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Research article

Finite-block power identities for generalized derivations on semiprime rings

Abu Zaid Ansari*, **Mohammad Habbad and Faarie Alharbi**

Department of Mathematics, Faculty of Science, Islamic University of Madinah, Saudi Arabia

* **Correspondence:** Email: ansari.abuzaid@gmail.com.

Abstract: Let S be a unital semiprime ring. We establish a finite-block theorem for additive maps satisfying a power identity whose terms are separated into blocks of prescribed sizes. Under transparent torsion-free assumptions on the additive group of S, the associated map is a derivation and the principal map is a generalized derivation. The four-block identity is obtained as a corollary. We also prove an automatic-continuity consequence for unital semisimple complex Banach algebras: the associated derivation is continuous, and it vanishes in the commutative case. A finite-difference coefficient lemma and the torsion obstruction in the degenerate positive-block case are made explicit.

Keywords: Banach algebra; semiprime ring; algebraic identities; generalized derivation
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1. Introduction

Derivations and generalized derivations are important tools in the study of functional identities on rings and algebras. Classical results in prime ring theory include Posner's work on derivations in prime rings and Herstein's theorem on Jordan derivations of prime rings [1, 2]. A central question is whether an additive map satisfying a prescribed algebraic identity must arise from a derivation.

Throughout the paper, S denotes an associative ring with identity element e. For an integer $p \geq 1$, the ring S is called p-torsion-free if $pa = 0$ implies $a = 0$ for every $a \in S$. The condition for $p = 1$ is automatic. The ring S is prime if $zSy = \{0\}$ implies $z = 0$ or $y = 0$, and it is semiprime if $zSz = \{0\}$ implies $z = 0$.

An additive map $d : S \to S$ is a derivation if

$$d(xy) = d(x)y + xd(y) \qquad (x, y \in S),$$

and it is a Jordan derivation if

$$d(x^2) = d(x)x + xd(x) \qquad (x \in S).$$

An additive map $G : S \to S$ is called a generalized derivation associated with a derivation g if

$$G(xy) = G(x)y + xg(y) \qquad (x, y \in S).$$

Generalized derivations and generalized Jordan derivations have been studied in rings, Banach algebras, and standard operator algebras [3, 4]. Related results concerning left Jordan derivations were obtained by Vukman [5], while centralizers of semiprime rings were studied by Zalar [6].

Every derivation is a Jordan derivation. A fundamental result of Herstein states that every Jordan derivation on a prime ring of characteristic different from 2 is a derivation [2]. This result was extended to semiprime rings by Cusack and Brešar [7, 8]. Moreover, Lanski [9] also presented a result on generalized derivations and n^{th} power maps in semiprime rings. The torsion assumptions are essential in these results. In particular, Lee and Lin described Jordan derivations on prime rings of characteristic 2, where additional additive central-valued terms vanishing on squares may occur [10].

Several power identities for additive mappings have already been considered. Ansari and Rehman studied the two-block identity

$$F(x^{m+n}) = F(x^m)x^n + x^m d(x^n)$$

in semiprime rings [11]. Ansari investigated the three-block relation

$$\Delta(x^{m+n+p}) = \Delta(x^m)x^{n+p} + x^m \partial(x^n)x^p + x^{m+n}\partial(x^p)$$

in [12]. A related symmetrized identity and its consequences for semisimple Banach algebras were studied by Shujat and Ansari [13]. In the operator-algebra setting, Zhu and Xiong considered generalized derivations and P-preserving kernel-to-range mappings on von Neumann algebras [14].

The preceding results motivate a unified finite-block formulation. Let $r \geq 2$, let $n_1, n_2 \geq 1$, and let $n_3, \ldots, n_r \geq 0$. Put

$$N = \sum_{j=1}^{r} n_j.$$

We prove that the finite-block identity

$$G(a^N) = G(a^{n_1})a^{N-n_1} + \sum_{j=2}^{r} a^{n_1+\cdots+n_{j-1}} g(a^{n_j}) a^{n_{j+1}+\cdots+n_r}$$

forces g to be a derivation and G to be a generalized derivation under separate torsion assumptions corresponding to the coefficient cancellations in the proof. The four-block identity is obtained as a direct corollary.

In all Banach-algebra statements in this paper, derivations are assumed to be complex-linear; continuity is not assumed unless expressly stated. We also derive a Banach-algebra consequence. On a unital semisimple complex Banach algebra, the associated derivation is continuous by the Johnson–Sinclair theorem [15]. If the algebra is commutative, the Singer–Wermer theorem implies that every continuous derivation has range in the Jacobson radical [16]. Thomas proved that the same conclusion holds without assuming continuity [17]. Consequently, the associated derivation vanishes in the commutative case.

The paper is organized as follows. Section 2 contains the finite-difference lemma and the main finite-block theorem, followed by the four-block corollary. Section 3 discusses the torsion obstruction in the degenerate positive-block case. Section 4 gives the semisimple Banach-algebra and $C(K)$ consequences. The final section summarizes the results.

2. A finite-block theorem

The following lemma is a finite-difference version of a coefficient lemma used in the proof by Chung and Luh [18, Lemma 1].

Lemma 1 (Integer-polynomial coefficient lemma)**.** *Let B be an abelian group and let $n \geq 0$. Assume that B is $n!$-torsion-free. If*

$$P(t) = b_0 + tb_1 + \cdots + t^n b_n, \qquad b_0, \ldots, b_n \in B,$$

vanishes for every positive integer t, then $b_0 = \cdots = b_n = 0$.

Proof. Let $\Delta P(t) = P(t+1) - P(t)$. Since P vanishes on the positive integers, every finite difference of P also vanishes there. Moreover,

$$\Delta^n P(t) = n! b_n$$

for every integer t. Hence, $n!b_n = 0$, and the torsion-free assumption gives $b_n = 0$. If $n \geq 1$, then $n!$-torsion-freeness implies $(n-1)!$-torsion-freeness, because $(n-1)!b = 0$ implies $n!b = 0$. Repeating the argument proves that all coefficients vanish. The case $n = 0$ is immediate. □

When an expression has been expanded as a polynomial in t with coefficients in an abelian group, $[t^k]$ denotes its coefficient of t^k. The lemma justifies equating such coefficients whenever the difference of the two sides has degree at most n and vanishes at every positive integer.

Lemma 2 ([7])**.** *Every Jordan derivation on a 2-torsion-free semiprime ring is a derivation.*

Theorem 1. *Let S be a unital semiprime ring with identity element e. Let $r \geq 2$, let $n_1, n_2 \geq 1$, and let $n_3, \ldots, n_r \geq 0$ be fixed integers. Put*

$$N = \sum_{j=1}^{r} n_j, \qquad A = \sum_{1 \leq i < j \leq r} n_i n_j,$$

and, for $1 \leq j \leq r$, put

$$P_{j-1} = \sum_{i=1}^{j-1} n_i, \qquad Q_j = \sum_{k=j+1}^{r} n_k,$$

where $P_0 = Q_r = 0$. Assume that the additive group of S is $(r-1)$-torsion-free, 2-torsion-free, A-torsion-free, and $(N-1)!$-torsion-free. Suppose that additive maps $G, g : S \to S$ satisfy

$$G(a^N) = G(a^{n_1}) a^{N-n_1} + \sum_{j=2}^{r} a^{P_{j-1}} g(a^{n_j}) a^{Q_j} \tag{2.1}$$

for every $a \in S$, where $a^0 = e$. Then g is a derivation on S, and G is a generalized derivation associated with g. In particular,

$$G(ab) = G(a)b + ag(b) \qquad (a, b \in S).$$

Proof. Putting $a = e$ in (2.1) gives

$$G(e) = G(e) + (r-1)g(e).$$

Since the additive group of S is $(r-1)$-torsion-free, we have

$$g(e) = 0. \tag{2.2}$$

Fix $a \in S$ and replace a in (2.1) by $x = a + te$, where t is a positive integer. Since e is the identity, for every integer $m \geq 0$, we have

$$x^m = \sum_{i=0}^{m} \binom{m}{i} t^i a^{m-i}.$$

Because G and g are additive, they are $\mathbb{Z}$-linear, hence

$$G(x^m) = \sum_{i=0}^{m} \binom{m}{i} t^i G(a^{m-i}), \qquad g(x^m) = \sum_{i=0}^{m} \binom{m}{i} t^i g(a^{m-i}).$$

The difference between the two sides of (2.1), after this substitution, is an S-valued polynomial in t. Its coefficient of t^N is zero: the left-hand side and the first term on the right-hand side both contribute $G(e)$, while every other top-degree contribution is a multiple of $g(e) = 0$. Therefore, the difference polynomial has degree at most $N-1$. By Lemma 1, with $B = (S, +)$ and $n = N-1$, its coefficients may be compared.

Set $M = N - n_1$. The coefficient of t^{N-1} on the left-hand side is

$$[t^{N-1}]G(x^N) = NG(a).$$

For the first term on the right-hand side,

$$[t^{N-1}](G(x^{n_1})x^M) = n_1 G(a) + MG(e)a.$$

For $j \geq 2$, the coefficient of t^{N-1} in the jth summand is

$$[t^{N-1}](x^{P_{j-1}} g(x^{n_j}) x^{Q_j}) = n_j g(a).$$

Indeed, all contributions in which one of the outer factors loses a power of t contain $g(e)$ and vanish, while the contribution in which the middle factor loses one power is $n_j g(a)$. If $n_j = 0$, then $g(x^{n_j}) = g(e) = 0$, and the same formula remains valid.

Consequently,

$$NG(a) = n_1 G(a) + MG(e)a + \sum_{j=2}^{r} n_j g(a) = n_1 G(a) + M\big(G(e)a + g(a)\big).$$

Thus,

$$M\big(G(a) - G(e)a - g(a)\big) = 0.$$

Since $1 \leq M \leq N-1$ and $M \mid (N-1)!$, the $(N-1)!$-torsion-free assumption gives

$$G(a) = G(e)a + g(a) \qquad (a \in S). \tag{2.3}$$

Applying (2.3) to a^N and a^{n_1}, respectively, we obtain

$$G(a^N) = G(e)a^N + g(a^N)$$

and

$$G(a^{n_1}) = G(e)a^{n_1} + g(a^{n_1}).$$

Substituting these expressions into (2.1) gives

$$G(e)a^N + g(a^N) = (G(e)a^{n_1} + g(a^{n_1}))a^M + \sum_{j=2}^{r} a^{P_{j-1}} g(a^{n_j}) a^{Q_j},$$

where $M = N - n_1$. Hence,

$$g(a^N) = g(a^{n_1})a^M + \sum_{j=2}^{r} a^{P_{j-1}} g(a^{n_j}) a^{Q_j}.$$

Since

$$P_0 = 0, \qquad Q_1 = \sum_{k=2}^{r} n_k = N - n_1 = M,$$

we have

$$a^{P_0} g(a^{n_1}) a^{Q_1} = e\, g(a^{n_1}) a^M = g(a^{n_1}) a^M.$$

Therefore,

$$g(a^N) = \sum_{j=1}^{r} a^{P_{j-1}} g(a^{n_j}) a^{Q_j}. \tag{2.4}$$

Now, we replace a by $x = a + te$ in (2.4). Since (2.4) holds for every element of S, we obtain

$$g(x^N) = \sum_{j=1}^{r} x^{P_{j-1}} g(x^{n_j}) x^{Q_j}, \quad \text{for every positive integer } t.$$

Define

$$H_a(t) = g(x^N) - \sum_{j=1}^{r} x^{P_{j-1}} g(x^{n_j}) x^{Q_j}.$$

Then, $H_a(t) = 0$ for every positive integer t.

Using

$$x^m = \sum_{i=0}^{m} \binom{m}{i} t^i a^{m-i},$$

we have

$$g(x^N) = \sum_{i=0}^{N} \binom{N}{i} t^i g(a^{N-i}).$$

The coefficient of t^N in this expression is

$$g(e) = 0.$$

For each j, we have

$$x^{P_{j-1}} g(x^{n_j}) x^{Q_j} = \left(\sum_{u=0}^{P_{j-1}} \binom{P_{j-1}}{u} t^u a^{P_{j-1}-u} \right) \times \left(\sum_{v=0}^{n_j} \binom{n_j}{v} t^v g(a^{n_j-v}) \right) \times \left(\sum_{w=0}^{Q_j} \binom{Q_j}{w} t^w a^{Q_j-w} \right).$$

Since

$$P_{j-1} + n_j + Q_j = N,$$

the largest possible power of t is t^N. This power occurs only when

$$u = P_{j-1}, \qquad v = n_j, \qquad w = Q_j.$$

Therefore, the coefficient of t^N is

$$e\, g(e)\, e = g(e) = 0.$$

Thus, the coefficient of t^N in every term of $H_a(t)$ is zero. Although each term has degree at most N, its t^N-coefficient vanishes. Hence,

$$\deg H_a(t) \leq N - 1.$$

So, Lemma 1 permits comparison of the coefficient of t^{N-2}. Using $\binom{m}{k} = 0$ for $k < 0$ or $k > m$, we have

$$[t^{N-2}](x^{P_{j-1}} g(x^{n_j}) x^{Q_j}) = \binom{n_j}{2} g(a^2) + n_j P_{j-1} a g(a) + n_j Q_j g(a) a.$$

Hence,

$$\binom{N}{2} g(a^2) = \sum_{j=1}^{r} \binom{n_j}{2} g(a^2) + \left(\sum_{j=1}^{r} n_j P_{j-1} \right) a g(a) + \left(\sum_{j=1}^{r} n_j Q_j \right) g(a) a.$$

Since

$$\sum_{j=1}^{r} n_j P_{j-1} = A, \qquad \sum_{j=1}^{r} n_j Q_j = A, \qquad \binom{N}{2} - \sum_{j=1}^{r} \binom{n_j}{2} = A,$$

we get

$$A(g(a^2) - a g(a) - g(a) a) = 0.$$

The additive group of S is A-torsion-free. Therefore,

$$g(a^2) = a g(a) + g(a) a \qquad (a \in S).$$

Thus, g is a Jordan derivation. Since S is semiprime and 2-torsion-free, Lemma 2 implies that g is a derivation.

Finally, by (2.3),

$$\begin{aligned} G(ab) &= G(e)ab + g(ab) \\ &= G(e)ab + g(a)b + ag(b) \\ &= G(a)b + ag(b). \end{aligned}$$

Hence, G is a generalized derivation associated with g. □

Corollary 1. *Let S be a unital semiprime ring with identity element e. Let $\ell_1, \ell_2 \geq 1$ and $\ell_3, \ell_4 \geq 0$, and put*

$$L = \ell_1 + \ell_2 + \ell_3 + \ell_4, \qquad A = \sum_{1 \leq i < j \leq 4} \ell_i \ell_j.$$

Assume that the additive group of S is 3-torsion-free, 2-torsion-free, A-torsion-free, and $(L-1)!$-torsion-free. If additive maps $G, g : S \to S$ satisfy

$$\begin{aligned} G(a^L) = G(a^{\ell_1})a^{\ell_2+\ell_3+\ell_4} + a^{\ell_1} g(a^{\ell_2}) a^{\ell_3+\ell_4} \\ + a^{\ell_1+\ell_2} g(a^{\ell_3}) a^{\ell_4} + a^{\ell_1+\ell_2+\ell_3} g(a^{\ell_4}) \end{aligned} \tag{2.5}$$

for every $a \in S$, then g is a derivation and G is a generalized derivation associated with g.

Proof. Apply Theorem 1 with $r = 4$ and $(n_1, n_2, n_3, n_4) = (\ell_1, \ell_2, \ell_3, \ell_4)$. □

Remark 1. *The separate torsion assumptions in Corollary 2.4 are equivalent to $6A(L-1)!$-torsion-freeness and to $\mathrm{lcm}(3, 2, A, (L-1)!)$-torsion-freeness. We state them separately to indicate their respective uses in the proof. The analogous equivalence holds in Theorem 2.3. We do not claim that these equivalent sufficient conditions are necessary for every choice of the exponents.*

3. The torsion obstruction

The additional $(r-1)$-torsion condition is not a cosmetic part of the proof. It is needed to deduce $g(e) = 0$ from the identity evaluated at the identity element.

Example 1 (A degenerate-block counterexample)**.** *Let $r \geq 3$, let $\mathbb{F}$ be a field whose characteristic divides $r-1$, and choose*

$$n_1 = n_2 = 1, \qquad n_3 = \cdots = n_r = 0.$$

Let $S = \mathbb{F}$, let $G = 0$, and let $g = \mathrm{id}_S$. Then, $N = 2$, and for every $a \in S$ the right-hand side of the finite-block identity is

$$\sum_{j=2}^{r} a^{P_{j-1}} g(a^{n_j}) a^{Q_j} = (r-1)a^2 = 0.$$

Thus, the identity holds, but $g(e) = e \neq 0$, so g is not a derivation. Taking $r = 4$ and $F = \mathbb{F}_3$ gives a counterexample to the four-block assertion without the 3-torsion-free assumption. It does not contradict Theorem 1, because $\mathbb{F}_3$ is not 3-torsion-free.

The same issue occurs in the degenerate case of the three-block statement cited from [12, Theorem 2.1]. Its statement excludes the case where $m = n = 1$ and $p = 0$; however, if allowed, its condition of being $(m+n+p-1)!$-torsion-free would reduce to 1!-torsion-free. Taking $S = \mathbb{F}_2$, $G = 0$, and $g = \mathrm{id}_{\mathbb{F}_2}$ gives

$$G(a^2) = 0, \qquad G(a)a + ag(a) + a^2 g(e) = a^2 + a^2 = 0,$$

for every $a \in \mathbb{F}_2$, while $g(e) = 1$. Thus, that parameter range requires an additional torsion hypothesis. These examples show the obstruction caused by the missing $(r-1)$ factor; they do not prove that the full product or least-common-multiple condition is sharp.

Example 2 (The necessity of the 2-torsion-free assumption)**.** *Let* $S = \mathbb{F}_2[x]$, $r = 2$, $n_1 = n_2 = 1$. *Then,* $N = 2$, $A = n_1 n_2 = 1$. *For* $a = \sum_{k=0}^{m} c_k x^k \in S$, $c_k \in \mathbb{F}_2$, *define additive maps* $G, g : S \to S$ *by* $G = g$, $g(a) = c_1$. *Since the characteristic of* S *is* 2,

$$a^2 = \left(\sum_{k=0}^{m} c_k x^k\right)^2 = \sum_{k=0}^{m} c_k x^{2k}.$$

Thus, the coefficient of x *in* a^2 *is zero, and consequently*

$$G(a^2) = g(a^2) = 0.$$

Moreover, since S *is commutative and has characteristic* 2,

$$G(a)a + ag(a) = g(a)a + ag(a) = 2ag(a) = 0.$$

Therefore,

$$G(a^2) = G(a)a + ag(a),$$

which is precisely (2.1) *for* $r = 2$ *and* $n_1 = n_2 = 1$.

However, g *is not a derivation. Indeed,* $g(x^3) = 0$, *whereas*

$$g(x)x^2 + xg(x^2) = 1 \cdot x^2 + x \cdot 0 = x^2 \neq 0.$$

Hence,

$$g(x \cdot x^2) \neq g(x)x^2 + xg(x^2).$$

The additive group of S *is* 1*-torsion-free, and both* $A = 1$ *and* $(N-1)! = 1$ *cause no torsion restriction. Thus, this example shows that the* 2*-torsion-free assumption is essential.*

The two constructions in Example 3.1 exhibit the obstruction caused by omitting the $(r-1)$-torsion-free condition. Example 3.2 shows that the 2-torsion-free assumption is also essential.

The following example shows that the A-torsion-free assumption cannot be omitted from the general statement. It also gives a counterexample to [12, Theorem 2.1] with all three block sizes positive.

Example 3. *Take*

$$S = \mathbb{F}_3[x], \quad r = 3, \quad n_1 = n_2 = n_3 = 1,$$

and let $G = g$, *where*

$$g\left(\sum_{k\geq 0} c_k x^k\right) = c_1.$$

Here $N = 3$, $A = 3$, *and* $g(e) = 0$. *In characteristic* 3, *the cube of any polynomial has no term of degree* 1, *and hence*

$$G(a^3) = g(a^3) = 0.$$

Since the ring is commutative,

$$G(a)a^2 + ag(a)a + a^2 g(a) = 3a^2 g(a) = 0.$$

Thus, the three-block identity holds. Nevertheless,

$$g(x \cdot x^2) = 0 \neq x^2 = g(x)x^2 + xg(x^2).$$

The ring is semiprime, $(r-1)$*-torsion-free,* 2*-torsion-free, and* $(N-1)!$*-torsion-free, but it is not A-torsion-free. This supplies a counterexample when only the latter assumption is dropped. Consequently, this is a counterexample to [12, Theorem 2.1] for* $m = n = p = 1$ *under its stated* $2!$*-torsion-free hypothesis.*

4. Semisimple Banach-algebra consequences

The continuous Singer–Wermer theorem states that every continuous derivation on a commutative Banach algebra has range contained in the Jacobson radical [16]. Derivations are assumed to be complex-linear, though continuity is not assumed unless expressly stated. An analogous assumption applies to Thomas's theorem, which establishes the same conclusion without requiring a continuity assumption [17]. The Johnson–Sinclair theorem states that every derivation on a semisimple Banach algebra is continuous [15].

Proposition 1. *Let* $\mathfrak{A}$ *be a unital semisimple complex Banach algebra with identity element* e*, not necessarily commutative. Let* $r, n_1, \dots, n_r, N, P_{j-1}$*, and* Q_j *be as in Theorem 1. Suppose that complex-linear maps* $H, h : \mathfrak{A} \to \mathfrak{A}$ *satisfy*

$$H(x^N) = H(x^{n_1})x^{N-n_1} + \sum_{j=2}^{r} x^{P_{j-1}} h(x^{n_j}) x^{Q_j} \tag{4.1}$$

for every $x \in \mathfrak{A}$*. Then,* h *is a continuous derivation and*

$$H(x) = H(e)x + h(x) \qquad (x \in \mathfrak{A}).$$

In particular, H *is continuous. If* $\mathfrak{A}$ *is commutative, then*

$$h = 0, \qquad H(x) = H(e)x \qquad (x \in \mathfrak{A}).$$

Proof. Since the underlying additive group is that of a complex vector space, multiplication by any positive integer is injective. Moreover, a semisimple Banach algebra is semiprime. Theorem 1 therefore applies to (4.1) and shows that h is a derivation and

$$H(xy) = H(x)y + xh(y) \qquad (x, y \in \mathfrak{A}).$$

Putting $x = e$ gives

$$H(y) = H(e)y + h(y) \qquad (y \in \mathfrak{A}).$$

By Johnson–Sinclair, h is continuous. Left multiplication by the fixed element $H(e)$ is bounded, so the displayed representation proves that H is continuous.

If $\mathfrak{A}$ is commutative, Thomas's theorem gives

$$h(\mathfrak{A}) \subseteq \operatorname{rad}(\mathfrak{A}) = \{0\},$$

and hence $h = 0$. The representation of H then reduces to $H(x) = H(e)x$. □

Corollary 2 ($C(K)$ consequence)**.** *Let K be a compact Hausdorff space and let $C(K)$ be the Banach algebra of continuous complex-valued functions on K with pointwise operations. Let $H, h : C(K) \to C(K)$ be complex-linear maps satisfying the finite-block identity* (4.1)*, with powers and products taken pointwise. Then*

$$h = 0, \qquad H(f) = uf \qquad (f \in C(K)),$$

where $u = H(1) \in C(K)$. In particular, H is continuous.

Proof. The algebra $C(K)$ is unital, commutative, and semisimple. Apply Proposition 1 and put $u = H(1)$. □

5. Conclusions

We established a finite-block power-identity theorem for additive maps on unital semiprime rings. Under separate torsion-free assumptions corresponding to the identity element, the coefficient comparison, the Jordan-derivation step, and the final cancellation, the associated map is a derivation and the principal map is a generalized derivation. The four-block identity follows as a direct corollary.

For unital semisimple complex Banach algebras, the associated derivation is continuous, and the principal map has the representation $H(x) = H(e)x + h(x)$. In the commutative case, the derivation vanishes, and the principal map is multiplication by the fixed element $H(e)$. The degenerate-block examples explain why the additional $(r-1)$-torsion condition cannot simply be omitted. No claim of sharpness is made for the complete least-common-multiple condition.

Author contributions

Abu Zaid Ansari: Conceptualization, methodology, supervision; Mohammad Habbad and Faarie Alharbi: Investigation, validation, formal analysis; and Faarie Alharbi: Writing–original draft and editing. All authors collectively contributed to every part of the work and have reviewed and approved the submitted manuscript. All authors have also read and approved the final version for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest related to this manuscript.

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