

MBE, 18(5): 6806–6818. DOI: 10.3934/mbe.2021338 Received: 27 May 2021 Accepted: 28 July 2021 Published: 13 August 2021

http://www.aimspress.com/journal/MBE

Research article

Total, average and marginal rates of basal heat production during

human growth

Michael R. Murphy^{1,*} and Bruce M. Hannon²

- ¹ Department of Animal Sciences and Division of Nutritional Sciences, University of Illinois at Urbana-Champaign, USA
- ² Department of Geography, University of Illinois at Urbana-Champaign, USA

* Correspondence: Email: mrmurphy@illinois.edu; Tel: +2173330093; Fax: +217333-5044.

Abstract: Our goal was to examine how total, average (heat production rate per unit mass) and marginal (the increase in the heat production rate per unit increase in mass) rates of basal heat production changed as mass increased in growing humans. Specifically, our hypotheses were that the marginal basal heat production rate did not decrease monotonically as humans grew; and that an energetically optimal mass, one at which the average basal heat production rate of a growing human was minimal, existed. Marginal rates of heat production were estimated and six potential models to describe the effect of mass during human growth on basal heat production rate were evaluated using a large, meticulously curated, dataset from the literature. Marginal rates of heat production were quadratically related to body mass during growth; they declined initially, reached a minimum, and then increased. This suggested that the relationship between basal heat production rate and mass was cubic. Of the six potential models evaluated, a three-parameter cubic polynomial best described the data. Marginal rates of heat production were minimal for 56-kg females and 62-kg males. Basal heat production rates per unit mass of a growing human were minimal (i.e., energetically optimal) for 83-kg females and 93-kg males; the average masses of U.S. adults have been increasing and approaching these optima over the last 60 yr.

Keywords: body mass; energy metabolism; heat production rate; basal metabolism; optimal size

1. Introduction

The relationship between basal heat production rates associated with respiration (R, megajoules per day) and a human's body mass (M, kilograms) has been an important topic of research for over 100 years [1]. This focus on intraspecies effects has resulted in various, often linear, functions being employed piecemeal to describe the effect of growing human M on R for each sex [2–5].

Across species, variation in the energy requirements for maintenance of mature animals at M equilibrium has been described as being directly proportional to metabolic body size [6,7], usually defined as $M^{\frac{3}{4}}$ power. Unfortunately, this relationship has also been applied to growing animals within a species [8,9]; implying that their average (R/M) and marginal (dR/dM) rates of heat production can only decrease with growth regardless of how large the animal becomes. In a previous study [10] we used a simple empirical function (the cubic $aM^3 + bM^2 + cM$) to test whether animals eventually grew large enough that the heat production rate for their next unit of growth was larger than the previous one. Our results indicated that both R/M and dR/dM declined initially but then increased with continued growth of fish, rats, chickens, goats, sheep, swine, cattle, and horses [10]. We ascribed this phenomenon to increased size and metabolic activity of organs supporting the physiological state of growth, primarily the liver and gastrointestinal tract, and changes in body composition as animals grew [10].

How R/M and dR/dM change as M increases in growing humans is also relevant to understanding the biological processes involved. The latter seems to have been only addressed indirectly; whereas, the former has occasionally been explicitly discussed [1,2,11]. Current models agree that R increases, but suggest that both R/M and dR/dM only decrease, as M increases during human growth. In contrast, Hannon [12] developed theory indicating that R/M should reach a minimum as an individual grows. The d(R/M)/dM = 0 and dR/dM = R/M at that M, which would also be energetically optimal because a growing human of that size would require the least R per unit of M.

Benedict [1] averred that there was "no possibility of finding a general relationship" between R/M and M in data for 77 women and 97 men. Holliday et al. [2] concluded that R/M only decreased as humans grew. More recently, Prentice et al. [11] reported that R/M "was similar for lean and obese women".

Our objectives were: to determine whether growing humans represented an exception to our theory and data for the 8 other species [10,12]; and, more generally, to examine how R, R/M, and dR/dM changed as M increased in growing humans.

2. Materials and methods

2.1. Description of compiled human data

Human basal metabolism data for both females (n = 2364) and males (n = 4811) were from the meticulously curated dataset summarized in Appendix 3 of Schofield [3]. Schofield [13] provided an annotated bibliography for studies included in Schofield [3]. Individual observations are not available but Schofield [3] tabulated means for approximately 1-kg *M* intervals. Mean *M* ranged from 2 to 96 kg for females and from 3 to 109 kg for males. These means exhibited a bimodal frequency distribution and were each based on 1 to 81 observations for females and 1 to 189 for males (Figure 1).

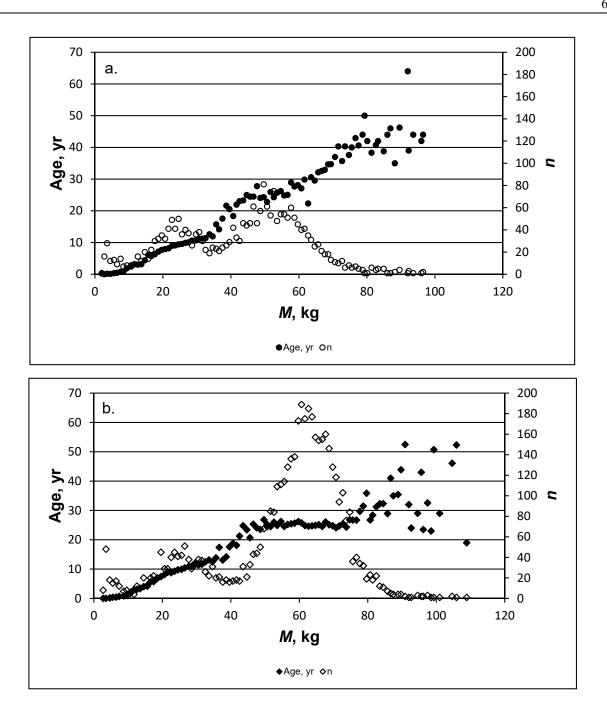


Figure 1. Compiled age (years), observations per mean (*n*) and body mass (*M*, kilograms) data from Appendix 3 of Schofield [3]. The simple correlation coefficients between age and *M* were 0.975 (P = 0.001) for females and 0.904 (P = 0.001) for males, indicating that growth (i.e., *M* increasing with age) was essentially continuous for the average person. a. Females (total n = 2364); b. Males (total n = 4811).

Compiled age (years) and M data of Schofield [3] are also in Figure 1. The simple correlation coefficients between M and age were 0.975 (P = 0.001) for females and 0.904 (P = 0.001) for males. These indicated that growth (i.e., M increasing with age) was essentially continuous for the average person, a conclusion based on the fact that each sample mean represents a point estimate of the population mean.

Height was also closely correlated with both M [0.844 (P = 0.001) for females and 0.891 (P = 0.001) for males] and age [0.811 (P = 0.001) for females and 0.847 (P = 0.001) for males] in these data [3]. This multicollinearity among M, age, and height indicated that these variables contained significant redundant information. The variance inflation factors ranged from 3 to 20 and make their simultaneous inclusion in regression models, as some have done [3,4,14,15], problematic. Indeed, inclusion of height in addition to M in the regression model did not improve prediction of R significantly [4].

2.2. Estimation of dR/dM and dM/dt

Differences between means in Appendix 3 of Schofield [3] were utilized to estimate dR/dM as M increased. A 3-point central difference formula [16], modified because the reported mean M were not equally spaced, was used to estimate dR/dM as $(R_{i+1} - R_{i-1})/(M_{i+1} - M_{i-1})$ at an M_i of $(M_{i+1} + M_{i-1})/2$. Although unbiased, individual estimates of derivatives using this technique are usually quite variable [17]; therefore, a 3-point median filter was applied to help distinguish between signal and noise [18]. We did this to determine if an inflection point existed for R versus M in the data of Schofield [3]. Existence of such a point would verify that, although dR/dM decreased initially as M increased, dR/dM eventually began to rise; i.e., it was not monotonic. A cubic or similar function would then be required to describe R versus M in a growing human.

Similarly, dM/dt was estimated from differences between means in the data [3]; this was done to ascertain how growth status may have changed as M increased.

2.3. Evaluation of potential models to describe the relationship between R and M

Standard *n*-weighted least-squares regression [19] was used to evaluate the ability of six potential models [linear, quadratic, cubic, and quartic polynomials; and two power (or allometric) functions] to describe the overall relationship between *R* and *M* in data for growing human females and males [3]. Intercepts were not included for the polynomials because *R* must be 0 when *M* is 0. Power functions were of the form $R = uM^{\nu}$. In one case ν was 0.75 to test scaling to metabolic body size; whereas, ν was best-fit to the data in another. The approach is statistically equivalent to conducting similar analyses using the original, but now unavailable, datasets.

Goodness-of-fit statistics included adjusted coefficients of determination, and Akiake and Bayesian information criteria [20,21]. These techniques consider, but compensate differently for, the fact that the number of estimated parameters differed among models.

2.4. Orthogonal regression estimates of cubic model parameters for R vs. M

Because compiled data [3] violated the critical assumption of standard least-squares regression that error is only present in the dependent variable [22], orthogonal nonlinear (*n*-weighted) least-squares regression [23] in the program R version 3.4.1 was also used to estimate coefficients of our cubic function.

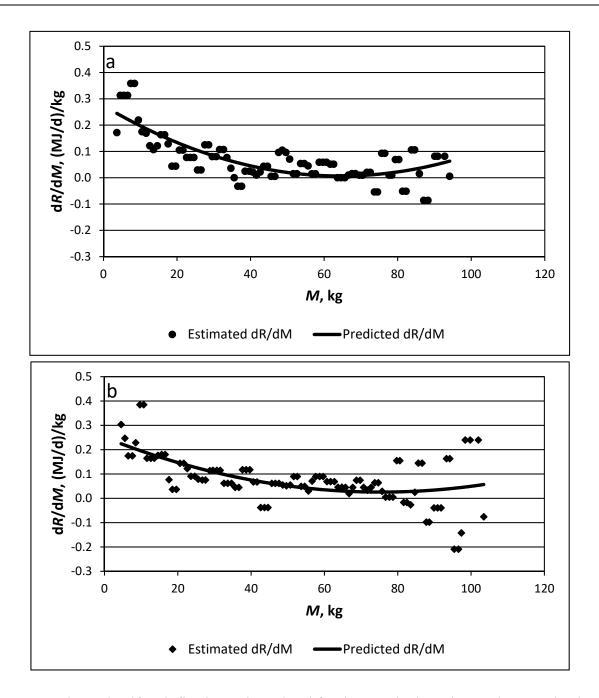


Figure 2. To determine if an inflection point existed for the marginal respiratory heat production rate (dR/dM, megajoules per day per kilogram) in the data of Schofield [3], differences between means (*M*, kilograms; *R*, megajoules per day) were utilized to estimate dR/dM as *M* increased. Although unbiased, individual estimates of derivatives using this technique are usually quite variable [17]; therefore, a 3-point median filter was applied to help distinguish between signal and noise [18]. A quadratic relationship existed between dR/dM and *M*. All coefficients differed from 0 for both females (*P* = 0.00001) and males (*P* = 0.00091) [$dR/dM = 6.419E-5*M^2 - 8.281E-3*M + 2.740E-1$, $dR/dM = 3.952E-5*M^2 - 5.962E-3*M + 2.506E-1$; SE of the estimates were 9.704E-6, 9.578E-4, 2.007E-2, 1.154E-5, 1.258E-3, and 2.917E-2; and the coefficients of determination were 0.603 and 0.337, respectively]. This means that the relationship between *R* and *M*, the integral of dR/dM, was cubic. a. Females; b. Males.

3. Results and discussion

3.1. dR/dM vs. M

For growing humans, a quadratic relationship existed between dR/dM and M (Figure 2). All coefficients differed from 0 for both females (P = 0.00001) and males (P = 0.00091) [dR/dM = 6.419E $-5*M^2 - 8.281E-3*M + 2.740E-1$, $dR/dM = 3.952E-5*M^2 - 5.962E-3*M + 2.506E-1$; SE of the estimates were 9.704E-6, 9.578E-4, 2.007E-2, 1.154E-5, 1.258E-3, and 2.917E-2; and the coefficients of determination were 0.603 and 0.337, respectively]. These results indicated that dR/dM decreased initially as human grew, reached a minimum, and then increased with continued growth; i.e., dR/dM was not monotonic. They also demonstrated that the relationship between R and M in growing humans, the integral of dR/dM, was cubic.

3.2. R vs. M

Parameter estimates for all models differed from 0, P = 0.00001 (Table 1). All goodness-of-fit criteria ranked the models in a consistent order for both females and males. Best to worst was: quartic, cubic, best-fit power, quadratic, power with an exponent of 0.75, and linear. A linear model has constant slope (R/M); however, Schofield [3] needed six line segments with initially decreasing slopes to describe adequately the overall relationship between R and M. This means that a single line function cannot explain satisfactorily the relationship between R and M in humans.

For the quadratic model fit to the datasets [3], predicted R peaked at an M of 64 kg for females and 80 kg for males. For the quartic equation, predicted R peaked at an M of 88 kg for females and 96 kg for males. Within a species, and for a given sex and age, larger animals are expected to have higher rates of heat production than smaller animals in the same physiological state. Quadratic and quartic relationships between R and M, then, are physiologically unrealistic.

Among remaining models, the cubic was superior to both power functions. Power functions with an exponent < 1 also predict ever-increasing efficiency of energy use as an individual grows, an unphysiological result. Our conclusion was that a three-parameter cubic function best described the overall relationship between *R* and *M* in a growing human (Figure 3; $R = aM^3 + bM^2 + cM$). The cubic explained 97% to 98% of variation in *R* across the full spectrum of human growth in the data of Schofield [3] (Table 1). Given that compiled experimental data must include some degree of random variation, consideration of additional variables was apparently not required. Mean absolute prediction errors for the cubic equation fit to these data were 4.0% for females and 1.5% for males. These findings were consistent with our previous results for 8 other animal species [10] and several of their implications will now be explored.

Both R/M ($aM^2 + bM + c$) and dR/dM ($3aM^2 + 2bM + c$) are quadratically related to M when the relationship between R and M has cubic form. Minimum dR/dM occurs at 2/3 of the M at which R/M is minimal; the R/M minimum is at M = -b/(2a). Marginal heat production rates (dR/dM) were always positive as M increased, but minimal for 56-kg human females and 62-kg human males. A consistently positive value for dR/dM was expected and indicated that R always increased as M increased. The energetically optimal M, where the average heat production rate (R/M) was minimal and equal to the marginal heat production rate (dR/dM), was 83.4(\pm 0.5) kg for human females and 92.8(\pm 0.7) kg for human males.

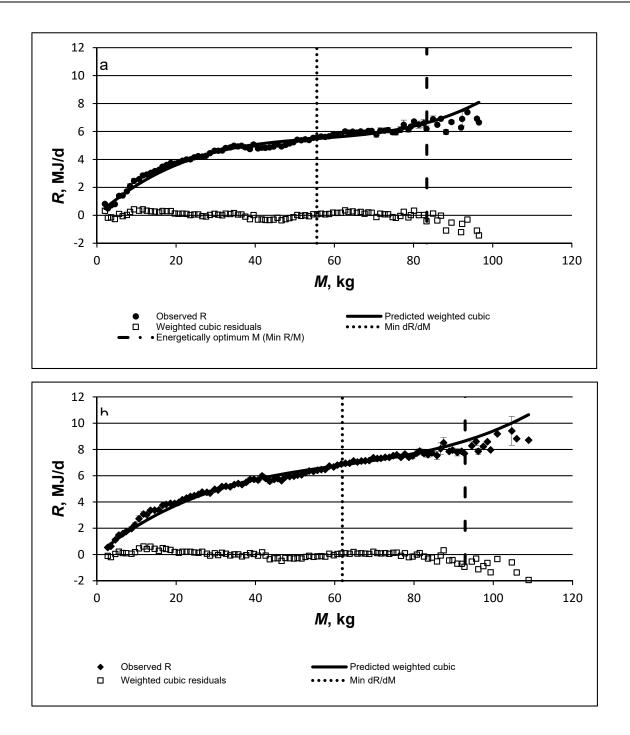


Figure 3. Predicted cubic regression equation (weighted, solid line) describing basal metabolism (heat production rate, R in megajoules per day) versus body mass (M, kilograms) for the data of Schofield [3]. Closed circles or diamonds denote observed means and open squares indicate weighted residuals. Standard error bars are indicated for observed means with n > 1. The energetically optimum M (minimum R/M) and M at which dR/dM was minimal are also shown. Coefficients of the cubic equations are in Table 1. They differed from 0, P = 0.00001, for both females and males. a. Females; b. Males.

Sex	Model	Equation	ACD [†]	AIC [‡]	BIC§
Female	Linear	R = 1.043 E - 1 * M	0.1170	6954	6965
	Quadratic	$R = -1.426E - 3*M^2 + 1.834E - 1*M$	0.8465	2818	2835
	Cubic¶	$R = 2.511E - 5*M^3 - 4.188E - 3*M^2 + 2.541E - 1*M$	0.9681	-892	-869
	Quartic	$R = -4.344E - 7*M^4 + 9.715E - 5*M^3 - 7.846E - 3*M^2 + 3.094E - 1*M$	0.9854	-2741	-2712
	иM ^v	$R = 1.016 \text{E}0^* M^{4.236\text{E}-1}$	0.9506	137	154
	$uM^{0.75}$	$R = 2.839 \text{E} - 1 * M^{7.500 \text{E} - 1}$	0.6378	4847	4859
Male	Linear	R = 1.102 E - 1 * M	0.5971	12012	12025
	Quadratic	$R = -1.152E - 3*M^2 + 1.841E - 1*M$	0.9361	3154	3174
	Cubic¶	$R = 1.783E - 5*M^3 - 3.310E - 3*M^2 + 2.468E - 1*M$	0.9820	-2940	-2914
	Quartic	$R = -3.170E - 7*M^4 + 7.481E - 5*M^3 - 6.499E - 3*M^2 + 3.007E - 1*M$	0.9916	-6597	-6565
	иM ^v	$R = 8.639 \text{E} - 1 * M^{5.013 \text{E} - 1}$	0.9816	-2847	-2828
	$uM^{0.75}$	$R = 3.111 \text{E} - 1 * M^{7.500 \text{E} - 1}$	0.8673	6668	6681

Table 1. The relationship between basal heat production rate (R, megajoules per day) and mass (M, kilograms). All coefficients differed from 0, P < 0.00001.

Note: † Adjusted coefficient of determination.

‡ Akaike information criterion.

§ Bayesian information criterion.

As an aside, we found that these were the same optima estimated by orthogonal nonlinear (n-weighted) least-squares regression. This method is sensitive to the units employed [24] and results were different when R was expressed in kilocalories per day; therefore, all reported parameter estimates and goodness-of-fit statistics are based on standard n-weighted least-squares regression.

For comparison, we analyzed the original R/M versus M data of Benedict [1]. Masses ranged from 46 to 109 kg for males, ages unspecified (Chart III); and from 35 to 94 kg for females, ages unspecified (Chart IV). These data had been included among the compiled data [3,13] we used. We found that a quadratic relationship existed between R/M and M. All coefficients differed from 0 for both females (P = 0.030) and males (P = 0.035) [$R/M = 9.739E-6*M^2 - 2.142E-3*M + 1.926E-1$, $R/M = 9.482E-6*M^2 - 1.952E-3*M + 1.931E-1$; SE of the estimates were 4.416E-6, 5.616E-4, 1.724E-2, 4.425E-6, 6.268E-4, and 2.181E-2; and the coefficients of determination were 0.667 and 0.426, respectively]. The corresponding M optima, where R/M was minimal, for this small subsample were 110 kg for females and 103 kg for males. Apparently there was a relationship between R/M and M in the data of Benedict [1] after all.

Our cubic equation represents an extremely parsimonious model of how R changes with M during human growth. Only 3 parameters are required instead of the 12 for each sex given in Table 1 of

[¶] Female: estimated standard errors of the parameters were: 2.648E-7, 2.976E-5, and 8.241E-4; these were 1.0, 0.7, and 0.3% of the parameter estimates themselves, respectively. Male: estimated standard errors of the parameters were: 1.610E-7, 1.986E-5, and 6.185E-4; these were 0.9, 0.6, and 0.2% of the parameter estimates themselves, respectively.

Schofield [3], Table 13 of Henry [4], and Table 5.3 of Das and Roberts [5]. Confidence intervals for the line segments of Schofield [3] varied because each was based on a different sample size (n varied 22-fold for females and 58-fold for males); whereas, the cubic function was statistically weighted to account for variation in n across M. The cubic equation also demonstrated the interrelationships between dR/dM and R/M, and identified the points where heat production rates per kilogram of body mass were minimal.

The *R* data were underestimated by the cubic between mean *M* of 10 to 20 kg (approximately ages 1 to 7 yr) in both females and males (Figure 3). This discrepancy may be explained by the difficulty in meeting the strict protocol required to determine basal *R* in children via indirect calorimetry. Its measurement should be performed with the subject in a "complete resting posture, post 8-h sleep (typically performed after an overnight in-patient stay), post 12-h fast, in a thermal neutral temperature setting, and with darkened or dimmed lighting and quiet ambient conditions" [25]. Children become restless during such an evaluation period and this increases measured *R* above basal. Over the *M* range of 10 to 20 kg, the difference between reported and predicted *R* decreased from approximately 20% to 5%. This also suggests that use of Schofield's equations [3] to predict the basal metabolic rates of adolescents, as Silva et al. [26] did recently for 10- to 17-yr olds, does not introduce appreciable error.

The apparent overestimation of *R* at large *M* (> 80 kg for females, > 90 kg for males) likely indicates that rate of gain has slowed significantly, stopped, or was perhaps even negative in some people (Figure 3). Data in this *M* range are sparse, often only single observations (6 of 14 means for females and 7 of 13 means for males). The growth status of individuals is also unknown. That said, differences in *M* and age between means in the data [3] indicated that dM/dt were positive 82% of the time in females (for 73 of 89 intervals) and 78% of the time in males (for 76 of 97 intervals). In contrast, for females > 80 kg, only 58% of dM/dt estimates were positive (7 of 12 intervals, including 37 observations); whereas, for males > 90 kg, only 27% of dM/dt estimates were positive (3 of 11 intervals, including 21 observations). Schofield [3] also noted that variation in *R* was greater for older (and larger) people compared to younger (and smaller) people, with line segments explaining an average of 46% of variation in *R* for ≥ 18-yr olds but 78% for those < 18 yr old. The same was found for the even larger "Oxford" database [4], where line segments explained an average of 55% of variation in *R* for ≥ 18-yr olds but 75% for those < 18 yr old.

Basal metabolic rate at equilibrium M (i.e., not growing) can be much less than that of a growing animal of the same M. In order to hold a growing animal at its current M energy intake declines exponentially [10,27]. This phenomenon is likely related to changes in the masses and functions of metabolically active internal organs, especially the liver and gastrointestinal tract [2,10]. The growth status of humans in energy metabolism studies is an important variable that should always be assessed and reported because it affects R at any given M.

Although similar to basal metabolic rate, protocols for estimating resting metabolic rate are less stringent [28]. McMurray et al. [28] compiled and analyzed resting metabolic rate data for 11,951 adults. We were unable to include these data in our study because *n*-weighted linear regression indicated that the average adult was not growing; i.e., *M* and age were not related significantly (P > 0.8 for both females and males).

3.3. Implications of changes in average body mass over time on efficiency of energy use

The relationships between R/M and M for human females and males, based on our fitted cubic equations, are shown in Figure 4. Also indicated in Figure 4 are the average M for females and males, aged 20 to 74 yr, in the five National Health Examination and NHANES studies conducted between 1960 and 2002 [29]. The mean M of 20 to 74 yr-old females and males in the U.S. increased from 64 to 75 kg and from 76 to 87 kg, respectively, over this time; i.e., linearly at 0.28 kg/yr and 0.28 kg/yr [standard *n*-weighted least-squares regressions; P = 0.01]. There is some evidence that this rate of increase in M may have moderated recently, to about 0.22 kg/yr for females and 0.20 kg/yr for males (linear; females P = 0.0095, males P = 0.0036) [30]; however, these 1999 through 2016 data were based on slightly different criteria (age ≥ 20). By 2015 to 2016 females averaged 77 kg and males 90 kg. From 1971 to 2000, energy intakes [31] have been 109 to 128 % of R at respective M in the NHANES studies for females and 134 to 143% of R for males. Energy consumption in excess of R(Figure 3) and decreasing R/M with growth (Figure 4) help explain M increases over this period.

The energetically optimal masses, where R/M was minimal in Figure 4, are here re-designated as "attractors." This term was adopted because it is used to describe a stable fixed point "to which all neighboring trajectories converge" in a deterministic dynamic system [32]. We show that R/M is moving toward these attractors through time because R per unit M declines with growth towards the attractor. Once M exceeds that associated with the attractors, rising R/M (i.e., decreasing efficiency of energy use) begins to reduce M increases for the same excess energy intake. This would tend to move R/M back towards its minimum.

Energetically optimal masses of 83 kg (females) and 93 kg (males) correspond to body mass indices (BMI) of 31 kg/m² and 29 kg/m² in the data of Schofield [3], respectively. Between 1960 and 2002, mean BMI of 20- to 74-yr olds increased from 25 to 28 kg/m² in both females and males [29]. By 2015 to 2016 the BMI were 29.6 kg/m² and 29.1 kg/m² for those \geq 20 yr old [30].

Adults having a BMI > 30 kg/m² are generally considered at risk for developing obesity-related diseases [33]. The BMI associated with the lowest all-cause mortality in this meta-analysis of 239 prospective studies was 20.0 to 25.0 kg/m². This suggests that body masses minimizing R/M, the "attractors", are larger than those maximizing long-term health and minimizing risk of mortality. We note that obesity-related diseases manifesting at ages well past those required for reproduction and rearing of offspring are not expected to influence genetic fitness significantly.

The apparent physiological condition of minimal R/M at relatively large M, 83 kg for females and 93 kg for males, may have evolved as an adaptation to address the potentially devastating effects of seasonal food availability [34] by allowing use of energy accumulated in one season to survive another with much more limited food. Marginal heat production rates (dR/dM) are minimal at 2/3 of those values, 56 kg for females and 62 kg for males; thereafter, the R required for each additional kilogram of gain increases. Between M of 56 and 83 kg for females and between 62 and 93 kg for males, the average rate of heat production (R/M) is still decreasing; however, beyond those M the energetic advantage of continued growth is exhausted. A secular trend of increasing M with urbanization may account for the fact that our estimates of minimal dR/dM at 56 kg for females and 62 kg for males are larger than the median adult sizes for 22 small-scale societies (48 kg for females and 56 kg for males) [35].

The objective of many health-intervention programs is to reduce body mass. For this to occur, energy intake must obviously be less than total energy expenditure. As M decreases, R also decreases, making further reductions in M through dieting that much more difficult.

4. Conclusions

Growing humans share the same cubic relationship between basal metabolic rate and body mass that we found applied to 8 other species of growing animals [10]. Contrary to previous assertions [1, 2, 11], R/M in growing humans was found to be quadratically related to M. The R/M was minimal (i.e., energetically optimal) in females and males at particular masses which seem to serve as attractors. The average masses of adults in the U.S. have been approaching these attractors over the last 60 yr. Few data on the energy metabolism of growing very large humans are available. This makes additional studies of such people important, if perhaps more ethically challenging than conducting them in other species.

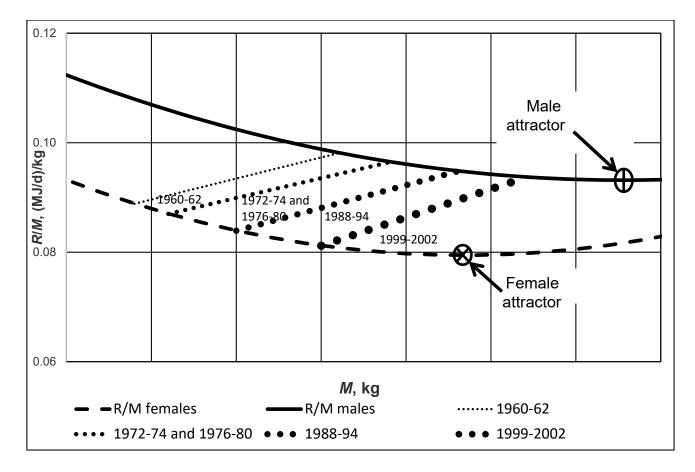


Figure 4. Average heat production rate [R/M, (megajoules/day)/kilogram] versus body mass (*M*, kilograms) for females and males based on our cubic regression equation fit to the data of Schofield [3] in Figure 3. The two curves were obtained by dividing cubic equations in Table 1 by *M*. Dotted lines indicate mean *M* for 20 to 74 year-old females and males in five national studies between 1960 and 2002 [29]. The female and male attractors (points of minimum R/M and energetically optimal *M*, see text) are also shown.

Acknowledgments

We thank Ignacio Sarmiento Barbieri, Assistant Professor of Economics, Universidad de Los Andes, Bogotá, Colombia; and John I. Marden, Professor Emeritus of Statistics, University of Illinois at Urbana-Champaign for assistance with our statistical analyses.

Conflict of interest

The authors declare that they have no conflict of interest.

References

- 1. F. G. Benedict, Factors affecting basal metabolism, J. Biol. Chem., 20 (1915), 263–299.
- 2. M. A. Holliday, D. Potter, A. Jarrah, S. Bearg, The relation of metabolic rate to body weight and organ size, *Pediatr. Res.*, **1** (1967), 185–195.
- 3. W. N. Schofield, Predicting basal metabolic rate, new standards and review of previous work, *Clin. Nutr.*, **39** (1985), 5–41.
- 4. C. J. D. Henry, Basal metabolic rate studies in humans: measurement and development of new equations, *Public Health Nutr.*, **8** (2005), 1133–1152.
- 5. S. K. Das, S. B. Roberts, Energy metabolism in fasting, fed, exercise, and re-feeding states, in *Present Knowledge in Nutrition* (eds. J. W. Erdman, Jr., I. A. Macdonald and S. H. Zeisel), Wiley-Blackwell, (2012), 58–68.
- 6. M. Kleiber, The Fire of Life: An Introduction to Animal Energetics, Krieger, (1975).
- 7. Z. Wang, J. Zhang, Z. Ying, S. B. Heymsfield, Organ-tissue level model of resting energy expenditure across mammals: New insights into Kleiber's Law, *ISRN Zool.*, **2012** (2012), 1–9.
- 8. National Research Council, Nutrient Requirements of Swine, National Academies Press, (2012).
- 9. National Research Council, *Nutrient Requirements of Beef Cattle*, National Academies Press, (2016).
- 10. B. M. Hannon, M. R. Murphy, Toward a theory of energetically optimal body size in growing animals, *J. Anim. Sci.*, **94** (2016), 2532–2541.
- A. M. Prentice, A. E. Black, P. R. Murgatroyd, G. R. Goldberg, W. A. Coward, Metabolism or appetite: questions of energy balance with particular reference to obesity, *J. Hum. Nutr. Diet.*, 2 (1989), 95–104.
- 12. B. Hannon, Biological time value, Math. Biosci., 100 (1990), 115–140.
- 13. C. Schofield, An annotated bibliography of source material for basal metabolic rate data, *Clin. Nutr.*, **39** (1985), 42–91.
- 14. M. Kleiber, Metabolic Rate and Food Utilization as a Function of Body Size, (1961).
- M. D. Mifflin, S. T. St Jeor, L. A. Hill, B. J. Scott, S. A. Daugherty, Y. O. Koh, A new predictive equation for resting energy expenditure in healthy individuals, *Am. J. Clin. Nutr.*, **51** (1990), 241–247.
- 16. K. G. Tay, S. L. Kek, R. Abdul-Kahar, Numerical differentiation spreadsheet calculator, *Proc. National Symp. Appl. Sci. Math.*, **2013** (2013), 111–120.
- 17. D. Rogosa, Myths and methods: "Myths about longitudinal research" plus supplemental questions, in *The Analysis of Change* (ed. J. M. Gottman), Lawrence Erlbaum, (1995), 3–65.

- 18. J. W. Tukey, Exploratory Data Analysis, Reading, Addison-Wesley, (1977).
- 19. NLREG, Available from: http://NLREG.com.
- 20. K. P. Burnham, D. R. Anderson, *Model Selection and Multimodel Inference: A practical information-theoretic approach*, Springer-Verlag, 2002.
- A. N. Spiess, N. Neumeyer, An evaluation of R² as an inadequate measure for nonlinear models in pharmacological and biochemical research: A Monte Carlo approach, *BMC Pharmacol.*, 10 (2010), 1–11.
- S. D. Hodges, P. G. Moore, Data uncertainties and least squares regression, J. R. Stat. Soc. C Appl., 21 (1972), 185–195.
- 23. A. N. Spiess, Orthogonal nonlinear least-squares regression, (2015). Available from: https://rdrr.io/cran/onls/man/onls.html.
- 24. R. L. Branham Jr, Astronomical data reduction with total least squares, *New Astron. Rev.*, **45** (2001), 649–661.
- 25. T. Psota, K. Y. Chen, Measuring energy expenditure in clinical populations: rewards and challenges, *Eur. J. Clin. Nutr.*, **67** (2013), 436–442.
- 26. A. L. Silva, J. Teles, L. F. Olivares, I. Fragoso, Energy intake and expenditure in children and adolescents, contributions of biological maturity, *Am. J. Hum. Biol.*, **2020** (2020), 1–14.
- 27. B. M. Hannon, M. R. Murphy, Progressive limit feeding to maximize profit in the feedlot, *J. Anim. Sci.*, **97** (2019), 1600–1608.
- R. G. McMurray, J. Soares, C. J. Caspersen, T. McCurdy, Examining variations of resting metabolic rate of adults: A public health perspective, *Med. Sci. Sports Exerc.*, 46 (2014), 1352– 1358.
- C. L. Ogden, C. D. Fryar, M. D. Carroll, K. M. Flegal, *Mean body weight, height, and body mass index: United States 1960-2002*, Department of Health and Human Services, Centers for Disease Control and Prevention, National Center for Health Statistics, 2004.
- C. D. Fryar, D. Kruszon-Moran, Q. Gu, C. L. Ogden, *Mean body weight, height, waist circumference, and body mass index among adults: United States, 1999-2000 through 2015-2016, 2018.* Available from: https://stacks.cdc.gov/view/cdc/61430.
- J. D. Wright, J. Kennedy-Stephenson, C. Y. Wang, M. A. McDowell, C. L. Johnson, Trends in intake of energy and macronutrients – United States, 1971-2000, *MMWR Morb. Mortal. Wkly Rep.*, 53 (2004), 80–82.
- 32. S. H. Strogatz, Nonlinear Dynamics and Chaos, CRC Press, 2015.
- E. Di Angelantonio, S. N. Bhupathiraju, D. Wormser, P. Gao, S. Kaptoge, A. Berrington de Gonzalez et al., Body-mass index and all-cause mortality: Individual-participant-data metaanalysis of 239 prospective studies in four continents, *Lancet*, 388 (2016), 776–786.
- 34. F. Branca, P. D'Acapito, Seasonality, in *Encyclopedia of Human Nutrition* (ed. B Caballero), Academic Press, (2013), 178–185.
- 35. R. Walker, M. Gurven, K. Hill, A. Migliano, N. Chagnon, R. De Souza, et al., Growth rates and life histories in twenty-two small-scale societies. *Am. J. Hum. Biol.*, **18** (2006), 295–311.



Mathematical Biosciences and Engineering

AIMS Press ©2021 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0)