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Research article

Modeling and analyzing an opinion network dynamics considering the environmental factor

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Abstract: With the development of Internet technology, social media has gradually become an important platform where users can express opinions about hot events. Research on the mechanism of public opinion evolution is beneficial to guide the trend of opinions, making users' opinions change in a positive direction or reach a consensus among controversial crowds. To design effective strategies for public opinion management, we propose a dynamic opinion network susceptible-forwarding-immune model considering environmental factors (NET-OE-SFI), which divides the forwarding nodes into two types: support and opposition based on the real data of users. The NET-OE-SFI model introduces environmental factors from infectious diseases into the study of network information transmission, which aims to explore the evolution law of users' opinions affected by the environment. We attempt to combine the complex media environmental factors in social networks with users' opinion information to study the influence of environmental factors on the evolution of public opinion. Data fitting of real information transmission data fully demonstrates the validity of this model. We have also made a variety of sensitivity analysis experiments to study the influence of model parameters, contributing to the design of reasonable and effective strategies for public opinion guidance.

Keywords: complex network; opinion dynamics; environmental factors; dynamic model; information propagation

1. Introduction

Social media in the Internet age has gathered more users than ever before and is a major channel for sharing public opinions. Especially during the COVID-19 pandemic, social media use has reached unprecedented scales and the Internet is awash with information on a wide range of opinions due to government control for traveling. Generally speaking, opinion polarity can be divided into two categories: support and opposition. Take opposition as an example, users tend to express their opinions on social media after a controversial topic is published by mainstream media. When the number of opposing opinions increases, it will inevitably lead to the large-scale spread of polarized emotions among users. It is easy to form a negative emotional atmosphere on the Internet platform. If the government and relevant departments find out the opinion trend of the Internet platform in time, they can control the relevant public opinion situation, which is conducive to the construction of a positive network environment. Therefore, it is of great significance to study the dissemination of opinions in the process of network information propagation.

Figure 1 shows the opinion dissemination process involving environmental factors on social media networks. In the information transmission network of a single platform, nodes of different colors represent users with different transmission states, in which forwarding user nodes and environment effect nodes are both divided into supporting or opposing opinion nodes based on their forwarding text content. When users forward information on a certain platform, they are likely to be influenced by public opinion information on some environmental factors outside this social platform. For instance, when the users who have already forwarded information on Chinese Sina Microblog browse a similar topic on TV, newspaper, Instagram or other media platforms again, their ability to influence and spread the information to susceptible users will be enhanced. If a forwarding node is affected by the environment of multi-platform information transmission, it will be transformed into the environment effect node, besides, an indirect transmission path will be generated in the information transmission net, to promote the effect of information dissemination.

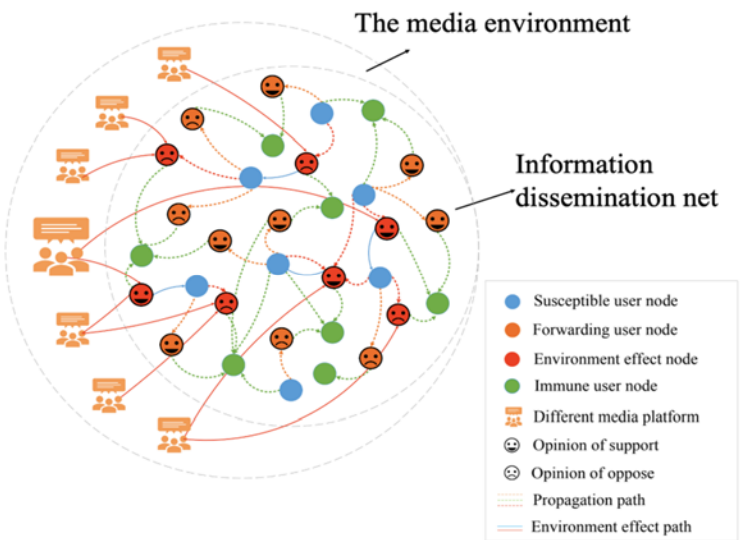

Figure 1. Opinion dissemination process involving environmental factors in social media networks.

The dissemination of public opinion is often accompanied by the information content and the public diffuses the publisher's information through forwarding behavior. For some controversial topics, forwarding users often derive two polarizing opinions: support and opposition. At the same time, there are various kinds of information communication media. Therefore, active users on one platform may coincide with active users on the mass media of the dissemination environment. This also leads to more complex information dissemination processes on social networks. Thus, the research of information dissemination in a sophisticated media environment is becoming quite important. Therefore, this paper constructs an opinion network dynamic model using real data and studies the public opinion's direct transmission process on the complex network and the indirect transmission process affected by the sophisticated media environment, which is conducive to the research of opinion evolution direction affected by environmental factors in social media network.

2. Related work

Since the Internet age and the massive growth of information, the infectious disease model has been widely applied to information dissemination based on social networks successfully by many scholars [1–3], promoting the flourishing development of research on the dynamics of information dissemination. Many studies have shown that social networks can be regarded as complex networks with features of small-world and scale-free [4,5]. There are some limitations in analyzing the process of information dissemination using the traditional mixed homogeneous dynamic models [6,7], while the dynamic models of complex networks can reveal the mechanism of information dissemination more effectively. Therefore, more and more scholars have begun to simulate the process of network information transmission from the perspective of a BA scale-free network or small-world network [8–11].

Considering the characteristics and related influencing factors of rumor propagation in the real world, Sun et al. [12] proposed a rumor propagation model with a non-uniform propagation rate to describe different propagation rates of different nodes. For complex networks in the real world, it is necessary to develop the traditional susceptible-infected-recovered (SIR) model by considering the dependence of rumor spread rate on weight-based connection strength between different nodes. Therefore, Singh et al. [13] studied the effect of degree-degree correlation on rumor propagation and the effectiveness of dissemination strategies in real social networks. Nian et al. [14] developed a new SSIR information propagation model by dividing the susceptible state into two parts, thus constructing a dynamic BA scale-free network to study the evolution of node impact based on secondary propagation experiments. Since the fact that online information in complex networks has played a more and more significant impact on real society, Zhang et al. [15] established a modified network public opinion dissemination model under public crisis. The model uses mean field theory based on BA scale-free network, updating the traditional susceptible-exposed-infected-recovered (SEIR) infectious disease model from a novel perspective. Regarding the dissemination of behavior-related information as the influencing medium, Dang et al. [16] constructed a behavior propagation and confrontation competition model based on the energy of information and carried out the simulation in a small-world network. To investigate the impact of dimension on the process of information dissemination more conveniently, Wang et al. [17] defined an effective model with a higher dimensional small-world network and characterized the dependence of structural properties of the network on dimension. Currently, the online public opinion environment is complex and ever-changing and the self-organizing network information dissemination of social platforms is universal. However, the existing dynamic research based on complex networks mostly starts from the perspective of network topology. There is little work using real data from social platforms to study the information dissemination mode in complex networks, and so much simulation research instead.

In order to explore the opinion evolution of social groups, opinion dynamic is being developed. According to different view descriptions, opinion dynamic models can be divided into discrete models [18,19] and continuous models [20,21]. In recent years, many scholars have paid close attention to using infectious disease models in the research of social network dissemination, including opinion dynamics [22,23]. Liang et al. [24] proposed an absorption law and the modified trust propagation method to describe the opinion evolution process between the three types of people in social networks, based on the infectious disease transmission model. Considering the significant effect of people's subjective communication tendencies on the propagation of different opinions, Wang [25] proposed a SIDR compartment model, which divided the public into four types: susceptible individuals, irrational individuals, doubters and rational individuals. Mitsutsuji et al. [26] developed a new model of public opinion dynamics to describe the phenomenon that citizen agents have dual attitudes, specifically in terms of attitudes toward war. In addition, some scholars have studied the key role of opinion leaders in opinion evolution. Based on the limited confidence model, Chen et al. [27] analyzed the influence of competitive opinion leaders on attracting followers of social groups from the four characteristics of reputation, stubbornness, attractiveness, and extreme. Zhao et al. [28] divided individuals in the social network into opinion leaders and their followers and established a new dynamic model based on bounded confidence to simulate the opinion evolution between the groups. Research on the evolution of opinions on social networks is conducive to the promotion of positive opinions and the supervision of negative opinions by governments and enterprises. However, the equal possibility of all individual contacts in the uniformly mixed network ignores the influence of the individual contact process and group mixing mode. Using real data to construct an opinion dynamic model based on the complex network is helpful to master the evolution law of opinions in social platforms.

With the deep development of the research on infectious disease models in recent years, some scholars have further studied the process of indirect transmission of pathogens in a polluted environment [29,30]. Richard et al. [31] considered that the main transmission mode of infectious diseases is indirect and mediated by contact with a polluted reservoir. Based on previous works on the research of cholera dynamics, they developed a group of reservoir-mediated SIR models with the infection threshold for pathogen density. Shuai et al. [32] established a general compartmental model for cholera, which incorporated the two ways of transmission, namely direct and indirect in the condition of polluted water. Shu et al. [33] promoted global dynamics from a mathematical perspective, namely, constructing a class of mathematical epidemiological models formulated by systems of differential equations. David et al. [34] considered that susceptible individuals would become infected at some rate whenever they contact with infectious pathogens (indirect transmission), studying the transmission of infectious diseases by constructing and analyzing a class of coupled partial and ordinary differential equations (PDE-ODE). In the dynamics of information transmission, there have been relatively mature studies on the direct transmission model of infectious diseases, but few studies on indirect transmission related to environmental factors. The information transmission mechanism of the new social network is complex, facing many problems such as a large number of social platforms, high mobility of social users and wide spread of social information. Therefore, studying the indirect transmission mechanism of network information in the complex media environment is conducive to improving the current network information ecology.

3. Model description

This paper constructs an opinion network dynamic model, which combines various environmental factors and opinion dynamics on information transmission in the social network. Based on the traditional SFI model [35] developed from the SIR model, we introduce our net-opinion-environment susceptible-forwarding-immune model (NET-OE-SFI), in order to describe the evolution of forwarding quantities of two opinions affected by environmental factors, as shown in Figure 2. Compared with the traditional information transmission dynamic model, the NET-OE-SFI model researches the propagation rules of information on user nodes with different degrees based on genuine data. The model also divides users into forwarding groups with different opinions and studies the direct information transmission on the single platform and the indirect information transmission in the environment.

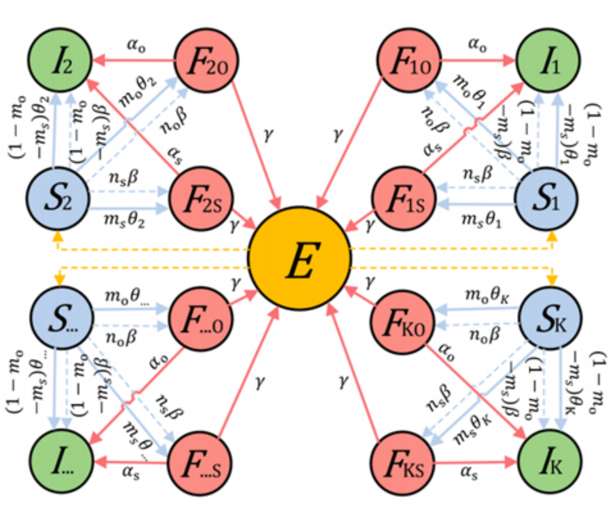

Figure 2. A schematic illustration of information dissemination of different opinions in the network under the multiplatform environment.

We assume that individuals participate in the information dissemination and opinion evolution in a closed and uneven mixed complex social network, where individuals can be represented by vertices, connections between individuals can be represented by edges, and the total number of nodes is N (N remains unchanged). According to the degree k of nodes, the N individuals in the complex social network can be divided into four states:

- susceptible state (S_k): individuals have not yet been exposed to information and may be affected by it
- opposing forwarding state (F_{kO}): individuals have forwarded the information with opposing opinion and have the ability to influence other susceptible individuals to forward the information
- supporting forwarding state (F_{kS}): individuals have forwarded the information with supportive opinion and have the ability to influence other susceptible individuals to forward the information
- immune state (I_k): individuals in the susceptible state or forwarding state (including opposing and

supporting forwarding states) have read or forwarded the information, but no longer forward the information even if receive it again

Therefore, we define F_k as the forwarding state, including the opposing (F_{kO}) and supporting (F_{kS}) forwarding states. The node degree k represents the number of nodes that an individual can contact per unit of time in the social network and the max degree is K. After spreading information on the Chinese Sina Microblog platform, some individuals may browse related information from other social media platforms, resulting in generating individuals (E) in the environment, in which the users have the ability to influence the information propagation.

To sum up, $S_k(t)$, $F_{kO}(t)$, $F_{kS}(t)$, $I_k(t)$ and $E(t)$ denote the numbers of users in respective states at time t. Thus, $S_k(t)+F_{kO}(t)+F_{kS}(t)+I_k(t)+E(t)=N$ and $F_k(t)=F_{kO}(t)+F_{kS}(t)$. Therefore, introducing the parameters in Table 1, our NET-OE-SFI model involving users in the above five states can be characterized by the following system of differential equations:

$$\frac{d}{dt}S_k(t)=-kS_k(t)\theta(t)-\beta S_k(t)E(t) \tag{1}$$

$$\frac{d}{dt}F_{kO}(t)=m_O kS_k(t)\theta(t)+n_O\beta S_k(t)E(t)-\alpha_O F_{kO}(t) \tag{2}$$

$$\frac{d}{dt}F_{kS}(t)=m_S kS_k(t)\theta(t)+n_S\beta S_k(t)E(t)-\alpha_S F_{kS}(t) \tag{3}$$

$$\begin{aligned}\frac{d}{dt}I_k(t)&=(1-m_O-m_S)kS_k(t)\theta(t)+(1-n_O-n_S)\beta S_k(t)E(t)\\&\quad+\alpha_O F_{kO}(t)+\alpha_S F_{kS}(t)\end{aligned} \tag{4}$$

$$\frac{d}{dt}E(t)=\gamma\sum_{k=1}^{K}F_{kO}(t)+\gamma\sum_{k=1}^{K}F_{kS}(t) \tag{5}$$

where $\theta(t)$ represents the connection probability between a susceptible node of degree k and the forwarding nodes F, which can be obtained from Eq (6).

$$\theta(t)=\sum_{l=1}^{K}P(l|k)\,F_{lS}(t)/N_l+\sum_{l=1}^{K}P(l|k)\,F_{lO}(t)/N_l \tag{6}$$

$$P(l|k)=lp(l)/\langle k\rangle \tag{7}$$

$$p(l)=N_l/N \tag{8}$$

$$\langle k\rangle=\sum_{k=1}^{K}kp(k) \tag{9}$$

$F_{lS}(t)$ and $F_{lO}(t)$ represent the numbers of supporting and opposing forwarding nodes with node degree l, respectively. N_l represents the total number of the nodes with degree l. $P(l|k)$ represents the conditional probability of the connection between the node with degree k and the node with degree l. According to the degree distribution characteristics of network nodes, the degree-degree correlations can be written as Eq (7), where $p(l)$ is the probability of nodes with degree l in the network structure and $\langle k\rangle$ is the average degree of the complex network. $p(l)$ can be expressed as the ratio of the total number N_l of nodes with degree l to the total number N of nodes in the complex network, while $\langle k\rangle$ can be calculated by Eq (9). Then, $\theta_S(t)=\sum_{l=1}^{K}P(l|k)\,F_{lS}(t)/N_l$ and $\theta_O(t)=\sum_{l=1}^{K}P(l|k)\,F_{lO}(t)/N_l$ represent the probabilities of connection between a susceptible node of degree k and the supporting or opposing forwarding nodes at time t, respectively. Moreover,

$\theta(t)$, $\theta_S(t)$ and $\theta_O(t)$ can also be represented by Eqs (10)–(12) respectively, which are obtained from Eqs (6), (7) and (9).

$$\theta(t) = \sum_{k=1}^{K} [F_{kS}(t) + F_{kO}(t)] kp(k)/\langle k \rangle N_k \quad (10)$$

$$\theta_S(t) = \sum_{k=1}^{K} F_{kS}(t) kp(k)/\langle k \rangle N_k \quad (11)$$

$$\theta_O(t) = \sum_{k=1}^{K} F_{kO}(t) kp(k)/\langle k \rangle N_k \quad (12)$$

Table 1. NET-OE-SFI model parameters interpretation.

Parameter	Interpretation
k	The degree of a node, which indicates the number of connections between a node and other nodes in the complex network.
m_O	The opposing forwarding rate under the precondition that the susceptible node contact the forwarding nodes, $m_O \in [0,1]$.
m_S	The supporting forwarding rate under the precondition that the susceptible node contact the forwarding nodes, $m_S \in [0,1]$.
n_O	The opposing forwarding rate under the precondition that the susceptible node contact the effect nodes of the environment, $n_O \in [0,1]$.
n_S	The supporting forwarding rate under the precondition that the susceptible node contact the effect nodes of the environment, $n_S \in [0,1]$.
α_O	The average immune rate of opposing forwarding users, i.e., the average rate that an opposing forwarding user becomes inactive after influencing other users, $\frac{1}{\alpha_O} \in [0,48h]$.
α_S	The average immune rate of supporting forwarding users, i.e., the average rate that a supporting forwarding user becomes inactive after influencing other users, $\frac{1}{\alpha_S} \in [0,48h]$.
β	The average exposure rate that a susceptible node can expose to the effect nodes of the environment, $\beta \in [0,1]$.
γ	The ratio of forwarding nodes with environmental impact capability, which can be used to measure the complexity of the environment in the current propagation situation, $\gamma \in [0,1]$.

In our model, a susceptible user can contact an average of k users per unit of time, and the ratio that a susceptible user can contact active forwarding users is $\theta(t)$. Thus, a susceptible user will contact $k\theta(t)$ forwarding users per unit time. In particular, this paper creatively divides forwarding users into two different opinion types according to their text content. Therefore, susceptible users will have two different forwarding directions after accessing to information. After $kS_k(t)\theta(t)$ susceptible users being exposed to messages, $m_O kS_k(t)\theta(t)$ susceptible users will choose to forward the information with an opposing opinion, while $m_S kS_k(t)\theta(t)$ susceptible users will choose to forward the information with a supporting opinion. Similarly, $\beta S_k(t)E(t)$ susceptible users are exposed to the multiplatform environmental individuals $E(t)$ at the same time, so that $n_O\beta S_k(t)E(t)$ susceptible users will choose to forward the information with an opposing opinion and $n_S\beta S_k(t)E(t)$ susceptible users will choose to forward the information with a supporting opinion. Meanwhile, $(1-m_O-m_S)kS_k(t)\theta(t)$ and $(1-n_O-n_S)\beta S_k(t)E(t)$ susceptible users will no longer forward the information, hence becoming immune users, namely transferring from the S_k state to the I_k state after accessing to the information. Besides, $\alpha_O F_{kO}(t)$ opposing forwarding users and $\alpha_S F_{kS}(t)$ supporting forwarding users will become immune users when being out of active time, transferring from the F_k state to the I_k state.

We develop the variable $\Re_0$, which denotes the average number of individuals infected by a patient during the average infection period in the epidemic model [2], namely the basic information propagation reproduction ratio, to judge public opinion will erupt. When $dF_k(0)/dt<0$, namely, the growth rate of the forwarding users at the initial time is less than 0, the public opinion will never burst. Based on this fact, we can obtain the mathematical derivation process of the public opinion reproduction ratio $\Re_0$. Combining Eqs (2) and (3) with $F_k(t)=F_{kO}(t)+F_{kS}(t)$, we can obtain:

$$\frac{d}{dt}F_k(t)=(m_O+m_S)kS_k(t)\theta(t)+(n_O+n_S)\beta S_k(t)E(t)-\alpha_O F_{kO}(t)-\alpha_S F_{kS}(t) \quad (13)$$

Then, multiplying Eq (13) with $kp(k)/\langle k\rangle N_k$, we can obtain:

$$\begin{aligned}\frac{dF_k(t)}{dt}[kp(k)/\langle k\rangle N_k]=[&(m_O+m_S)kS_k(t)\theta(t)+(n_O+n_S)\beta S_k(t)E(t)\\&-\alpha_O F_{kO}(t)-\alpha_S F_{kS}(t)][kp(k)/\langle k\rangle N_k]\end{aligned} \quad (14)$$

Sum over k on both sides of Eq (14) to obtain:

$$\begin{aligned}\sum_{k=1}^{K}\frac{dF_k(t)}{dt}[kp(k)/\langle k\rangle N_k]=\sum_{k=1}^{K}[&(m_O+m_S)kS_k(t)\theta(t)+(n_O+n_S)\beta S_k(t)E(t)\\&-\alpha_O F_{kO}(t)-\alpha_S F_{kS}(t)][kp(k)/\langle k\rangle N_k]\end{aligned} \quad (15)$$

So that we can conclude Eq (16) from Eqs (2), (3), (10) and (15):

$$\sum_{k=1}^{K}\frac{dF_k(t)}{dt}[kp(k)/\langle k\rangle N_k]=\frac{d}{dt}\theta(t) \quad (16)$$

Because $\sum_{k=1}^{K}k^2p(k)=\langle k^2\rangle$, $\sum_{k=1}^{K}kp(k)=\langle \mathrm{k}\rangle$, we can get Eq (17) from Eq (15):

$$\begin{aligned}\frac{d}{dt}\theta(t)=\theta(t)&\frac{(m_O+m_S)}{<k>}\sum_{k=1}^{K}\frac{k^2p(k)S_k(t)}{N_k}+\beta\frac{E(t)(n_O+n_S)}{<k>}\sum_{k=1}^{K}\frac{kp(k)S_k(t)}{N_k}\\&-[\alpha_O\sum_{k=1}^{K}F_{kO}(t)kp(k)/\langle k\rangle N_k+\alpha_S\sum_{k=1}^{K}F_{kS}(t)kp(k)/\langle k\rangle N_k]\end{aligned} \quad (17)$$

Substitute Eqs (11) and (12) into Eq (17):

$$\begin{aligned}\frac{d}{dt}\theta(t)=\theta(t)&\frac{(m_O+m_S)}{<k>}\sum_{k=1}^{K}\frac{k^2p(k)S_k(t)}{N_k}+\beta\frac{E(t)(n_O+n_S)}{<k>}\sum_{k=1}^{K}\frac{kp(k)S_k(t)}{N_k}\\&-[\alpha_O\theta_O(t)+\alpha_S\theta_S(t)]\end{aligned} \quad (18)$$

Only when $\frac{d}{dt}F_k(0)=\frac{d}{dt}F_k(t)|_{t=0}>0$, can the public opinion will burst in social network. Because $kp(k)/\langle k\rangle N_k>0$, we can educe that $\frac{d}{dt}\theta(t)|_{t=0}>0$ according to Eq (16). When $t=$

0 , $S_0 = S_k(0) = N_k$. Defining $\theta_0 = \theta(t)|_{t=0}$, $E_0 = E(t)|_{t=0}$, $\theta_{O0} = \theta_O(t)|_{t=0}$ and $\theta_{S0} = \theta_S(t)|_{t=0}$, we obtain:

$$\frac{\theta_0(m_O+m_S)\langle k^2\rangle}{<k>} + (n_O + n_S)\beta E_0 > \alpha_O\theta_{O0} + \alpha_S\theta_{S0} \tag{19}$$

Divide $(\alpha_O\theta_{O0} + \alpha_S\theta_{S0})$ both sides of Eq (19) simultaneously:

$$\frac{\theta_0(m_O+m_S)\langle k^2\rangle}{<k>(\alpha_O\theta_{O0}+\alpha_S\theta_{S0})} + \frac{(n_O+n_S)\beta E_0}{\alpha_O\theta_{O0}+\alpha_S\theta_{S0}} > 1 \tag{20}$$

Therefore, we define the left side of Eq (20) as the public opinion reproduction ratio $\mathfrak{R}_0$:

$$\mathfrak{R}_0 = \frac{\theta_0(m_O+m_S)\langle k^2\rangle}{<k>(\alpha_O\theta_{O0}+\alpha_S\theta_{S0})} + \frac{(n_O+n_S)\beta E_0}{\alpha_O\theta_{O0}+\alpha_S\theta_{S0}} \tag{21}$$

When $\mathfrak{R}_0 > 1$, the number of forwarding users will increase exponentially and only in this case can public opinion burst in the network. Apparently, the larger the value of $\mathfrak{R}_0$ is, the faster the speed of the outbreak will be.

4. Numerical implementation

In order to analyze the effectiveness and rationality of the net-opinion-environment susceptible-forwarding-immune model of this paper, we select the genuine data as a research case in this section. The model parameters are estimated and fitted with genuine data to verify the model performance. We selected the information dissemination process of "Chinese Union Medical College Hospital has found that skipping dinner is good for metabolic health" on Chinese Sina Microblog as the case of this paper and we obtained users' forwarding time data and text content (shown in Figure 3) from application program interface (API), aiming to estimate the model parameters, the initial susceptible population S_0 and the initial cross-platform population of the environment E_0.

This is a message posted by state media on Sina Microblog at 10:31 on March 13, 2022. Analyzing the collected data, we found that the information did not cause a wide range of transmission within 12 hours after the release. And up to 22:26 of the day, the cumulative number of forwarding is only 303 times. Subsequently, the microblog broke out with more than 200 forwarding times every ten minutes and rapidly became a trending topic on the Chinese Sina Microblog. This information has aroused intense discussion among users of social platforms and the polarization of users' opinions was severe. Many users believe that there is a saying in Buddhism that "no food after noon" and they argue that eating before going to bed is not conducive to digestion, so they hold supporting opinions. However, some users believe that not eating on time is not conducive to their health. Therefore, the spread of this information has the polarization characteristics of users' views. Furthermore, after 22:00 every night, people have sufficient time to contact this information on other social platforms, which is highly likely to lead to a cross-platform population in the environment of the information propagation. To summarize, this event is suitable for the NET-OE-SFI model in this paper.

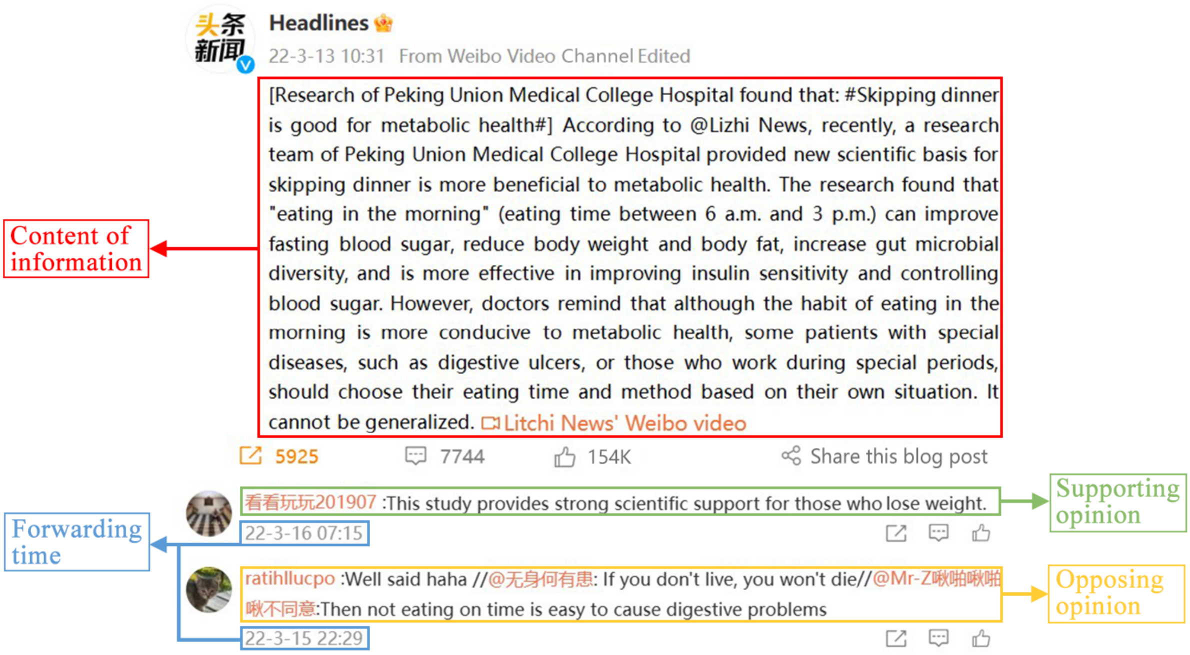

Figure 3. The forwarding structure and the genuine information example on the Chinese Sina Microblog.

Table 2. The cumulative numbers of forwarding users with different opinions in the case.

$t(10\,min)$	0	1	2	3	4	5	6	7	8	9
C_{Opp}	79	112	172	257	347	416	502	562	624	676
C_{Sup}	224	322	467	670	894	1083	1277	1475	1637	1770
$t(10\,min)$	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
C_{Opp}	723	768	794	826	850	871	881	891	903	912
C_{Sup}	1889	2006	2139	2221	2305	2389	2448	2486	2515	2549
$t(10\,min)$	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
C_{Opp}	917	923	927	933	939	943	947	951	956	959
C_{Sup}	2573	2588	2604	2622	2635	2649	2658	2672	2687	2693
$t(10\,min)$	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
C_{Opp}	959	960	960	962	964	964	965	965	968	968
C_{Sup}	2701	2708	2712	2717	2722	2727	2731	2736	2738	2740
$t(10\,min)$	**40**	**41**	**42**	**43**	**44**					
C_{Opp}	970	973	973	974	974					
C_{Sup}	2742	2743	2743	2743	2743					

For this incident, we collected the forwarding text content and forwarding time from Chinese Sina Microblog as shown in Figure 3. According to the opinion information of the forwarding text, we divided the data into opposing and supporting opinions. Because of the particularity of the outbreak time of this event, we don't have to filter the raw data to avoid the interference from information stagnation period due to physiological needs. After these pretreatment processes, we obtained the accurate forwarding time and corresponding copywriting of the supporting message and opposing message. The above results were used to calculate the cumulative numbers of forwarding users ($C_{Sup}(t)$, $C_{Opp}(t)$) as shown in Table 2. Here, we set the beginning time to 0 and the sampling interval to ten minutes.

In order to fit our model with the real data collected from the Chinese Sina Microblog, we use the LS method to estimate the model parameters, the initial susceptible population S_0, and the initial cross-platform population of the environment E_0. The parameter vector can be set as $\Theta = (\alpha_O, \alpha_S, m_O, m_S, S_0, N, K, n_O, n_S, \gamma, \beta, E_0)$ and according to the parameter vector, the corresponding numerical calculation for $C_{Opp}(t)$ and $C_{Sup}(t)$ are denoted by $f_{C_{Opp}}(i, \Theta)$ and $f_{C_{Sup}}(i, \Theta)$, respectively.

The LS error function

$$LS = \sum_{i=0}^{T} \left| f_{C_{Opp}}(i, \Theta) - C_{Opp\_i} \right|^2 + \sum_{i=0}^{T} \left| f_{C_{Sup}}(i, \Theta) - C_{Sup\_i} \right|^2 \tag{22}$$

is used in our calculation, where C_{Opp_i} and C_{Sup_i} denote the actual cumulative numbers of the forwarding users with opposing and supporting opinion given in Table 2 and $i = 0, 1, 2, \dots T$ is the sampling time. To discover the optimal data fitting, Θ is set to minimize the Eq (22).

As shown in Figure 4, we perform data fitting of the genuine data given in Table 2 and the fitting curve is approximately consistent with the actual value. We use curves of different shapes to display the actual cumulative forwarding quantities and the predicted cumulative forwarding quantities by the model, which consists of opposing and supporting opinions. As we can see in Figure 4, users with supporting opinions play a key role in the information propagation of this case, including that the initial propagation speed of supporting opinions is faster than opposing opinion, and the outbreak of the supporting messages lasts for a longer time. In contrast, the cumulative number of forwarding users with opposing opinions goes to a steady state earlier. The fitting curves of the two opinions in Figure 4 closely coincide with the actual values, which fully verifies the feasibility and accuracy of our NET-OE-SFI model.

Table 3 has shown the estimated results of our NET-OE-SFI model parameters. We can conclude from the table that the average immune rate of opposing forwarding users α_O is greater than that of supporting forwarding users α_S. In other words, more users change from the opposing forwarding state F_{kO} into the immune state I_k. The supporting forwarding rate under the precondition that the susceptible node contact the forwarding nodes m_S and the supporting forwarding rate under the precondition that the susceptible node contact the effect nodes of environment n_S are both greater than those of opposing forwarding rates, which indicates that more users change from the susceptible state S_k into the supporting forwarding state F_{kS}. This is also consistent with the trends of the cumulative numbers of forwarding users shown in Figure 4.

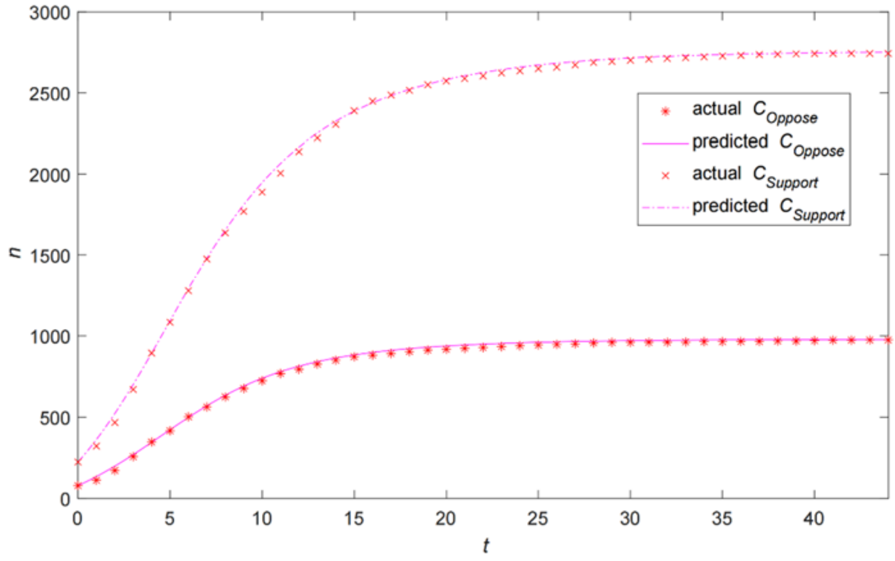

Figure 4. Numerical data fitting results of the case.

Table 3. The estimated results of our NET-OE-SFI model parameters in the case.

α_O	α_S	m_O	m_S	S_0	N
0.09843	0.09516	0.10256	0.23662	9266	21,510
K	n_O	n_S	γ	β	E_0
51	0.07790	0.40157	0.00249	0.00142	154

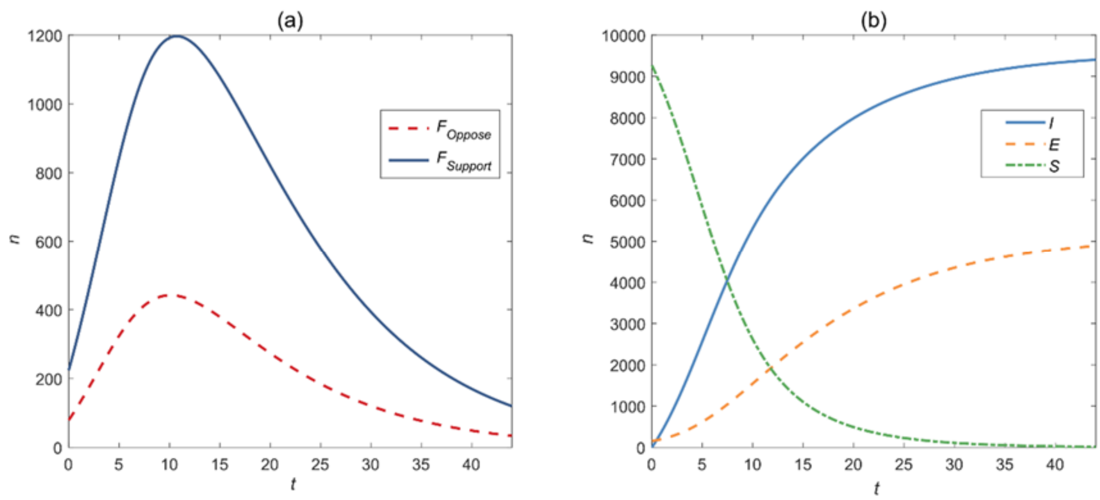

Figure 5. The change results of users with different states in the case.

Figure 5(a),(b) shows the change process of the total number of people in the event, in which F_{Oppose}, $F_{Support}$, I, E and S are represented by different curves. Here, the people of the same order of magnitude are shown in Figure 5(a). Both the two types of forwarding curves show an overall upward trend at the beginning of the outbreak of public opinion. When public opinion tends to be gentle, the number of forwarding individuals newly added gradually decreases. Therefore, the changes of the two curves in Figure 5(a) both show a bell shape. The difference is, $F_{Support}$ plays a more important role in the information dissemination process of the event: "Skipping dinner is good for your metabolic health." Considering that all groups will be gradually immune to the event, the curve of overall immune groups always shows an upward trend. Accordingly, the curve of the total number of susceptible groups shows a downward trend as a whole.

E groups represent some individuals who will browse the related information from other social platforms and have an environmental impact. At the beginning of the outbreak of public opinion, the initial number of E groups E_0 stands for the potential individuals in the susceptible state who will browse other platforms at the same time. We can draw from Table 3 that the value of E_0 is 154, which is much less than $\mathrm{S}_0 = 9266$. However, with the spread of information, more and more forwarding users begin to browse the relevant information on other platforms after forwarding the information. The individuals in forwarding states are transformed into E individuals in proportion $\gamma = 0.00249$, so the individuals with spatial overlap in the Sina Microblog social network will further promote the dissemination of information.

5. Sensitivity analysis and discussion

To further analyze how the different parameters of our NET-OE-SFI model play a role in the information propagation process, we conduct parameter sensitivity analysis using the partial rank correlation coefficients (PRCCs) method. PRCCs is a significant method for the comprehensive analysis of parameter sensitivity, which carries out repetitive experiments within the parameter boundary range through 1000 groups of samples and finally gives the average parameter sensitivity results.

Figure 6 shows how the values of $\Re_0$, $F_{Opp(max)}$ and $F_{Sup(max)}$ are affected by the model parameters. According to the PRCCs histogram, the forwarding rates (m_O and m_S) under the precondition that the susceptible node contact the forwarding nodes, the initial value of the susceptible users S_0, the max value of degree K, the opposing forwarding rate n_S under the precondition that the susceptible node contact the effect nodes of environment, the proportion γ of forwarding nodes with environment information dissemination capability, the average exposure rate β, the initial value E_0 of E groups have a positive effect on $\Re_0$, in which m_O, m_S, K have a strong influence and S_0, K, n_S, γ, β, E_0 have a weak influence. As for the average immune rates (α_O and α_S) with different opinions have a strong negative effect on $\Re_0$. Thus, it can be seen that, increasing the immune rate in the process of information dissemination, that is to say, accelerating transformation from the forwarding state to the immune state will help suppress the outbreak of public opinion.

In order to better study the differences of opinions in the information dissemination process, we research the influence of model parameters on the maximum number of current forwarding users F_{max} with different opinions ($F_{Opp(max)}$, $F_{Sup(max)}$). In Figure 6, the parameter α_O has a strong negative effect on $F_{Opp(max)}$, while α_S has a weak negative effect on it. The corresponding is α_S has a strong negative influence on $F_{Sup(max)}$, while α_O has a weak negative influence on it. The same conclusion can be applied to m_O and m_S. For example, m_O has a strong positive effect on $F_{Opp(max)}$, while α_S has a weak positive effect on it. For the maximum number of current forwarding users F_{max}, the parameters of the same opinion have greater influence than those of the opposite opinion.

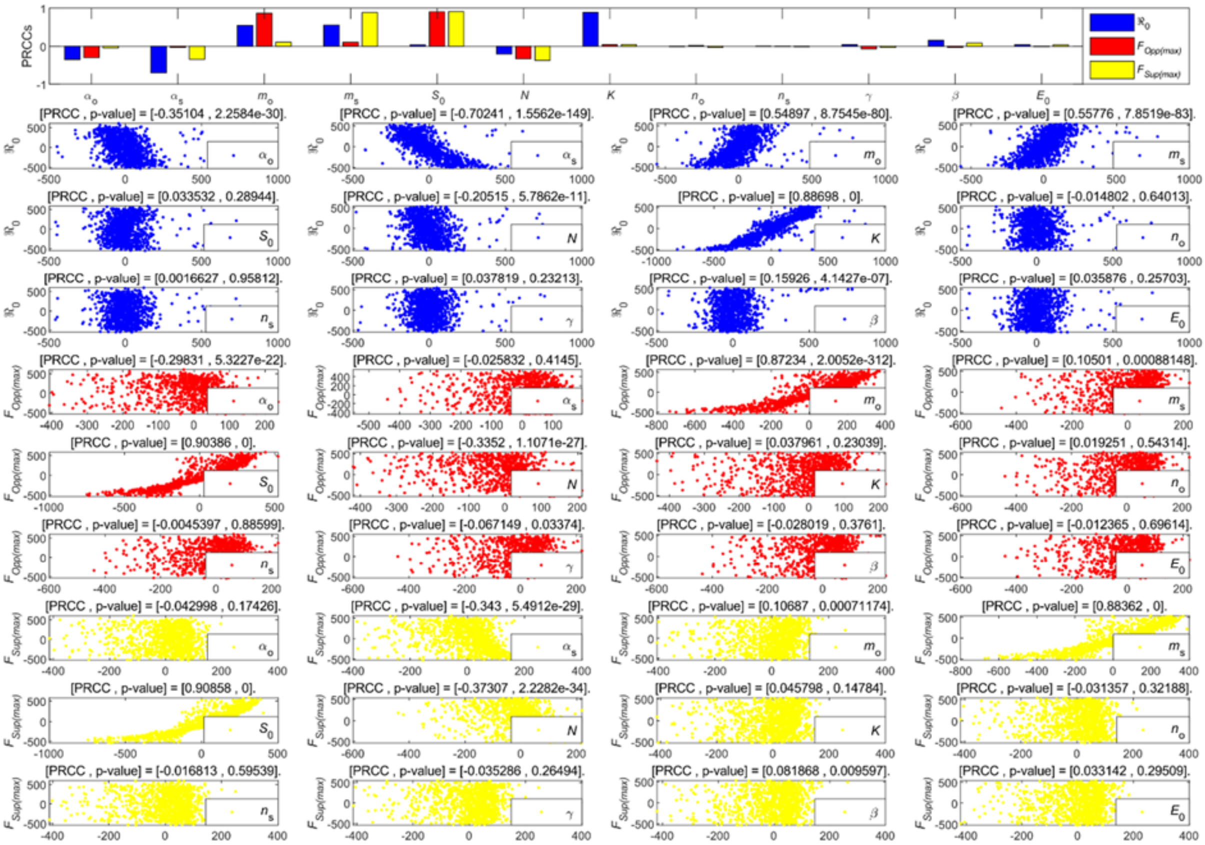

Figure 6. PRCC results and PRCC scatter plots with the model parameters of $\Re_0$, $F_{Opp(max)}$, $F_{Sup(max)}$.

Considering the effects of n_O, n_S, γ, β and E_0 are not obvious compared with other parameters in Figure 6, we conduct analysis on them by group separately. Figures 7–10 show the effects of five parameters on values of the maximum numbers of current forwarding users with different opinions $F_{Opp(max)}$, $F_{Sup(max)}$ and the stable numbers of the cumulative forwarding users with different opinions $C_{Opp(s)}$, $C_{Sup(s)}$ respectively. The four figures show that, on one hand, the conclusions of the maximum current and stable cumulative numbers of forwarding users are highly similar. On the other hand, there are also specific patterns of conclusions between information with different opinions.

As can be seen from Figure 7, the increase of γ is always going to obviously promote $F_{Opp(max)}$, $F_{Sup(max)}$. The increase of n_O will obviously promote $F_{Opp(max)}$ and the increase of n_S has little impact on $F_{Opp(max)}$. However, the impacts on $F_{Sup(max)}$ are contrary. In Figure 9, as β increases within $(0,0.2)$, $F_{Opp(max)}$ will increase gradually but E_0 has little influence on it. On the contrary, $F_{Sup(max)}$ will decrease in the same range. In response to the stable numbers of cumulative forwarding users, the conclusions of parameter sensitivity analysis are similar to the corresponding maximum numbers of current forwarding users, as shown in Figures 8 and 10. For example, the conclusions of $C_{Opp(s)}$ are similar to those of $F_{Opp(max)}$.

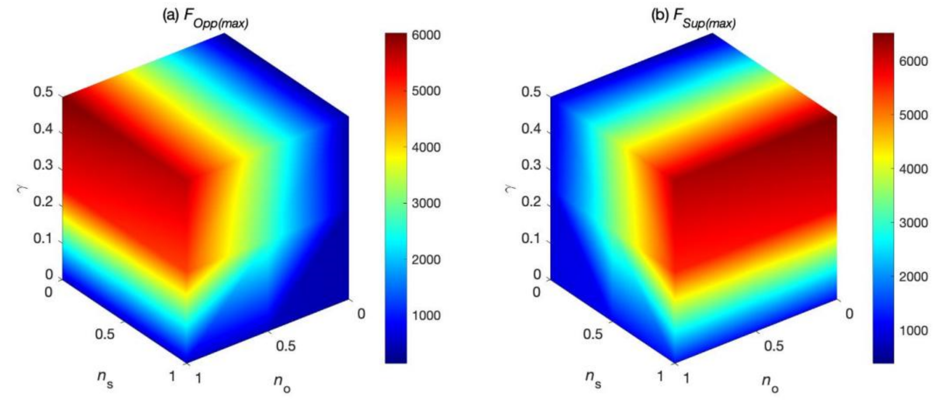

Figure 7. The comprehensive influence of multiple parameter variations on $F_{Opp(max)}$ and $F_{Sup(max)}$: (a) γ, n_O and n_S change for $F_{Opp(max)}$; (b) γ, n_O and n_S change for $F_{Sup(max)}$.

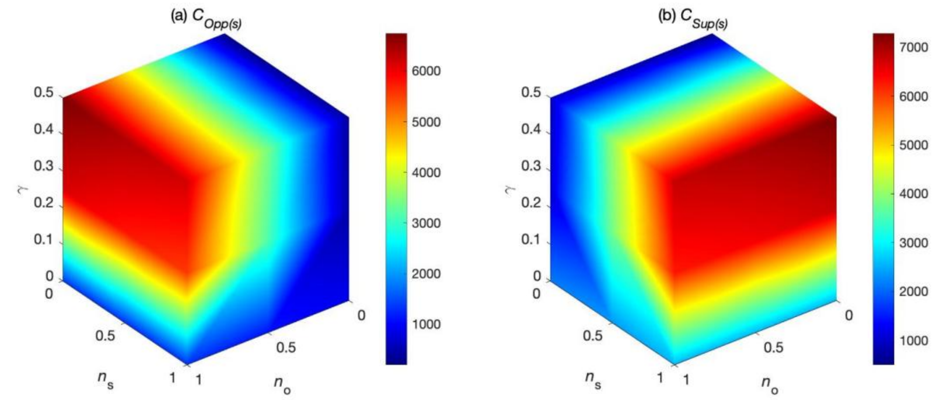

Figure 8. The comprehensive influence of multiple parameter variations on $C_{Opp(s)}$ and $C_{Sup(s)}$: (a) γ, n_O and n_S change for $C_{Opp(s)}$; (b) γ, n_O and n_S change for $C_{Sup(s)}$.

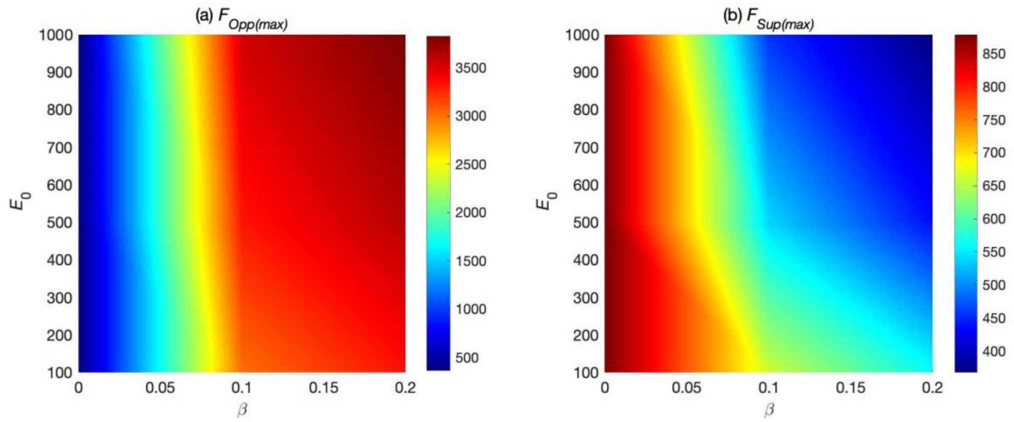

Figure 9. The comprehensive influence of multiple parameter variations on $F_{Opp(max)}$ and $F_{Sup(max)}$: (a) E_0 and β change for $F_{Opp(max)}$; (b) E_0 and β change for $F_{Sup(max)}$.

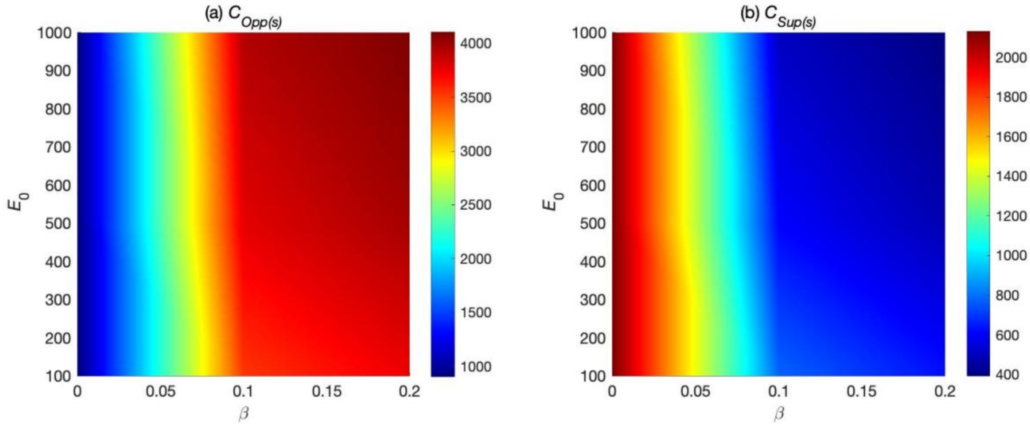

Figure 10. The comprehensive influence of multiple parameter variations on $C_{Opp(s)}$ and $C_{Sup(s)}$: (a) E_0 and β change for $C_{Opp(s)}$; (b) E_0 and β change for $C_{Sup(s)}$.

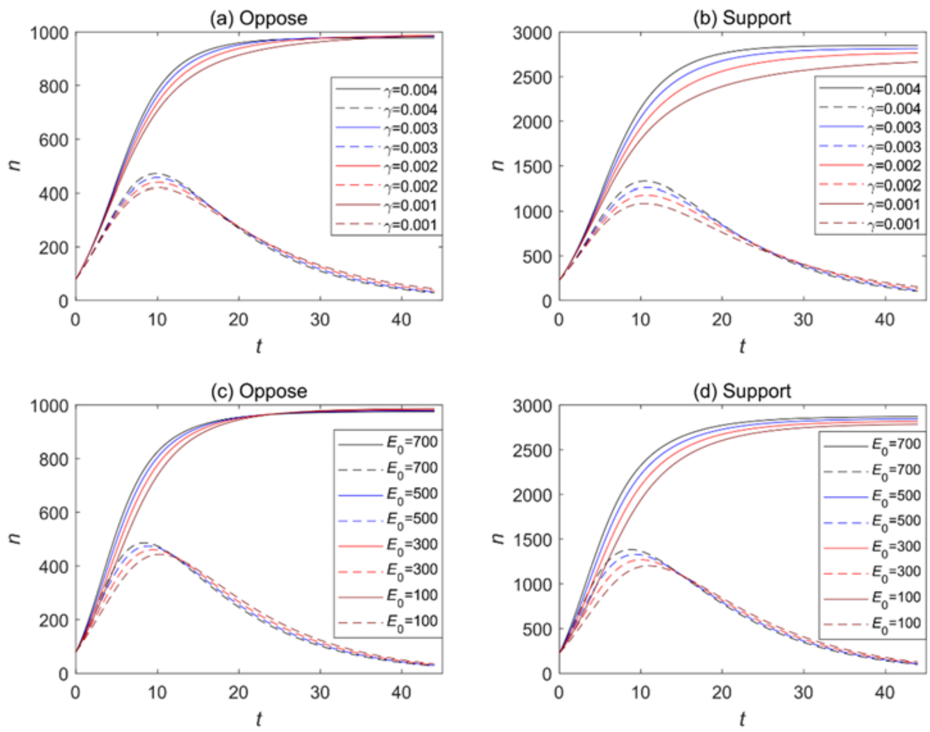

Figure 11. The influences of key environment-related parameters on $F(t)$ and $C(t)$: (a) only γ changes in the opposing opinion dissemination; (b) only γ changes in the supporting opinion dissemination; (c) only E_0 changes in the opposing opinion dissemination; (d) only E_0 changes in the supporting opinion dissemination.

To analyze the effects of the environment-related factors on the current and cumulative number of forwarding users with different opinions, we change γ and E_0 respectively in Figure 11, where the full lines denote the change of the cumulative number of forwarding users and the dotted lines denote the change of the current number of forwarding users. Figure 11(a),(b) show that at the outbreak stage, when the ratio of forwarding nodes with environmental impact capability (γ) decreases, both $F(t)$ and $C(t)$ decrease with it. Figure 11(c),(d) show that when the initial value of E nodes (E_0)

decreases, $F(t)$ and $C(t)$ decrease with it in a period of time. Take Figure 11(c) as an example, with the decrease of E_0, $F_{Opp}(t)$ and $C_{Opp}(t)$ decrease first but after a period of time, $F_{Opp}(t)$ and $C_{Opp}(t)$ are negatively related to E_0. Figure 11 fully shows that indirect dissemination process affected by the complex media environment plays an important role in the diffusion of public opinion. Consequently, to control public opinion, social platforms can choose to intervene in cross-platform information forwarding by formulating relevant public opinion strategies.

6. Conclusions

Public opinion is a concentrated expression of public attitudes, emotions, opinions and views. Research on the propagation laws of public opinion is helpful to give the strategy guidance of public opinion evaluation. Based on the complex network, this paper constructs an opinion dynamic model combining opinion and information dissemination, which also considers the indirect dissemination of information under the influence of the media environment. We use real data to carry out data fitting and parameter sensitivity analysis on the model to provide a significant theoretical basis for network public opinion analysis. The model proposed in this paper not only improves the disadvantage of the traditional dynamics model that ignores the different contact probabilities between user nodes, but also simulates the model well based on the real social platform data. In addition, this model also introduces the concept of environment in epidemiology into the field of information transmission, which can research the laws of opinion transmission more comprehensively.

The experimental results of parameter sensitivity analysis show that the forwarding rates (m_O and m_S) under the precondition that the susceptible nodes contact the forwarding nodes and the max value of degree K have a strong influence on the outbreak of public opinion. In the information dissemination network, different network structures have diverse effects on information dissemination. In order to study the differences of opinions in the information dissemination process better, we investigate the influence of model parameters on the variables about forwarding users with different opinions. Moreover, the indirect propagation path affected by the complex media environment also plays a certain role in promoting information dissemination. We hope that our NET-OE-SFI model can promote the development of research on opinions dissemination of network information under the influence of a complex media environment and provide control strategies support for the dissemination of opinions on the Internet from the perspective of a mathematical model.

Data availability

The raw data not involving users' privacy issues of the case in this paper is available to download from https://github.com/wwwangjinxia/NET-OE-SFI, which is collected from the Chinese Sina Microblog using the application program interface (API).

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest. Jianhong Wu is an editorial board member for Mathematical Biosciences and Engineering and was not involved in the editorial review or the decision to publish this article.

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