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Research article

Implementation and validation of a new method to model voluntary departures from emergency departments

Running Title: Modeling Voluntary departures from emergency departments

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Abstract: In the literature, several organizational solutions have been proposed for determining the probability of voluntary patient discharge from the emergency department. Here, the issue of self-discharge is analyzed by Markov theory-based modeling, an innovative approach diffusely applied in the healthcare field in recent years. The aim of this work is to propose a new method for calculating the rate of voluntary discharge by defining a generic model to describe the process of first aid using a "behavioral" Markov chain model, a new approach that takes into account the satisfaction of the patient. The proposed model is then implemented in MATLAB and validated with a real case study from the hospital "A. Cardarelli" of Naples. It is found that most of the risk of self-discharge occurs during the wait time before the patient is seen and during the wait time for the final report; usually, once the analysis is requested, the patient, although not very satisfied, is willing to wait longer for the results. The model allows the description of the first aid process from the perspective of the patient. The presented model is generic and can be adapted to each hospital facility by changing only the transition probabilities between states.

Keywords: Markov Chain; emergency department; simulation; voluntary departure

1. Introduction

In recent decades, the subject of health management has seen growing interest from both political and social points of view since the conspicuous consumption and waste of public health resources have resulted in low quality and poor access to health care services [1]. As shown by the studies of Nulty and Ferlie, health care management takes shape in a highly political and complex organizational context characterized by the coexistence of different professional groups and numerous regulatory systems. These features make it difficult to apply management techniques that have been successfully employed in other areas to the health care system [2,3].

In particular, hospital emergency departments (EDs) consist of complex organizational structures that can be analyzed, managed and improved by pursuing the objectives of effectiveness, efficiency, and reductions in waste production and resource use. Good management of the hospital ED is crucial because it forms the interface between resources and health [4,5].

Despite its vital role in providing first, prompt aid to patients and in giving them access to healthcare facilities and services, the ED often faces management and organizational issues. Most of these issues involve patient identification, acceptance, monitoring, and handling [6,7], while others involve increasing maintenance costs, difficulties in ensuring the safety and service quality of service [8–12], and overcrowding and delays due to increasing wait times and queues [13–16]. The latter in particular can cause a decrease in patient satisfaction and in the perceived quality of care, thereby increasing the number of patients who leave without being seen (LWBS) [17], i.e., those who depart the ED before being visited by a clinician. This voluntary dropout represents a considerable indication of low quality and malfunction of the ED [18–23] and reflects inefficiencies, bottlenecks, constraints, and risks in the management of an ED as well as in the emergency medicine field itself [24].

Different studies have investigated the problem of voluntary departure from EDs in recent years. The work of Hsia et al. [25] studied the percentage of LWBS in various American hospitals, finding a wide range of leaving percentages that depended on the insurance level and income of the patients. An interesting work by Goodacre [26] evaluated patient characteristics to determine whether they were associated with a higher probability of leaving the ED without being seen. Similarly, Pham and colleagues [27] proposed a model to predict the percentage of LWBS in the ED. Other attempts to decrease the LWBS involve the adoption of team triage, fast tracking, streaming, and point-of-care testing [28,29]. Furthermore, different methods have been proposed to forecast and manage patient flow in EDs [30]. Weng et al. [31] determined an optimal allocation of resources in the ED using simulation to smooth the flow within the ED. The simulation model was based on the National Emergency Department Overcrowding Scale (NEDOCS) and OptQuest and developed using the Simul8 platform. Abo-Hamad et al. [32] proposed an integrated framework to analyze patient flow through an ED and used a simulation model based on a decision support framework [32]. Di Leva et al. [33] used business process modeling to identify solutions to ED bottlenecks.

Other interesting results have been obtained from the application of Markov chain (MC) models to EDs. Indeed, MC-based theories, already successfully applied in several contexts, such as production planning and control [34–37], shown to be useful in numerous biomedical and health-related applications, such as surgical procedures [38], biological models [39], and, indeed, ED management [40–42].

In 2013, Zhu et al. [41] proposed an MC-based approach to model and predict patient flow through the ED. Further studies used MC models to support the decision-making process in EDs [43–45], to control and manage the problem of overcrowding [46], and to analyze patient transitions between the ED and the intensive or critical care unit [38].

While other modeling techniques cannot reliably represent the complex ED environment since they require a high degree of simplification and the posing of unrealistic assumptions, the main advantage of the MC in the context of the ED lies in the possibility of modeling the first aid process and capturing its peculiarities and dynamics without reducing its complexity.

The models proposed to date to reduce the risk of LWBS in the ED, while carefully and precisely describing the process, do not take into account patient satisfaction, which is not only an indicator of the perceived quality of the healthcare services but also a crucial factor determining the voluntary dropout from the ED. Indeed, dissatisfaction due to long waits and a lack of medical attention can lead to LWBS [47]. Therefore, there is a need for models examining the ED not only from the perspective of hospital managers and clinicians but also from the patient side. Such a patient-centered approach to model development could ensure a more reliable prediction of the LWBS behavior and a better informed optimization of ED resources.

Within this framework, this work aims to predict the probability of voluntary departure from the ED starting from an assessment of the degree of patient satisfaction. Thus, by assessing the probability that the patient will reach a certain level of dissatisfaction, the probability of voluntary departure from the ED can be evaluated. To do so, we model the first aid process by using MC theory and propose a behavioral MC (BMC) approach for ED management, taking into account both the process characteristics and patient satisfaction.

2. Methods

2.1. The proposed Markov chain-based approach

The proposed approach follows these three steps:

1) Step 1: We construct a description of the first aid process, i.e., the path of the patient from initial identification to discharge, starting from an MC flowchart. The system outputs a vector containing the probability of patient self-discharge, as reported elsewhere [41].

2) Step 2: We obtain a more complex representation of the first aid process, called the "behavioral Markov chain", which takes into consideration the level of patient satisfaction. The approach is the same as that used for MCs elsewhere [39]: different degrees of satisfaction are achieved with transition probabilities that depend on the wait time. To define the time limits within which the patient feels satisfied, not very satisfied, or dissatisfied, a questionnaire-based approach is adopted.

3) Step 3: The Markov model is validated based on data supplied by the "Centro di Elaborazione Dati" (CED) of the hospital "A. Cardarelli" of Naples and is simulated using MATLAB software.

2.2. Step 1: System description

2.2.1 First aid process description with flowchart

Figure 1 illustrates the aid process flowchart, which highlights the events involved in a patient's voluntary self-discharge from the ED, from the patient's identification by a triage nurse to departure from the ED.

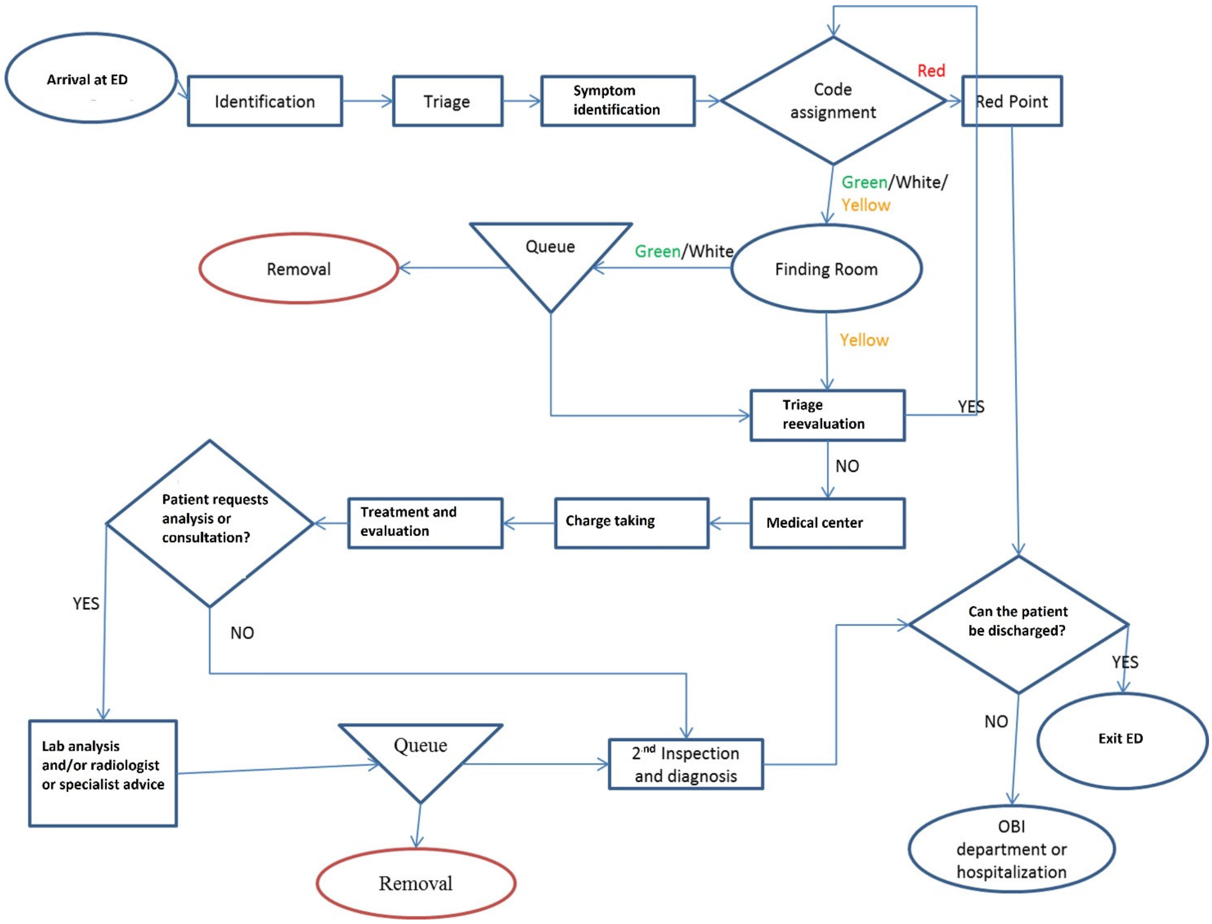

Figure 1. First aid flow chart. OBI represents the short intensive observation room where at-risk patients wait before eventually being hospitalized.

Nurses ("Triage"), based on an analysis of vital signs (consciousness, breathing, and blood circulation), assign a priority code and a medical resource (medical, surgical, orthopedic) to each patient depending on the urgency and type of disease. If the patient presents with a deficiency in one of the three vital signs, a red code is assigned, indicating the need for immediate action. The patient is immediately led to a dedicated area ("Red Point") where he or she has the right of first refusal; this area is also used as an extra waiting room for patients of any other code whenever resources are otherwise occupied. If the patient shows signs such that medical help may be delayed, he or she will be assigned a priority code of yellow, green, or white and will be led back to the waiting room. If necessary, patients receive first aid (such as a bandage) or undergo some initial checks before entering the queue and waiting in a room monitored by triage nurses ("Triage re-evaluation") so that any increase in severity can be detected immediately. As soon as resources are available and according to priority, patients receive a first medical consultation ("Charge taking"), following a first-in-first-out (FIFO) rule for that color code. During this first inspection, the staff establishes the patient's condition and assigns a treatment regimen.

Then, the patient has to decide among a request for further analysis ("Lab analysis and/or radiologist or specialist advice", laboratory tests such as urinalysis, hematologic-chemical tests, or imaging such as X-rays and ultrasound), discharge or hospitalization. This step is followed by a second inspection after waiting for the results of the analyses, and finally, the patient decides whether to be discharged or admitted to the hospital. Due to overcrowding, congestion, or unavailability of resources, wait times can be long, thereby generating patient dissatisfaction and a voluntary and untraceable discharge from the ED. Normally, the main cause of long wait times in the emergency department is the time needed to obtain the test results, particularly those from the laboratory. Another typical cause of long wait times is an incoming "red codes" patient, who has the right to be assisted without any waiting and often consumes all the medical and paramedical resources of the department. If the clinician realizes that a specialist visit is required, time can pass waiting for the specialist, which can be another cause of leaving. Wait times also vary depending on priority code and type of resource consumption; generally speaking, patients with the lowest priority codes tend to wait longer because they are judged as being less urgent than patients with other codes.

Twelve kinds of codes can be assigned: white orthopedic, surgical, or medical code; green orthopedic, surgical, or medical code; yellow orthopedic, surgical, or medical code; red surgical, medical, or orthopedic code. Patients with the highest priority code (red) should not be made to wait long because, as has already been explained, such patients have the right of first refusal over other patients.

2.2.2 First aid process modeling with MC

The chain in Figure 2 shows the first aid process previously described in Figure 1 as seen from an MC perspective, where each step of the process is modeled as a state in which the patient stays before moving to the next one. Each state in MC represents a step in the first aid flowchart. Thus, the number of states is equal to the number of blocks in the first aid workflow. Therefore, the MC defines the states affecting the dynamics of the system. Since the model focuses on the risk of patient self-discharge from the ED, the states related to this event are highlighted in red.

This general model could be applied to any ED. The differences among particular cases only depend on the transition probabilities connecting the states of the model ("arrows" in Figure 2). The assumptions underlying the model are as follows:

1) Each hospital has a unique model defined by the transition probabilities, which are placed on the arrows.

2) The probabilities on each of the arrows are obtained by the dynamics of the patient arrival and resource usage rates.

3) The model is valid for each priority code separately.

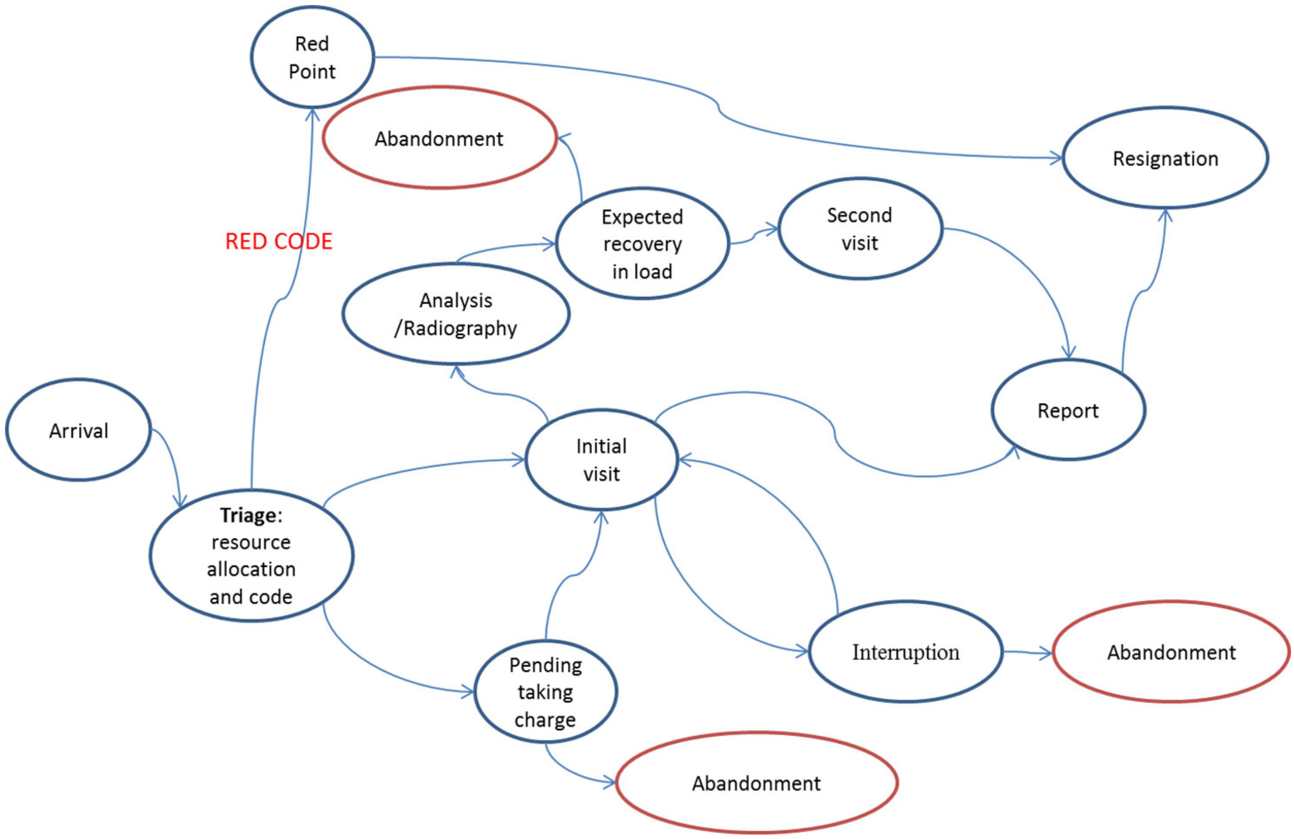

Figure 2. First aid process Markov chain.

2.3. Step 2: The behavioral Markov chain

Once the generic MC model of the ED is obtained, the transition probabilities must be included on each of the arrows connecting one state to another. However, prior to calculating the transition probabilities, we introduce additional states to the previously built model. These added states are used to model the degree of patient satisfaction based on one of three possible responses: (1) satisfied; (2) not very satisfied; and (3) self-discharge. These three states are coded with probabilities as follows: Satisfied: = 0.62; Not very satisfied = 0.375; Self-discharge = 0.005 (please also see "Results" section). To determine the transition probabilities due to the degree of patient satisfaction, a questionnaire was administered to patients awaiting first aid (those with white, green or yellow codes, as patients with red codes were immediately moved to the red point). The questionnaire consists of 16 questions addressing the time the patients are willing to wait to receive assistance and/or lab analysis results.

The following analysis is therefore based on the assumption that the patient's self-discharge is the result of a high level of dissatisfaction and discontent, mainly due to a long wait time. When passing from one state to another, the patient can be satisfied, not very satisfied, or dissatisfied. In the last case, the only result is self-discharge; if the patient is satisfied, he or she will go to the next state, likely without complaint. Poor satisfaction creates a greater likelihood of dissatisfaction (and then transition to the state of self-discharge) in the following stages. The complexity of the Markov chain described here is thus increased by the presence of these states representing the degree of satisfaction. A representation of the BMC is shown in Figure 3.

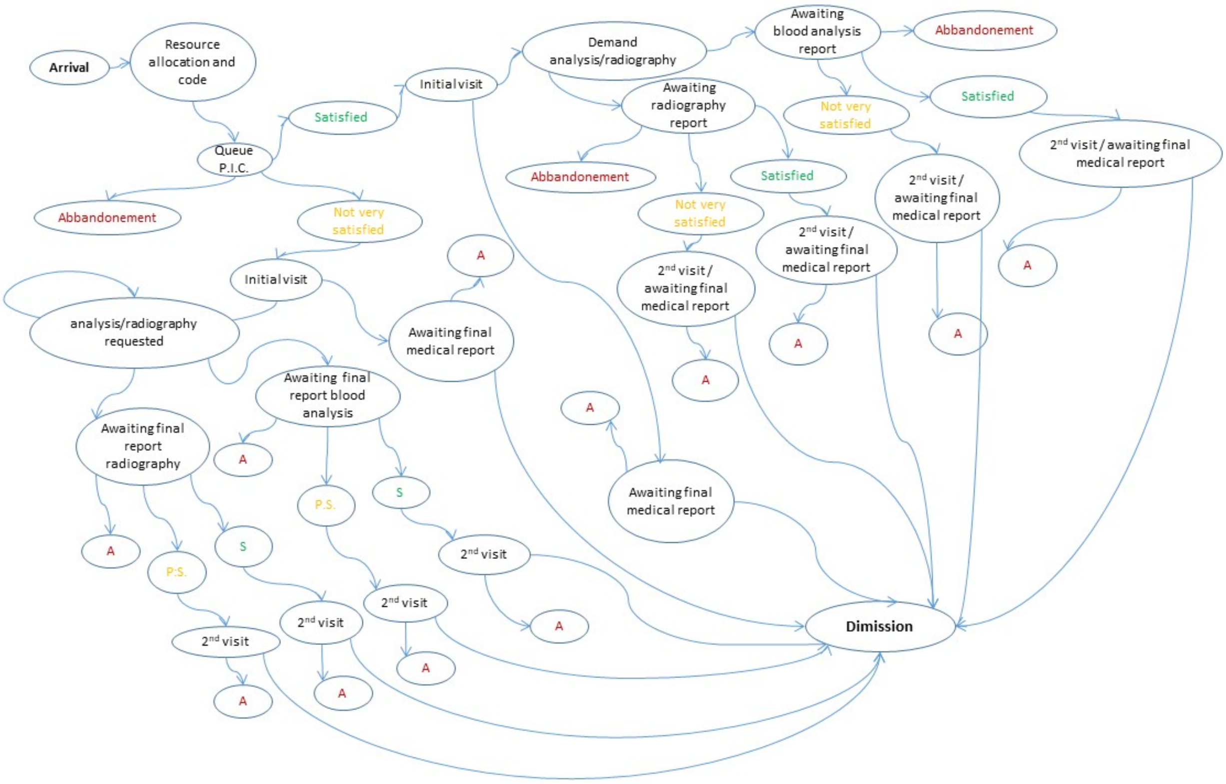

Figure 3. Behavioral Markov chain.

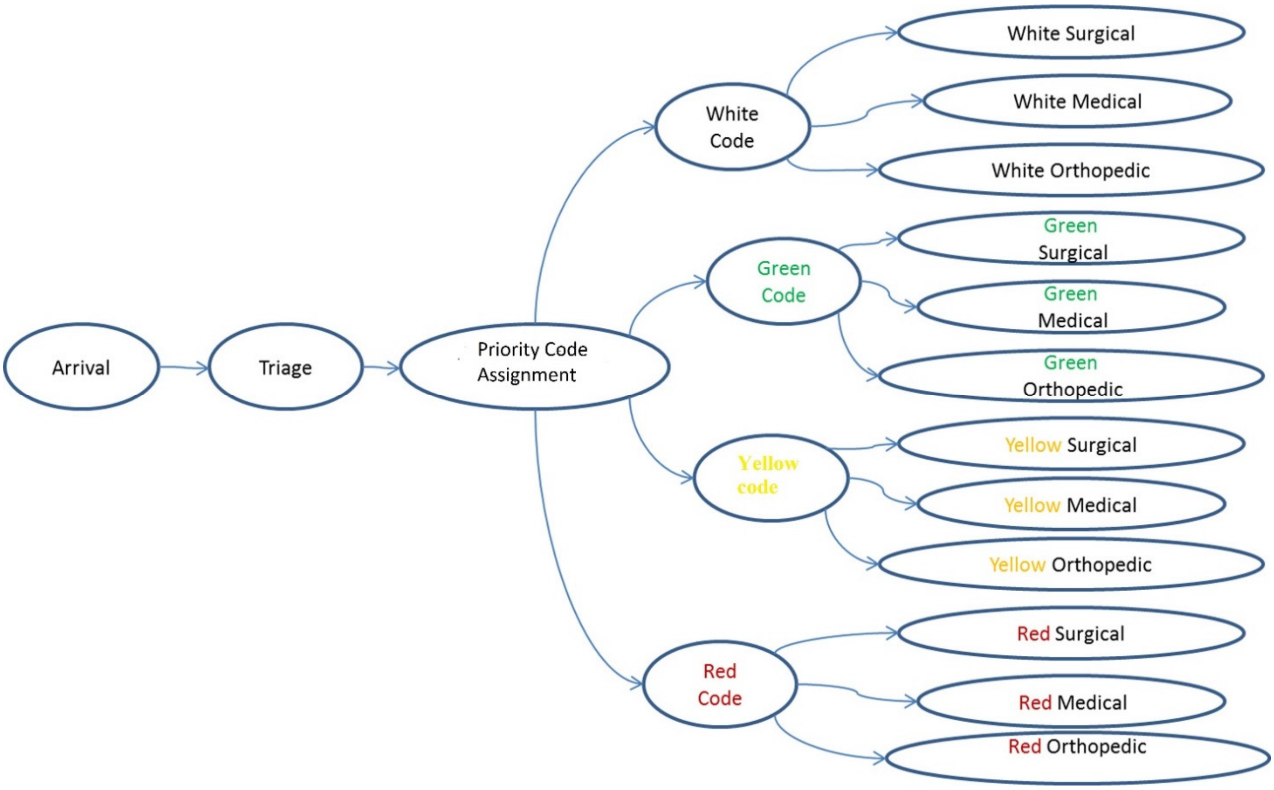

Figure 4. Resource assignment and triage coding.

To simplify the representation displayed in Figure 3, at the end of the chain, the state of self-discharge is indicated with a red “A” and low satisfaction with the ED with the letters “N V S” in yellow.

The model in Figure 3 can be applied to each of the 12 cases considered in this work and summarized in Figure 4.

Figure 4 depicts the 12 possible starting points of the BMC described in Figure 3. The triage codes (color codes) represent the severity of the pathology of the patients arriving at the ED: red codes imply patients greatly at risk, while white codes represent patients who did not need to come to the ED.

The transition probabilities in both Figure 3 and Figure 4 will change because each code will feature different priorities, different availabilities of resources, and, consequently, different wait times. To compute these probabilities, it is necessary to obtain first aid process data, arrival rates, wait times, and the average number of ED patients who need blood and/or radiographic analysis.

In particular, we can distinguish between two types of transition probability:

- the probabilities that derive from the arrival rates and the average number of patients who require further examinations; and
- the probabilities that derive from wait times and consequently define the transition toward a state of satisfaction.

For the first type of transition probability, we assume that the arrival rates correspond to the transition probability; for example, in Figure 4, the transition probability from the code assignment state to a priority code state is calculated as the percentage of patients assigned to that specific priority code (e.g., the probability for a white code is the ratio between the number of white codes and the total number of arrivals). The same assumption is proposed for medical resource assignment and for transitions that depend on analysis requests and wait times for analysis/radiography results (calculated as the percentage of patients assigned to a specific code and to a particular medical resource for the requested analyses).

For the second type of transition probability, the procedure consists of transforming a satisfaction index into a transition probability. The chosen approach is derived from two assumptions:

- The available data are the patient's waiting time in the ED; and
- Dissatisfaction is generated by excessively long wait times.

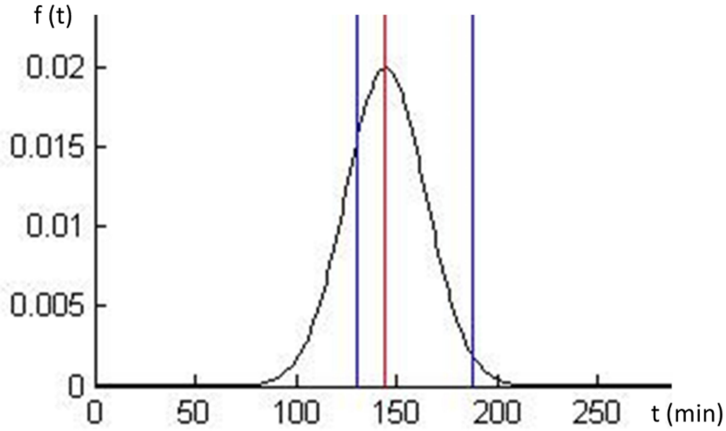

Figure 5. Normally distributed laboratory report waiting time. Satisfaction limits are displayed in blue, while the mean is displayed in red.

The wait time is considered to possess a normal distribution, and the limits in terms of satisfaction are defined as follows: (1) the lower limit represents the maximum waiting time for which a patient will be satisfied and (2) the upper limit represents the maximum waiting time beyond which the patient will feel unsatisfied and be more likely to leave the ED. Between the two limits, there is an area that corresponds to low satisfaction, in which the patient still waits but with an increasing likelihood of self-discharge. After these limits are defined, the transition probabilities for the BMC are computed by evaluating the area under the curve bounded by the satisfaction limits (Figure 5).

Calculation of the probabilities from the distribution and the limits for the chains was carried out in MATLAB as follows:

```
x = 0:480; % choosing a representative interval on the x-axis ranging from 0 to 480 minutes

mi = 87.4; % defining the mean of the waiting time, which was previously computed with the "mean"
command.

sigma = 8.73; % defining the standard deviation of the waiting time, which was previously computed
with the "std" command.

y =normpdf (x,mi,sigma); % defining the normal distribution using mi and sigma as the mean and
standard deviation, respectively

% graph representing the distribution
plot (x,y,'k');
hold on

min = 90; % defining the lower limit
max = 110; % defining the upper limit

% drawing limits and mean on the figure
y= linspace (0,0.047);
xmin= min*ones (size (y));
plot (xmin,y,'b')
hold on
y = linspace (0,0.047);
media = mi*ones (size (y));
plot (media,y,'r')
xmax = max*ones (size (y));
plot (xmax,y,'b')

% computing the areas under the curves
PrSodd = normcdf (min,mi,sigma)
PrInsodd = 1- normcdf (max,mi,sigma)
PrPSodd = normcdf (max,mi,sigma)-normcdf (min,mi,sigma)
```

2.4. Step 3: Validation

For validation, a case study with real ED data is considered. Data were supplied by the CED of the hospital "A. Cardarelli" of Naples.

The transition probabilities of the MC model, which as stated above are the probabilities of going from one state to another, are calculated based on data on the number of patients that arrived at the ED per triage code provided by the CED (Table 1).

Table 1. Number of patients who arrived at the ED per triage code.

Priority code	Medical resources	Orthopedic resources	Surgical resources	Total
White Code	181	58	18	257
Green Code	21703	7909	7873	37485
Yellow Code	21919	262	1832	24013
Red Code	717	7	114	838
TOTAL	44520	8236	9837	62593

The values in Table 1 represent the absolute numbers of patients who came to the ED to receive first aid during an 11-month period (data from February to December 2013). The transition probabilities related to the number of patients who underwent laboratory tests and/or radiographic examination are obtained by considering the data and percentages reported in Table 2.

Table 2. Percentages of patients who underwent analyses and examinations in three different cases: surgical, medical and orthopedic.

Priority code	Laboratory analysis	%	Diagnostic imaging	%	No analysis	%	Arrivals
a) Surgical cases							
White Code	0	0	4	0.22	14	0.78	18
Green Code	180	0.02	4394	0.56	3299	0.42	7873
Yellow Code	267	0.15	1292	0.70	273	0.15	1832
Red Code	43	0.38	71	0.62	0	0	114
TOTAL	**490**	**0.05**	**5761**	**0.59**	**3586**	**0.36**	**9837**
b) Medical cases							
White Code	0	0	18	0.10	163	0.90	181
Green Code	2236	0.10	4073	0.19	15394	0.71	21703
Yellow Code	4648	0.21	3954	0.18	13317	0.61	21919
Red Code	375	0.52	342	0.48	0	0	717
TOTAL	**7284**	**0.16**	**8412**	**0.19**	**28874**	**0.65**	**44520**
c) Orthopedic cases							
White Code	0	0	9	0.16	49	0.84	58
Green Code	10	0.01	3100	0.39	4799	0.60	7909
Yellow Code	7	0.03	133	0.51	122	0.46	262
Red Code	1	0.14	1	0.14	5	0.72	7
TOTAL	**18**	**0.01**	**3243**	**0.39**	**4975**	**0.60**	**8236**

3. Results

Among the 120 questionnaires collected, only those that related to medical cases were considered, and then the limits were defined according to the times that would lead to different states of satisfaction for each phase and for each code. A total of 75 questionnaires were related to medical cases, of which 23 were white codes, 32 were green codes, and 20 were yellow codes.

The times used to define the limits of satisfaction are described as follows: the lower limit is the maximum time before which the patient feels satisfied, and the upper limit is the maximum time after which the patient declares himself dissatisfied and leaves the emergency department. These limits are determined from the questionnaires completed by waiting patients, using the arithmetic mean of the data collected; "wait time adjusted" refers to the lower bounds, whereas "maximum wait time for customer satisfaction" refers to the upper limit. We computed the lower limit by averaging the times from the questionnaires for which patients stated “acceptable wait time”; a similar computation was performed for the upper limit.

The distribution of patient satisfaction for the green medical code is shown in Figure 6, which reports the limits of the patient’s satisfaction in terms of wait times from the arrival to the emergency room to the admission procedure. More specifically, if the waiting time is below the lower limit, the patient is satisfied; if the waiting time is above the upper limit, the patient is unsatisfied and leaves the emergency room without being seen.

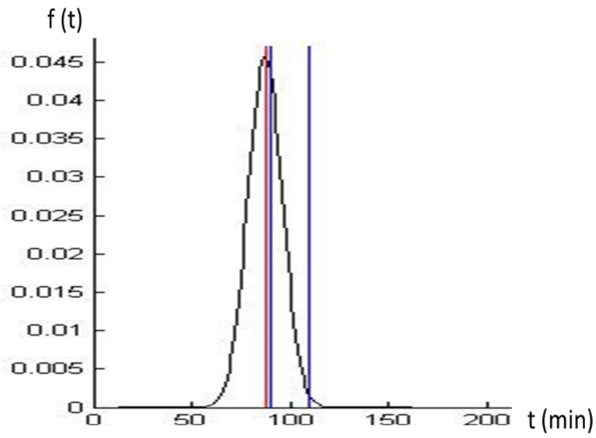

Figure 6. Waiting time distribution.

In Figure 6, the three vertical lines represent the average waiting time before a green-code patient is admitted (red line) and the upper and lower limits, in terms of waiting time, for patient satisfaction (blue lines, right and left, respectively).

For each specific transition state, the probability of the patient being satisfied (Pr_{Sat}), not very satisfied (Pr_{notSat}), or dissatisfied (Pr_{Dissat}) is equal to the area under a certain portion of the distribution curve, bounded by the minimum and maximum wait times and computed by using MATLAB software as follows:

- Pr_{Sat} = 0.62 (area to the left of the lower limit);
- Pr_{notSat} = 0.375 (area between the lower and the upper limits); and
- Pr_{Dissat} = 0.005 (area to the right of the upper limit).

Transition probability chains representing three cases (white, green and yellow codes) are presented in Figures 7 8 and 9, respectively. For each figure, the transition probabilities are reported in bold next to each arrow connecting different states of the process, from patient arrival to final discharge.

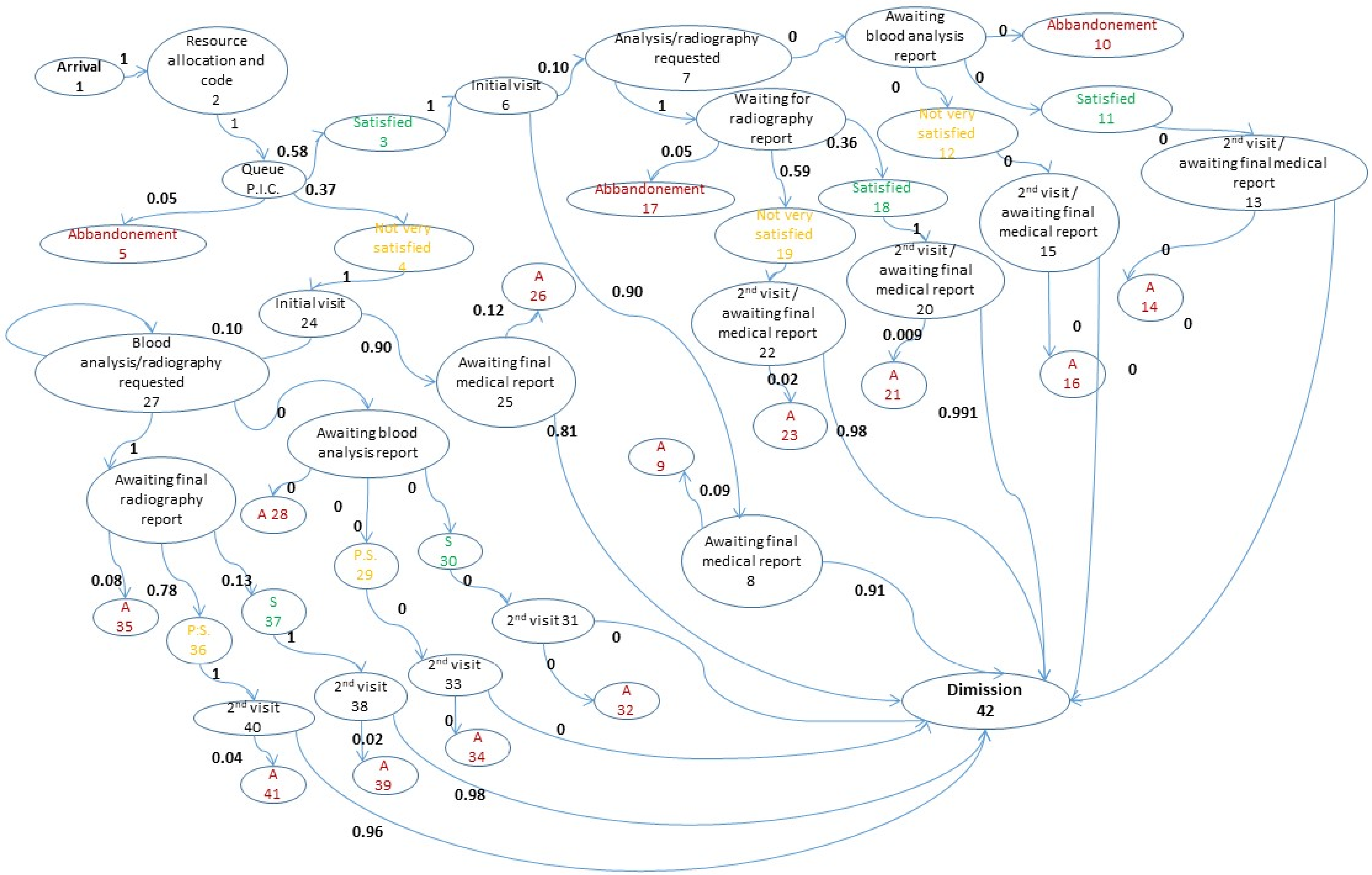

Figure 7. Probability chain: white code.

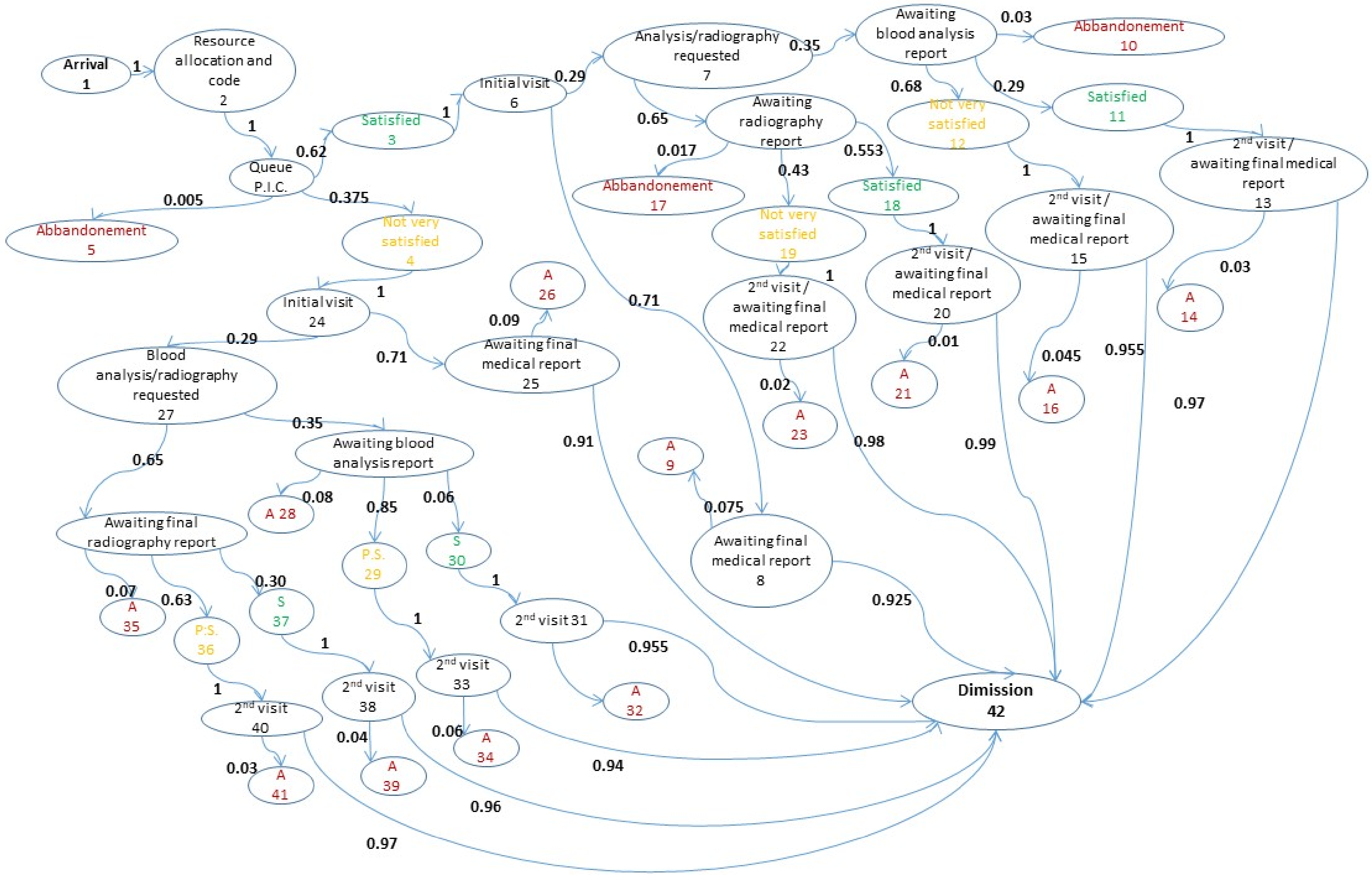

Figure 8. Probability chain: green code.

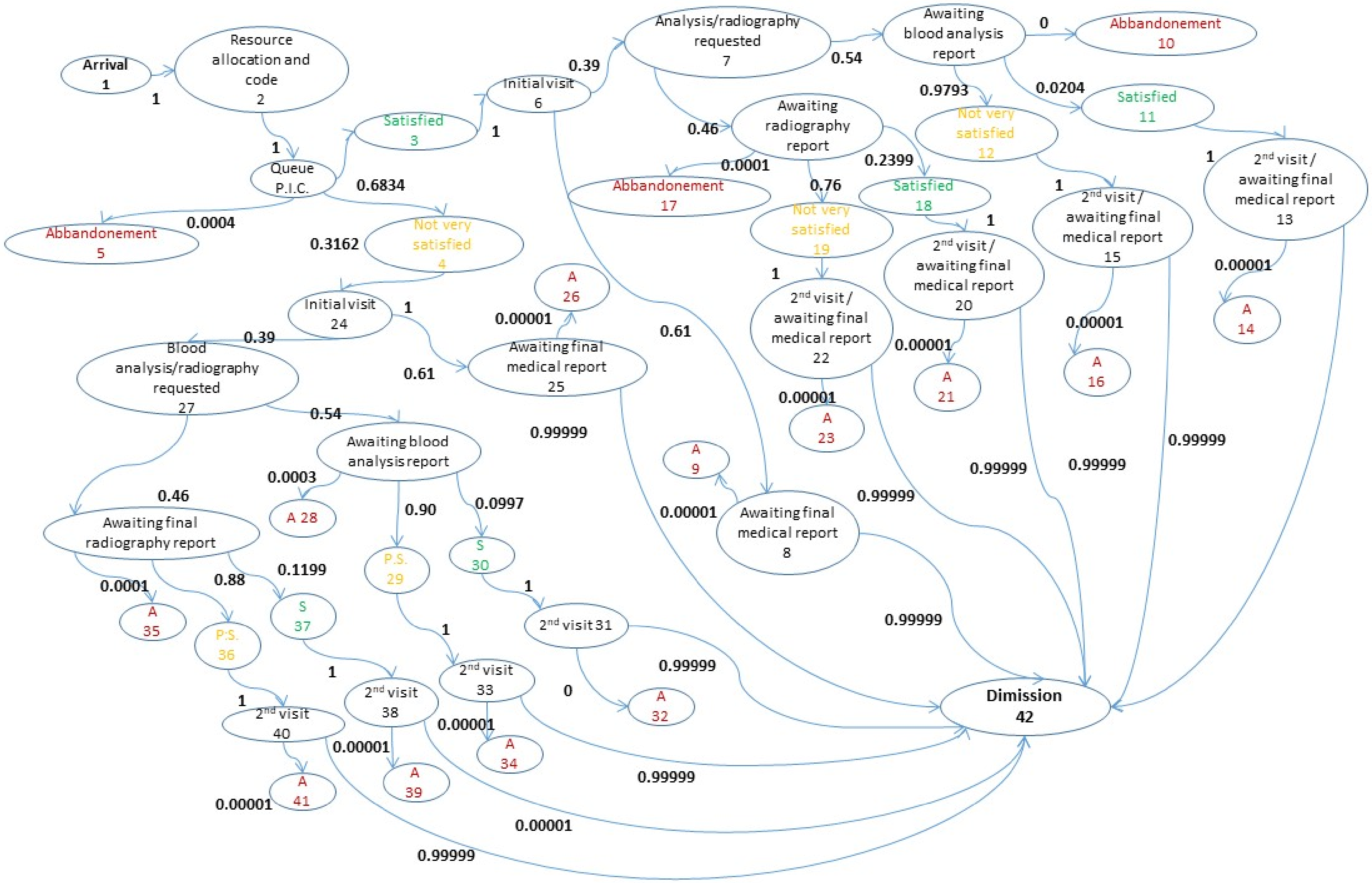

Figure 9. Probability chain: yellow code.

Some peculiarities can be seen in the figures*:*

- Since patients with white codes do not undergo laboratory analysis, all the transitions from "awaiting blood test report" nodes have a probability equal to zero;
- For the yellow code, the probability of dissatisfaction that leads to self-discharge is particularly low because the higher gravity of the patient's condition makes him or her more likely to wait even longer to be treated. Nevertheless, yellow-code patients are almost always not very satisfied with the wait times until discharge.

3.1. Chain behavior under present conditions

To compute the probability of a patient voluntarily departing from the ED without being seen by a doctor, we calculate the transition probabilities of each of the chains under nominal conditions by setting the arrival of the patient as the initial condition. The sum of the computed probabilities gives the rate of departure from the emergency department throughout the profiled processes.

- For the white code, the voluntary departure rate is 2.88%.
- For the green code, the voluntary departure rate is 1.35%.
- Finally, for the yellow code, the voluntary departure rate is 0.0009%.

As expected, the likelihood of self-discharge is higher for lower-priority codes, with the value for the yellow code being practically zero because yellow-code patients are willing to wait a very long time just to be seen. The number of patients who are at risk of self-discharge during the period in question is obtained by multiplying the probabilities for each code by the corresponding total number of patients with that code: from a total of 31 white-code patients, one patient is at risk of self-discharge, and from a total of approximately 4073 green-code patients, 55 patients are at risk of self-discharge. The likelihood of self-discharge due to awaiting the final report has a substantial impact on the overall self-discharge percentage. In general, the interviewed patients expect the time to discharge to be lower than the average wait time; for yellow-code patients, high dissatisfaction is caused by awaiting the outcome of laboratory tests.

3.2. Sensitivity analysis

Once the model was validated and its operation observed in present conditions, we wanted to see how the percentages of self-discharge could be changed. As stated above, of the three cases presented, the probability of a yellow-code patient self-discharging is remarkably low; thus, we chose to evaluate the positive changes that can be achieved for lower priority cases (white and green codes). In particular, we aimed to analyze the degree to which a decrease in the average time required to discharge a patient and in the average time taken to provide patients with the outcomes of their radiographic analysis could impact the likelihood of self-discharge. The main results regarding this sensitivity analysis are summarized in Table 3 and discussed in the following sections for each code and hypothesis.

For the white code, the sample mean discharge time is 40.4 minutes. A decrease of 10% in the discharge time means that patients will experience a mean wait time of 36.4 minutes. If the chain starts from "satisfied with waiting time for admission", the stated limits are 45 to 60 minutes for satisfaction and dissatisfaction, respectively, and patients transition with the previous probabilities for self-discharge (Table 3).

If the chain starts from "not very satisfactory with waiting time for admission not very satisfied", the stated limits are 35 minutes for satisfaction and 55 for self-discharge (Table 3).

The sample mean time to report the results of radiographic analysis is 97.2 minutes; a decrease of 10% means that patients experience an average wait time of 87.5 minutes. However, if the chain starts from "satisfied with waiting time for admission", the satisfaction and dissatisfaction limits are 90 and 130 minutes, respectively (Table 3).

Table 3. Summary of the main results of the sensitivity analysis.

	Decrease in	Probability	Chain started from: Satisfied with waiting time for admission	Chain started from: Not very satisfied with waiting time for admission	New self-discharge percentage (%)
White code	Discharge time	Pr_{Sat}	0.7849	0.4489	1.60
		Pr_{notSat}	0.0152	0.0440	
		Pr_{Dissat}	0.1999	0.5071	
	Radiographic reporting time	Pr_{Sat}	0.5497	0.2660	2.70
		Pr_{notSat}	0.0168	0.0304	
		Pr_{Dissat}	0.4335	0.7036	
Green code	Discharge time	Pr_{Sat}	0.4338	0.2556	1.00
		Pr_{notSat}	0.0364	0.0473	
		Pr_{Dissat}	0.5298	0.6971	
	Radiographic reporting time	Pr_{Sat}	0.6041	0.2309	1.34
		Pr_{notSat}	0.0029	0.0152	
		Pr_{Dissat}	0.3930	0.7539	

If the chain starts at "not very satisfied with waiting time for admission", the stated limits are 75 minutes for satisfaction and 125 minutes for self-discharge (Table 3).

For the green code, the sample mean time to discharge is 70.45 minutes. A decrease in the mean time of 10% results in an average wait time of 63.4 minutes. If the chain starts from "satisfied with waiting time for admission", the limits are 60 minutes for satisfaction and 100 minutes for dissatisfaction; the patients transition with the previous probabilities of 0.075 for dropping and 0.925 for discharge (Table 3).

If the chain starts at "not very satisfied with waiting time for admission", the stated limits are 50 minutes for satisfaction and 97.5 for dropping. The previous transition probabilities, 0.09 for dropping and 0.91 for discharge, are used for the patients (Table 3).

The sample mean laboratory reporting time is 160.8 minutes. A decrease in the mean time of 10% results in an average wait time of 144.72 minutes. If the chain starts from "satisfied with waiting time for admission", the stated limits are 150 minutes for satisfaction and 200 minutes for dissatisfaction (Table 3).

If the chain starts at "not very satisfied with waiting time for admission", the stated limits are 130 min for satisfaction and 188 for dissatisfaction (Table 3).

Simulating the chain again with the new transition probabilities for 650 steps, we achieve a new deployment limit in which the sum of the probabilities of transitions toward self-discharge provide the values shown in Table 3. For the white code, we achieved a good reduction in the rate of voluntary departure, demonstrating the great importance of the transitions in the self-discharge percentage by reducing the wait time for the final report (1.6%), while the decrease in the wait time for radiographic analysis did not result in a substantial change in the self-discharge percentage. For the green code, we obtained a decrease in the rate of voluntary departure similar to that of the white-code priority (1.0%), while the decrease in the dropout percentage obtained by reducing the wait time for radiographic analysis was not very high; this is because, as in the case of the white code, the patient is more willing to wait longer to receive the results of the tests. Nevertheless, the level of dissatisfaction is lower, indicating an opportunity to improve the perception of quality of the department in the eyes of the patient.

In conclusion, most of the risk of self-discharge occurs during the wait time before the patient is seen and during the wait time for the final test report; generally, once the analysis (either radiographic or laboratory) is requested, the patient, although not very satisfied, is willing to wait longer for the results. To continuously improve the quality of service, it is necessary to eliminate waste throughout the process, thus reducing the time required to carry out individual tasks and, therefore, the patient wait time. In this way, as evidenced by the sensitivity analysis, the level of dissatisfaction and, consequently, the percentage of voluntary departure from the ED could be reduced.

4. Discussion and conclusions

This paper describes a new methodological approach for describing and predicting the probability of voluntary patient departure from the ED. In particular, MC theory is applied to the first aid process (Figure 1) and modified to take into account patient satisfaction. Then, a BMC (Figure 3) model is developed, compared to the MC model (Figure 2), and adopted as a novel strategy to model the ED from the patient's perspective and to offer a potential way to predict the probability of self-discharge from the ED.

In the BMC model, states represent the level of patient satisfaction and can be achieved with transition probabilities that depend on wait times. A method to calculate transition probabilities related to patient satisfaction is proposed (Figure 5). The proposed model is then applied and validated in a real case study (Figures 6–9).

The work tackles an issue of great interest in the field of process management and analysis; it focuses on the processes that take place in the ED, whose organization and proper functioning are fundamental to every hospital, as this department is, in most cases, the first contact between the patient and his or her health care. In the literature, there are a remarkable number of articles addressing the topic of patients who leave the hospital before being discharged, in which several organizational solutions are proposed and/or adopted. The adoption of a patient-tracking system, such as radio-frequency identification (RFID) and electronic bracelets, is most frequently preferred.

In this paper, the problem of patients discharging themselves against medical advice is analyzed as a risk that the researchers sought to examine as a model with the help of Markov theory, resulting in one of the most innovative approaches in health care. To our knowledge, indeed, the application of MC to health care processes has not been described in the literature.

This work aims to describe and validate a new method for calculating the percentage of patient self-discharge from the ED, obtained by presenting a generic model, describing the process of first aid using MCs, and taking into account patient satisfaction.

It is necessary to identify the conditions leading to patient satisfaction and then calculate the relative transition probabilities that depend on patient expectation. Therefore, the first result is the implementation of a BMC to describe the first aid process, which is formed by states as seen from the perspective of the patient. The model is presented as generic and thus can be adapted to each health facility by changing only the transition probabilities between states. In the final analysis, the probability of patient self-discharge from hospital "A. Cardarelli" of Naples is calculated according to Markov theory or by calculating the wait time distribution and limits. The self-discharge percentage corresponds to the result obtained from the sum of the transition probabilities associated with the self-discharge state at different phases of the process. This allows us to analyze the impact of each activity and the wait time of each stage based on the calculated probability. The self-discharge percentage is evaluated only for the emergency categories, shown as white, green, and yellow codes, in terms of medical resource use only; it is then subjected to a sensitivity analysis in which the wait times are improved. The proposed approach is novel when compared with the current models in the literature, where the satisfaction of the patient is not considered, or only marginally if so. The possibility of generating a probability of leaving for each step in the ED procedure, in fact, can help in modifying only crucial aspects of the process to reduce the percentage of LWBS.

In summary, the proposed approach provides another perspective in the modeling of healthcare processes since it implements a patient-centered design, where the optimization of healthcare workflows and processes takes into account the patient's perspective. This is particularly relevant and useful in modeling complex processes such as those of an ED. Indeed, EDs are the first access to healthcare facilities and the departments where patients suffer the most from organizational inefficiencies (e.g., long wait times, lack of staff, inappropriate triage coding). Therefore, the ED is the ideal environment for the application of the proposed model, since it acts like a resonance chamber for both the patients' perception of quality (degree of satisfaction) and process inefficiencies (wait times), which are the main factors taken into consideration by the proposed BMC. Among other simulation/modeling approaches, the BMC is chosen for its ability to take into account not only the transition probabilities from one state to another within the ED process but also patient satisfaction, which is one of the main drivers of voluntary departure from the ED.

Furthermore, the simulation and modeling of self-discharge percentages could also be extended or adapted to other cases, situations, and environments.

4.1. Novelty of the study

The novelty of the work proposed here consists of a new way to apply Markov chain theory to the healthcare process based on a patient perspective and taking into consideration patient satisfaction. In particular, a behavioral Markov chain model is developed and proposed as a tool to model the first aid process and predict the probability of voluntary patient departure from the ED.

4.2. Limitations of the study

First, although the proposed model is a promising tool for modeling the flow in an ED and calculating the probability of patient self-discharge from an ED, a validation step should be implemented to test the validity of the model with real data. Second, the model is valid for the hospital where the data were collected, and different hospitals require their own model to be computed, as different hospitals may have different priority codes and associated probabilities.

Finally, the variation in the patient flow in the ED over time is a concern. In this work, a new method to reduce the percentages of patients leaving the department without receiving appropriate assistance was developed. From this perspective, our interest was to compare our method with existing methods and explain how our approach can be considered novel. The time-varying nature of the emergency department can change the waiting time, but we did not truly examine this relationship in depth; rather, we focused on the level of patient satisfaction, correlating it with the decision to leave.

Conflict of interests

The authors declare no potential conflict of interest.

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