, viruses, and fungi, as well as abnormal or damaged cells. It comprises two primary components: The innate and adaptive immune systems. The innate immune system serves as the body's immediate defense, employing barriers like the skin, chemical agents such as enzymes, and immune cells like neutrophils and macrophages to combat threats. On the other hand, the adaptive immune system delivers a more precise response by identifying specific pathogens. Key players in this system include T-cells and B-cells, with B-cells generating antibodies to neutralize invaders and T-cells targeting infected cells and orchestrating immune activities. Adaptive immunity also involves memory cells, which enable a faster and stronger response upon re-exposure to the same pathogen. That is, the immune system is a complex network of organs, cells, and proteins that defends the body against infections while protecting its own cells. It keeps a record of every germ (microbe) it has ever defeated, enabling it to quickly recognize and destroy the microbe if it re-enters the body. A healthy immune system can effectively defeat invading pathogens. Working together, the whole system protects the body from infections, promotes healing, and maintains balance to prevent over-activation, which could lead to some autoimmune disorders.

One can strengthen their immune system through healthy habits such as eating well, being physically active, maintaining a healthy weight, getting enough sleep, not smoking, and avoiding excessive alcohol use. A strong immune system across a large population can help reduce the total infectious individuals by increasing the rate of natural recovery. This effect is illustrated in Figure 3(b).

For certain diseases, once an individual is infected and subsequently recovers, they acquire lifelong immunity against that specific pathogen. This lifelong protection arises because the immune system generates antibodies that remain effective throughout the individual's life. However, this phenomenon does not apply universally to all diseases. In some cases, antibodies gradually lose their effectiveness, while in others, they naturally disappear after a certain duration. Consequently, individuals may become reinfected by the same disease multiple times. In certain situations, the duration of antibody effectiveness can be prolonged through a healthy lifestyle or the use of specific medications. This extension in immunity duration has a direct impact on the number of infected individuals. When the period of immunity is extended, reinfections occur less frequently, thereby decreasing the overall rate of infection within the population. This effect is illustrated in Figure 4.

The waning immunity rate ρ critically influences malaria's epidemiology

$$\text{Reinfection frequency} = \rho R_h(t) \quad (3.1)$$

where $R_h(t)$ represents the recovered population. This leads to

- Continuous replenishment of the susceptible pool (S_h);
- Sustained transmission even in endemic areas;
- Age-dependent infection patterns.

Immunity modulation takes place through:

- Seasonal chemoprevention,

- Transmission-blocking vaccines,
- Antimalarial prophylaxis.

These factors can effectively reduce ρ , thereby decreasing prevalence $I_h(t)$ as shown in Figure 4.

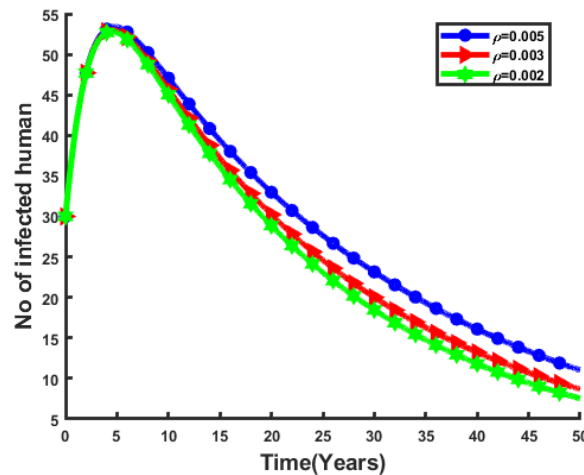


Figure 4. Effect of different rates of losing immunity on humans.

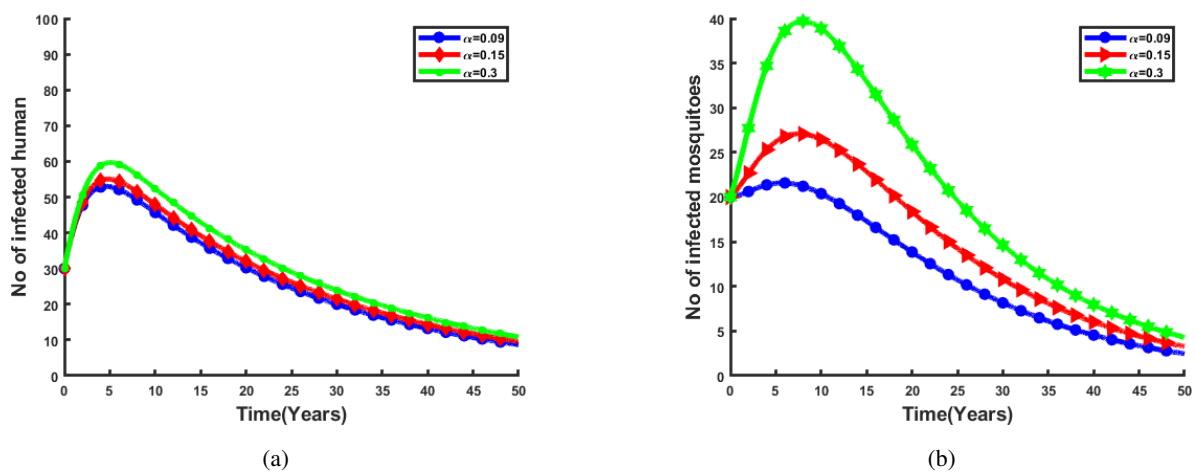


Figure 5. The effect of different rates of infection of mosquitoes.

A parasite that frequently infects a particular kind of mosquito—the female *Anopheles* mosquito—that feeds on people is the source of malaria, a dangerous and occasionally lethal illness. Malaria is often contracted by a bite from an infected *Anopheles* mosquito. Only after consuming a blood meal from an infected individual can these insects spread malaria. A mosquito ingests a tiny quantity of blood containing microscopic malaria parasites when it bites an infected human. These parasites combine with the mosquito's saliva and are injected into the victim of the bite approximately a week later when the mosquito feeds on blood. Since the root cause of malaria is infected mosquitoes, the rate of mosquito infection is a crucial factor to consider. If the rate of mosquito infection increases,

the number of infected mosquitoes will also rise. Consequently, this leads to a greater number of infected individuals. This relationship is depicted in Figure 5.

The mortality rate of mosquitoes (η) plays a crucial role in disease dynamics. A higher mosquito death rate reduces their average lifespan, consequently decreasing the following:

- (1) The probability of infected mosquitoes transmitting the pathogen to humans;
- (2) The opportunities for susceptible mosquitoes to acquire the infection from infected humans.

This dual effect leads to a reduction in both the infected mosquito population (I_m), and subsequently, the infected human population (I_h). The quantitative relationship between mosquito mortality and infection prevalence is demonstrated in Figure 6.

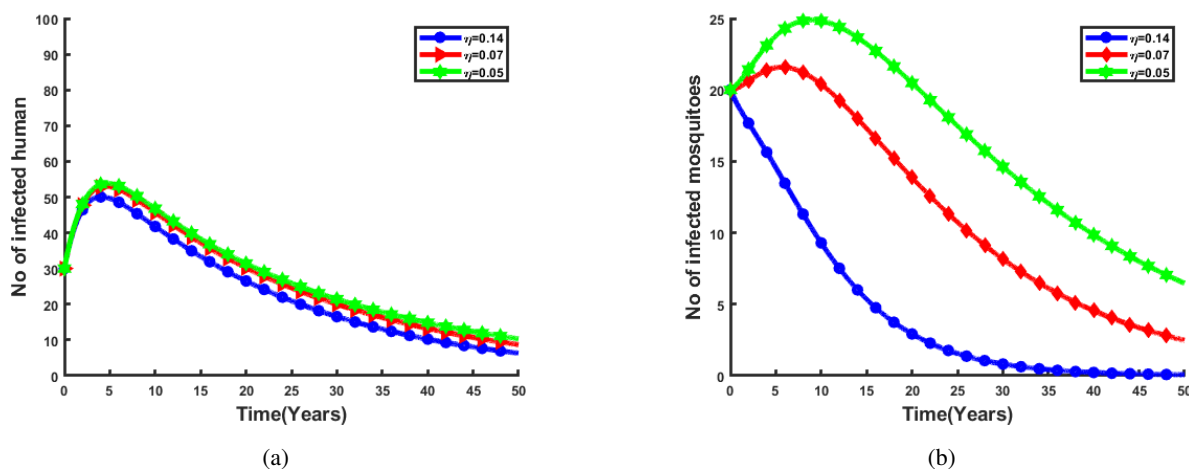


Figure 6. The effect of different natural death rates of mosquitoes.

Increasing mosquito mortality plays a key role in reducing the spread of malaria by interfering with the transmission process of the *Plasmodium* parasite, which requires mosquitoes for its life cycle. Female *Anopheles* mosquitoes need to live long enough for the parasite to mature and become capable of infecting humans. By enhancing mosquito death rates, fewer mosquitoes survive to carry and transmit the parasite, thereby decreasing malaria transmission. Furthermore, reducing the mosquito population lowers the frequency of mosquito bites, diminishing the likelihood of infection. This approach is particularly effective when combined with other control measures, such as insecticide-treated bed nets and spraying, as well as managing mosquito breeding sites. However, achieving lasting results requires careful management to prevent resistance to insecticides and to minimize unintended environmental effects. When used alongside other prevention methods, increasing mosquito mortality can play a vital role in malaria control efforts.

3.2. Sensitivity analysis

The latin hypercube sampling (LHS) and partial rank correlation coefficient (PRCC) methods are widely used techniques in sensitivity analysis to evaluate the influence of input parameters on model outputs. LHS is an efficient sampling method that divides the range of each input parameter into intervals of equal probability, ensuring comprehensive exploration of the parameter space with fewer

samples compared with random sampling. The PRCC, on the other hand, assesses the monotonic relationship between inputs and outputs by calculating rank-based correlations while accounting for the influence of other variables. Together, the LHS and PRCC provide a robust framework for identifying key parameters and understanding complex model behaviors efficiently. According to McKay et al. [49], the LHS method generates an unbiased approximation of the average model output.

The magnitude and statistical significance of a parameter's PRCC value indicate its contribution to the imprecision in the model's prediction. The closer the PRCC value is to +1 or -1, the more strongly the LHS parameter influences the outcome measure.

In Table 3, we have determined the maximum and minimum values for each of the LHS parameters. The baseline values for each parameter are set to the mean or a close approximation of the mean value of the parameter's range of minimum and maximum values. For some parameters, the baseline values are assumed, while for others, they are derived from the references presented in Table 2.

Table 3. Minimum values, maximum values, and baseline values used in the LHS analysis.

Parameter	Minimum	Baseline	Maximum
τ_1	0.0	0.0000548	0.0001
β	0.8009	1.52	2.24
ρ	0.0014	$\frac{1}{183}$	0.0041
γ_1	0.03	0.07	0.1
γ_2	0.0	$\frac{1}{21}$	0.1
ω	0.0	0.6	1.0
μ	0.0	0.0000548	0.0001
δ	0.0	0.001	0.002
τ_2	0.03	$\frac{1}{15}$	0.1
α	0.04	0.09	0.14
η	0.03	$\frac{1}{15}$	0.1

We will now perform an in-depth analysis to assess the sensitivity of the parameters in our proposed model. Sensitivity analysis is a crucial technique used to evaluate how variations in the values of independent variables influence a specific dependent variable within a predefined framework of assumptions. This process allows us to identify which parameters have the most significant impact on the model's behavior and outputs.

In essence, sensitivity analysis provides a systematic approach to investigating the contributions of various sources of uncertainty within a mathematical model to the overall uncertainty of its predictions. By analyzing these effects, we can better understand the robustness of the model, prioritize parameters for refinement, and enhance the reliability of its predictions. This step is particularly important in complex systems, such as those involving disease dynamics, where parameter uncertainty can significantly affect the outcomes and effectiveness of proposed interventions.

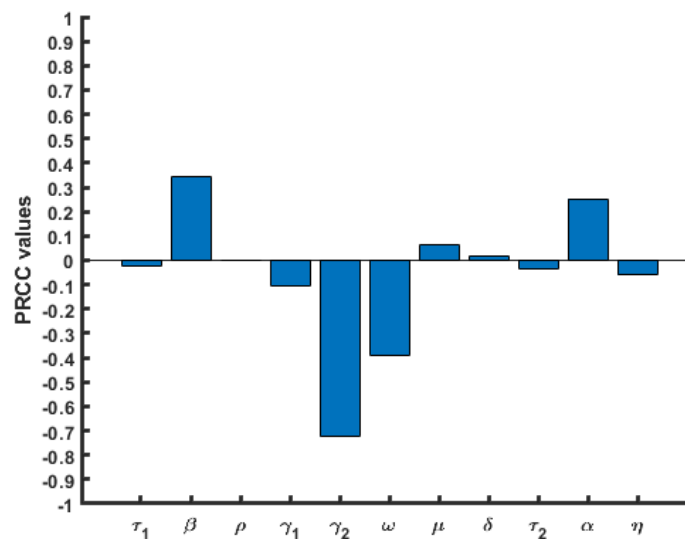


Figure 7. PRCC plot of the basic reproduction number R_0 .

The PRCC values in Figure 7 illustrate the sensitivity of each parameter in the model. Specifically, four parameters significantly impact changes in the basic reproductive number (R_0). These parameters are the rate of transmission to humans due to infected mosquito bites (β), the rate of natural recovery (γ_2), the rate of vaccination (ω), and the rate of transmission to mosquitoes from biting infected humans α . The parameters β and α have a positive sensitivity index, while γ_2 and ω have a negative sensitivity index.

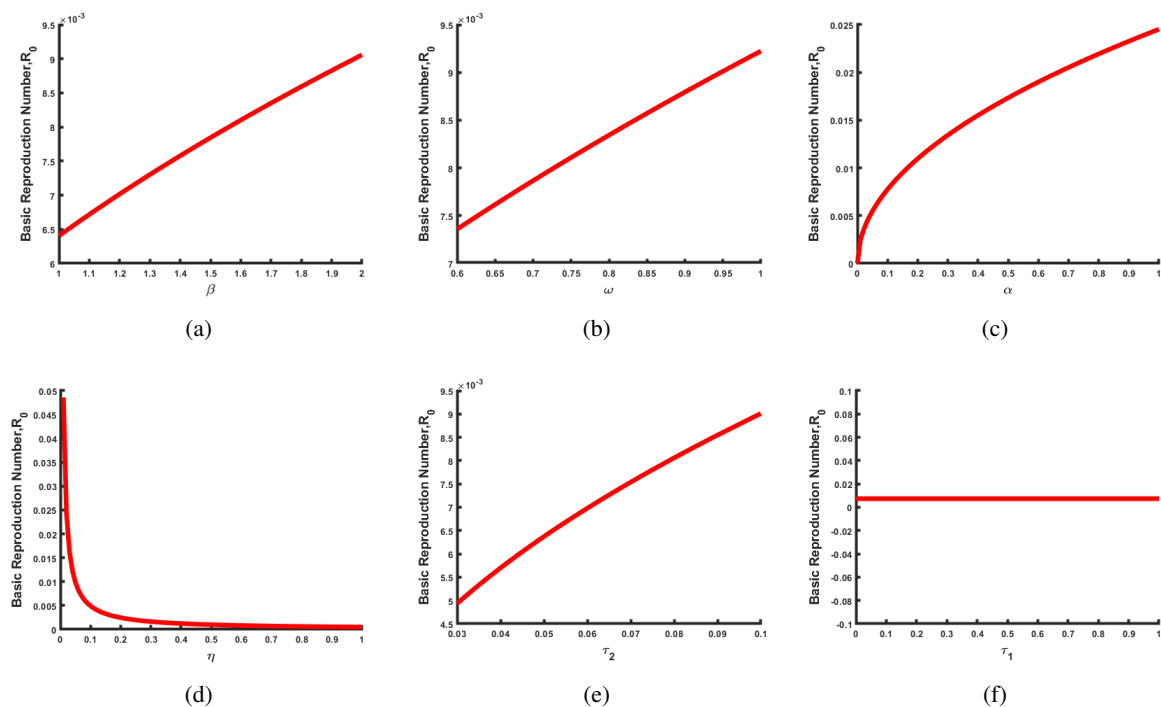


Figure 8. The effect of different parameters on the basic reproduction number, R_0 .

A positive sensitivity index indicates that increasing the value of the parameter while keeping other parameters constant will lead to an increase in the basic reproductive number (R_0). Conversely, a negative sensitivity index means that increasing the parameter's value will lead to a decrease in R_0 . Therefore, parameters with a positive sensitivity index contribute positively to the increase in R_0 , and those with a negative sensitivity index contribute negatively. In Figure 8, we illustrate the relationship between several parameters and the basic reproduction number, R_0 . For the remaining parameters, the results are similar to those observed for τ_1 .

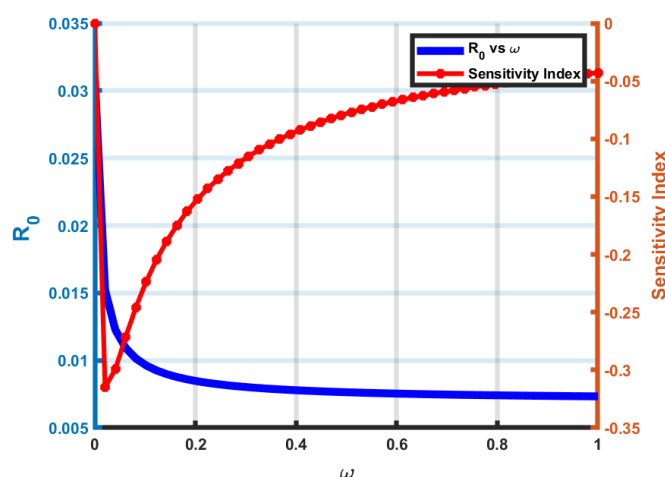


Figure 9. Effect for different rates of vaccination on R_0 and the sensitivity index of R_0 .

In Figure 9, we illustrate the relationship between ω and the basic reproduction number, R_0 along with the sensitivity index of R_0 , using an idea from [50]. As the vaccination rate (ω) increases, the basic reproduction number (R_0) decreases, indicating reduced disease transmission. However, the sensitivity index (SI_ω) increases, meaning the relative impact of vaccination on (R_0) becomes stronger at higher vaccination levels. This happens because the sensitivity index (SI_ω) is proportional to $\frac{\omega}{R_0}$, and as R_0 decreases, this fraction grows. The sensitivity index remains strictly negative for all $\omega > 0$; this indicates that R_0 decreases monotonically as the vaccination rate increases. As ω increases, the sensitivity index becomes less negative, approaching zero. This implies a diminishing marginal effect of vaccination on R_0 . At high vaccination rates, further increases in ω contribute less significantly to reducing R_0 , which reflects the saturation of herd immunity. The rigorous decline in the magnitude of sensitivity with an increasing ω demonstrates the efficacy of early vaccination efforts. From a public health perspective, this supports prioritizing the rapid scale-up of the initial vaccination coverage, as the return on investment is highest when coverage is low.

Geometrically, Figure 10 presents contour plots of the basic reproduction number R_0 as a function of two parameters. The color bar on the right indicates the value of R_0 , essentially showing the height of the R_0 surface. The interpretation of Figure 10 involves observing how changes in parameter values affect R_0 , specifically whether these changes cause R_0 to approach or move away from 1. The figure highlights combinations of the four parameters with the highest PRCC values, as shown in Figure 7; the parameters are α , β , ω , and γ_2 .

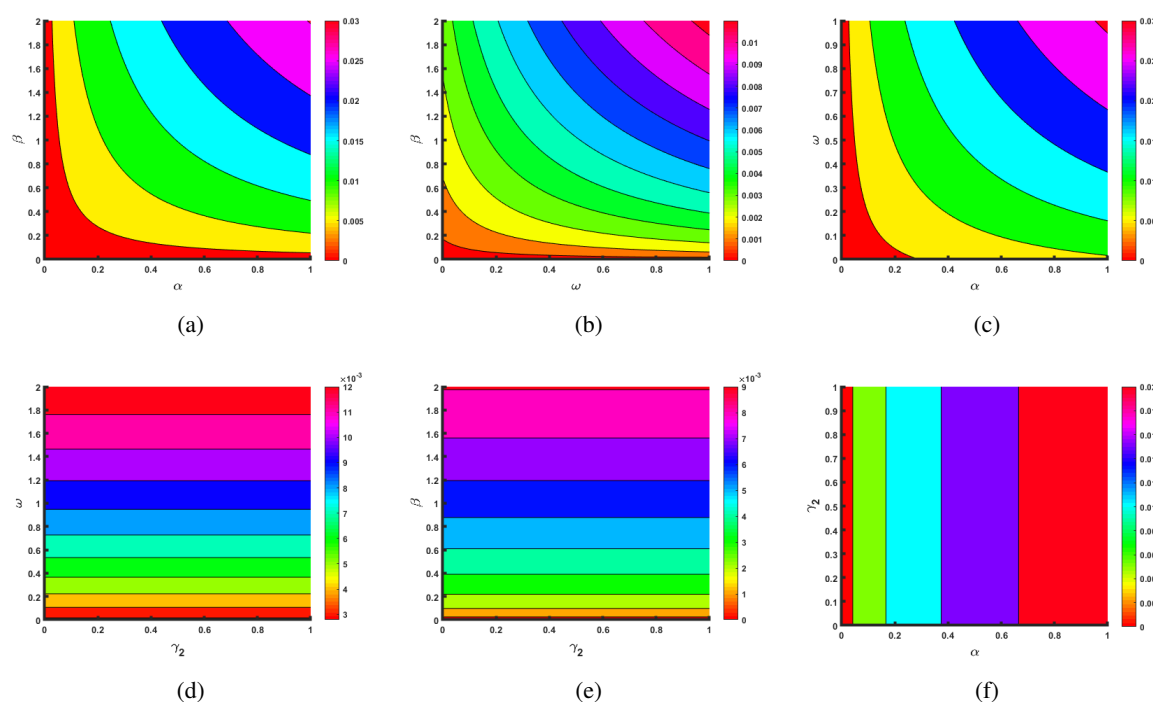


Figure 10. Contour plot of the basic reproduction number for the all possible combinations of the parameters α , β , ω , and γ_2 .

The plots above indicate that none of the parameter combinations with higher PRCC values lead to the basic reproduction number R_0 exceeding the threshold value of 1. This result arises from the assumption that a portion of the total population is vaccinated. Therefore, the mathematical analysis of the proposed model suggests that the vaccination program is highly effective in controlling the spread of malaria.

4. Optimal control

Optimal control theory offers a valuable approach for efficiently managing and reducing malaria transmission by determining the most effective allocation of resources and interventions. In malaria control, the goal is to optimize the use of strategies such as vaccination, insecticide-treated bed nets, insecticide spraying, and antimalarial treatments to achieve the largest possible decrease in infection rates while considering limitations in resources. By developing a mathematical model that captures the dynamics of malaria spread, optimal control provides a systematic way to assess the impact of various interventions and balance their effectiveness against the associated costs and trade-offs. This section delves into the application of optimal control to malaria, emphasizing the need for combining different intervention measures to enhance their collective impact. Through the use of simulations and model analysis, we aimed to identify the most efficient set of strategies to reduce malaria transmission in regions where the disease is endemic, while also accounting for practical challenges and resource constraints.

We introduced time-dependent prevention (c_1 , c_3) and treatment (c_2) as control measures that contribute to preventing the transmission of malaria into the proposed model (2.1). In this case, (2.1)

becomes as follows:

$$\begin{cases} S'_h(t) = \tau_1 - (1 - c_1)\beta\frac{S_h I_m}{N} + \rho R_h - (\omega + \mu)S_h, \\ V'_h(t) = \omega S_h - 0.7\beta\frac{V_h I_m}{N} - (\mu + \gamma_1)V_h, \\ I'_h(t) = (1 - c_1)\beta\frac{S_h I_m}{N} + 0.7\beta\frac{V_h I_m}{N} - (c_2 + \gamma_2 + \delta + \mu)I_h, \\ R'_h(t) = \gamma_1 V_h + (\gamma_2 + c_2)I_h - (\rho + \mu)R_h, \\ S'_m(t) = \tau_2 - (1 - c_1)\alpha\frac{S_m I_h}{N} - \eta S_m - c_3(1 - p)S_m, \\ I'_m(t) = (1 - c_1)\alpha\frac{S_m I_h}{N} - c_3(1 - p)I_m - \eta I_m. \end{cases} \quad (4.1)$$

- **Bed net usage (c_1):** The control function $c_1 \in [0, 1]$ represents the compliance rate for insecticide-treated net (ITN) usage, where
 - $c_1 = 1$ denotes perfect adherence to bed net protection;
 - $c_1 = 0$ indicates no net usage;
 - ITNs provide both physical barrier and insecticidal effects.
- **Treatment efficacy (c_2):** The treatment control is $c_2 \in [0, a_2]$
 - a_2 represents the maximum drug efficacy;
 - c_2/a_2 gives the relative effectiveness of the treatment;
 - c_2 affects recovery rate of infected individuals.
- **Insecticide spraying (c_3):** The spraying control $c_3 \in [0, a_3]$ impacts the mosquito population through:

$$\text{Mosquito mortality rate} = c_3(1 - p), \quad (4.2)$$

where:

- p is the proportion of mosquitoes in protected areas;
- $(1 - p)$ is the fraction exposed to insecticides;
- a_3 is the maximum insecticide efficacy.

Combined impact These controls collectively reduce transmission by;

- Decreasing the human-vector contact rate ($\propto 1 - c_1$);
- Enhancing recovery ($\propto c_2$);
- Depleting the mosquito population ($\propto c_3$).

The effectiveness is constrained by the maximum efficacies a_2 (drugs) and a_3 (insecticides), with the following objective function:

$$J(c_1, c_2, c_3) = \int_0^{t_f} (x_1 I_h + x_2 c_1^2 + x_3 c_2^2 + x_4 c_3^2) dt, \quad (4.3)$$

where t_f is the final time and the coefficients x_1 , x_2 , x_3 , and, x_4 are the weights used to balance the factors which are assumed to be positive. Our aim is to decrease the size of the infected population $I_h(t)$ while lowering the expense of $c_1(t)$, $c_2(t)$, and, $c_3(t)$. So we want c_1^* , c_2^* , and, c_3^* , say optimal control, such that

$$J(c_1^*, c_2^*, c_3^*) = \min_{c_1, c_2, c_3} \{J(c_1, c_2, c_3) | c_1, c_2, c_3 \in U\} \quad (4.4)$$

where the control set $U = \{(c_1, c_2, c_3) | c_i : [0, t_f] \rightarrow [0, 1], \text{ Lebesgue measurable, } i = 1, 2, 3\}$.

Here, $x_1 I_h$ represents the cost of infection, whereas $x_2 c_1^2$, $x_3 c_2^2$, and, $x_4 c_3^2$ represent the expenses of employing bed nets, treatment efforts, and pesticides, respectively. Pontryagin's maximum principle [51] transforms (4.1)–(4.3) into a problem of minimizing pointwise a Hamiltonian H , with respect to (c_1, c_2, c_3)

$$\begin{cases} H = x_1 I_h + x_2 c_1^2 + x_3 c_2^2 + x_4 c_3^2 + \\ \lambda_{Sh}(\tau_1 - (1 - c_1)\beta \frac{S_h I_m}{N} + \rho R_h - (\omega + \mu)S_h) + \\ \lambda_{Vh}(\omega S_h - 0.7\beta \frac{V_h I_m}{N} - (\mu + \gamma_1)V_h) + \\ \lambda_{Ih}((1 - c_1)\beta \frac{S_h I_m}{N} + 0.7\beta \frac{V_h I_m}{N} - (c_2 + \gamma_2 + \delta + \mu)I_h) + \\ \lambda_{Rh}(\gamma_1 V_h + (\gamma_2 + c_2)I_h - (\rho + \mu)R_h) + \\ \lambda_{Sm}(\tau_2 - (1 - c_1)\alpha \frac{S_m I_h}{N} - \eta S_m - c_3(1 - p)S_m) + \\ \lambda_{Im}((1 - c_1)\alpha \frac{S_m I_h}{N} - c_3(1 - p)I_m - \eta I_m) \end{cases} \quad (4.5)$$

where the the adjoint variables are λ_{Sh} , λ_{Vh} , λ_{Ih} , λ_{Rh} , λ_{Sm} , and λ_{Im} .

Now, we apply the Filippov–Cesari existence theorem to our model (4.1). The Filippov–Cesari existence theorem provides the conditions under which optimal control exists for control systems governed by differential equations. It is widely used in optimal control theory to ensure that a solution to an optimal control problem is not just theoretically defined but also achievable..

We consider our optimal control problem (4.1) where the state variables are S_h , V_h , I_h , R_h , S_m , and I_m and the control variables are (c_1, c_2, c_3) , representing personal protection, treatment, and (4.3), where x_1, x_2, x_3, x_4 are positive weighting parameters. To prove the existence of an optimal control, we check the following conditions:

- (1) The control variables c_1, c_2, c_3 satisfy $0 \leq c_1, c_2, c_3 \leq 1$. Since this is a compact, convex set, the condition is satisfied.
- (2) The right-hand sides of the system equations are continuous in the state variables S_h, V_h, I_h, R_h, S_m , and I_m . The control functions c_1, c_2 , and c_3 appear in measurable form, meaning that the system satisfies standard Caratheodory conditions i.e., continuous in the states, and measurable in the controls. Thus, the measurability and continuity condition is satisfied.
- (3) The initial conditions $S_h(0), V_h(0), I_h(0), R_h(0), S_m(0)$, and $I_m(0)$ are given and finite. The equations involve bounded parameters, and the system models two populations, so the state variables remain non-negative and bounded. Thus, the state space is bounded.
- (4) Since the state variables evolve in a bounded domain and satisfy Lipschitz continuity, their trajectories belong to a compact subset of $C([0, t_f], \mathbb{R}^6)$ (the space of continuous functions). Thus, the trajectory set is compact.
- (5) The cost function (4.3) is continuous in the state and control variables. Since quadratic terms are convex and lower semi-continuous, this condition also holds.

Since all the Filippov–Cesari conditions are satisfied, we conclude that an optimal control exists for the system (4.1).

The following results were produced using Pontryagin's maximum principle [52] and the existence outcome of optimal control [51].

Theorem 3. *Given an optimal control c_1^* , c_2^* , and c_3^* and the solutions S_h^* , V_h^* , I_h^* , R_h^* , S_m^* , I_m^* of the system (4.1) that minimizes $J(c_1, c_2, c_3)$ over U , there are the adjoint variables λ_{Sh} , λ_{Vh} , λ_{Ih} , λ_{Rh} , λ_{Sm} , and λ_{Im} satisfying*

$$\begin{cases} -\frac{\lambda_{Sh}}{dt} = \lambda_{Sh}(-(1-c_1)\frac{\beta I_m}{N}) - (\omega + \mu) + \lambda_{Vh}\omega + \lambda_{Ih}(1-c_1)\frac{\beta I_m}{N}, \\ -\frac{\lambda_{Vh}}{dt} = \lambda_{Vh}(-0.7\frac{\beta I_m}{N} - (\gamma_1 + \mu)) + 0.7\lambda_{Ih}(\frac{\beta I_m}{N}) + \lambda_{Rh}\gamma_1, \\ -\frac{\lambda_{Ih}}{dt} = \lambda_{Ih}(-(\gamma_2 + \delta + \mu + c_2)) + \lambda_{Rh}(\gamma_2 + c_2) + \lambda_{Sm}(-(1-c_1)\frac{\alpha S_m}{N}) + \lambda_{Im}((1-c_1)\frac{\alpha S_m}{N}) + x_1, \\ -\frac{\lambda_{Rh}}{dt} = \lambda_{Sh}\rho - \lambda_{Rh}(\rho + \mu), \\ -\frac{\lambda_{Sm}}{dt} = \lambda_{Sm}(-(1-c_1)\frac{\alpha I_h}{N} - c_3(1-p) - \eta) + \lambda_{Im}(1-c_1)\frac{\alpha I_h}{N}, \\ -\frac{\lambda_{Im}}{dt} = \lambda_{Sh}(-(1-c_1)\frac{\beta S_h}{N}) - \lambda_{Vh}\frac{0.7\beta V_h}{N} + \lambda_{Ih}((1-c_1)\frac{\beta S_h}{N} + \frac{0.7\beta V_h}{N}) + \lambda_{Im}(-c_3(1-p) - \eta). \end{cases} \quad (4.6)$$

and with the transversality conditions

$$\lambda_{Sh}(t_f) = \lambda_{Vh}(t_f) = \lambda_{Ih}(t_f) = \lambda_{Rh}(t_f) = \lambda_{Sm}(t_f) = \lambda_{Im}(t_f) = 0, \quad (4.7)$$

and the controls c_1^* , c_2^* , c_3^* satisfy the optimal condition

$$\begin{cases} c_1^* = \max\{0, \min(1, \frac{\alpha S_m^* I_h^* \lambda_{Im} + \beta S_h^* I_m^* \lambda_{Ih} - \alpha I_h^* S_m^* \lambda_{Sm} - \beta S_h^* I_m^* \lambda_{Sh}}{2Nx_2})\}, \\ = \max\{0, \min(1, \frac{\alpha S_m^* I_h^* (\lambda_{Im} - \lambda_{Sm}) + \beta S_h^* I_m^* (\lambda_{Ih} - \lambda_{Sh})}{2Nx_2})\}, \\ c_2^* = \max\{0, \min(1, \frac{(\lambda_{Ih} - \lambda_{Rh}) I_h^*}{2x_3})\}, \\ c_3^* = \max\{0, \min(1, \frac{(1-p)(\lambda_{Sm} S_m^* + \lambda_{Im} I_m^*)}{2x_4})\}. \end{cases} \quad (4.8)$$

Proof. The adjoint system can be expressed as

$$\begin{cases} -\frac{\lambda_{Sh}}{dt} = \frac{\partial H}{\partial S_h} = \lambda_{Sh}(-(1-c_1)\frac{\beta I_m}{N}) - (\omega + \mu) + \lambda_{Vh}\omega + \lambda_{Ih}(1-c_1)\frac{\beta I_m}{N}, \\ -\frac{\lambda_{Vh}}{dt} = \frac{\partial H}{\partial V_h} = \lambda_{Vh}(-\frac{0.7\beta I_m}{N} - (\gamma_1 + \mu)) + \lambda_{Ih}(\frac{0.7\beta I_m}{N}) + \lambda_{Rh}\gamma_1, \\ -\frac{\lambda_{Ih}}{dt} = \frac{\partial H}{\partial I_h} = \lambda_{Ih}(-(\gamma_2 + \delta + \mu + c_2)) + \lambda_{Rh}(\gamma_2 + c_2) + \lambda_{Sm}(-(1-c_1)\frac{\alpha S_m}{N}) + \lambda_{Im}((1-c_1)\frac{\alpha S_m}{N}) + x_1, \\ -\frac{\lambda_{Rh}}{dt} = \frac{\partial H}{\partial R_h} = \lambda_{Sh}\rho - \lambda_{Rh}(\rho + \mu), \\ -\frac{\lambda_{Sm}}{dt} = \frac{\partial H}{\partial S_m} = \lambda_{Sm}(-(1-c_1)\frac{\alpha I_h}{N} - c_3(1-p) - \eta) + \lambda_{Im}(1-c_1)\frac{\alpha I_h}{N}, \\ -\frac{\lambda_{Im}}{dt} = \frac{\partial H}{\partial I_m} = \lambda_{Sh}(-(1-c_1)\frac{\beta S_h}{N}) - \lambda_{Vh}\frac{0.7\beta V_h}{N} + \lambda_{Ih}((1-c_1)\frac{\beta S_h}{N} + \frac{0.7\beta V_h}{N}) + \lambda_{Im}(-c_3(1-p) - \eta). \end{cases} \quad (4.9)$$

with

$$\lambda_{Sh}(t_f) = \lambda_{Vh}(t_f) = \lambda_{Ih}(t_f) = \lambda_{Rh}(t_f) = \lambda_{Sm}(t_f) = \lambda_{Im}(t_f) = 0.$$

Within the interior of the control set, where $0 < c_i < 1$, for $i = 1, 2, 3$, we obtain

$$\begin{cases} 0 = \frac{\partial H}{\partial c_1} = 2x_2 c_1^* + \lambda_{Sh}(\frac{\beta S_h^* I_m^*}{N}) - \lambda_{Ih}(\frac{\beta S_h^* I_m^*}{N}) + \lambda_{Sm}(\frac{\alpha S_m^* I_h^*}{N}) - \lambda_{Im}(\frac{\alpha S_m^* I_h^*}{N}), \\ 0 = \frac{\partial H}{\partial c_2} = 2x_3 c_2^* - \lambda_{Ih} I_h^* + \lambda_{Rh} I_h^*, \\ 0 = \frac{\partial H}{\partial c_3} = 2x_4 c_3^* - \lambda_{Sm}(1-p) S_m^* - \lambda_{Im}(1-p) I_m^*. \end{cases}$$

Hence, we obtain

$$\begin{cases} c_1^* = \frac{\alpha S_m^* I_h^* (\lambda_{Im} - \lambda_{Sm}) + \beta S_h^* I_m^* (\lambda_{Ih} - \lambda_{Sh})}{2Nx_2}, \\ c_2^* = \frac{(\lambda_{Ih} - \lambda_{Rh}) I_h^*}{2x_3}, \\ c_3^* = \frac{(1-p)(\lambda_{Sm} S_m^* + \lambda_{Im} I_m^*)}{2x_4}. \end{cases}$$

and

$$\begin{cases} c_1^* = \max\{0, \min(1, \frac{\alpha S_m^* I_h^* \lambda_{Im} + \beta S_h^* I_m^* \lambda_{Ih} - \alpha I_h^* S_m^* \lambda_{Sm} - \beta S_h^* I_m^* \lambda_{Sh}}{2Nx_2})\}, \\ c_2^* = \max\{0, \min(1, \frac{\alpha S_m^* I_h^* (\lambda_{Im} - \lambda_{Sm}) + \beta S_h^* I_m^* (\lambda_{Ih} - \lambda_{Sh})}{2Nx_2})\}, \\ c_3^* = \max\{0, \min(1, \frac{(\lambda_{Ih} - \lambda_{Rh}) I_h^*}{2x_3})\}, \\ c_3^* = \max\{0, \min(1, \frac{(1-p)(\lambda_{Sm} S_m^* + \lambda_{Im} I_m^*)}{2x_4})\} \end{cases}$$

The following section covers the solutions numerically for the optimal system and optimal control pairs, as well as parameter selection and interpretations of diverse scenarios.

4.1. Numerical results

We will now conduct a numerical analysis of the malaria model's optimal transmission parameter control. The optimal control can be found by solving the optimality system, which consists of six ordinary differential equations (ODEs) generated from the state and adjoint equations. Our study examines the impact of control strategies on malaria transmission using a basic model that includes both prevention and treatment.

(1) Single control measures:

- Only preventive measures as personal protection;
- Only treatment measures;
- Only insecticide spraying.

(2) Combinations of two/three control measures:

- Preventive and treatment measures together;
- Preventive measures combined with another control;
- Treatment measures combined with another control;
- Combination of all three control measures.

Through these simulations, we aim to identify the most effective control strategies and understand how different control combinations measure the impact on the transmission dynamics of malaria. The estimated parameters values are shown in Table 4.

Table 4. Values used for numerical simulations.

Notation	Value	Notation	Value
S_{h0}	600	V_{h0}	300
I_{h0}	0	R_{h0}	100
S_{m0}	5000	I_{m0}	50
N	1000	δ	0.001
τ_1	0.0000548	τ_2	$\frac{1}{15}$
μ	0.0000548	η	$\frac{1}{15}$
β	1.32	α	0.09
ω	0.6	γ_1	$\frac{1}{15}$
γ_2	$\frac{1}{21}$	ρ	$\frac{1}{183}$
x_1	20	x_4	10
x_2	65	x_3	10

In Figures 11–17, we suppose that the control c_2 has a higher weight factor (x_3) than the controls c_1 and c_3 . This is assumed by considering the fact that the costs related to c_1 and c_3 account for the cost of specialized bed nets for personal protection as well as insecticide spraying, whereas the cost related to c_2 will account for the cost of drug treatment and medical procedures. For the numerical simulations, we have used the following values to demonstrate the effect of different optimal control strategies on the transmission of the disease in a population. Therefore, we have considered the transmission of the disease in an endemic population.

4.1.1. Optimal personal protection

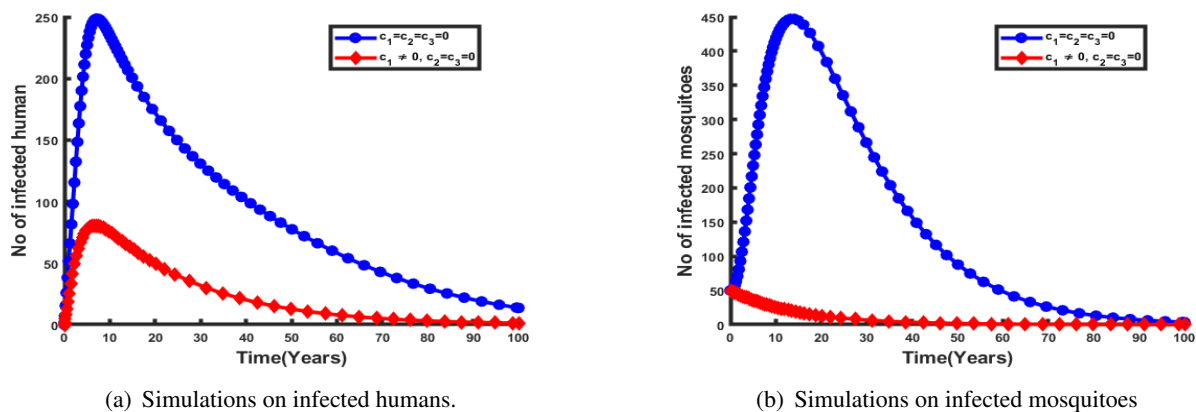


Figure 11. The human and mosquito populations (infected) with the effect of using personal protection.

First, we focus on optimizing the objective function J using the control variable for personal protection (c_1), while keeping the controls for treatment (c_2) and pesticide spraying (c_3) set to zero. Figure 11 illustrates the significant impact of this strategy by comparing the dynamics of infected humans

I_h and mosquitoes I_m under the optimal control and no control scenarios. In the first panel, it is evident that implementing personal protection as a control measure effectively reduces the number of infected humans I_h , in contrast to the unmanaged scenario, where the number of infections increases steadily. Similarly, the second panel shows that the control strategy leads to a significant decrease in the number of infected mosquitoes I_m , whereas the uncontrolled scenario results in a notable rise in mosquito infections. These findings highlight the effectiveness of personal protection in mitigating disease transmission in the absence of additional interventions.

4.1.2. Optimal treatment

This approach focuses on optimizing J using the treatment control (c_2), while the personal protection (c_1) and pesticide spray (c_3) controls are set to zero. Figure 12 highlights the significant differences in the dynamics of infected humans I_h and mosquitoes I_m between the optimal control strategy and the no-control scenario. The results clearly show that implementing treatment as a control measure has a substantial impact on reducing the disease burden among the human population. Although the reduction in infected mosquito numbers I_m is less pronounced, this technique emphasizes the critical role of treatment in controlling malaria transmission and mitigating its effects on humans.

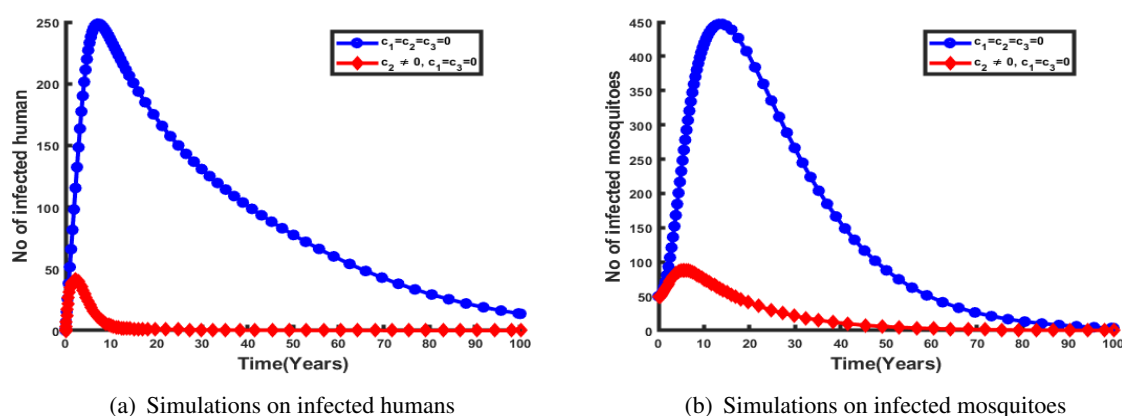


Figure 12. The human and mosquito populations (infected) with the effect of treatment after infection.

4.1.3. Optimal insecticide spraying strategy

In this control strategy, we exclusively employ insecticide spraying ($c_3 > 0$) to optimize the objective functional J , while maintaining the treatment ($c_2 = 0$) and personal protection ($c_1 = 0$) controls inactive. Figure 13 reveals striking epidemiological contrasts between the optimally controlled and uncontrolled scenarios. The left panel demonstrates that targeted insecticide application produces a rapid decline in infected human prevalence (I_h), contrasting sharply with the uncontrolled case where infections grow monotonically. Correspondingly, the right panel shows the reduced density of infected mosquitoes (I_m) under of the control implementation, as opposed to unimpeded growth in the baseline scenario. These dual effects, namely – a direct reduction in the mosquito population and the consequent breaking of transmission cycles, highlight the intervention's efficacy. The results quantitatively validate that strategic insecticide deployment can significantly disrupt malaria's transmission dynamics

by simultaneously impacting both the vector and human infection compartments.

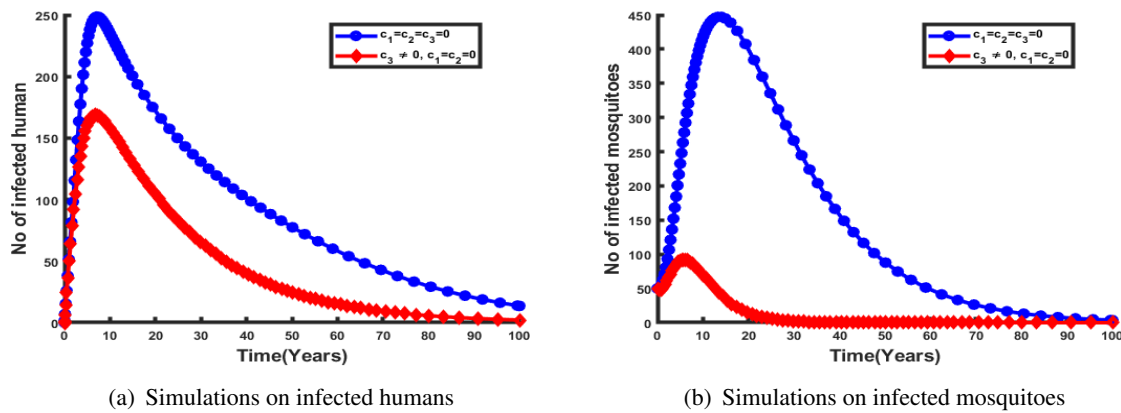


Figure 13. The human and mosquito populations (infected) with the effect of spraying insecticides.

4.1.4. Optimal insecticide spraying and treatment

In this technique, both treatment (c_2) and insecticide spraying (c_3) are employed to optimize J , while personal protection (c_1) is not included as a control measure. The results, as depicted in the figure below, reveal significant differences in the dynamics of infected humans I_h and mosquitoes I_m between the optimal control strategy and the no-control scenario. Figure 14(a) demonstrates that the combined control measures effectively reduce the number of infected humans I_h compared with the uncontrolled condition, where infections increase. Similarly, Figure 14(b) shows that the uncontrolled scenario leads to a rise in the number of infected mosquitoes I_m , whereas the application of treatment and insecticide spraying as control measures significantly reduces mosquito infections. These results highlight the synergistic effect of combining treatment and insecticide spraying in mitigating malaria transmission.

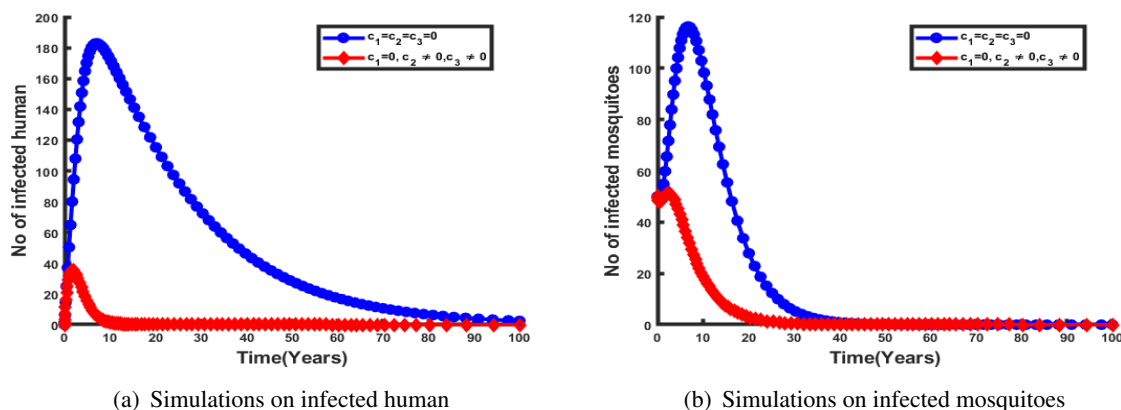


Figure 14. The human and mosquitoes populations (infected) with the effect of treatment and spraying insecticides.

4.1.5. Optimal strategies for personal protection and insecticide spraying

The objective functional J is optimized through the simultaneous implementation of personal protection ($c_1 > 0$) and insecticide spraying ($c_3 > 0$), while treatment remains inactive ($c_2 = 0$). Figure 15 demonstrates the epidemiological impact of this strategy: panel (a) reveals a significant reduction in the prevalence of human infection (I_h) under control measures compared with uncontrolled exponential growth, while panel (b) shows the corresponding suppression of the infected mosquito population (I_m). This synergistic effect – combining direct vector reduction through c_3 with bite prevention via c_1 – proves to be particularly effective in disrupting malaria transmission, as evidenced by the simultaneous decline in both the human and mosquito infection compartments. The results quantitatively validate that integrated vector control strategies outperform single-intervention approaches.

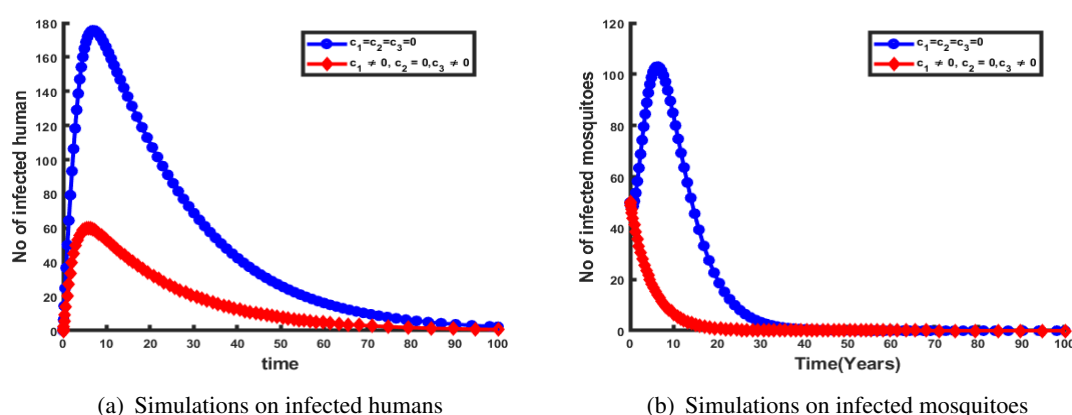


Figure 15. The human and mosquito populations (infected) with the effect of personal protection and spraying insecticides.

4.1.6. Optimal techniques for personal protection and treatment

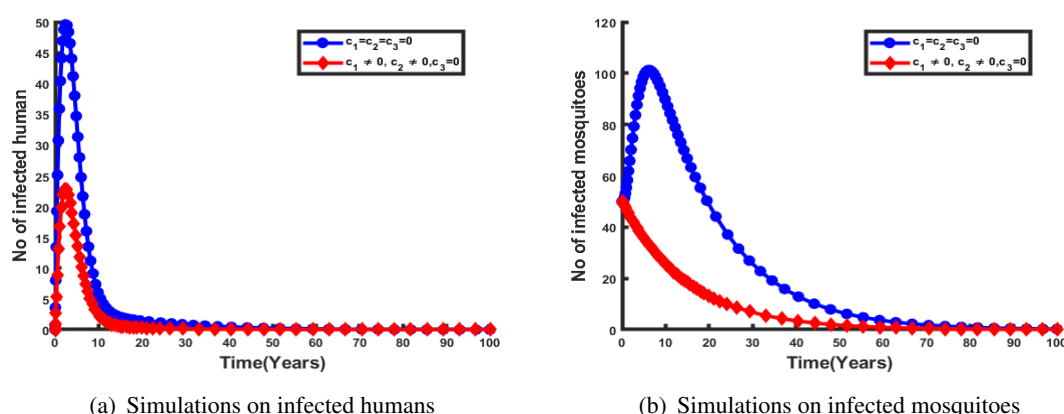
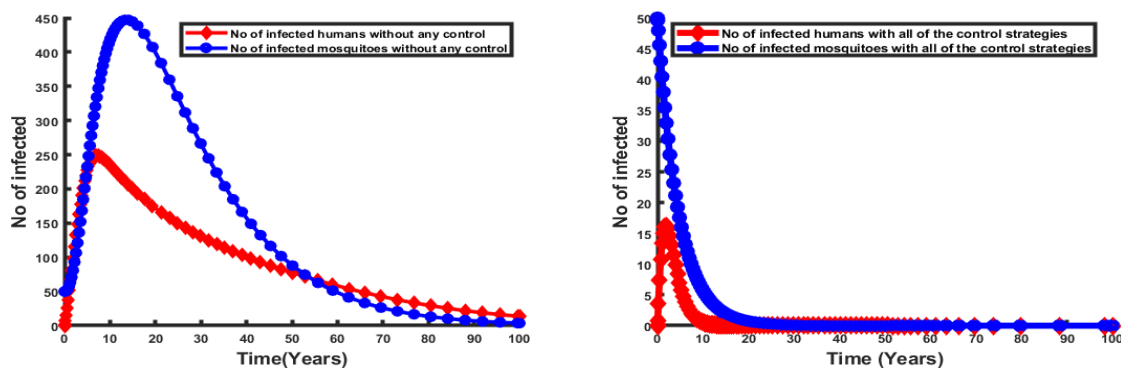


Figure 16. The human and mosquito populations (infected) with the effect of treatment and personal protection.

The function J is optimized using protection c_1 and spraying (c_3) as control measures, while the treatment control remains inactive ($c_2 = 0$). As shown in Figure 16, the number of infected humans (I_h) and mosquitoes (I_m) differs significantly between the controlled and uncontrolled scenarios. Figure 16(a) illustrates that the number of infected humans (I_h) is notably lower when the control measures are applied, compared with the scenario without any controls. Similarly, Figure 16(b) demonstrates that the controlled strategy effectively reduces the number of infected mosquitoes (I_m) relative to the uncontrolled strategy. These results underscore the effectiveness of combining personal protection and insecticide spraying in controlling malaria transmission.

4.1.7. Optimal strategies for personal protection, treatment, and insecticide spraying

Here, the objective function J is optimized using all three controls (c_1, c_2, c_3), with the weight factors $x_1 = 92$, $x_2 = 20$, $x_3 = 65$, and $x_4 = 10$. The figures in Figure 17(a),(b) illustrate that the implementation of these control strategies result in a reduction in the number of infected mosquitoes (I_m) and infected humans (I_h) compared with the uncontrolled scenario, where both variables showed an increase.



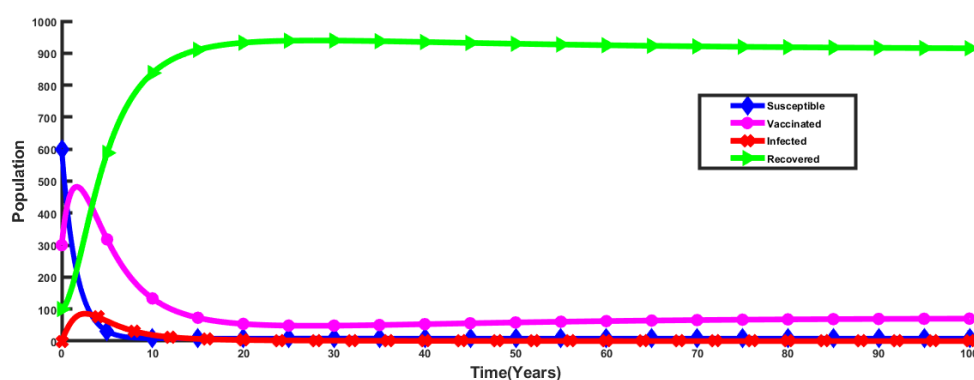
(a) Simulation of infected humans and mosquitoes without any control strategies. (b) Simulation of infected humans and mosquitoes with all of the control strategies.

Figure 17. The effect of control strategies on humans and mosquitoes (infected).

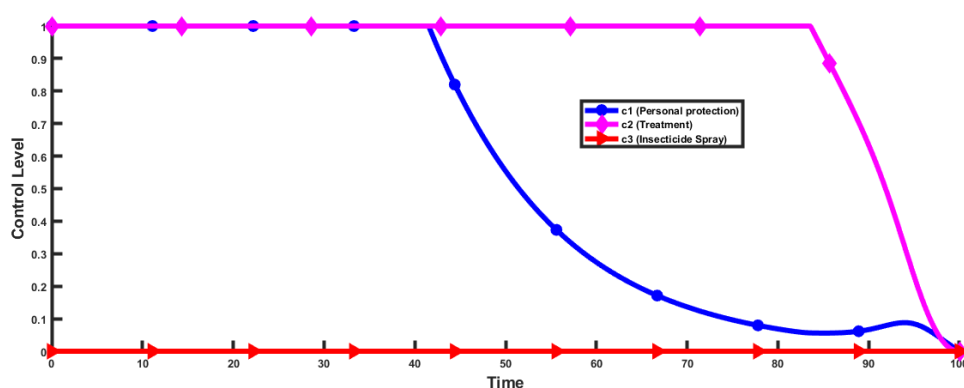
In all the figures presented above, it is evident that the populations of both infected humans and infected mosquitoes naturally decline over time, even in the absence of any control interventions. However, a closer examination reveals that the implementation of control measures significantly accelerates the reduction in infection levels. The use of optimal control strategies—such as personal protection, treatment, or insecticide spraying—not only expedites the decline in infections in both the human and mosquito populations but also leads to a more rapid stabilization of the system. This suggests that although the disease may eventually subside without intervention, the adoption of appropriate control strategies can substantially shorten the time required to mitigate the outbreak and achieve disease containment.

Figure 18(a) depicts the time evolution of the four compartments of the human population under the impact of optimal control strategies. Initially, the number of infected people grows rapidly, peaking within the first few years. However, with the introduction of control measures, the infection

begins to drop steadily and is nearly eradicated within 10–15 years. As a result, the vulnerable population rapidly declines, whereas the vaccinated population first increases before stabilizing. The regenerated population grows significantly and takes over the system, stabilizing at a high level over time. This demonstrates the long-term efficacy of vaccination and treatment in moving the system toward disease-free equilibrium.



(a) Effect of using control strategies in different human compartments.



(b) Control level in a specified time interval.

Figure 18. Control strategies and their effects in different time intervals.

Figure 18(b) represents the optimal time-dependent control measures for the three control strategies: c_1 (personal protection), c_2 (treatment), and c_3 (insecticide spraying). During the scenario, both c_1 and c_2 are fully utilized, emphasizing the need for immediate personal protection and treatment to control the epidemic. As infection levels fall, the intensity of c_1 (personal protection) gradually declines, indicating that it is no longer necessary. The control c_2 (treatment) remains high for a longer period of time and begins to taper down after approximately 80 years, indicating that treatment continues to play an important role for a large portion of the time horizon. The control c_3 (insecticide spraying) remains negligible, indicating that it makes little contribution to disease control under the current parameter settings. The combined control measures successfully reduce the infected population and drive the system toward herd immunity. The graphs illustrate that early and robust intervention is critical for disease control, and that the system can then maintain a disease-free condition with minimal effort.

5. Discussion

Addressing global health issues in the present day necessitates a multidisciplinary strategy that incorporates several specialties. Because they provide important insights into the behavior and evolution of infectious diseases, mathematical models have become essential for comprehending their transmission and management. These models aid in the creation of focused disease management and preventive strategies by mimicking transmission dynamics. In order to investigate its possible value in guiding decision-making processes, this debate centers on developing a conceptual mathematical framework for malaria transmission. This method emphasizes how crucial predictive tools are to creating successful public health policy, especially in areas where malaria is still a major problem.

The SVIR model has served as our foundational paradigm. In our model, we have utilized the vaccine compartment. We conducted our research using hypothetical data because it is extremely difficult to find real malaria vaccination data. Some of the values are assumed, some are collected from various publications, and some have been computed for the parameters. We have demonstrated how several factors affect the spread of malaria. Additionally, we have demonstrated how the parameter affects the basic reproduction number as it varies within a given range. Which parameters are more effective than others in the malaria transmission process are displayed by the PRCC plot. Following that, we suggested some control measures to control the spread of malaria. We have demonstrated that, in comparison with scenarios without control measures, transmission is reduced and under control when control mechanisms are in place. Since vaccination improves immunity, we may conclude from our findings that if a suitable immunization program is maintained and appropriate control measures are implemented, the spread of malaria will be contained and the likelihood of an outbreak is extremely low.

Through numerical simulations, this work offers insightful information about the dynamics of malaria transmission and the effects of different control strategies; nevertheless, it has certain limitations. A significant limitation is the dependence on theoretical simulations, which, however instructive, might not adequately represent the intricacies of actual situations, such as changes in a population's behavior, environmental influences, or practical difficulties in putting the control mechanisms into place. The suggested vaccination coverage is predicated on consistent availability and acceptability, which may not be possible in many endemic areas because of infrastructure or socioeconomic constraints. Furthermore, possible pesticide or treatment resistance is not taken into consideration in the model, which could have a big influence on how well the suggested control measures work. The interplay of multiple elements in disease transmission is oversimplified by the assumption that all therapies are equally effective. Lastly, the results' practical usefulness is limited by the lack of real-world data validation, highlighting the necessity for future research to include field data for improved model calibration and accuracy.

6. Concluding remarks

A numerical solution to the proposed model was obtained, and the results were simulated for different scenarios of the disease situation. From these simulations, we concluded that if a vaccination program can be successfully introduced to cover at least half of the population, the basic reproduction number (R_0) will be less than 1. This means that the disease's spread will decline over time and even-

tually come to an end. In our optimal control problem, we investigated the effects of using single forms of control, followed by combinations. Our results demonstrated that personal protection significantly affects both the number of infected humans and mosquitoes, while treatment and spraying insecticides also play important roles in reducing the size of the infected human, susceptible mosquito, and infected mosquito populations. Furthermore, all combinations of two control measures together showed a significant reduction in the spread of the infection between humans and mosquitoes. Ultimately, when all control measures were considered to be non-zero, there was the most substantial reduction in the spread of the infection among both populations. The simulation results in the optimal control section indicate that the number of infected individuals in both the human and mosquito populations tends to decline over time, even in the absence of any intervention strategies. Nevertheless, a detailed analysis reveals that the application of control measures considerably enhances the rate at which the infection diminishes. Interventions such as personal protection, treatment, and insecticide spraying significantly accelerate the decline in the infection's prevalence. Therefore, it can be concluded that although the disease may eventually wane without active intervention, the implementation of targeted control strategies effectively reduces the time required to suppress the outbreak and achieve epidemiological stability. By combining vaccination with other malaria control techniques, this study offers a complete approach that guarantees a more successful approach to malaria prevention and mitigation. Policymakers can prioritize important control measures by using sensitivity analysis to further determine the parameters that have the greatest influence on malaria transmission. Furthermore, combining several approaches can maximize resource allocation, resulting in more economical malaria prevention. In the end, the study offers helpful recommendations for creating integrated control initiatives and focused immunization campaigns, which will improve public health results. Notwithstanding its benefits, the study has several drawbacks. The availability and dependability of epidemiological data, which can differ by location, are critical factors in determining the correctness of mathematical models. Because of data uncertainty, questions over vaccine efficacy, and logistical difficulties in organizing various control measures, the study's dependence on mathematical models adds constraints. Furthermore, outside variables like climate change, public compliance, and insect resistance could weaken the efficacy and resilience of the suggested tactics. In order to dynamically enhance vaccine and vector control methods, future research could investigate adaptive control strategies that use real-time data. The goal is to create an age-structured model for this issue in the future. The efficacy and scalability of malaria - preventive initiatives could also be improved by looking into the socioeconomic effects of malaria interventions and using machine learning for predictive modeling.

Use of AI tools declaration

The authors declare they have not used artificial intelligence (AI) tools in the creation of this article.

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Author contributions

Khadiza Akter Eme: Data curation, formal analysis, methodology, software, validation, writing – original draft. Md Kamrujjaman: Conceptualization, formal analysis, investigation, software, supervision, writing – original draft, writing – review, and editing. Muntasir Alam: Methodology, software, validation, writing – review, and editing. Md Afsar Ali: Investigation, funding acquisition, resources, writing – review, and editing.

Conflict of interest

The authors declare no conflict of interest.

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