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*Research article*

## **Analysis of COVID-19 transmission in Shanxi Province with discrete time imported cases**

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**Abstract:** Since December 2019, an outbreak of a novel coronavirus pneumonia (WHO named COVID-19) swept across China. In Shanxi Province, the cumulative confirmed cases finally reached 133 since the first confirmed case appeared on January 22 2020, and most of which were imported cases from Hubei Province. Reasons for this ongoing surge in Shanxi province, both imported and autochthonous infected cases, are currently unclear and demand urgent investigation. In this paper, we developed a SEIQR difference-equation model of COVID-19 that took into account the transmission with discrete time imported cases, to perform assessment and risk analysis. Our findings suggest that if the lock-down date in Wuhan is earlier, the infectious cases are fewer. Moreover, we reveal the effects of city lock-down date on the final scale of cases: if the date is advanced two days, the cases may decrease one half (67, 95% CI: 66–68); if the date is delayed for two days, the cases may reach about 196 (95% CI: 193–199). Our investigation model could be potentially helpful to study the transmission of COVID-19, in other provinces of China except Hubei. Especially, the method may also be used in countries with the first confirmed case is imported.

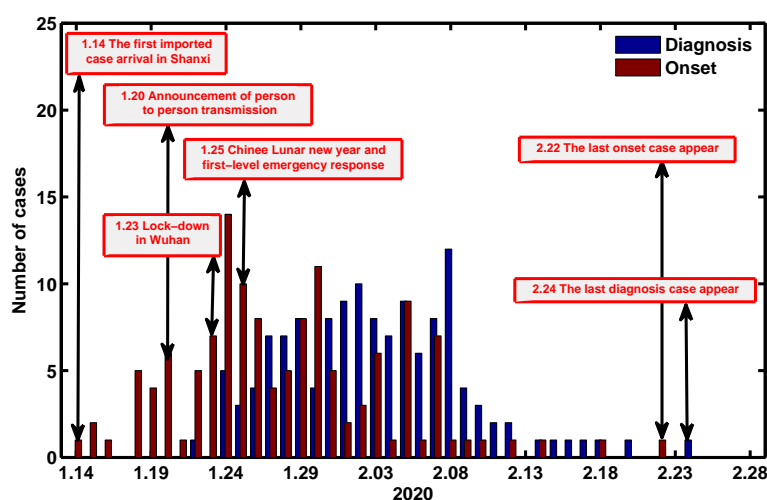
**Keywords:** COVID-19; imported cases; SEIQR model; difference equation; city lock-down strategy

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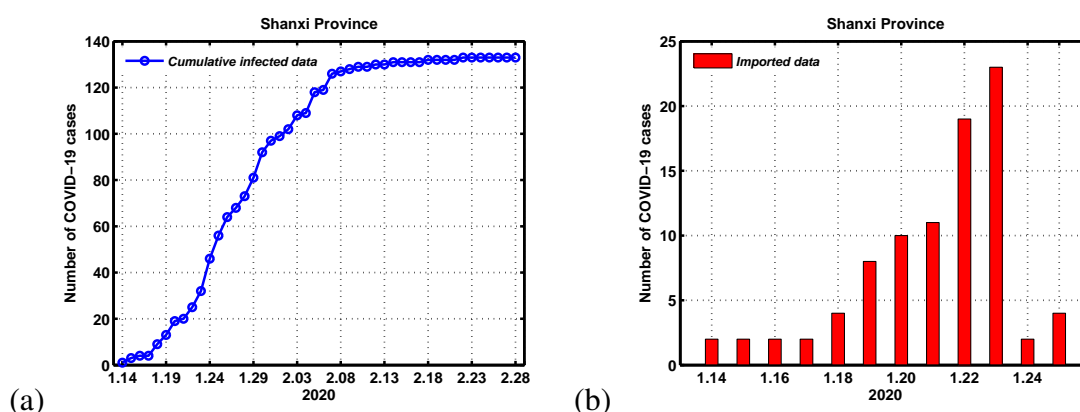
## 1. Introduction

Since December 2019, an outbreak of a novel coronavirus pneumonia (WHO named COVID-19) swept across China [1]. Subsequently, on January 20, 2020, the Chinese government has revised the law provisions concerning infectious diseases to add the COVID-19 as a notifiable infectious disease (Category B infectious diseases). Considering the seriousness of the disease, public health officials have announced COVID-19 are managed according to Category A infectious diseases, the same as SARS in 2003. COVID-19 is mainly transmitted through respiratory droplets and can also be transmitted through intimate contact, etc. The average incubation period is 5.2 days (95% CI, 4.1 to 7.0) before clinical symptoms [1]. In addition, related researches found that there exists the infectivity in the incubation period, and most infection are mild or asymptomatic, only partly infection clinical manifestations are pneumonia. The primary symptoms are fever, cough, and weakness, and pulmonary imaging CT showed significantly changes in the lung [2].

With the arrival of Spring Festival of 2020, passengers travelled from Hubei Province go to all the rest parts of mainland China, which may increase the risk of imported case in other provinces. On January 20, the National Health Commission of China announced that there existed person-to-person transmission, which caused the panicked people to hasten away from Wuhan. From January 13, confirmed cases started to appear in other provinces of mainland China. In order to control the transmission of COVID-19, the Chinese authorities implemented the lock-down strategy in Wuhan to restrict the movement on January 23. As of January 25, a total of 30 provincial governments (except Tibet) initiated a first-level emergency response. Meanwhile, some non-pharmaceutical interventions include strict travel restriction [3], extensive monitoring of suspected cases, mandatory registration of all people arriving from Hubei Province, and the registered individuals with home quarantine for at least 14 days [4]. The surveillance for new cases, and the tracing and management of contacts have been strictly performed. People are suggested to stay at home as much as possible. All of these measures are designed to early detection, early reporting and early quarantine for infectious cases.



**Figure 1.** Time differences for between disease onset and confirmed diagnosis of all patients ( $n = 133$ ) from January 14 to February 28, 2020.



**Figure 2.** (a) Number of cumulative infected COVID-19 cases in Shanxi province. (b) The arrival date and number of the confirmed COVID-19 cases in Shanxi Province.

Using information published by the Health Commission of Shanxi Province [5], we constructed a dataset of COVID-19 patients that are laboratory-confirmed in Shanxi Province as the end of February 28, 2020. The detailed information includes the cumulative and daily laboratory-confirmed cases, and life track of these laboratory-confirmed cases. By means of statistical analysis the life track of all laboratory-confirmed cases, we obtain the data of symptom onset (the self-reported date of symptoms such as fever, cough, or other respiratory symptoms) for all laboratory-confirmed cases, and arrival date of the confirmed COVID-19 cases in Shanxi Province. The first confirmed case was reported on January 22 (notice that the arrival date of the first confirmed imported case and the first symptom onset case in Shanxi province was January 14), then confirmed cases continued to emerge and the cumulative confirmed cases reached 133 at the end of February, most of which were imported case from Hubei Province and their local family members in Shanxi province (89 cases). The time series of differences between disease symptom onset and confirmed diagnosis COVID-19 cases in Shanxi Province is shown in Figure 1. Cumulative infected COVID-19 cases and the actual time of arrival in Shanxi province with confirmed COVID-19 cases are shown in Figure 2a,b, respectively. To our knowledge, some works have done on transmission dynamics of COVID-19 in mainland China or Wuhan city [6–13], but few study has focused on Shanxi Province. Therefore, understanding the factors influencing COVID-19 outbreaks has become a major provincial public health priority in Shanxi Province.

In this paper, to make predictions and perform assessment and risk analysis for COVID-19 outbreak of Shanxi Province, we developed a SEIQR difference-equation model of COVID-19 that takes into account the transmission with discrete time imported cases. Then the model parameters were estimated by the extensive MCMC method, and numerical simulations support the data reasonably well. Finally, sensitivity analyses are conducted to inform the earlier with Wuhan city lock-down, the fewer cases in Shanxi and other imported provinces.

## 2. The mathematical model

In this study, we developed a SEIQR difference-equation model to describe the transmission of COVID-19 in Shanxi Province. The model classifies the human population ( $N(t)$ ) into susceptible

cases  $S(t)$ , exposed cases  $E(t)$ , infectious cases  $I(t)$ , confirmed cases  $Q(t)$  and recover cases  $R(t)$  compartments. Human population birth and death rates are ignored, and the human host incubation period is  $1/\delta$  days.  $A(t)$  and  $B(t)$  are the imported COVID-19 cases at discrete time  $t$ , which are considered entering the exposed compartment and infected compartment. Infectious cases at the rate  $m$  revert to confirmed cases and the recover rate is  $\gamma$ . Susceptible humans acquire COVID-19 through direct contact with exposed cases and infected cases at rates  $\frac{\beta S(t)E(t)}{N(t)}$  and  $\frac{\beta_1 S(t)I(t)}{N(t)}$ , respectively. The transmission dynamic of COVID-19 diagram in Shanxi Province is shown in Figure 3. Hence, the following difference equations are considered to describe the COVID-19 transmission model:

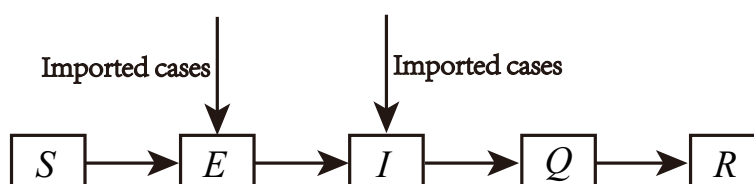
$$\begin{cases} S(t+1) = S(t) - \frac{\beta S(t)E(t) + \beta_1 S(t)I(t)}{N(t)}, \\ E(t+1) = E(t) + A(t) + \frac{\beta S(t)E(t) + \beta_1 S(t)I(t)}{N(t)} - \delta E(t), \\ I(t+1) = I(t) + B(t) + \delta E(t) - mI(t), \\ Q(t+1) = Q(t) + mI(t) - \gamma Q(t), \\ R(t+1) = R(t) + \gamma Q(t), \\ N(t) = S(t) + E(t) + I(t) + Q(t) + R(t). \end{cases} \quad (2.1)$$

All parameters of system are assumed to be nonnegative. If there does not exist imported cases  $A(t)$  and  $B(t)$ , system (2.1) will become the following difference equations:

$$\begin{cases} S(t+1) = S(t) - \frac{\beta S(t)E(t) + \beta_1 S(t)I(t)}{N(t)}, \\ E(t+1) = E(t) + \frac{\beta S(t)E(t) + \beta_1 S(t)I(t)}{N(t)} - \delta E(t), \\ I(t+1) = I(t) + \delta E(t) - mI(t), \\ Q(t+1) = Q(t) + mI(t) - \gamma Q(t), \\ R(t+1) = R(t) + \gamma Q(t), \\ N(t) = S(t) + E(t) + I(t) + Q(t) + R(t). \end{cases} \quad (2.2)$$

It is easy to obtain that the reproduction number of system (2.2) by using the method of paper [14, 15], and which is

$$\mathcal{R}_0 = \frac{\beta}{\delta} + \frac{\beta_1}{m}.$$



**Figure 3.** Transmission diagram of COVID-19 in Shanxi Province.

### 3. Results

By means of statistical analysis the life track of all laboratory-confirmed cases, we found the last infected case appeared at February 10 (the other four cases after this time were asymptomatic patients transferred to diagnosed patients). One can conclude that the last infected case was infected on February 05, when the incubation period is about 5.2 days (95% confidence interval [CI], 4.1 to 7.0) [1]. Hence, the transmission of COVID-19 in Shanxi Province can be divided into two stages. The first stage is used to give parameter estimation from January 14 to February 05, and the second stage is used to validate the accuracy of proposed model from February 05 to February 28 without any transmissions.

#### 3.1. Parameter estimation and Fitting results

The human population is assumed to be fully susceptible to the virus. Hence, suppose that the initial value of the susceptible population  $S$  equals to  $3.718 \times 10^7$ , which is Year-end population of 2018 for Shanxi province [17], and while all others (including  $E, I, Q, R$ ) are 0. In previous paper [1], they obtained that the mean incubation period is 5.2 days (95% confidence interval [CI], 4.1 to 7.0), so we choose the value of  $\delta$  is  $\frac{1}{5.2}$ . By means of statistical analysis of all laboratory-confirmed cases, we obtain the period between symptom onset and laboratory confirmation of the first stage is about 9 days, and the value of  $m$  is 0.11. For parameter  $\gamma$ , it depends on the actual situation of infected cases, and also does not influence the model simulation. Therefore, we assume  $\gamma = 1/14$ . In Shanxi province, Shanxi provincial government initiated a first-level emergency response on January 25, after that all people from Hubei Province, whose family members, and related contacts were registered and complied with home quarantine for at least 14 days. In this situation, if someone has a fever, he or she will avoid contact with others. Hence, the transmission rate  $\beta_1$  will be less than  $\beta$ , and we assume that the proportion between  $\beta_1$  and  $\beta$  is  $1/14$  (which also means  $\beta_1 = 1/14 * \beta$ ). In order to estimate the value of parameter  $\beta$ , we use extensive Markov-chain Monte-Carlo (MCMC) simulations based on the adaptive combination Delayed rejection and Adaptive Metropolis (DRAM) algorithm [18, 19] for system (2.1). Using 100000 sample realizations, we can acquire the parameter value for  $\beta$  with 1D parameter MCMC chain in Figure 4. Then we further get the mean value, the standard deviation, MCMC error and Geweke for parameter  $\beta$ , which are shown in Table 1. From Figure 4 and the value of Geweke, it is easy to see that the Markov-chain of parameter  $\beta$  is convergent. If there does not exist the imported cases for this model, we can obtain the mean value and standard deviation of  $\mathcal{R}_0$  are 0.4494 and 0.0165, respectively. And which also means the epidemic will disappear in the future with current control measures.

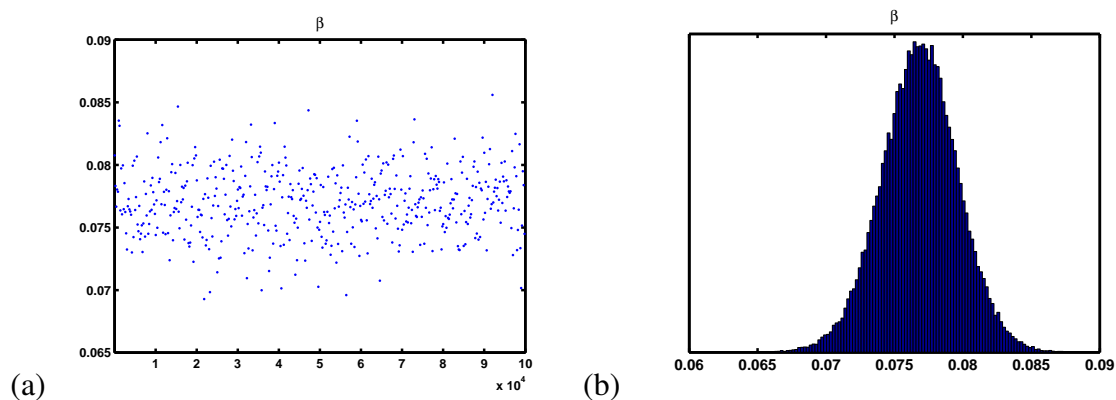
For the first stage, the time evolution of both theoretical infectious cases and comparison with the infected of COVID-19 cases in Shanxi Province is shown in Figure 5, which also shows the 95% percent interval for all 100000 passing simulation trajectories and the median of these 100000 simulation outputs. It is clear that the theoretical prediction is nearly full agreement with real data, which also well validates the accuracy of proposed model. Then, system (2.2) was used to fit the infectious cases without any transmissions from February 05 to February 28. The adaptive Metropolis-Hastings algorithm was also used to carry out extensive Markov-chain Monte-Carlo (MCMC) simulations, and the mean, standard deviation, MCMC error and Geweke value of  $\delta$  are shown in Table 2.

**Table 1.** Parameter estimation for parameter  $\beta$  with the method of MCMC.

| Notation | Mean Value | Standard Deviation | MCMC error | Geweke |
|----------|------------|--------------------|------------|--------|
| $\beta$  | 0.0768     | 0.0028             | 1.6453e-05 | 0.9998 |

**Table 2.** Parameter estimation for parameter  $\delta$  with the method of MCMC.

| Notation | Mean Value | Standard Deviation | MCMC error | Geweke |
|----------|------------|--------------------|------------|--------|
| $\delta$ | 0.2860     | 0.0215             | 1.4622e-05 | 0.9970 |



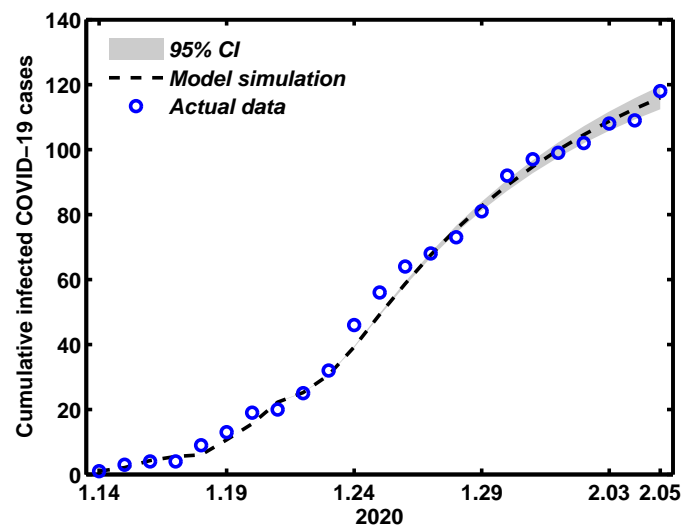
**Figure 4.** (a) Simulation results for parameter  $\beta$  of MCMC chain with 100000 sample realizations. (b) The histogram of MCMC chain for parameter  $\beta$ . The Geweke convergence diagnostic method was employed to assess convergence of the chains [16], and the Geweke values of these two parameters are shown in Table 1.

### 3.2. The prediction with the model

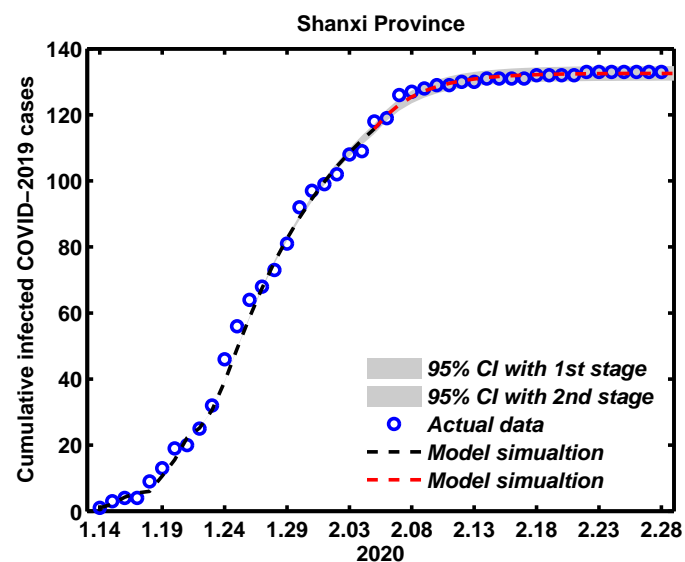
With the uncertainty for estimated parameters values, Monte Carlo simulation runs are then conducted to assess the performance of the model by using the available model parameter in Tables 1 and 2. Figure 6 unveils the prospect of the epidemic in Shanxi province with cumulative number of infected COVID-19 cases in Shanxi Province, shows that the 95% percent interval for all 100000 passing simulation trajectories and the median of these 100000 simulation outputs. The disease will disappear in late February and the mean final scale of cases will reach 133 (95% CI: 131–135).

### 3.3. The situation with different proportions between $\beta_1$ and $\beta$

Due to the assumed model parameter proportion between  $\beta_1$  and  $\beta$  with substantial uncertainty in their values, we gave another six different proportions between  $\beta_1$  and  $\beta$  to estimate parameter  $\beta$  by using same actual data and same method of MCMC, and the results were shown in Table 3. Table 3 show that the value of  $\beta$  decreases huge as the proportion between  $\beta_1$  and  $\beta$  increasing from 0.01 to 1. From previous assumption we know that the proportion value between  $\beta_1$  and  $\beta$  is also meaning the control intensity for infected cases, which are inverse proportional relationship. The more control intensity, the less value of  $\mathcal{R}_0$  by using same data for parameter estimation. Figure 7 shows the prospect



**Figure 5.** Fitting results of theoretical cumulative number of infected COVID-19 cases with its actual reported number.

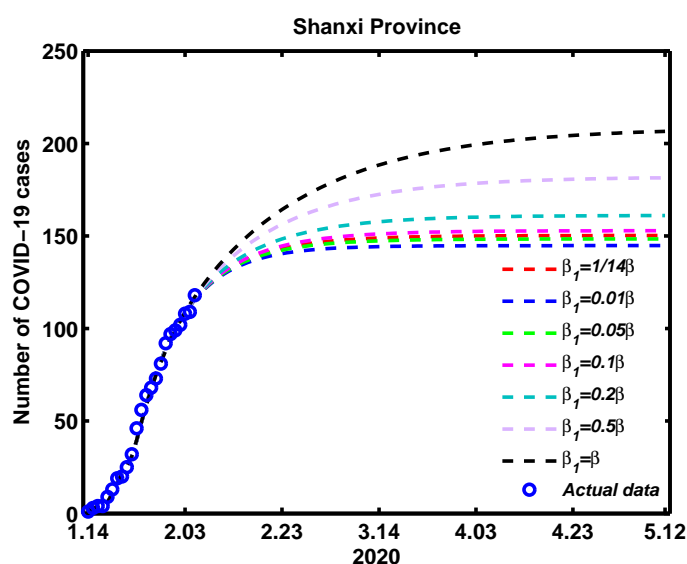


**Figure 6.** The simulation result of cumulative number of infected COVID-19 cases in Shanxi Province.

of the epidemic for different proportions between  $\beta_1$  and  $\beta$  in Shanxi Province without any other control measures from February 05. From the figure we can obtain that the more control intensity for infectious people, the less final size of epidemic.

**Table 3.** Parameter estimates of  $\beta$  and  $m$  with different situation for  $\beta_1$  and  $\beta$ .

| Parameter       | $\beta_1 = 0.01\beta$ |        | $\beta_1 = 0.05\beta$ |        | $\beta_1 = 0.1\beta$ |        |
|-----------------|-----------------------|--------|-----------------------|--------|----------------------|--------|
| Notation        | Mean                  | Std    | Mean                  | Std    | Mean                 | Std    |
| $\beta$         | 0.0815                | 0.0030 | 0.0784                | 0.0029 | 0.0749               | 0.0028 |
| $\mathcal{R}_0$ | 0.4311                | 0.0159 | 0.4435                | 0.0162 | 0.4573               | 0.0169 |
| Parameter       | $\beta_1 = 0.2\beta$  |        | $\beta_1 = 0.5\beta$  |        | $\beta_1 = \beta$    |        |
| Notation        | Mean                  | Std    | Mean                  | Std    | Mean                 | Std    |
| $\beta$         | 0.0687                | 0.0025 | 0.0549                | 0.0021 | 0.0412               | 0.0016 |
| $\mathcal{R}_0$ | 0.4819                | 0.0178 | 0.5354                | 0.0201 | 0.5885               | 0.0229 |



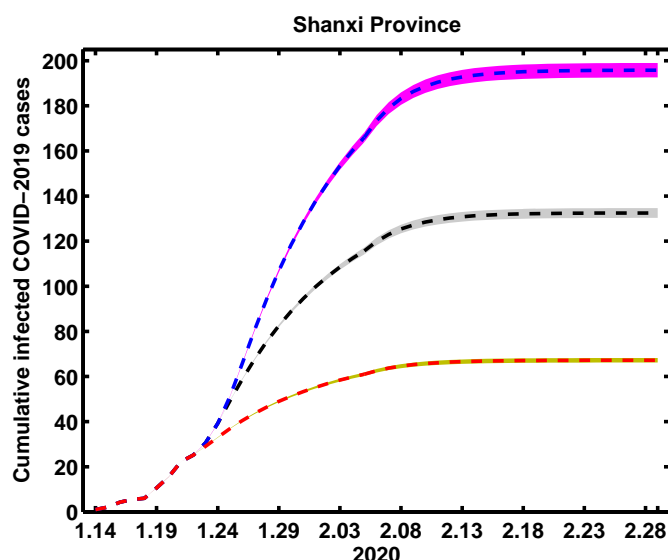
**Figure 7.** The variations of cumulative number of infected COVID-19 cases with different proportions between  $\beta_1$  and  $\beta$ .

### 3.4. Simulation results with different dates of lock-down strategy

As we know, the outbreak of COVID-19 was reported in Wuhan, then the confirmed cases from Wuhan started to appear in other Chinese provinces until January 13, 2020. In order to control the transmission of COVID-19, the Chinese authorities introduced the implementation of the city lock-down strategy in Wuhan to shut down the movement on January 23, 2020. And the time of lock-down strategy in Wuhan affects the number of imported cases in other provinces. In this section, we will explore that the impact of different Wuhan city lock-down dates for the cumulative infected COVID-19 cases in Shanxi Province. On January 20, the leader of the high-level expert group of the National Health Commission announced that there existed person-to-person transmission, then more and more



people left Wuhan City as far as possible. Hence, we will give simulation results with three different Wuhan city lock-down dates, which is shown in Figure 8. The dates of Wuhan city lock-down were crucial in producing the outbreak pattern in Shanxi Province. It is obtained that the earlier with city lock-down, the fewer cases, and the later with city lock-down, the more cases. If the city lock-down date ahead two days, the final scale of cases will decrease one half (67, 95% CI: 66–68). And if the city lock-down date delay for two days, the mean final scale of cases will reach about 196 (95% CI: 193–199).



**Figure 8.** Simulation results for cumulative number of infected COVID-19 cases in Shanxi Province with different dates of lock-down strategy in Wuhan.

#### 4. Discussions and conclusions

In 2020, an outbreak of COVID-19 epidemic was reported in mainland China. Since January 23, 2020, government has been seeking various prevention and control measures to drastically reduce within-population contact rates and transmission. These measures included travel restriction and quarantine of both suspected individuals and subjects who have had close contacts with suspected cases, mandatory registration of all people arriving from Hubei Province, the registered individuals with home quarantine for at least 14 days, and extending the Spring Festival holiday. By the end of February 2020, the COVID-19 epidemic of China has been basically controlled in all provinces and cities except Hubei province. In Shanxi Province, by organizing the detailed information of confirmed cases, the cumulative confirmed cases arrived 133 at the end of February, and most of which were imported case from Hubei Province (89 cases). To the best of our knowledge, few study has focused on Shanxi Province to study the transmission of COVID-19. To investigate the underlying dynamics of COVID-19 transmission in Shanxi province, we developed a SEIQR difference-equation model of COVID-19 that took into account the transmission with discrete time imported cases, to make predictions and perform assessment and risk analysis.

Statistical analysis of the detailed cases demonstrates that the time from symptom to diagnosis in Shanxi Province is about 9 days. The detailed infectious cases from January 14 to February 05 were used to give parameter estimation for model (2.1). If there does not exist the imported cases for this model, we can obtain the mean value and standard deviation of  $\mathcal{R}_0$  are 0.4494 and 0.0165, respectively. In our simulation, the disease disappeared in late February and the mean final scale of cases reached 133 (95% CI: 131–135). Our findings suggested that the earlier with Wuhan city lock-down, the fewer cases, and the later with Wuhan city lock-down, the more cases. If the city lock-down date ahead two days, the final scale of cases will decrease one half (67, 95% CI: 66–68). And if the city lock-down date delay for two days, the mean final scale of cases will reach about 196 (95% CI: 193–199). Hence, the travel restrictions could be instrumental to national and international agencies for public health response planning, and the earlier the better for other provinces except Hubei in mainland China.

Compared to previous ordinary differential equation models, our model is a difference equation model. The advantage of our model is that it is not necessary to estimate the initial value of the model, and the Markov-chain of model parameter  $\beta$  is convergent (See Figure 4). It should be noted that the dynamical model developed in this study could increase the understanding of the spread and control of COVID-19, which could be potentially helpful to study the transmission of COVID-19, in other provinces of China except Hubei. We hope our work is a good studying point for the countries with the first confirmed case is imported.

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## Conflict of interest

All authors declare no conflicts of interest in this paper.

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