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Theory article

Planar p-Laplacian Schrödinger-Poisson system with Choquard-type nonlinearity term

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Abstract: Via variational methods, we establish the existence of nontrivial solutions to a class of Schrödinger-Poisson systems with Choquard-type nonlinearity and the p-Laplacian term

$$\begin{cases} -\Delta_p u + |u|^{p-2}u + \phi u = (I_\alpha * |u|^q)|u|^{q-2}u, & \text{in } \mathbb{R}^2, \\ \Delta\phi = u^2, & \text{in } \mathbb{R}^2, \end{cases}$$

where $\frac{4}{3} \le p < 2$, $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < \frac{p(2+\alpha)}{2(2-p)}$, $\alpha \in (0,2)$, and $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian. To overcome the main difficulties arising from the presence of logarithmic kernels and the nonlinear p-Laplacian operator $-\Delta_p$, we introduce a suitable function space for the variational analysis of the system, and employ Jeanjean's technique to prove the boundedness of Palais-Smale sequences.

Keywords: Schrödinger-Poisson system; p-Laplacian; planar system; Choquard term; variational method
Mathematics Subject Classification: 35J50, 35Q40, 35J20

1. Introduction

We consider nontrivial solutions to the planar Schrödinger-Poisson system with Choquard-type nonlinearity and the p-Laplacian term

$$\begin{cases} -\Delta_p u + |u|^{p-2}u + \phi u = (I_\alpha * |u|^q)|u|^{q-2}u, & \text{in } \mathbb{R}^2, \\ \Delta\phi = u^2, & \text{in } \mathbb{R}^2, \end{cases} \tag{1.1}$$

where $\frac{4}{3} \le p < 2$, $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < \frac{p(2+\alpha)}{2(2-p)} < p^* = \frac{2p}{2-p}$, $\alpha \in (0,2)$, and $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian. I_α denotes the Riesz potential defined by

$$I_\alpha(x) = \frac{\Gamma(\frac{2-\alpha}{2})}{\Gamma(\frac{\alpha}{2})2^\alpha \pi |x|^{2-\alpha}} := \frac{A_\alpha}{|x|^{2-\alpha}}, \ x \in \mathbb{R}^2 \setminus \{0\}.$$

System (1.1) can be regarded as an extension of the Schrödinger-Poisson system

$$\begin{cases} -\Delta u + u^{p-2}u + \lambda\phi u = g(x,u), & \text{in } \mathbb{R}^N, \\ \Delta\phi = u^2, & \text{in } \mathbb{R}^N, \end{cases} \tag{1.2}$$

which originates from Hartree-Fock models in quantum mechanics as well as electromagnetic field models, and has important applications in physical fields such as quantum mechanics and semiconductor theory [1, 2]. The solution of the Poisson equation is $\phi_{N,u} = \Gamma_N * u^2$, where

$$\Gamma_N = \begin{cases} \frac{1}{2\pi}\ln|x|, & N = 2, \\ \frac{1}{N(2-N)\omega_N}|x|^{2-N}, & N \geq 3, \end{cases}$$

and ω_N denotes the volume of the unit ball in $\mathbb{R}^N$. Substituting the solution $\phi_{N,u}$ of the Poisson equation into the first equation in (1.2), we obtain the associated energy functional defined by

$$I_N(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2\,dx + \frac{1}{p}\int_{\mathbb{R}^N} |u|^p\,dx + \frac{\lambda}{4}\int_{\mathbb{R}^N} \phi_{N,u}u^2\,dx - \int_{\mathbb{R}^N} G(x,u)\,dx,$$

where $G(x,t) = \int_0^t g(x,s)\,ds$. In the case $N \geq 3$, the energy functional I_N is well defined and of class C^1 in $H^1(\mathbb{R}^N)$. Numerous results are available for three-dimensional and higher-dimensional cases, and they can be grouped by the type of result. For existence, see [3, 4]. For nonexistence, see [5]. For multiplicity, see [6]. Further specific results can be found in the references cited in these works. In contrast to the classical case $p = 2$, when $p \neq 2$, the p-Laplacian is nonlinear and does not share the linear property of $-\Delta$. Meanwhile, the underlying Sobolev space $W^{1,p}(\mathbb{R}^N)$ is not a Hilbert space, and the well-known favorable properties of Hilbert spaces cannot be applied. For $p \neq 2$, Du et al. [7] first extended system (1.2) to the case involving the p-Laplacian, investigating the Schrödinger-Poisson system with the p-Laplacian in $\mathbb{R}^3$. They prove the existence of nontrivial solutions by employing the mountain pass theorem, combined with the Jeanjean minimax technique [8]. Later, extending their results to a more general nonlinearity, Chen and Tang [9] investigated sign-changing solutions to the p-Laplacian Schrödinger-Poisson equation. As the study of p-Laplacian equations has deepened, further progress has been made in this field. Liu and He [10] studied normalized solutions for the Sobolev critical Schrödinger-Poisson system with the p-Laplacian under a prescribed L^p-mass constraint. Next, Liang et al. [11] extended the p-Laplacian to the fractional double-critical Schrödinger-Poisson system and obtained the multiplicity and concentration behavior of normalized solutions. In [12], Ma et al. investigated the Schrödinger-Poisson system with doubly critical growth on the first Heisenberg group and established the existence of multiple nontrivial solutions. Moreover, He et al. [13] investigated a class of doubly critical fractional p-Schrödinger-Poisson systems with electromagnetic fields, and established the existence and multiplicity of weak solutions for the system. In particular, in [14], Benhana and Zhang combined the p-Laplacian with the Choquard nonlocal term and the Schrödinger-Poisson system, and extended system (1.2) to

$$\begin{cases} -\Delta_p u + \mu V(x)|u|^{p-2}u + \phi|u|^{r-2}u = (I_\alpha * |u|^q)|u|^{q-2}u + \nu f(x)|u|^{s-2}u, & \text{in } \mathbb{R}^N, \\ -\Delta\phi = |u|^r, & \text{in } \mathbb{R}^N, \end{cases} \tag{1.3}$$

where $N \geq 3$, $1 < s < 2 \leq p < N$, $\alpha \in (0, N)$, and $\frac{p}{2}(\frac{N+\alpha}{N}) < q < \frac{p}{2}(\frac{N+\alpha}{N-p})$, $\frac{p}{2} < r \leq q$. They proved that the system admits at least two nontrivial solutions via variational methods. For more references, see [15–17].

In recent years, increasing attention has been paid to the Schrödinger-Poisson system in the planar case $\mathbb{R}^2$. Compared with the case $N \geq 3$, the logarithmic integral kernel $\frac{1}{2\pi}\log|x|$ is sign-changing and unbounded, and the energy functional is not well defined on the natural Hilbert space $H^1(\mathbb{R}^2)$. To overcome this difficulty, Stubbe [18] introduced the small Hilbert space

$$\mathcal{X} = \left\{ u \in H^1(\mathbb{R}^2) : \int_{\mathbb{R}^2} \log(1+|x|)u^2(x)\,dx < \infty \right\},$$

equipped with the norm $\|u\|;_{\mathcal{X}} := \left(\int_{\mathbb{R}^2}(|\nabla u|^2 + [1+\log(1+|x|)]u^2)\,dx\right)^{\frac{1}{2}}$. This guarantees that the associated energy functional is well defined and of class C^1 on $\mathcal{X}$. Subsequently, inspired by [18], a growing number of scholars have obtained extensive results concerning system (1.2) in the two-dimensional setting within the variational framework of Stubbe. See [19–21] and the references therein. In particular, Chen and Pan [22] further studied the two-dimensional case where the nonlinearity is given by $g(x,u) = (I_\mu * F(x,u))f(x,u)$ and V(x) is axially symmetric. They introduced a weighted Hilbert space and applied variational methods to prove the existence of ground state solutions for this axially symmetric system. In the two-dimensional case, the Schrödinger-Poisson system involving the p-Laplacian has also received increasing attention, and several results have been established. Bucur et al. [23] studied the following strongly coupled system of quasilinear Schrödinger equations and higher-order fractional Poisson equations:

$$\begin{cases} -\Delta_m u + V(x)|u|^{m-2}u = f(u)v, & \text{in } \mathbb{R}^N,\ N \geq 2, \\ -\Delta^{\frac{\alpha}{2}} v = F(u), \end{cases} \tag{1.4}$$

where $m \geq 2$, $N \geq 2$. They proved the existence of a nontrivial mountain pass solution. Albuquerque et al. [24] studied a class of zero-mass Schrödinger-Poisson systems involving the (p,q)-Laplacian operator

$$\begin{cases} -\Delta u - \Delta_q u + \phi|u|^{r-2}u = \lambda|u|^{p-2}u, & \text{in } \mathbb{R}^2, \\ \Delta\phi = 2\pi|u|^r, & \text{in } \mathbb{R}^2, \end{cases} \tag{1.5}$$

where $\lambda \geq 0$, $1 < q < 2$, $q^* = \frac{2q}{2-q} < r < \infty$, and $p \geq 2r$, and they established the existence of nonnegative least energy solutions. Subsequently, they [25] extended system (1.5) to the case of a system of equations. Moreover, Cassani et al. [26] studied a class of nonlinear Schrödinger equations involving the fractional p-Laplacian strongly coupled with the Poisson equation

$$\begin{cases} (-\Delta)^{\frac{s}{2}}_{\frac{s}{2}} u + |u|^{\frac{2}{s}-2}u = \phi f(u), & \text{in } \mathbb{R}^2, \\ -\Delta\phi = F(u), & \text{in } \mathbb{R}^2, \end{cases} \tag{1.6}$$

where $s \in (0,1)$ and $p = \frac{2}{s}$. Clearly, $p > 2$. By using the variational approximation method, they proved the existence of positive solutions to the system, and by the moving planes method, they established their symmetry.

In spite of these developments, naturally, one may ask whether solutions exist for the p-Laplacian Schrödinger-Poisson system in the two-dimensional case with $p < 2$. To the best of our knowledge, for $p < 2$, the planar Schrödinger-Poisson system with the p-Laplacian and a Choquard-type nonlocal

term has not yet been studied in the literature. In this paper, we establish the existence of nontrivial solutions for system (1.1). The corresponding energy functional is given by

$$I(u) = \frac{1}{p}\int_{\mathbb{R}^2}(|\nabla u|^p + |u|^p)\,dx + \frac{1}{8\pi}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(|x-y|)u^2(x)u^2(y)\,dxdy - \frac{1}{2q}\int_{\mathbb{R}^2}(I_\alpha * |u|^q)|u|^q\,dx. \quad (1.7)$$

Undoubtedly, the following difficulties will be encountered: (i) the energy functional is not well defined on $W^{1,p}(\mathbb{R}^2)$; (ii) the combined effect of the sign-changing two-dimensional logarithmic kernel and the Choquard-type nonlinearity makes it more difficult to establish the boundedness of Palais-Smale sequences; (iii) the problem under consideration on the whole space involves a loss of compactness.

To overcome these issues, inspired by Stubbe [18], we first construct a new function space on which the energy functional is well defined, and establish a rigorous variational framework. Moreover, we propose a new scaling argument, construct an auxiliary functional, and employ the Hardy-Littlewood-Sobolev inequality to prove the boundedness of Palais-Smale sequences. In addition, by exploiting the symmetry of the Choquard term and the bounded linearity of the Riesz potential operator, we control the Choquard term in the limiting sense. Finally, motivated by the approach in [7], we combine the Boccardo-Murat gradient almost-everywhere convergence result with the vanishing lemma in [27] to establish the existence of a nontrivial solution to system (1.1).

We extend the existing results in the literature [7, 22]. Our main result can be stated as follows.

Theorem 1.1. *Assume that* $\frac{4}{3} \le p < 2$, $0 < \alpha < 2$, *and* $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < \frac{p(2+\alpha)}{2(2-p)}$. *Then problem* (1.1) *has at least one nontrivial solution.*

The paper is organized as follows. In Section 2, we give the variational framework for (1.1) and verify that the corresponding energy functional admits mountain-pass geometry. In Section 3, using suitable arguments, we establish the boundedness of the Palais-Smale sequence and give the main proof of the existence of nontrivial solutions for problem (1.1).

2. Variational framework

In this section, we build the variational framework for (1.1) and present some useful preliminary results. Let $W^{1,p} := W^{1,p}(\mathbb{R}^2)$ be the Sobolev space with the norm $\|u\|_{W^{1,p}} := \left(\int_{\mathbb{R}^2}(|\nabla u|^p + |u|^p)\,dx\right)^{\frac{1}{p}}$.

Lemma 2.1. *[28, Theorem 6.4.4] If* $\frac{4}{3} \le p < 2$, *the embedding of* $W^{1,p}(\mathbb{R}^2)$ *into* $L^s(\mathbb{R}^2)$ *is continuous for any* $p \le s < p^*$.

For any measurable function $u : \mathbb{R}^2 \to \mathbb{R}$, we define

$$\|u\|_* = \left(\int_{\mathbb{R}^2}\log(1+|x|)u^2\,dx\right)^{\frac{1}{2}} \in (0,\infty].$$

We shall work in the Banach space

$$E := \left\{u \in W^{1,p}(\mathbb{R}^2) : \|u\|_* < \infty\right\},$$

equipped with the norm $\|u\|_E := \|u\|_{W^{1,p}} + \|u\|_*$. In view of Lemma 2.1 and the definition of the space E, we have the embedding

$$E \hookrightarrow W^{1,p}(\mathbb{R}^2) \hookrightarrow L^s(\mathbb{R}^2), \quad \text{for all } \ p \le s < p^*. \quad (2.1)$$

Lemma 2.2. *The embedding $E \hookrightarrow L^s(\mathbb{R}^2)$ is compact for $2 \le s < p^*$.*

Proof. The proof procedure is analogous to that of Lemma 2.2 in reference [25]. Let $\{u_n\} \subset E$ be a bounded sequence such that $u_n \rightharpoonup 0$ in E. For any $R > 0$, we can write

$$\int_{\mathbb{R}^2} |u_n|^2 \, dx = \int_{B_R(0)} |u_n|^2 \, dx + \int_{\mathbb{R}^2 \setminus B_R(0)} |u_n|^2 \, dx.$$

Since $E \hookrightarrow W^{1,p}(\mathbb{R}^2) \hookrightarrow L^s(\mathbb{R}^2)$, we have

$$\int_{B_R(0)} |u_n|^2 \, dx = o_n(1), \text{as } n \to \infty.$$

Moreover,

$$\int_{\mathbb{R}^2 \setminus B_R(0)} |u_n|^2 \, dx \le \frac{1}{\log(1+R)} \int_{\mathbb{R}^2 \setminus B_R(0)} \log(1+|x|)|u_n|^2 \, dx \le \frac{1}{\log(1+R)} \|u_n\|_*^2. \tag{2.2}$$

Since $\{u_n\} \subset E$ is a bounded sequence, for any $\varepsilon > 0$ and sufficiently large $R > 0$ we have

$$\int_{\mathbb{R}^2 \setminus B_R(0)} |u_n|^2 \, dx \le \varepsilon.$$

Hence, the embedding $E \hookrightarrow L^2(\mathbb{R}^2)$ is compact. Using interpolation and (2.1), and choosing $t \in \mathbb{R}$ with $2 < s < t < \frac{2p}{2-p}$ such that $\frac{1}{s} = \frac{\theta}{2} + \frac{1-\theta}{t}$, we have

$$\|u_n\|_{L^s} \le \|u_n\|_{L^2}^{\theta} \|u_n\|_{L^t}^{1-\theta} \to o_n(1).$$

Therefore, the embedding $E \hookrightarrow L^s(\mathbb{R}^2)$ is compact for any $2 \le s < p^*$. □

For notational simplicity, we adopt the notation from [29, 30] and define

$$N_0(u) = A_0(u^2, u^2) = \frac{1}{2\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \log(|x-y|) u^2(x) u^2(y) \, dx \, dy.$$

Since $\log r = \log(1+r) - \log(1+\frac{1}{r})$ for all $r > 0$, we have $N_0(u) = N_1(u) - N_2(u)$, with

$$N_1(u) = A_1(u^2, u^2) = \frac{1}{2\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \log(1+|x-y|) u^2(x) u^2(y) \, dx \, dy,$$

and

$$N_2(u) = A_2(u^2, u^2) = \frac{1}{2\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \log(1+\frac{1}{|x-y|}) u^2(x) u^2(y) \, dx \, dy.$$

Note that, using $0 \le \log(1+r) \le r$ for $r > 0$, together with the Hardy-Littlewood-Sobolev inequality [31] and (2.1), we obtain

$$|N_2(u)| \le C_0 \Big| \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{1}{|x-y|} u^2(x) u^2(y) \, dxdy \Big| \le C_1 \|u\|_{\frac{4}{3}}^4, \quad \text{for a constant } C_1 > 0, \tag{2.3}$$

and

$$\int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^q \, dx \le C_4 \left(\int_{\mathbb{R}^2} |u|^{\frac{4q}{2+\alpha}} dx \right)^{\frac{2+\alpha}{2}} \le C_5 \|u\|^{2q}_{W^{1,p}}. \tag{2.4}$$

Moreover, the Hardy-Littlewood-Sobolev inequality, states that if $k \in (1, \frac{2}{\alpha})$, then for every $v \in L^k(\mathbb{R}^2)$, $I_\alpha * v \in L^{\frac{2k}{2-\alpha k}}(\mathbb{R}^2)$ and

$$\int_{\mathbb{R}^2} |I_\alpha * v|^{\frac{2k}{2-\alpha k}} dx \le C(\int_{\mathbb{R}^2} |v|^k dx)^{\frac{2}{2-\alpha k}}. \tag{2.5}$$

By using the same argument as in Lemma 2.2 of [32], we have that N_0, N_1, and N_2 are of class C^1 on E, and

$$\langle N_i'(u), v \rangle = 4A_i(u^2, uv) \ \ \forall\, u, v \in E,\ i = 1, 2. \tag{2.6}$$

From Eqs (2.1), (2.5), and the definition of the space E, we deduce that the functional

$$I(u) = \frac{1}{p} \int_{\mathbb{R}^2} (|\nabla u|^p + |u|^p) \, dx + \frac{1}{4} N_0(u) - \frac{1}{2q} \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^q \, dx,$$

is well defined and C^1 in E. Moreover, for any $u, v \in E$,

$$I'(u)[v] = \int_{\mathbb{R}^2} (|\nabla u|^{p-2} \nabla u \nabla v + |u|^{p-2} uv) dx + \int_{\mathbb{R}^2} \phi_{2,u} uv \, dx - \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^{q-2} uv \, dx.$$

Hence, the solutions of (1.1) are the critical points of the reduced functional (1.7).

Lemma 2.3. *[27, Lemma I.1] Let $1 < p \le \infty$, $1 \le r < \infty$ with $r \ne \frac{Np}{N-p}$ if $p < N$. Assume that $\{u_n\}$ is bounded in $L^r(\mathbb{R}^N)$, ∇u_n is bounded in $L^p(\mathbb{R}^N)$, and*

$$\sup_{y \in \mathbb{R}^N} \int_{B_R(r)} |u_n|^r \, dx \to 0, \quad \text{for some } R > 0.$$

Then $u_n \to 0$ in $L^\alpha(\mathbb{R}^N)$ as $n \to \infty$, for any $r < \alpha < \frac{Np}{N-p}$.

We add some observations on the functional geometry of I.

Lemma 2.4. *Suppose that q satisfies $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < \frac{p(2+\alpha)}{2(2-p)}$. Then the following hold:*
(i) Let $u \in E \setminus \{0\}$, and let $u_t \in E$ for $t > 0$. Then we have $I(u_t) < 0$ as $t \to \infty$;
(ii) There exists $\rho > 0$ such that

$$n_\beta = \inf \{I(u) : u \in E, \|u\|_{W^{1,p}} = \beta\} > 0 \quad \text{for } 0 < \beta \le \rho \tag{2.7}$$

and

$$m_\beta = \inf \{I'(u)u : u \in E, \|u\|_{W^{1,p}} = \beta\} > 0 \quad \text{for } 0 < \beta \le \rho. \tag{2.8}$$

Proof. (*i*) If $2 \le q < \frac{p(2+\alpha)}{2(2-p)}$, we consider $u \in E \setminus \{0\}$ and test the functional at $u_t = t^{\frac{4}{4-p}} u(tx)$. Then we have that

$$I(u_t) = \frac{t^{K_1}}{p} \int_{\mathbb{R}^2} |\nabla u|^p \, dx + \frac{t^{K_2}}{p} \int_{\mathbb{R}^2} |u|^p \, dx + \frac{t^{K_3}}{4} N_0(u) - \frac{t^{K_3}}{8\pi} \log t \|u\|_2^4 - \frac{t^{K_4}}{2q} \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^q \, dx,$$

where $K_1 = \frac{4p}{4-p} - 2 + p$, $K_2 = \frac{4p}{4-p} - 2$, $K_3 = \frac{16}{4-p} - 4$, and $K_4 = \frac{8q}{4-p} - (2+\alpha)$. As $\frac{4}{3} \le p < 2$ and $2 \le q < \frac{p(2+\alpha)}{2(2-p)}$, it follows that

$$K_2 < K_1 < K_3 < K_4. \tag{2.9}$$

Hence there exists a sufficiently large $t_0 > 0$ such that $I(u_{t_0}) < 0$.

Let $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < 2$. We consider $u \in E \setminus \{0\}$ and denote $u_t = t^{\frac{2-\alpha}{4-2q}} u(tx)$. Then we have that

$$I(u_t) = \frac{t^{R_1}}{p} \int_{\mathbb{R}^2} |\nabla u|^p \, dx + \frac{t^{R_2}}{p} \int_{\mathbb{R}^2} |u|^p \, dx + \frac{t^{R_3}}{4} N_0(u) - \frac{t^{R_3}}{8\pi} \log t \|u\|_2^4 - \frac{t^{R_4}}{2q} \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^q \, dx,$$

where $R_1 = \frac{p(2-\alpha)}{4-2q} - 2 + p, R_2 = \frac{p(2-\alpha)}{4-2q} - 2, R_3 = \frac{4(2-\alpha)}{4-2q} - 4, R_4 = \frac{2q(2-\alpha)}{4-2q} - 2 - \alpha$. Since $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < 2$, it follows that

$$R_2 < R_1 < R_3 = R_4; \tag{2.10}$$

therefore, there exists a sufficiently large $t_1 > 0$ such that $I(u_{t_1}) < 0$.

(*ii*) For each $u \in E$, by Eqs (2.1), (2.3), and (2.4), we derive that

$$\begin{aligned} I(u) &= \frac{1}{p} \int_{\mathbb{R}^2} (|\nabla u|^p + |u|^p) \, dx + \frac{1}{4} N_1(u) - \frac{1}{4} N_2(u) - \frac{1}{2q} \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^q \, dx \\ &\ge \frac{1}{p} \int_{\mathbb{R}^2} (|\nabla u|^p + |u|^p) \, dx - \frac{1}{4} N_2(u) - C_6 \frac{1}{2q} \Big(\int_{\mathbb{R}^2} |u|^{\frac{4q}{2+\alpha}} \, dx \Big)^{\frac{2+\alpha}{2}} \\ &\ge \frac{1}{p} \|u\|_{W^{1,p}}^p - C_7 \|u\|_{W^{1,p}}^4 - C_8 \|u\|_{W^{1,p}}^{2q}. \end{aligned} \tag{2.11}$$

As $\frac{4}{3} \le p < 2$ and $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q$, it is clear that $2q > p$. This implies that (2.7) holds for $\rho > 0$ sufficiently small.

Since

$$\langle I'(u), u \rangle = \int_{\mathbb{R}^2} (|\nabla u|^p + |u|^p) \, dx + N_1(u) - N_2(u) - \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^q \, dx,$$

for $u \in E$, similarly, to obtain the validity of Eq (2.8), we need to take $\rho > 0$ sufficiently small. □

Lemma 2.4 shows that the functional I satisfies the mountain-pass geometry. Accordingly, we may consider the mountain-pass level

$$c = \inf_{\gamma \in \Gamma} \max_{t \in [0,1]} I(\gamma(t)) > 0,$$

where

$$\Gamma = \{\gamma \in C([0,1], E) | \gamma(0) = 0, \ I(\gamma(1)) < 0\}.$$

3. Proof of main results

In the following lemma, we use the approach of [32] and the general minimax principle from Proposition 2.8 in [33] to construct an auxiliary functional and establish a Cerami sequence $\{u_n\}$ at the energy level c.

Lemma 3.1. *Set* $\frac{4}{3} \le p < 2$, $0 < \alpha < 2$, *and* $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < \frac{p(2+\alpha)}{2(2-p)}$. *Then there exists a sequence* $\{u_n\}$ *in* E *such that*

$$I(u_n) \to c > 0,\ \|I'(u_n)\|_{E^*}(1 + \|u_n\|_E) \to 0, \tag{3.1}$$

for which $J(u_n) \to 0$ *as* $n \to \infty$. *If* $2 \le q < \frac{p(2+\alpha)}{2(2-p)}$,

$$J(u_n) = \frac{K_1}{p}\int_{\mathbb{R}^2} |\nabla u_n|^p\,dx + \frac{K_2}{p}\int_{\mathbb{R}^2} |u_n|^p\,dx + \frac{K_3}{4}N_0(u_n) - \frac{1}{8\pi}\|u_n\|_2^4 - \frac{K_4}{2q}\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^q\,dx, \tag{3.2}$$

and, if $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < 2$,

$$J(u_n) = \frac{R_1}{p}\int_{\mathbb{R}^2} |\nabla u_n|^p\,dx + \frac{R_2}{p}\int_{\mathbb{R}^2} |u_n|^p\,dx + \frac{R_3}{4}N_0(u_n) - \frac{1}{8\pi}\|u_n\|_2^4 - \frac{R_4}{2q}\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^q\,dx. \tag{3.3}$$

Proof. The proof is analogous to that provided in Lemma 3.2 of [32]. We consider the space $\widetilde{E} := \mathbb{R}\times E$, equipped with the standard product norm $\|(t,v)\|_{\mathbb{R}\times E} := (|t|^2 + \|v\|_E^2)^{\frac{1}{2}}$, for $t \in \mathbb{R}$, $v \in E \setminus \{0\}$. Moreover, we define the continuous map η with

$$\eta : \mathbb{R}\times E \to E,\ \eta(t,v)(x) = \begin{cases} e^{\frac{4t}{4-p}}v(e^t x), & \text{if } 2 \le q < \frac{p(2+\alpha)}{2(2-p)}, \\ e^{\frac{(2-\alpha)t}{4-2q}}v(e^t x), & \text{if } 2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < 2. \end{cases} \tag{3.4}$$

Indeed, consider the function $\psi = I \circ \eta$. Then for any $(t,v) \in \mathbb{R}\times E$, if $2 \le q < \frac{p(2+\alpha)}{2(2-p)}$, it is obtained that

$$\psi(t,v) = \frac{e^{K_1 t}}{p}\int_{\mathbb{R}^2} |\nabla v|^p\,dx + \frac{e^{K_2 t}}{p}\int_{\mathbb{R}^2} |v|^p\,dx + \frac{e^{K_3 t}}{4}N_0(v) - \frac{e^{K_3 t}}{8\pi}t\|v\|_2^4 - \frac{e^{K_4 t}}{2q}\int_{\mathbb{R}^2} (I_\alpha * |v|^q)|v|^q\,dx, \tag{3.5}$$

for $t \in \mathbb{R}$ and $v \in E$. If $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < 2$, we deduce

$$\psi(t,v) = \frac{e^{R_1 t}}{p}\int_{\mathbb{R}^2} |\nabla v|^p\,dx + \frac{e^{R_2 t}}{p}\int_{\mathbb{R}^2} |v|^p\,dx + \frac{e^{R_3 t}}{4}N_0(v) - \frac{e^{R_3 t}}{8\pi}t\|v\|_2^4 - \frac{e^{R_4 t}}{2q}\int_{\mathbb{R}^2} (I_\alpha * |v|^q)|v|^q\,dx. \tag{3.6}$$

In any case, since for any $v \in E \setminus \{0\}$, (2.9) and (2.10) hold, from Eqs (3.5) and (3.6), we have $\lim_{t\to\infty} \psi(t,v) = -\infty$. Combining this with $(I \circ \eta)(0,0) = 0$, we define the minimax level

$$\bar{c} = \inf_{\widetilde{\gamma}\in\widetilde{\Gamma}} \max_{t\in[0,1]} (I \circ \eta)(\widetilde{\gamma}(t)), \tag{3.7}$$

where

$$\widetilde{\Gamma} = \{\widetilde{\gamma} \in C([0,1], \mathbb{R}\times E) : \widetilde{\gamma}(0) = (0,0), (I \circ \eta)(\widetilde{\gamma}(1)) < 0\}.$$

This allows us to repeat the arguments in Lemma 3.2 of [32] and conclude. □

In the following key step, we shall prove that there exists a bounded Palais-Smale sequence in the space E.

Lemma 3.2. *Let* $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < \frac{p(2+\alpha)}{2(2-p)}$, *and consider the sequence* $\{u_n\} \subset E$ *obtained in Lemma 3.1 satisfying* (3.1)–(3.3). *Then we obtain that the translated sequence of* $\{u_n\}$ *is bounded in the space* E, *and this translated sequence is a Palais-Smale sequence for the original functional* (1.7).

Proof. We distinguish two cases separately. If $2 \le q < \frac{p(2+\alpha)}{2(2-p)}$, by (3.1) and (3.2), we derive that for n large enough,

$$I(u_n) - \frac{1}{K_3}J(u_n) = \frac{K_3 - K_1}{pK_3}\int_{\mathbb{R}^2} |\nabla u_n|^p\,dx + \frac{K_3 - K_2}{pK_3}\int_{\mathbb{R}^2} |u_n|^p\,dx + \frac{1}{8\pi K_3}\|u_n\|_2^4 - \frac{K_3 - K_4}{2qK_3}\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^q\,dx.$$

If $2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < 2$, by (3.1) and (3.3), we derive that for n large enough,

$$I(u_n) - \frac{1}{R_3}J(u_n) = \frac{R_3 - R_1}{pR_3}\int_{\mathbb{R}^2} |\nabla u_n|^p\,dx + \frac{R_3 - R_2}{pR_3}\int_{\mathbb{R}^2} |u_n|^p\,dx + \frac{1}{8\pi R_3}\|u_n\|_2^4.$$

Since (2.9) and (2.10) hold, we can deduce that the sequence $\{u_n\}$ is bounded in $W^{1,p}(\mathbb{R}^2)$ and $L^s(\mathbb{R}^2)$ in both cases, for any $p \le s < p^*$.

Next, we shall prove that there exists a bounded Palais-Smale sequence in the space E. The proof makes reference to Lemma 3.1 in [25]. We first show that there exists $\delta > 0$ such that

$$\sup_{y\in\mathbb{R}^2}\int_{B_y(r)} |u_n|^2\,dx \ge \delta > 0, \quad n \to \infty. \tag{3.8}$$

Otherwise, we can apply Lemma 2.3 to get $u_n \to 0$ in $L^\alpha(\mathbb{R}^2)$ for any $2 < \alpha < \frac{2p}{2-p}$. By Eqs (2.3), (2.4), and the Sobolev inequality, we have

$$|N_2(u_n)| \le C_1\|u_n\|_{\frac{8}{3}}^4 \to o(1),\ |\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^q\,dx| \le C_4\|u_n\|_{L^{\frac{4q}{2+\alpha}}}^{2q} \to o(1), \quad \text{as } n \to \infty,$$

and we further obtain from (3.1) that

$$\int_{\mathbb{R}^2} (|\nabla u_n|^p + |u_n|^p)\,dx \to o(1),$$

and

$$\begin{aligned} c + o(1) &= I(u_n) - \frac{1}{4}I'(u_n)[u_n] \\ &= (\frac{1}{p} - \frac{1}{4})\int_{\mathbb{R}^2} (|\nabla u_n|^p + |u_n|^p)\,dx - (\frac{1}{2q} - \frac{1}{4})\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^q\,dx \to o(1), \end{aligned} \tag{3.9}$$

which is a contradiction. Thus, (3.8) holds. Hence, there exists $\{y_n\} \subset \mathbb{R}^2$ such that the shifted sequence $\widetilde{u}_n(x) = u_n(x - y_n)$ satisfies

$$\int_{B_R(0)} |\widetilde{u}_n|^2 dx \ge \frac{\delta}{2} > 0. \tag{3.10}$$

We claim that the sequence $\{\widetilde{u}_n\}$ is bounded in E. Since the functional I is invariant under $\mathbb{Z}^2$ – *translations*, $I(\widetilde{u}_n) = c + o(1)$. Since $\{\widetilde{u}_n\}$ is bounded in $W^{1,p}(\mathbb{R}^2)$, we can deduce that $N_1(\widetilde{u}_n)$ is bounded, and we can prove that $|\widetilde{u}_n|_* \le C_6$ for $C_6 > 0$ and any $n \in \mathbb{N}$.

Note that for any $x \in \mathbb{R}^2 \setminus B_{2R}(0)$ and $y \in B_R(0)$,

$$1 + |x - y| \geq 1 + |x| - |y| \geq 1 + |x| - |R| \geq 1 + \frac{|x|}{2} \geq (1 + |x|)^{\frac{1}{2}}.$$

There exists $C_7 > 0$ satisfying

$$\begin{aligned} C_7 \geq N_1(\widetilde{u}_n) &\geq \frac{1}{2\pi} \int_{\mathbb{R}^2 \setminus B_{2R}(0)} \int_{B_R(0)} \log(1 + |x|)\widetilde{u}_n^2(x)\widetilde{u}_n^2(y)\, dxdy \\ &\geq \frac{1}{2\pi} \int_{\mathbb{R}^2 \setminus B_{2R}(0)} \log(1 + |x|)\widetilde{u}_n^2(x)\, dx \cdot \frac{\delta}{2}. \end{aligned} \tag{3.11}$$

Applying (2.1), (3.10), and (3.11), we can conclude that

$$\int_{\mathbb{R}^2} \log(1 + |x|)\widetilde{u}_n^2(x)\, dx \leq \frac{4C_7}{\delta} + \int_{B_{2R}(0)} \log(1 + |x|)\widetilde{u}_n^2(x)\, dx \leq C_8. \tag{3.12}$$

Hence, the sequence $\{\widetilde{u}_n\}$ is bounded in the space E.

We can easily see that $I(\widetilde{u}_n) \to c > 0$. It remains to prove that for any test function $\eta \in E$, we have $\langle I'(\widetilde{u}_n), \eta\rangle \to 0$. By Lemma 3.1, we have a sequence $\{u_n\}$ that satisfies

$$I(u_n) \to c > 0 \text{ and } \|I'(u_n)\|_{E^*}(1 + \|u_n\|_E) \to 0. \tag{3.13}$$

For any $\{y_n\} \subset \mathbb{R}^2$, since (3.10) holds, we can obtain

$$\begin{aligned} [1 + \log(1 + |y_n|)]\delta &\leq \int_{B_R(0)} [1 + \log(1 + |y_n|)]\widetilde{u}_n^2(x)dx \\ &\leq \int_{B_R(0)} [1 + \log(1 + R) + \log(1 + |x - y_n|)]\widetilde{u}_n^2(x)dx \\ &\leq [1 + \log(1 + R)]\|\widetilde{u}_n\|_2^2 + \int_{\mathbb{R}^2} \log(1 + |x - y_n|)\widetilde{u}_n^2(x)dx \\ &\leq [1 + \log(1 + R)]\|u_n\|_2^2 + \int_{\mathbb{R}^2} \log(1 + |z|)u_n^2(z)dz \\ &\leq C[1 + \log(1 + R)]\|u_n\|_E^2. \end{aligned} \tag{3.14}$$

Thus, we can obtain that

$$\sqrt{1 + \log(1 + |y_n|)} \leq C_{15}\|u_n\|_E. \tag{3.15}$$

Since (3.15) holds, for any test function $\eta \in E$,

$$\begin{aligned} \|\eta(x + y_n)\|_E^2 &\leq 2(\|\eta(x + y_n)\|_{W^{1,p}(\mathbb{R}^2)}^2 + \int_{\mathbb{R}^2} \log(1 + |x|)\eta^2(x + y_n)dx) \\ &\leq 2(\|\eta\|_{W^{1,p}(\mathbb{R}^2)}^2 + \int_{\mathbb{R}^2} \log(1 + |z - y_n|)\eta^2(z)dz) \\ &\leq 2(\|\eta\|_{W^{1,p}(\mathbb{R}^2)}^2 + \|\eta\|_*^2 + \log(1 + |y_n|)\|\eta\|_2^2) \\ &\leq 2(\|\eta\|_E^2 + \log(1 + |y_n|)\|\eta\|_2^2) \\ &\leq C_{16}[1 + \log(1 + |y_n|)]\|\eta\|_E^2. \end{aligned} \tag{3.16}$$

Combining this with (3.15), we have

$$\|\eta(x+y_n)\|_E \le C_{17}\|u_n\|_E\|\eta\|_E. \tag{3.17}$$

Using (3.13) and (3.17), we can infer that

$$\begin{aligned} |\langle I'(\widetilde{u}_n),\eta\rangle| &= |\langle I'(u_n),\eta(x+y_n)\rangle| \\ &\le \|I'(u_n)\|_{E^*}\cdot\|\eta(x+y_n)\|_E \\ &\le C_{17}\|I'(u_n)\|_{E^*}\cdot\|u_n\|_E\cdot\|\eta\|_E \to 0. \end{aligned} \tag{3.18}$$

We can conclude $\langle I'(\widetilde{u}_n),\eta\rangle \to 0$. Thus, we obtain a bounded Palais-Smale sequence $\{\widetilde{u}_n\}$ satisfying

$$I(\widetilde{u}_n)\to c, I'(\widetilde{u}_n)\to 0. \tag{3.19}$$

Next, since the Palais-Smale sequence $\{\widetilde{u}_n\}$ is bounded, there exists $u\in E$ such that $\widetilde{u}_n \rightharpoonup u$ in E. We claim that $u\neq 0$. Otherwise, $u=0$, which implies that $\widetilde{u}_n \rightharpoonup 0$ in E, and by Lemma 2.2, (2.1), and (2.3), we can deduce that

$$|N_2(\widetilde{u}_n)|\to 0 \ \text{ and } \ \Big|\int_{\mathbb{R}^2}(I_\alpha * |\widetilde{u}_n|^q)|\widetilde{u}_n|^q\,dx\Big|\to 0 \quad \text{as } n\to\infty.$$

Moreover, since

$$N_1(\widetilde{u}_n)=\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)\widetilde{u}_n^2(x)\widetilde{u}_n^2(y)dxdy \ge \log 1\cdot\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\widetilde{u}_n^2(x)\widetilde{u}_n^2(y)dxdy\ge 0,$$

applying (3.19), we can easily infer that

$$\int_{\mathbb{R}^2}(|\nabla\widetilde{u}_n|^p+|\widetilde{u}_n|^p)dx\to 0 \ \text{ and } \ N_1(\widetilde{u}_n)\to 0.$$

It follows that

$$I(\widetilde{u}_n)=\frac{1}{p}\int_{\mathbb{R}^2}(|\nabla\widetilde{u}_n|^p+|\widetilde{u}_n|^p)\,dx+\frac{1}{4}N_1(\widetilde{u}_n)-\frac{1}{4}N_2(\widetilde{u}_n)-\frac{1}{2q}\int_{\mathbb{R}^2}(I_\alpha*|\widetilde{u}_n|^q)|\widetilde{u}_n|^q\,dx\to 0,$$

which contradicts the fact that $I(\widetilde{u}_n)\to c>0$. Therefore, $u\neq 0$. □

By Lemma 3.2, $\{\widetilde{u}_n\}$ is bounded in E. For convenience, we denote $\{\widetilde{u}_n\}$ simply as $\{u_n\}$. According to Lemma 3.2, it follows that

$$I(u_n)\to c, I'(u_n)\to 0. \tag{3.20}$$

Finally, to establish the existence of nontrivial solutions for system (1.1), we present the following lemma. In particular, $\{u_n\}$ used in the subsequent lemmas is the shorthand notation for $\{\widetilde{u}_n\}$.

Lemma 3.3. *Let $\{u_n\}$ be a bounded sequence in E satisfying $I(u_n)\to c\in\mathbb{R}$ and $I'(u_n)\to 0$, as $n\to\infty$. Then, up to a subsequence, there exists $u\in E$ such that*

$$\nabla u_n(x)\to\nabla u(x) \ \text{ a.e. in } \mathbb{R}^2, \text{ for } n\to\infty.$$

Proof. Let $\{u_n\} \subset E$ be a Palais-Smale sequence for the functional (1.7). By (2.1), there exists a subsequence, still denoted by $\{u_n\}$, and $u \in E$ such that

$$u_n \rightharpoonup u \text{ in } E,\ u_n \rightharpoonup u \text{ in } W^{1,p}(\mathbb{R}^2),\ u_n \rightharpoonup u \text{ in } L^s(\mathbb{R}^2)(p \le s < p^*),\ \nabla u_n \rightharpoonup \nabla u \text{ in } (L^p(\mathbb{R}^2))^2.$$

Since $I'(u_n) \to 0$, setting $v_n = u_n - u$, with $v_n \in E$, we have $I'(u_n)[u_n - u] \to 0$, and

$$\begin{aligned} I'(u_n)[u_n - u] = &\int_{\mathbb{R}^2} |\nabla u_n|^{p-2}\nabla u_n \nabla(u_n - u)\,dx + \int_{\mathbb{R}^2} |u_n|^{p-2}u_n(u_n - u)\,dx + \frac{1}{4}N_1'(u_n)(u_n - u) \\ &- \frac{1}{4}N_2'(u_n)(u_n - u) - \int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^{q-2}u_n(u_n - u)\,dx = o(1). \end{aligned} \tag{3.21}$$

Using Eq (3.21), we can obtain that

$$\begin{aligned} &\int_{\mathbb{R}^2} (|\nabla u_n|^{p-2}\nabla u_n - |\nabla u|^{p-2}\nabla u)\nabla(u_n - u)\,dx + \int_{\mathbb{R}^2} (|u_n|^{p-2}u_n - |u|^{p-2}u)(u_n - u)\,dx + \frac{1}{4}N_1'(u_n)(u_n - u) \\ &= -\int_{\mathbb{R}^2} |\nabla u|^{p-2}\nabla u \nabla(u_n - u)\,dx - \int_{\mathbb{R}^2} |u|^{p-2}u(u_n - u)\,dx + \frac{1}{4}N_2'(u_n)(u_n - u) \\ &\quad + \int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^{q-2}u_n(u_n - u)\,dx + o(1). \end{aligned} \tag{3.22}$$

By the Hardy-Littlewood-Sobolev inequality and Lemma 2.2, we have

$$|N_2'(u_n)(u_n - u)| \le C\|u_n\|_{\frac{8}{3}}^3 \|u_n - u\|_{\frac{8}{3}} = o(1). \tag{3.23}$$

Moreover, adapting part of the argument from Lemma 3.3 in [34], we have

$$\begin{aligned} N_1'(u_n)(u_n - u) &= \frac{2}{\pi}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1 + |x - y|)u_n^2(x)u_n(y)(u_n(y) - u(y))\,dx\,dy \\ &= \frac{2}{\pi}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1 + |x - y|)u_n^2(x)\,(u_n(y) - u(y))^2\,dx\,dy \\ &\quad + \frac{2}{\pi}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1 + |x - y|)u_n^2(x)u(y)\,(u_n(y) - u(y))\,dx\,dy \\ &\ge \frac{2}{\pi}\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1 + |x - y|)u_n^2(x)u(y)\,(u_n(y) - u(y))\,dx\,dy = o(1). \end{aligned} \tag{3.24}$$

Since Eq (2.5) holds, the Riesz potential is a bounded linear map from $L^{\frac{4}{2+\alpha}}(\mathbb{R}^2)$ to $L^{\frac{4}{2-\alpha}}(\mathbb{R}^2)$, and we have $I_\alpha * |u_n|^q \in L^{\frac{4}{2-\alpha}}(\mathbb{R}^2)$. Applying the Hölder inequality and Lemma 2.2, we obtain that

$$\begin{aligned} &\Big|\int_{\mathbb{R}^2} |u_n|^{\frac{4(q-1)}{2+\alpha}}|u_n - u|^{\frac{4}{2+\alpha}}dx\Big| \\ &\le \left(\int_{\mathbb{R}^2} |u_n|^{\frac{4q(q-1)}{(2+\alpha)(q-1)}}dx\right)^{\frac{q-1}{q}} \left(\int_{\mathbb{R}^2} |u_n - u|^{\frac{4q}{2+\alpha}}dx\right)^{\frac{1}{q}} \\ &\le C\|u_n - u\|_{\frac{4q}{2+\alpha}}^{\frac{4}{2+\alpha}} \to o(1), \end{aligned} \tag{3.25}$$

and

$$|u_n|^{q-2}u_n(u_n-u)\to 0 \text{ in } L^{\frac{4}{2+\alpha}}(\mathbb{R}^2). \tag{3.26}$$

By (3.26) and $I_\alpha * |u_n|^q \in L^{\frac{4}{2-\alpha}}(\mathbb{R}^2)$, using the Hölder inequality, we deduce the following:

$$\left|\int_{\mathbb{R}^2}(I_\alpha * |u_n|^q)|u_n|^{q-2}u_n(u_n-u)\,dx\right| \le \|(I_\alpha * |u_n|^q)\|_{L^{\frac{4}{2-\alpha}}}\||u_n|^{q-2}u_n(u_n-u)\|_{L^{\frac{4}{2+\alpha}}} \to o(1). \tag{3.27}$$

Moreover, by $\nabla u_n \rightharpoonup \nabla u$ in $(L^p(\mathbb{R}^2))^2$ and $u_n \rightharpoonup u$ in $L^p(\mathbb{R}^2)$, we have

$$\int_{\mathbb{R}^2}|\nabla u|^{p-2}\nabla u\nabla(u_n-u)\,dx \to o(1), \tag{3.28}$$

and

$$\int_{\mathbb{R}^2}|u|^{p-2}u(u_n-u)\,dx \to o(1). \tag{3.29}$$

Since (3.22)–(3.24) and (3.27)–(3.29) hold, we obtain

$$\int_{\mathbb{R}^2}(|\nabla u_n|^{p-2}\nabla u_n - |\nabla u|^{p-2}\nabla u)\nabla(u_n-u)\,dx \to o(1),$$

and by applying Theorem 2.1 of Boccardo and Murat in [35], we conclude that

$$\nabla u_n(x)\to\nabla u(x) \text{ a.e. in } \mathbb{R}^2, \text{ for } n\to\infty. \qquad \square$$

Lemma 3.4. *[29, Lemma 2.6] Let $\{u_n\}$, $\{v_n\}$, and $\{w_n\}$ be bounded sequences in E such that $u_n \rightharpoonup u$ weakly in E. Then, for any $\varphi \in E$, we have*

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)v_n(x)w_n(x)\varphi(y)\,(u_n(y)-u(y))\,dxdy\to 0, \text{ as } n\to\infty.$$

Lemma 3.5. *For any bounded sequence $\{u_n\}\subset E$ satisfying*

$$I(u_n)\to c>0, \quad I'(u_n)\to 0, \quad \text{as } n\to\infty,$$

there is $u\in E\setminus\{0\}$ such that $I'(u)=0$.

Proof. From (2.1) and Lemmas 2.2 and 3.2, we deduce that $\{u_n\}$ is bounded in E. Then there exists $u\in E$ such that up to a subsequence,

$$u_n\rightharpoonup u \text{ in } E,\ u_n\rightharpoonup u \text{ in } W^{1,p}(\mathbb{R}^2),\ u_n\to u \text{ in } L^s(\mathbb{R}^2)\ (2\le s<p^*),\ u_n\to u \text{ a.e. } \mathbb{R}^2. \tag{3.30}$$

Let $\varphi\in C_c^\infty(\mathbb{R}^2)$ be an arbitrary test function. $C_c^\infty(\mathbb{R}^2)$ is dense in the space E; see [18, 29]. Then by (3.30) and Lemma 3.3, using [28, Proposition 5.4.7], we have

$$\int_{\mathbb{R}^2}|\nabla u_n|^{p-2}\nabla u_n\nabla\varphi dx\to\int_{\mathbb{R}^2}|\nabla u|^{p-2}\nabla u\nabla\varphi dx \quad\text{and}\quad \int_{\mathbb{R}^2}|u_n|^{p-2}u_n\varphi dx\to\int_{\mathbb{R}^2}|u|^{p-2}u\varphi dx.$$

Next, since $\{u_n\}$ is bounded in the space E, in view of Lemmas 2.2, 2.3, and 3.4, borrowing the ideas from the proof of Lemma 2.6 in reference [29], we can prove

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)u_n^2(x)\varphi(y)u_n(y)\,dxdy\to\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)u^2(x)\varphi(y)u(y)\,dxdy, \tag{3.31}$$

and

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+\frac{1}{|x-y|})u_n^2(x)\varphi(y)u_n(y)\,dxdy\to\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+\frac{1}{|x-y|})u^2(x)\varphi(y)u(y)\,dxdy. \quad (3.32)$$

First, we prove (3.31). Let

$$B_n=\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)u_n^2(x)\varphi(y)u_n(y)\,dxdy, B=\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)u^2(x)\varphi(y)u(y)\,dxdy;$$

then

$$\begin{aligned}B_n-B&=\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)\varphi(y)u_n(y)\left(u_n^2(x)-u^2(x)\right)dxdy\\&\quad+\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)\varphi(y)u^2(x)\left(u_n(y)-u(y)\right)dxdy\\&=E_1+D_1.\end{aligned} \quad (3.33)$$

We need to prove $E_1, D_1\to o(1)$, as $n\to\infty$. For E_1,

$$u_n^2(x)-u^2(x)=\left(u_n(x)+u(x)\right)\left(u_n(x)-u(x)\right).$$

The sequence $\{u_n\}$ is bounded in the space E and $u_n\to u$ in $L^2(\mathbb{R}^2)$. Since $1+|x-y|\le(1+|x|)\cdot(1+|y|)$, we have

$$\begin{aligned}|E_1|&=|\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)\varphi(y)u_n(y)(u_n^2(x)-u^2(x))dxdy|\\&\le\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)|\varphi(y)u_n(y)(u_n^2(x)-u^2(x))|dxdy\\&\le\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x|)|\varphi(y)u_n(y)(u_n^2(x)-u^2(x))|dxdy\\&\quad+\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|y|)|\varphi(y)u_n(y)(u_n^2(x)-u^2(x))|dxdy.\end{aligned} \quad (3.34)$$

Let

$$\begin{aligned}E_{n,1}&=\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x|)|\varphi(y)u_n(y)(u_n^2(x)-u^2(x))|dxdy,\\E_{n,2}&=\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|y|)|\varphi(y)u_n(y)(u_n^2(x)-u^2(x))|dxdy.\end{aligned}$$

We will prove that $E_{n,1}\to 0$ and $E_{n,2}\to 0$ as $n\to\infty$. First, we prove $E_{n,2}\to 0$.

$$\begin{aligned}E_{n,2}&=\int_{\mathbb{R}^2}\log(1+|y|)|\varphi(y)u_n(y)|dy\int_{\mathbb{R}^2}|(u_n(x)-u(x))(u_n(x)+u(x))|dx\\&\le\|\varphi\|_*\|u_n\|_*\|u_n(x)-u(x)\|_2\|u_n(x)+u(x)\|_2\to 0, \text{as } n\to\infty.\end{aligned} \quad (3.35)$$

Next, we prove that $E_{n,1}\to 0$. Using the Hölder inequality, we know that

$$\begin{aligned}E_{n,1}&=\int_{\mathbb{R}^2}\varphi(y)|u_n(y)|dy\int_{\mathbb{R}^2}\log(1+|x|)|(u_n(x)-u(x))(u_n(x)+u(x))|dx\\&\le C_{14}\int_{\mathbb{R}^2}\log(1+|x|)|(u_n(x)-u(x))(u_n(x)+u(x))|dx.\end{aligned} \quad (3.36)$$

For $\int_{\mathbb{R}^2} \log(1+|x|)|(u_n(x)-u(x))(u_n(x)+u(x))|dx$, we treat separately the regions inside and outside the ball. First, we deal with the case inside the ball; for any $R > 0$,

$$\int_{B_R} \log(1+|x|)|(u_n(x)-u(x))(u_n(x)+u(x))|dx \le \log(1+|R|)\|(u_n(x)-u(x))\|_2\|u_n(x)+u(x)\|_2 \to 0. \quad (3.37)$$

Next, we deal with the case outside the ball. Applying the Hölder inequality, we can obtain

$$\begin{aligned} &\int_{\mathbb{R}^2\setminus B_R} \log(1+|x|)|(u_n(x)-u(x))(u_n(x)+u(x))|dx \\ &\le (\int_{\mathbb{R}^2\setminus B_R} \log(1+|x|)(u_n(x)-u(x))^2dx)^{\frac{1}{2}}(\int_{\mathbb{R}^2\setminus B_R} \log(1+|x|)(u_n(x)+u(x))^2dx)^{\frac{1}{2}} \\ &\le C_{15}(\int_{\mathbb{R}^2\setminus B_R} \log(1+|x|)(u_n(x)-u(x))^2dx)^{\frac{1}{2}}. \end{aligned} \quad (3.38)$$

Thus, we need to show that $\|u_n(x)-u(x)\|_* \to 0$ as $n \to \infty$. Combining (3.21), (3.23), (3.24), and (3.27), we obtain that

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x-y|)u_n^2(x)u_n(y)(u_n(y)-u(y))dxdy \to 0.$$

Since

$$\begin{aligned} &\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x-y|)u_n^2(x)u_n(y)(u_n(y)-u(y))dxdy \\ =& \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x-y|)u_n^2(x)(u_n(y)-u(y))^2dxdy \\ &+ \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x-y|)u_n^2(x)u(y)(u_n(y)-u(y))dxdy, \end{aligned} \quad (3.39)$$

we can easily deduce that

$$\begin{aligned} &|\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x-y|)u_n^2(x)u(y)(u_n(y)-u(y))dxdy| \\ \le& \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x-y|)|u_n^2(x)u(y)(u_n(y)-u(y))|dxdy \\ \le& \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x|)|u_n^2(x)u(y)(u_n(y)-u(y))|dxdy \\ &+ \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|y|)|u_n^2(x)u(y)(u_n(y)-u(y))|dxdy. \end{aligned} \quad (3.40)$$

Using the Hölder inequality, we can easily obtain that

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \log(1+|x|)|u_n^2(x)u(y)(u_n(y)-u(y))|dxdy \to 0. \quad (3.41)$$

Using Hölder's inequality, noting that $u_n \to u$ in $L^2(\mathbb{R}^2)$, and taking an arbitrary $R > 0$, we treat the

problem separately for the interior and exterior of the ball.

$$\begin{aligned}
&\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|y|)|u_n^2(x)u(y)(u_n(y)-u(y))|dxdy\\
&=\int_{\mathbb{R}^2}u_n^2(x)dx\int_{\mathbb{R}^2}\log(1+|y|)|u(y)(u_n(y)-u(y))|dy\\
&\le C_{17}(\int_{B_R}\log(1+|y|)|u(y)(u_n(y)-u(y))|dy+\int_{\mathbb{R}^2\setminus B_R}\log(1+|y|)|u(y)(u_n(y)-u(y))|dy)\to 0.
\end{aligned}\tag{3.42}$$

Thus, by (3.39)–(3.42), we know that

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x-y|)u_n^2(x)(u_n(y)-u(y))^2dxdy\to 0.$$

Using Lemma 2.15 in [34], we obtain that $\|u_n-u\|_*\to 0$. Thus, (3.38) yields

$$\int_{\mathbb{R}^2\setminus B_R}\log(1+|x|)|(u_n(x)-u(x))(u_n(x)+u(x))|dx\to 0.\tag{3.43}$$

Combining (3.37) and (3.43), we can conclude that $E_{n,1}\to 0$. Since $E_{n,2}\to 0$ holds, we can infer that $E_1\to 0$.

Further, we claim that $D_1\to o(1)$ as $n\to\infty$. The argument follows the ideas in the proof of Lemma 2.6 of reference [29]. Indeed, using the boundedness of the subsequence $\{u_n\}$ in the space E, together with Lemma 2.3 and Eq (2.1), we have

$$\begin{aligned}
|D_1|\le&\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x|)|u^2(x)\varphi(y)\,(u_n(y)-u(y))\,|dxdy\\
&+\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|y|)|u^2(x)\varphi(y)\,(u_n(y)-u(y))\,|dxdy,
\end{aligned}\tag{3.44}$$

where

$$\begin{aligned}
&\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|x|)|u^2(x)\varphi(y)(u_n(y)-u(y))|dxdy\\
&=\int_{\mathbb{R}^2}\log(1+|x|)u^2(x)dx\int_{\mathbb{R}^2}|\varphi(y)(u_n(y)-u(y))|dy\\
&=\|u\|_*^2\int_{\mathbb{R}^2}|\varphi(y)(u_n(y)-u(y))|dy\\
&\le C_{12}\|\varphi(y)\|_2\|u_n-u\|_2\to o(1).
\end{aligned}\tag{3.45}$$

There exists an arbitrarily large radius $R>0$, using (2.1) and $\varphi(y)\in C_c^\infty(\mathbb{R}^2)$, such that

$$\begin{aligned}
&\int_{\mathbb{R}^2}\int_{\mathbb{R}^2}\log(1+|y|)|u^2(x)\varphi(y)\,(u_n(y)-u(y))\,|dxdy\\
&=C_{12}\int_{B_R(0)}\log(1+|y|)|\varphi(y)\,(u_n(y)-u(y))\,|dy+C_{13}\int_{\mathbb{R}^2\setminus B_R(0)}\log(1+|y|)|\varphi(y)\,(u_n(y)-u(y))\,|dy\\
&\le o(1)+\left(\int_{\mathbb{R}^2\setminus B_R(0)}\log(1+|y|)\varphi^2(y)\,dy\right)^{\frac{1}{2}}\left(\int_{\mathbb{R}^2\setminus B_R(0)}\log(1+|y|)|u_n-u|^2\,dy\right)^{\frac{1}{2}}\\
&\to o(1).
\end{aligned}\tag{3.46}$$

Thus, we obtain that $D_1 \to o(1)$ as $n \to \infty$, and conclude that (3.31) holds. The proof of (3.32) is similar to that of (3.31).

Moreover, using Eqs (2.1), (2.5), and Lemma 2.2, we employ a proof procedure similar to that of Theorem 1.1 in reference [36] to show that

$$\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^{q-2}u_n\varphi\, dx \to \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^{q-2}u\varphi\, dx.$$

Indeed, we know that $\{u_n\}$ is bounded in the space E, $u_n \rightharpoonup u$ in E, and $u_n(x) \to u(x)$ a.e. in $\mathbb{R}^2$. By Lemma 2.1, $\{u_n\}$ is bounded in $L^{\frac{4q}{2+\alpha}}(\mathbb{R}^2)$; therefore, the sequence $\{|u_n|^q\}$ is bounded in $L^{\frac{4}{2+\alpha}}(\mathbb{R}^2)$. Applying Lemma 2.2, we obtain that $u_n \to u$ strongly in $L_{loc}^{\frac{4q}{2+\alpha}}(\mathbb{R}^2)$ and $|u_n|^{q-2}u_n \to |u|^{q-2}u$ strongly in $L_{loc}^{\frac{4q}{(2+\alpha)(q-1)}}(\mathbb{R}^2)$ for $\frac{2+\alpha}{2} < 2 + \frac{(\alpha-2)(4-p)}{2(p+2)} < q < \frac{p(2+\alpha)}{2(2-p)}$ (see Theorem A.2 in [37]). Hence, $|u_n|^{q-2}u_n\varphi \to |u|^{q-2}u\varphi$ in $L^{\frac{4}{2+\alpha}}(\mathbb{R}^2)$ for any $\varphi \in C_c^\infty(\mathbb{R}^2)$. Since Eq (2.5) holds, the Riesz potential is a bounded linear map from $L^{\frac{4}{2+\alpha}}(\mathbb{R}^2)$ to $L^{\frac{4}{2-\alpha}}(\mathbb{R}^2)$; we have

$$I_\alpha * (|u_n|^{q-2}u_n)\varphi \to I_\alpha * (|u|^{q-2}u)\varphi$$

in $L^{\frac{4}{2-\alpha}}(\mathbb{R}^2)$. Since $C_c^\infty(\mathbb{R}^2)$ is dense in the space E, we can obtain

$$\begin{aligned}
&\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^{q-2}u_n\varphi dx - \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^{q-2}u\varphi dx \\
&= \int_{\mathbb{R}^2} |u_n|^q (I_\alpha * (|u_n|^{q-2}u_n\varphi)dx - \int_{\mathbb{R}^2} |u|^q (I_\alpha * (|u|^{q-2}u\varphi)dx \\
&= \int_{\mathbb{R}^2} |u_n|^q (I_\alpha * (|u_n|^{q-2}u_n\varphi) - I_\alpha * (|u|^{q-2}u\varphi))dx + \int_{\mathbb{R}^2} (|u_n|^q - |u|^q)(I_\alpha * (|u|^{q-2}u\varphi))dx \\
&= F + G.
\end{aligned}$$

Next we prove that $F, G \to o(1)$ as $n \to \infty$.

Since $\{u_n\}$ is bounded in the space E, the Hölder inequality yields

$$\begin{aligned}
F &= \int_{\mathbb{R}^2} |u_n|^q (I_\alpha * (|u_n|^{q-2}u_n\varphi) - I_\alpha * (|u|^{q-2}u\varphi))dx \\
&\le \left(\int_{\mathbb{R}^2} |u_n^q|^{\frac{4}{2+\alpha}} dx \right)^{\frac{2+\alpha}{4}} ||(I_\alpha * (|u_n|^{q-2}u_n\varphi) - I_\alpha * (|u|^{q-2}u\varphi))||_{\frac{4}{2-\alpha}} \\
&\to o(1).
\end{aligned}$$

Using the strong convergence $u_n \to u$ in $L^{\frac{4q}{2+\alpha}}(\mathbb{R}^2)$ and Eq (2.5), we obtain

$$\begin{aligned}
G &= \int_{\mathbb{R}^2} (|u_n|^q - |u|^q)(I_\alpha * (|u|^{q-2}u\varphi))dx \\
&\le ||||u_n|^q - |u|^q||_{\frac{4}{2+\alpha}} ||I_\alpha(*|u|^{q-2}u\varphi)||_{\frac{4}{2-\alpha}} \to o(1).
\end{aligned}$$

That is,

$$\int_{\mathbb{R}^2} (I_\alpha * |u_n|^q)|u_n|^{q-2}u_n\varphi\, dx \to \int_{\mathbb{R}^2} (I_\alpha * |u|^q)|u|^{q-2}u\varphi\, dx.$$

As a result, we deduce that $I'(u)[\varphi] = 0$ for any $\varphi \in C_c^\infty(\mathbb{R}^2)$. □

Proof of Theorem 1.1. First, from Lemma 2.4, we prove that the associated energy functional (1.7) admits the classical mountain-pass structure, and we construct a Cerami sequence $\{u_n\}$ by virtue of Lemma 3.1. Second, in Lemma 3.2, we prove that the Cerami sequence $\{u_n\}$ constructed in Lemma 3.1 is bounded in $W^{1,p}(\mathbb{R}^2)$. After that, we show that the translated sequence $\{\widetilde{u}_n\}$ of this Cerami sequence $\{u_n\}$ is a bounded Palais-Smale sequence in the space E, satisfying

$$I(\widetilde{u}_n) \to c,\ I'(\widetilde{u}_n) \to 0.$$

Furthermore, we denote the Palais-Smale sequence $\{\widetilde{u}_n\}$ simply by $\{u_n\}$. Finally, by Lemma 3.5, there exists a function $u \in E$ with $u \neq 0$ such that the Palais-Smale sequence satisfies $u_n \rightharpoonup u$ in E and $I'(u)[\varphi] = 0$, so we conclude that u is a nontrivial solution to (1.1). The proof is completed. □

4. Conclusions

In this paper, we employ the variational method to investigate the existence of nontrivial solutions for a two-dimensional p-Laplacian Schrödinger-Poisson system (1.1) with a Choquard-type nonlocal term and $p < 2$. The key to the proof is to construct an appropriate variational framework. By combining the properties of the Choquard-type nonlocal term and the logarithmic kernel with scaling arguments, we build an auxiliary functional ((3.2) and (3.3)) and further establish the boundedness of Palais-Smale sequences. Next, combining the Boccardo-Murat gradient almost everywhere convergence result and the vanishing lemma, we obtain the existence of nontrivial solutions for system (1.1).

Author contributions

Jin Deng: Conceptualization, methodology, supervision, writing-review, and editing. Yuqing Guo: Theoretical derivation, analysis, and verification, validation, writing-original draft. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that no conflicts of competing interests exist.

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