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Research article

A characterization of upper triangular matrix algebras generated by idempotents

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Abstract: In the present paper, our main result is a characterization of upper triangular matrix algebras generated by idempotents. We prove that every upper triangular matrix algebra over a unital simple algebra with a nontrivial idempotent is generated by its idempotents. We give two examples in which upper triangular matrix algebras are not generated by idempotents. Finally, using our main result, we improve a result on local Lie derivations of upper triangular matrix algebras obtained by Liu in 2020.

Keywords: upper triangular matrix algebra; matrix algebra; idempotents; local Lie derivation
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1. Introduction

Let $\mathcal{B}$ be an associative algebra over a commutative ring R. By $\mathcal{J}(\mathcal{B})$ we denote the subalgebra of $\mathcal{B}$ generated by all idempotents in $\mathcal{B}$.

In 2007, Brešar [1] proved that every matrix algebra over a unital algebra is generated by idempotents. In 2016, Brešar [2] proved that every finite-dimensional zero product determined algebra is generated by idempotents. In 2021, Hu and Xiao [3] discussed the conditions under which every algebra over an algebraically closed field is generated by idempotents.

We remark that the algebra generated by idempotents is an important type of algebra, which is closely related to generalized inverses (see [4] for details).

In the present paper, we give the main result as follows.

Theorem 1.1. *Let S be a unital algebra over a commutative ring R. By $T_n(S)$ we denote the set of all $n \times n$ upper triangular matrices over S, where $n \geq 2$. Then*

$$\mathcal{J}(S) = S \quad \text{if and only if} \quad \mathcal{J}(T_n(S)) = T_n(S).$$

It follows from Theorem 1.1 that S being generated by idempotents is equivalent to $T_n(S)$ being generated by idempotents.

In Section 2, we give the proof of the main result. In Section 3, we give some corollaries of the main result. Moreover, we present two examples in which upper triangular matrix algebras cannot be generated by idempotents. In Section 4, as an application of the main result we give a description of local Lie derivations of upper triangular matrix algebras over a commutative ring.

2. The proof of Theorem 1.1

Let S be a unital algebra over a commutative ring R. By $T_n(S)$ we denote the set of all $n \times n$ upper triangular matrices over S, where $n \geq 2$. By e_{ij}, where $1 \leq i \leq j \leq n$, we denote the matrix unit in $T_n(S)$, meaning the element at the (i, j)-position is 1 and the elements at other positions are 0. For any $A \in T_n(S)$, we can write

$$A = \sum_{1 \leq i \leq j \leq n} c_{ij} e_{ij},$$

where $c_{ij} \in S$. If $A \in \mathcal{J}(T_n(S))$, we mean that there exist the idempotents $e_1, \ldots, e_l$ in $T_n(S)$, where $l \geq 1$, such that A is generated by $e_1, \ldots, e_l$. That is, there exists a polynomial $f(x_1, \ldots, x_l)$ with zero constant term in the polynomial algebra $R[x_1, \ldots, x_l]$ with noncommutative variables $x_1, \ldots, x_l$, such that

$$A = f(e_1, \ldots, e_l).$$

We begin with the following technical result.

Lemma 2.1. *Let S be a unital algebra over a commutative ring R. By $T_n(S)$ we denote the set of all $n \times n$ upper triangular matrices over S, where $n \geq 2$. Then every idempotent in $T_n(S)$ is of the following form*

$$c_{11} e_{11} + \sum_{\substack{1 \leq i \leq j \leq n \\ (i,j) \neq (1,1)}} c_{ij} e_{ij}, \tag{2.1}$$

where $c_{ij} \in S$ with $c_{11}^2 = c_{11}$.

Proof. Take an idempotent e in $T_n(S)$. We write

$$e = c_{11} e_{11} + \sum_{\substack{1 \leq i \leq j \leq n \\ (i,j) \neq (1,1)}} c_{ij} e_{ij}, \tag{2.2}$$

where $c_{ij} \in S$. It is easy to check that

$$e^2 = c_{11}^2 e_{11} + \sum_{\substack{1 \leq i \leq j \leq n \\ (i,j) \neq (1,1)}} c'_{ij} e_{ij}, \tag{2.3}$$

where $c'_{ij} \in S$. Since $e^2 = e$, comparing (2.2) with (2.3) yields $c_{11}^2 = c_{11}$. This proves the result. □

The following result will be used in the proof of Theorem 1.1.

Lemma 2.2. *Let S be a unital algebra over a commutative ring R. By $T_n(S)$ we denote the set of all $n \times n$ upper triangular matrices over S, where $n \geq 2$. For any $A = \sum_{1\leq i\leq j\leq n} a_{ij}e_{ij} \in \mathcal{J}(T_n(S))$, where $a_{ij} \in S$, we have that $a_{11} \in \mathcal{J}(S)$.*

Proof. For any $A = \sum_{1\leq i\leq j\leq n} a_{ij}e_{ij} \in \mathcal{J}(T_n(S))$, we assume that A is generated by the idempotents $e_1, \ldots, e_l$ in $T_n(S)$, where $l \geq 1$. It follows that there exists a polynomial $f(x_1, \ldots, x_l)$ with zero constant term in the polynomial algebra $R[x_1, \ldots, x_l]$ with noncommutative variables $x_1, \ldots, x_l$, such that

$$A = f(e_1, \ldots, e_l). \tag{2.4}$$

In view of Lemma 2.1, we can write

$$e_k = c_{11}^{(k)}e_{11} + \sum_{\substack{1\leq i\leq j\leq n\\(i,j)\neq(1,1)}} c_{ij}^{(k)}e_{ij},$$

where $c_{ij}^{(k)} \in S$, $1 \leq i \leq j \leq n$, and $c_{11}^{(k)}$ is an idempotent in S, $k = 1, \ldots, l$. Since $f(x_1, \ldots, x_l)$ is a polynomial with zero constant term, we get from (2.4) that

$$\begin{aligned} A &= f(e_1, \ldots, e_l)\\ &= f(\sum_{1\leq i\leq j\leq n} c_{ij}^{(1)}e_{ij}, \ldots, \sum_{1\leq i\leq j\leq n} c_{ij}^{(l)}e_{ij})\\ &= f(c_{11}^{(1)}e_{11}, \ldots, c_{11}^{(l)}e_{11}) + \sum_{\substack{1\leq i\leq j\leq n\\(i,j)\neq(1,1)}} d_{ij}e_{ij}\\ &= f(c_{11}^{(1)}, \ldots, c_{11}^{(l)})e_{11} + \sum_{\substack{1\leq i\leq j\leq n\\(i,j)\neq(1,1)}} d_{ij}e_{ij} \end{aligned} \tag{2.5}$$

for some $d_{ij} \in S$. It follows from (2.5) that the $(1,1)$-entry of the matrix $A = f(e_1, \ldots, e_l)$ is $f(c_{11}^{(1)}, \ldots, c_{11}^{(l)})$. Since $A = \sum_{1\leq i\leq j\leq n} a_{ij}e_{ij}$, we get

$$a_{11} = f(c_{11}^{(1)}, \ldots, c_{11}^{(l)}).$$

Since $c_{11}^{(k)}$ is an idempotent in S, $k = 1, \ldots, l$, we get that

$$a_{11} = f(c_{11}^{(1)}, \ldots, c_{11}^{(l)}) \in \mathcal{J}(S).$$

This proves the result. □

We are ready to give the proof of Theorem 1.1.

Proof of Theorem 1.1. Suppose that $\mathcal{J}(S) = S$. We claim that $\mathcal{J}(T_n(S)) = T_n(S)$. It is clear that $\mathcal{J}(T_n(S)) \subseteq T_n(S)$. We prove that $T_n(S) \subseteq \mathcal{J}(T_n(S))$.

Take any $\alpha e_{ii} \in T_n(S)$, where $\alpha \in S$ and $1 \leq i \leq n$. Since $\mathcal{J}(S) = S$, we assume that α is generated by the idempotents $\alpha_1, \ldots, \alpha_k$ in S, where $k \geq 1$. We have that there exists a polynomial $f(x_1, \ldots, x_k) \in R[x_1, \ldots, x_k]$ with zero constant term in the noncommutative variables $x_1, \ldots, x_k$, such that

$$\alpha = f(\alpha_1, \ldots, \alpha_k). \tag{2.6}$$

It is clear that

$$(\alpha_t e_{ii})(\alpha_t e_{ii}) = \alpha_t^2 e_{ii} = \alpha_t e_{ii}$$

for $t = 1, \ldots, k$. This implies that $\alpha_t e_{ii}$ is an idempotent in $T_n(S)$, where $t = 1, \ldots, k$. Since $f(x_1, \ldots, x_l)$ is a polynomial with zero constant term, we get from (2.6) that

$$\alpha e_{ii} = f(\alpha_1 e_{ii}, \ldots, \alpha_k e_{ii}).$$

This implies that αe_{ii} is generated by the idempotents $\alpha_1 e_{ii}, \ldots, \alpha_k e_{ii}$ in $T_n(S)$. Hence, $\alpha e_{ii} \in \mathcal{J}(T_n(S))$ for all $1 \leq i \leq n$ and $\alpha \in S$.

Take any $\alpha e_{ij} \in T_n(S)$, where $\alpha \in S$ and $1 \leq i < j \leq n$. Note that

$$\begin{aligned}(e_{ii} - \alpha e_{ij})(e_{ii} - \alpha e_{ij}) &= e_{ii} - \alpha e_{ii} e_{ij} - \alpha e_{ij} e_{ii} + \alpha^2 e_{ij} e_{ij} \\ &= e_{ii} - \alpha e_{ij}.\end{aligned}$$

This implies that $e_{ii} - \alpha e_{ij}$ is an idempotent in $T_n(S)$. Since e_{ii} is an idempotent in $T_n(S)$, we get

$$\alpha e_{ij} = e_{ii} - (e_{ii} - \alpha e_{ij}) \in \mathcal{J}(T_n(S)).$$

Take any $A = \sum\limits_{1 \leq i \leq j \leq n} \alpha_{ij} e_{ij} \in T_n(S)$, where $\alpha_{ij} \in S$. We get

$$A = \sum_{1 \leq i \leq n} \alpha_{ii} e_{ii} + \sum_{1 \leq i < j \leq n} \alpha_{ij} e_{ij} \in \mathcal{J}(T_n(S)).$$

We obtain that $T_n(S) \subseteq \mathcal{J}(T_n(S))$, as desired. Hence, $\mathcal{J}(T_n(S)) = T_n(S)$.

Suppose next that $\mathcal{J}(T_n(S)) = T_n(S)$. We claim that $\mathcal{J}(S) = S$. Since $\mathcal{J}(S) \subseteq S$, it suffices to prove that $S \subseteq \mathcal{J}(S)$.

Take any $\alpha \in S$. Since $\mathcal{J}(T_n(S)) = T_n(S)$, we get that $\alpha e_{11} \in \mathcal{J}(T_n(S))$. In view of Lemma 2.2 we can write

$$\alpha e_{11} = c_{11} e_{11} + \sum_{\substack{1 \leq i \leq j \leq n \\ (i,j) \neq (1,1)}} c_{ij} e_{ij}, \tag{2.7}$$

where $c_{ij} \in S$ with $c_{11} \in \mathcal{J}(S)$. It follows from (2.7) that $\alpha = c_{11} \in \mathcal{J}(S)$. We obtain that $S \subseteq \mathcal{J}(S)$. Hence, $S = \mathcal{J}(S)$. The proof of the result is complete. □

3. Some corollaries of the main result

We begin with the following technical result.

Lemma 3.1. *Let $\mathcal{A}_i$ be an algebra generated by idempotents, where $i = 1, \ldots, t$. Let $\mathcal{A} = \mathcal{A}_1 \oplus \cdots \oplus \mathcal{A}_t$. Then $\mathcal{A}$ is generated by idempotents.*

Proof. Take any $(a_1, \ldots, a_t) \in \mathcal{A}$. We assume that $a_i \in \mathcal{A}_i$ is generated by the idempotents $e_{i1}, \ldots, e_{it_i}$ in $\mathcal{A}_i$, where $i = 1, \ldots, t$. We set

$$e'_{ij} = (0, \ldots, 0, e_{ij}, 0, \ldots, 0)$$

for $i = 1, \ldots, t$ and $j = 1, \ldots, t_i$. It is easy to check that e'_{ij} is an idempotent in $\mathcal{A}$ and

$$(0, \ldots, 0, a_i, 0, \ldots, 0)$$

is generated by $e'_{i1}, \ldots, e'_{it_i}$ for $i = 1, \ldots, t$. We have that

$$(a_1, \ldots, a_t) = \sum_{i=1}^{t} (0, \ldots, 0, a_i, 0, \ldots, 0)$$

is generated by the set of idempotents

$$\{e'_{ij} \mid i = 1, \ldots, t, \quad j = 1, \ldots, i_t\}.$$

Hence, $\mathcal{J}(\mathcal{A}) = \mathcal{A}$. □

Corollary 3.1. *Let $\mathcal{A}$ be a finite-dimensional semisimple algebra over an algebraically closed field $\mathcal{F}$. Then $\mathcal{J}(\mathcal{A}) = \mathcal{A}$ and $\mathcal{J}(T_n(\mathcal{A})) = T_n(\mathcal{A})$, where $n \geq 2$.*

Proof. It is well known that $\mathcal{A}$ is a direct sum of some matrix algebras over $\mathcal{F}$. Noting that every matrix algebra over a unital algebra is generated by idempotents (see [1, Section 2, Example (C)]), we get from Lemma 3.1 that $\mathcal{A}$ is generated by idempotents. Then the result follows from Theorem 1.1. □

Corollary 3.2. *Let $\mathcal{A}$ be a unital simple algebra with a nontrivial idempotent. Then $\mathcal{J}(T_n(\mathcal{A})) = T_n(\mathcal{A})$, where $n \geq 2$.*

Proof. Note that $\mathcal{J}(\mathcal{A}) = \mathcal{A}$ (see [1, Section 2, Example (A)]). By Theorem 1.1, we obtain the desired result. □

We say that an algebra $\mathcal{A}$ over a field F is zero product determined if for every bilinear map $f : \mathcal{A} \times \mathcal{A} \to X$, where X is an arbitrary vector space over F, with the property that for all $x, y \in \mathcal{A}$,

$$xy = 0 \Longrightarrow f(x, y) = 0,$$

there exists a linear map $\Phi : \mathcal{A} \to X$ such that

$$f(x, y) = \Phi(xy)$$

for all $x, y \in \mathcal{A}$ (see [2] for details).

Corollary 3.3. *Let $\mathcal{A}$ be a finite-dimensional zero product determined algebra. Then $\mathcal{J}(T_n(\mathcal{A})) = T_n(\mathcal{A})$, where $n \geq 2$.*

Proof. Note that $\mathcal{J}(\mathcal{A}) = \mathcal{A}$ (see [2, Theorem 3.7]). By Theorem 1.1, we obtain the desired result. □

The following two results give two examples in which upper triangular matrix algebras are not generated by idempotents.

Corollary 3.4. *Let $\mathcal{R}$ be the real field. Let $\mathcal{Q}$ be the rational field. If we regard $\mathcal{R}$ as a $\mathcal{Q}$-algebra, then $\mathcal{J}(T_n(\mathcal{R})) \neq T_n(\mathcal{R})$, where $n \geq 2$.*

Proof. It is clear that the only nonzero idempotent in $\mathcal{R}$ is 1. We get

$$\mathcal{J}(\mathcal{R}) = \{r \cdot 1 \mid r \in \mathcal{Q}\} \neq \mathcal{R}.$$

By Theorem 1.1, we get that $\mathcal{J}(T_n(\mathcal{R})) \neq T_n(\mathcal{R})$. □

Corollary 3.5. *Let $\mathcal{F}$ be a field. Let $\mathcal{A} = \mathcal{F}[x]/(x^2)$. Then $\mathcal{J}(T_n(\mathcal{A})) \neq T_n(\mathcal{A})$, where $n \geq 2$.*

Proof. Note that

$$\mathcal{F}[x]/(x^2) = \{a + bx \mid a, b \in \mathcal{F}\}.$$

We claim that the only nonzero idempotent in $\mathcal{F}[x]/(x^2)$ is 1. Indeed, suppose that $a + bx$ is a nonzero idempotent in $\mathcal{F}[x]/(x^2)$. Since $x^2 = 0$, we get

$$(a + bx)^2 = a^2 + 2abx = a + bx.$$

It follows that $a^2 = a$ and $2ab = b$. We get that $a = 1$ and $b = 0$. Hence, $a + bx = 1$, as desired. We have

$$\mathcal{J}(\mathcal{A}) = \{a \cdot 1 \mid a \in \mathcal{F}\} \neq \mathcal{A}.$$

Then the result follows from Theorem 1.1. □

4. Local Lie derivations of upper triangular matrix algebras

Let $\mathcal{B}$ be an associative algebra over a commutative ring R with $\frac{1}{2} \in R$. By $Z(\mathcal{B})$ we denote the center of $\mathcal{B}$. Set $[x, y] = xy - yx$.

An R-linear map $f : \mathcal{B} \to \mathcal{B}$ is said to be a *Lie derivation* if

$$f([x, y]) = [f(x), y] + [x, f(y)]$$

for all $x, y \in \mathcal{B}$. It is clear that $f = d + h$ is a Lie derivation of $\mathcal{B}$, where d is a derivation of $\mathcal{B}$ and $h : \mathcal{B} \to Z(\mathcal{B})$ is an R-linear map vanishing on each commutator, which is said to be *proper*.

An R-linear map $f : \mathcal{B} \to \mathcal{B}$ is said to be a *local Lie derivation* if for every $x \in \mathcal{B}$ there exists a Lie derivation f_x of $\mathcal{B}$, depending on x, such that $f(x) = f_x(x)$.

In 2015, Chen and Lu [5] investigated local Lie derivations of nest algebras. In 2022, An, Zhang, He, and Qian [6] gave a characterization of local Lie derivations of von Neumann algebras. In 2020, Liu [7] gave a description of local Lie derivations of triangular algebras. More precisely, he proved the following result.

Theorem 4.1. *[7, Theorem 3.2] Let $\mathcal{T} = \begin{pmatrix} A & M \\ 0 & B \end{pmatrix}$ be a triangular algebra over a commutative ring R with $\frac{1}{2} \in R$. Suppose that*

(i) $A = \mathcal{J}(A)$ and $B = \mathcal{J}(B)$,
(ii) $Z(A) = \pi_A(Z(\mathcal{T}))$ and $Z(B) = \pi_B(Z(\mathcal{T}))$.

Then every local Lie derivation of $\mathcal{T}$ is a proper Lie derivation.

As a consequence of Theorem 4.1, Liu gave the following result.

Corollary 4.1. *[7, Corollary 3.3] Let $\mathcal{R}$ be the real field. Then every local Lie derivation of $T_n(\mathcal{R})$, where $n > 3$, is a proper Lie derivation.*

For more results related to local Lie derivations, we refer the reader to [8–10].

Let S be a unital algebra over a commutative ring R with $\frac{1}{2} \in R$. By $T_n(S)$ we denote the set of all $n \times n$ upper triangular matrices over S.

We now give the main result of the section.

Theorem 4.2. *Let S be a unital algebra over a commutative ring R with $\frac{1}{2} \in R$. Suppose that $\mathcal{J}(S) = S$. Then every local Lie derivation of $T_n(S)$, where $n \geq 2$, is a proper Lie derivation.*

Proof. Write

$$T_n(S) = \begin{pmatrix} A & M \\ 0 & B \end{pmatrix},$$

where $A = T_s(S)$, $M = M_{s,t}(S)$, and $B = T_t(S)$, with $s + t = n$ and $s, t \geq 1$. Then $T_n(S)$ is a triangular algebra (see [11, 12] for details).

We claim that $\mathcal{J}(A) = A$. If $s = 1$, we have

$$A = S = \mathcal{J}(S) = \mathcal{J}(A).$$

Suppose that $s > 1$. In view of Theorem 1.1, we get that $\mathcal{J}(A) = A$. Similarly, we can obtain $\mathcal{J}(B) = B$. This implies that $T_n(S)$ satisfies the assumption (i) in Theorem 4.1.

Note that

$$\begin{cases} Z(T_n(S)) = \left\{ \begin{pmatrix} z \cdot 1_A & 0 \\ 0 & z \cdot 1_B \end{pmatrix} \mid z \in Z(S) \right\}; \\ Z(A) = Z(S) \cdot 1_A; \\ Z(B) = Z(S) \cdot 1_B. \end{cases}$$

This implies that the projection π_A from $Z(T_n(S))$ to $Z(A)$ is surjective. We get that $Z(A) = \pi_A(Z(\mathcal{T}))$. Similarly, we get that $Z(B) = \pi_B(Z(\mathcal{T}))$. This implies that the assumption (ii) in Theorem 4.1 is satisfied. Thus, $T_n(S)$ satisfies all assumptions of Theorem 4.1. By Theorem 4.1, we obtain that every local Lie derivation of $T_n(S)$, where $n \geq 2$, is a proper Lie derivation. □

As a consequence of Theorem 4.2, we have the following result.

Corollary 4.2. *Let R be a commutative ring with $\frac{1}{2} \in R$. Then every local Lie derivation of $T_n(R)$, where $n \geq 2$, is a proper Lie derivation.*

Proof. It is clear that

$$\mathcal{J}(R) = \{a \cdot 1 \mid a \in R\} = R.$$

In view of Theorem 4.2, we can obtain the desired result. □

We remark that Corollary 4.2 improves Corollary 4.1.

Corollary 4.3. *Let $\mathcal{B}$ be a finite-dimensional semisimple algebra over an algebraically closed field $\mathcal{F}$ with $\frac{1}{2} \in \mathcal{F}$. Then every local Lie derivation of $T_n(\mathcal{B})$, where $n \geq 2$, is a proper Lie derivation.*

Proof. In view of Corollary 3.1, we get that $\mathcal{J}(\mathcal{B}) = \mathcal{B}$. Then the result follows from Theorem 4.2. □

Corollary 4.4. *Let R be a commutative ring with $\frac{1}{2} \in R$. Let $\mathcal{B}$ be a unital simple algebra over R. If $\mathcal{B}$ contains a nontrivial idempotent, then every local Lie derivation of $T_n(\mathcal{B})$, where $n \geq 2$, is a proper Lie derivation.*

Proof. Suppose that $\mathcal{B}$ contains a nontrivial idempotent. In view of [1, Section 2, Example (A)], we note that $\mathcal{J}(\mathcal{B}) = \mathcal{B}$. Then the result follows from Theorem 4.2. □

Corollary 4.5. *Let $\mathcal{B}$ be a finite-dimensional zero product determined algebra over a field $\mathcal{F}$ with $\frac{1}{2} \in \mathcal{F}$. Then every local Lie derivation of $T_n(\mathcal{B})$, where $n \geq 2$, is a proper Lie derivation.*

Proof. Note that $\mathcal{J}(\mathcal{B}) = \mathcal{B}$ (see [2, Theorem 3.7]). Then the result follows from Theorem 4.2. □

Finally, we remark that Theorem 4.2 can be applied not only to local Lie derivations on upper triangular matrix algebras, but also to other local mapping problems on upper triangular matrix algebras.

5. Conclusions

The main result is that $\mathcal{J}(S) = S$ iff $\mathcal{J}(T_n(S)) = T_n(S)$. As an application of the main result, we prove that $T_n(S)$, where S is a finite-dimensional semisimple algebra over an algebraically closed field, is generated by idempotents. As another application of the main result, we give two examples in which upper triangular matrix algebras are not generated by idempotents. Finally, we give a description of local Lie derivations of upper triangular matrix algebras over a commutative ring, which improves a result obtained by Liu.

Author contributions

Qian Chen: Conceptualization, writing-original draft. Yu Wang: Investigation, validation. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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