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Research article

An efficient numerical algorithm for solving fractional SIRC model with salmonella bacterial infection

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Abstract: This paper revisits the study of numerical approaches for fractional SIRC model with Salmonella bacterial infection (FSIRC-MSBI). This model is investigated by the aid of fully shifted Jacobi's collocation method for temporal discretization. It is concluded that the method of the current paper is far more efficient and reliable for the considered model. Numerical results illustrate the performance efficiency of the algorithm. The results also point out that the scheme can lead to spectral accuracy of the studied model.

Keywords: fractional SIRC model; spectral collocation method; Gauss-Radau quadrature; Shifted Jacobi polynomials; Caputo fractional derivative

1. Introduction

In the last decades, fractional calculus theory [1–4] has been developed rapidly. It has been applied in several scientific areas like physics, economic, diffusion processes, serology, engineering, etc. [5–10]. Actually, fractional calculus theory was spotted as a veritable development to classical calculus theory. Large amount of work on modelling biological systems has been restricted to fractional order ordinary differential equations [11,12]. Therefore, the urgent necessity to find the exact solutions or merely the approximate ones to these problems has emerged. Whereas obtaining the exact solution is complicated to get, the numerical solution as an alternative was appeared and its methods were developed.

Mathematical modelling of infectious diseases has been studied for a long time. The classical susceptible-Infected-Recovered (SIR) model has been introduced in [13] and several studies have investigated the dynamical behaviors of SIR model [14,15]. Casagrandi et al. [16] developed SIR model by introducing a new compartment called cross-immune compartment. The new development of SIR model named by SIRC model which describes the case between the fully susceptible and the fully protected one. More recently, the fractional-order SIRC model, a disease in human population, was discussed in [17,18,19].

Salmonella infection [20] is a common disease caused by the Salmonella bacteria. It affects on the intestinal tract in which develop consequently diarrhea, fever, and abdominal cramps. Salmonella infection is usually considered as the public health issue. The objective of this work is to introduce an efficient numerical algorithm for solving FSIRC-MSBI.

In many areas of sciences such as engineering, biology, economics, physics and others, several high-order numerical methods have been developed to deal with the related problems. Recently, spectral methods are known as an efficient and highly accurate schemes. Exponential rate of convergence and a high level of accuracy are the main characteristic of spectral methods. The spectral method is classified into four kinds namely collocation [20], tau [21], Galerkin [22] and Petrov Galerkin [23] methods. Here, shifted Jacobi Gauss-Radau collocation (SJ-GR-C) method is developed to approximate the system of a FSIRC-MSBI.

The paper is organized as follows. We present some mathematical preliminaries in section 2. In sections 3, we propose a numerical technique for solving system of a FSIRC-MSBI. Section 4 implements the proposed method on an example to show its accuracy and efficiency. Finally, in section 5 we outline the main conclusions.

2. Preliminaries and notation

2.1. *Shifted Jacobi polynomials*

For shifted Jacobi polynomial $\mathcal{P}_{\mathfrak{L},k}^{(\rho,\sigma)}(x) = \mathcal{P}_{k}^{(\rho,\sigma)}(\frac{2x}{\mathfrak{L}} - 1), \mathfrak{L} > 0$, where $\mathcal{P}_{k}^{(\rho,\sigma)}(x)$ is the standard Jacobi polynomial [24,25], the analytic formula is

$$\begin{aligned} \mathcal{P}_{\mathfrak{L},k}^{(\rho,\sigma)}(x) &= \sum_{j=0}^{k} (-1)^{k-j} \frac{\Gamma(k+\sigma+1)\Gamma(j+k+\rho+\sigma+1)}{\Gamma(j+\sigma+1)\Gamma(k+\rho+\sigma+1)(k-j)!j!\mathfrak{L}^j} x^j \\ &= \sum_{j=0}^{k} \frac{\Gamma(k+\rho+1)\Gamma(k+j+\rho+\sigma+1)}{j!(k-j)!\Gamma(j+\rho+1)\Gamma(k+\rho+\sigma+1)\mathfrak{L}^j} (x-\mathfrak{L})^j. \end{aligned} \tag{1}$$

Taking $w_{\mathfrak{L}}^{(\rho,\sigma)}(x) = (\mathfrak{L} - x)^{\rho} x^{\sigma}$, for the weighted space $\mathrm{L}^2_{w_{\mathfrak{L}}^{(\rho,\sigma)}}[0,\mathfrak{L}]$, we get

$$(\phi,\varphi)_{w_{\mathfrak{L}}^{(\rho,\sigma)}} = \int_0^{\mathfrak{L}} \phi(x)\varphi(x) w_{\mathfrak{L}}^{(\rho,\sigma)}(x)dx,$$

$$\| \varphi \|^2_{w_{\mathfrak{L}}^{(\rho,\sigma)}} = (\varphi,\varphi)_{w_{\mathfrak{L}}^{(\rho,\sigma)}}, \| \mathcal{P}_{\mathfrak{L},k}^{(\rho,\sigma)} \|^2_{w_{\mathfrak{L}}^{(\rho,\sigma)}} = \left(\frac{\mathfrak{L}}{2}\right)^{\rho+\sigma+1} h_k^{(\rho,\sigma)}. \tag{2}$$

We used $\mathrm{t}_{\mathcal{S},s}^{(\rho,\sigma)}$, and $\varpi_{\mathcal{S},s}^{(\rho,\sigma)}, 0 \le s \le \mathcal{S}$, for the nodes and Christoffel numbers of the Jacobi Gauss Radau interpolation. Related to the shifted Jacobi polynomials, we list

$$t_{\mathfrak{L},\mathcal{S},s}^{(\rho,\sigma)} = \frac{\mathfrak{L}}{2}(t_{\mathcal{S},s}^{(\rho,\sigma)} + 1),$$

$$\mathfrak{w}_{\mathfrak{L},\mathcal{S},s}^{(\rho,\sigma)} = \left(\frac{\mathfrak{L}}{2}\right)^{\rho+\sigma+1} \mathfrak{w}_{\mathcal{S},s}^{(\rho,\sigma)}, \; 0 \le s \le \mathcal{S}.$$

Given $\mathcal{R} \ge 0$, $\Omega \in \mathbb{S}_{2\mathcal{R}+1}[0,\mathfrak{L}]$ and by means of Jacobi-Gauss quadrature property, we obtain

$$\begin{aligned} \int_0^{\mathfrak{L}} (\mathfrak{L}-t)^{\rho} t^{\sigma} \Omega(t)dt &= \left(\frac{\mathfrak{L}}{2}\right)^{\rho+\sigma+1} \int_{-1}^{1} (1-t)^{\rho}(1+t)^{\sigma} \Omega\left(\frac{\mathfrak{L}}{2}(t+1)\right)dt \\ &= \left(\frac{\mathfrak{L}}{2}\right)^{\rho+\sigma+1} \sum_{s=0}^{\mathcal{S}} \mathfrak{w}_{\mathcal{S},s}^{(\rho,\sigma)} \Omega\left(\frac{\mathfrak{L}}{2}(t_{\mathcal{S},s}^{(\rho,\sigma)}+1)\right) \\ &= \sum_{s=0}^{\mathcal{S}} \mathfrak{w}_{\mathfrak{L},\mathcal{S},s}^{(\rho,\sigma)} \Omega\left(t_{\mathfrak{L},\mathcal{S},s}^{(\rho,\sigma)}\right). \end{aligned} \tag{3}$$

2.2. The fractional integration in the Caputo sense

The fractional derivative is occurred in various formulas which are not generally equal, see [26]. The two most famous definitions are Riemann-Liouville and Caputo ones.

Definition 2.1. For $\nu > 0$, the Riemann-Liouville fractional integral is

$$\begin{aligned} J^{\nu}\mathfrak{F}(x) &= \frac{1}{\Gamma(\nu)} \int_0^{\xi} (\xi-\zeta)^{\nu-1} \mathfrak{F}(\zeta)d\zeta, \nu > 0, \xi > 0, \\ J^{0}\mathfrak{F}(\xi) &= f(\xi), \end{aligned} \tag{4}$$

where

$$\Gamma(\nu) = \int_0^{\infty} x^{\nu-1} e^{-x} dx,$$

is gamma function.

Definition 2.2. For $\nu > 0$, the Caputo fractional derivatives is

$$D^{\nu}\mathfrak{F}(x) = \frac{1}{\Gamma(m-\nu)} \int_0^{\xi} (\xi-\zeta)^{m-\nu-1} \frac{d^m}{dt^m} f(\zeta)d\zeta, m-1 < \nu \le m, \; \xi > 0, \tag{5}$$

where m is the ceiling function of ν.

3. Jacobi spectral collocation scheme

In this work, we want to numerically solve the FSIRC-MSBI. The classical disease model is given as

$$\begin{aligned} \dot{\mathcal{S}} = \; & \Omega_1(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) = \mu N + \eta \mathcal{C}(t) - (\beta \mathcal{I}(t) + \mu)\mathcal{S}(t), \\ \dot{\mathcal{I}} = \; & \Omega_2(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) = \beta \mathcal{S}(t)\mathcal{I}(t) + \sigma\beta\mathcal{I}(t)\mathcal{C}(t) - (\theta + m + \mu)\mathcal{I}(t), \\ \dot{\mathcal{R}} = \; & \Omega_3(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) = (1-\sigma)\beta\mathcal{C}(t)\mathcal{I}(t) + \theta\mathcal{I}(t) - (\mu+\delta)\mathcal{R}(t), \\ \dot{\mathcal{C}} = \; & \Omega_4(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) = \delta\mathcal{R}(t) - \beta\mathcal{C}(t)\mathcal{I}(t) - (\mu+\eta)\mathcal{C}(t), \end{aligned} \tag{6}$$

where θ^{-1}, μ, η^{-1}, δ^{-1}, β, ρ are the infectious period, the mortality rate in every compartment, crossimmune period, the total immune period, the contact rate and the fraction of the exposed cross immune individuals, respectively. While, $\mathcal{S}$, $\mathcal{I}$, $\mathcal{R}$, $\mathcal{C}$ are the proportion of susceptible individuals, the proportion of infected individuals, the proportion of recovered individuals and the proportionthe total number of herd animals is $N = \mathcal{S} + \mathcal{I} + \mathcal{R} + \mathcal{C}$. The more general class of the previous system is called FSIRC-MSBI and is given by:

$$\begin{aligned} D^{\nu_1}\mathcal{S}(t) &= \Omega_1(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}), \mathcal{S}(0) = \mathcal{S}_0, \\ D^{\nu_2}\mathcal{I}(t) &= \Omega_2(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}), \mathcal{I}(0) = 0, \\ D^{\nu_3}\mathcal{R}(t) &= \Omega_3(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}), \mathcal{R}(0) = 0, \\ D^{\nu_4}\mathcal{C}(t) &= \Omega_4(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}), \mathcal{C}(0) = 0. \end{aligned} \tag{7}$$

We use SJ-GR-C technique to solve FSIRC-MSBI. The main idea is to reduce FSIRC-MSBI to a system of algebraic equations that easily solved. To do this, Shifted Jacobi polynomials are appointed for the temporal discretization. The efficiency of our technique will examine via a numerical test problem.

Firstly, we rewrite the system (7), as

$$D^{\nu}\Psi(t) = F(t,\Psi(t)), \tag{8}$$

where

$$D^{\nu}\Psi = \begin{pmatrix} D^{\nu_1}\mathcal{S}(t) \\ D^{\nu_1}\mathcal{I}(t) \\ D^{\nu_1}\mathcal{R}(t) \\ D^{\nu_1}\mathcal{C}(t) \end{pmatrix} = \begin{pmatrix} \Omega_1(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) \\ \Omega_2(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) \\ \Omega_3(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) \\ \Omega_4(t,\mathcal{S},\mathcal{C},\mathcal{I},\mathcal{R}) \end{pmatrix}.$$

Via SJ-GR-C method, we approximate the independent variable at $t^{(\rho,\sigma)}_{\mathfrak{L},\mathcal{N},j}$, where $t^{(\rho,\sigma)}_{\mathfrak{L},\mathcal{N},i}$ is a Jacobi-Gauss-Radau collocation nodes.

The approximate solution of (8) is

$$\psi_i(t) = \sum_{j=0}^{N} a_{ij} P^{(\rho,\sigma)}_{\mathfrak{L},j}(t), \ i = 1,2,3,4. \tag{9}$$

the fractional derivative $D^{\nu}\psi_i(t)$ is

$$D^{\nu}\psi_i(t) = \sum_{j=0}^{N} a_{ij} D^{\nu}(P^{(\rho,\sigma)}_{\mathfrak{L},j}(t)), \ i = 1,2,3,4. \tag{10}$$

And using analytical form of shifted Jacobi polynomial, we find

$$\begin{aligned} D^{\nu}P^{(\rho,\sigma)}_{\mathfrak{L},j}(x) = P^{(\rho,\sigma,\nu)}_{\mathfrak{L},j}(t) = &\sum_{k=0}^{j} (-1)^{j-k} \frac{\Gamma(j+\sigma+1)\Gamma(k+j+\rho+\sigma+1)}{\Gamma(k+\sigma+1)\Gamma(j+\rho+\sigma+1)(j-k)!k!\mathfrak{L}^k} D^{\nu}t^k \\ &= \sum_{k=0}^{j} (-1)^{j-k} \frac{(\Gamma(k+1)\Gamma(j+\sigma+1)\Gamma(j+k+\rho+\sigma+1))}{k!\mathfrak{L}^k(j-k)!\Gamma(k+\sigma+1)\Gamma(k+\nu)\Gamma(j+\rho+\sigma+1)} t^{k+\nu-1}. \end{aligned} \tag{11}$$

Then

$$D^{\nu}\psi_i(t) = \sum_{j=m}^{N} a_{ij} D^{\nu}(P^{(\rho,\sigma)}_{\mathfrak{L},j}(t)) = \sum_{j=m}^{N} a_{ij} P^{(\rho,\sigma,\nu)}_{\mathfrak{L},j}(t). \tag{12}$$

Consequently

$$\Psi^{(\nu)}(\mathrm{t}) = \mathcal{F}(\mathrm{t}), \tag{13}$$

where

$$\Psi^{(\nu)}(t) = \begin{pmatrix} \sum_{j=m}^{N} a_{1j} P_{\mathfrak{L},j}^{(\rho,\sigma,\nu_1)}(t) \\ \sum_{j=m}^{N} a_{2j} P_{\mathfrak{L},j}^{(\rho,\sigma,\nu_2)}(t) \\ \sum_{j=m}^{N} a_{3j} P_{\mathfrak{L},j}^{(\rho,\sigma,\nu_3)}(t) \\ \sum_{j=m}^{N} a_{4j} P_{\mathfrak{L},j}^{(\rho,\sigma,\nu_4)}(t) \end{pmatrix}$$

$$\mathcal{F}(t) = \begin{pmatrix} \Omega_1(t, \sum_{j=0}^{N} a_{1j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{2j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{3j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{4j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t)) \\ \Omega_2(t, \sum_{j=0}^{N} a_{1j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{2j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{3j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{4j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t)) \\ \Omega_3(t, \sum_{j=0}^{N} a_{1j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{2j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{3j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{4j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t)) \\ \Omega_4(t, \sum_{j=0}^{N} a_{1j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{2j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{3j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t), \sum_{j=0}^{N} a_{4j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(t)) \end{pmatrix} \tag{14}$$

Let $t = t_{\mathfrak{L},\mathcal{N},r}^{(\rho,\sigma)}$, to get

$$\Psi^{(\nu)}(t_{\mathfrak{L},\mathcal{N},r}^{(\rho,\sigma)}) = \mathcal{F}(t_{\mathfrak{L},\mathcal{N},r}^{(\rho,\sigma)}), \qquad r = 1,2,\dots,\mathcal{N}. \tag{15}$$

Also, we have

$$\sum_{j=0}^{N} a_{1j} P_{\mathfrak{L},j}^{(\rho,\sigma)}(0) = S_0, \qquad \sum_{j=0}^{N} a_{ij} P_{\mathfrak{L},j}^{(\rho,\sigma)}(0) = 0, \qquad i = 2,3,4, \tag{16}$$

merge Eqs (15) and (16), to build a system of $(4N + 4)$ algebraic equations that easily solved. The existence and uniqueness are realized by the following theories.

Theorem 3.1. Let $0 \le \mu < \nu < 1$ and let $F: [0, L] \times \mathbb{R} \to \mathbb{R}$ be a given function continuous in $(0, L] \times \mathbb{R}$. Assume that $t^{\mu} F(t, \psi)$ is a continuous function on $(0, L] \times \mathbb{R}$. Then the fractional differential equation

$$D^{\nu}\Psi(t) = F(t, \Psi(t)), \tag{17}$$

has at least a continuous solution defined on $[0, L]$, d for a suitable $d \le L$.

Theorem 3.2. Let $0 \le \mu < \nu < 1$ and assume $t^{\mu} F(t, \psi)$ is continuous on $(0, L] \times \mathbb{R}$. Assume further

$$\| F(t, \phi) - F(t, \varphi) \| \le \frac{T}{t^{\mu}} \| \phi - \varphi \|, \tag{18}$$

for some positive constant T independent of $\phi, \varphi \in \mathbb{R}$ and $t \in [0, L]$. Then the equation

$$D^{\nu}\Psi(t) = F(t, \Psi(t)), \tag{19}$$

has a unique solution $\phi \in C^0[0, L]$. The proofs of the previous theories can directly obtain from [27].

4. Error analysis

Using Lagrange interpolation polynomials, we introduce an upper bound of the absolute errors.

Theorem 4.1. Suppose that $D^k\chi(x) \in C[0,\mathfrak{L}]$ for $k = 0,1,\ldots,N-1, (3+2N+\sigma) > 0$. If $\chi_{\mathcal{N}}(x)$ is the best approximation to $\chi(x)$ from F_N, then the error bound is presented as follows

$$\| \chi(x) - \chi_{\mathcal{N}}(x) \|_{\mathcal{W}_{\mathfrak{L},f}^{(\rho,\sigma)}(x)} \le \frac{E\gamma(\rho+1)}{\Gamma((\mathcal{N}+1)+1)} \sqrt{\frac{\mathfrak{L}^{(2\mathcal{N}+\rho+3)}\Gamma(3+2\mathcal{N}+\sigma)}{\Gamma(4+2\mathcal{N}+\rho+\sigma)}} \tag{20}$$

Proof. Since $\chi_{\mathcal{N}}(x)$ is the best approximation to $\chi(x)$ from $\mathcal{F}_{\mathcal{N}}$,then by the definition of the best approximation, we have

$$\forall \mathcal{V}_{\mathcal{N}}(x) \in F_N, \| \chi(x) - \chi_{\mathcal{N}}(x) \|_{\mathcal{W}_{\mathfrak{L},f}^{(\rho,\sigma)}(x)} \le \| \chi(x) - \mathcal{V}_{\mathcal{N}}(x) \|_{\mathcal{W}_{\mathfrak{L},f}^{(\rho,\sigma)}(x)} \tag{21}$$

Based on the generalized Taylors formula [28], we obtain $\mathcal{V}_{\mathcal{N}}(x) = \sum_{k=0}^{\mathcal{N}} \frac{x^k}{\Gamma(k+1)} D^k\chi(0^+)$, then

$$|\chi(x) - \sum_{k=0}^{\mathcal{N}} \frac{x^k}{\Gamma(k+1)} D^k\chi(0^+)| \le E \frac{x^{(\mathcal{N}+1)}}{\Gamma((\mathcal{N}+1)+1)}. \tag{22}$$

Then, we conclude the following

$$\begin{aligned} \| \chi(x) - \chi_{\mathcal{N}}(x) \|^2_{\mathcal{W}_{\mathfrak{L},f}^{(\rho,\sigma)}(x)} &\le \| \chi(x) - \sum_{k=0}^{\mathcal{N}} \frac{x^k}{\Gamma(k+1)} D^k\chi(0^+) \|^2_{\mathcal{W}_{\mathfrak{L},f}^{(\rho,\sigma)}(x)} \\ &\le \frac{E^2}{(\Gamma((\mathcal{N}+1)+1))^2} \int_0^{\mathfrak{L}} x^{2(\mathcal{N}+1)} \mathcal{W}_{\mathfrak{L},f}^{(\rho,\sigma)}(x)dx \\ &\le \frac{E^2}{(\Gamma((\mathcal{N}+1)+1))^2} \int_0^{\mathfrak{L}} x^{2(\mathcal{N}+1)} (\mathfrak{L}-x)^\rho x^\sigma dx \\ &\le \frac{\mathfrak{L}^{(2\mathcal{N}+\rho+3)}E^2}{(\Gamma((\mathcal{N}+1)+1))^2} \int_0^1 x^{2(\mathcal{N}+1)+\sigma} (1-x)^\rho dx \\ &\le \frac{\mathfrak{L}^{(2\mathcal{N}+\rho+3)}E^2\gamma(\rho+1)\Gamma(3+2\mathcal{N}+\sigma)}{\Gamma(4+2\mathcal{N}+\rho+\sigma)(\Gamma((\mathcal{N}+1)+1))^2}. \end{aligned} \tag{23}$$

Thus, an upper bound of the absolute error is acquired.

5. Numerical results

Using the algorithm presented in the previous section, we give in this section some numerical results. We discuss the FSIRC-MSBI (8) with the following values of parameters:

$$\mu = 0.11, \quad \sigma = 0.15, \quad \theta = 0.16, \quad m = 0.41,$$

$$\eta = 0.5, \quad \sigma = 0.6, \quad \delta = 0.5, \quad S_0 = N_t = 345.$$

Using the previous algorithm, we numerically treat with the previous equation and the related conditions. We plot the numerical solutions curves of FSIRC-MSBI with values of parameters listed above in Figure 1, where $\nu_1 = 0.9$, $\nu_2 = 0.5$, $\nu_3 = 0.4$, $\nu_4 = 0.8$ and $n = 20$. We observe that, the numerical solutions curves of FSIRC-MSBI are matching with the increment of $\mathcal{N}$. Taking $\nu_1 = 0.9$, $\nu_2 = 0.5$, $\nu_3 = 0.4$, $\nu_4 = 0.8$ and $\mathcal{N} = 20$, we obtain the numerical solutions curves of FSIRC-MSBI as:

$$s(t) = 2.2666315109313093^{-14}t^{20} - 4.518750617539102^{-12}t^{19} + 4.163173290734993^{-10}t^{18} - 2.3516540612935937^{-8}t^{17} + 9.111514822946975^{-7}t^{16} - 0.0000256664t^{15} + 0.000543643t^{14} - 0.00883285t^{13} + 0.111316t^{12} - 1.09319t^{11} + 8.35991t^{10} - 49.4985t^{9} + 224.439t^{8} - 766.155t^{7} + 1920.76t^{6} - 3413.07t^{5} + 4082.39t^{4} - 3039.73t^{3} + 1242.89t^{2} - 254.087t + 345,$$

$$i(t) = -8.88178 * 10^{-}16 + 13.1658t - 139.698t^{2} + 485.673t^{3} - 828.835t^{4} + 829.952t^{5} - 539.16t^{6} + 242.091t^{7} - 78.3949t^{8} + 18.8519t^{9} - 3.43483t^{10} + 0.480438t^{11} - 0.0519666t^{12} + 0.00435508t^{13} - 0.000281732t^{14} + 0.0000139229t^{15} - 5.15539 \times 10^{-7}t^{16} + 1.38363 * 10^{-8}t^{17} - 2.54027 \times 10^{-10}t^{18} + 2.8527 \times 10^{-12}t^{19} - 1.47738 * 10^{-}14t^{20},$$

$$r(t) = 5.55112 * 10^{-}16 + 3.23093t - 28.4521t^{2} + 88.0133t^{3} - 138.078t^{4} + 129.504t^{5} - 79.7545t^{6} + 34.2339t^{7} - 10.6626t^{8} + 2.47775t^{9} - 0.437851t^{10} + 0.0595744t^{11} - 0.00628352t^{12} + 0.00051453t^{13} - 0.0000325786t^{14} + 1.57814 \times 10^{-6}t^{15} - 5.73525 \times 10^{-8}t^{16} + 1.51241 \times 10^{-9}t^{17} - 2.73096 \times 10^{-11}t^{18} + 3.01896 \times 10^{-13}t^{19} - 1.54027 \times 10^{-15}t^{20},$$

$$c(t) = 5.55112 \times 10^{-17} + 2.24331t - 14.3887t^{2} + 36.4142t^{3} - 49.226t^{4} + 40.958t^{5} - 22.7984t^{6} + 8.96153t^{7} - 2.58105t^{8} + 0.558842t^{9} - 0.0925786t^{10} + 0.0118684t^{11} - 0.00118452t^{12} + 0.0000921207t^{13} - 5.55759 * 10^{-6}t^{14} + 2.57246 \times 10^{-7}t^{15} - 8.95598 \times 10^{-9}t^{16} + 2.26773 \times 10^{-10}t^{17} - 3.94014 \times 10^{-12}t^{18} + 4.19918 \times 10^{-14}t^{19} - 2.06912 \times 10^{-16}t^{20}.$$

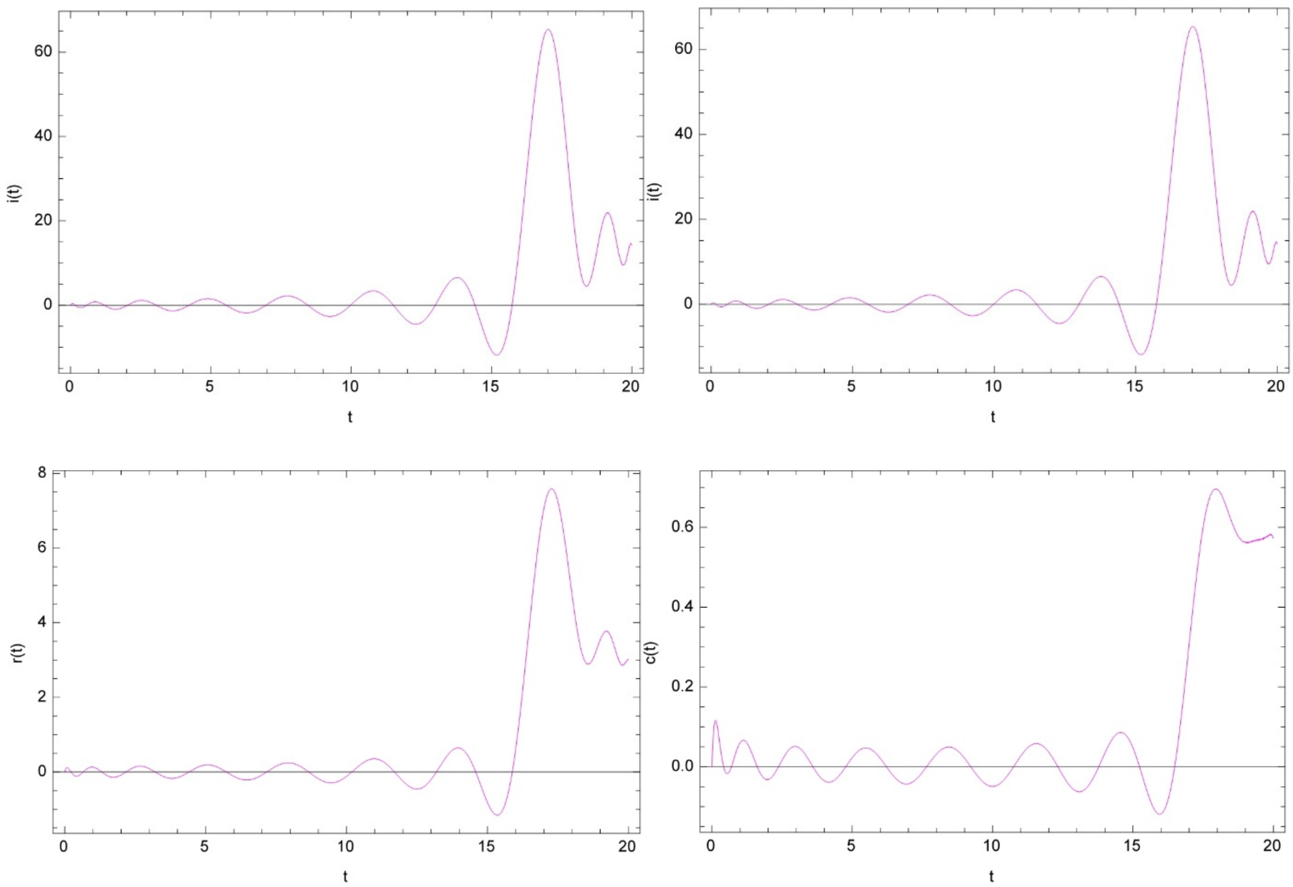

Figure 1. Numerical solutions curves of FSIRC-MSBI, where $\nu_1 = 0.9,\ \nu_2 = 0.5,\ \nu_3 = 0.4,\ \nu_4 = 0.8$ and $\mathcal{N} = 20$.

FSIRC-MSBI is only solved by Rihan et al. [29]. The numerical technique [29] is mentioned as local technique. However, they face complicated treatment due to the nonlocal fractional operator. Otherwise, these methods obtained the approximate solution at given points, whereas the global ones give it in entire domain, consequently, these techniques surely be the preferable. Collocation methods have exponential convergence rates as well as a high accuracy level. Thus, collocation technique surely is the preferable.

6. Conclusions

Expanding and development Collocation method for solving FSIRC-MSBI is our aspired. Our aspired was achieved via SJ-GR-C method. We listed an illustrative example to appear the effectiveness and applicability of our method. The results demonstrated that the spectral collection method is effective. The results clarified that the accuracy is achieved even use comparatively few nodes and then lower computational operations. We must emphasize that this method also excelled over other approximated methods. By word, if we have a problem with not smooth solution, the accuracy of the majority techniques may be deteriorated. That would be stopped merely exchanging fractional order Jacobi instead of the Jacobi polynomial [30], also could using smoothing mapping. Finally, we indicated that our algorithm can be used to handle various biological models like novel nonlinear fractal coronavirus (COVID-19) [31].

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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