AIMS Mathematics, 11(11): 37158–37185.
DOI:10.3934/math.20261436
Received: 10 July 2026
Revised: 27 September 2026
Accepted: 14 October 2026
Published: 10 November 2026

https://www.aimspress.com/journal/Math

Research article

Semantics of target argumentation: a calculation of target strength based on attack paths

Yunjie Zhang and Jiachao Wu*

School of Mathematics and Statistics, Shandong Normal University, No.1 University Road, Jinan 250358, China

* **Correspondence:** Email: wujiachao1981@163.com.

Abstract: Traditional gradual semantics in argumentation frameworks typically relies on global computation, resulting in significant computational redundancy and topological instability. To address these limitations, we focused on the evaluation of a specific target argument and established the semantics for a localized Target Argumentation Framework (TAF). First, we formulated a set of core local axioms for TAF to regulate localized gradual semantics. Second, we decomposed the direct attacks on the target into independent evaluation dimensions and employed backward tracing along specific attack paths. Finally, we designed a path-based exact algorithm to calculate the strength of the target argument. By restricting the evaluation to relevant ancestral subgraphs, this algorithm successfully simplified the computation from global iterations to local path tracing. Furthermore, the proposed TAF theory provided solid theoretical guarantees for semantic rationality, system robustness, and reasoning interpretability.

Keywords: abstract argumentation; gradual semantics; target argumentation; attack paths; dimensional decomposition
Mathematics Subject Classification: 68T27

Notations	
$AF = \langle Ar, att \rangle$	Classical Abstract Argumentation Framework
$TAF = \langle Ar, att, w, Trg \rangle$	Target Argumentation Framework with target set $Trg \subseteq Ar$
$w(x) \in [0, 1]$	Initial base weight (degree of belief) of argument x
$Att(t) = \{d \in Ar \mid (d, t) \in att\}$	Set of direct attackers of target argument t
$Dim(d_i, t)$	Dimension sub-framework induced by direct attacker d_i
$P = (x_k, x_{k-1}, \ldots, x_1, t)$	Simple attack path of length $L(P) = k$ terminating at t
$P_i = (x_i, \ldots, x_1, t)$	Terminal suffix sub-path of length i originating at x_i and ending at t

Continued on the next page

Notations	
$Paths(d_i, t)$	Set of all valid attack paths in dimension $Dim(d_i, t)$
$Paths_{pruned}(d_i, t)$	Set of attack paths truncated at upstream target arguments
$F(x_k, t, P) \in \mathbb{R}$	Signed influence of path P on target argument t
$\|F(x_i, t, P_i)\|$	Absolute recursive influence along suffix path P_i of length i
$Imp(d_i, t) \in \mathbb{R}$	Cumulative dimensional net influence of $Dim(d_i, t)$ on t
$Val(t) \in [0, w(t)]$	Final evaluated acceptability strength of target argument t
$\alpha, \gamma \in (0, 1)$	Basic decay factor and sequential dampening factor
b, D	Maximum ancestor branching factor (in-degree) and maximum path depth

1. Introduction

As a core formal tool in non-monotonic and uncertainty reasoning, abstract argumentation frameworks have become a crucial foundation for modeling conflicting reasoning and supporting intelligent decision-making in artificial intelligence since they were first proposed by Dung [1]. In early studies, researchers primarily focused on extension-based semantics proposed by [2, 3], which yield binary or discrete reasoning outcomes by determining the acceptability of argument sets. To accommodate more complex scenarios of reasoning under uncertainty, related research has gradually evolved toward a fine-grained, quantitative direction, giving rise to the sub-branch of gradual semantics [4–6]. Gradual semantics can output continuous strength values [7–9] for each argument, finding applications in domains such as legal reasoning [10–12], multi-agent negotiation [13, 14]. Furthermore, advances in semantic-level reasoning and agentic AI have demonstrated strong potential in distributed intelligent networks and cross-domain optimization [15, 16], further highlighting the necessity of efficient and localized uncertainty evaluation and evidence-based medicine [17]. In particular, [18] explicitly noted that argumentation frameworks can provide the core formal support for clinical decision-making within EBM scenarios. Additionally, [10, 19, 20] studied the inverse problem of gradual semantics.

Despite their expressiveness, gradual semantics typically adhere to a global computation paradigm [4, 5, 21]: Even if a user is solely interested in evaluating the strength of a few specific target arguments, a comprehensive traversal of all nodes across the argumentation graph remains mandatory. This global computational mode suffers from two key limitations:

First, it incurs significant computational redundancy. Even when many arguments carry no direct relevance to the final target decision, their strengths must be fully computed merely because they reside in the broader network, leading to unnecessary computational overhead. This complexity issue becomes particularly pronounced in large-scale argumentation graphs containing hundreds or thousands of nodes, which are increasingly common in modern argument mining and evaluation applications [22, 23].

Second, the global computation process lacks sufficient stability. Most classic abstract argumentation frameworks [1, 24, 25] are inherently non-monotonic and highly sensitive to topological changes; introducing even a single new argument can trigger a cascading effect that alters the status of distant nodes. Under gradual semantics [5, 26, 27], accommodating such dynamic updates requires re-evaluating the entire framework. This yields redundant strength values for numerous intermediate nodes that are of no interest to the end-user, thereby wasting valuable computational resources and cognitive effort on interpreting irrelevant outcomes.

To illustrate this within a clinical decision-making scenario in medicine, let us consider the following set of arguments regarding whether to perform surgery on Jack:

- a: Jack should undergo surgery.
- b: Jack has a history of allergy to essential surgical medications.
- c: Jack has recently consumed alcohol, and this surgery is not recommended post-alcohol consumption.
- d: Jack consumed only a small amount of alcohol, and it was several days ago.
- e: Jack's current allergy test shows no reaction to the medication.
- f: The recent allergy test has been reported to have issues.

In reasoning about whether to perform the surgery, the structure constitutes an argumentation framework $AF = (\{a, b, c, d, e, f\}, \{(b, a), (c, a), (d, c), (e, b), (f, e)\})$, as depicted in Figure1. In this scenario, the sole core objective of clinical decision-making is to determine the acceptability strength of argument a, while all other arguments serve merely as auxiliary nodes to support the calculation of a's strength. For instance, argument f does not directly impact the final decision; instead, it attacks e, which in turn influences the credibility of b, thereby acting on a. Moreover, global computation paradigms in gradual semantics require sequentially calculating the strengths of f, e, b, d, and c before obtaining the final result for a. Moreover, they output the strength values for all of these intermediate nodes. However, these intermediate outcomes are of no practical value for clinical decision-making, unnecessarily increasing computational overhead and the cost of result interpretation. This issue has become one of the primary obstacles to deploying argumentation frameworks in real-world applications such as evidence-based medicine.

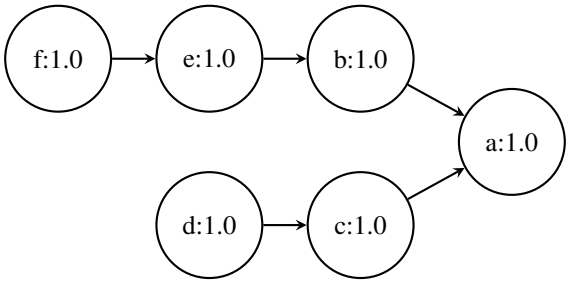


Figure 1. The TAF argumentation graph for Jack's clinical surgery decision.

To address these issues, researchers have conducted preliminary explorations. Regarding the demand for target-oriented argumentation evaluation, the researchers in [28] focused on the contribution of arguments to target outcomes and proposed a Shapley value-based method to quantify the influence of each argument on the target argument's strength. This provides an important perspective for non-global argumentation evaluation; however, computing Shapley values inherently requires enumerating all subsets of arguments, and its exponential computational complexity makes it difficult to scale to medium-to-large-scale argumentation frameworks in practical scenarios. On the other hand, research on the rationality of gradual semantics has established a mature axiomatic foundation. For instance, [24, 29, 30] systematically reviewed the core principles that gradual semantics should satisfy, clarifying

the basic rules governing how argument strength varies under attack and defense relations [14, 27, 31], which offers a unified validation standard for the rationality of various gradual semantics and some articles on the structural analysis and labeling of arguments [2, 32, 33]. Nevertheless, axiomatic systems are designed for global computation scenarios involving all arguments across the graph. They lack adaptive designs tailored to target-oriented, lightweight computation, rendering them directly inapplicable for guiding the development of target-oriented semantics. Despite the existence of studies on improving efficiency [34–36], there remains a lack of a progressive semantic approach capable of simultaneously ensuring semantic plausibility and achieving efficient computation of target arguments with low redundancy.

To fill this research gap, we focus on the demand for evaluating the strength of target arguments, proposing a target-oriented gradual semantics computation method based on dimensional attack paths. We first construct a target argumentation framework and outline the relevant theories; subsequently, we propose a multidimensional strategy utilizing the direct attack relations within that framework; finally, based on this multidimensional approach, we present an algorithm applicable to acyclic argumentation graphs.

The main contributions of this paper are twofold:

- The goal-based argumentation framework and its axiomatic system proposed in this paper enrich and extend classical abstract argumentation theory, while ensuring semantic soundness, system robustness, and interpretability in the evaluation process.

- We introduce a dimensional decomposition mechanism and a path-based exact algorithm. By strictly limiting the evaluation scope to the subgraph of the goal's ancestors and decomposing attack relations into mutually independent, traceable paths, the method completely avoids redundant global graph iterations. This local tracing paradigm simplifies the evaluation process, significantly reducing computational overhead while enhancing the transparency of the reasoning process.

The remainder of this paper is organized as follows: In Section 2, we review the basic definitions of classical abstract argumentation frameworks and weighted argumentation frameworks, and introduce the relevant definitions for target argumentation frameworks. In Section 3, we explore the boundaries of rationality for target argumentation semantics, define an influence function, and propose an axiomatic system. In Section 4, we instantiate a specific exponential decay influence function and provide a formula for calculating target argumentation strength. In Section 5 , we systematically analyze and prove several theoretical properties satisfied by the proposed semantics. Finally, in Section 6 and 7, we summarize the work presented in the paper.

2. Preliminaries

In this section, we review the basic concepts of the Abstract Argumentation Framework and the Weighted Argumentation Framework. On this basis, we formally introduce the core structure of this paper: the Target Argumentation Framework (TAF), along with its related definitions of paths and distance. Throughout this paper, the term target argument (denoted by $t \in Trg$) is strictly used in all formal semantics, axiomatic principles, and theoretical statements, emphasizing its nature as an epistemic reason in abstract argumentation. We reserve the graph-theoretic term **node** (or vertex)

exclusively when discussing structural graph properties and algorithmic graph traversals (e.g., in-degree, ancestor subgraphs, and DFS simple-path enumeration).

2.1. Dung's abstract argumentation framework

The cornerstone of computational argumentation is the Abstract Argumentation Framework proposed by P.M. Dung in 1995 [1]. In this framework, the internal structure of arguments is ignored, and the focus is on the attack relations between arguments.

Definition 2.1 (Abstract Argumentation Framework AF)**.** An Abstract Argumentation Framework (AF) is a tuple $AF = \langle Ar, att \rangle$, where Ar is a finite set of arguments and $att \subseteq Ar \times Ar$ is a binary attack relation on Ar. If $(a, b) \in att$, we say that the argument a attacks the argument b.

In the interactions within an argumentation graph, because the attack relations are directional, when multiple attack relations are linked head-to-tail, the semantic concept of defense (*Defend*) arises:

Definition 2.2 (Defend)**.** Given an argumentation framework $AF = \langle Ar, att \rangle$ and arguments $a, b, c \in Ar$. If $(b, c) \in att$ and $(a, b) \in att$, i.e., argument a attacks the argument b that attacks argument c, then argument a is said to defend argument c.

According to the aforementioned definition of defense, we can derive the parity transformation properties of argumentation relations during propagation. For any target argument t:

- Nodes at a distance of 1 from t (such as $b \to t$) constitute a direct attack on t.
- Nodes at a distance of 2 from t (such as $a \to b \to t$) constitute a defense of t by defending against the attacker.

In a branchless argumentation chain, arguments at an odd distance from the target argument always perform the role of an **attack (negative influence)**, whereas arguments at an even distance always perform the role of a **defense (positive influence)**.

A set of arguments $S \subseteq Ar$ is conflict-free if there are no $a, b \in S$ such that $(a, b) \in att$. An argument a is defended by S if S attacks all attackers of a. Classical extension-based semantics (such as grounded, complete, preferred, and stable extensions) evaluate acceptable sets of arguments based on these notions [1]. Since this work focuses on continuous and gradual semantics in weighted settings, we refer readers to [1, 3] for exhaustive formulations of extension semantics and proceed directly to weighted and targeted frameworks.

2.2. Weighted argumentation framework and target argumentation framework

In the classical Dung framework, all arguments are treated as equally credible in their initial state. However, in real-world scenarios such as medical decision-making, different pieces of evidence (arguments) often possess different initial strengths or degrees of belief. To address this, researchers have introduced the Weighted Argumentation Framework.

Definition 2.3 (Weighted Argumentation Framework)**.** A Weighted Argumentation Framework (WAF) is a triple $WAF = \langle Ar, att, w \rangle$, where $\langle Ar, att \rangle$ is a standard abstract argumentation framework, and $w : Ar \to [0, 1]$ is a weight mapping function that assigns an initial weight (or basic strength) to each argument.

In WAFs, gradual semantics typically require calculating the final acceptability of every argument in the graph using iterative equations. However, when the argumentation graph is extremely large and complex, such global computation is not only resource-intensive but also prone to triggering cascading fluctuations in global strength when new arguments are introduced at remote ends of the graph. Furthermore, in practical applications, decision-makers are usually concerned only with the acceptability of specific arguments (such as "whether to undergo surgery"). Auxiliary arguments serve merely as supporting tools for reasoning about this target argument. To address these issues, we introduce the target-oriented argumentation framework.

Definition 2.4 (Target Argumentation Framework TAF)**.** A Target Argumentation Framework (TAF) is a quadruple $TAF = \langle Ar, att, w, Trg \rangle$, where:

- Ar is a finite set of arguments;
- $att \subseteq Ar \times Ar$ represents the attack relations between arguments;
- $w : Ar \to [0, 1]$ represents the initial weights of the arguments;
- $Trg \subseteq Ar$ represents the set of target arguments (Target Arguments), which contains the arguments that are primarily of interest and whose final strengths need to be evaluated in the system.

When $Trg = Ar$, every argument is selected for evaluation. The underlying weighted framework is $\langle Ar, att, w \rangle$; this choice does not imply equivalence with an existing WAF semantics.

Unless an asymptotic extension is explicitly discussed, the semantics and algorithms developed below are restricted to finite directed acyclic attack graphs. Thus, (Ar, att) is a DAG. Cyclic frameworks are outside the scope of this evaluation procedure.

In TAF, to compute only the strength of the target arguments without forcing the calculation of irrelevant subgraphs, we need to formally define the "attack paths" leading to the targets.

Definition 2.5 (Attack Path)**.** Given a TAF and a target argument $t \in Trg$, an attack path leading to t is a sequence $P = (x_k, x_{k-1}, \ldots, x_1, x_0)$ with $k \geq 1$, where:

1) $x_0 = t$ (the path must end at the target argument);
2) for any $1 \leq i \leq k$, there exists an attack relation $(x_i, x_{i-1}) \in att$;
3) there are no duplicate nodes in the sequence (i.e., the path is a simple path to prevent infinite loops).

We refer to the argument x_k as the starting point of this path.

Definition 2.6 (Path Length)**.** For a given attack path $P = (x_k, x_{k-1}, \ldots, x_1, t)$, the length of this path is denoted as $L(P)$ and defined as the number of attack relations contained in the path. Thus, $L(P) = k$.

Combining the concepts of attack and defense in Dung's semantics [1] with the propagation of uncertainty in practical scenarios, the influence of an argument on the target argument along an attack path decays as $L(P)$ increases. Furthermore, when $L(P)$ is odd, it produces a negative influence (attack); when $L(P)$ is even, it produces a positive influence (defense). The path-dependent influence function $F(x, t, P)$ is defined below.

Definition 2.7 (Tree Unfolding)**.** Given a finite DAG-based TAF and a target t, its tree unfolding is a rooted tree whose root represents t and whose other vertices represent attack paths ending at t. For a vertex representing $P = (x_k, \ldots, x_1, t)$, its parent represents the suffix path $(x_{k-1}, \ldots, x_1, t)$; when $k = 1$, its parent is the root. The attack edge is directed from the vertex to its parent.

The vertex representing P inherits the weight $w(x_k)$. Different paths may therefore produce distinct occurrences of the same original argument. Each occurrence retains the identity of its original argument, including its membership in Trg. The unfolding is a representation of paths; duplicated occurrences are not independent pieces of evidence.

3. Axiomatic system of the influence function

In classical gradual semantics, Amgoud et al. proposed a series of core principles (such as Directionality, Weakening, and Proportionality) to regulate the strength of arguments [5]. However, these principles are designed for the final absolute strength of arguments in a global graph. To achieve local and efficient computation within a Target Argumentation Framework (TAF), we shift our perspective: Instead of directly evaluating the absolute strength of each intermediate node, we evaluate the degree of direct or indirect influence of source arguments on the target argument.

To this end, we first formally define the "influence function" and propose the axiomatic principles that the influence function should satisfy.

Definition 3.1 (Influence Function)**.** Given a target argumentation framework $TAF = \langle Ar, att, w, Trg \rangle$, let $t \in Trg$ be the target argument, $x \in Ar$ be any argument in the framework, and P be a valid attack path from x to t. An influence function, denoted as $F(x, t, P) \in \mathbb{R}$, represents the degree of influence that argument x exerts on the target argument t along the path P. Specifically, $F(x, t, P) > 0$ indicates a positive influence (defense/support); $F(x, t, P) < 0$ indicates a negative influence (attack/weakening); and $F(x, t, P) = 0$ indicates no influence.

To guarantee the rationality of the evaluation, the influence function $F(x, t, P)$ must satisfy several core principles:

The first principle guarantees the independence of influence among arguments: If an argument in an argumentation framework has no path connecting it to the target argument, then this argument should not exert any influence on the target.

Principle 3.1 (Independence of No Influence)**.** *An influence function F satisfies the independence of no influence if and only if, given a $TAF = \langle Ar, att, w, Trg \rangle$, if there is no attack path P between argument $x \in Ar$ and the target $t \in Trg$, then $F(x, t, P) = 0$.*

The next principle considers the distinction between arguments sharing the same attack path structure. When two arguments have different initial weights but share the identical attack structure, the argument with the larger initial weight should exert a greater influence on the target argument than the one with the smaller initial weight. For instance, in the introductory example shown in Figure1, regarding the arguments that affect whether the surgery is conducted, let argument x be "Jack has an allergy to the medication based on historical medical records" and argument y be "Jack had an itch after taking cold medicine in his childhood." The initial weights of these two arguments should satisfy $w(x) > w(y)$. Consequently, the argument with the larger initial weight should release an influence with a greater absolute value.

Principle 3.2 (Weak Monotony)**.** *For two arguments x and y sharing the identical attack path structure P of length $k = L(P)$ to target t, if $w(x) \geq w(y)$, then:*

$$|F(x, t, P)| \geq |F(y, t, P)|. \tag{3.1}$$

Furthermore, strict inequality $|F(x, t, P)| > |F(y, t, P)|$ holds if and only if the influence transmission is not saturated by downstream bottleneck, i.e., when $\alpha^k \cdot w(y) < \gamma \cdot |F(x_{k-1}, t, P_{k-1})|$.

Principle 3.3 (Parity of Attack and Defense)**.** *A direct attacker should weaken the strength of the target, whereas an attacker of an attacker should strengthen (reinforce) it. Mapping this to the path length $L(P)$:*

- *If $L(P)$ is odd, then $F(x, t, P) < 0$ (acting as an attack).*
- *If $L(P)$ is even, then $F(x, t, P) > 0$ (acting as a defense).*

To prevent the framework structure from collapsing due to the addition of peripheral nodes in large-scale argumentation graphs, information transmission along long paths must be accompanied by a loss of uncertainty.

Principle 3.4 (Distance Decay)**.** *The influence of an argument over an attack path decays with distance in two rigorous senses:*

1) ***Potential Distance Decay:*** *For any source argument x placed at distance L from target t along path P, its theoretical maximum potential influence is strictly bounded by:*

$$|F(x, t, P)| \leq \bar{F}(L) = \alpha \cdot \gamma^{L-1} \cdot w(x). \tag{3.2}$$

The upper bound $\bar{F}(L)$ strictly decreases as path length L increases, and asymptotically vanishes: $\lim_{L\to\infty} \bar{F}(L) = 0$.

2) ***Controlled Distance Decay:*** *Let P_1 and P_2 be two attack paths from x to t with lengths $L(P_1) < L(P_2)$. Under identical intermediate node confidence baselines (i.e., all intermediate arguments carry uniform maximal weights $w = 1.0$), the transmitted influence strictly decreases:*

$$|F(x, t, P_1)| > |F(x, t, P_2)|. \tag{3.3}$$

In real world non-monotonic reasoning, an intermediate argument with near-zero confidence acts as an information bottleneck. Moreover, if a short path traverses an extremely unreliable mediator ($w \approx 0$), its propagated influence is naturally choked and can be quantitatively smaller than that of a longer path supported by fully substantiated intermediate evidence. Rather than a flaw, this property faithfully reflects the epistemic intuition that the reliability of a deductive chain cannot exceed its weakest link.

Principle 3.5 (Sequential Distance Decay)**.** *Let $Dim(d, t)$ be a dimension sub-framework and $P = (x_k, x_{k-1}, \ldots, x_1, t)$ be a simple attack path in $Dim(d, t)$ leading to the target t, where $x_1 = d$ and $(x_{i+1}, x_i) \in att$ for all $1 \leq i \leq k-1$. For any argument x_i on the path, let $P_i = (x_i, x_{i-1}, \ldots, x_1, t)$ denote its sub-path to t. The absolute influence exerted by a node along the argumentation chain strictly decreases as its distance to the target increases:*

$$|F(x_{i+1}, t, P_{i+1})| < |F(x_i, t, P_i)|, \quad \text{for all } i \geq 1. \tag{3.4}$$

Definition 3.2 (Dimension Sub-framework)**.** Given a TAF $\langle Ar, att, w, Trg\rangle$ and a target $t \in Trg$, let

$$Att(t) = \{d \in Ar \mid (d, t) \in att\}.$$

For each $d_i \in Att(t)$, define

$$Paths(d_i, t) = \{P = (x_k, \ldots, x_1, t) \,\big|\, P \text{ is an attack path and } x_1 = d_i\}.$$

Let $V(P)$ and $E(P)$ denote the vertices and directed edges of P, respectively. The dimension induced by d_i is

$$Dim(d_i, t) = \left\langle Ar_{Dim(d_i,t)}, att_{Dim(d_i,t)}, w|_{Ar_{Dim(d_i,t)}} \right\rangle,$$

where

$$Ar_{Dim(d_i,t)} = \{t\} \cup \bigcup_{P \in Paths(d_i,t)} V(P),$$

and

$$att_{Dim(d_i,t)} = \bigcup_{P \in Paths(d_i,t)} E(P).$$

Different dimensions may share upstream arguments and attack edges in the original graph.

When the original TAF is decomposed into different sub-frameworks based on dimensions, the total impact within each sub-framework is determined by the net impact of the dimension.

Definition 3.3 (Dimensional Net Influence)**.** Given a $TAF = \langle Ar, att, w, Trg\rangle$, a target argument $t \in Trg$, and a direct attacker $d_i \in Att(t)$. The dimensional net influence of the dimension sub-framework $Dim(d_i, t)$ on t, denoted as $\mathrm{Imp}(d_i, t) \in \mathbb{R}$, is defined as the algebraic sum of the path influences of all valid attack paths within this dimension:

$$\mathrm{Imp}(d_i, t) = \sum_{P \in Paths(d_i,t)} F(\mathrm{start}(P), t, P), \tag{3.5}$$

where $Paths(d_i, t)$ is the set of all valid attack paths within the sub-framework $Dim(d_i, t)$, and $\mathrm{start}(P)$ denotes the starting node of the path P. This unpruned expression applies when

$$(Ar_{Dim(d_i,t)} \setminus \{t\}) \cap Trg = \emptyset.$$

When upstream target arguments are present, the pruned formulation in Definition 4.2 is used instead.

Lemma 3.1. *Let $T(G)$ be the tree unfolding of the TAF. If $d_1, d_2 \in Att(t)$ and $d_1 \neq d_2$, then the path sets within the dimension sub-frameworks $Dim(d_1, t)$ and $Dim(d_2, t)$ in $T(G)$ are mutually disjoint except for the endpoint t, i.e.:*

$$\forall P_1 \in Paths(d_1, t), \forall P_2 \in Paths(d_2, t) \implies P_1 \cap P_2 = \{t\}.$$

Principle 3.6 (Dimensional Isolation)**.** *Let $t \in Trg$ be a target argument with distinct dimensions $Dim(x_i, t)$ and $Dim(x_j, t)$ ($x_i, x_j \in Att(t), x_i \neq x_j$).*

1) ***Disjoint Dimensional Isolation:*** *If the ancestral subgraphs of $Dim(x_i, t)$ and $Dim(x_j, t)$ share no intermediate target arguments (i.e., $Ar_{Dim(x_i,t)} \cap Ar_{Dim(x_j,t)} \cap Trg = \emptyset$), then any topological or weight modification in $Dim(x_i, t)$ strictly preserves $Imp(x_j, t)$.*

2) ***Conditional Interface Isolation:*** *If the two dimensions share an upstream target argument $u \in Trg$, the pruning rule in Definition 4.2 establishes u as a modular interface. Conditioned on a fixed evaluated strength $Val(u)$, any private topological modification within $Dim(x_i, t)$ (not passing through u) leaves $Imp(x_j, t)$ invariant.*

The epistemic justification for target pruning is semantic encapsulation: Because $Val(u)$ already synthesizes the totality of upstream attacks and defenses rooted in $u's$ ancestral subgraph, traversing beyond u during downstream evaluation would commit the fallacy of double-counting upstream evidence. Thus, treating u as a calibrated modular interface is mathematically necessary and conceptually sound.

Example 3.1. In the introductory example concerning whether Jack should undergo surgery (argument a), the target argument has two dimensions: Dimension 1 is c (recent alcohol consumption), and Dimension 2 is b (history of allergies). As such, if a physician, by reviewing medical records, introduces a new argument g ("Jack went into anaphylactic shock due to this medication 10 years ago," where g attacks e) into Dimension 2, according to the principle of dimensional isolation, this newly introduced piece of information will alter only $\mathrm{Imp}(b, a)$ and will never cause any numerical interference to the net influence of Dimension 1 $\mathrm{Imp}(c, a)$.

Remark. In traditional gradual semantics (h-Categorizer global graph iterations), any minor change in the graph propagates to all nodes across the graph through iterations. In contrast, in a TAF, the Dimensional Isolation principle mathematically guarantees computational modularity. This enables a clear understanding of how changes in each dimension affect the target argument: "The risk within the allergy history dimension has increased, but the risk within the recent behavior dimension remains unchanged."

Within any dimension $Dim(x_i, t)$, even if an attack chain of infinite length is added to this dimension, the increment of cumulative influence it exerts on the target argument t remains strictly bounded and eventually converges to a constant.

Principle 3.7 (Explicit Dimensional Boundedness)**.** *For any target argument $t \in Trg$ with maximum ancestor branching factor (in-degree) $b = \max_v |Att(v)|$, the cumulative absolute influence exerted by all attack paths in any dimension sub-framework $Dim(d_i, t)$ must be strictly bounded by a closed-form function of the decay factors (α, γ) and the graph topological parameters (b, D):*

1) ***Finite Path Bound:*** *For an ancestor subgraph with maximum attack path depth D, the total cumulative influence is bounded by:*

$$\sum_{P \in Paths(d_i,t)} |F(start(P), t, P)| \leq M(b, D, \alpha, \gamma) = \begin{cases} \alpha \cdot \dfrac{1 - (b\gamma)^D}{1 - b\gamma}, & \text{if } b\gamma \neq 1, \\ \alpha D, & \text{if } b\gamma = 1. \end{cases} \tag{3.6}$$

2) ***Asymptotic Convergence (Infinite Tree Unfolding):*** *When extended to unbounded depth ($D \to \infty$), the cumulative influence series converges to a finite bound:*

$$\lim_{D \to \infty} \sum_{k=1}^{D} \sum_{\substack{P \in Paths(d_i,t) \\ L(P)=k}} |F(start(P), t, P)| \leq M_\infty = \frac{\alpha}{1 - b\gamma} < \infty, \tag{3.7}$$

if and only if the dampening factor satisfies the spectral stability condition: $\gamma < \frac{1}{b}$.

Example 3.2. Suppose in an argumentation structure, each counterargument is recursively attacked by $b = 2$ counterarguments. Thus, if we set $\alpha = 0.5$ and select the dampening factor $\gamma = 0.4$ such that $b\gamma = 0.8 < 1$, then despite the combinatorial explosion of 2^{k-1} attack paths at depth k, the aggregate dimensional influence from this infinite debate is strictly bounded by $M_\infty = \frac{0.5}{1-0.8} = 2.5$. The total influence converges rather than diverging to infinity.

Remark 3.1. This principle establishes the growth-decay equilibrium in argumentation topologies: the combinatorial path-count growth ($O(b^k)$) must be neutralized by exponential per-path dampening ($O(\gamma^{k-1})$). It provides an operational criterion for setting hyperparameters: enforcing $\gamma < 1/b$ guarantees system robustness, preventing remote information cascades or infinite rumor branches from destabilizing local target evaluation.

In addition, we address a fundamental principle in gradual argumentation semantics: Whether the protection of an argument, formed by other arguments attacking its direct attackers, can enhance the acceptability of the original argument. In the literature, through global computation over all arguments, the value of an argument never exceeds its assigned initial weight. However, as we consider target-oriented scenarios, we introduce Principle 8.

Principle 3.8 (Dimensional Weakening)**.** *Let $TAF = \langle Ar, att, w, Trg\rangle$ be a TAF and $t \in Trg$ be a target argument. Let $TAF' = \langle Ar', att', w', Trg\rangle$ be the updated framework obtained by introducing a new direct attacker $d_{new} \notin Att(t)$ and its corresponding dimension sub-framework $Dim(d_{new}, t)$, such that $Ar' = Ar \cup Ar_{Dim(d_{new},t)}$, $att' = att \cup att_{Dim(d_{new},t)} \cup \{(d_{new}, t)\}$, and $w'|_{Ar} = w$. If the current strength of the target argument satisfies $Val(t) > 0$, then the introduction of the new argument dimension strictly decreases its final strength:*

$$Val(t)' < Val(t), \tag{3.8}$$

where $Val(t)'$ is the strength of t in TAF'.

4. Influence function and target argumentation semantics

The four aforementioned principles establish the theoretical boundaries of the TAF influence mechanism. In this section, we instantiate this influence function using a concrete exponential decay function and formulate the final strength computation of the target argument based on the previously defined concept of dimensions. For any attack path $P = (x_k, \ldots, x_1, t)$ of length k, we explicitly denote by $P_i = (x_i, \ldots, x_1, t)$ the terminal suffix sub-path of length i ($1 \le i \le k$) terminating at t.

Definition 4.1. Let $P = (x_k, x_{k-1}, \ldots, x_1, t)$ be an attack path to target t. Let $\alpha \in (0, 1)$ be the basic decay factor, and $\gamma \in (0, 1)$ be the sequential dampening factor (typically we set $\gamma = \alpha$). The absolute influence $|F(x_i, t, P_i)|$ of the sub-path $P_i = (x_i, \ldots, t)$ of length i ($1 \le i \le k$) is recursively defined as:

$$|F(x_i, t, P_i)| = \begin{cases} \alpha \cdot w(x_1) & \text{if } i = 1, \\ \min\left(\alpha^i \cdot w(x_i),\ \gamma \cdot |F(x_{i-1}, t, P_{i-1})|\right) & \text{if } i > 1. \end{cases} \tag{4.1}$$

The signed path influence is then given by:

$$F(x_i, t, P_i) = (-1)^i \cdot |F(x_i, t, P_i)|. \tag{4.2}$$

Proposition 4.1. *The exponential decay influence function satisfies the principles of* ***Independence of No Influence***, ***Weight Monotony***, ***Parity of Attack and Defense***, *and* ***Distance Decay***.

Proof. **Satisfaction of Principle 1:** Let $\mathcal{P}(x,t)$ be the set of all simple paths from node x to the target node t. If there is no connecting attack path between x and t, i.e., $\mathcal{P}(x,t)=\emptyset$, then no path P satisfying the definition exists. According to our formulation, when there is no valid input path, the influence function is directly assigned its baseline value:

$$F(x,t,P)=0. \tag{4.3}$$

Therefore, nodes with no connecting paths exert neither direct nor indirect influence on the target node, satisfying Principle 1.

Satisfaction of Principle 2: Let P_x and P_y share the identical sub-path of length $k-1$ terminating at t, with downstream absolute influence $V_{k-1}=|F(x_{k-1},t,P_{k-1})|$. Suppose $w(x)\geq w(y)$.

- If $k=1$: $|F(x,t,P_x)|=\alpha w(x)\geq \alpha w(y)=|F(y,t,P_y)|$.
- If $k>1$: By Definition 4.1,

$$|F(x,t,P_x)|=\min\left(\alpha^k w(x),\gamma V_{k-1}\right),\quad |F(y,t,P_y)|=\min\left(\alpha^k w(y),\gamma V_{k-1}\right). \tag{4.4}$$

  Since $\alpha^k>0$ and $w(x)\geq w(y)$, we have $\alpha^k w(x)\geq \alpha^k w(y)$. By the monotonicity of the minimum function, $\min(a,c)\geq\min(b,c)$ for $a\geq b$, we directly obtain:

$$|F(x,t,P_x)|\geq |F(y,t,P_y)|. \tag{4.5}$$

  This rigorously satisfies Weak Monotony. Furthermore, when $\alpha^k w(x)\geq\gamma V_{k-1}$ and $\alpha^k w(y)\geq\gamma V_{k-1}$, both paths saturate at γV_{k-1}, yielding $|F(x,t,P_x)|=|F(y,t,P_y)|$. If $\alpha^k w(y)<\gamma V_{k-1}$ and $w(x)>w(y)$, then $|F(x,t,P_x)|>|F(y,t,P_y)|$, establishing conditional strict monotony.

Satisfaction of Principle 3: We first show by induction that if the initial weights of all nodes along the path $P=(x_k,\ldots,x_1,t)$ satisfy $w(x_i)>0$, and the decay factors satisfy $\alpha,\gamma\in(0,1)$, then the absolute influence $|F(x_i,t,P_i)|>0$ for all $1\leq i\leq k$:

- **Base case** ($i=1$)**:** $|F(x_1,t,P_1)|=\alpha\cdot w(x_1)>0$ because both $\alpha>0$ and $w(x_1)>0$.
- **Inductive step:** Assume $|F(x_{i-1},t,P_{i-1})|>0$. Then $\gamma\cdot|F(x_{i-1},t,P_{i-1})|>0$ since $\gamma>0$. Combined with $\alpha^i\cdot w(x_i)>0$, both arguments inside the minimum operator are strictly positive. Thus, their minimum is also strictly positive:

$$|F(x_i,t,P_i)|=\min\left(\alpha^i\cdot w(x_i),\ \gamma\cdot|F(x_{i-1},t,P_{i-1})|\right)>0. \tag{4.6}$$

By induction, the absolute influence of the starting node $|F(x,t,P)|=|F(x_k,t,P)|>0$.

Now, we discuss the parity of the path length $L(P)=k$ in two cases:

- When k is odd (i.e., $k=2m+1$ for $m\in\mathbb{N}$), we have $(-1)^k=-1$. The signed influence becomes:

$$F(x,t,P)=-|F(x,t,P)|<0. \tag{4.7}$$

  This indicates that nodes at an odd distance exert a negative influence (an attacking/weakening effect) on the target.

- When k is even (i.e., $k = 2m$ for $m \in \mathbb{N}^+$), we have $(-1)^k = 1$. The signed influence becomes:

$$F(x, t, P) = |F(x, t, P)| > 0. \tag{4.8}$$

  This indicates that nodes at an even distance exert a positive influence (a protective/defensive effect) on the target.

In summary, the polarity of the influence function is determined by the parity of the path length, satisfying Principle 3. Satisfaction of Principle 4 & 5: For any simple attack path $P = (x_k, \ldots, x_1, t)$, we analyze sequential distance decay (Principle 5), distance decay (Principle 4):

- **Sequential Distance Decay (Principle 5):** For any node x_i on the path ($i > 1$), by definition of the recursive minimum operator:

$$|F(x_i, t, P_i)| = \min\left(\alpha^i \cdot w(x_i),\ \gamma \cdot |F(x_{i-1}, t, P_{i-1})|\right) \leq \gamma \cdot |F(x_{i-1}, t, P_{i-1})|. \tag{4.9}$$

  When all arguments along the chain carry non-zero weights ($w(x_j) > 0$ for $1 \leq j \leq i$), we have $|F(x_{i-1}, t, P_{i-1})| > 0$. Since $\gamma \in (0, 1)$, it follows strictly that:

$$\gamma \cdot |F(x_{i-1}, t, P_{i-1})| < |F(x_{i-1}, t, P_{i-1})|. \tag{4.10}$$

  Therefore, the absolute influence strictly decreases sequentially along the chain:

$$|F(x_i, t, P_i)| < |F(x_{i-1}, t, P_{i-1})|. \tag{4.11}$$

  (If an intermediate node satisfies $w(x_j) = 0$, the transmitted influence is choked to zero, satisfying weak sequential decay $\leq$).

- **Potential Distance Decay (Principle 4, Part 1):** Applying the inequality recursively from the starting node $x_k = x$ down to the direct attacker x_1, we obtain:

$$|F(x, t, P)| \leq \gamma \cdot |F(x_{k-1}, t, P_{k-1})| \leq \cdots \leq \gamma^{k-1} \cdot |F(x_1, t, P_1)|. \tag{4.12}$$

  Substituting $|F(x_1, t, P_1)| = \alpha \cdot w(x_1) \leq \alpha$ yields the explicit potential upper bound:

$$|F(x, t, P)| \leq \bar{F}(k) = \alpha \cdot \gamma^{k-1} \cdot w(x_1). \tag{4.13}$$

  Since $\gamma \in (0, 1)$, $\bar{F}(k)$ strictly decreases with respect to path length k. Taking the limit as $k = L(P) \to \infty$:

$$\lim_{L(P)\to\infty} \bar{F}(L(P)) = \lim_{k\to\infty} \alpha \cdot \gamma^{k-1} \cdot w(x_1) = 0. \tag{4.14}$$

  By the Squeeze Theorem, $\lim_{L(P)\to\infty} |F(x, t, P)| = 0$, ensuring asymptotic vanishing.

- **Controlled Distance Decay under Uniform Baseline (Principle 4, Part 2):** Consider two paths P_1 and P_2 originating from the same source argument x with $L(P_1) < L(P_2)$. Under a standardized baseline where intermediate arguments provide uninhibited transmission (i.e., uniform maximal weights $w = 1.0$ and canonical $\gamma = \alpha$), induction over Eq (4.1) yields:

$$|F(x, t, P)| = \alpha^{L(P)} \cdot w(x). \tag{4.15}$$

Since $\alpha \in (0,1)$ and $w(x) > 0$, $L(P_1) < L(P_2)$ strictly implies:

$$\alpha^{L(P_1)} \cdot w(x) > \alpha^{L(P_2)} \cdot w(x) \implies |F(x,t,P_1)| > |F(x,t,P_2)|. \tag{4.16}$$

This establishes strict distance decay under controlled intermediate baselines, completing the proof.

□

We next aggregate path influences within the dimension induced by each direct attacker $d_i \in Att(t)$. These dimensions group paths by their final attack edge and may share upstream arguments.

Definition 4.2 (Target Strength Computation)**.** Let $t \in Trg$ be a target in a finite DAG-based TAF. Assume that the strengths of all upstream target arguments have been evaluated.

For each $d_i \in Att(t)$, define

$$Paths_{\text{pruned}}(d_i,t) = \left\{ P = (x_k,\ldots,x_1,t) \in Paths(d_i,t) \;\middle|\; x_j \notin Trg \text{ for } 1 \le j < k \right\}.$$

Thus, a path may start at an upstream target, but it cannot continue beyond that target when traced backward from t. All shorter paths satisfying this condition are also included, and each distinct path is counted once. For $x \neq t$, define the effective weight

$$W_t(x) = \begin{cases} w(x), & x \notin Trg, \\ Val(x), & x \in Trg. \end{cases}$$

For an admissible path $P_i = (x_i,\ldots,x_1,t)$, its absolute pruned influence is

$$|F_{\text{pruned}}(x_i,t,P_i)| = \begin{cases} \alpha W_t(x_1), & i = 1, \\ \min\left(\alpha^i W_t(x_i), \gamma |F_{\text{pruned}}(x_{i-1},t,P_{i-1})|\right), & i > 1, \end{cases} \tag{4.17}$$

where $P_{i-1} = (x_{i-1},\ldots,x_1,t)$ for $i > 1$. Its signed influence is

$$F_{\text{pruned}}(x_i,t,P_i) = (-1)^i |F_{\text{pruned}}(x_i,t,P_i)|.$$

The dimensional net influence is

$$Imp(d_i,t) = \sum_{P \in Paths_{\text{pruned}}(d_i,t)} F_{\text{pruned}}(\text{start}(P),t,P). \tag{4.18}$$

The final target strength is

$$Val(t) = \min\left(w(t), \max\left(0,\ w(t) + \sum_{d_i \in Att(t)} Imp(d_i,t)\right)\right). \tag{4.19}$$

Through this dimensional aggregation computation based on a concrete influence function, we not only guarantee theoretical rigor but also completely discard inefficient global iterations. Consequently, the algorithm exhibits remarkable robustness and interpretability when processing massive knowledge graphs and dynamically evolving medical or legal evidence chains.

Proposition 4.2. *Let $d_1, d_2 \in Att(t)$ be two distinct direct attackers of the target t. Let $TAF = \langle Ar, att, w, Trg\rangle$ and $TAF' = \langle Ar, att, w', Trg\rangle$ be two target argumentation frameworks that differ only in the initial weight of node $y \in Ar$ (i.e., $\forall z \in Ar \setminus \{y\}, w(z) = w'(z)$). If no path from y to t passes through d_2, then the change in the initial weight of y does not affect the net influence of the dimension $Dim(d_2, t)$:*

$$\mathrm{Imp}_{TAF}(d_2, t) = \mathrm{Imp}_{TAF'}(d_2, t). \tag{4.20}$$

Proof. Under the frameworks TAF and TAF', the net influence of the dimension $Dim(d_2, t)$ is represented as:

$$\mathrm{Imp}_{TAF}(d_2, t) = \sum_{P \in Paths(d_2,t)} F_w(\mathrm{start}(P), t, P). \tag{4.21}$$

$$\mathrm{Imp}_{TAF'}(d_2, t) = \sum_{P \in Paths(d_2,t)} F_{w'}(\mathrm{start}(P), t, P). \tag{4.22}$$

For any path $P \in Paths(d_2, t)$, let $Nodes(P)$ be the set of nodes contained in P. Since it is known that no path from y to t can reach t through d_2, y can never belong to any path $P \in Paths(d_2, t)$, which means $y \notin Nodes(P)$. Since TAF and TAF' differ only in the initial weight of y, for any node $z \in Nodes(P)$ on path P, we have $w(z) = w'(z)$. Consequently, the path influence of P remains identical under both frameworks:

$$F_w(\mathrm{start}(P), t, P) = F_{w'}(\mathrm{start}(P), t, P). \tag{4.23}$$

Substituting this equation into the net influence formulation yields:

$$\mathrm{Imp}_{TAF}(d_2, t) = \sum_{P \in Paths(d_2,t)} F_{w'}(\mathrm{start}(P), t, P) = \mathrm{Imp}_{TAF'}(d_2, t). \tag{4.24}$$

This completes the proof. □

Proposition 4.3 (Dimensional Isolation under Multi-Target DAGs)**.** *Let $d_1, d_2 \in Att(t)$ be distinct direct attackers of target t, and let TAF and TAF' differ only by topological or weight changes restricted to $Dim(d_1, t) \setminus Dim(d_2, t)$.*

1) *If $Paths_{pruned}(d_2, t)$ contains no node whose evaluated strength $Val(\cdot)$ is altered in TAF', then $Imp_{TAF}(d_2, t) = Imp_{TAF'}(d_2, t)$.*

2) *For any shared target node $u \in Ar_{Dim(d_1,t)} \cap Ar_{Dim(d_2,t)} \cap Trg$, the pruning operation in Definition 4.2 truncates all paths at u. Consequently, the net influence $Imp(d_2, t)$ depends on the upstream subgraph of u strictly through the scalar interface $Val(u)$, preserving conditional modularity.*

Proof. By Definition 4.2, the net influence is given by:

$$Imp(d_2, t) = \sum_{P \in Paths_{pruned}(d_2,t)} (-1)^{L(P)} \cdot |F_{pruned}(start(P), t, P)|. \tag{4.25}$$

For case (1), since the modifications are confined to $Dim(d_1, t) \setminus Dim(d_2, t)$, every pruned path $P \in Paths_{pruned}(d_2, t)$ contains identical nodes and initial weights across both frameworks. Because no evaluated target strengths along these paths are altered, each path influence $(-1)^{L(P)}|F_{pruned}(start(P), t, P)|$ remains identical. Hence $Imp_{TAF}(d_2, t) = Imp_{TAF'}(d_2, t)$.

For case (2), let $P \in Paths_{pruned}(d_2, t)$ be any path passing through $u \in Trg$. By Definition 4.2, P is truncated at u, meaning no nodes upstream of u enter $Paths_{pruned}(d_2, t)$. By Eq (4.17), the absolute influence $|F_{pruned}(u, t, P)|$ is solely a function of $Val(u)$, α, γ, and the downstream suffix path from u to t. Thus, given a fixed value $Val(u)$, variations in the internal topology of $Dim(d_1, t)$ cannot enter the summation of $Imp(d_2, t)$. This establishes conditional interface isolation. □

In the study of gradual semantics for abstract argumentation, researchers (such as Amgoud et al.) have proposed a series of axiomatic principles. However, these principles cannot be satisfied simultaneously, as some classical principles are logically incompatible. Below, we analyze these classical conflicts and present the corresponding solutions of TAF.

When faced with a choice between "one extremely strong attacker" and "multiple weaker attackers", different semantics adopt different strategies, which constitutes the most famous ternary opposition conflict in the literature:

- **Quality Precedence (QP):** This strategy posits that the maximum quality (i.e., the strongest attacker) determines everything. The threat posed by any number of weak attackers cannot compare to that of a single strong attacker.
- **Cardinality Precedence (CP):** This strategy posits that the number of attackers determines everything. As long as the number of attackers is sufficiently large, their weakening effect will inevitably exceed that of a single attacker.
- **Compensation:** This strategy posits that quality and quantity can make compromises with each other. That is, although a single weak attacker is not as influential as a single strong attacker, the collective influence of multiple weak attackers can compensate for and eventually surpass that of a single strong attacker.

The Choice and Rationality of TAF: Our TAF framework adopts the Compensation strategy in the summation of cumulative influence in Definition 4.2. This is because, in medical clinical decision-making (such as Jack's surgery case in the Introduction) or legal proceedings, the accumulated credibility of "multiple low-risk drug reactions" or "multiple indirect suspicious pieces of evidence" can fully offset or surpass "a single counter-argument of medium strength".

5. Theoretical properties and algorithms

In this section, we first establish several theoretical properties of the proposed TAF. Subsequently, we present an exact algorithm that faithfully implements the path-based semantics introduced in Definition 4.2. Finally, we discuss a dynamic programming optimization for special graph structures.

Unlike traditional gradual semantics based on global fixed-point iterations, the proposed TAF evaluates only the ancestor subgraph of the target argument. The exact semantics is defined by aggregating the influence of all pruned attack paths. Consequently, the computational complexity depends on the number of relevant paths rather than the size of the argumentation graph.

5.1. Argumentative properties

Analogous to the axiomatic studies in quantitative bipolar argumentation, a rational target-oriented semantics should intuitively conform to the core principles of argumentative interactions. The following

theorems demonstrate that the TAF target strength computation (Eq (4.19)) and the pruning-bounded influence function satisfy several desirable theoretical properties:

Theorem 5.1. *Let $TAF = \langle Ar, att, w, Trg\rangle$ be a target argumentation framework. For any target argument $t \in Trg$, if t is not attacked in the graph (i.e., $Att(t) = \emptyset$), then its final strength equals its initial weight $Val(t) = w(t)$.*

Proof. When $Att(t) = \emptyset$, the set of pruned paths $Paths_{\text{pruned}}(d_i, t)$ is empty for any potential direct attacker. Consequently, for any d_i, the dimensional net influence is $Imp(d_i, t) = 0$. Substituting this into the target strength aggregation formula:

$$Val(t) = \min\left(w(t),\ \max\left(0,\ w(t) + \sum_{d_i \in \emptyset} 0\right)\right) = \min(w(t), \max(0, w(t))). \tag{5.1}$$

Since the initial weight $w(t) \in [0, 1]$, we have $\max(0, w(t)) = w(t)$, which yields $Val(t) = w(t)$. Thus, the theorem holds. □

Theorem 5.2. *Let $t \in Trg$ be a target argument. If all pruned attack paths $P \in \bigcup Paths_{pruned}(d_i, t)$ leading to t are of odd length (i.e., there are only attackers and no defenders in the evaluation subgraph), then the final strength of the target argument is bounded by its initial weight $Val(t) \leq w(t)$.*

Proof. If all paths P are of odd length, then $(-1)^{L(P)} = -1$. For any pruned path P, its signed influence is:

$$F_{\text{pruned}}(\text{start}(P), t, P) = -|F_{\text{pruned}}(\text{start}(P), t, P)| \leq 0. \tag{5.2}$$

Since the dimensional net influence $Imp(d_i, t)$ is the summation of these negative influences, we have $Imp(d_i, t) \leq 0$ for all $d_i \in Att(t)$. Consequently, the sum of dimensional net influences satisfies $\sum_{d_i \in Att(t)} Imp(d_i, t) \leq 0$. According to Eq (4.19):

$$Val(t) = \min\left(w(t),\ \max\left(0,\ w(t) + \sum Imp(d_i, t)\right)\right) \leq w(t). \tag{5.3}$$

The theorem holds.
Remark: In a pure attack-based graph, the dual case where all paths are of even length is topologically impossible when $Att(t) \neq \emptyset$, because each direct attacker $d_i \in Att(t)$ always introduces a path (d_i, t) of length 1 (odd). □

Theorem 5.3. *Let $x \in Ar$ be an ancestor node of the target argument $t \in Trg$.*

1) *If all paths from x to t are of even length (i.e., x is a pure defender of t), then $Val(t)$ is non-decreasing with respect to $w(x)$ (or $Val(x)$ if $x \in Trg$);*

2) *If all paths from x to t are of odd length (i.e., x is a pure attacker of t), then $Val(t)$ is non-increasing with respect to $w(x)$ (or $Val(x)$ if $x \in Trg$).*

Proof. By definition, the target strength computation formula, the pruning-bounded path influence (using the min operator), and the boundary capping functions ($\max(0, \cdot)$ and $\min(w(t), \cdot)$) are non-decreasing monotonic functions. Since the composition of non-decreasing functions is also non-decreasing, the overall monotonicity is determined by the parity signs $(-1)^{L(P)}$. If all paths from x to t are even, the signs

of their influences are positive, implying that an increase in $w(x)$ (or $Val(x)$) results in a non-decreasing increase of positive influences, thereby making $Val(t)$ non-decreasing. Conversely, if all paths are odd, the signs are negative, meaning that an increase in $w(x)$ leads to a non-decreasing increase of negative influences, making $Val(t)$ non-increasing. □

Theorem 5.4. *As the decay factors $\alpha \to 0$ and $\gamma \to 0$, the final strength of any target argument under any topology smoothly converges to its initial weight:*

$$\lim_{\alpha\to 0,\gamma\to 0} Val(t) = w(t). \tag{5.4}$$

Proof. For any pruned path P of length $L(P) \geq 1$, because $\alpha \to 0$ and $\gamma \to 0$, the absolute path influence satisfies:

$$|F_{\text{pruned}}(\text{start}(P), t, P)| \leq \alpha^{L(P)} W(\text{start}(P)) \to 0. \tag{5.5}$$

Consequently, the dimensional net influence of each direct attacker converges to zero: $\lim_{\alpha\to 0,\gamma\to 0} Imp(d_i, t) = 0$. Substituting this into the strength aggregation formula yields:

$$\lim_{\alpha\to 0,\gamma\to 0} Val(t) = \min\left(w(t),\ \max(0,\ w(t) + 0)\right) = w(t). \tag{5.6}$$

The theorem holds. This indicates that the decay parameters govern the propagation of uncertainty, degenerating into isolated local decisions as uncertainty increases, which aligns with robust control intuitions. □

Theorem 5.5. *Let $TAF = \langle Ar, att, w, Trg\rangle$ be a target argumentation framework, $t \in Trg$, and $d_i \in Att(t)$ be a direct attacker. In the tree unfolding $T(G)$, let b denote the maximum branching factor of the ancestor subgraph of t, and let $\alpha, \gamma \in (0, 1)$ be the decay parameters. Then:*

1) For any finite maximum path depth D, the dimensional net influence magnitude $|Imp(d_i, t)|$ satisfies:

$$|Imp(d_i, t)| \leq \sum_{P\in Paths(d_i,t)} |F_{pruned}(start(P), t, P)| \leq M(b, D, \alpha, \gamma). \tag{5.7}$$

2) When $b\gamma < 1$, the cumulative path influence remains finite even on infinite tree unfoldings, and $\lim_{D\to\infty} M(b, D, \alpha, \gamma) = \frac{\alpha}{1-b\gamma}$.

3) The final target strength $Val(t)$ is unconditionally bounded within $[0, w(t)] \subseteq [0, 1]$.

Proof. For any attack path $P = (x_k, x_{k-1}, \ldots, x_1, t) \in Paths(d_i, t)$ of length $L(P) = k$, we have $x_1 = d_i$. By Principle 4 (Potential Distance Decay, Eq (3.2)), the absolute influence transmitted along P is bounded by:

$$|F(x_k, t, P)| \leq \alpha \cdot \gamma^{k-1} \cdot w(x_1) \leq \alpha \cdot \gamma^{k-1}, \tag{5.8}$$

since the argument weight $w(x_1) \in [0, 1]$. When target pruning (Definition 4.2) is applied, $|F_{\text{pruned}}|$ is further capped by $\alpha^k Val(x_k) \leq \alpha^k \leq \alpha\gamma^{k-1}$. Thus, Eq (5.8) holds universally.

In the tree unfolding $T(G)$, the direct attacker d_i is at distance $k = 1$, yielding exactly $1 = b^0$ path. For any distance $k \geq 2$, because each intermediate node has at most b incoming attack edges, the number of simple attack paths of length k terminating at t through d_i is at most:

$$N(k) \leq b^{k-1}. \tag{5.9}$$

By the triangle inequality, the cumulative absolute influence over all paths up to depth D is bounded by the sum of individual path bounds:

$$\begin{aligned}\sum_{P\in Paths_{\leq D}(d_i,t)} |F_{\text{pruned}}(\text{start}(P),t,P)| &\leq \sum_{k=1}^{D} \sum_{\substack{P\in Paths(d_i,t)\\ L(P)=k}} |F(x_k,t,P)| \\ &\leq \sum_{k=1}^{D} N(k)\cdot\left(\alpha\gamma^{k-1}\right) \\ &\leq \sum_{k=1}^{D} b^{k-1}\cdot\alpha\gamma^{k-1} = \alpha\sum_{j=0}^{D-1}(b\gamma)^j. \end{aligned} \tag{5.10}$$

Evaluating the geometric series in Eq (5.10):

- If $b\gamma \neq 1$, $\alpha\sum_{j=0}^{D-1}(b\gamma)^j = \alpha\frac{1-(b\gamma)^D}{1-b\gamma} = M(b,D,\alpha,\gamma)$.
- If $b\gamma = 1$, $\alpha\sum_{j=0}^{D-1} 1^j = \alpha D$.

Since any finite DAG has a finite maximum topological depth $D \leq |Ar|$, the cumulative influence $M(b,D,\alpha,\gamma)$ is always finite, proving (1).

For (2), taking the limit $D\to\infty$, the geometric series $\sum_{j=0}^{\infty}(b\gamma)^j$ converges if and only if the common ratio satisfies $|b\gamma|<1$. Because $b\geq 1$ and $\gamma>0$, this condition is strictly $b\gamma<1$ (i.e., $\gamma<1/b$), which yields:

$$M_\infty = \lim_{D\to\infty}\alpha\frac{1-(b\gamma)^D}{1-b\gamma} = \frac{\alpha}{1-b\gamma} < \infty. \tag{5.11}$$

For (3), by Eq (4.19), the target evaluation formula applies explicit clamping:

$$Val(t) = \min\left(w(t), \max\left(0, w(t) + \sum_{d_i\in Att(t)} Imp(d_i,t)\right)\right). \tag{5.12}$$

Because the outer function is $\min(w(t),\cdot)$ and the inner function is $\max(0,\cdot)$, we strictly have $0\leq Val(t)\leq w(t)\leq 1$, irrespective of the magnitude of $\sum Imp(d_i,t)$. This completes the proof. □

5.2. Exact path-based computation

The semantic definition introduced in Section 4 computes the final strength of a target argument by aggregating the influence of every pruned attack path. Therefore, the exact computation naturally follows a depth-first traversal over the ancestor subgraph of the target argument.

For each direct attacker, all admissible simple attack paths leading to the target are enumerated. The influence of each path is recursively computed according to Definition 4.1 and accumulated into the corresponding argumentation dimension. Finally, the strengths of all dimensions are aggregated using Definition 4.2.

Algorithm 1: Exact Path-based Target Strength Computation

Input: A finite DAG-based TAF $\langle Ar, att, w, Trg\rangle$, a target $t \in Trg$, and cached strengths $Val(u)$ for all $u \in Trg \cap Anc(t)$

Output: Final strength $Val(t)$

```
Extract the ancestor subgraph of t;
totalImp ← 0;
foreach direct attacker d ∈ Att(t) do
    Imp(d) ← DFS(d, t, ⟨d, t⟩);
    totalImp ← totalImp + Imp(d);
end
Return Val(t) = min(w(t), max(0, w(t) + totalImp));
```

Let $\mathcal{P}_{\text{pruned}}$ denote the set of pruned attack paths enumerated during the evaluation of a target argument, and let $L(P_i)$ denote the length of an individual path P_i. Since the influence of each path is recursively evaluated along its nodes, the computational cost of processing P_i is $O(L(P_i))$. Therefore, the total running time is characterized by

$$T = O\left(\sum_{P_i \in \mathcal{P}_{\text{pruned}}} L(P_i)\right). \tag{5.13}$$

Let $P = |\mathcal{P}_{\text{pruned}}|$ denote the number of enumerated pruned paths and let $\bar{L}$ denote their average length. Then,

$$T = O(P\bar{L}). \tag{5.14}$$

Thus, the proposed exact algorithm is output-sensitive: its running time is linear in the total amount of enumerated path information rather than in the total number of arguments or attacks in the complete graph.

For a requested target t, let $Anc(t)$ denote its strict ancestors. Process the targets in $(Trg \cap Anc(t)) \cup \{t\}$ in a topological order consistent with the attack edges, so that upstream targets are evaluated first. Apply Algorithm 1 to each target in this order and cache the resulting strength.

The exact algorithm computes the target strength exactly as defined in Definition 4.2. Let $\mathcal{P}_{\text{pruned}}$ denote the set of enumerated pruned attack paths. Since each path is processed along its sequence of nodes, the running time is $O\left(\sum_{P_i \in \mathcal{P}_{\text{pruned}}} L(P_i)\right) = O(P\bar{L})$, where $P = |\mathcal{P}_{\text{pruned}}|$ and $\bar{L}$ is the average path length. Hence, the algorithm should not be interpreted as having a graph-size-linear worst-case complexity. Instead, its complexity is linear with respect to the amount of path information enumerated for the target.

The recursion stack and the current simple-path sequence require $O(D)$ auxiliary space, where D is the maximum depth of the traversed ancestor subgraph.

Although the output-sensitive characterization in Eq (5.14) describes the actual computational workload, the number of admissible paths P is not generally linear in the size of the ancestor subgraph. In particular, simple-path enumeration in a directed acyclic graph can grow exponentially with the graph depth and branching factor.

Let D denote the maximum path length from an ancestor argument to the target and let b denote the maximum number of predecessors of any argument in the ancestor subgraph. A conservative upper bound on the number of admissible paths is

$$P \leq \sum_{k=1}^{D} b^k = \begin{cases} \dfrac{b^{D+1} - b}{b - 1}, & b > 1, \\ D, & b = 1. \end{cases} \tag{5.15}$$

Therefore,

$$P = O(b^D). \tag{5.16}$$

Algorithm 2: Recursive Dimensional Net Influence with Pruning

Function DFS(*curr, target, path*)**:**
  Input: Current argument *curr*, target argument *target*, current attack path sequence $path = (curr, \ldots, target)$
  Output: Cumulative net influence of paths rooted in the sub-tree at *curr*
  $L \leftarrow |path| - 1$;
  // Length of the path from curr to target
  // Check Target Pruning Condition according to Definition 4.2
  if $curr \in Trg \setminus \{target\}$ **then**
    // Truncate traversal and use the pre-evaluated target strength Val(curr)
    $inf \leftarrow (-1)^L \cdot |F_{pruned}(curr, target, path)|$;
    return inf;
    // Prune further upstream branches
  end
  // Compute signed influence of the sub-path originating at intermediate/leaf node curr
  $inf \leftarrow (-1)^L \cdot |F(curr, target, path)|$;
  $dimSum \leftarrow inf$;
  // Recursively accumulate influences from upstream attackers
  foreach $v \in Att(curr)$ **do**
    if $v \notin path$ **then**
      // Ensure simple path to prevent cycles
      $dimSum \leftarrow dimSum + \mathtt{DFS}(v, target, \langle v \rangle \circ path)$;
    end
  end
  return $dimSum$;

Combining Eq (5.16) with the output-sensitive complexity in Eq (5.14), the worst-case running time satisfies

$$T = O(b^D D). \tag{5.17}$$

Equation (5.17) makes explicit that the proposed exact algorithm does not provide a graph-size-linear worst-case guarantee. Rather, its computational advantage arises from target-oriented localization and path pruning, which can substantially reduce the actual number of enumerated paths relative to the structural worst case.

5.3. Illustrative example and semantic comparison

To highlight the conceptual differences and computational advantages of the proposed TAF compared with traditional gradual semantics, we analyze the weighted graph G_2 in Figure 2.

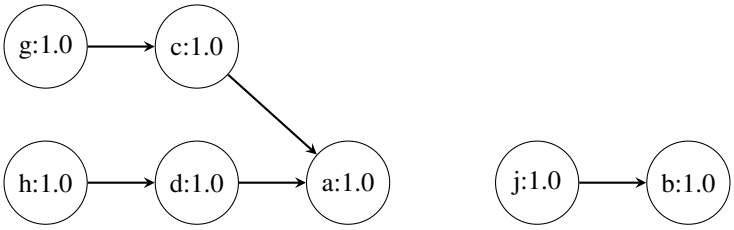


Figure 2. Weighted graph $\mathbf{G_2}$.

The primary differences between TAF and traditional gradual semantics (such as h-Categorizer) lie in the *evaluation scope* and the *modular computation mechanism*:

- **Selective Local Evaluation**: Traditional semantics like h-Categorizer are inherently global. To evaluate the strength of a single node a, the graph must be traversed and computed. In contrast, TAF evaluates strengths selectively. For instance, when $Trg = \{a\}$, TAF ignores the disconnected component $\{j, b\}$ and avoids computing intermediate strengths for c and d.

- **Topological Path Pruning**: When multiple target arguments are defined (e.g., $Trg = \{a, c, d\}$), TAF computes the upstream targets first and propagates their evaluated strengths to downstream nodes. This avoids redundant pathway traversal and enables efficient local updates.

We present a numerical comparison of the evaluated strengths between the classical h-Categorizer and TAF under two different target configurations, namely $Trg = \{a\}$ and $Trg = \{a, c, d\}$, on graph G_2.

For the numerical evaluation, we set the decay factor and damping factor to $\alpha = \gamma = 0.5$. The initial argument weights are those specified for graph G_2, and the same parameter setting is used for both target configurations in Table 1.

As shown in Table 1, under this parameter setting, the evaluated strength of the target argument a is 0.5 for both target configurations, which coincides with the value obtained by the classical h-Categorizer. This agreement is specific to the considered graph G_2, its initial weights, and the parameter setting $\alpha = \gamma = 0.5$. It should therefore be interpreted as an instance-level numerical consistency, rather than as a general semantic equivalence between TAF and the classical h-Categorizer.

In general, the strength produced by TAF depends on the attack structure, the initial argument weights, the selected target set, and parameters α and γ. Different parameter values or graph structures may consequently lead to different numerical results.

Table 1. Numerical strength comparison and evaluation scopes on graph G_2.

Argument	***h*-Categorizer Strength**	**TAF Strength** ($Trg = \{a\}$)	**TAF Strength** ($Trg = \{a, c, d\}$)
g	1.0	N/A (Ignored)	N/A (Ignored)
c	0.5	N/A (Ignored)	0.5
h	1.0	N/A (Ignored)	N/A (Ignored)
d	0.5	N/A (Ignored)	0.5
a	0.5	0.5	0.5
j	1.0	N/A (Ignored)	N/A (Ignored)
b	0.5	N/A (Ignored)	N/A (Ignored)

The numerical agreement ($Val(a) = 0.5$) in Table 1 arises because, under uniform weights $w = 1.0$ and $\alpha = \gamma = 0.5$, the first-order linear dampening of TAF along a single attack edge ($1 - \alpha \cdot w = 0.5$) matches the rational fractional evaluation of h-Categorizer ($\frac{1}{1+1} = 0.5$). However, their underlying mathematical mechanisms differ fundamentally:

- **Global Coupling vs. Dimensional Modularity:** h-Categorizer defines argument acceptability via a system of coupled non-linear equations $Deg(x) = \frac{w(x)}{1+\sum_{(y,x)\in att} Deg(y)}$, necessitating global iterative convergence across the network. Any peripheral change ripples through the graph. In contrast, TAF decomposes interactions into independent dimensional sub-frameworks $Dim(d_i, t)$ via linear summation $\sum Imp(d_i, t)$, mathematically guaranteeing that updates in one dimension remain strictly isolated from others.
- **Computational Paradigm:** While h-Categorizer provides a holistic global valuation for every argument, TAF restricts computation exclusively to the target's ancestor subgraph, bypassing disconnected or downstream components (e.g., $\{j, b\}$ in Figure 2) and providing direct path-traceable transparency.

Crucially, the table highlights the computational efficiency of TAF:

1) When $Trg = \{a\}$, only final target a is evaluated. Its ancestors g, c, h, d are traced only as path flows rather than being evaluated as target strengths, and the disconnected component $\{j, b\}$ is pruned.
2) When $Trg = \{a, c, d\}$, the intermediate targets c and d are evaluated alongside a. However, the unrelated component $\{j, b\}$ remains untouched.

This example illustrates that, for the considered graph and parameter setting, TAF can produce numerical strengths that coincide with those of the classical gradual semantics for the evaluated target.

6. Discussion

Our proposed Target Argumentation Framework introduces a localized, target-oriented paradigm to compute gradual semantics based on dimensional attack paths. In contrast to traditional gradual semantics that often necessitate a comprehensive graph traversal, TAF restricts its evaluation scope to the relevant ancestral subgraphs of specified target arguments. This localization can substantially reduce redundant computation when the target-relevant subgraph is small relative to the complete argumentation

graph. However, as established in Section 5.2, exact path enumeration remains potentially exponential in the maximum branching factor and depth. Therefore, the computational scalability of TAF should be understood in terms of the size and structure of the target-relevant path space rather than the global graph size alone.

This work maintains a close relationship with foundational global gradual semantics, particularly the h-Categorizer and trust-based possibilistic models [4, 5]. These global approaches excel in providing a mathematically rigorous, holistic evaluation of all arguments across the network by solving a system of global fractional equations. While TAF shares the same core qualitative properties (such as monotonicity and distance decay) and yields identical target strength values in isolated topologies, it departs from their global computation scheme. The primary advantage of TAF over [4, 5] lies in its elimination of redundant evaluations on disconnected components and irrelevant ancestors, though it does not compute the absolute strengths of all non-target nodes, which might still be of secondary interest in fully collaborative multi-agent settings.

While the proposed localized paradigm offers notable benefits, its scope presents certain areas for further exploration. The precise path-tracing and dimensional decomposition mechanisms are most naturally aligned with Directed Acyclic Graphs, which are highly typical of structured evidence chains. In scenarios involving highly dense networks with complex cyclic structures, the enumeration of simple paths could scale rapidly, and the interplay of cyclic feedback loops may require more nuanced dampening protocols. Refining the path-filtering criteria and establishing adaptive convergence thresholds for cyclic topologies represent promising avenues to further broaden the general applicability of TAF.

7. Conclusions

To address the computational redundancy and topological instability in traditional gradual semantics, we propose the Target Argumentation Framework (TAF). By decomposing goal-oriented argumentation graphs into independent evaluation dimensions and employing an exponential-decay path influence function, TAF isolates irrelevant subgraphs to focus evaluation strictly on critical paths. From a computational perspective, the proposed exact algorithm has an output-sensitive running time of $O(P\bar{L})$, where P is the number of enumerated pruned attack paths and $\bar{L}$ is their average length. The theoretical analysis further establishes that P may grow as $O(b^D)$ in the worst case, where b is the maximum branching factor and D is the maximum path depth. Consequently, the worst-case running time is bounded by $O(b^D D)$. This characterization makes explicit the computational advantage and the scalability boundary of the proposed exact path-based formulation: Target-oriented localization and pruning can substantially reduce the practical path space, but exact enumeration does not eliminate the inherent worst-case complexity of simple-path traversal.

Beyond its algorithmic execution, this work carries profound significance for the broader field of computational argumentation. On a theoretical level, TAF challenges the long-standing assumption in gradual semantics that argument strength must be evaluated as a global, absolute property. By establishing localized, target-oriented semantics supported by a dedicated axiomatic system, we open a promising subfield of "influence-based" abstract argumentation, demonstrating that semantic rationality and mathematical rigor can be fully preserved locally. On a practical level, our framework bridges the historical gap between abstract argumentation theory and real-world AI deployment. Abstract

argumentation has often faced criticism for being computationally intractable and presenting "black-box" numerical outputs in gradual settings. By enabling dimensional decomposition and localized computation, TAF transforms argument evaluation into an interpretable, computationally lightweight decision support tool, making the formal integration of argumentation in high-stakes, dynamic domains, such as evidence-based medicine and legal reasoning, empirically viable.

In future work, we plan to generalize the TAF paradigm to cyclic structures and more expressive bipolar argumentation models, further expanding its theoretical properties and practical utility in complex decision-making domains. Furthermore, while we establish the foundational validity of the proposed local axioms, conducting a complete axiomatic characterization, including the formal proof of axiom independence and exhaustive mapping to global gradual principles, remains a dedicated objective of our future theoretical work.

Author contributions

Yunjie Zhang: Conceptualization, Methodology, Formal analysis, Software, Investigation, Visualization, Writing–original draft; Jiachao Wu: Conceptualization, Methodology, Validation, Supervision, Project administration, Writing–review and editing. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no competing interests.

References

1. P. M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games, *Artif. Intell.*, **77** (1995), 321–357. http://doi.org/10.1016/0004-3702(94)00041-X
2. T. J. M. Bench-Capon, P. E. Dunne, Argumentation in artificial intelligence, *Artif. Intell.*, **171** (2007), 619–641. http://doi.org/10.1016/j.artint.2007.05.001
3. P. Baroni, M. Caminada, M. Giacomin, An introduction to argumentation semantics, *Knowl. Eng. Rev.*, **26** (2011), 365–410. http://doi.org/10.1017/S0269888911000166
4. C. da Costa Pereira, A. G. B. Tettamanzi, S. Villata, Changing one's mind: Erase or rewind? possibilistic belief revision with fuzzy argumentation based on trust, In: *Proceedings of the 22nd international joint conference on artificial intelligence*, Barcelona, Catalonia, Spain, July 16–22, 2011, 164–171. http://doi.org/10.5591/978-1-57735-516-8/IJCAI11-039
5. L. Amgoud, D. Doder, S. Vesic, Evaluation of argument strength in attack graphs: foundations and semantics, *Artif. Intell.*, **302** (2022), 103607. http://doi.org/10.1016/j.artint.2021.103607

6. P. E. Dunne, A. Hunter, P. McBurney, S. Parsons, M. Wooldridge, Weighted argument systems: basic definitions, algorithms, and complexity results, *Artif. Intell.*, **175** (2011), 457–486. http://doi.org/10.1016/j.artint.2010.09.005
7. L. Amgoud, J. Ben-Naim, Ranking-based semantics for argumentation frameworks, In: *Scalable uncertainty management*, Berlin, Heidelberg: Springer, 2013, 134–147. http://doi.org/10.1007/978-3-642-40381-1_11
8. C. Cayrol, M.-C. Lagasquie-Schiex, Graduality in argumentation, *J. Artif. Intell. Res.*, **23** (2005), 245–297. http://doi.org/10.1613/jair.1411
9. Z. Wang, Y. Shen, Convergent semantics for weighted bipolar argumentation, *Proceedings of the 40th AAAI Conference on Artificial Intelligence*, **40** (2026), 19415–19423. http://doi.org/10.1609/aaai.v40i23.39019
10. N. Oren, B. Yun, S. Vesic, M. Baptista, Inverse problems for gradual semantics, In: *Proceedings of the 31st international joint conference on artificial intelligence*, 2022, 2719–2725. http://doi.org/10.24963/ijcai.2022/377
11. I. Ahmad, M. N. Khan, R. Jan, N. N. A. Razak, Efficient numerical method for pricing multi-asset options with the time-fractional Black-Scholes model: focus on American and digital options, *Math. Model. Control*, **5** (2025), 147–163. http://doi.org/10.3934/mmc.2025011
12. Z. Wang, Y. Shen, Fuzzy labelling semantics for quantitative argumentation, *J. Log. Comput.*, **35** (2025), exaf009. http://doi.org/10.1093/logcom/exaf009
13. L. Dupuis de Tarlé, E. Bonzon, N. Maudet, Multiagent dynamics of gradual argumentation semantics, In: *Proceedings of the 21st international conference on autonomous agents and multiagent systems*, 2022, 363–371.
14. M. Haripriya, A. Manivannan, S. Dhanasekar, S. Lakshmanan, Finite-time synchronization of delayed complex dynamical networks via sampled-data controller, *Math. Model. Control*, **5** (2025), 73–84. http://doi.org/10.3934/mmc.2025006
15. Z. Feng, Z. Lin, H. Wang, R. Ma, Y. Long, K. An, et al., Large AI models for empowering semantic communications in 6G non-terrestrial networks, *IEEE Netw.*, in press. http://doi.org/10.1109/MNET.2026.3704846
16. Y. He, Z. Lin, Z. Feng, D. Yan, LEO satellites meet movable antennas: an SEEI entropy driven agentic AI cross-domain optimization framework, *Sci. China Inf. Sci.*, **69** (2026), 200302. http://doi.org/10.1007/s11432-026-5056-5
17. T. Mayer, S. Marro, E. Cabrio, S. Villata, Enhancing evidence-based medicine with natural language argumentative analysis of clinical trials, *Artif. Intell. Med.*, **118** (2021), 102098. http://doi.org/10.1016/j.artmed.2021.102098
18. J. Fuller, B. Chin-Yee, R. E. G. Upshur, The argument framework is a flexible approach to evidence in healthcare, *Nat. Med.*, **30** (2024), 2113–2116. http://doi.org/10.1038/s41591-024-02930-x
19. S. Guedim, A. Benkerrouche, M. S. Souid, A. Amara, R. Jan, I. Ahmad, Initial value problem for fractional differential equations of variable order, *Math. Model. Control*, **5** (2025), 379–389. http://doi.org/10.3934/mmc.2025026

20. Z. Wang, Y. Shen, On SCC-recursiveness in quantitative argumentation, In: *Logic and argumentation*, Singapore: Springer, 2025, 309–326. http://doi.org/10.1007/978-981-96-7956-0_19

21. J. Wu, Relative attack strength and its application in argumentation frameworks, *AIMS Math.*, **10** (2025), 16355–16370. http://doi.org/10.3934/math.2025731

22. L. Malmqvist, T. Yuan, P. Nightingale, Approximating problems in abstract argumentation with graph convolutional networks, *Artif. Intell.*, **336** (2024), 104209. http://doi.org/10.1016/j.artint.2024.104209

23. N. Mirzakhmedova, M. Gohsen, C. H. Chang, B. Stein, Are large language models reliable argument quality annotators?, In: *Robust argumentation machines*, Cham: Springer, 2024, 129–146. http://doi.org/10.1007/978-3-031-63536-6_8

24. P. Baroni, F. Toni, B. Verheij, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games: 25 years later, *Argument Comput.*, **11** (2020), 1–14. http://doi.org/10.3233/AAC-200901

25. E. Bonzon, J. Delobelle, S. Konieczny, N. Maudet, A comparative study of ranking-based semantics for abstract argumentation, *Proceedings of the 30th AAAI Conference on Artificial Intelligence*, **30** (2016), 914–920. http://doi.org/10.1609/aaai.v30i1.10116

26. D. M. Gabbay, Equational approach to argumentation networks, *Argument Comput.*, **3** (2012), 87–142. http://doi.org/10.1080/19462166.2012.704398

27. C. Cayrol, M.-C. Lagasquie-Schiex, Gradual valuation for bipolar argumentation frameworks, In: *Symbolic and quantitative approaches to reasoning with uncertainty*, Berlin, Heidelberg: Springer, 2005, 366–377. http://doi.org/10.1007/11518655_32

28. X. Yin, N. Potyka, F. Toni, Explaining arguments' strength: unveiling the role of attacks and supports, In: *Proceedings of the 33rd international joint conference on artificial intelligence*, 2024, 3622–3630. http://doi.org/10.24963/ijcai.2024/401

29. L. Amgoud, Axiomatic characterisations of argumentation semantics, In: *Proceedings of the 22nd international conference on principles of knowledge representation and reasoning*, Melbourne, Australia, November 11–17, 2025, 36–45. http://doi.org/10.24963/kr.2025/4

30. P. Teovanović, S. Vesic, B. Yun, Human compliance with computational argumentation principles, *Artif. Intell.*, **351** (2026), 104457. http://doi.org/10.1016/j.artint.2025.104457

31. J. Wu, H. Li, N. Oren, T. J. Norman, Gödel fuzzy argumentation frameworks, In: *Computational models of argument*, IOS Press, 2016, 447–458. http://doi.org/10.3233/978-1-61499-686-6-447

32. S. Modgil, H. Prakken, The ASPIC+ framework for structured argumentation: a tutorial, *Argument Comput.*, **5** (2014), 31–62. http://doi.org/10.1080/19462166.2013.869766

33. M. Berthold, L. Blümel, A. Rapberger, On strong and weak admissibility in non-flat assumption-based argumentation, arXiv:2508.11182. http://doi.org/10.48550/arXiv.2508.11182

34. S. Zhao, J. Wu, An efficient algorithm of fuzzy reinstatement labelling, *AIMS Math.*, **7** (2022), 11165–11187. http://doi.org/10.3934/math.2022625

35. M. Bernreiter, W. Dvořák, A. Rapberger, S. Woltran, The effect of preferences in abstract argumentation under a claim-centric view, *J. Artif. Intell. Res.*, **81** (2024), 203–262. http://doi.org/10.1613/jair.1.15932

36. J. Dauphin, M. Cramer, L. van der Torre, A dynamic approach for combining abstract argumentation semantics, In: *Dynamics, uncertainty and reasoning*, Singapore: Springer, 2019, 21–43. http://doi.org/10.1007/978-981-13-7791-4_2



AIMS Press

© 2026 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0)