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*Research article*

# State-dependent event-triggered impulsive synchronization of stochastic delayed networks under countably infinite-state Markovian switching

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**Abstract:** This work focused on the synchronization of stochastic complex networks with time-varying delays and state-dependent countably infinite-state Markovian switching under event-triggered impulsive control. A drive–response framework was adopted, and the synchronization problem was reduced to the $p$-th moment exponential stability of an associated error system. The response network was regulated by a state-dependent impulsive mechanism whose triggering instants were generated by the evolution of the synchronization error, thereby avoiding unnecessary control updates. To handle the countably infinite switching state space, Lyapunov methods were combined with uniform strong exponential ergodicity of the switching process and mode-uniform coefficient bounds. Within this framework, a general Lyapunov-type criterion was first established for the error dynamics, and a more explicit coefficient-based criterion was then derived to facilitate verification in applications. These results yield sufficient conditions for $p$-th moment exponential synchronization of the drive and response networks. In addition, an explicit positive lower bound on the inter-event intervals was obtained, which excludes Zeno behavior and ensures implementability of the proposed control law. Numerical experiments on a coupled Chua circuit network with finite-state switching were presented as a special-case illustration of the theory. The simulations showed that the proposed event-triggered impulsive strategy effectively suppresses the synchronization error while remaining consistent with the theoretical lower-bound estimate for the triggering intervals.

**Keywords:** event-triggered impulse control; countably infinite-state; synchronization; Markovian switching; complex networks
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## 1. Introduction

Synchronization is a central collective phenomenon in complex networks and underlies coordinated behavior in neural, cyber-physical, and engineered interconnected systems. For instance, synchronization plays an important role in cooperative unmanned aerial vehicle networks, where it supports coordinated formation tracking, improves tracking accuracy, and reduces energy consumption under external disturbances [1]. Synchronization phenomena have also been investigated in brain and inter-brain networks [2, 3]. In realistic networked systems, however, communication delays, stochastic perturbations, and abrupt changes in operating conditions may jointly affect both transient and asymptotic behaviors [4]. These features make the synchronization problem intrinsically hybrid and call for analytical tools that combine delay-system arguments, stochastic stability theory, and switching system techniques.

Among the available frameworks for modeling abrupt environmental or structural variation, Markovian switching is particularly effective. It has been widely employed in multi-agent systems [5], connected and automated vehicles [6], and power networks to characterize abrupt changes in communication topologies and operating conditions [7]. In addition, the study in [8] examined the dynamic balance and reliability of a stochastic predator–prey system and revealed the effects of switching frequency and noise intensity on stochastic bifurcation and failure behavior. Important progress has been made for stochastic coupled systems without switching [9], and synchronization criteria are also available for networks with finite-state Markovian switching under intermittent control [10]. Related stability and control results for finite-state Markov jump systems further illustrate the analytical convenience of a finite mode set [11]. That convenience is structural rather than superficial: when the switching state space is finite, mode-dependent coefficients can be bounded through finite maxima, and switching effects can often be handled by finite-dimensional inequalities. Once the switching process evolves on the countable state space $\mathbb{Z}_+$, these arguments are no longer routine, because the Lyapunov analysis must accommodate infinitely many mode-dependent coefficients and the recurrence properties of the switching mechanism become part of the stability analysis itself.

Control design introduces another challenge. Impulsive control is appealing for network synchronization because corrective actions are imposed only at discrete instants, thereby reducing the continuous actuation burden [12, 13]. However, periodic or preassigned impulsive instants do not reflect the real-time evolution of the synchronization error and may therefore produce unnecessary control updates. Event-triggered control addresses this issue by activating the controller only when a prescribed state-dependent condition is satisfied [14, 15]. For example, an event-triggered impulsive strategy has been applied to trajectory tracking of underactuated underwater vehicles to reduce communication and control updates while excluding Zeno behavior [16]. For delayed stochastic networks, the main challenge is therefore to guarantee synchronization while proving that the triggering instants do not accumulate within a finite time interval [17]. Recent studies have also investigated prescribed-time and prescribed-performance control for nonlinear systems [18–20]. Unlike these methods, which focus on convergence within a prescribed time or performance envelope, the present study addresses $p$-th moment exponential synchronization of delayed stochastic complex networks with state-dependent countably infinite-state Markovian switching. A positive lower bound on the inter-event intervals is further derived to exclude Zeno behavior.

The above observations reveal a clear gap in the literature. The most relevant synchronization results either concern stochastic coupled systems without Markovian switching [9], or focus on networks with finite-state switching under intermittent regulation [10]. By contrast, available studies on countably infinite-state regime-switching systems mainly address stability, regularity, or well-posedness of individual stochastic dynamics [21, 22]. A unified synchronization framework that simultaneously incorporates time-varying delays, stochastic perturbations, countably infinite-state Markovian switching, and state-dependent event-triggered impulsive control is still lacking.

Motivated by this gap, this paper investigates the synchronization of stochastic complex networks with time-varying delays and state-dependent countably infinite-state Markovian switching under an event-triggered impulsive control scheme. A drive–response framework is adopted, and the synchronization problem is transformed into the $p$-th moment exponential stability of the associated error system. This framework preserves the autonomous evolution of the drive network while allowing the response network to be regulated independently, thereby facilitating stability analysis and controller design. By incorporating uniform strong exponential ergodicity into the Lyapunov analysis, a general Lyapunov-type criterion and an explicit coefficient-type criterion are established. Moreover, a positive lower bound on the inter-event intervals is derived to exclude Zeno behavior. Finally, a coupled Chua-circuit network is presented to illustrate the theoretical results.

The main contributions of this paper are summarized as follows:

1) A drive–response synchronization framework is formulated for delayed stochastic complex networks with state-dependent countably infinite-state Markovian switching. The synchronization problem is transformed into the $p$-th moment exponential stability problem of the associated error system.

2) Different from the control methods in [4, 10, 23, 24], a state-dependent event-triggered impulsive control scheme is proposed. The impulsive instants are determined adaptively according to the evolution of the synchronization error, thereby avoiding unnecessary control updates while preserving the autonomous evolution of the drive network. Moreover, a positive lower bound on the inter-event intervals is derived to exclude Zeno behavior.

3) State-dependent countably infinite-state Markovian switching is incorporated into the synchronization model. In contrast to the finite-state switching systems considered in [11, 25–27], the switching process evolves over a countably infinite mode space. By incorporating uniform strong exponential ergodicity into the Lyapunov analysis, a general synchronization criterion and an explicit coefficient-type criterion are established.

The structure of the paper is organized as follows: Section 2 presents the model and assumptions; Section 3 derives the main synchronization criteria and provides the corresponding theorems; Section 4 validates the theoretical results through simulations; Section 5 concludes the paper and discusses future work.

## 2. Preliminaries and model description

### 2.1. Preliminaries

Unless otherwise specified, the following notation is used throughout this paper. Let $\mathbb{Z}_+ = \{1, 2, \dots\}$ and define $\mathbb{L} = \{1, 2, \dots, N\}$ as the index set of network nodes, where $N \geq 2$ and $N \in \mathbb{Z}_+$. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ is the $n$-dimensional Euclidean space, and $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$. Denote by $C([-\tau, 0]; \mathbb{R}^n)$ the space of continuous functions $X(\mathrm{t}) : [-\tau, 0] \to \mathbb{R}^n$, equipped with the norm $\|X(\mathrm{t})\| := \sup_{-\tau \leq u \leq 0} |X(u)|$. The upper-right Dini derivative of $X$ at $\mathrm{t}$ is defined by $\mathbb{D}^+ X(\mathrm{t}) := \limsup_{h \to 0^+} \frac{X(\mathrm{t}+h) - X(\mathrm{t})}{h}$. Moreover, $C^2(\mathbb{R}^n \times \mathbb{Z}_+; \mathbb{R}_+)$ denotes the class of twice continuously differentiable, nonnegative, real-valued functions $V(X, k)$ defined on $\mathbb{R}^n \times \mathbb{Z}_+$, while $L^2([-\tau, 0]; \mathbb{R}^n)$ stands for the space of square-integrable functions on $[-\tau, 0]$ with values in $\mathbb{R}^n$, which is used to characterize the initial conditions of the system. In addition, $tr(\cdot)$ denotes the trace of a matrix, $|\cdot|$ denotes the Euclidean norm of a vector, and the superscript "$\top$" represents the transpose of a vector or matrix. The symbol $\vee$ denotes the maximum operator, i.e., $a \vee b := \max\{a, b\}$.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_\mathrm{t}\}_{\mathrm{t} \geq 0}, \mathbb{P})$ be a complete filtered probability space, where the filtration $\{\mathcal{F}_\mathrm{t}\}_{\mathrm{t} \geq 0}$ satisfies the usual conditions, and let $\mathbb{E}[\cdot]$ denote the expectation with respect to $\mathbb{P}$. Suppose that $\alpha(\mathrm{t})$, $\mathrm{t} \geq 0$, is a right-continuous state-dependent Markovian switching process defined on this probability space and taking values in the countably infinite state space $\mathbb{Z}_+$. Its state-dependent generator is denoted by $\mathcal{S}(x) = (q_{\mathfrak{M}\mathfrak{N}}(x))_{\mathfrak{M}, \mathfrak{N} \in \mathbb{Z}_+}$, where $x$ denotes the current continuous state variable. In addition, let $B(\mathrm{t})$ be a one-dimensional standard Brownian motion defined on the same probability space. The transition probabilities of $\alpha(\mathrm{t})$ are given by

$$\mathbb{P}\{\alpha(\mathrm{t} + \Delta\mathrm{t}) = \mathfrak{N} \mid \alpha(\mathrm{t}) = \mathfrak{M},\ X(\mathrm{t}) = x\} = \begin{cases} q_{\mathfrak{M}\mathfrak{N}}(x)\Delta\mathrm{t} + o(\Delta\mathrm{t}), & \mathfrak{N} \neq \mathfrak{M}, \\ 1 + q_{\mathfrak{M}\mathfrak{M}}(x)\Delta\mathrm{t} + o(\Delta\mathrm{t}), & \mathfrak{N} = \mathfrak{M}. \end{cases}$$

Here $q_{\mathfrak{M}\mathfrak{N}} \geq 0$ for $\mathfrak{N} \neq \mathfrak{M}$, $q_{\mathfrak{M}\mathfrak{M}} = -\sum_{\mathfrak{N} \neq \mathfrak{M}} q_{\mathfrak{M}\mathfrak{N}}$, and $\sum_{\mathfrak{N} \neq \mathfrak{M}} q_{\mathfrak{M}\mathfrak{N}} < \infty$ for each $\mathfrak{M} \in \mathbb{Z}_+$. Throughout this paper, the $Q$-matrix $\mathbb{S}$ is assumed to be conservative and non-explosive.

Next, we will introduce some concepts of digraph. Let $\mathcal{H} = \{\mathcal{G}, A\}$ be a digraphs, where $\mathcal{G}$ is the set of nodes and $A$ is the set of arcs $(i, j)$ leading from initial node $i$ to terminal node $j$ in digraph $\mathcal{H}$. The digraph $\mathcal{H}$ is said to be strongly connected if, for every pair of distinct nodes $i, j \in \mathcal{G}$, there exists a directed path from $i$ to $j$. For fixed $k \in \mathbb{Z}_+$, denote the weight matrix of $\mathcal{H}$ as $\mathbb{W}(Q) = (a_{ij})_{n \times n}$. In addition $a_{ij} > 0$ if there exists an arc from node $i$ to node $j$, and $a_{ij} = 0$ otherwise. Throughout this paper, we write a weighted digraph as $(\mathcal{H}, \mathbb{W}(Q))$, of which the weighted matrix of $\mathcal{H}$ is $\mathbb{W}(Q)$. The Laplacian matrix of $(\mathcal{H}, \mathbb{W}(Q))$ is defined as $L = (p_{ij})_{n \times n}$, where $p_{ij} = -a_{ij}$ for $i \neq j$ and $p_{ij} = \sum_{h \neq j} a_{ih}$ for $i = j$.

### 2.2. Model description

We consider the following hybrid stochastic coupled systems consisting of $N$ identical coupled nodes. Each node $i$ is governed by the dynamical system

$$\mathrm{d}\mathcal{Y}_i(\mathrm{t}) = \Big[f_i(\mathcal{Y}_i(\mathrm{t}), \mathcal{Y}_i(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t})) + \sum_{j=1, j \neq i}^{n} a_{ij}(\alpha(\mathrm{t}))\Gamma_{ij}(\mathcal{Y}_i(\mathrm{t}), \mathcal{Y}_j(\mathrm{t}), \alpha(\mathrm{t}))\Big]\mathrm{dt}$$

$$+ g_i(\mathcal{Y}_i(\mathrm{t}), \mathcal{Y}_i(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t}))\mathrm{d}B(\mathrm{t}), i \in \mathbb{L}, \mathrm{t} \geq 0, \tag{2.1}$$

where $\mathcal{Y}_i(\mathrm{t}) \in \mathbb{R}^n$ is the state vector associated with the $i$-th node, and the function $f_i(\cdot,\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{Z}_+ \to \mathbb{R}^n$ is a continuous vector-valued function. Let $\tau(\mathrm{t})$ be a time-varying delay satisfying $0 \leq \tau(\mathrm{t}) \leq \tau$ for some constant $\tau > 0$. The Markovian switching process is denoted by $\alpha(\mathrm{t})$. Let $a_{ij}(\cdot)$ indicate the coupling strength. The inner coupling function is represented by $\Gamma_{ij}(\cdot,\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{Z}_+ \to \mathbb{R}^n$, which indicates the connections between nodes. The $g_i(\cdot,\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{Z}_+ \to \mathbb{R}^n$ represents the stochastic perturbation. Let $B(\mathrm{t})$ be a one-dimensional standard Brownian motion.

If system (2.1) is taken as the driving system, then its corresponding response system is

$$\begin{aligned} \mathrm{d}\tilde{\mathcal{Y}}_i(\mathrm{t}) = \Big[& f_i(\tilde{\mathcal{Y}}_i(\mathrm{t}), \tilde{\mathcal{Y}}_i(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t})) + U_i(\mathrm{t}) + \sum_{j=1, j\neq i}^{N} a_{ij}(\alpha(\mathrm{t}))\Gamma_{ij}(\tilde{\mathcal{Y}}_i(\mathrm{t}), \tilde{\mathcal{Y}}_j(\mathrm{t}), \alpha(\mathrm{t}))\Big]\mathrm{d}\mathrm{t} \\ &+ g_i(\tilde{\mathcal{Y}}_i(\mathrm{t}), \tilde{\mathcal{Y}}_i(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t}))\, \mathrm{d}B(\mathrm{t}), i \in \mathbb{L}, \mathrm{t} \geq 0, \end{aligned} \tag{2.2}$$

where $\tilde{\mathcal{Y}}_i(\mathrm{t}) \in \mathbb{R}^n$ is the state vector associated with the $i$-th node and $U_i(\cdot)$ is the control input of the $i$-th node.

Define the synchronization error by $X_i(\mathrm{t}) = \tilde{\mathcal{Y}}_i(\mathrm{t}) - \mathcal{Y}_i(\mathrm{t}), X_i(\mathrm{t}) \in \mathbb{R}^n$. For notational simplicity, let $\mathcal{Y}_i^\tau = \mathcal{Y}_i(\mathrm{t} - \tau(\mathrm{t})), \tilde{\mathcal{Y}}_i^\tau = \tilde{\mathcal{Y}}_i(\mathrm{t} - \tau(\mathrm{t})), X_i^\tau = X_i(\mathrm{t} - \tau(\mathrm{t}))$. To regulate the response system according to the synchronization error, an event-triggered impulsive control strategy is introduced. The control input is applied only at a sequence of impulsive instants $\{\mathrm{t}_z\}_{z\in\mathbb{Z}_+}$, with $\mathrm{t}_0 = 0$. Between two successive impulsive instants, the response system evolves continuously. At each impulsive instant $\mathrm{t}_z$, the impulsive action produces the following jump of the error state: $X_i(\mathrm{t}_z) = \mathfrak{d}_i^{(z)} X_i(\mathrm{t}_z^-), i \in \mathbb{L},\ z \in \mathbb{Z}_+$, where $\mathfrak{d}_i^{(z)}$ denotes the impulsive gain.

The impulsive instants are generated by the following event-triggered rule:

$$\mathrm{t}_z = \inf\left\{\mathrm{t} > \mathrm{t}_{z-1} : \mathbb{E}\Omega(\mathrm{t}) \geq \omega_z \mathbb{E}(\Omega(\mathrm{t}_{z-1}) \vee \Omega_0)\right\}, \tag{2.3}$$

where $\omega_z > 1$ is the event-triggering parameter. Moreover, $\Omega(\mathrm{t}) = e^{m\mathrm{t}} \sum_{i=1}^N \sigma_i V_i(X_i(\mathrm{t}), k), \Omega_0 = \sum_{i=1}^N \sigma_i V_i^0$, with $m > 0$, $V_i^0 = \sup_{\zeta\in[-\tau,0]} V_i(X_i(\zeta), k)$. Here $V_i$ is the Lyapunov function associated with the $i$-th node and $\sigma_i$ are topology-dependent positive constants to be specified later.

In the error system, the symbols $f_i$, $g_i$, and $\Gamma_{ij}$ are used to denote the corresponding difference mappings generated by subtracting the drive system from the response system. More precisely, for $k = \alpha(\mathrm{t})$, define $f_i(X_i, X_i^\tau, k) = f_i(\tilde{\mathcal{Y}}_i, \tilde{\mathcal{Y}}_i^\tau, k) - f_i(\mathcal{Y}_i, \mathcal{Y}_i^\tau, k), g_i(X_i, X_i^\tau, k) = g_i(\tilde{\mathcal{Y}}_i, \tilde{\mathcal{Y}}_i^\tau, k) - g_i(\mathcal{Y}_i, \mathcal{Y}_i^\tau, k), \Gamma_{ij}(X_i, X_j, k) = \Gamma_{ij}(\tilde{\mathcal{Y}}_i, \tilde{\mathcal{Y}}_j, k) - \Gamma_{ij}(\mathcal{Y}_i, \mathcal{Y}_j, k)$. Then, by subtracting the drive system (2.1) from the response system (2.2), the error system is obtained as

$$\begin{cases} \mathrm{d}X_i(\mathrm{t}) = \Big[f_i\left(X_i(\mathrm{t}), X_i(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t})\right) + \displaystyle\sum_{j=1,\, j\neq i}^{N} a_{ij}(\alpha(\mathrm{t}))\Gamma_{ij}\left(X_i(\mathrm{t}), X_j(\mathrm{t}), \alpha(\mathrm{t})\right)\Big]\mathrm{d}\mathrm{t} \\ \qquad\qquad + g_i\left(X_i(\mathrm{t}), X_i(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t})\right)\mathrm{d}B(\mathrm{t}), \quad \mathrm{t} \neq \mathrm{t}_z, \\ X_i(\mathrm{t}_z) = \mathfrak{d}_i^{(z)} X_i(\mathrm{t}_z^-), \quad z \in \mathbb{Z}_+. \end{cases} \tag{2.4}$$

For the sake of rigor, it is assumed throughout this paper that $f(\cdot,\cdot,\cdot)$, $g(\cdot,\cdot,\cdot)$, and $\Gamma_{ij}(\cdot,\cdot,\cdot)$ satisfy the usual local Lipschitz and linear growth conditions. Under these conditions, system (2.4) admits a unique solution. The subsequent analysis will show that, under the proposed event-triggered impulsive mechanism, the error system is $p$-th moment exponentially stable and the triggering sequence admits no finite accumulation point.

To clarify the proposed control strategy, the drive–response synchronization framework is shown in Figure 1. The drive network evolves autonomously, while the synchronization error $X_i(t) = \widetilde{Y}_i(t) - Y_i(t)$ is monitored by the event-triggering mechanism. Once the triggering condition is satisfied, an impulsive control action is applied to the response network. The effects of Markovian switching, time-varying delay, and stochastic disturbance are incorporated into the error system.

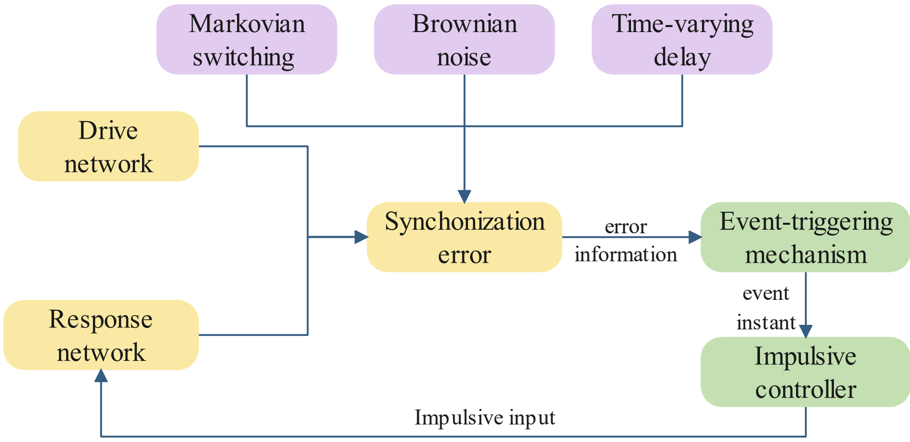


**Figure 1.** Control flow of the proposed framework.

Before deriving the main results, we introduce some definitions and auxiliary lemmas that will be used in the subsequent Lyapunov analysis. These include the infinitesimal operator, the node-Lyapunov function, and a graph-theoretic identity associated with the weighted digraph. They provide the technical foundation for establishing the moment stability of the error system and the synchronization of the drive-response systems.

**Definition 1.** *Let* $\mathcal{S}(x) = (q_{\mathfrak{M}\mathfrak{N}}(x))_{\mathfrak{M},\mathfrak{N}\in\mathbb{Z}_+}, x \in \mathbb{R}^{Nn}$, *be the state-dependent generator of the countably infinite-state Markovian switching process* $\alpha(\mathrm{t})$. *For each fixed* $x \in \mathbb{R}^{Nn}$, *denote by* $p_{\mathfrak{M}\mathfrak{N}}(\mathrm{t}) = \mathbb{P}\{\alpha(\mathrm{t}) = \mathfrak{N} \,|\, \alpha(0) = \mathfrak{M}\}$ *the transition probabilities of the continuous-time Markov chain generated by* $\mathcal{S}(x)$.

*Assume that there exists a probability measure* $\nu = (\nu_1, \nu_2, \dots)$ *such that* $\nu\mathcal{S}(x) = 0, x \in \mathbb{R}^{Nn}$. *The state-dependent Markovian switching mechanism is said to be strongly exponentially ergodic if there exist constants* $C > 0$ *and* $\eth > 0$ *such that*

$$\sup_{\mathfrak{M}\in\mathbb{Z}_+} \sum_{\mathfrak{N}\in\mathbb{Z}_+} |p_{\mathfrak{M}\mathfrak{N}}(x) - \nu_{\mathfrak{N}}| \le Ce^{-\eth \mathrm{t}}, \qquad \mathrm{t} \ge 0.$$

**Definition 2** ( [28]). *Let* $V_i(\cdot, k) \in C^2(\mathbb{R}^n; \mathbb{R}_+)$ *for each* $k \in \mathbb{Z}_+$. *The infinitesimal operator* $\mathscr{L}$ *applied to* $V_i(X_i, k)$ *is given by*

$$(\mathscr{L}V_i)(X_i, k) = \sum_{\mathfrak{N}\in\mathbb{Z}_+, \mathfrak{N}\neq k} q_{k\mathfrak{N}}(X(\mathrm{t}))\Big[V_i(X_i, \mathfrak{N}) - V_i(X_i, k)\Big] + V_{i,x}(X_i, k)\Big[f_i(X_i(\mathrm{t}), X_i(\mathrm{t}-\tau(\mathrm{t})), k)$$
$$+ \sum_{j=1,\, j\neq i}^{N} a_{ij}(k)\Gamma_{ij}(X_i(\mathrm{t}), X_j(\mathrm{t}), k)\Big] + \frac{1}{2}\operatorname{tr}\Big[g_i^\top(X_i(\mathrm{t}), X_i(\mathrm{t}-\tau(\mathrm{t})), k)V_{i,xx}(X_i, k)g_i(X_i(\mathrm{t}), X_i(\mathrm{t}-\tau(\mathrm{t})), k)\Big],$$

*where $V_{i,x}(X_i,k)$ and $V_{i,xx}(X_i,k)$ denote the gradient and Hessian of $V_i$ with respect to $X_i$, respectively.*

**Definition 3.** *For any $i \in \mathbb{L}$, a function $V_i(X_i,k) \in C^2(\mathbb{R}^n \times \mathbb{Z}_+; \mathbb{R}_+)$ is called the i-th node-Lyapunov function for* (2.4) *if the following conditions are satisfied:*

*V1: There exist positive constants $\pi_1^{(k)}$, $\pi_2^{(k)}$, and $p > 0$ such that, for all $i \in \mathbb{L}$ and $k \in \mathbb{Z}_+$,*

$$\pi_1^{(k)}|X_i|^p \le V_i(X_i,k) \le \pi_2^{(k)}|X_i|^p. \tag{2.5}$$

*Throughout this paper, we assume that $0 < \underline{\pi}_1 := \inf_{k\in\mathbb{Z}_+} \pi_1^{(k)}, \overline{\pi}_2 := \sup_{k\in\mathbb{Z}_+} \pi_2^{(k)} < \infty$.*

*V2: There exist positive constants $\Lambda_i^{(k)}$, $\Pi_i^{(k)}$, $\mathcal{D}_{ij}^{(k)}$, and functions $\Upsilon_{ij}^{(k)}(\mathfrak{L}_j(X_j), \mathfrak{L}_i(X_i))$ such that, for all $i \in \mathbb{L}$, $j \ne i$, and $k \in \mathbb{Z}_+$,*

$$\mathbb{E}(\mathscr{L}V_i)(X_i,k) \le \Lambda_i^{(k)}\mathbb{E}V_i(X_i(\mathrm{t}),k) + \Pi_i^{(k)}\mathbb{E}V_i(X_i(\mathrm{t}-\tau(\mathrm{t})),k) + \sum_{j=1,\, j\ne i}^{N} \mathcal{D}_{ij}^{(k)}\mathbb{E}\Upsilon_{ij}^{(k)}(\mathfrak{L}_j(X_j), \mathfrak{L}_i(X_i)). \tag{2.6}$$

*Moreover, assume that $\bar{\Lambda} := \sup_{k\in\mathbb{Z}_+} \max_{i\in\mathbb{L}} \Lambda_i^{(k)} < \infty, \bar{\Pi} := \sup_{k\in\mathbb{Z}_+} \max_{i\in\mathbb{L}} \Pi_i^{(k)} < \infty$. In addition, there exists a constant $\rho > 0$ such that, for all $\mathrm{t} \ge 0$ and $k \in \mathbb{Z}_+$,*

$$\mathbb{E} \sup_{\zeta\in[-\tau,0]} \sum_{i=1}^{N} \sigma_i V_i(X_i(\mathrm{t}+\zeta),k) \le \rho\mathbb{E}\sum_{i=1}^{N} \sigma_i V_i(X_i(\mathrm{t}),k). \tag{2.7}$$

*V3: For every directed cycle C of the weighted digraph $(\mathcal{H}, \mathbb{W}(Q))$, the following condition holds:*

$$\sum_{j=1,\, j\ne i}^{N} \Upsilon_{ij}^{(k)}(\mathfrak{L}_j(X_j), \mathfrak{L}_i(X_i)) \le 0, X_j, X_i \in \mathbb{R}^n. \tag{2.8}$$

**Lemma 1.** *[29] Let $c_k$ denote the cofactor of the k-th diagonal element of the matrix L. Then the following identity holds:*

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i(k)\varkappa_{ij}(k)\left(M_j(\mathrm{e}_j) - M_i(\mathrm{e}_i)\right) = \sum_{Q\in\mathbb{Q}} \mathbb{W}(Q) \sum_{(s,r)\in E(C_Q)} (M_r(\mathrm{e}_r) - M_s(\mathrm{e}_s)),$$

*where $\mathbb{Q}$ denotes the set of all spanning unicyclic subgraphs of the directed graph $(\mathcal{H}, \mathbb{W}(Q))$, $\mathbb{W}(Q)$ represents the weight of the subgraph $\tilde{C}$, and $C_Q$ is the unique directed cycle within $\tilde{C}$.*

**Lemma 2.** *If the Markov chain $\alpha(\mathrm{t})$ with generator Q is strongly exponentially ergodic and admits the invariant probability measure $\nu_{\mathfrak{M}} = (\nu_1, \nu_2, \ldots)^\top$, then for any bounded vector $b_{\mathfrak{M}} = (b_1, b_2, \ldots)^\top$ satisfying $\sum_{\mathfrak{M}} \nu_{\mathfrak{M}} b_{\mathfrak{M}} = 0$, there exists a bounded vector $c_{\mathfrak{N}} = (c_1, c_2, \ldots)^\top$ such that*

$$b_{\mathfrak{M}} = \sum_{\mathfrak{N}} q_{\mathfrak{M}\mathfrak{N}} c_{\mathfrak{N}}, \qquad \mathfrak{M}, \mathfrak{N} \in \mathbb{Z}_+.$$

**Remark 1.** *The synchronization problem for the drive-response systems is transformed into the stability analysis of the error system* (2.4). *Indeed, since* $X_i(\mathrm{t}) = \tilde{\mathcal{Y}}_i(\mathrm{t}) - \mathcal{Y}_i(\mathrm{t})$, *the p-th moment exponential stability of* (2.4) *directly implies*

$$\lim_{\mathrm{t}\to\infty} \mathbb{E} \sum_{i=1}^{N} \left|\tilde{\mathcal{Y}}_i(\mathrm{t}) - \mathcal{Y}_i(\mathrm{t})\right|^p = 0.$$

*Thus, the driving system and the response system achieve synchronization in the p-th moment exponential sense. Therefore, the subsequent analysis is carried out on the error dynamics rather than on the two original systems separately.*

**Remark 2.** *The strong exponential ergodicity condition in Definition 1 is essential for treating the countably infinite mode space. In finite-state Markovian switching systems, mode-dependent coefficients can generally be bounded by taking maxima over finitely many modes. Such finite-mode maximization is no longer applicable when the switching state space is* $\mathbb{Z}_+$. *The uniform ergodic estimate enables the long-term effects of infinitely many modes to be incorporated into the Lyapunov analysis and provides the probabilistic foundation for the Poisson-equation representation employed in the subsequent coefficient-based criterion.*

Theorem 1 gives a general Lyapunov-type criterion for the existence of a synchronization. However, its assumptions are expressed in terms of abstract node-Lyapunov conditions and may be difficult to verify directly. Therefore, we next introduce explicit coefficient-type conditions on the drift, diffusion, and coupling terms, which lead to a more tractable sufficient criterion.

## 3. Main results

In this section, we examine whether system (2.4) admits a synchronization. Sufficient conditions for its existence are obtained by using Lemma 2.1 in [30].

**Theorem 1.** *Assume that the state-dependent countably infinite-state Markovian switching mechanism generated by* $\mathcal{S}(x) = (q_{\mathfrak{M}\mathfrak{N}}(x))_{\mathfrak{M},\mathfrak{N}\in\mathbb{Z}_+}$, $x \in \mathbb{R}^{Nn}$, *is strongly exponentially ergodic with common invariant probability measure* $\nu = (\nu_1, \nu_2, \ldots)$ *in the sense of Definition 1. Suppose that the following conditions hold:*

- *A1: The directed graph* $(\mathcal{H}, \mathbb{W}(Q))$, *with weight matrix* $\mathbb{W}(Q) = (a_{ij})_{N\times N}$, *is strongly connected.*
- *A2: For each* $i \in \mathbb{L}$, *there exists an i-th node-Lyapunov function* $V_i(X_i, k)$ *for the error system* (2.4) *in the sense of Definition 3.*
- *A3: For each impulsive instant* $\mathrm{t} = \mathrm{t}_z$, *there exists* $\mu_i^{(z)} > 0$ *such that, for all* $i \in \mathbb{L}$ *and* $k \in \mathbb{Z}_+$,

$$\mathbb{E}V_i(X_i(\mathrm{t}_z), k) \le \mu_i^{(z)} \mathbb{E}V_i(X_i(\mathrm{t}_z^-), k), \tag{3.1}$$

*where* $\mu_z := \max_{i\in\mathbb{L}} \mu_i^{(z)}$.

*Furthermore, assume that there exist constants* $m > 0$ *and* $\bar{\rho} > 0$ *such that* $\bar{\rho}e^{m\tau} \le \rho, \omega_z > 1, \omega_z \prod_{\ell=1}^{z-1} \omega_{z-\ell}\mu_{z-\ell} \le \bar{\rho}, z \in \mathbb{Z}_+$, *where* $\rho$, $\bar{\Lambda}$, *and* $\bar{\Pi}$ *are given in Definition 3. Then, under the event-triggered condition* (2.3), *error system* (2.4) *is p-th moment exponentially stable. Consequently, driving system* (2.1) *and response system* (2.2) *achieve synchronization in the p-th moment exponential sense.*

*Moreover, if there exists a constant* $\varepsilon > 0$ *such that*

$$\frac{\ln \omega_z}{m + \bar{\Lambda} + \rho\bar{\Pi}} \geq \varepsilon, z \in \mathbb{Z}_+, \tag{3.2}$$

*then* $\mathrm{t}_z - \mathrm{t}_{z-1} \geq \varepsilon, z \in \mathbb{Z}_+$. *Hence,* $\lim_{z\to\infty} \mathrm{t}_z = +\infty$ *and the Zeno phenomenon is excluded.*

*Proof.* Define the following weighted Lyapunov function:

$$V(X, k) = \sum_{i=1}^{N} \sigma_i V_i(X_i(\mathrm{t}), k),$$

where $\sigma_i = [\sigma_1, \ldots, \sigma_N]^\top$ denotes the cofactor corresponding to the $i$-th diagonal entry of the Laplacian matrix associated with $(\mathcal{H}, \mathbb{W}(Q))$.

This estimate will now be used in analyzing all possible event-triggering patterns. To ensure a complete analysis, we consider all possible triggering patterns, classified into the following three cases:
**Case 1. (No events triggered)** If the triggering condition (2.3) is not met, then

$$\mathbb{E}\Omega(\mathrm{t}) < \omega_1 \mathbb{E}(\Omega(0) \vee \Omega_0) < \omega_1 \mathbb{E}\Omega_0,$$

for all $\mathrm{t} \geq 0$. Hence no impulses occur and event accumulation is impossible.
**Case 2. (Finitely many triggers occur)** Assume that events are triggered only up to index $\bar{z}$, i.e., $\mathrm{t}_z$ exists for $z = 0, 1, 2, \ldots, \bar{z}, \bar{z} \in \mathbb{Z}_+$. At each triggering instant,

$$\mathbb{E}\mathit{\Omega}(\mathrm{t}) = \ \omega_z \mathbb{E}(\mathit{\Omega}(\mathrm{t}_{z-1}) \vee \mathit{\Omega}_0) \leq \omega_z \prod_{\ell=1}^{\bar{n}} \mu_{z-1} \omega_z \mathbb{E}\mathit{\Omega}_0, \quad \mathrm{t} \in [\mathrm{t}_{z-1}, \mathrm{t}_z), \tag{3.3}$$

where $\mu_z = \max_{i \in N}\{\mu_i^{(z)}\}, \bar{n} \in N_z, N_z = \{1, 2, \ldots, z-1\}$. In addition, when $\mathrm{t} = \mathrm{t}_\xi$, by assumption (3.1), the following can be deduced:

$$\mathbb{E}\Omega(\mathrm{t}_z) = \mathbb{E}\sum_{i=1}^{N} \sigma_i V_i(X_i(\mathrm{t}_z), k) e^{m\mathrm{t}_z} \leq \mathbb{E}\sum_{i=1}^{N} \sigma_i \mu_i^{(z)} V_i(X_i(\mathrm{t}_z^-), k) e^{m\mathrm{t}_z} \leq \mu_z \mathbb{E}\Omega(\mathrm{t}_z^-) \leq \mu_z \omega_z \prod_{\ell=1}^{\bar{n}} \mu_{z-1} \omega_z \mathbb{E}\Omega_0.$$

Since the number of impulsive instants is finite, no impulse is generated when $\mathrm{t} > \mathrm{t}_{\bar{z}}$, where $\bar{z} \in \mathbb{Z}_+$. It then follows from (2.3) that

$$\mathbb{E}\Omega(\mathrm{t}) < \omega_{\bar{z}+1} \mathbb{E}(\Omega(\mathrm{t}_{\bar{z}}) \vee \Omega_0) < \omega_{\bar{z}+1} \prod_{\ell=1}^{\bar{n}} \mu_{\bar{z}-\ell+1} \omega_{\bar{z}-\ell+1} \mathbb{E}\Omega_0, \tag{3.4}$$

where $\bar{n} \in N_z, N_z = \{1, 2, \ldots, z, \ldots, \bar{z}\}, \mathrm{t} \in [\mathrm{t}_\xi, \infty)$. On the basis of (3.3) and (3.4), we obtain the following conclusions for the case where only finitely many impulses occur.

$$\mathbb{E}\Omega(\mathrm{t}) \leq \bar{\rho}\mathbb{E}\Omega_0, \ \mathrm{t} \geq \mathrm{t}_0.$$

**Case 3. (Infinitely many triggers)** Suppose that impulsive events occur infinitely many times. Then the corresponding sequence of triggering instants satisfying (2.3) is denoted by $\{\mathrm{t}_\xi\}_{\xi \in \mathbb{Z}^+}$, where

$$0 = \mathrm{t}_0 < \mathrm{t}_1 < \mathrm{t}_2 < \cdots < \mathrm{t}_\xi < \cdots.$$

It remains to rule out the possibility of Zeno behavior.

Since $\mathbb{E}V_i(X_i(\mathrm{t}),k)$ is continuous on $[\mathrm{t}_{z-1},\mathrm{t}_z)$, there exists $\bar{\mathrm{t}}_{z-1}$ such that

$$\bar{\mathrm{t}}_{z-1}=\sup\left\{\ \mathrm{t}\in[\mathrm{t}_{z-1},\mathrm{t}_z):\mathbb{E}\Omega(\mathrm{t})\le\mathbb{E}(\Omega(\mathrm{t}_{z-1})\vee\Omega_0)\ \right\} \tag{3.5}$$

and

$$\mathbb{E}\Omega(\bar{\mathrm{t}}_{z-1})=\mathbb{E}(\Omega(\mathrm{t}_{z-1})\vee\Omega_0).$$

In addition, for every $\mathrm{t}\in[\bar{\mathrm{t}}_{z-1},\mathrm{t}_z)$, one has

$$\mathbb{E}\Omega(\bar{\mathrm{t}}_{z-1})\le\mathbb{E}\Omega(\mathrm{t})\le\mathbb{E}\Omega(\mathrm{t}_z^-).$$

Moreover, for any $\mathrm{t}\in[\bar{\mathrm{t}}_{z-1},\mathrm{t}_z]$, if $(\mathrm{t}+\zeta)\in[\mathrm{t}_{z-1},\mathrm{t}_z]$, then

$$\mathbb{E}\Omega(\mathrm{t}+\zeta)\le\mathbb{E}\Omega(\mathrm{t}_z)\le\omega_z\mathbb{E}(\Omega(\mathrm{t}_{z-1})\vee\Omega_0)=\omega_z\mathbb{E}\Omega(\bar{\mathrm{t}}_{z-1})\le\omega_z\mathbb{E}\Omega(\mathrm{t}). \tag{3.6}$$

Likewise, for any $\mathrm{t}\in[\bar{\mathrm{t}}_{z-1},\mathrm{t}_z]$, if $(\mathrm{t}+\zeta)\in[\mathrm{t}_{z-2},\mathrm{t}_{z-1}]$, then

$$\mathbb{E}\Omega(\mathrm{t}+\zeta)\le\mathbb{E}\Omega(\mathrm{t}_{z-1})\le\mathbb{E}\omega_{z-1}(\Omega(\mathrm{t}_{z-2})\vee\Omega_0)\le\omega_{z-1}\prod_{\ell=1}^{z-2}\mu_{z-\ell-1}\omega_{z-\ell-1}\mathbb{E}\Omega(\mathrm{t}). \tag{3.7}$$

Repeating the same argument, for any $\mathrm{t}\in[\bar{\mathrm{t}}_{z-1},\mathrm{t}_z]$, if $(\mathrm{t}+\zeta)\in[\mathrm{t}_0,\mathrm{t}_1]$, then we obtain

$$\mathbb{E}\Omega(\mathrm{t}+\zeta)\le\mathbb{E}\Omega(\mathrm{t}_z)\le\omega_1\mathbb{E}\Omega(\mathrm{t}).$$

Similarly, if $(\mathrm{t}+\zeta)\in[\mathrm{t}_0-\tau,\mathrm{t}_0]$, then

$$\mathbb{E}\Omega(\mathrm{t}+\zeta)\le\mathbb{E}\Omega_0\le\mathbb{E}\Omega(\mathrm{t}). \tag{3.8}$$

Combining (3.6)–(3.8), it follows that for any $(\mathrm{t})\in[\bar{\mathrm{t}}_{z-1},\mathrm{t}_z]$, the following inequality holds:

$$\mathbb{E}\Omega(\mathrm{t}+\zeta)\le[\omega_z\vee\omega_z\prod_{\ell=1}^{z-2}\mu_{z-\ell-1}\omega_{z-\ell-1}\vee\cdots\vee\omega_1\vee 1]\mathbb{E}\Omega(\mathrm{t})\le\bar{\rho}\mathbb{E}\Omega(\mathrm{t}).$$

It also means that

$$\mathbb{E}\sup_{\zeta\in[-\tau,0]}\sum_{i=1}^{N}\sigma_i\,V_i(X_i(\mathrm{t})+\zeta,k)\le\ \rho\mathbb{E}\sum_{i=1}^{N}\sigma_i\,V_i(X_i(\mathrm{t}),k),$$

where $\bar{\rho}e^{m\tau}\le\rho$ . Taking the upper-right-hand Dini derivative and applying the linearity of expectation yields

$$D^+\mathbb{E}\Omega(\mathrm{t})=me^{m\mathrm{t}}\sum_{i=1}^{N}\mathbb{E}\sigma_iV_i(X_i(\mathrm{t}),k)+e^{m\mathrm{t}}\sum_{i=1}^{N}\sigma_i\mathbb{E}\mathscr{L}V_i(X_i(\mathrm{t}),k).$$

Using inequality (2.6) and Lemma 1, we obtain

$$D^+\mathbb{E}\Omega(\mathrm{t})\le me^{m\mathrm{t}}\sum_{i=1}^{N}\sigma_i\mathbb{E}V_i(X_i(\mathrm{t}),k)+e^{m\mathrm{t}}\sum_{i=1}^{N}\sigma_i\mathbb{E}\Big[\Lambda_i^{(k)}V_i(X_i(\mathrm{t}),k)$$

$$
\begin{aligned}
&+ \Pi_i^{(k)} V_i(X_i(\mathrm{t}-\tau(\mathrm{t})),k) + \mathbb{E}\sum_{j=1,j\neq i}^{N} \mathcal{D}_{ij}^{(k)} \Upsilon_{ij}^{(k)}(\mathfrak{L}_j(X_j),\mathfrak{L}_i(X_i))\Big] \\
&\leq me^{m\mathrm{t}} \sum_{i=1}^{N} \sigma_i \mathbb{E} V_i(X_i(\mathrm{t}),k) + e^{m\mathrm{t}} \sum_{i=1}^{N} \sigma_i \mathbb{E}\Big[\Lambda_i^{(k)} V_i(X_i(\mathrm{t}),k) + \Pi_i^{(k)} V_i(X_i(\mathrm{t}-\tau(\mathrm{t})),k)\Big] \\
&\leq \mathbb{E}\Omega(\mathrm{t})(m+\bar{\Lambda}+\rho\bar{\Pi}).
\end{aligned}
$$

Therefore,

$$
\mathbb{E}\Omega(\mathrm{t}_z^-) \leq e^{(m+\bar{\Lambda}+\rho\bar{\Pi})(\mathrm{t}_z-\bar{\mathrm{t}}_{z-1})} \mathbb{E}\Omega(\bar{\mathrm{t}}_{z-1}). \tag{3.9}
$$

Combining (3.5) with (3.9) gives

$$
\mathbb{E}\Omega(\mathrm{t}_z^-) = \omega_z \mathbb{E}(\Omega(\mathrm{t}_{z-1}) \vee \Omega_0) = \omega_z \mathbb{E}\Omega(\bar{\mathrm{t}}_{z-1}) \leq e^{(m+\bar{\Lambda}+\rho\bar{\Pi})(\mathrm{t}_z-\bar{\mathrm{t}}_{z-1})} \mathbb{E}\Omega(\bar{\mathrm{t}}_{z-1}). \tag{3.10}
$$

Since $\mathbb{E}\Omega(\bar{\mathrm{t}}_{z-1}) > 0$, it follows from (3.10) that

$$
\frac{\ln \omega_z}{m+\bar{\Lambda}+\rho\bar{\Pi}} \leq \mathrm{t}_z - \bar{\mathrm{t}}_{z-1} \leq \mathrm{t}_z - \mathrm{t}_{z-1}.
$$

By (3.2), one obtains $\mathrm{t}_z - \mathrm{t}_{z-1} \geq \varepsilon, z \in \mathbb{Z}_+$. Consequently,

$$
\mathrm{t}_z = \mathrm{t}_0 + \sum_{\ell=1}^{z} (\mathrm{t}_\ell - \mathrm{t}_{\ell-1}) \geq \mathrm{t}_0 + z\varepsilon.
$$

Therefore, $\lim_{z\to\infty} \mathrm{t}_z = +\infty$, which shows that the triggering instants cannot accumulate within any finite time interval. Hence, the Zeno phenomenon is excluded.

Next, it is shown that the $p$-th moment exponential stability of the error system implies synchronization between the driving system and the response system.

Deriving differential inequalities for a Lyapunov function demonstrates synchronization in the $p$-th moment exponential. From (2.5), we derive upper and lower bounds for $V_i(X_i(\mathrm{t}),k)$ as follows

$$
V(X(\mathrm{t}),k) \leq \max_{i\in\mathbb{L}} \left(\sigma_i \pi_2^{(k)}\right) \sum_{i=1}^{N} |X_i(\mathrm{t})|^p, \quad V(X(\mathrm{t}),k) \geq \min_{i\in\mathbb{L}} \left(\sigma_i \pi_1^{(k)}\right) \sum_{i=1}^{N} |X_i(\mathrm{t})|^p.
$$

Let $\underline{\pi}_1 := \inf_{k\in\mathbb{Z}_+} \pi_1^{(k)} > 0, \quad \overline{\pi}_2 := \sup_{k\in\mathbb{Z}_+} \pi_2^{(k)} < \infty$. Then, for all $\mathrm{t} \geq 0$ and $k \in \mathbb{Z}_+$,

$$
\min_{i\in\mathbb{L}} \sigma_i \underline{\pi}_1 \, \|X(\mathrm{t})\|^p \leq V(X(\mathrm{t}),k) \leq \max_{i\in\mathbb{L}} \sigma_i \overline{\pi}_2 \, \|X(\mathrm{t})\|^p,
$$

where $\|X(\mathrm{t})\|^p := \sum_{i=1}^{N} |X_i(\mathrm{t})|^p$.

From the previous estimates, it follows that

$$
\mathbb{E}\|X_i(\mathrm{t})\|^p \leq \frac{\max_{i\in\mathbb{L}} \sigma_i \overline{\pi}_2}{\min_{i\in\mathbb{L}} \sigma_i \underline{\pi}_1} \mathbb{E}\Omega(\mathrm{t}) \leq \frac{\max_{i\in\mathbb{L}} \sigma_i \overline{\pi}_2}{\min_{i\in\mathbb{L}} \sigma_i \underline{\pi}_1} \bar{\omega} \, \mathbb{E}\Omega(0) \exp\{-m\mathrm{t}\} = \frac{\widetilde{\pi_2}}{\underset{\sim}{\pi_1}} \bar{\omega} \, \mathbb{E}\Omega(0) \exp\{-m\mathrm{t}\},
$$

where $\widetilde{\pi_2} = \max_{i\in\mathbb{L}} \sigma_i \overline{\pi}_2, \underset{\sim}{\pi_1} = \min_{i\in\mathbb{L}} \sigma_i \underline{\pi}_1$. Since $X_i(\mathrm{t}) = \widetilde{\mathcal{Y}}_i(\mathrm{t}) - \mathcal{Y}_i(\mathrm{t})$, the above estimate implies $\lim_{\mathrm{t}\to\infty} \mathbb{E} \sum_{i=1}^{N} |\widetilde{\mathcal{Y}}_i(\mathrm{t}) - \mathcal{Y}_i(\mathrm{t})|^p = 0$. Hence, the driving system and the response system achieve synchronization in the $p$-th moment exponential sense.

This completes the proof. □

**Remark 3.** *The lower bound condition*

$$\frac{\ln \omega_z}{m + \bar{\Lambda} + \rho\bar{\Pi}} \geq \varepsilon > 0$$

*has a direct interpretation in the event-triggered impulsive mechanism. It ensures that the weighted Lyapunov functional requires a nonzero amount of time to grow from its post-event level to the next triggering threshold. Hence, successive triggering instants cannot accumulate in any finite time interval. This condition provides an explicit Zeno-exclusion criterion and guarantees the implementability of the proposed impulsive control strategy.*

Although Theorem 1 provides a general Lyapunov-type criterion for the existence of a synchronization and the exclusion of Zeno behavior, its conditions may not be directly verifiable in applications. Therefore, we next impose additional structural conditions on the drift, diffusion, and coupling terms, and derive a more explicit coefficient-type criterion. These conditions ensure that the node-Lyapunov requirements in Definition 3 are satisfied, so that the conclusion of Theorem 1 can be applied.

**Theorem 2.** *Assume that the state-dependent countably infinite-state Markovian switching mechanism* $\alpha(\mathrm{t})$*, generated by* $\mathcal{S}(x) = (q_{\mathfrak{M}\mathfrak{N}}(x))_{\mathfrak{M},\mathfrak{N}\in\mathbb{Z}_+}$*, is strongly exponentially ergodic with the common invariant probability measure* $\nu = (\nu_1, \nu_2, \ldots)$ *in the sense of Definition 1. Assume that, for every* $i, j \in \mathbb{L}$ *with* $j \neq i$ *and* $k \in \mathbb{Z}_+$*, the following conditions hold:*

*B1: For any* $X_i, G_i \in \mathbb{R}^n$*, there exist positive constants* $d_i^{(k)}$*,* $h_i^{(k)}$*, and* $\mathscr{X}_{ij}^{(k)}$ *such that*

$$(X_i)^\top f_i(X_i, G_i, k) \leq d_i^{(k)}|X_i|^2 + h_i^{(k)}|G_i|^2 \tag{3.11}$$

*and*

$$(X_i)^\top \Gamma_{ij}(X_i, X_j, k) \leq \mathscr{X}_{ij}^{(k)}(|X_i|^2 - |X_j|^2). \tag{3.12}$$

*B2: For any* $X_i, E_i \in \mathbb{R}^n$*, there exist nonnegative constants* $\delta_i^{(k)}$ *and* $\varepsilon_i^{(k)}$ *such that*

$$\mathrm{tr}(g_i(X_i, E_i, k)^\top g_i(X_i, E_i, k)) \leq \delta_i^{(k)}|X_i|^2 + \varepsilon_i^{(k)}|E_i|^2. \tag{3.13}$$

*B3: The weighted digraph* $(\mathcal{H}, \mathbb{W}(Q))$ *is strongly connected, where* $\mathbb{W}(Q) = (a_{ij})_{N\times N}$*.*

*Assume further that, for each impulsive instant* $\mathrm{t} = \mathrm{t}_z$*, the impulsive effect satisfies*

$$\mathbb{E}V_i(X_i(\mathrm{t}_z), k) \leq \mu_i^{(z)}\mathbb{E}V_i(X_i(\mathrm{t}_z^-), k), \tag{3.14}$$

*where* $\mu_z := \max_{i\in\mathbb{L}} \mu_i^{(z)}$*.*

*Let* $\bar{\Lambda}$ *and* $\bar{\Pi}$ *be the constants induced by* (3.11)–(3.13) *in the Lyapunov estimate, assume that* $\bar{\Lambda} := \sup_{k\in\mathbb{Z}_+} \Lambda^{(k)} < \infty, \bar{\Pi} := \sup_{k\in\mathbb{Z}_+} \Pi^{(k)} < \infty$*. Moreover, suppose that there exist positive constants m,* $\rho$*, and* $\bar{\rho}$ *such that* $\bar{\rho}e^{m\tau} \leq \rho, \omega_z > 1, \omega_z \prod_{\ell=1}^{z-1} \omega_{z-\ell}\mu_{z-\ell} \leq \bar{\rho}, z \in \mathbb{Z}_+$*.*

*Then, under the event-triggered condition* (2.3)*, the error system* (2.4) *is p-th moment exponentially stable. Consequently, the driving system* (2.1) *and the response system* (2.2) *achieve synchronization in the p-th moment exponential sense.*

*Furthermore, if there exists a constant* $\varepsilon > 0$ *such that* $\frac{\ln \omega_z}{m+\bar{\Lambda}+\rho\bar{\Pi}} \geq \varepsilon, z \in \mathbb{Z}_+$*, then* $\mathrm{t}_z - \mathrm{t}_{z-1} \geq \varepsilon, z \in \mathbb{Z}_+$*. Hence,* $\lim_{z\to\infty} \mathrm{t}_z = +\infty$ *and the Zeno phenomenon is excluded.*

*Proof.* For each $h > 0$, denote by $B_h \subseteq \mathbb{R}^n$ the open ball centered at the origin with radius $h$. Let

$$\chi = -\sum_{\mathfrak{M}\in\mathbb{Z}_+} c_{\mathfrak{M}}\nu_{\mathfrak{M}}, \quad c_{\mathfrak{M}} < 0, \quad |c_{\mathfrak{M}}| \geq \frac{3\chi}{4}.$$

Since $\sum_{\mathfrak{M}\in\mathbb{Z}_+} \nu_{\mathfrak{M}} = 1$, it follows that $\sum_{\mathfrak{M}\in\mathbb{Z}_+}(c_{\mathfrak{M}} + \chi)\nu_{\mathfrak{M}} = 0$. Given the strong exponential ergodicity of $\alpha(\mathrm{t})$, Lemma 2 guarantees the existence of a bounded sequence $\{b_i^{(\mathfrak{M})} : \mathfrak{M} \in \mathbb{Z}_+,\ i \in \mathbb{L}\}$ satisfying

$$\sum_{\mathfrak{N}\in\mathbb{Z}_+} q_{\mathfrak{M}\mathfrak{N}}(0)\, b_i^{(\mathfrak{N})} = \chi + c_{\mathfrak{M}}, \quad \mathfrak{M} \in \mathbb{Z}_+,\ i \in \mathbb{L}. \tag{3.15}$$

Since $\sum_{\mathfrak{N}\in\mathbb{Z}_+} q_{\mathfrak{M}\mathfrak{N}}(0) = 0$ for any $\mathfrak{M} \in \mathbb{Z}_+$ , it follows from (3.15) that

$$\sum_{\mathfrak{N}\in\mathbb{Z}_+} q_{\mathfrak{M}\mathfrak{N}}(0)b_i^{(\mathfrak{M})} = \sum_{\mathfrak{N}\in\mathbb{Z}_+} q_{\mathfrak{M}\mathfrak{N}}(0)(1 - \eta b_i^{(\mathfrak{M})}) = -\eta(\chi + c_{\mathfrak{M}}), \mathfrak{M} \in \mathbb{Z}_+,\ i \in \mathbb{L}. \tag{3.16}$$

Given the boundedness of $\{b_i^{(\mathfrak{M})}\}$, we can choose $\eta \in (0, 1), \forall x \in B_h$ such that

$$\eta|b_i^{(\mathfrak{M})}| \leq \min\left\{\frac{\chi}{4}, \frac{1}{2}\right\}. \tag{3.17}$$

In view of Definition 1 and (3.17), there is an $h > 0$ sufficiently small such that

$$\sum_{\mathfrak{N}\in\mathbb{Z}_+} (1 - \eta b_i^{(\mathfrak{M})})|q_{\mathfrak{M}\mathfrak{N}}(x) - q_{\mathfrak{M}\mathfrak{N}}(0)| < \frac{\eta\chi}{4}. \tag{3.18}$$

Define the function $V(X(\mathrm{t}), k) : B_h \times \mathbb{Z}_+ \to \mathbb{R}_+$ by $V(X(\mathrm{t}), k) = \sum_{i=1}^N \sigma_i V_i(X_i(\mathrm{t}), k)$. Since $(\mathcal{H}, \mathbb{W}(Q))$ is strongly connected, we have $\sigma_i > 0$ for all $i \in \mathbb{L}$, and $\sigma_i$ is the cofactor of the $i$-th diagonal element of the Laplacian matrix of $(\mathcal{H}, \mathbb{W}(Q))$:

$$\sup_{\zeta\in[-\tau,0]} \mathbb{E}\sum_{i=1}^N V_i(X_i(\mathrm{t} + \zeta), k) \leq \rho\mathbb{E}\sum_{i=1}^N V_i(X_i(\mathrm{t}), k).$$

Then

$$\sup_{\zeta\in[-\tau,0]} \mathbb{E}\sum_{i=1}^N \sigma_i V_i(X_i(\mathrm{t} + \zeta), k) \leq \rho\mathbb{E}\sum_{i=1}^N \sigma_i V_i(X_i(\mathrm{t}), k).$$

Define the function $V_i(X_i(\mathrm{t}), k) : B_h \times \mathbb{Z}_+ \to \mathbb{R}_+$ by $V_i(X_i(\mathrm{t}), k) = (1 - \eta b_i^{(\mathfrak{M})})U^\eta(X_i(\mathrm{t})), U(X_i(\mathrm{t})) = |X_i(\mathrm{t})|^q, p = \eta q \geq 2$. By Itô's formula, we have

$$\begin{aligned} \mathcal{L}V_i(X_i(\mathrm{t}), k) = {}& \eta(1 - \eta b_i^{(\mathfrak{M})})U^{\eta-1}(X_i(\mathrm{t}))\mathcal{L}U_i(X_i(\mathrm{t})) + U^\eta(X_i(\mathrm{t}))\sum_{\mathfrak{N}\in\mathbb{Z}_+}(1 - \eta b_i^{(\mathfrak{N})})q_{\mathfrak{M}\mathfrak{N}}(x) \\ & - \frac{\eta(1-\eta)}{2}U^{\eta-2}(X_i(\mathrm{t}))|U_x(X_i(\mathrm{t}))g(X_i(\mathrm{t}), X_i(\mathrm{t} - \tau(\mathrm{t})), k)|^2. \end{aligned} \tag{3.19}$$

For simplicity of notation and computation, the following analysis will be carried out separately in several parts.

The application of Conditions (B1) and (B2) leads to

$$\begin{aligned}\mathcal{L}U_i(X_i(\mathrm{t})) &= q|X_i(\mathrm{t})|^{q-2}(X_i(\mathrm{t}))^\top\Big[f_i(X_i(\mathrm{t}),X_i(\mathrm{t}-\tau(\mathrm{t})),k)\\ &\quad+\sum_{j=1,j\neq i}^{N}a_{ij}(k)\Gamma_{ij}(X_i(\mathrm{t}),X_j(\mathrm{t}),k)\Big]+\frac{q(q-1)}{2}|X_i(\mathrm{t})|^{q-2}|g_i(X_i(\mathrm{t}),X_i(\mathrm{t}-\tau(\mathrm{t})),k)|^2\\ &\le |X_i(\mathrm{t})|^q(qd_i^{(k)}+\frac{q(q-1)}{2}\delta_i^{(k)})+(qh_i^{(k)}+\frac{q(q-1)}{2}\epsilon_i^{(k)})|X_i(\mathrm{t})|^{q-2}|X_i(\mathrm{t}-\tau(\mathrm{t}))|^2\\ &\quad+\sum_{j=1,j\neq i}^{N}a_{ij}(k)\mathscr{X}_{ij}^{(k)}(|X_i(\mathrm{t})|^q-|X(t_i)|^{q-2}|X_j(\mathrm{t})|^2).\end{aligned}\tag{3.20}$$

In that way, we can introduce

$$\begin{aligned}\eta(1-\eta b_i^{(\mathfrak{M})})U^{\eta-1}(X_i(\mathrm{t}))\mathcal{L}U_i(X_i(\mathrm{t})) &\le \eta(1-\eta b_i^{(\mathfrak{M})})|X_i(\mathrm{t})|^{\eta q-q}\Big[|X_i(\mathrm{t})|^q(qd_i^{(k)}+\tfrac{q(q-1)}{2}\delta_i^{(k)})+(qh_i^{(k)}\\ &\quad+\tfrac{q(q-1)}{2}\epsilon_i^{(k)})|X_i(\mathrm{t})|^{q-2}|X_i(\mathrm{t}-\tau(\mathrm{t}))|^2+\sum_{j=1,j\neq i}^{N}a_{ij}(k)\mathscr{X}_{ij}^{(k)}(|X_i(\mathrm{t})|^q-|X_i(\mathrm{t})|^{q-2}|X_j(\mathrm{t})|^2)\Big]\\ &\le \eta(1-\eta b_i^{(\mathfrak{M})})|X_i(\mathrm{t})|^{\eta q}\Big[qd_i^{(k)}+\tfrac{q(q-1)}{2}\delta_i^{(k)}+(h_i^{(k)}+\tfrac{q-1}{2}\epsilon_i^{(k)})\tfrac{\eta q-2}{\eta}\Big]+(1-\eta b_i^{(\mathfrak{M})})(2h_i^{(k)}\\ &\quad+(q-1)\epsilon_i^{(k)})|X_i(\mathrm{t}-\tau(\mathrm{t}))|^{\eta q}+\tfrac{2}{q}(1-\eta b_i^{(\mathfrak{M})})\sum_{j=1,j\neq i}^{N}a_{ij}(k)\mathscr{X}_{ij}^{(k)}(|X_i(\mathrm{t})|^{\eta q}-|X_j(\mathrm{t})|^{\eta q}).\end{aligned}\tag{3.21}$$

Moreover, by making use of Formulas(3.16) and (3.18), we can deduce that

$$\begin{aligned}U^\eta(X_i(\mathrm{t}))\sum_{\mathfrak{N}\in\mathbb{Z}_+}(1-\eta b_i^{(\mathfrak{M})})q_{\mathfrak{M}\mathfrak{N}}(x) &= U^\eta(X_i(\mathrm{t}))\sum_{\mathfrak{N}\in\mathbb{Z}_+}(1-\eta b_i^{(\mathfrak{M})})q_{\mathfrak{M}\mathfrak{N}}(0)\\ &\quad+U^\eta(X_i(\mathrm{t}))\sum_{\mathfrak{N}\in\mathbb{Z}_+}(1-\eta b_i^{(\mathfrak{M})})|q_{\mathfrak{M}\mathfrak{N}}(x)-q_{\mathfrak{M}\mathfrak{N}}(0)|\\ &\le (1-\eta b_i^{(\mathfrak{M})})U^\eta(X_i(\mathrm{t}))\left(\frac{-3\eta\chi-4\eta c_{\mathfrak{M}}}{4(1-\eta b_i^{(\mathfrak{M})})}\right)\le V_i(X_i(\mathrm{t}),k)\left(\frac{-3\eta\chi-4\eta c_{\mathfrak{M}}}{4(1-\eta b_i^{(\mathfrak{M})})}\right).\end{aligned}\tag{3.22}$$

Hence, by synthesizing points (3.20) and (3.22), we can derive the final result:

$$\begin{aligned}\mathcal{L}V_i(X_i(\mathrm{t}),k) &\le V_i(X_i(\mathrm{t}),k)\Big[\eta qd_i^{(k)}+\frac{1}{2}\eta q(q-1)\delta_i^{(k)}+(h_i^{(k)}+\frac{1}{2}(q-1)\epsilon_i^{(k)})(\eta q-2)+\frac{-3\eta\chi-4\eta c_{\mathfrak{M}}}{4(1-\eta b_i^{(\mathfrak{M})})}\Big]\\ &\quad+V(X_i(\mathrm{t}-\tau(\mathrm{t})),z)(2h_i^{(k)}+\ (q-1)\epsilon_i^{(k)})+\sum_{j=1,j\neq i}^{N}a_{ij}(k)\frac{2}{q}(1-p\eta b_i^{(\mathfrak{M})})\mathscr{X}_{ij}^{(k)}(|X_i(\mathrm{t})|^{\eta q}-|X_j(\mathrm{t})|^{\eta q}))\\ &=\Lambda_i^{(k)}V_i(X_i(\mathrm{t}),k)+\Pi_i^{(k)}V_i(X_i(\mathrm{t}-\tau(\mathrm{t})),k)+\sum_{j=1,j\neq i}^{N}\mathcal{D}_{ij}^{(k)}\Upsilon_{ij}^{(k)}(\mathfrak{L}_j(X_j),\mathfrak{L}_i(X_i)),\end{aligned}$$

where

$$\Lambda_i^{(k)}=\eta qd_i^{(k)}+\frac{1}{2}\eta q(q-1)\delta_i^{(k)}+\frac{-3\eta\chi-4\eta c_{\mathfrak{M}}}{4(1-\eta b_i^{(\mathfrak{M})})}+(h_i^{(k)}+\frac{1}{2}(q-1)\epsilon_i^{(k)})(\eta q-2)>0,$$

$$\mathcal{D}_{ij}^{(k)} = \frac{2}{q}(1 - \eta b_i^{(\mathfrak{M})})\mathscr{X}_{ij}^{(k)} > 0, \quad \Pi_i^{(k)} = (2h_i^{(k)} + (q-1)\epsilon_i^{(k)}) > 0,$$
$$\Upsilon_{ij}^{(k)}(\mathfrak{L}_j(X_j), \mathfrak{L}_i(X_i)) = |X_i(\mathrm{t})|^{\eta q} - |X_j(\mathrm{t})|^{\eta q}.$$

Hence, the V2 is satisfied.

At each impulsive instant $\mathrm{t} = \mathrm{t}_z$, we have

$$\mathbb{E}V_i(X_i(\mathrm{t}_z), k) = (1 - \eta b_i^{(k)})\mathbb{E}|X_i(\mathrm{t}_z)|^{\eta q} = (1 - \eta b_i^{(k)})(\mathfrak{d}_i^{(k)})^{\eta q}\mathbb{E}|X_i(\mathrm{t}_z^-)|^{\eta q} \leq \mu_i^{(z)}\mathbb{E}V_i(X_i(\mathrm{t}_z^-), k).$$

Thus, the impulsive condition in A3 of Theorem 1 is satisfied. Hence, the node-Lyapunov conditions in Definition 3 are verified. Combining the above estimates with the impulsive condition and the event-triggered condition (2.3), all the assumptions of Theorem 1 are fulfilled. Therefore, the error system (2.4) is $p$-th moment exponentially stable. Consequently, the driving system (2.1) and the response system (2.2) achieve synchronization in the $p$-th moment exponential sense. Moreover, since $\lim_{z\to\infty} \mathrm{t}_z = +\infty$, the triggering instants do not accumulate in any finite time interval. Hence, the Zeno phenomenon is excluded.

This completes the proof. □

**Remark 4.** *The stability in Theorem 2 is ensured by the strong exponential ergodicity of the Markovian process and Lyapunov-based analysis. The event-triggered impulsive control framework guarantees stable synchronization, excludes Zeno behavior, and ensures a minimum inter-event interval. This approach aligns with [31] on stochastic system stabilization, [32] on event-triggered synchronization, and [33] on the exponential stability of fuzzy networks under intermittent control.*

## 4. Application

### 4.1. Application to a network of coupled Chua circuits

To illustrate the above theoretical results, Chua's circuit is chosen as an application example. This circuit is not only one of the simplest circuits that can be implemented experimentally, but also a classical representative nonlinear chaotic circuit.

Consider the following internally interconnected Chua's circuit system. The single Chua's circuit depicted in Figure 2 is described by the following differential equations:

$$\begin{cases} C_1\dot{v}_1(\mathrm{t}) = \dfrac{1}{R}(-v_1(\mathrm{t}) + v_2(\mathrm{t})) - f(v_1(\mathrm{t})), \\ C_2\dot{v}_2(\mathrm{t}) = \dfrac{1}{R}(v_1(\mathrm{t}) - v_2(\mathrm{t})) + i_3(\mathrm{t}), \\ \quad L\dot{i}_3(\mathrm{t}) = -(v_2(\mathrm{t}) + R_0 i_3(\mathrm{t})), \end{cases}$$

where $v_1$ and $v_2$ are the voltages across the capacitors $C_1$ and $C_2$, respectively, and the term $f(v_1)$ represents the current through the nonlinear resistor $N_R$.

$f(v_1)$ satisfies the following formula: $f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|v_1 + E| - |v_1 - E|)$, where $G_a$ and $G_b$ are the slopes of the inner and outer regions.

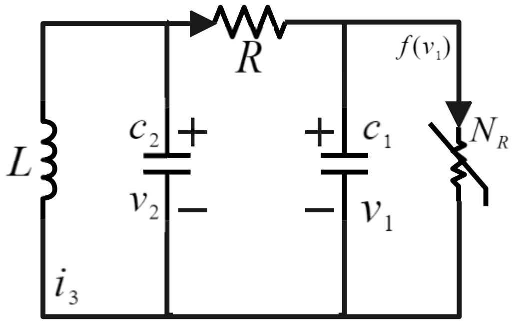


**Figure 2.** A single Chua's circuit.

We now proceed to describe the system dynamics for the Chua's circuit in a more generalized form:

$$\begin{aligned}\begin{pmatrix}\dot{Y}_{i1}\\ \dot{Y}_{i2}\\ \dot{Y}_{i3}\end{pmatrix} &= \sum_{j=1}^{N}\bar{C}_{ij}(\alpha(\mathrm{t}))M_{ij}(Y_i,Y_j,\alpha(\mathrm{t})) + \begin{pmatrix}-\mu(k) & \mu(k) & 0\\ \Delta(k) & -\Delta(k) & z(k)\\ 0 & -\psi(k) & -\gamma(k)\end{pmatrix}\begin{pmatrix}Y_{i1}(\mathrm{t}-\tau(\mathrm{t}))\\ Y_{i2}(\mathrm{t})\\ Y_{i3}(\mathrm{t})\end{pmatrix}\\ &+\begin{pmatrix}l_1 & 0 & 0\\ 0 & l_2 & 0\\ 0 & 0 & l_3\end{pmatrix}\dot{B}(\mathrm{t}) + \begin{pmatrix}-w(\alpha(\mathrm{t}))\check{f}(Y_{i1}(\mathrm{t}-\tau(\mathrm{t})),\alpha(\mathrm{t}))\\ 0\\ 0\end{pmatrix}, \quad i\in\mathbb{L}, \qquad (4.1)\end{aligned}$$

where $\dot{Y}_{i1} = \dot{v}_1(t), \dot{Y}_{i2} = \dot{v}_2(t), \dot{Y}_{i3} = \dot{i}_3(t), \mu(k) = 1/(R(\alpha(\mathrm{t}))C_1(\alpha(\mathrm{t})))$, $\Delta(k) = 1/R(\alpha(\mathrm{t}))C_2(\alpha(\mathrm{t}))$, $z(k) = 1/C_2(\alpha(\mathrm{t}))$, $\psi(k) = 1/L(\alpha(\mathrm{t}))$, $\gamma(k) = R_0(\alpha(\mathrm{t}))/L(\alpha(\mathrm{t})$, $\ni (\alpha(\mathrm{t})) = 1/C_1(\alpha(\mathrm{t}))$, and $\tau$ is the inner time-varying delay. Let $l_1 = \bar{v}_{i1}(Y_{i1}(\mathrm{t}), Y_{i1}(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t}))$, $l_2 = \bar{v}_{i2}(Y_{i2}(\mathrm{t}), Y_{i2}(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t}))$, $l_3 = \bar{v}_{i3}(Y_{i3}(\mathrm{t}), Y_{i3}(\mathrm{t} - \tau(\mathrm{t})), \alpha(\mathrm{t}))$, and $l_1, l_2, l_3 \neq 0$. Moreover, $\check{f}(Y_{i1}(\mathrm{t}), \alpha(\mathrm{t})) = f(Y_{i1}(\mathrm{t}))$, and $\tau$ will denote the time delay.

Let $Y_i(t) = (\dot{Y}_{i1}, \dot{Y}_{i2}, \dot{Y}_{i3})^\top$. Define the following:

$$\Xi(Y_i(\mathrm{t}), Y_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t})) = \begin{pmatrix}-\mu(k) & \mu(k) & 0\\ \Delta(k) & -\Delta(k) & z(k)\\ 0 & -\psi(k) & -\gamma(k)\end{pmatrix}Y_i(\mathrm{t}) + \begin{pmatrix}-\ni(\alpha(\mathrm{t}))\check{f}(Y_{i1}(-\tau(\mathrm{t})),\alpha(\mathrm{t}))\\ 0\\ 0\end{pmatrix},$$

$$\bar{G}_i(Y_i(\mathrm{t}), Y_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t})) = \begin{pmatrix}l_1 & 0 & 0\\ 0 & l_2 & 0\\ 0 & 0 & l_3\end{pmatrix}.$$

Then, system (4.1) can be modified as

$$\begin{aligned}\mathrm{d}Y_i(\mathrm{t}) &= \Big[\Xi_i(Y_i(\mathrm{t}), Y_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t})) + \sum_{j=1}^{N}\bar{C}_{ij}(\alpha(\mathrm{t}))M_{ij}(Y_i(\mathrm{t}), Y_j(\mathrm{t}), \alpha(\mathrm{t}))\Big]\mathrm{dt}\\ &+ g_i(Y_i(\mathrm{t}), Y_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t}))\mathrm{d}B(\mathrm{t}), \quad i\in\mathbb{L}. \qquad (4.2)\end{aligned}$$

Consider system (4.2) as the driving system and the corresponding response system is in the form of system (2.2).

Define the error vector $E_i(\mathrm{t}) = \bar{Y}_i(\mathrm{t}) - Y_i(\mathrm{t})$ and the error system with event-triggered impulsive control is defined as follows.

$$\begin{cases} \mathrm{d}E_i(\mathrm{t}) = \Big[\Xi_i(E_i(\mathrm{t}), E_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t})) + \sum_{j=1}^{N} \bar{C}_{ij}(\alpha(\mathrm{t}))M_{ij}(E_i(\mathrm{t}), E_j(\mathrm{t}), \alpha(\mathrm{t}))\Big]\mathrm{d}t \\ \qquad\qquad + g_i(E_i(\mathrm{t}), E_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t}))\mathrm{d}B(\mathrm{t}), \\ E_i(\mathrm{t}) = \mathfrak{d}_i^{(z)} E_i(\mathrm{t}_z^-), \quad i \in \mathbb{L}. \end{cases} \tag{4.3}$$

Next, the following theorem will be presented, which guarantees the existence of a synchronization for the Chua's circuit with Markovian switching under event-triggered impulsive control. In addition, according to [34], we have $\check{f}(E_{k1}(\mathrm{t}-\tau(\mathrm{t})) = \sigma_{k1}(\bar{Y}_{i1}(\mathrm{t}-\tau(\mathrm{t}))) - \nu_1(\mathrm{t}-\tau(\mathrm{t})) \triangleq \sigma_{k1}E_{k1}(\mathrm{t}-\tau(\mathrm{t}))$ with $G_a \leq \sigma_{k1} \leq G_b$.

**Theorem 3.** *For all $i, j \in \mathbb{L}$ and $k \in \mathbb{Z}_+$, let the digraph$(\mathcal{H}, \mathbb{W}(Q))$ be strongly connected. Assume the following conditions hold:*

*C1: For any $X_i, X_j \in \mathbb{R}^n$ and $k \in \mathbb{Z}_+$, there exist positive constants $\kappa_{ij}^{(k)}$ such that*

$$(X_i)^\top \bar{C}_{ij}(X_i, X_j, k) \leq \kappa_{ij}^{(k)}(|X_i|^2 - |X_j|^2). \tag{4.4}$$

*C2: For any $Y_i, E_i \in \mathbb{R}^n$, there exist non-negative constants $\delta_i^{(k)}, \epsilon_i^{(k)}$ such that*

$$\mathrm{tr}[\,\bar{G}_i^\top(Y_i, E_i, k)\,\bar{G}_i(Y_i, E_i, k)] \leq \delta_i^{(k)}|Y_i|^2 + \epsilon_i^{(k)}|E_i|^2. \tag{4.5}$$

*C3: There exist $\ni$ and $\sigma_{k1}$ such that*

$$\frac{1}{2} \ni \sigma_{k1} + \bar{M} > 0, \tag{4.6}$$

*where $\bar{M} = \max\{\check{A}(k), \check{B}(k), \check{C}(k)\}$ and $\check{A}(k), \check{B}(k), \check{C}(k)$ will be introduced later. In addition, there exist positive constants satisfying $\bar{\rho}$: $\bar{\rho}e^{m\tau} \leq \rho$, $\omega_z \prod_{\ell=1}^{z-1} \omega_{z-\ell}\mu_{z-\ell} \leq \bar{\rho}$, $m+\Lambda^{(k)}+\rho\Pi^{(k)} \leq \ln \omega_z$, and then, under the event-triggered condition* (2.3)*, system* (4.2) *exhibits a synchronization with no Zeno behavior.*

*Proof.* We establish the equivalence of the two sets of conditions by taking the inner product of the system's dynamic equations with the transpose of the corresponding state vector. For the error system, we have

$$\mathrm{d}E_i(\mathrm{t}) = (E_i(\mathrm{t}))^\top\Big[\Xi(E_i(\mathrm{t}), E_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t})) + \sum_{j=1}^{N} \bar{C}_{ij}(\alpha(\mathrm{t}))M_{ij}(E_i(\mathrm{t}), E_j(\mathrm{t}), \alpha(\mathrm{t}))\Big]\mathrm{d}t$$
$$+ (E_i(\mathrm{t}))^\top g_i(E_i(\mathrm{t}), E_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t}))\mathrm{d}B(\mathrm{t}).$$

This leads to inequalities involving the norm of the system state. The derivation proceeds as follows.

We can perform the following inner product operation:

$$(E_i(\mathrm{t}))^\top\,\Xi\,(E_i(\mathrm{t}), E_i(\mathrm{t}-\tau(\mathrm{t})), k) = \Big(E_i^{(k)}(\mathrm{t})\Big)^\top \Bigg[\begin{pmatrix} -w(k)\,\check{f}(E_{i1}^{(k)}(\mathrm{t}-\tau(\mathrm{t})), k) \\ 0 \\ 0 \end{pmatrix}$$

$$+\begin{pmatrix} -\mu(k) & \mu(k) & 0 \\ \Delta(k) & -\Delta(k) & z(k) \\ 0 & -\psi(k) & -\gamma(k) \end{pmatrix} E_i^{(k)}(\mathrm{t}) \Bigg]$$

$$\leq \max\left\{\check{A}(k), \check{B}(k), \check{C}(k)\right\}\left(E_i^{(k)}(\mathrm{t})\right)^2 + \frac{1}{2} \ni \sigma_{k1}(E_i^{(k)}(\mathrm{t}-\tau(\mathrm{t})))^2 + \frac{1}{2} \ni \sigma_{k1}(E_{i1}^{(k)}(\mathrm{t}))^2,$$

where

$$\check{A}(k) = -\mu(k) - \frac{\Delta(k)}{2} - \frac{\mu(k)}{2}, \quad \check{B}(k) = \frac{\mu(k)}{2} - \frac{\Delta(k)}{2} + \frac{\psi(k)+z(k)}{2}, \quad \check{C}(k) = \frac{\psi(k)+z(k)}{2} - \gamma(k).$$

So, we can get

$$(E_i^{(k)}(\mathrm{t}))^\top \Xi\,(E_i(\mathrm{t}), E_i(\mathrm{t}-\tau(\mathrm{t})), \alpha(\mathrm{t})) \leq (\frac{1}{2} \ni \sigma_{k1} + \bar{M})(E_i^{(k)}(\mathrm{t}))^2 + \frac{1}{2} \ni \sigma_{k1}(E_i^{(k)}(\mathrm{t}-\tau(\mathrm{t})))^2.$$

This implies that (3.11) is satisfied.

Therefore, condition (4.4) and condition (3.11) jointly satisfy the first condition of Theorem 2. Conditions (3.13) is checked using conditions (4.5) and (4.6), respectively, while strongly connected directed graphs are the same. So, all the conditions for Theorem 2 have been fulfilled. Therefore, driving system (4.2) and response system (2.2) exhibit a synchronization according to Theorem 2. □

## *4.2. Numerical simulations*

To describe the network topology of the coupled Chua's circuits, a directed graph consisting of 17 nodes is employed. Each node represents an individual Chua's circuit, while the directed edges characterize the influence from node $j$ to node $i$. The Markovian modes are distinguished by edge colors, with red and blue corresponding to $\alpha(\mathrm{t}) = 1$ and $\alpha(\mathrm{t}) = 2$, respectively. In addition, solid edges and dashed edges denote strong and weak coupling strengths, respectively. As shown in Figure 3, the digraph $(\mathcal{G}, A)$ is strongly connected. The nonzero entries of the corresponding weighted matrix $\mathbb{W}(Q) = (a_{ij})_{N\times N}$ are given in Table 1.

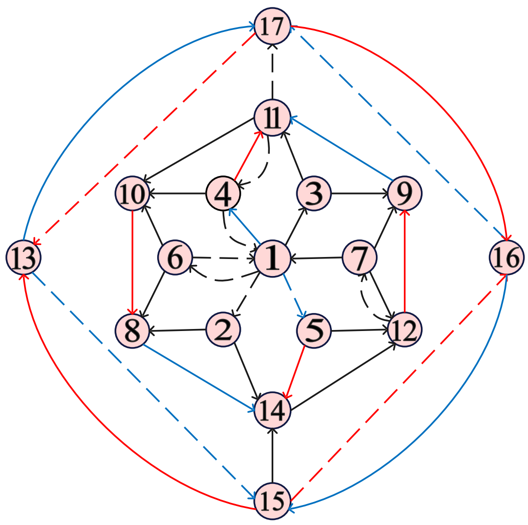


**Figure 3.** Coupling structure of 17 coupled Chua's circuits.

**Table 1.** The nonzero elements of matrix $\mathbb{W}(Q) = (a_{ij})_{17\times 17}$.

| The Values of the Non-zero Weights | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $a_{1,2} = 0.42$ | $a_{3,5} = 1.05$ | $a_{4,7} = 0.81$ | $a_{6,8} = 0.63$ | $a_{7,9} = 0.43$ | $a_{2,4} = 0.56$ | $a_{5,6} = 1.12$ | $a_{8,10} = 0.74$ | $a_{9,11} = 0.38$ |
| $a_{10,12} = 0.59$ | $a_{11,13} = 0.95$ | $a_{12,14} = 0.62$ | $a_{13,15} = 0.77$ | $a_{14,16} = 1.21$ | $a_{15,17} = 0.36$ | $a_{16,1} = 0.84$ | $a_{17,2} = 0.53$ | $a_{1,3} = 0.42$ |
| $a_{2,5} = 0.96$ | $a_{3,7} = 0.64$ | $a_{4,9} = 1.15$ | $a_{5,11} = 0.39$ | $a_{6,13} = 0.47$ | $a_{7,15} = 0.82$ | $a_{8,17} = 1.06$ | $a_{9,2} = 0.55$ | $a_{10,4} = 0.93$ |
| $a_{11,6} = 0.74$ | $a_{12,8} = 0.29$ | $a_{13,10} = 0.68$ | $a_{14,12} = 0.44$ | $a_{15,14} = 1.03$ | $a_{16,3} = 0.72$ | $a_{17,5} = 0.26$ | $a_{1,7} = 0.58$ | $a_{2,9} = 0.88$ |
| $a_{3,11} = 0.41$ | $a_{4,13} = 0.66$ | $a_{5,15} = 0.92$ | $a_{6,17} = 0.35$ | $a_{7,2} = 0.73$ | $a_{8,4} = 1.08$ | $a_{9,6} = 0.54$ | $a_{10,8} = 1.25$ | $a_{11,10} = 0.77$ |
| $a_{12,16} = 0.42$ | $a_{13,1} = 0.69$ | $a_{14,3} = 0.91$ | $a_{15,5} = 0.37$ | $a_{16,7} = 0.82$ | $a_{17,9} = 1.04$ | $a_{1,11} = 0.48$ | $a_{2,13} = 0.72$ | $a_{3,15} = 0.95$ |
| $a_{4,17} = 0.33$ | $a_{5,2} = 0.84$ | $a_{6,4} = 1.09$ | $a_{7,6} = 0.28$ | $a_{8,12} = 0.64$ | $a_{9,14} = 1.12$ | $a_{10,16} = 0.53$ | $a_{11,1} = 0.85$ | $a_{12,3} = 0.44$ |
| $a_{13,5} = 0.67$ | $a_{14,7} = 1.06$ | $a_{15,9} = 0.36$ | $a_{16,11} = 0.59$ | $a_{17,13} = 0.92$ | $a_{1,14} = 0.41$ | $a_{2,16} = 1.15$ | $a_{3,17} = 0.68$ | $a_{4,2} = 0.73$ |

We define the Markovian generating source, state space of Chua's circuit network, and transition probability matrix as follows:

$$Q = \begin{bmatrix} -1.5 & 1.5 \\ 1 & -1 \end{bmatrix}, S = [1, 2].$$

We set $C_1 = 0.08, C_2 = 0.86, R = 1.01, R_0 = 0.66, L = 1.08, G_a = -1.12, G_b = -0.41$, $\tau(\mathrm{t}) = \sin^2(\mathrm{t})$, $\Delta t = 0.01$, and

$$\bar{G}_i(Y_i(\mathrm{t}), Y_i(\mathrm{t} - \tau(\mathrm{t})), t) = 0.001\,[\sin Y_i(\mathrm{t}) + \sin Y_i(\mathrm{t} - \tau(\mathrm{t}))]\,,$$

$$M_{ij}(Y_i, Y_j, t) = \left[\sin Y_i(\mathrm{t}) - \sin Y_j(\mathrm{t})\right].$$

According to the parameters of the Chua circuit, one obtains $\overline{M} = 6.66, \widehat{d} = 13.66, \widehat{h} = 7.00$. Moreover, the diffusion coefficients satisfy $\delta_i^{(k)} = \epsilon_i^{(k)} = 2.00 \times 10^{-6}$. Therefore, the uniform coefficients in the Lyapunov estimate can be selected as $\bar{\Lambda} = 2\widehat{d} + \delta_i^{(k)} = 27.31, \bar{\Pi} = 2\widehat{h} + \epsilon_i^{(k)} = 14.00$. The control parameters are chosen as $m = 0.10, \omega_z = 2.00, d_i^{(z)} = -0.60, \bar{\rho} = 2.00, \rho = 2.30$. Since $p = 2$, the corresponding impulsive contraction coefficient is $\mu_z = |d_i^{(z)}|^2 = 0.36$. Consequently, $\omega_z \prod_{\ell=1}^{z-1} \omega_{z-\ell}\mu_{z-\ell} = 2.00(0.72)^{z-1} \le 2.00 = \bar{\rho}$ and $\bar{\rho}e^{m\tau} = 2.00e^{0.10} \approx 2.21 \le 2.30 = \rho$. Furthermore, $m + \bar{\Lambda} + \rho\bar{\Pi} = 0.10 + 27.31 + 2.30 \times 14.00 = 59.61$, and hence $\frac{\ln \omega_z}{m+\bar{\Lambda}+\rho\bar{\Pi}} = \frac{\ln 2.00}{59.61} \approx 0.01$. More precisely, the above ratio is greater than 0.01. Therefore, the Zeno-exclusion condition is satisfied by taking $\varepsilon = 0.01$, which yields $t_z - t_{z-1} \ge 0.01 > 0$.

Figure 4 presents the state trajectories of the uncontrolled drive and response networks. Figure 4(a)–(c) show the three state components of the drive network, whereas Figures 4(d)–(f) display the corresponding state components of the response network. Persistent differences can be observed between the two groups of trajectories, indicating that the drive and response networks cannot achieve synchronization without control. Figures 5 compares the synchronization errors before and after the proposed event-triggered impulsive control is applied. Figures5(a)–(c) show the uncontrolled error trajectories, which remain nonzero and oscillatory. By contrast, the controlled error trajectories in Figures 5(d)–(f) converge to zero after a transient period. These results demonstrate that the proposed control scheme achieves synchronization. To further evaluate the control performance, a variable-interval impulsive control scheme is considered for comparison, whose impulsive intervals satisfy

$$0.15 \le \Delta t_z \le 0.65.$$

As shown in Figure 6, both control schemes achieve synchronization under the considered simulation setting. However, the proposed event-triggered scheme drives the synchronization errors into a neighborhood of the origin within a shorter time than the variable-interval impulsive control scheme. Finally, Figures 7 and 8 present the impulsive intervals generated by the proposed event-triggered scheme and the variable-interval scheme, respectively. The proposed scheme generates fewer impulsive actions, while its observed inter-event intervals remain strictly positive throughout the simulation horizon. This observation is consistent with the theoretical Zeno-exclusion result.

**Remark 5.** *For numerical implementation, a finite-state Markov chain is adopted in the simulation. This setting should be understood as a finite-state special case of the proposed countably infinite-state framework. The purpose of the simulation is to illustrate the effectiveness of the event-triggered impulsive control strategy and the applicability of the obtained synchronization criteria. The theoretical results, however, are formulated for switching processes with a countably infinite state space.*

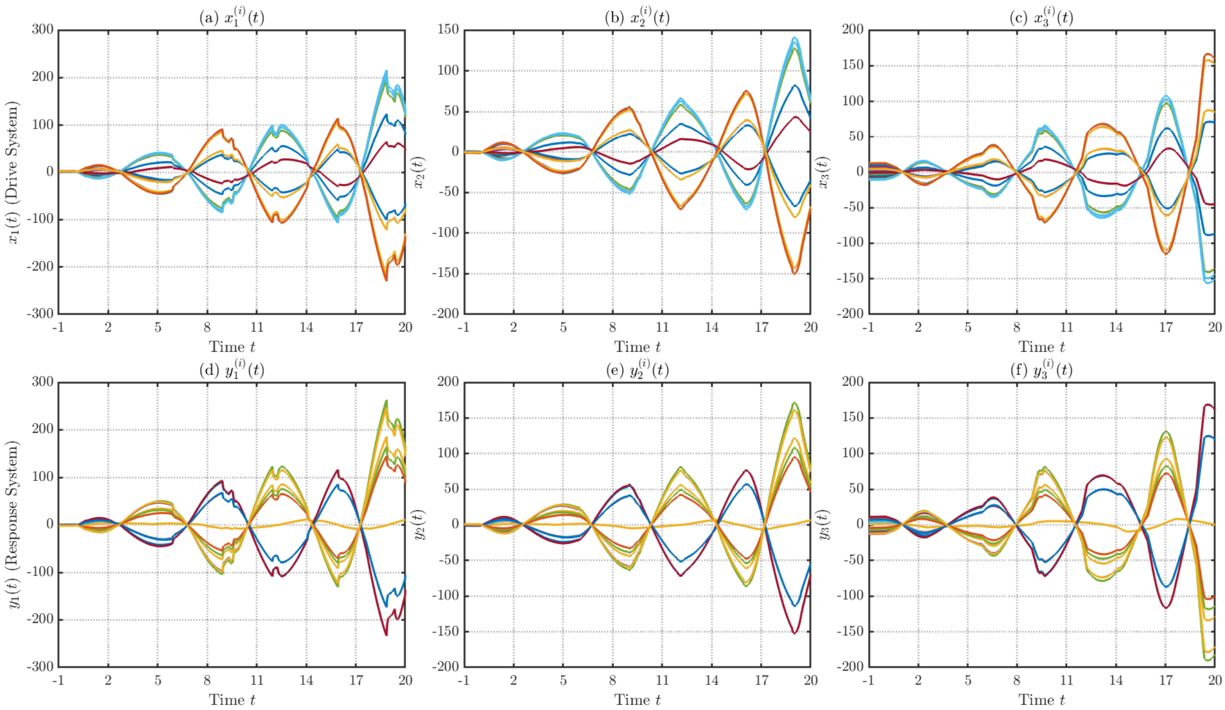


**Figure 4.** State trajectories of the uncontrolled drive and response networks.

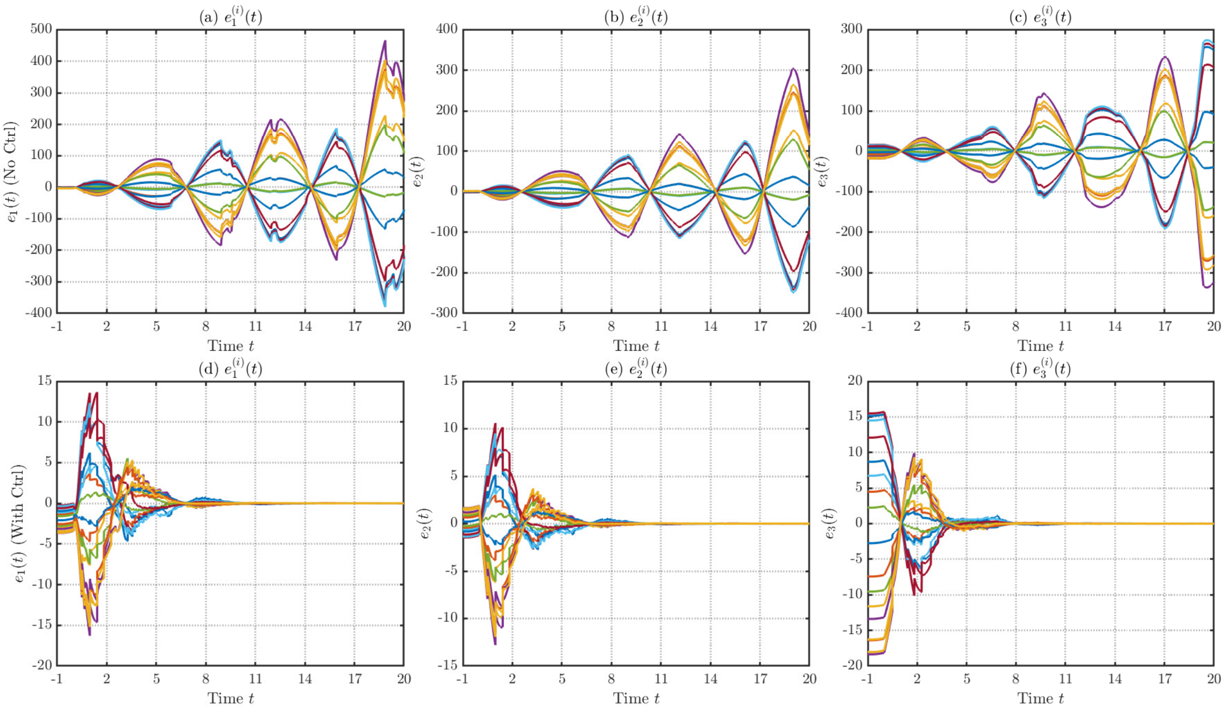


**Figure 5.** Synchronization errors before and after event-triggered impulsive control.

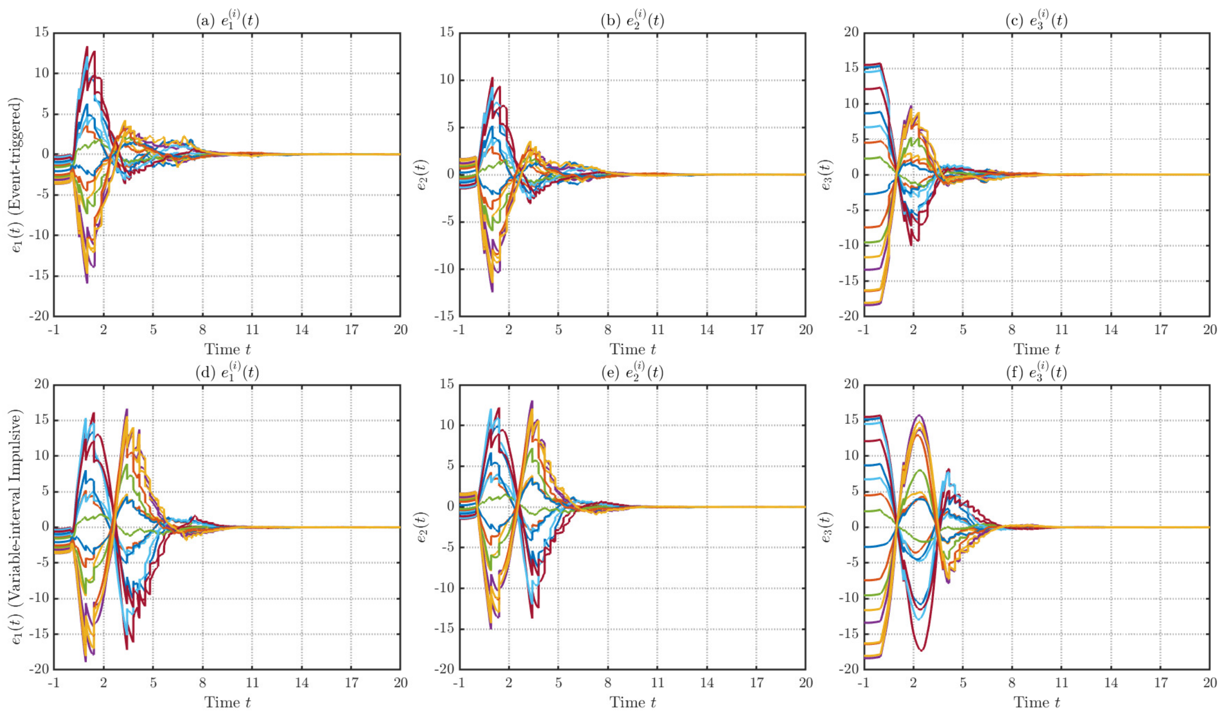


**Figure 6.** Comparison of synchronization errors under two impulsive control schemes.

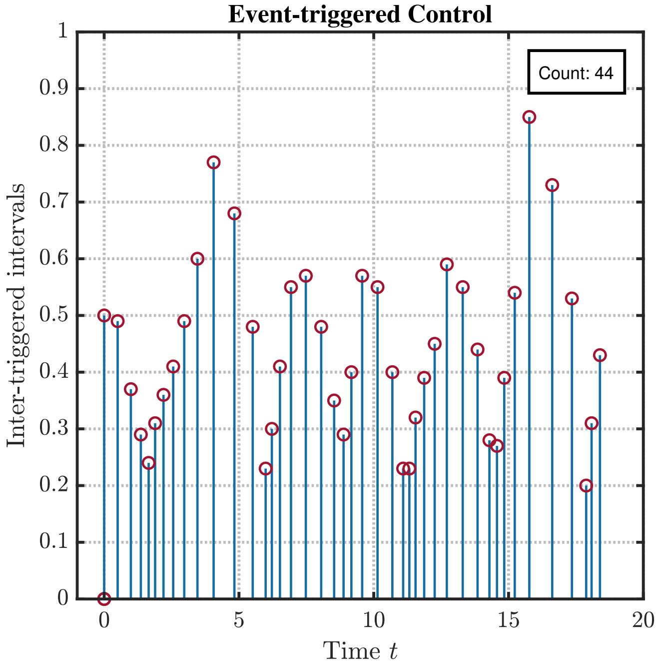


**Figure 7.** Event-triggered intervals.

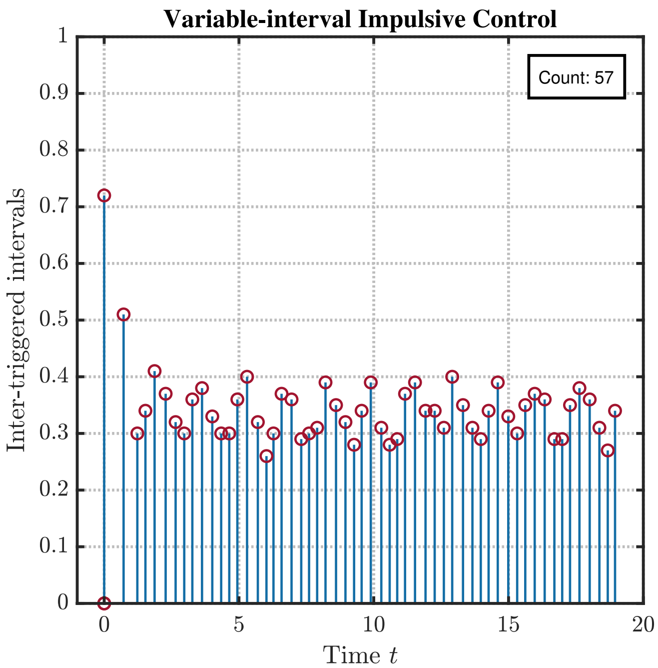


**Figure 8.** Variable impulsive intervals.

## 5. Conclusions

This paper considers the synchronization problem of stochastic coupled systems with time delay under event-triggered impulsive control in the presence of countably infinite-state Markovian switching, while excluding the Zeno phenomenon. By combining the Lyapunov method with graph theory, two new criteria for synchronization are established. Compared with existing results, the proposed model is more general. In addition, the effectiveness of the obtained results is verified through numerical simulations on a directed graph with 17 nodes. Since practical directed networks are not always strongly connected, future work will focus on synchronization conditions under more general network structures.

## Author contributions

Zezhen Feng: Conceptualization, Methodology, Investigation, Writing–Reviewing and Editing, Visualization; Sihan Zhao: Simulation, Writing–Original draft preparation; Jiqiang Feng: Funding acquisition, Supervision, Investigation; Chen Xu: Supervision, Investigation.

## Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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## Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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