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Research article

Mathematical modeling for *Hypothenemus hampei* and *Colletotrichum kahawae* co-dynamics with optimal control strategies

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Abstract: Several pests and diseases are major factors challenging the coffee industry worldwide. Particularly, production of *Coffee Arabica* in many African countries has been affected by *Hypothenemus hampei* and *Colletotrichum kahawae* in a coffee farm. Pest(s) and disease(s) are commonly inter-related and can interact, because pests and pathogens have the same biophysical requirements in ecosystems. Assessment of coffee berries damage due to multiple pests and diseases is a necessary step in designing appropriate control strategies. In this paper, we developed a mathematical model describing the co-dynamics of *Hypothenemus hampei* (coffee berry borer, CBB) and *Colletotrichum kahawae* (coffee berry disease, CBD). The model used a system of nonlinear ordinary differential equations to capture the interactions among the CBB pest population, the CBD fungal pathogen, and the healthy and infected coffee berry populations. Optimal control strategies were also incorporated to assess effective management approaches. Optimal control strategies were obtained by minimizing the number of pests and fungal pathogen population by incorporating two control variables such as biological control and cultural practices. The existence of optimal controls was examined using Pontryagin's minimum principle. The Hamiltonian was constructed, and adjoint equations were solved to minimize the cost functional. Lastly, from different scenarios, the numerical simulations were performed to illustrate the model's co-dynamics with and without optimal control strategies.

Keywords: co-dynamics; coffee berry; *Hypothenemus hampei*; *Colletotrichum kahawae*; optimal control; numerical simulation

1. Introduction

The coffee plant is well adapted to tropical and subtropical highland regions. Coffee is an essential agricultural commodity worldwide. It is a favored beverage with significant contribution to international trade, ranking as the second most traded commodity after crude oil [1]. However, the coffee production industry faces persistent challenges from diseases, weeds, and numerous pests. Among these, the coffee berry borer (CBB), caused by *H. hampei*, is considered the most serious pest affecting commercial coffee in nearly all coffee-producing countries [2, 3]. Other major insect pests impacting coffee production include the Antestia bug and the coffee blotch miner [4].

The *H. hampei* completes its entire developmental cycle within green coffee berries. Female *H. hampei* undergo two larval stages, whereas males go through only one; each larval stage lasts approximately 10–26 days. During this time, the larvae feed on the endosperm of the coffee bean. Dispersal is primarily carried out by the females, who emerge from infested berries, create galleries in new berries, and lay their eggs there. In contrast, males and immature stages remain inside the original berries throughout their development [5]. This pest causes both direct losses (i.e., reduced coffee yield) and indirect losses (i.e., diminished quality of the coffee berries) [6]. Infestation levels can reach up to 95% on some farms, though they vary considerably from year to year [7].

Coffee berry disease (CBD), caused by *C. kahawae*, is a fungal plant pathogen that significantly affects the production of *Coffee Arabica* in many African countries [8]. The pathogen disrupts the development of coffee berries-the harvestable portion of the crop-results in production loss [9]. While most infected berries fall prematurely, those that remain attached to the branches serve as the primary source of secondary inoculum.

As shown in Figure 1, the female beetle bores into the coffee berries through the navel region, creating tunnels and feeding on the internal contents as well as associated fungal pathogens. This is evidenced by the concentric rings surrounded by emerging black acervuli within the lesion [8, 10].

(a) (b)

Figure 1. (a) Coffee borer entering a coffee cherry and (b) Coffee berries infected with CBD [8].

The *C. kahawae* fungus is locally dispersed among coffee trees and branches primarily through wind and rain [11]. However, long- and medium-distance dispersal is commonly facilitated by coffee berry harvesters, insects, and birds [12]. Rainfall plays a critical role in the fungus's life cycle, being essential for conidial production, dispersion, germination, and subsequent infection [13]. Infection begins when the conidia (asexual spores) germinate, which can occur within 24 hours of contact with host plant tissue. While optimal rainfall levels of around 10 mm are effective for dispersing most conidia, heavy rainfall exceeding 700 mm tends to wash the spores from the coffee tree canopy into the soil, reducing their potential to infect [13].

Numerous mathematical models have been developed to study the effects of preventive and control strategies on the co-dynamics of plant pests and disease spread (see [14–17]). For example, a study by [14] investigated the transmission dynamics of plant diseases with and without the implementation of roguing mechanisms. The findings showed that roguing significantly reduces disease transmission. In [18], Nyaberi et al. formulated and analyzed a mathematical model for the co-infection of CBD and coffee leaf rust (CLR), incorporating various control strategies to conduct an optimal control analysis. Their numerical simulations suggest that a combination of all available interventions offers the most effective approach to mitigating the spread of CBD-CLR co-infection.

Control measures for CBD include chemical control [13], the use of resistant genetic varieties [19], biological control methods [20], and improved cultivation practices [21]. Similarly, CBB management strategies involve enhanced cultural practices, insecticidal chemical control, biological agents, and traps [22–24]. In many African countries, including Ethiopia, the primary approach to managing CBB has been through cultural practices [21].

The aim of this study is to provide quantitative and qualitative explanations for a co-dynamical mathematical model with optimal control strategies to manage the interacting populations of *H. hampei* and *C. kahawae* in coffee plantations. Unlike previous models, our framework explicitly accounts for the bidirectional feedback between pest infestation and fungal infection where *H. hampei* damages coffee berries, facilitating *C. kahawae* invasion, while high fungal prevalence reduces healthy berries available for pest infestation, thereby influencing pest dynamics. Additionally, we integrate optimal control strategies such that biological and cultural practices explore effective management interventions for mitigating the co-infestation of pests and pathogens. This work advances agricultural systems modeling by offering a mechanistic understanding of pest-pathogen coexistence and actionable strategies for sustainable crop protection.

2. CBB-CBD co-dynamical model with controls

In this model, the interaction of populations among coffee berries, female *H. hampei* pests and *C. kahawae* pathogens are considered. In a closed system with a fixed number of coffee plants, both pests and pathogens interact through the coffee berries. The *H. hampei* facilitates the spread of *C. kahawae* by creating entry points for the fungus. To investigate the dynamics of *H. hampei* and *C. kahawae* co-infection, we divided the coffee berry population in the coffee farm into five classes at any time t, namely, susceptible coffee berry $S(t)$, infested coffee berry due to *H. hampei* females $I_h(t)$, infected coffee berry due to *H. hampei* $I_c(t)$, co-infected coffee berry due to both *H. hampei* and *C. kahawae* $I_{hc}(t)$ and removed coffee berries $R(t)$ with two control measures are incorporated, namely, u_1 and u_2. The first control u_1 represents biological control strategies (i.e., natural predators, Antagonistic microorganisms, Endophytic fungi, trapping), and the second control u_2 represents cultural control strategies (i.e., field sanitation, pruning, shade management, crop rotation). The biological control strategy (u_1) limits the infection caused by CBB by reducing the transitions from S to I_h and from I_c to I_{hc}, while the cultural control strategy (u_2) mitigates the infection caused by CBD by reducing the transitions from S to I_c and from I_h to I_{hc}.

Susceptible coffee berries are recruited at rate Λ. This depends on the growth cycle of the coffee plant and environmental conditions. The coffee berries drop naturally at rate μ. This is the rate at which berries fall off the plant due to natural causes (e.g., maturation, wind, or other environmental factors). Susceptible coffee berries contact with *H. hampei* and *C. kahawae* at a rate of β_h and β_c, respectively. The induced mortality rate due to *H. hampei* infestation and *C. kahawae* infection for removed coffee berries is γ_h and γ_c, respectively.

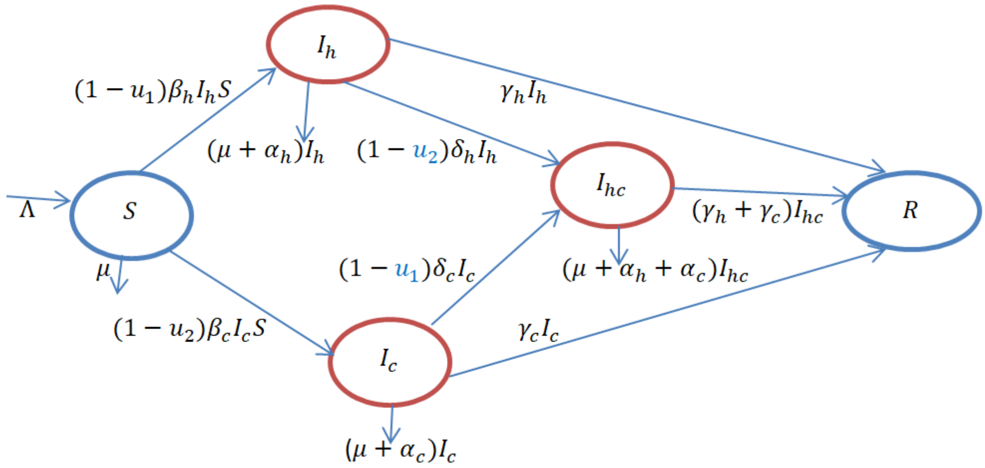


Figure 2. Flow diagram for *H. hampei* and *C. kahawae* co-dynamics.

Table 1. Description of state variables.

State variables	Descriptions
S	Susceptible coffee berries
I_h	Coffee berries infested by *H. hampei*
I_c	Coffee berries infected by *C. kahawae*
I_{hc}	Coffee berries co-infected by both *H. hampei* and *C. kahawae*
R	Removed coffee berries

Table 2. Description of proposed model parameters and their estimated values.

Parameters	Descriptions	Estimated	Source
Λ	Rate of new berry formation	0.00056	Assumed
β_h	Transmission rate of *H. hampei*	0.0002	Assumed
β_c	Transmission rate of *C. kahawae*	0.000795455	[16]
γ_h	Induced mortality rate for *H. hampei* infestation	0.01	Assumed
γ_c	Induced mortality rate for *C. kahawae* infection	0.005	Assumed
μ	Natural berry drop rate	0.002	[15]
α_h	Additional berry drop rate due to *H. hampei*	0.001	Assumed
α_c	Additional berry drop rate due to *C. kahawae*	0.0001	[16]
δ_h	Rate of co-infection from *H. hampei*	0.01	Assumed
δ_c	Rate of co-infection from *C. kahawae*	0.02	Assumed

The coffee berries drop additionally due to *H. hampei* and *C. kahawae* at a rate of α_h and α_c, respectively. The rate at which coffee berries are simultaneously infested by the borer and infected by the fungus are δ_h and δ_c, respectively. The model is schematically described in the Figure 2. The description of state variables and parameters are summarized in Tables 1 and 2, respectively.

Based on the above assumptions, the co-dynamics of *H. hampei* and *C. kahawae* with control variables is described by the following system of differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta_h(1-u_1)SI_h - \beta_c(1-u_2)SI_c - \mu S,\\
\frac{dI_h}{dt} &= \beta_h(1-u_1)SI_h - \delta_h(1-u_2)I_h - (\mu+\alpha_h)I_h,\\
\frac{dI_c}{dt} &= \beta_c(1-u_2)SI_c - \delta_c(1-u_1)I_c - (\mu+\alpha_c)I_c,\\
\frac{dI_{hc}}{dt} &= \delta_h(1-u_2)I_h + \delta_c(1-u_1)I_c - (\mu+\gamma_h+\gamma_c+\alpha_h+\alpha_c)I_{hc},\\
\frac{dR}{dt} &= \gamma_h I_h + \gamma_c I_c + (\gamma_h+\gamma_c)I_{hc},
\end{aligned} \tag{2.1}$$

with corresponding initial data:

$$S(0) > 0,\ I_h(0) \geq 0,\ I_c \geq 0,\ I_{hc}(0) \geq 0,\ R(0) \geq 0. \tag{2.2}$$

3. The optimal control problem and existence of optimal controls

Optimal control can be effectively applied to manage infestation and disease spread. The primary objective is to minimize the number of infectious coffee berries caused by *H. hampei*, those infected by *H. kahawae*, and the number of co-infected coffee berries resulting from both *H. hampei* and *H. kahawae*. To achieve this goal, we construct an optimal control problem with two control variables u_1 and u_2. With this in mind, we define the objective functional for the minimization problem as

$$J = \min_{(u_1,u_2)} \int_0^T \left(B_h I_h(t) + B_c I_c(t) + B_{hc} I_{hc}(t) + \frac{1}{2}C_1u_1^2 + \frac{1}{2}C_2u_2^2 \right) dt, \tag{3.1}$$

subject to state system (2.1) with initial data (2.2). The fixed constant T denotes the final intervention time while the coefficients B_h, B_c, and B_{hc} are positive constants to keep a balance in the cost coefficients for infestation or infection: $I_h(t), I_c(t)$, and $I_{hc}(t)$, respectively. That means B_h is the cost associated with infected coffee berries I_h, B_c is the cost associated with CBB females I_c, while B_{hc} the cost that corresponds to co-infected coffee berries $I_{hc}(t)$. The term $0.5C_1u_1^2$ and $0.5C_2u_2^2$ represent the cost coefficients for control efforts.

The current cost at time t can be obtained using the integrand $B_hI_h(t)+B_cI_c(t)+B_{hc}I_{hc}(t)+0.5C_1u_1^2+0.5C_2u_2^2$. For our model (2.1), we need to compute the optimal control $(u_1^*(t), u_2^*(t))$ such that

$$J(u_1^*(t), u_2^*(t)) = \min(J(u_1(t), u_2(t)) : u_1(t), u_2(t) \in U). \tag{3.2}$$

where the set of admissible control functions is defined by

$$U = \{(u_1(t), u_2(t)) : u_1(t), u_2(t) \text{ are measurable for } 0 \leq t \leq T\}. \tag{3.3}$$

The existence of the optimal controls can be showed by using an approach of [25]. Here, the boundedness of solutions to state system (2.1) in the finite time interval is very crucial to determine the existence and uniqueness of optimal control to our model. Clearly, from Eq (2.1) we have $N(t) \leq \Lambda/\mu$ as $t \to \infty$, where $N = S + I_c + I_h + I_{hc} + R$. This implies that the Λ/μ is an upper bound for the total coffee berry population N, which dominates the state variables S, I_c, I_h, I_{hc}, and R. This result can be used to prove the existence of optimal controls. For a detailed proof, see [17] and [26].

4. The Hamiltonian and optimality system

In this subsection, we briefly present the characterization of optimal control. Assume that $(u_1^*(t), u_2^*(t)) \in U$ is an optimal solution for problems (2.1)–(3.1) with fixed final time T. Then there exists a nontrivial absolutely continuous adjoint vector $\lambda : [0, T] \to \mathbb{R}^5$ that satisfies the following conditions.

1) The Hamiltonian function associated to our problem is given by

$$\begin{aligned} H(K, u, \lambda, t) = {} & B_h I_h(t) + B_c I_c(t) + B_{hc} I_{hc}(t) + \frac{1}{2} C_1 u_1^2 + \frac{1}{2} C_2 u_2^2 \\ & + \lambda_1(\Lambda - \beta_h(1 - u_1) S I_h - \beta_c(1 - u_2) S I_c - \mu S) \\ & + \lambda_2(\beta_h(1 - u_1) S I_h - \delta_h(1 - u_2) I_h - (\mu + \alpha_h) I_h) \\ & + \lambda_3(\beta_c(1 - u_2) S I_c - \delta_c(1 - u_1) I_c - (\mu + \alpha_c) I_c) \\ & + \lambda_4(\delta_h(1 - u_2) I_h + \delta_c(1 - u_1) I_c - (\mu + \gamma_h + \gamma_c + \alpha_h + \alpha_c) I_{hc}) \\ & + \lambda_5(\gamma_h I_h + \gamma_c I_c + (\gamma_h + \gamma_c) I_{hc}), \end{aligned} \tag{4.1}$$

where $K = (S, I_c, I_h, I_{hc}, R)$ are state variables, and $\lambda = (\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t), \lambda_5(t))$ are adjoint state variables.

2) The control system:

$$\frac{dS}{dt} = \frac{\partial H}{\partial \lambda_1}, \quad \frac{dI_h}{dt} = \frac{\partial H}{\partial \lambda_2}, \quad \frac{dI_c}{dt} = \frac{\partial H}{\partial \lambda_3}, \quad \frac{dI_{hc}}{dt} = \frac{\partial H}{\partial \lambda_4}, \quad \frac{dR}{dt} = \frac{\partial H}{\partial \lambda_5}. \tag{4.2}$$

3) The adjoint system:

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial S}, \quad \frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial I_h}, \quad \frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial I_c}, \quad \frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial I_{hc}}, \quad \frac{d\lambda_5}{dt} = -\frac{\partial H}{\partial R}. \tag{4.3}$$

4) The optimality condition:

$$\frac{\partial H}{\partial u_1} = 0, \quad \frac{\partial H}{\partial u_2} = 0. \tag{4.4}$$

5) The transversality condition:

$$\lambda_i(T) = 0, \ i = 1, ..., 5. \tag{4.5}$$

To obtain the adjoint variables $\lambda_i, \ i = 1, ..., 5$, we follow the classical result of Pontryagin [25]. Pontryagin's Maximum Principle (PMP) stated in [26] gives the required necessary optimality condition as given in the following theorem. Note that since we look for minimization, we use PMP.

Theorem 1. *Let $u^* = (u_1^*, u_2^*)$ be the optimal control and $(S^*, I_c^*, I_h^*, I_{hc}^*, R^*)$ be the associated unique solutions of the optimal control problems* (2.1)–(3.3) *with fixed final time T. Then, there exist adjoint variables $\lambda_i,\ i = 1, ..., 5$ which satisfy the following adjoint system:*

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= (\lambda_1 - \lambda_2)\beta_h(1 - u_1)I_h + \lambda_1(\beta_c(1 - u_2)I_c + \mu),\\
\frac{d\lambda_2}{dt} &= -\beta_h + (\lambda_1 - \lambda_2)\beta_h(1 - u_1)S + (\lambda_2 - \lambda_4)\delta_h(1 - u_2) + \lambda_2(\delta_h(1 - u_2) + \mu + \alpha_h) - \lambda_5\gamma_h,\\
\frac{d\lambda_3}{dt} &= -\beta_c + (\lambda_1 - \lambda_3)\beta_c(1 - u_2)S + (\lambda_3 - \lambda_4)\delta_c(1 - u_1) + \lambda_3(\mu + \alpha_c) - \lambda_5\gamma_c,\\
\frac{d\lambda_4}{dt} &= -\beta_{hc} + (\lambda_4 - \lambda_5)(\gamma_h + \gamma_c) + \lambda_4(\mu + \alpha_h + \alpha_c),\\
\frac{d\lambda_5}{dt} &= 0
\end{aligned} \tag{4.6}$$

together with transversality condition: $\lambda_i(T) = 0,\ i = 1, ..., 5$.
Also, we get optimal controls $u_1^*(t)$ and $u_2^*(t)$ which are characterized by

$$\begin{aligned}
u_1^*(t) &= min\left\{max\left\{0, \frac{(\lambda_2 - \lambda_1)\beta_h S^* I_h^* + (\lambda_4 - \lambda_3)\delta_c I_c^*}{C_1}\right\}, 1\right\},\\
u_2^*(t) &= min\left\{max\left\{0, \frac{(\lambda_3 - \lambda_1)\beta_c S^* I_c^* + (\lambda_4 - \lambda_2)\delta_h I_h^*}{C_2}\right\}, 1\right\}.
\end{aligned} \tag{4.7}$$

Proof. The adjoint equations are computed by differentiating the Hamiltonian (4.1) with respect to the corresponding state variables: $S,\ I_c,\ I_h,\ I_{hc},\ R$ as Eq (4.3). We assume that the final states $S(T),\ I_c(T),\ I_h(T),\ I_{hc}(T)$ and $R(T)$ are free, then we obtain the transversality condition: $\lambda_i(T) = 0,\ i = 1, ..., 5$.

The optimal controls: $u_1^*(t)$ and $u_2^*(t)$ are computed from the optimality condition (4.4) as follows.

(i) $\dfrac{\partial H}{\partial u_1} = C_1 u_1 + (\lambda_2 + (\lambda_1 - \lambda_2)\beta_h S I_h + (\lambda_3 - \lambda_4)\delta_c I_c = 0$ at $u_1(t) = u_1^*(t)$,

$$u_1^*(t) = \frac{(\lambda_2 - \lambda_1)\beta_h S^* I_h^* + (\lambda_4 - \lambda_3)\delta_c I_c^*}{C_1}.$$

(ii) $\dfrac{\partial H}{\partial u_2} = C_2 u_2 + (\lambda_1 - \lambda_3)\beta_c S I_c + (\lambda_2 - \lambda_4)\delta_h I_h = 0$ at $u_2(t) = u_2^*(t)$,

$$u_2^*(t) = \frac{(\lambda_3 - \lambda_1)\beta_c S^* I_c^* + (\lambda_4 - \lambda_2)\delta_h I_h^*}{C_2}.$$

5. Numerical simulation

We performed numerical simulations which illustrate the effectiveness of the proposed control strategies in reducing CBB and CBD populations using MATLAB software as performed in [27]. The fourth-order Runge-Kutta algorithm is used to solve the state equation (2.1) and adjoint equation (4.6) using the parameter values in Table 2 and assumed initial data: $S(0) = 200,\ I_h(0) = 50,\ I_c(0) = 10,\ I_{hc}(0) = 20,\ R(0) = 10$. The assumed weight constants are $\beta_h = 10$, $\beta_c = 10$, $\beta_{hc} = 1000$, $C_1 = 20$, $C_2 = 250$.

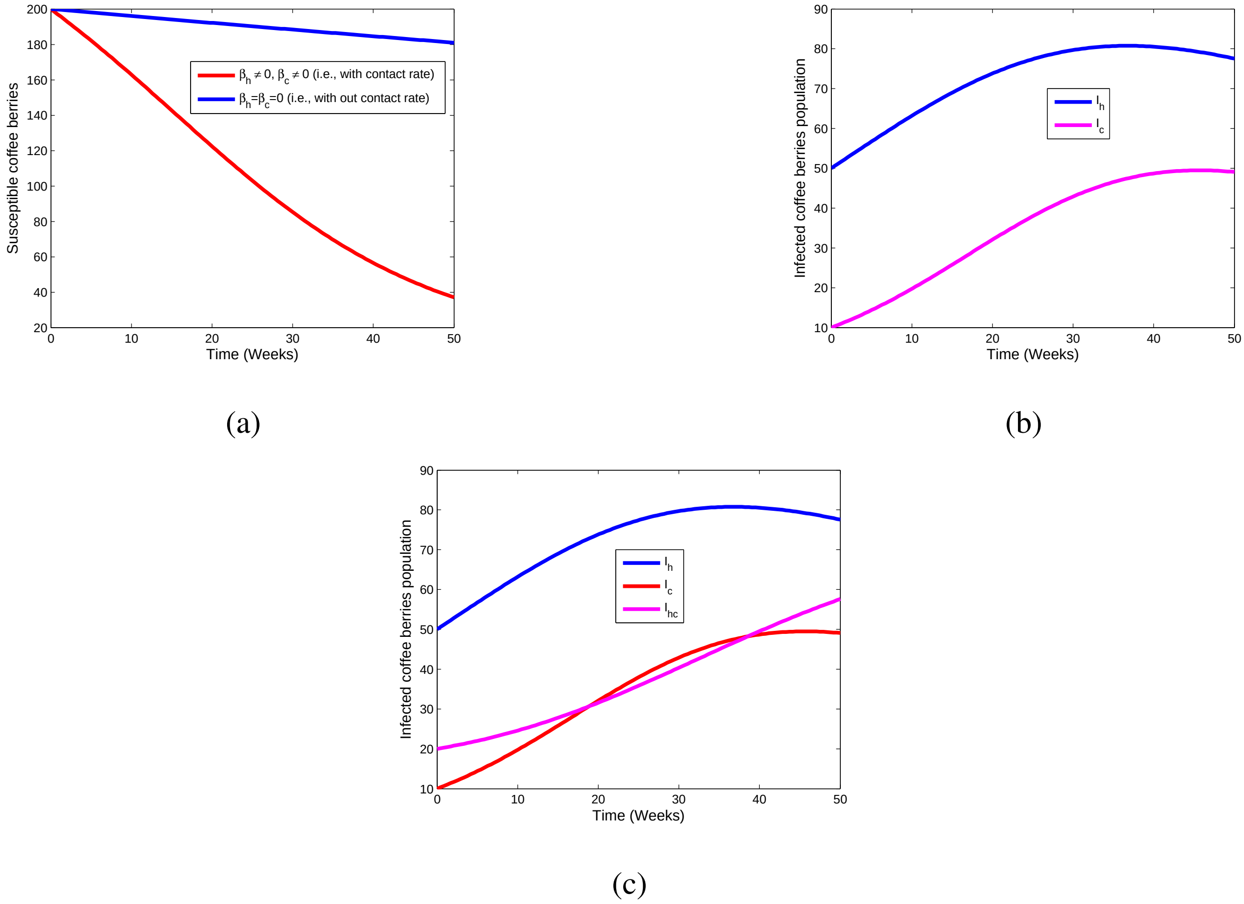


Figure 3. Simulation results of (a) susceptible coffee berries with and without contact rate, (b) Infected coffee berries I_h, I_c, and (c) Infected coffee berries I_h, I_c, I_{hc} together.

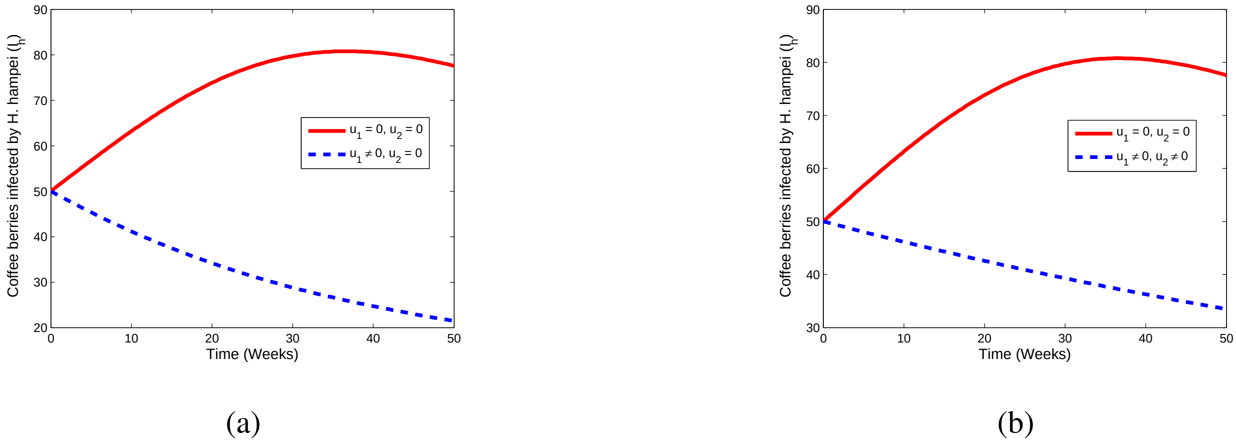
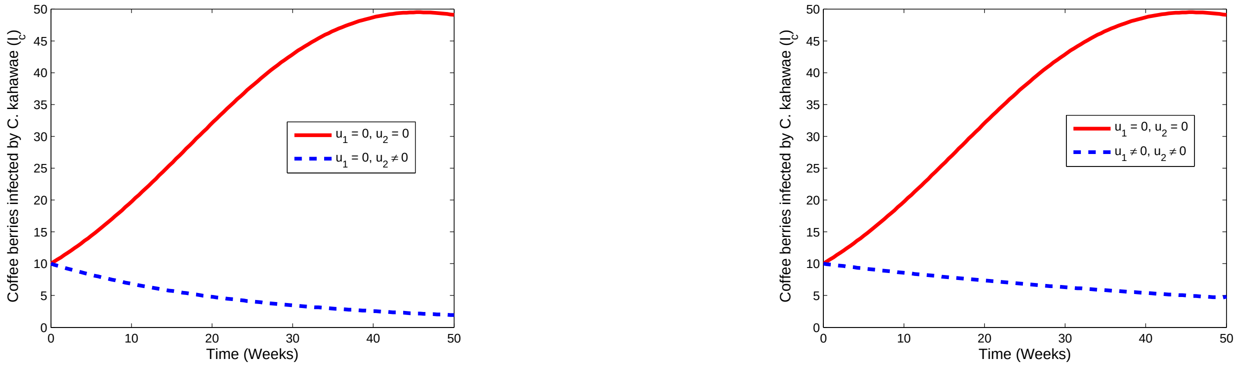
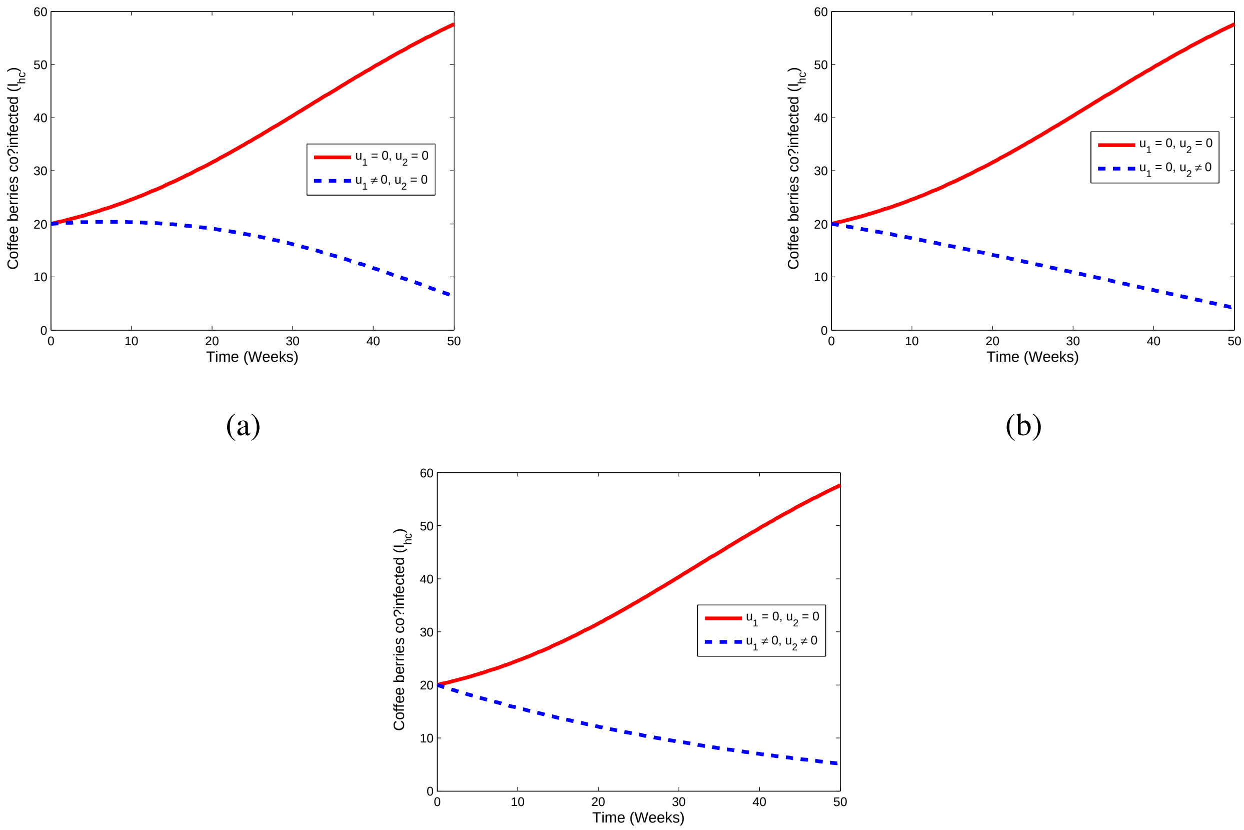


Figure 4. Simulation results of coffee berries infected by *H. hampei* without control(s) and with (a) control u_1, (b) controls u_1, u_2.



(a) (b)

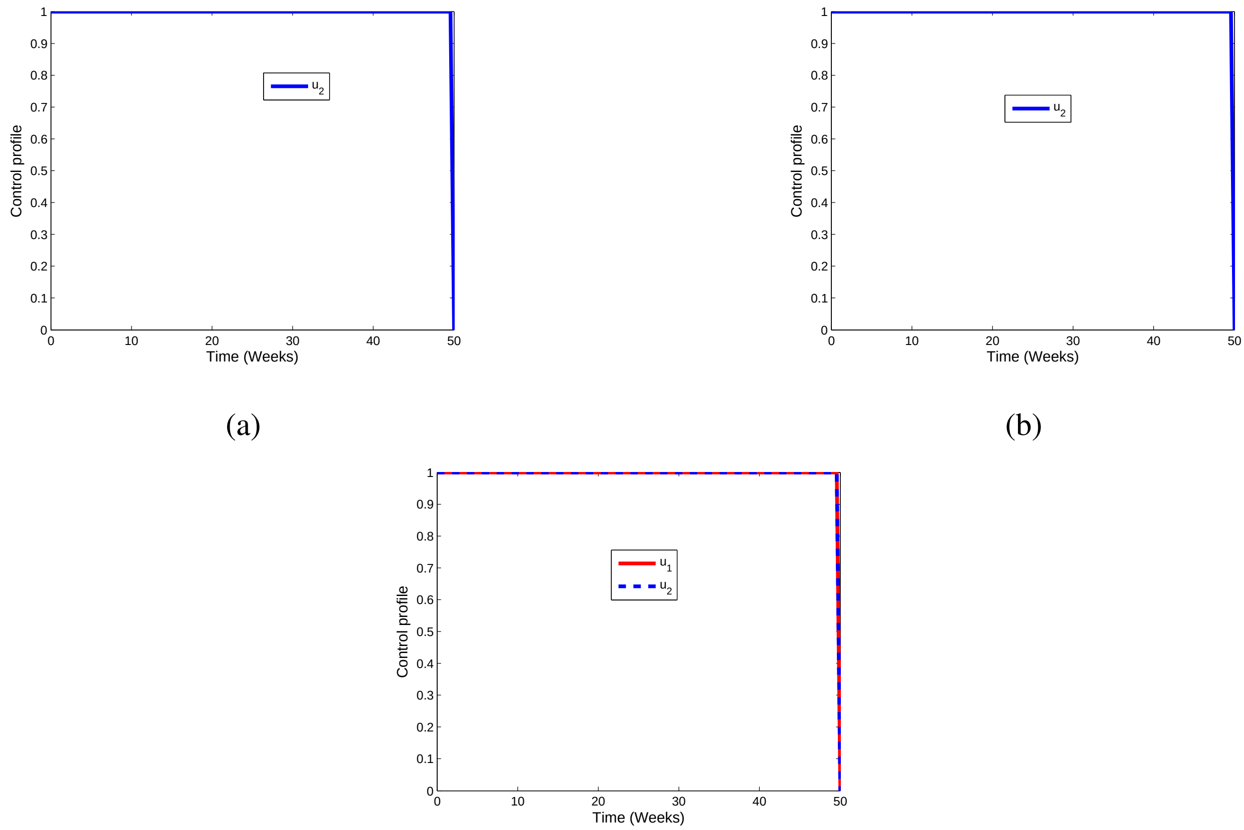
Figure 5. Simulation results of coffee berries infected by *C. kahawae* without control(s) and with (a) control u_2, (b) controls u_1, u_3.



(a) (b)

(c)

Figure 6. Simulation results of coffee berries co-infected without control(s) and with (a) control u_1, (b) control u_2, (c) controls u_1, u_2.



(a) (b)

(c)

Figure 7. Simulation results of control profile (a) control u_1, (b) control u_2, and (c) controls u_1, u_2.

Figure 3(a) shows a significant increase in the population of susceptible coffee berries when there is no contact with *H. hampei* and *C. kahawae*. However, this population decreases upon contact with these pests, as *H. hampei* infests healthy berries and *C. kahawae* spreads through fungal spore dispersal. Figure 3(b),(c) illustrates an increase in the population of infected coffee berries, attributed to the high reproduction rates of the pest and pathogen in the environment, which facilitates disease transmission.

Figure 4(a),(b) illustrates a significant decrease in coffee berries infected by H. hampei as a result of various control strategies. In particular, a greater reduction is observed in Figure 4(a), which corresponds to the application of biological control strategies such as natural predators, antagonistic microorganisms, endophytic fungi, and trapping. A similar reduction is also observed in Figure 4(b), where both biological and cultural control practices are applied to the system.

Figure 5(a),(b) shows a significant decrease in coffee berries infected by C. kahawae following the application of various control strategies. In Figure 5(a), the reduction is associated with cultural control strategies such as field sanitation, pruning, shade management, and crop rotation. A similar reduction is also observed in Figure 4(b), attributed to the combined effect of both biological and cultural control strategies.

Figure 6(a)–(c) shows a significant decrease in coffee berries co-infected by H. hampei and C.

kahawae as a result of the applied control strategies. In particular, a greater reduction in co-infected berries is observed in Figure 6(b) corresponding to the implementation of control strategies u_2.

Figure 7(a)–(c) presents the simulation results of the control profiles. As indicated in these figures, the two control strategies applied in the system are shown to be highly effective.

6. Conclusions

This study presents a comprehensive mathematical modeling approach to analyze the co-dynamics of *H. hampei* (CBB) and *C. kahawae* (CBD) in coffee plantations, incorporating optimal control strategies for effective management. An optimal control problem is formulated to maximize the yield of healthy coffee berries at the end of the cropping season while minimizing CBB and CBD populations in the subsequent season. The existence of optimal controls and their characterization using PMP are investigated. Numerical simulations are then performed to assess the impact of control strategies on reducing CBB and CBD populations. Numerical results suggest that implementing effective control interventions on specific parameters can substantially mitigate disease and pest impacts. These interventions include biological and cultural control strategies. Our comparative analysis highlights that cultural practices such as field sanitation, pruning, shade management, and crop rotation are directly implementable by farmers. In contrast, biological control methods, including the use of natural predators, antagonistic microorganisms, endophytic fungi, and trapping techniques, typically require professional support from agricultural experts. From an epidemiological perspective, both CBB and CBD incidences can be significantly reduced through the implementation of appropriate optimal control measures. Thus, the findings of this study demonstrate that mathematical optimization in integrated pest and disease management can significantly reduce CBB and CBD impacts, enhancing coffee production sustainability. Since both CBB and CBD are highly influenced by climatic factors, we are working on a paper that incorporates climatic variability into the model for future research.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest in this article.

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