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Research article

The novel stochastic solutions for the Maccari system driven by Wiener perturbations

Hesham G. Abdelwahed[1,*] **and Mahmoud A. E. Abdelrahman**[2]

[1] Department of Physics, College of Science and Humanities, Al-Kharj, Prince Sattam bin Abdulaziz University, Al- Kharj11942, Saudi Arabia

[2] Department of Mathematics, College of Science, Taibah University, Madinah, Saudi Arabia

* **Correspondence:** Email: h.abdelwahed@psau.edu.sa.

Abstract: In this paper, we look at novel stochastic solutions for the coupled Maccari system through the Wiener process. The incorporation of random perturbations provides a rigorous framework for modeling physically relevant phenomena in which noise-induced effects play a significant role. Such effects naturally arise in a wide range of applications, including signal propagation in optical fibers, plasma dynamics, and the evolution of fluid interfaces, where stochastic fluctuations can substantially influence the system behavior. To generate explicit analytical solitary wave solutions, we use the extended tanh function method (ETFM), which allows for the systematic derivation of accurate stochastic wave structures, such as soliton-like, blow up, periodic, and rational-type solutions under stochastic influence. To illustrate the propagation behavior of solitary waves in the stochastic Maccari model, 2D graphical representations of selected solutions were generated using MATLAB software. The obtained solutions demonstrate complex interactions between deterministic nonlinear dynamics and stochastic fluctuations, shedding light on the modulation and stability of wave propagation in noisy environments. These discoveries not only contribute to a better theoretical understanding of stochastic nonlinear systems, but they also have potential applications in sectors where random disturbances have a large impact on wave evolution.

Keywords: Wiener process; stochastic Maccari system; noise-driven dynamics; random perturbations; stochastic wave solutions
Mathematics Subject Classification: 35C07, 60H15, 35Q55, 35Q70

1. Introduction

One of the most comprehensive and adaptable mathematical frameworks for explaining intricate dynamical events in science and engineering is nonlinear partial differential equations (NPDEs) [1–3]. These types of equations, in contrast to their linear counterparts, capture underlying nonlinear interactions that result in amazing behaviors like turbulence, chaos, solitons, and pattern generation. The mathematical foundation for simulating complex processes in fluid dynamics, plasma physics, nonlinear optics, quantum mechanics, and biological systems is provided by these equations [4–6]. The study of NPDEs has been transformed by recent advances in analytical and computational tools, such as symmetry analysis, stochastic perturbation methods, and machine learning-assisted solvers, which allow researchers to find new classes of exact, approximate, and data-driven solutions. The NPDEs are a fundamental component of contemporary mathematical physics and applied sciences because of their nonlinear nature, which not only challenges conventional mathematical tools but also opens the door to the discovery of emergent structures and hidden symmetries that control the evolution of complex systems.

One of the most amazing examples of nonlinear dynamics is a solitary wave, which perfectly balances dispersion and nonlinearity. Solitary waves have particle-like robustness in continuous media, maintaining their shape and speed during propagation and even after mutual interaction, in contrast to conventional waves that dissipate or deform over time [7]. Their findings revolutionized our knowledge of nonlinear systems by demonstrating how complicated interactions can spontaneously produce coherence and stability. Solitary waves are not just theoretical concepts in contemporary study; they are essential to technical advancements in quantum information, fluid transport, optical communication, and plasma confinement. Their creation, modulation, and stability under external perturbations or random noise have been better understood thanks to developments in analytical methods, numerical simulations, and stochastic modeling. Furthermore, the investigation of multi-dimensional, coupled, and stochastic solitary waves keeps revealing novel physical phenomena, connecting basic mathematics with state-of-the-art uses in nonlinear metamaterials, and photonics [8].

A potent mathematical framework for expressing the intrinsic uncertainty and unpredictability that characterize real-world dynamical systems is stochastic partial differential equations (SPDEs) [9]. The SPDEs allow the modeling of complicated phenomena where deterministic techniques fail, such as turbulent fluid flows, fluctuating interfaces, and noise-driven wave propagation, by including stochastic processes, such as the Wiener process into traditional PDE frameworks [10, 11]. By demonstrating how randomness can lead to order, synchronization, or even novel ways of pattern generation, these equations skillfully connect probability theory with nonlinear dynamics. Recent advances in stochastic analysis, numerical approximation, and data-driven inference have transformed the area, enabling researchers to investigate multiscale dynamics and quantify uncertainty with remarkable precision. The SPDEs today play a critical role in a wide range of disciplines, from mathematical finance and climate modeling to quantum field theory and biological systems, where the interaction of randomness and structure governs the evolution of complex events [12, 13]. The Wiener process is a cornerstone of stochastic analysis, modeling the continuous yet unpredictable evolution of random phenomena in time [9]. It serves as the mathematical foundation for modern probability theory, stochastic calculus, and noise-driven systems, bridging microscopic randomness with macroscopic behavior. Its innovative strength lies in capturing the essence of natural fluctuations—whether in particle motion, financial markets, or quantum systems—through a continuous, nowhere-differentiable path that embodies pure randomness. By driving stochastic differential and partial differential equations, the Wiener process transforms randomness from a source of uncertainty into a fundamental tool for understanding and predicting complex dynamical behaviors [14–16].

A basic integrable model that describes the complex interactions between long-wave and short-wave components in nonlinear dispersive media is the Maccari system [17,18]. It offers a flexible framework for examining energy exchange, modulation instability, and the development of coherent structures including solitons, breather modes, and periodic wave patterns. It is distinguished by its linked evolution equations. The balance of nonlinearity, dispersion, and cross-phase interactions gives it mathematical richness and enables precise solutions using methods like the inverse scattering method, Hirota bilinear forms, and other sub-ordinary differential equations (sub-ODE) reduction strategies. Physically, the Maccari system appears in a variety of settings, including fluid interfaces, nonlinear optics, and plasma physics. It provides profound insights into multi-scale wave propagation and the dynamical principles behind stability, pattern generation, and wave localization. The development of exact solutions, such as multi-solitons, breathers, explosive, and rogue wave solutions, has been the focus of recent research [19–21]. The nonlinear Maccari system is given by [22]

$$\begin{aligned} &iU_t + U_{xx} + \Psi U = 0\,, \\ &i P_t + P_{xx} + \Psi P = 0, \\ &\Psi_t + \Psi_y + (|\, U + P\,|^2)_x = 0, \end{aligned} \tag{1.1}$$

where $U = U(x, y, t)$ and $P = P(x, y, t)$ are complex-valued short-wave envelope functions that describe the evolution of interacting wave packets and $\Psi = \Psi(x, y, t)$ is a real-valued long-wave function that accounts for the low-frequency mode induced by the interaction of the short waves.

The stochastic Maccari system extends the conventional deterministic framework by integrating random fluctuations commonly represented by the Wiener process into its coupled nonlinear evolution equations, expressing the fundamentally unpredictable behavior of real-world wave environments. This stochastic perturbation introduces randomness into amplitude, phase, and interaction dynamics, significantly affecting coherence, stability, and soliton persistence. As a result, solutions evolve not just under nonlinear and dispersive balances but also under probabilistic modulation, which can cause phenomena like noise-induced transitions, stochastic resonance, and fluctuating energy exchange between wave modes. In this work, we examine model (1.1) within the framework of a Wiener process, which is described as follows:

$$\begin{aligned} &iU_t + U_{xx} + \Psi U + \sigma\Omega_t U = 0, \\ &i P_t + P_{xx} + \Psi P + \sigma\Omega_t P = 0, \\ &\Psi_t + \Psi_y + (|\, U + P\,|^2)_x = 0, \end{aligned} \tag{1.2}$$

where $\Omega_t = \frac{d\Omega}{dt}$ denotes the time derivative of the Wiener process $\{\Omega(t)\}_{t\geq 0}$. The stochastic model (1.2) provides a powerful and physically realistic framework for modeling the interaction between short-wave and long-wave modes in nonlinear dispersive media when random effects are unavoidable. In many practical settings, such as nonlinear optical fibers, stratified fluids, and plasma environments, wave propagation is intrinsically influenced by stochastic perturbations arising from thermal noise, external forcing, structural inhomogeneities, and turbulent fluctuations. This framework is especially useful for explaining pulse propagation under amplifier noise and random dispersion control in nonlinear optics, and it describes surface–internal wave interactions in turbulent or fluctuating environments in fluid dynamics. Similarly, in plasma physics, the model (1.2) can be used to investigate the interaction of high-frequency Langmuir waves with low-frequency ion-acoustic waves under noisy conditions.

In this study, we employ the extended tanh function method (ETFM) [23] to create novel stochastic solitary wave solutions for the Maccari system under the influence of the Wiener process in the Stratonovich sense. This method constructs stochastic rational, hyperbolic, trigonometric, and hybrid solutions. The ETFM provides various advantages over earlier sophisticated techniques, including the elimination of complicated and time-consuming computations and the ability to get exact results with physical materials. It will also include built-in functions for solving various equations in applied science. The employed method is straightforward, reliable, and robust over long-term applications. We also explain how stochastic processes affect the success of the given stochastic solutions. To the best of our knowledge, the suggested stochastic model has never before been addressed using the ETFM.

The remaining sections of this work are organized as follows. Section 2 provides the ODE form of the coupled Maccari system (1.2). Section 3 presents various stochastic solutions for the stochastic coupled Maccari system. Section 4 provides a description of the suggested critical solutions. Several 2D graphs of the solutions have been created for suitable values of the free parameters.

Section 5 provides a conclusion based on the results gathered along with suggestions for additional research.

2. Mathematical analysis

We use the wave transformation

$$\begin{aligned} &U(x,y,t) = u(\xi)\, e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))};\, P(x,y,t) = p(\xi)\, e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}; \\ &\Psi(x,y,t) = \Psi(\xi); \xi = x + \delta y - 2kt, \end{aligned} \tag{2.1}$$

where k, μ, λ, and δ denote the wave number in the x-direction, transverse wave number in the y-direction, temporal frequency parameter, and direction parameter, respectively. Then, Eq (1.2) transfers into

$$\begin{aligned} &u'' - (\lambda + k^2)u + u\Phi = 0, \\ &w'' - (\lambda + k^2)p + \Psi\, p = 0, \\ &(\delta - 2k)\Psi' + ((u + p)^2)' = 0. \end{aligned} \tag{2.2}$$

When the third equation of Eq (2.2) is integrated and the integration constant is set to zero, the result is

$$\Psi = \left(\frac{-1}{\delta - 2k}\right)(u + p)^2. \tag{2.3}$$

Substituting Eq (2.3) into the first equation of (2.2) produces:

$$\begin{aligned} &u'' - \frac{1}{\delta - 2k}u(u + p)^2 - (\lambda + k^2)u = 0, \\ &p'' - \frac{1}{\delta - 2k}p(u + p)^2 - (\lambda + k^2)p = 0. \end{aligned} \tag{2.4}$$

Even though the two equations cannot be solved directly, we can derive a clear relation between p and u. We thus set

$$p = \alpha\, u, \tag{2.5}$$

where α is an arbitrary constant. Substituting (2.5) into the Eq (2.4) yields

$$u'' - \frac{1}{\delta - 2c} u(u + \alpha\, u)^2 - (\lambda + k^2)u = 0, \tag{2.6}$$

thus we have

$$(\delta - 2k)u'' - (1 + \alpha)^2\, u^3 - (\delta - 2k)(\lambda + k^2)u = 0. \tag{2.7}$$

3. The stochastic solutions

We employ the ETFM to solve Eq (2.7). By balancing the highest nonlinear term, u^3, with the highest-order derivative, u'', we obtain $N = 1$.

Based on the ETFM [23], the solution of Eq (2.7) is assumed to have the following form:

$$\begin{aligned} u(\xi) &= \sum_{j=0}^{j=N} a_j \varphi^j(\xi) + \sum_{j=1}^{j=N} b_j \varphi^{-j}(\xi) \\ &= a_0 + a_1\varphi(\xi) + \frac{b_1}{\varphi(\xi)}, \end{aligned} \tag{3.1}$$

where the function $\varphi(\xi)$ satisfies the Riccati equation

$$\varphi' = \varrho + \varphi^2(\xi), \tag{3.2}$$

and ϱ is a constant.

$$u'(\xi) = a_1\varrho + a_1\varphi^2 - \frac{b_1\varrho}{\varphi^2} - b_1. \tag{3.3}$$

A set of algebraic equations is obtained by substituting Eq (3.1) and its derivative into Eq (2.7), then grouping all terms according to their respective powers of φ^3, φ^2, φ, φ^0, φ^{-1}, φ^{-2}, and φ^{-3}. The following solutions are obtained by using Maple to solve these equations:

Family I: For $\lambda + k^2 < 0$, the solutions of (2.7) are

$$\begin{aligned} u_{1,2}(\xi) &= \pm\sqrt{\frac{-(\delta - 2k)(\lambda + k^2)}{(1 + \alpha)^2}}\, tanh(\sqrt{\frac{-(\lambda + k^2)}{2}}\,\xi), \quad \delta > 2k, \\ u_{3,4}(\xi) &= \pm\sqrt{\frac{-(\delta - 2k)(\lambda + k^2)}{(1 + \alpha)^2}}\, coth(\sqrt{\frac{-(\lambda + k^2)}{2}}\,\xi), \quad \delta > 2k. \end{aligned} \tag{3.4}$$

Thus, the solutions of (1.2) are

$$\begin{aligned} U_{1,2}(x, y, t) &= \pm\sqrt{\frac{-(\delta - 2k)(\lambda + k^2)}{(1 + \alpha)^2}}\, tanh(\sqrt{\frac{-(\lambda + k^2)}{2}}\,(x + \delta y - 2kt))\, e^{i(kx+\mu\, y+\lambda\, t+\sigma\Omega(t))}, \\ U_{3,4}(x, y, t) &= \pm\sqrt{\frac{-(\delta - 2k)(\lambda + k^2)}{(1 + \alpha)^2}}\, coth(\sqrt{\frac{-(\lambda + k^2)}{2}}\,(x + \delta y - 2kt))\, e^{i(kx+\mu\, y+\lambda\, t+\sigma\Omega(t))}, \end{aligned} \tag{3.5}$$

$\delta > 2k$.

$$P_{1,2}(x,y,t) = \pm a\sqrt{\frac{-(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,tanh(\sqrt{\frac{-(\lambda+k^2)}{2}}\,(x+\delta y-2kt))\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))},$$
$$P_{3,4}(x,y,t) = \pm a\sqrt{\frac{-(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,coth(\sqrt{\frac{-(\lambda+k^2)}{2}}\,(x+\delta y-2kt))\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.6}$$

$\delta > 2k$.

$$\Psi_1(x,y,t) = (\lambda+k^2)\,tanh^2(\sqrt{\frac{-(\lambda+k^2)}{2}}\,(x+\delta y-2kt)),$$
$$\Psi_2(x,y,t) = (\lambda+k^2)\,coth^2(\sqrt{\frac{-(\lambda+k^2)}{2}}\,(x+\delta y-2kt)). \tag{3.7}$$

For $\lambda + k^2 > 0$, the solutions of (2.7) are

$$u_{5,6}(\xi) = \pm\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,tan(\sqrt{\frac{\lambda+k^2}{2}}\,\xi),\quad \delta > 2k,$$
$$u_{7,8}(\xi) = \pm\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,cot(\sqrt{\frac{\lambda+k^2}{2}}\,\xi),\quad \delta > 2k. \tag{3.8}$$

Thus, the solutions of (1.2) are

$$U_{5,6}(x,y,t) = \pm\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,tan(\sqrt{\frac{\lambda+k^2}{2}}\,(x+\delta y-2kt))\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))},$$
$$U_{7,8}(x,y,t) = \pm\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,cot(\sqrt{\frac{\lambda+k^2}{2}}\,(x+\delta y-2kt))\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.9}$$

$\delta > 2k$.

$$P_{5,6}(x,y,t) = \pm a\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,tan(\sqrt{\frac{\lambda+k^2}{2}}\,(x+\delta y-2kt))\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))},$$
$$P_{7,8}(x,y,t) = \pm a\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,cot(\sqrt{\frac{\lambda+k^2}{2}}\,(x+\delta y-2kt))\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.10}$$

$\delta > 2k$.

$$\Psi_3(x,y,t) = -(\lambda+k^2)\, tan^2\Big(\sqrt{\frac{\lambda+k^2}{2}}(x+\delta y-2kt)\Big),$$
$$\Psi_4(x,y,t) = -(\lambda+k^2)\, cot^2\Big(\sqrt{\frac{\lambda+k^2}{2}}(x+\delta y-2kt)\Big), \tag{3.11}$$

For $\lambda + k^2 = 0,\ \delta > 2k$, the solutions of (2.7) are

$$u_{9,10}(\xi) = \pm\sqrt{\frac{2(\delta-2k)}{(1+\alpha)^2}}\,\frac{1}{\xi}. \tag{3.12}$$

Thus, the solutions of (1.2) are

$$U_{9,10}(x,y,t) = \pm\sqrt{\frac{2(\delta-2k)}{(1+\alpha)^2}}\,\frac{1}{x+\delta y-2kt}\; e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.13}$$

$$P_{9,10}(x,y,t) = \pm\, a\,\sqrt{\frac{2(\delta-2k)}{(1+\alpha)^2}}\,\frac{1}{x+\delta y-2kt}\; e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.14}$$

$$\Psi_5(x,y,t) = \frac{-2}{(x+\delta y-2kt)^2}. \tag{3.15}$$

Family II: For $\lambda + k^2 > 0,\ \delta > 2k$, the solutions of (2.7) are

$$u_{11,12}(\xi) = \pm\sqrt{\frac{2(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\; csch\Big(\sqrt{\lambda+k^2}\,\xi\Big). \tag{3.16}$$

Thus, the solutions of (1.2) are

$$U_{11,12}(x,y,t) = \pm\sqrt{\frac{2(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\; csch\Big(\sqrt{\lambda+k^2}\,(x+\delta y-2kt)\Big)\; e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.17}$$

$$P_{11,12}(x,y,t) = \pm\, a\,\sqrt{\frac{2(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\; csch\Big(\sqrt{\lambda+k^2}\,(x+\delta y-2kt)\Big)\; e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.18}$$

$$\Psi_6(x,y,t) = -2(\lambda+k^2)\, csch^2\Big(\sqrt{\lambda+k^2}\,(x+\delta y-2kt)\Big). \tag{3.19}$$

For $\lambda + k^2 < 0,\ \delta > 2k$, the solutions of (2.7) are

$$u_{13,14}(\xi) = \pm\sqrt{\frac{-2(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\; csc\Big(\sqrt{-(\lambda+k^2)}\,\xi\Big). \tag{3.20}$$

Thus, the solutions of (1.2) are

$$U_{13,14}(x,y,t) = \pm\sqrt{\frac{-2(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\; csc\Big(\sqrt{-(\lambda+k^2)}\,(x+\delta y-2kt)\Big)\; e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \tag{3.21}$$

$$P_{13,14}(x,y,t) = \pm\, a\,\sqrt{\frac{-2(\delta-2k)(\lambda+k^2)}{(1+\alpha)^2}}\,csc\left(\sqrt{-(\lambda+k^2)}\,(x+\delta y-2kt)\right)\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}, \quad (3.22)$$

$$\Psi_7(\xi) = 2(\lambda+k^2)\,csc^2\left(\sqrt{-(\lambda+k^2)}\,(x+\delta y-2kt)\right). \quad (3.23)$$

Family III: For $\lambda + k^2 > 0,\ \delta > 2k$, the solutions of (2.7) are

$$u_{15,16}(\xi) = \pm\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{4(1+\alpha)^2}}\left(tan\left(\sqrt{\frac{\lambda+k^2}{8}}\,\xi\right) - cot\left(\sqrt{\frac{\lambda+k^2}{8}}\,\xi\right)\right). \quad (3.24)$$

Thus, the solutions of (1.2) are

$$U_{15,16}(x,y,t) = \pm\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{4(1+\alpha)^2}}\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}\times$$
$$\left(tan\left(\sqrt{\frac{\lambda+k^2}{8}}\,(x+\delta y-2kt)\right) - cot\left(\sqrt{\frac{\lambda+k^2}{8}}\,(x+\delta y-2kt)\right)\right). \quad (3.25)$$

$$P_{15,16}(x,y,t) = \pm a\,\sqrt{\frac{(\delta-2k)(\lambda+k^2)}{4(1+\alpha)^2}}\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}\times$$
$$\left(tan\left(\sqrt{\frac{\lambda+k^2}{8}}\,(x+\delta y-2kt)\right) - cot\left(\sqrt{\frac{\lambda+k^2}{8}}\,(x+\delta y-2kt)\right)\right). \quad (3.26)$$

$$\Psi_8(x,y,t) = \frac{-1}{4}(\lambda+k^2)\times$$
$$\left(tan\left(\sqrt{\frac{\lambda+k^2}{8}}\,(x+\delta y-2kt)\right) - cot\left(\sqrt{\frac{\lambda+k^2}{8}}\,(x+\delta y-2kt)\right)\right)^2. \quad (3.27)$$

For $\lambda + k^2 < 0,\ \delta > 2k$, the solutions of (2.7) are

$$u_{17,18}(\xi) = \pm\sqrt{\frac{-(\delta-2k)(\lambda+k^2)}{4(1+\alpha)^2}}\left(tanh\left(\sqrt{\frac{-(\lambda+k^2)}{8}}\,\xi\right) + coth\left(\sqrt{\frac{-(\lambda+k^2)}{8}}\,\xi\right)\right). \quad (3.28)$$

Thus, the solutions of (1.2) are

$$U_{17,18}(x,y,t) = \pm\sqrt{\frac{-(\delta-2k)(\lambda+k^2)}{4(1+\alpha)^2}}\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}\times$$
$$\left(tanh\left(\sqrt{\frac{-(\lambda+k^2)}{8}}\,(x+\delta y-2kt)\right) + coth\left(\sqrt{\frac{-(\lambda+k^2)}{8}}\,(x+\delta y-2kt)\right)\right). \quad (3.29)$$

$$P_{17,18}(x,y,t) = \pm a\sqrt{\frac{-(\delta-2k)(\lambda+k^2)}{4(1+\alpha)^2}}\,e^{i(kx+\mu y+\lambda t+\sigma\Omega(t))}\times$$
$$\left(tanh\left(\sqrt{\frac{-(\lambda+k^2)}{8}}(x+\delta y-2kt)\right)+coth\left(\sqrt{\frac{-(\lambda+k^2)}{8}}(x+\delta y-2kt)\right)\right). \tag{3.30}$$

$$\Psi_9(x,y,t) = \frac{1}{4}(\lambda+k^2)\times$$
$$\left(tanh\left(\sqrt{\frac{-(\lambda+k^2)}{8}}(x+\delta y-2kt)\right)+coth\left(\sqrt{\frac{-(\lambda+k^2)}{8}}(x+\delta y-2kt)\right)\right)^2. \tag{3.31}$$

4. Physical interpretation

We use the ETFM to find robust stochastic solutions for the stochastic Maccari system in the Stratonovich sense. This method is an effective analytical technique for getting solitary wave solutions to NPDEs and nonlinear evolution equations. In contrast to established methodologies, the ETFM effectively handles severe nonlinearities and parameter-dependent effects, providing direct insight into how noise terms and physical factors influence wave propagation. It enhances the classic tanh-function method by including new terms and flexibility, enabling it to handle more complex nonlinearities.

The stochastic Maccari system extends the traditional deterministic framework by including a Wiener process to simulate random fluctuations that occur naturally in complicated physical systems. By incorporating the Wiener process into the nonlinear evolution equations, the system incorporates the effects of external noise and microscopic errors on wave dynamics. This stochastic formulation maintains the core integrable structure while revealing novel phenomena like noise-induced modulation, stochastic resonance, and fluctuating soliton interactions. As a result, the stochastic Maccari system provides a more realistic and adaptable platform for studying wave propagation in optical fibers, plasma media, and fluid interfaces where randomness plays an important role in determining dynamical behavior.

In actual nonlinear systems subject to random perturbations, the various solution structures of stochastic Maccari system admit a straightforward and physically meaningful interpretation. In optical waveguides, water-wave channels, and plasma media, where stochastic fluctuations show up as empirically observed phase jitter and amplitude wandering, localized soliton-type solutions correspond to stable energy-carrying wave packets. Oscillatory energy localization, pulse breathing in mode-locked lasers, intermittent wave amplification in hydrodynamics, and collective excitations in Bose-Einstein condensates under thermal noise are all described by periodic and breather-type solutions. Solutions with rapid amplitude amplification are commonly related with rogue-wave phenomena in oceanography and nonlinear optics, where stochastic forcing serves as a trigger mechanism for severe events. Dispersive and decaying wave patterns, on the other hand, indicate coherence loss regimes associated with signal deterioration in heterogeneous media, such as ultrasonic propagation and turbulent plasmas. The coupled mean-field potential in the stochastic Maccari system also accounts for long-short wave interactions, such as wave-induced currents in fluids and nonlinear refractive feedback in optics. Collectively, these links show that the solution spectrum of the stochastic Maccari system captures observable physical behaviors in noisy situations, showing that the model and its solutions are truly practical rather than just mathematical creations. A convincing physical explanation of how random perturbations, represented by a Wiener process, interact with nonlinear wave dynamics in real-world media can be found in the stochastic solutions of the Maccari system. Waves are continuously affected by thermal noise, environmental variations, and microscopic uncertainty in real-world environments including optical cables, plasma channels, and shallow-water interfaces. By showing how Wiener-driven disturbances modulate wave amplitude, change propagation velocity, and produce sporadic oscillations around otherwise stable solitary structures, the stochastic Maccari solutions capture these phenomena. Physically, these stochastic profiles represent the delicate balance between nonlinear self-organization and random driving, demonstrating how coherent patterns endure, distort, or evaporate in the presence of continuous noise.

The MATLAB software was used to create 2D charts of specific solutions to show how the solitary waves for the stochastic Maccari system propagate. The behavior of the generated solution $U_1(x, y, t)$ in the absence of the noise effect ($\sigma = 0$) is depicted in Figure 1 for $\lambda = -2, k = 1, \delta = 3.4, \alpha = 1$, and $\mu = 2$. The variations of periodic wave solution $U_1(x, y, t)$ with λ is plotted in Figure 2 for $k = 1, \delta = 3.4, \alpha = 1, \mu = 2$, and $\sigma = 0$. It was found that the decreasing of w increases the amplitudes of periodic solution form. Figure 3 depicts the 2D localized soliton wave solution $\Psi_1(x, y, t)$ for $\lambda = -2, k = 1, \delta = 3.4, \alpha = 1$, and $\mu = 2$. Figure 4 illustrates the 2D blow up wave solution $U_5(x, y, t)$ for $\lambda = 1.2, k = 0.5, \delta = 1.4, \alpha = 1, \mu = 1$, and $\sigma = 0$. Figure 5 shows the 2D explosive pulse wave solution $U_{13}(x, y, t)$ for $\lambda = -2, k = 1, \delta = 3.4, \alpha = 1, \mu = 2$, and $\sigma = 0$. The 2D periodic wave profile of $U_1(x, y, t)$ for various values of the noise term σ is shown in Figure 6 with $\lambda = -2, k = 1, \delta = 3.4, \alpha = 1$, and $\mu = 2$. The stochastic solution's evolution with respect to the spatial variable x and the noise intensity σ is shown in this image, which emphasizes a distinct periodic pattern with a constant phase shift and no propagation direction reversal. The qualitative and quantitative behavior of solitary waves, such as optical and hydrodynamic solitons, can be drastically changed by adding a stochastic forcing term to nonlinear wave models.

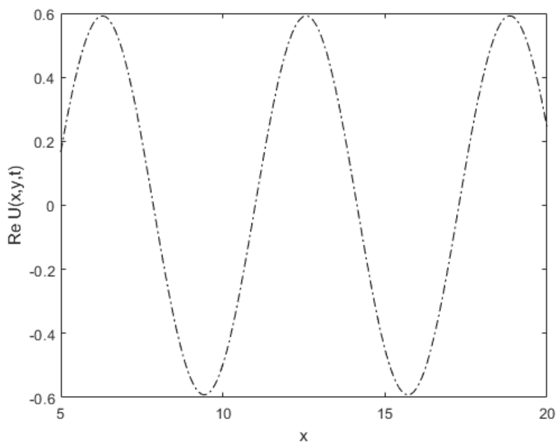

Figure 1. 2D periodic wave solution $U_1(x, y, t)$.

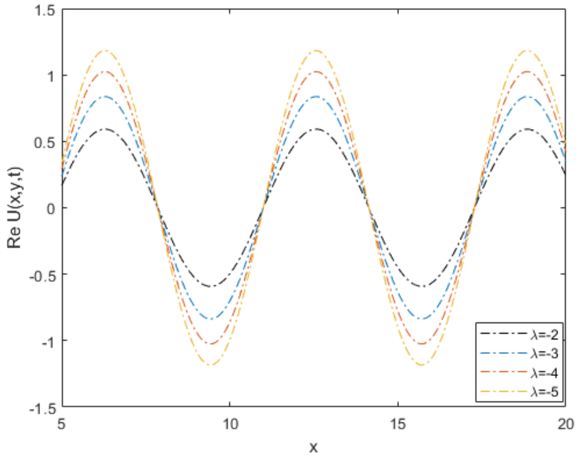

Figure 2. 2D periodic wave solution $U_1(x, y, t)$ with different values of λ.

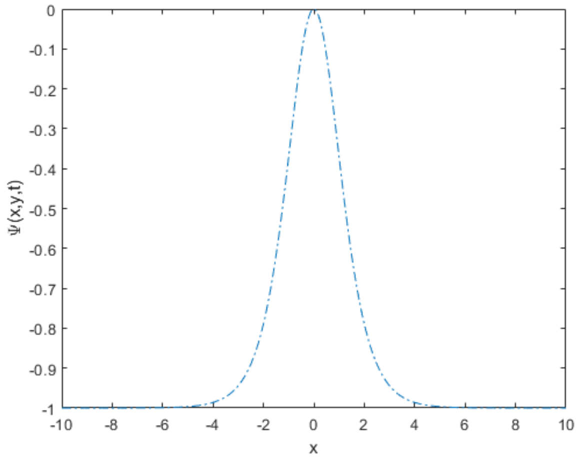

Figure 3. 2D localized soliton wave solution $\Psi_1(x, y, t)$.

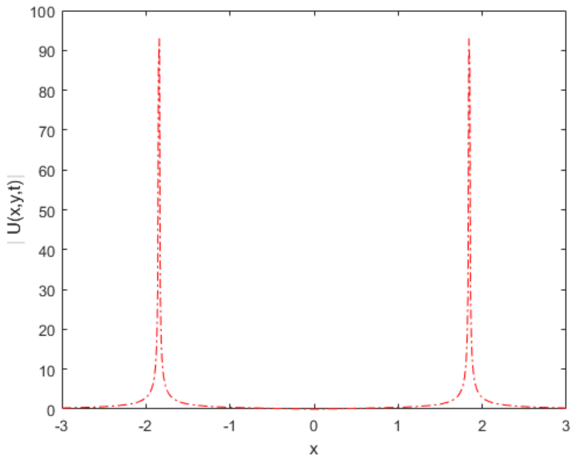

Figure 4. 2D blow up wave solution $U_5(x, y, t)$.

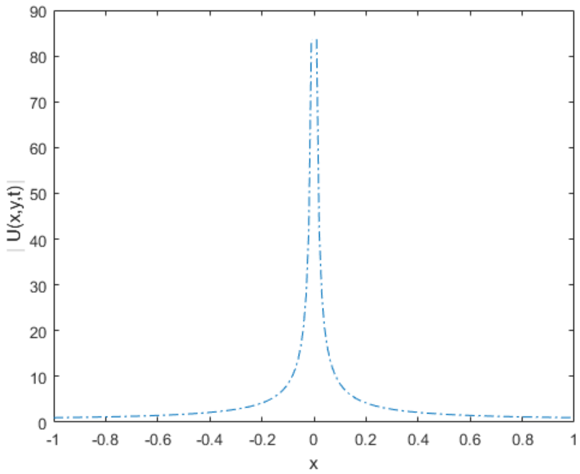

Figure 5. 2D explosive pulse wave solution $U_{13}(x, y, t)$.

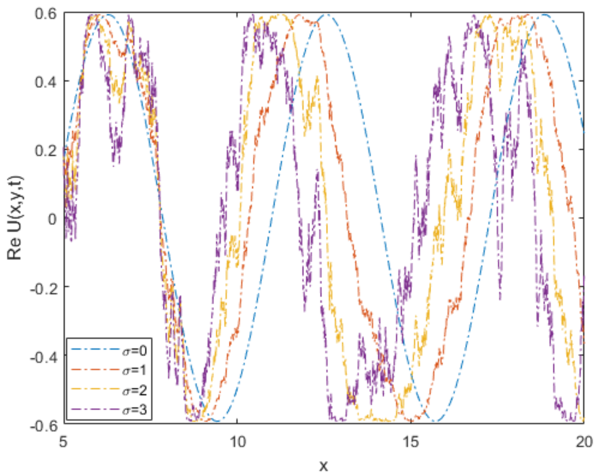

Figure 6. 2D periodic wave solution $U_1(x, y, t)$ with different values of σ.

5. Conclusions

We successfully developed innovative stochastic solutions for the two-coupled Maccari system by adding randomness via the Wiener process. Using the ETFM, we obtained explicit mathematical forms that represent the intricate interaction of nonlinear wave dynamics with stochastic fluctuations. The solutions include complex structures such as soliton-like, periodic, and rational-type waves, demonstrating how noise affects amplitude, form, and stability. These results provide light on the behavior of stochastic nonlinear systems and give a strong analytical foundation for modeling physical processes in noisy settings such as optical fibers, plasma waves, and fluid dynamics. This research lays the groundwork for further investigation into stochastic effects in multidimensional nonlinear systems and their practical applications.

Author contributions

H. G. Abdelwahed and M. A. E. Abdelrahman: Conception, investigation, resources, formal analysis, writing–original draft, writing–review & editing. All authors read and approved the final manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no competing interests.

References

1. Y. H. Liu, R. Guo, X. L. Li, Rogue wave solutions and modulation instability for the mixed nonlinear Schrödinger equation, *Appl. Math. Lett.*, **121** (2021), 107450. https://doi.org/10.1016/j.aml.2021.107450
2. H. Zhang, X. Yang, D. Xu, Unconditional convergence of linearized orthogonal spline collocation algorithm for semilinear subdiffusion equation with nonsmooth solution, *Numer. Meth. Part. D. E.*, **37** (2021), 1361–1373. https://doi.org/10.1002/num.22583
3. Y. Y. Wang, Y. P. Zhang, C. Q. Dai, Re-study on localized structures based on variable separation solutions from the modified tanh-function method, *Nonlinear Dynam.*, **83** (2016), 1331–1339. https://doi.org/10.1007/s11071-015-2406-5
4. X. H. Wu, Y. T. Gao, Generalized Darboux transformation and solitons for the Ablowitz-Ladik equation in an electrical lattice, *Appl. Math. Lett.*, **137** (2023), 108476. https://doi.org/10.1016/j.aml.2022.108476
5. M. A. E. Abdelrahman, G. Alshreef, Closed-form solutions to the new coupled Konno–Oono equation and the Kaup-Newell model equation in magnetic field with novel statistic application, *Eur. Phys. J. Plus*, **136** (2021), 1–10. https://doi.org/10.1140/epjp/s13360-021-01472-2
6. E. H. M. Zahran, M. S. M. Shehata, S. M. M. Alizamini, M. N. Alam, L. Akinyemi, Exact propagation of the isolated waves model described by the three coupled nonlinear Maccari's system with complex structure, *Int. J. Mod. Phys. B*, **35** (2021), 2150193. https://doi.org/10.1142/S0217979221501939
7. G. Arora, R. Rani, H. Emadifar, Soliton: A dispersion-less solution with existence and its types, *Heliyon*, **8** (2022), e12122. https://doi.org/10.1016/j.heliyon.2022.e12122

8. S. Turitsyn, A. Mikhailov, *Applications of solitons*, Scattering, Elsevier, 2002, 1741–1753. https://doi.org/10.1016/B978-012613760-6/50098-X
9. I. Karatzas, S. E. Shreve, *Brownian motion and stochastic calculus,* 2 Eds., Berlin: Springer-Verlag, 1991.
10. G. J. Lord, C. E. Powell, T. Shardlow, *An introduction to computational stochastic PDEs*, Cambridge University Press, 2014.
11. F. Flandoli, *Random perturbation of PDEs and fluid dynamic models*, Springer, 2011.
12. A. J. Majda, I. Timofeyev, E. V. Eijnden, A mathematical framework for stochastic climate models, *Commun. Pure Appl. Math.*, **54** (2002), 891–974. https://doi.org/10.1002/cpa.1014
13. J. M. Steele, *Stochastic calculus and financial applications*, Springer, 2001.
14. K. A. Alsatami, Analysis of solitary wave behavior under stochastic noise in the generalized schamel equation, *Sci. Rep.*, **15** (2025), 19157. https://doi.org/10.1038/s41598-025-04696-9
15. H. G. Abdelwahed, E. K. El-Shewy, M. A. E. Abdelrahman, S. S. Alghanim, The noise sense effects on the characteristic nonlinearly Schrödinger equation solitary propagations, *J. Low Freq. Noise V. A.*, **42** (2022), 818–825. https://doi.org/10.1177/14613484221122114
16. H. G. Abdelwahed, A. F. Alsarhan, E. K. El-Shewy, M. A. E. Abdelrahman, The stochastic structural modulations in collapsing Maccari's model solitons, *Fractal Fract.*, **7** (2023), 290. https://doi.org/10.3390/fractalfract7040290
17. T. Xu, Y. Chen, Z. Qiao, Multi-dark soliton solutions for the (2+1)-dimensional multi-component Maccari system, *Mod. Phys. Lett. B*, **33** (2019), 1950390. https://doi.org/10.1142/S0217984919503901
18. R. A. Alomair, S. Z. Hassan, M. A. E. Abdelrahman, A new structure of solutions to the coupled nonlinear Maccari's systems in plasma physics, *AIMS Math.*, **7** (2022), 8588–8606. https://doi.org/10.3934/math.2022479
19. A. Maccari, The Maccari system as model system for rogue waves, *Phys. Lett. A*, **384** (2020), 126740. https://doi.org/10.1016/j.physleta.2020.126740
20. C. Moser, J. LaBelle, I. H. Cairns, High bandwidth measurements of auroral Langmuir waves with multiple antennas, *Ann. Geophys.*, **40** (2022), 231–245. https://doi.org/10.5194/angeo-40-231-2022
21. A. Ciancio, H. M. Baskonus, T. A. Sulaiman, H. Bulut, New structural dynamics of isolated waves via the coupled nonlinear Maccari's system with complex structure, *Indian J. Phys.*, **92** (2018), 1281–1290. https://doi.org/10.1007/s12648-018-1204-6
22. D. Rostamy, F. Zabihi, Exact solutions for different coupled nonlinear Maccari's systems, *Nonlinear Stud.*, **9** (2012), 291–301.
23. A. M. Wazwaz, The extended tanh method for abundant solitary wave solutions of nonlinear wave equations, *Appl. Math. Comput.*, **187** (2007), 1131–1142. https://doi.org/10.1016/j.amc.2006.09.013