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Correction

Correction: Fractional behaviors analysis and modelling using DRT

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A correction on

Fractional behaviors analysis and modelling using DRT
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The author would like to make the following correction to the published paper [1]. Equation numbers refer to that manuscript.

1. Change of variable and corrected inversion

In Section 2.1, the change of variable $u = 1/\tau$ from Eq (2) to Eq (3) omits the factor $1/\tau$ already present in the integration measure. The Jacobian $d\tau = -du/u^2$ is correct, but the complete measure becomes:

$$\frac{d\tau}{\tau} = -\frac{du}{u}.$$

Reversing the integration limits gives the following replacement for Eq (3):

$$h(t) = \int_0^\infty \frac{G(1/u)}{u} e^{-tu}\, du, \quad t > 0. \qquad (3\ corrected)$$

Consequently, Eq (4) must be replaced by

$$\varphi(u) = \frac{G(1/u)}{u}, \quad u > 0. \qquad (4\ corrected)$$

The Laplace-transform interpretation in Eqs (5) to (7) remains valid with this definition. In particular, $\varphi = \mathcal{L}^{-1}\{h\}$. Substituting $u = 1/\tau$ gives $\varphi(1/\tau) = \tau G(\tau)$, rather than $\tau^2 G(\tau)$. Thus Eq (8) must be

$$G(\tau) \;=\; \frac{1}{\tau}[\mathcal{L}^{-1}\{h\}](1/\tau) \;=\; \frac{1}{2\pi i\tau}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{\frac{\zeta}{\tau}} h(\zeta)\; d\zeta. \qquad (8\ corrected)$$

Here $h(\zeta)$ denotes the analytic continuation of the impulse response in its time argument, whereas $H(s)$ is the transfer function. The vertical contour must lie in a domain where that continuation and Bromwich inversion are valid. If an ordinary inverse density does not exist, inversion must be understood in a suitable measure or distribution sense.

2. Exponential example and direct verification

For $h(t) = e^{-t/\tau_0}$, $\tau_0 > 0$, the inverse transform is $\varphi(u) = \delta(u - 1/\tau_0)$. Equation (9) contains an extraneous factor $h(s)$ after the exponential has already been substituted. Its corrected form is

$$G(u) \;=\; \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{su} e^{\frac{-s}{\tau_0}}\; ds \;=\; \delta(u \;-\; \frac{1}{\tau_0}). \qquad (9\ corrected)$$

Using the corrected Eq (4), Eq (10) remains valid:

$$G(1/u) \;=\; u\delta(u \;-\; \frac{1}{\tau_0}). \qquad (10)$$

The Dirac distribution must also be transformed with its Jacobian:

$$\delta(\frac{1}{\tau} \;-\; \frac{1}{\tau_0}) \;=\; {\tau_0}^2\delta(\tau \;-\; \tau_0).$$

and

$$G(\tau) \;=\; \frac{1}{\tau}\delta(\frac{1}{\tau} \;-\; \frac{1}{\tau_0}) \;=\; \tau_0\delta(\tau \;-\; \tau_0). \qquad (11, unchanged)$$

Substitution into the defining representations verifies the amplitude and time constant:

$$\int_0^\infty \frac{G(\tau)}{\tau} e^{\frac{-t}{\tau}}\; d\tau \;=\; e^{\frac{-t}{\tau_0}},$$

thus

$$H(s) \;=\; \int_0^\infty \frac{G(\tau)}{1+s\tau}\; d\tau \;=\; \frac{\tau_0}{1+s\tau_0}.$$

Equation (11) is therefore correct, although the preceding derivation was inconsistent. Applying the erroneous $1/\tau^2$ prefactor would instead produce $\delta(\tau \;-\; \tau_0)$, and hence $\left(e^{-t/\tau_0}\right)/\tau_0$, which has the wrong amplitude.

3. Consistency, interpretation and scope

The corrected φ is precisely the spectral density μ used in Section 2.2. **Equations (18) and (A.5) already contain the correct relation and need no change:**

$$\mu(u) = \varphi(u) = \frac{G(1/u)}{u}, \qquad G(\tau) = \frac{\mu(1/\tau)}{\tau}.$$

The density g used with the logarithmic measure in equation (A.6) differs from the density G used with $d\tau$. Their relation remains

$$g(\tau) = \tau G(\tau) = \mu(1/\tau).$$

As a further analytical check, the fractional integrator $H(s) = {\omega_0}^{\nu} s^{-\nu},\ 0 < \nu < 1$, has the impulse response and spectral density

$$h(t) = \frac{{\omega_0}^{\nu}}{\Gamma(\nu)} t^{\nu-1}, \qquad \varphi(u) = {\omega_0}^{\nu} \frac{sin(\pi\nu)}{\pi} u^{-\nu}.$$

The corrected inversion gives

$$G(\tau) = {\omega_0}^{\nu} \frac{sin(\pi\nu)}{\pi} \tau^{\nu-1},$$

in agreement with the CPE kernel in Appendix A.2.1.

The statement following Eq (8) concerning Bernstein's theorem also requires precision: Complete monotonicity implies a representation by a nonnegative measure, not necessarily by an ordinary density. The exponential example corresponds to an atomic measure. A nonnegative function G exists when the corresponding spectral measure has the required absolute continuity. A possible atom at zero relaxation rate must be treated separately as an integrator term.

These corrections restore consistency between Sections 2.1 and 2.2. The defining DRT representation, Eq (2), and the directly formulated identification Eqs (20)–(29) do not require a change as a consequence of this substitution.

Appendix A.2 also requires independent corrections in Sections A.2.5–A.2.7. These appendix corrections are separate from the reciprocal-substitution error addressed above.

4. Conclusions

The typographical omissions and transcription errors in the intermediate derivation addressed by this erratum do not alter the main mathematical result establishing the equivalence between the DRT and diffusive representations. The final relations (11), (18), and (A.5), as well as the CPE kernel verified above, remain valid and unchanged. The corrections restore consistency in the intermediate steps and do not modify these final results or the directly formulated identification relations (20)–(29).

The changes have no material impact on the conclusion of this article. The original manuscript will be updated [1]. We apologize for any inconvenience caused to our readers by this change.

Conflict of interest

The authors declare no conflicts of interest.

References

1. J. Sabatier, H. Hallil, Fractional behaviors analysis and modelling using DRT, *AIMS Math.*, **11** (2026), 26503–26537. https://doi.org/10.3934/math.20261063



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