Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/MBE

MBE, 18(4): 4761–4771.
DOI: 10.3934/mbe.2021242
Received: 18 March 2021
Accepted: 19 May 2021
Published: 31 May 2021

Research article

Research on online advertising attention evaluation decision based on the stability of delay differential equations and Hopf bifurcation analysis

Xinzhou Xie[1], Zhongru Wang[2,*], Li Tian[1], Hamoud Dekkiche[3] and Mohamad Salama[4]

[1] School of New Media, Peking University, Beijing, China
[2] Chinese Academy of Cyberspace Studies, Beijing, China
[3] Canadian University Dubai, Dubai, United Arab Emirates
[4] Mohamad Salama Applied Science University, Al Eker, Kingdom of Bahrain

* **Correspondence:** Email: wangzhongru@bupt.edu.cn.

Abstract: This paper uses the stability of the delay differential equation to study its impact on online advertising, helps analyze Hopf branch characteristics in a big data environment, helps companies make online advertising decisions, and maximizes the benefits of product sales. The thesis fully considers various factors such as advertising volume, advertising schedule, and advertising investment level, discusses the singularity types of the advertising delay differential equation, and gives the best decision for advertising investment.The stability of the time-lag differential equation studied in this paper is to study its impact on online advertising, to help analyze the Hopf branch characteristics in the big data environment, and to help companies make online advertising decisions. structure of this article is also from the amount of advertising, the time of advertising, Advertising investment level gradually expands with a certain degree of continuity.

Keywords: complex variable function; shallow buried circular tunnel surrounding rock; deformation analysis; stress finite element analysis

1. Introduction

Advertising is a paid, organized, and comprehensive non-personnel information dissemination activity about products (goods, services, ideas) carried out by identifiable investors through various media. Advertising has two main purposes: 1) to constantly tell consumers that they have multiple choices; 2) to create opportunities for businesses to more effectively fight for consumer money [1]. Advertising can stimulate competition (more sellers and buyers), and in countries where consumers earn more than their basic living needs, advertising can also lead to product innovation or innovation. The nature of advertising determines that advertising is not "complete information". It will show content that is conducive to advertisers or brands. People are psychologically prepared for this, and they will not care, but if the advertiser goes beyond simply expressing views and creating scams People began to oppose the limits of [2].

This paper uses the stability of the delay differential equation to study its impact on online advertising, helps analyze Hopf branch characteristics in a big data environment, helps companies make online advertising decisions, and maximizes the benefits of product sales. The thesis fully considers various factors such as advertising volume, advertising schedule, and advertising investment level, discusses the singularity types of the advertising delay differential equation, and gives the best decision for advertising investment. The stability of the time-lag differential equation studied in this paper is to study its impact on online advertising, to help analyze the Hopf branch characteristics in the big data environment, and to help companies make online advertising decisions. structure of this article is also from the amount of advertising, the time of advertising, Advertising investment level gradually expands with a certain degree of continuity.

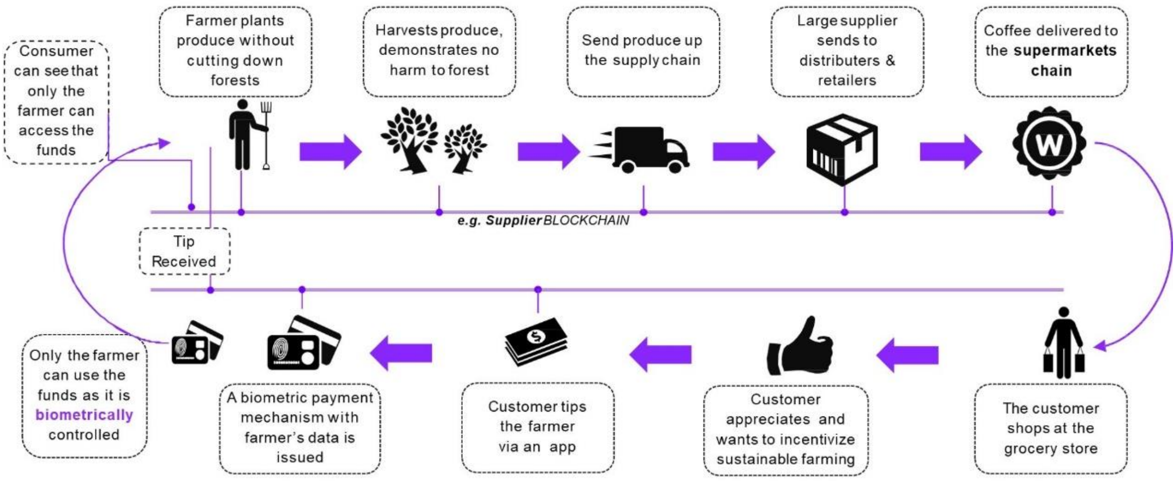

Figure 1. Supply chain where product advertising is located.

This article introduces the effect of time lag into supply chain advertising decisions, and studies the advertising strategies of supply chain members considering the dual effects of delay and memory. As shown in Figure 1, this is a supply chain model for product advertising. Generally, before the product enters the market, consumers do not understand the product. Manufacturers carry out a series of strategic planning in order to open the product market, so that the product quickly accumulates a certain brand reputation, that is, to obtain the initial value of the brand reputation; product entry After the market, manufacturers and retailers advertise products, and the brand reputation of products will change dynamically over time. The rate of change is directly affected by the manufacturer's advertising investment and the decline of brand reputation itself, not by the retailer's advertising. Retailer advertising directly affects product sales. Because the advertising effect is delayed and memorable, neither the positive effect of the manufacturer 's advertising investment on the brand reputation nor the negative effect of the decay of the brand reputation itself will occur immediately.

2. Modelling of delay differential equations

Consider a supply chain consisting of a manufacturer and a retailer, where the manufacturer and the retailer collaborate on advertising. Retailers conduct local advertising through a variety of advertising media. First, three types of advertising media are considered. In the model expansion, multiple advertising media will be considered. It is assumed that the local advertising input of the retailer in the three communication channels is $A_i(t), i = 1,2,3$, and the proportion of the manufacturer's advertising cost to the retailer is $\chi(t)$, and $0 \leq \chi(t) \leq 1$. Let the retailer's advertising cost function be $A_i^2(t)$ [3]. Assume that retailers can increase demand by advertising. According to the research in [4], a differential equation that reflects the relationship between demand and advertising expenditure and the three types of advertising synergy is as follows.

$$S(t) = \underset{i=1,2,3}{Lu(\rho_i A_i(t))} + \underset{i,j=1,2,3,i\neq j}{M(k_{ij}\sqrt{A_i(t)A_j(t)})} - \int_{t-x}^{t} u(t)d(t) \tag{1}$$

Suppose that the manufacturer and the retailer have the same and positive discount rate r. The goal of both parties is to seek to maximize their own profits in $[0, \infty)$. The marginal profits of both parties are π_m and π_r, which are constant. The objective functions of the manufacturer and retailer are

$$\max_{\chi(t)}\left\{J_m(S) = \int_0^\infty e^{-rt}\left[\pi_m S(t) - \chi(t)\left(A_1^2(t) + A_2^2(t) + A_3^2(t)\right)\right]dt\right\} \tag{2}$$

$$\max_{A_1(t),A_2(t),A_3(t)}\left\{J_r(S) = \int_0^\infty e^{-rt}\left[\pi_r S(t) - \left(1-\chi(t)\right)\left(A_1^2(t) + A_2^2(t) + A_3^2(t)\right)\right]dt\right\} \tag{3}$$

The constraint condition is (1). Formulas (2) and (3) indicate that the retailer decides the amount of advertising investment for various media, and the manufacturer determines the sharing ratio. This is because the retailer is close to the end customer, and it is more beneficial for the retailer to promote the advertising campaign to sell more products; the reason for the manufacturer to determine the share is that the manufacturer is the leader in the supply chain. A large number of literatures have adopted this decision model. References [5] and [6] show that the feedback control strategy and the empirical data fit better than the open-loop solution through empirical research. On the other hand, the feedback control strategy is also optimal for local countermeasures. Model feedback control strategy.

3. Centralized decision

Centralized decision-making means changing from two decision-makers under decentralized decision-making to one decision-maker under centralized decision-making. The following shows the optimal advertising investment and optimal supply chain profit for the three media under centralized decision-making. The superscript "C" indicates the optimal value under centralized decision-making. Figure 2 shows centralized decision making.

Figure 2. Centralized decision mode.

Proposition 1: Under centralized decision-making, the optimal advertising investment for the three media is

$$A_1^C = \frac{(\pi_m+\pi_r)\varpi_1}{2(r+\delta)}, A_2^C = \frac{(\pi_m+\pi_r)\varpi_2}{2(r+\delta)}, A_3^C = \frac{(\pi_m+\pi_r)\varpi_3}{2(r+\delta)} \tag{4}$$

The optimal profit of the supply chain is

$$V^C(S) = \frac{\pi_m+\pi_r}{r+\delta}S + \frac{(\pi_m+\pi_r)^2}{4r(r+\delta)^2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) \tag{5}$$

Where $\varpi_1, \varpi_2, \varpi_3$ is determined by the following equations

$$\begin{cases} \varpi_1 = \rho_1 + \frac{k_{12}}{2y_1} + \frac{k_{13}}{2y_2}, \varpi_2 = \rho_2 + \frac{k_{12}y_1}{2} + \frac{k_{23}}{2y_3} \\ \varpi_3 = \rho_3 + \frac{k_{13}y_2}{2} + \frac{k_{23}y_3}{2}, y_1^2 = \frac{\varpi_1}{\varpi_2}, y_2^2 = \frac{\varpi_1}{\varpi_3}, y_3^2 = \frac{\varpi_2}{\varpi_3} \end{cases} \tag{6}$$

Proof: Under centralized decision-making, the two parties will jointly determine the optimal advertising investment of the three media with the goal of maximizing the profit of the supply chain system. Then the objective function of the supply chain is

$$\max_{A_1,A_2,A_3} \left\{ J = J_m + J_r = \int_0^\infty e^{-rt} \left[(\pi_m + \pi_r)S - (A_1^2 + A_2^2 + A_3^2)\right]dt \right\} \tag{7}$$

Its optimal profit function $V(S)$ must satisfy the following Hamilton-Jacobi-Bellman (HJB) Eq (8)

$$rV(S) = \max_{A_1,A_2,A_3} \left\{ (\pi_m + \pi_r)S - (A_1^2 + A_2^2 + A_3^2) + V'(S)\dot{S} \right\} \tag{8}$$

Where $V'(S) = \frac{dV(S)}{dS}$ the first-order condition for the right end maximization of Eq (9) is

$$V'\left(\rho_i + \frac{k_{ij}}{2}\sqrt{\frac{A_j}{A_i}} + \frac{k_{ik}}{2}\sqrt{\frac{A_k}{A_i}}\right) - 2A_i = 0, i, j, k = 1,2,3, i \neq j \neq k \tag{9}$$

$2(A_1^2 + A_2^2 + A_3^2) = V'\left(\sum_{i=1,2,3}\rho_i A_i + \sum_{i,j=1,2,3,i\neq j} k_{ij}\sqrt{A_i A_j}\right)$ from Eq (9), which is substituted into the HJB equation of the supply chain.

$$rV(S) = (\pi_m + \pi_r - \delta V')S + (A_1^2 + A_2^2 + A_3^2) \tag{10}$$

Let $y_1 = \sqrt{\frac{A_1}{A_2}}, y_2 = \sqrt{\frac{A_1}{A_3}}, y_3 = \sqrt{\frac{A_2}{A_3}}$ be the optimal advertising input from Eq (9) to be $A_i = \frac{\varpi_i V'}{2}$, where $\varpi_i, i = 1,2,3$ is determined by the system of equations of Eq (6). Since Eq (6) does not contain unknown variables, $\varpi_1, \varpi_2, \varpi_3$ and y_1, y_2, y_3 are positive values. Substitute A_i into Eq (10), and combine to get $rV(S) = (\pi_m + \pi_r - \delta V')S + \frac{(V')^2(\varpi_1^2+\varpi_2^2+\varpi_3^2)}{4}$.

The linear optimal profit function on S is the solution of this HJB equation. Let $V(S) = l_1 S + l_2$, where C is the undetermined constant. Substituting $V(S)$ and its derivative for S into (10), find the parameter value

$$l_1 = \frac{\pi_m+\pi_r}{r+\delta},\ l_2 = \frac{(\pi_m+\pi_r)^2}{4r(r+\delta)^2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) \tag{11}$$

Then we get the optimal advertising investment and optimal profit function of the three media under centralized decision-making.

Under centralized decision-making, the higher the profit margin of the manufacturer (retailer), the higher the advertising investment and the total profit of the supply chain for the three media. It shows that the marginal profit is the driving force for the two sides of the channel to invest in advertising. The two sides can increase the marginal profit by reducing unit costs or operating costs. Substituting the optimal advertising investment of 3 media into (1)

$$\dot{S}(t) = \frac{(\pi_m+\pi_r)}{2(r+\delta)}\left(\sum_{i=1,2,3}\rho_i\varpi_i + \sum_{i,j=1,2,3,i\neq j}k_{ij}\sqrt{\varpi_i\varpi_j}\right) - \delta S(t) \tag{12}$$

The initial condition is $S(0) = S_0$. Solving the differential equation of formula (12) gives $S(t) = \frac{\bar{S}^C}{\delta} + e^{-\delta t}\left(S_0 - \frac{\bar{S}^C}{\delta}\right)$, where

$$\bar{S}^C = \frac{(\pi_m+\pi_r)}{2(r+\delta)}\left(\sum_{i=1,2,3}\rho_i\varpi_i + \sum_{i,j=1,2,3,i\neq j}k_{ij}\sqrt{\varpi_i\varpi_j}\right) \tag{13}$$

Then at $t \to \infty$, the stable sales volume is $\frac{\bar{S}^C}{\delta}$.

Proposition 2: Under the optimal advertising investment of centralized decision-making, (1) when $\delta > \frac{\bar{S}^C}{S_0}$, the sales volume decreases continuously over time, but is always greater than $\frac{\bar{S}^C}{\delta}$; when $\delta < \frac{\bar{S}^C}{S_0}$, the sales volume continues to increase over time, but is always less than $\frac{\bar{S}^C}{S_0}$; When $\delta = \frac{\bar{S}^C}{S_0}$, the sales volume is always S_0; (2) The greater the profit margin of the manufacturer or retailer, the more stable the sales volume; however, the greater the discount rate or the attenuation coefficient, the less the stable sales volume [8].

4. Decentralized decision

Under decentralized decision-making, the relationship between the manufacturer and the retailer is established as a sequential non-cooperative game model with the manufacturer as the leader and the retailer as the follower. From the manufacturer's point of view, the use of cooperative advertising strategies by manufacturers can motivate retailers to increase their local advertising spending. Under the Stickler game, the order of the game is as follows: first, the manufacturer determines the proportion of local advertising sharing for the retailer; then the retailer chooses the three-local media with the best local advertising investment. As shown in Figure 3, it is a decentralized decision mode.

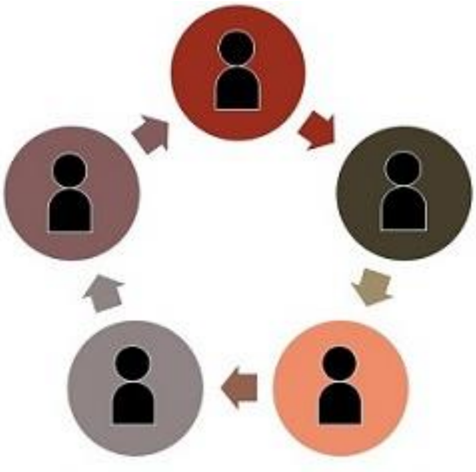

Figure 3. Decentralized decision mode.

Proposition 3: In the case of the Stickler master-slave game, the optimal advertising investment for the three media of the retailer is

$$A_1^* = \frac{(2\pi_m+\pi_r)\varpi_1}{4(r+\delta)}, A_2^* = \frac{(2\pi_m+\pi_r)\varpi_2}{4(r+\delta)}, A_3^* = \frac{(2\pi_m+\pi_r)\varpi_3}{4(r+\delta)} \tag{14}$$

The optimal advertising sharing ratio for manufacturers is $\chi^* = \frac{(2\pi_m-\pi_r)}{(2\pi_m+\pi_r)}$, and the optimal profits for manufacturers, retailers, and supply chains are

$$\begin{cases} V_m^*(S) = \frac{\pi_m}{r+\delta}S + \frac{(2\pi_m+\pi_r)^2}{16r(r+\delta)^2}(\varpi_1^2+\varpi_2^2+\varpi_3^2), V_r^*(S) = \frac{\pi_r}{r+\delta}S + \frac{\pi_r(2\pi_m+\pi_r)}{8r(r+\delta)^2}(\varpi_1^2+\varpi_2^2+\varpi_3^2) \\ V^*(S) = V_m^*(S) + V_r^*(S) = \frac{(\pi_m+\pi_r)}{r+\delta}S + \frac{(2\pi_m+\pi_r)(2\pi_m+3\pi_r)}{16r(r+\delta)^2}(\varpi_1^2+\varpi_2^2+\varpi_3^2) \end{cases} \tag{15}$$

Proof: In order to obtain the Stickler equilibrium of this game, the reverse induction method is used. First solve the retailer's optimal control problem. The optimal profit function $V_r(S)$ must satisfy the HJB equation.

$$rV_r(S) = \max_{A_1,A_2,A_3}\{\pi_r S - (1-\chi)(A_1^2+A_2^2+A_3^2) + V_r'(S)\dot{S}\} \tag{16}$$

The first-order condition for maximizing the right end of the formula (16) is

$$V_r'\left(\rho_i + \frac{k_{ij}}{2}\sqrt{\frac{A_j}{A_i}} + \frac{k_{ik}}{2}\sqrt{\frac{A_k}{A_i}}\right) - 2A_i(1-\chi) = 0, i,j,k = 1,2,3, i \neq j \neq k \tag{17}$$

From Eq (17), $2(A_1^2+A_2^2+A_3^2)(1-\chi) = V_r'\left(\sum_{i=1,2,3}\rho_i A_i + \sum_{i,j=1,2,3,i\neq j}k_{ij}\sqrt{A_iA_j}\right)$ and the optimal advertising input are $A_i(\chi) = \frac{V_r'\varpi_i}{(2(1-\chi))}$, which is substituted into the retailer HJB equation to obtain $rV_r(S) = (\pi_r - \delta V_r')S + (1-\chi)(A_1^2+A_2^2+A_3^2)$. The manufacturer's HJB equation at this time is

$$rV_m(S) = \max_{\chi}\{\pi_m S - \chi(A_1^2 + A_2^2 + A_3^2) + V_m'\dot{S}\} \tag{18}$$

Substituting $A_i(\chi)$ into (18) and simplifying

$$rV_m(S) = \max_{\chi}\left\{(\pi_m - \delta V_m')S + V_r'\frac{2(1-\chi)V_m' - \chi V_r'}{4(1-\chi)^2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2)\right\} \tag{19}$$

To maximize the right side of the above equation, get $\chi = \frac{(2V_m' - V_r')}{(2V_m' + V_r')}$. So, the retailer and manufacturer HJB equations are

$$rV_m(S) = (\pi_m - \delta V_m')S + \frac{(2V_m' + V_r')^2}{16}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) \tag{20}$$

$$rV_r(S) = (\pi_r - \delta V_r')S + \frac{V_r'(2V_m' + V_r')}{8}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) \tag{21}$$

From (20) and (21), we can know that the linear optimal profit function of S is the solution of the manufacturer's and retailer's HJB equation, respectively. Let $V_m(S) = g_1 S + g_2, V_r(S) = h_1 S + h_2$,where C is the undetermined constant. Substituting $V_m(S)$ and $V_r(S)$ their derivatives for S into Eqs (20) and (21), find the parameter value of the optimal profit function

$$g_1 = \frac{\pi_m}{r+\delta},\ g_2 = \frac{(2\pi_m+\pi_r)^2}{16r(r+\delta)^2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2),\ h_1 = \frac{\pi_r}{r+\delta},\ h_2 = \frac{\pi_r(2\pi_m+\pi_r)}{8r(r+\delta)^2}(\varpi_1^2 + \varpi_2^2 + \varpi_3^2) \tag{22}$$

Therefore, the optimal advertising investment of three media and the optimal profit function of manufacturers, retailers, and supply chains are obtained.

Proposition 4: (1) if $\frac{\pi_m}{\pi_r} > \frac{1}{2}$, then the manufacturer provides positive advertising subsidies to the retailer; if $\frac{\pi_m}{\pi_r} \le \frac{1}{2}$, then the manufacturer does not provide subsidies to the retailer; The more local advertising, and the higher the retailer 's marginal profit, the less local advertising shared by the manufacturer; the higher the marginal profit for both the manufacturer and the retailer, the more the local advertising investment of the retailer, At the same time, the more profits manufacturers and retailers have [9].

Similar to the centralized decision-making situation, the stable sales volume under distributed decision-making is $\frac{\bar{S}}{\delta}$, where

$$\bar{S} = \frac{(2\pi_m+\pi_r)}{4(r+\delta)}\left(\sum_{i=1,2,3}\rho_i\varpi_i + \sum_{i,j=1,2,3,i\neq j}k_{ij}\sqrt{\varpi_i\varpi_j}\right) \tag{23}$$

Proposition 5: When $\pi_m > \bar{\psi}\pi_r$, the profit of the manufacturer is higher than the profit of the retailer when it is stable; when $\pi_m < \bar{\psi}\pi_r$, the profit of the manufacturer is lower than the profit of the retailer when it is stable; when $\pi_m = \bar{\psi}\pi_r$, the profits of both parties are equal, where

$$\bar{\psi} = \frac{\left(\frac{\varpi_1^2+\varpi_2^2+\varpi_3^2}{r} + \frac{\bar{B}}{\delta}\right)}{\left(\frac{2(\varpi_1^2+\varpi_2^2+\varpi_3^2)}{r} + \frac{\bar{B}}{\delta}\right)} \in (0,1) \tag{24}$$

As long as the manufacturer's marginal profit is not very low, its profit will be higher than that of the retailer. This shows that whether manufacturers or retailers want to make more profits in cooperative advertising, it is necessary to improve their own marginal profits. Even when the marginal profits of the manufacturer and the retailer are equal, the profit of the manufacturer is greater than the profit of the retailer when stable. This is due to the leading position of the manufacturer in the Stickler game, the so-called "first mover advantage". The optimal advertising under centralized and decentralized decisions and the profit of the supply chain are related to ϖ_i, and (6) determines the value of ϖ_i. For Eq (6), it is difficult to give analytic solutions about ϖ_i and y_i. Now consider a special case-a symmetrical medium, such as $\rho_i = \rho, k_{ij} = k$. Then $\varpi_i = \varpi = \rho + k$ is obtained, and the optimal advertising investment is $A_i^* = A^* = \frac{(2\pi_m+\pi_r)(\rho+k)}{(4(r+\delta))}$. The optimal profits for manufacturers, retailers and supply chains are

$$\begin{cases} V_m^*(S) = \frac{\pi_m}{r+\delta}S + \frac{3(2\pi_m+\pi_r)^2(\rho+k)^2}{16r(r+\delta)^2}, V_r^*(S) = \frac{\pi_r}{r+\delta}S + \frac{3\pi_r(2\pi_m+\pi_r)(\rho+k)^2}{8r(r+\delta)^2} \\ V^*(S) = V_m^*(S) + V_r^*(S) = \frac{(\pi_m+\pi_r)}{r+\delta}S + \frac{3(2\pi_m+\pi_r)(2\pi_m+3\pi_r)(\rho+k)^2}{16r(r+\delta)^2} \end{cases} \quad (25)$$

5. Expansion of the model—multiple advertising media cooperative advertising model

The above studies the cooperative advertising model with the input of three types of advertising media. As an extension of the model, this section considers the cooperative advertising model of multiple advertising media. When retailer n type of advertising media is invested, Still set $A_i(i \in \boldsymbol{I} \equiv \{1,2,\cdots n\})$ as the advertising input of the i-th type of advertising media, and the sales volume satisfies the following differential equation

$$\dot{S}(t) = \sum_{i\in \boldsymbol{I}} \rho_i A_i + \sum_{i,j\in \boldsymbol{I}, i\neq j} k_{ij}\sqrt{A_i A_j} - \delta S, S(0) = S_0 \quad (26)$$

With n type of advertising media investment, the impact of advertising investment on demand is as follows: (1) the impact of a media advertising investment on demand, there are C_n^1 items; The impact has item C_n^2 [10]. First consider the asymmetry of n types of advertising media, then study the situation of symmetry. For n asymmetric advertising media, the objective function for manufacturers and retailers is

$$\max_{\chi}\left\{J_m = \int_0^\infty e^{-rt}\left(\pi_m S - \chi \sum_{i\in \boldsymbol{I}} A_i^2\right)dt\right\}, \max_{A_1\cdots A_n}\left\{J_r = \int_0^\infty e^{-rt}\left(\pi_r S - (1-\chi)\sum_{i\in \boldsymbol{I}} A_i^2\right)dt\right\} (27)$$

The constraint condition is (27).

Proposition 6: For n kind of symmetrical advertising media, such as: $\rho_i = \rho, k_{ij} = k$, the more the number of advertising media (n) chosen by the retailer, the retailer 's advertising, stable sales volume, and the optimal performance of the manufacturer, retailer, and supply chain. The more profits are made.

According to Proposition 9, in order to obtain more profits, retailers should choose as many media as possible. This is because the greater the number of media, the greater the demand for increased synergy between media, and the greater the profit. In fact, when a retailer chooses a media advertisement, it often incurs a fixed cost (mainly used for advertising design payment, such as: hiring celebrities to shoot advertisements, online advertising production costs, etc.) and this part of the cost does not depend on the amount of advertising Change. Once the retailer selects an advertising medium, this part of the advertising costs will become a sunk cost. Record this part as $F_i, i \in \boldsymbol{I}$. This part of the cost will only be incurred when the retailer first advertises in a media. So, the question arises, should the company choose a medium for advertising? Take the retailer as an example (the analysis of the manufacturer and the supply chain is similar), given the sales volume S, the profit when $i-1$ media is selected is $V_r^{**}(i-1|S)$, where $i \in \boldsymbol{I}, i \neq 1$, then the profit when adding an advertising media is $V_r^{**}(i|S)$, and the increased The fixed cost is F_i. Then when $V_r^{**}(i|S) - V_r^{**}(i-1|S) \geq F_i$, the retailer chooses to advertise on the media i; while $V_r^{**}(i|S) - V_r^{**}(i-1|S) < F_i$, the retailer should not advertise on the media i. The above analysis shows that although increasing the number of advertising media can increase profits, the size relationship between increasing profits and fixed costs must be considered. When the fixed cost of choosing a media is zero or low, retailers should invest in a larger number of media.

6. Conclusions

(1) The profit of the supply chain under centralized decision-making, the retailer 's advertising investment, and the stable sales volume are higher than the corresponding values under decentralized decision-making. The utility theory was used to divide the incremental profit under centralized decision-making. The stronger party will get the profit subsidy of the other party. The size of the subsidy fee depends on the relative value of the bargaining power of the two parties. (2) The stable sales volume of the retailer is related to the attenuation coefficient of the advertisement. Advertising cannot prevent the decrease in sales, but companies do not invest in advertising to reduce sales faster; and when the attenuation coefficient is relatively small, advertising will increase sales; (3) manufacturers continue to improve production methods and other means to reduce production Costs and the introduction of advanced management experience by retailers to reduce operating costs can improve supply chain performance, while the party with a high level of profit margins will gain more profit in the cooperative advertising strategy; (4) the multi-media cooperative advertising investment model of this article Compared with single-media cooperative advertising, it is found that both sides of the supply chain are invested by multiple media advertising Profits have increased. The stability of the time-lag differential equation studied in this paper is to study its impact on online advertising, to help analyze the Hopf branch characteristics in the big data environment, and to help companies make online advertising decisions. The structure of this article is also from the amount of advertising, the time of advertising, Advertising investment level gradually expands with a certain degree of continuity.

Acknowledgments

This work was supported in part by National Natural Science Foundation of China No.71633001 and supported in part by Guangdong Province Key Research and Development Plan under Grant No.2019B010136003 and No.2019B010137004.

Conflict of interest

The authors declare that they have no competing interests.

References

1. H. Harraga, M. Yebdri, Attractors for a nonautonomous reaction-diffusion equation with delay, *Appl. Math. Nonlinear Sci.*, **3** (2018), 127–150.

2. M. K. Ammar, D. A. Oda, Design of Gravity Assist trajectory from Earth to Jupiter, *Appl. Math. Nonlinear Sci.*, **3** (2018), 151–160.
3. P. K. Pandey, A new computational algorithm for the solution of second order initial value problems in ordinary differential equations, *Appl. Math. Nonlinear Sci.*, **3** (2018), 167–174.
4. S. Wu, A Traffic Motion Object Extraction Algorithm, *Int. J. Bifurcation Chaos*, **25** (2015), 1540039.
5. S. Wu, M. Wang, Y. Zou, Research on internet information mining based on agent algorithm, *Future Gener. Comput. Syst.*, **86** (2018), 598–602.
6. S. Wu, J. Liu, L. Liu, Modelling method of internet public information data mining based on probabilistic topic model, *J. Supercomput.*, **75** (2019), 5882–5897.
7. M. Martelli, Periodic solutions of some nonlinear delay-differential equations, *J. Math. Anal. Appl.*, **74** (1980), 494–503.
8. T. Su, H. Wu, J. Zhou, Stability of multi-dimensional uncertain differential equation, *Soft Comput.*, **20** (2016), 4991–4998.
9. X. Yang, J. Gao, Linear–quadratic uncertain differential game with application to resource extraction problem, *IEEE Trans. Fuzzy Syst.*, **24** (2016), 819–826.
10. X. Ji, J. Zhou, Multi-dimensional uncertain differential equation: existence and uniqueness of solution, *Fuzzy Optim. Decis. Making*, **14** (2015), 477–491.
11. I. Cebeci, A. Kantarci, N. Gürel, S. Adin, O. Tuncer, M. Carin, et al., Analysis of peripheral blood leukocytes in patients with cyclosporine a-induced gingival hyperplasia, *J. Periodontol.*, **69** (1998), 1435–1439.
12. D. Biernacki, S. Lenglet, P. Polesiuk, Proving soundness of extensional normal-form bisimilarities, *Electron. Notes Theor. Comput. Sci.*, **336** (2017), 41–56.
13. J. Frankle, P. M. Osera, D. Walker, S. Zdancewic, Example-directed synthesis: a type-theoretic interpretation, *ACM Sigplan Notices.*, **51** (2016), 802–815.
14. C. E. Dalazen, A. D. De Carli, R. A. Bomfim, M. L. Dos Santos, Contextual and individual factors influencing periodontal treatment needs by elderly brazilians: a multilevel analysis, *Plos One*, **11** (2016), e0156231.
15. G. Zoëga, K. A. Murphy, Life on the edge of the arctic: the bioarchaeology of the keldudalur cemetery in skagafjorour, Iceland, *Int. J. Osteoarchaeology*, **26** (2016), 574–584.
16. D. Peko, M. Vodanović, Dental health in antique population of vinkovci - cibalae in croatia (3rd-5th century), *AMHA-Acta Med. Hist. Adriat.*, **14** (2016), 41–56.
17. T. Xie, R. Liu, Z. Wei, Improvement of the Fast Clustering Algorithm Improved by K-Means in the Big Data, *Appl. Math. Nonlinear Sci.*, **5** (2020), 1–10.
18. Z. Cui, C. Yan, Deep Integration of Health Information Service System and Data Mining Analysis Technology, *Appl. Math. Nonlinear Sci.*, **5** (2020), 443–452.
19. X. Wang, S. Dong, Users' Sentiment Analysis of Shopping Websites Based on Online Reviews, *Appl. Math. Nonlinear Sci.*, **5** (2020), 493–502.
20. J. Yu, A Model Study Based on Social Network Relational Dimensions and Structural Dimensions, *Appl. Math. Nonlinear Sci.*, **5** (2020), 121–128.
21. F. Wan, D. Zhu, X. He, Q. Guo, D. Zhang, Z. Ren, et al., Agricultural Product Recommendation Model based on BMF, *Appl. Math. Nonlinear Sci.*, **5** (2020), 415–424.
22. J. Alidousti, Stability and bifurcation analysis for a fractional prey–predator scavenger model, *Appl. Math. Nonlinear Sci.*, **81** (2020), 342–355.

23. C. C. Wang, J. C. Hung, Comparative analysis of advertising attention to Facebook social network: Evidence from eye-movement data, *Comput. Hum. Behav.*, **100** (2019), 192–208.
24. C. Helmers, P. Krishnan, M. Patnam, Attention and saliency on the internet: Evidence from an online recommendation system, *J. Econ. Behav. Organ.*, **161** (2019), 216–242.
25. G R. Itovich, F S. Gentile and J L. Moiola, Hybrid Methods for Studying Stability and Bifurcations in Delayed Feedback Systems, *Int. J. Bifurcation Chaos*, **29** (2019), 1950167.
26. S. Panghal, M. Kumar, Neural network method: delay and system of delay differential equations, *Eng. Comput.*, **2021** (2021), 1–10.