



Research article

Event-triggered control of nonminimum phase-switched systems under denial-of-service attacks

Khalil Jouili^{1,*}, Mongi Ben Moussa² and Monia Charfeddine¹

¹ Laboratory of Advanced Systems, Polytechnic School of Tunisia (EPT), B. P. 743, 2078 Marsa, Tunisia

² Departement of Physics, College of Sciences, Umm Al-Qura University, 21955 Makkah, Saudi Arabia

* **Correspondence:** Email: khalil.jouili@ept.ucar.tn; Tel: +216 25544984.

Abstract: This work addresses the problem of robust stabilization for uncertain switched nonlinear systems with nonminimum phase behavior subject to aperiodic denial-of-service (DoS) attacks. A new event-triggered sliding mode control scheme is developed that simultaneously handles unstable internal dynamics and intermittent communication blockages. The method combines a mode-dependent event-triggered mechanism (MDET) with a sample-and-hold control input, substantially reducing communication requirements. Each nonminimum phase subsystem is equipped with a virtual sliding manifold designed to stabilize its unstable internal dynamics. Control updates are triggered only when a dynamic threshold—function of the sliding variable and the disturbance bound—is violated. Between triggers, the control signal is held constant. Using piecewise Lyapunov functions and the admissible edge-dependent average dwell time (AED-ADT) approach, semiglobal input-to-state stability (ISS) is established despite external disturbances and aperiodic DoS attacks. Unlike existing event-triggered sliding mode control schemes, which typically assume minimum-phase dynamics or neglect the internal dynamics, the proposed approach explicitly stabilizes the zero dynamics of nonminimum phase switched systems while remaining resilient to aperiodic DoS attacks. The mechanism also prevents Zeno behavior by enforcing a positive minimum interevent time. Simulation results on a benchmark system confirm the robustness and efficiency of the proposed strategy.

Keywords: event-triggered control; sliding mode control; switched nonlinear systems; nonminimum phase systems; denial-of-service (DoS) attacks; input-to-state stability; mode-dependent event-triggering mechanism; admissible edge-dependent average dwell time; Zeno-free behavior;

robust stabilization

Mathematics Subject Classification: 93C10, 93C30, 93D21

Notation

$\mathbb{R}$	Set of real numbers
$\mathbb{R}_n$	n -dimensional Euclidean space
$\ x\ $	Euclidean norm of vector x
$\ \theta\ _{[t_1, t_2]}$	Supremum norm of θ over $[t_1, t_2]$
$\mathcal{K}$	Class of strictly increasing functions with $\alpha(0)=0$
$\mathcal{K}_\infty$	Class $\mathcal{K}$ functions that are unbounded
$\mathcal{KL}$	Class of functions decreasing to zero in the second argument
$\sigma(t)$	Switching signal
$\theta(t)$	External disturbance
S_i	Virtual sliding variable for mode i
σ_i	Extended sliding variable for mode i
L_i	Gain matrix for the extended sliding manifold
$\mu_{j,i}$	Lyapunov ratio at switching from mode j to mode i
τ_D	DoS frequency bound
τ_d	DoS duration bound
$\Delta_{\max}$	Maximum allowable interevent time
$\Delta_{\min}$	Minimum interevent time

1. Introduction

Switched nonlinear systems constitute a fundamental class of hybrid dynamical systems that arise naturally in numerous engineering applications, including power electronics, chemical process control, robotic manipulators, and networked control systems [1–5]. Such systems consist of a finite family of nonlinear subsystems together with a switching signal that orchestrates the active mode at each time instant. A central challenge in switched-systems theory lies in the fact that stability of each individual subsystem does not guarantee stability of the overall switched system, and

conversely, a suitable switching strategy may render the closed loop stable even when all subsystems are unstable [6,7].

Among the many control objectives for switched nonlinear systems, robust stabilization in the presence of model uncertainties and external disturbances remains particularly demanding. This difficulty is amplified when the subsystems exhibit nonminimum phase (NMP) behavior, that is, when their internal dynamics are unstable [8,9]. NMP systems defy standard input-output linearization because the unstable internal dynamics cannot be canceled by feedback without risking instability. Classical approaches to NMP nonlinear systems rely on tools such as output redefinition, approximate linearization, or sliding mode control, yet their extension to the switched setting is far from trivial [10–12].

In a notable contribution, [13] addressed the robust stabilization of switched nonlinear systems where every subsystem may be NMP. The key idea was to construct, for each mode, a stabilizing sliding mode controller capable of forcing the unstable internal dynamics onto a stable sliding surface. Moreover, the author exploited the fact that the Byrnes–Isidori normal form admits infinitely many diffeomorphisms. A switching strategy was then devised to select, among this infinity of diffeomorphisms, the one that yields the best transient performance. The resulting closed-loop system was shown to be globally asymptotically stable under a suitable switching logic. Although effective, the method in [13] operates under a continuous-time control update paradigm, which imposes two major limitations in modern networked control environments.

First, continuous control updates consume significant communication bandwidth, a critical issue when the controller and plant are connected via shared or wireless networks [14,15]. Second, the continuous transmission of control signals makes the system highly vulnerable to cyber attacks, especially aperiodic denial-of-service (DoS) attacks that block communication channels intermittently [16,17]. Under such attacks, the continuous control signal may be completely unavailable during active attack intervals, potentially leading to instability. Furthermore, continuous sliding mode control is known to suffer from chattering and may exhibit accumulation of switching events if not properly regularized [18].

To overcome these limitations, event-triggered control (ETC) has emerged as a powerful paradigm for resource-aware networked systems [19–21]. In ETC, control updates are executed only when a predefined state-dependent condition is violated, thereby reducing the number of transmissions. Recent advances in event-triggered control have addressed various network constraints, including output-based triggering for nonlinear systems [21], periodic event-triggered schemes for resource-constrained networks [22], and event-triggered stabilization of switched systems [23]. However, these works generally assume minimum-phase dynamics or do not consider NMP behavior, and they do not address the combined challenge of unstable zero dynamics and aperiodic DoS attacks. More recently, event-triggered (ET) control has gained attention, where the control input is applied as an impulse at discrete triggering instants and remains constant between them [24,25]. This hybrid scheme is particularly well-suited for systems with discontinuous actuation and can inherently exclude Zeno behavior by establishing a positive lower bound on interevent intervals.

Crucially, the combination of ET control with sliding mode techniques for NMP switched systems remains largely unexplored. A recent work by [26] investigated ET control for switched systems under aperiodic DoS attacks using a mode-dependent event-triggered mechanism (MDET) and the admissible edge-dependent average dwell time (AED-ADT) approach. However, their

analysis assumed that the zero dynamics of each subsystem are stable (minimum phase), and they did not address the challenge of unstable internal dynamics. To clearly position the proposed approach, this subsection provides a comparative analysis against the most recent related methods.

First, although event-triggered sliding mode control has been extensively studied for nonlinear systems, most existing results, such as those in [20,27,28], assume minimum-phase dynamics or do not address the stabilization of unstable internal dynamics. In contrast, the proposed architecture explicitly handles nonminimum phase subsystems by designing an extended sliding manifold that stabilizes the zero dynamics.

Second, recent advances in control under DoS attacks have focused on multiagent systems and consensus problems [29,25]. However, these works typically do not consider switched nonlinear systems with nonminimum phase characteristics, nor do they address unstable zero dynamics. The proposed approach integrates an MDETM with sliding mode control, providing a unified architecture for DoS-resilient stabilization of switched nonminimum phase systems.

Third, the AED-ADT methodology has been applied to the stability analysis of switched systems [30,31], but its combination with event-triggered sliding mode control for nonminimum phase systems remains unexplored. This paper leverages AED-ADT to derive a novel input-to-state stability (ISS) condition that explicitly quantifies the trade-off between the decay rate, DoS parameters, and switching penalties.

Fourth, stabilization of nonminimum phase switched-systems has been addressed using continuous sliding mode control [13] and reduced-order sliding mode design [32]. However, these methods do not incorporate event-triggered mechanisms or DoS resilience. The proposed work provides the first event-triggered sliding mode solution for this class of systems, with rigorous ISS and Zeno-free guarantees.

The specific novelties of this paper are: (i) the first event-triggered sliding mode control approach for switched NMP systems with unstable zero dynamics; (ii) an MDETM that uses a full-state Lyapunov function to ensure stability of the complete state; (iii) a rigorous ISS analysis under aperiodic DoS attacks that explicitly incorporates switching penalties and AED-ADT bounds; (iv) an explicit positive lower bound on interevent times, proving Zeno-free behavior even in the presence of mode switches and DoS attacks; and (v) a practical design procedure with semiglobal ISS guarantees on a clearly specified domain.

Compared with the recent event-triggered results in [32–34], our work differs in three key aspects: (a) We explicitly address unstable zero dynamics, which are not treated in [32–34]; (b) we consider aperiodic DoS attacks that block some triggers, whereas [32–34] assume reliable communication; and (c) we provide a sample-and-hold implementation with a rigorous Zeno-free proof, which is not available in [32–34].

Motivated by the limitations of existing approaches, this work develops a new event-triggered sliding-mode control (ETISMC) strategy for the robust stabilization of uncertain switched nonlinear systems characterized by NMP behavior. The proposed strategy combines a mode-dependent event-triggered (ET) mechanism with sliding mode control (SMC), where control actions are updated only at discrete triggering instants determined by a dynamic threshold related to the active subsystem and disturbance level, thereby significantly reducing communication and computation requirements compared with continuous-time methods. To address the instability of internal dynamics in NMP subsystems, a dedicated virtual sliding manifold is designed for each mode, allowing the switched control law to ensure stability even under sporadic updates. Moreover, by

employing piecewise Lyapunov functions together with the AED-ADT technique, sufficient conditions are established to guarantee semiglobal ISS in the presence of external disturbances, modeling uncertainties, and aperiodic DoS attacks that may block certain transmissions. The proposed triggering mechanism also guarantees the exclusion of Zeno behavior through the derivation of a strictly positive lower bound on interevent intervals. Simulation studies conducted on a benchmark switched nonlinear system demonstrate that the proposed method preserves stabilization performance while considerably reducing the number of control updates and maintaining strong robustness against intermittent DoS attacks.

The remainder of this paper is organized as follows: Section 2 recalls the system model and the concept of NMP switched systems along with the MDETm and DoS model. Section 3 presents the design of the ETISMC. Section 4 provides the main stability theorems, including ISS analysis and Zeno-freeness. Section 5 presents numerical simulations, and Section 6 concludes the paper.

2. Preliminaries and problem formulation

This section establishes the mathematical foundations, notation, and key definitions required for the subsequent development. We describe the class of uncertain switched nonlinear systems under consideration, recall the concept of NMP dynamics, introduce the MDETm, and define the aperiodic DoS attack model together with the stability notion of ISS.

2.1. System description

Consider the following class of uncertain switched nonlinear systems:

$$\dot{x}(t) = F_{\sigma(t)}(x(t), \theta(t)) + G_{\sigma(t)}(x(t), \theta(t))u_{\sigma(t)}(t), \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\theta(t) \in \mathbb{R}^p$ is an unknown but bounded external disturbance or lumped uncertainty, and $u_i(t) \in \mathbb{R}$ is the control input associated with the active subsystem i . The switching signal $\sigma(t): \mathbb{R}_{\geq 0} \rightarrow M = \{1, 2, \dots, m\}$ is a right-continuous piecewise constant function that determines which subsystem is active at time t . The switching instants are denoted by $0 = t_0 < t_1 < t_2 < \dots$, with $\sigma(t)$ constant on each interval $[t_j, t_{j+1})$.

For each $i \in M$, the vector fields $F_i: \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$ and $G_i: \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$ are smooth (infinitely differentiable) functions satisfying $F_i(0, \theta) = 0$ and $G_i(0, \theta) = 0$ for all θ . The disturbance $\theta(t)$ is assumed to be locally Lipschitz and uniformly bounded that is, there exists a known constant $\bar{\theta} > 0$ such that $\|\theta(t)\| \leq \bar{\theta}$ for all $t \geq 0$.

The system (1) is assumed to be locally input-output linearizable with a well-defined relative degree $r_i \leq n$ for each mode i .

Specifically, for a chosen smooth output function $y = h_i(x)$, the Lie derivatives satisfy $L_{G_i} L_{F_i}^j h_i(x) = 0$ for all $0 \leq j < r_i - 1$ and $L_{G_i} L_{F_i}^{r_i-1} h_i(x) \neq 0$ in a neighborhood of the equilibrium $x = 0$. This assumption allows us to transform each subsystem into a normal form via a local diffeomorphism.

For each fixed mode i , there exists a diffeomorphism $\Phi_i(x)$ defined on a neighborhood of the origin that brings the subsystem into the Byrnes-Isidori normal form [8, 9]. Partition the transformed

coordinates as $\Phi_i(x) = (\xi_i, \eta_i)^T$; $\xi_i \in \mathbb{R}^{r_i}$ represents the input-output (IO) dynamics and $\eta_i \in \mathbb{R}^{n-r_i}$ the internal (zero) dynamics. In these coordinates, the subsystem dynamics read:

$$\begin{cases} \dot{\xi}_{i,1} = \xi_{i,2} \\ \dot{\xi}_{i,2} = \xi_{i,3} \\ \vdots \\ \dot{\xi}_{i,r_i} = a_i(\xi_i, \eta_i) + \phi_i(\xi_i, \eta_i, \theta_i) + b_i(\xi_i, \eta_i) u_i \\ \dot{\eta}_i = \Psi_i(\xi_i, \eta_i) \end{cases} \quad (2)$$

with $b_i(\xi_i, \eta_i) \neq 0$ for all (ξ_i, η_i) in the domain. The scalar function $a_i(\cdot)$ represents the known nominal part, and $\phi_i(\cdot)$ captures the effect of the disturbance $\theta(t)$ and possible modeling errors. The output is taken as $y = \xi_{i,1}$.

For each mode i , the diffeomorphism Φ_i is defined on an open neighborhood $D_i \subset \mathbb{R}^n$ of the origin. On this domain, the transformation is a local diffeomorphism, and the normal form (2) is valid. The domain D_i is assumed to be large enough to contain all reachable states under the proposed control law, starting from any initial condition in a given compact set $C_i \subset D_i$. The region where $b_i(\xi_i, \eta_i) \neq 0$ is specified as $B_i = \{(\xi_i, \eta_i) \in D_i : b_i(\xi_i, \eta_i) \neq 0\}$, and we assume that on the compact set of interest, b_i is bounded away from zero that is, there exists $\bar{b}_{\min} > 0$ such that $|b_i(\xi_i, \eta_i)| \geq \bar{b}_{\min}$ for all $(\xi_i, \eta_i) \in C_i$.

The i th subsystem is said to be NMP if its zero dynamics, obtained by setting $\xi_i = 0$ in (2) that is, $\dot{\eta}_i = \Psi_i(0, \eta_i)$, are unstable (i.e., not asymptotically stable at $\eta_i = 0$). Conversely, it is minimum phase if the zero dynamics are asymptotically stable. In this work, we consider the challenging scenario where each subsystem may be NMP. Consequently, the unstable internal dynamics cannot be simply ignored or canceled by feedback without a dedicated stabilization strategy.

A crucial observation exploited in [13] is that the normal form (2) is not unique. Indeed, for a given output function $h_i(x)$, the choice of the complementary coordinates $\chi_{j,i}(x)$ satisfying $L_{G_i} \chi_{j,i}(x) = 0$ for $j = 1, \dots, n - r_i$ is not unique; there exist infinitely many such functions, each yielding a different representation of the internal dynamics.

Formally, for each mode i , one can construct a family of diffeomorphisms $\{\Phi_i^{(k)}(x)\}_{k \in \mathbb{K}_i}$ indexed by an arbitrary smooth function. This degree of freedom can be exploited to select a representation that facilitates stabilization. In the proposed approach, we will use this flexibility to design a sliding surface that renders the reduced order dynamics on the sliding manifold exponentially stable, even when the original zero dynamics are unstable.

2.2. Mode-dependent event-triggered control

To reduce communication and enhance resilience against DoS attacks, we depart from continuous time control and adopt an event-triggered sample and hold strategy. The control input is updated only at discrete instants $\{t_k\}_{k=0}^{\infty}$ determined by a mode-dependent event-triggered (MDET) condition. Between updates, the control signal is held constant (zero-order hold). Moreover, when an update is blocked by a DoS attack, the previous control value is maintained. Let $\{t_k\}$ denote the sequence of successful triggering instants (i.e., instants at which a new control value is computed and, if not

blocked, applied). The control input is then defined as $u_i(t) = u_i(t_k)$ for all $t \in [t_k, t_{k+1})$. Because the state does not experience any instantaneous jump at the triggering instants, the proposed scheme is a genuine sample and hold implementation, not an system. The MDETm is defined as follows:

$$t_k = \inf \left\{ t > t_{k-1} \left\| S_{\sigma(t)} \right\| \geq e^{a_k} \left\| S_{\sigma(t_{k-1})}(t_{k-1}) \right\| + e^{b_k} \chi \left(\left\| \theta \right\|_{[t_{k-1}, t]} \right) \right\}, \quad (3)$$

where $S_i(t)$ is a suitably designed sliding variable associated with mode i , $a_k, b_k > 0$ are design parameters that may vary with k , $\chi \in \mathcal{K}_\infty$ is a known function that quantifies the influence of the disturbance, and $\left\| \theta \right\|_{[t_{k-1}, t]}$ is the supremum norm of the disturbance over the interval $[t_{k-1}, t]$.

The mechanism (3) is mode-dependent (MD) because the sliding variable $S_{\sigma(t)}(t)$ and the previous value $S_{\sigma(t_{k-1})}(t_{k-1})$ depend on the active subsystem via the switching signal.

When the condition is met, a new control value is computed using the current state and applied instantaneously. The control input then remains constant until the next triggering instant. If a DoS attack is active at time t_k , the update is blocked; the system continues to evolve with the previous control value, and the triggering mechanism remains enabled.

We consider aperiodic DoS attacks that intermittently block the communication channel from the controller to the actuator. Following the model in [16,26], the attack is characterized by intervals during which no control update can be transmitted.

Let $\{D_n\}_{n \in \mathbb{N}}$ be the sequence of DoS active intervals, where $D_n = [h_n, h_n + d_n)$ is the start time and $d_n > 0$ the duration. The complementary intervals $H_n = [h_n + d_n, h_{n+1})$ are the DoS sleep (attack-free) intervals. Over a time interval $[t_0, t)$, define:

$$\Theta_d(t_0, t) = \bigcup_n D_n \cap [t_0, t) \quad \text{total time under attack};$$

$$\Theta_h(t_0, t) = [t_0, t) \setminus \Theta_d(t_0, t) \quad \text{total attack free time}.$$

Assumption 1. There exist constants $N_0 \geq 0, \tau_D > 0, \theta_0 \geq 0$, and $\tau_d > 1$ such that for all $t \geq t_0$,

$$N_d(t_0, t) \leq N_0 + \frac{t - t_0}{\tau_D}, \quad (4)$$

$$|\Theta_d(t_0, t)| \leq \theta_0 + \frac{t - t_0}{\tau_d}, \quad (5)$$

where $N_d(t_0, t)$ denotes the number of DoS active intervals over $[t_0, t)$. Condition (4) limits the frequency of attacks, and (5) limits the total duration of attacks relative to the elapsed time. The constants τ_D and τ_d reflect the average dwell time of the attack and the average duty cycle, respectively. Because the system is subject to both external disturbances $\theta(t)$ and possible DoS-induced packet losses, we adopt the concept of ISS as the appropriate robustness notion.

System (1) under the ET control (3) is said to be ISS if there exist a class KL function β and a class K function γ such that for every initial state $x(t_0) = x_0$ and every bounded disturbance $\theta(\cdot)$, the corresponding solution satisfies:

$$\|x(t)\| \leq \beta(\|x_0\|, t - t_0) + \gamma\left(\left\| \theta \right\|_{[t_0, t]}\right), \quad \forall t \geq t_0. \quad (6)$$

ISS guarantees that the state remains bounded by a decaying function of the initial condition plus a term depending on the disturbance amplitude. In the absence of disturbances, it reduces to semiglobal asymptotic stability.

Switched systems with unstable subsystems may still be stabilizable by a suitable switching law if the average time spent in each mode respects certain constraints. For systems where the stability properties differ from one mode to another (e.g., some modes are stabilizable only on specific sliding surfaces), we employ the AEDADT strategy.

For a switched system with modes M , let $N_{j,i}^\sigma(t, t_0)$ denote the number of switches from subsystem j to subsystem i over the interval $[t_0, t)$, and let $T_{j,i}(t, t_0)$ denote the total time during which the system has been in mode i after a switch from mode j . If there exist constants $N_{j,i}^{\sigma,0}$ and $\tau_{j,i}^\sigma > 0$ such that

$$N_{j,i}^{\sigma,0}(t, t_0) \leq N_{j,i}^{\sigma,0} + \frac{T_{j,i}(t, t_0)}{\tau_{j,i}^0}, \quad \forall t \geq t_0, \quad (7)$$

then the switching signal σ is said to have admissible edge dependent average dwell time $\tau_{j,i}^\sigma$ with chatter bounds $N_{j,i}^{\sigma,0}$.

The AEDADT concept generalizes the classical average dwell time (ADT) by allowing different lower bounds on the dwell time depending on the ordered pair of consecutive modes. This flexibility is essential when the Lyapunov function gains $\mu_{j,i}$ (the ratio of Lyapunov functions before and after a switch) vary with the switching edge.

2.3. Problem formulation

Given the uncertain switched nonlinear system (1) where each mode $i \in M$ may be NMP, and subject to aperiodic DoS attacks satisfying (4) and (5), design an MDET sliding-mode control law of the form $u_i(t) = u_i(t_k)$ for $t \in [t_k, t_{k+1}]$, with t_k defined by (3), such that

- i. The closed-loop system is ISS with respect to the disturbance $\theta(t)$;
- ii. The control updates occur only when necessary, and Zeno behavior (infinitely many updates in finite time) is strictly excluded;
- iii. The system remains ISS even when a fraction of the impulses is blocked by DoS attacks, provided that the attack parameters satisfy (4) and (5), and a suitable ISS condition (to be derived) holds.

3. Design of the event-triggered sample and hold control

This section presents the detailed construction of the proposed ETISMC law. We first design a virtual sliding surface for each NMP subsystem, which is instrumental in stabilizing the unstable internal dynamics. Then, we introduce a coordinate transformation that reformulates the system dynamics in a form amenable to Lyapunov analysis. Subsequently, we define the MDETM that dictates when control impulses are generated. Finally, we synthesize the sliding mode controller and discuss its implementation under aperiodic DoS attacks.

For each mode $i \in M$, consider the normal form (2) with relative degree r_i . Following the classical sliding mode approach [18], we define a virtual sliding variable S_i as a linear combination of the IO states $\begin{bmatrix} \xi_{i,1} & \xi_{i,2} & \dots & \xi_{i,r_i} \end{bmatrix}^T$,

$$S_i = \xi_{i,r_i} + l_{r_i-1,i} \xi_{i,r_i-1} + \dots + l_{1,i} \xi_{i,1}, \quad (8)$$

where the coefficients $l_{1,i}, \dots, l_{r_i-1,i}$ are chosen such that the polynomial $p^{r_i-1} + l_{r_i-1,i} p^{r_i-2} + \dots + l_{2,i} p + l_{1,i}$ is Hurwitz (i.e., all its roots have strictly negative real parts). The variable S_i is called virtual because it will later be forced to zero via the sliding-mode controller, thereby imposing a stable linear dynamics on the part. However, in NMP systems, setting $S_i = 0$ alone does not guarantee stability of the internal states η_i . Therefore, we need to further modify the sliding manifold to also stabilize the zero dynamics.

To simultaneously stabilize both the IO dynamics and the unstable internal dynamics, we exploit the multiplicity of diffeomorphisms described in Section 2. For each mode i , we introduce a new set of coordinates (ζ_i, S_i) by augmenting the original normal form with an additional linear combination. Define

$$\zeta_i = \begin{bmatrix} \xi_{i,1} & \xi_{i,2} & \dots & \xi_{i,r_i-1} & \eta_i^T \end{bmatrix}^T \in \mathbb{R}^{n-1}. \quad (9)$$

The extended sliding manifold is then defined as

$$\sigma_i = S_i - L_i \zeta_i, \quad (10)$$

where $L_i \in \mathbb{R}^{1 \times (n-1)}$ is a constant gain matrix to be designed. The rationale is that when the system is confined to the manifold $\sigma_i = 0$ ($S_i = L_i \zeta_i$), the reduced order dynamics become

$$\dot{\zeta}_i = (A_{\zeta}^i + A_S^i L_i) \zeta_i + \bar{\delta}_i(\zeta_i, \sigma_i) + \text{high order terms}, \quad (11)$$

where A_{ζ}^i and A_S^i are the linear parts obtained from the Jacobian of the system in (ζ_i, S_i) coordinates. By choosing L_i such that the matrix $(A_{\zeta}^i + A_S^i L_i)$ is Hurwitz (i.e., all eigenvalues have negative real parts), we can ensure that the reduced dynamics (11) are exponentially stable in a neighborhood of the origin, provided the nonlinear terms are sufficiently small.

Consider the reduced dynamics (11) on the sliding manifold $\sigma_i = 0$. Suppose the pair (A_{ζ}^i, A_S^i) is controllable. Then, for any desired set of eigenvalues $\Lambda_i = \{\lambda_1, \dots, \lambda_{n-1}\}$ with strictly negative real parts, there exists a gain matrix $L_i \in \mathbb{R}^{1 \times (n-1)}$ such that the matrix $(A_{\zeta}^i + A_S^i L_i)$ has eigenvalues exactly Λ_i . Because (A_{ζ}^i, A_S^i) is controllable, there exists a state feedback gain L_i that assigns the eigenvalues of $(A_{\zeta}^i + A_S^i L_i)$ arbitrarily. A constructive procedure is given by the formula: $L_i [0 \ \dots \ 0 \ 1] C_i^{-1} \phi_i(A_{\zeta}^i)$, where $C_i = \begin{bmatrix} A_S^i & A_{\zeta}^i A_S^i & \dots & (A_{\zeta}^i)^{n-2} A_S^i \end{bmatrix}$ is the controllability matrix, and $\phi_i(\cdot)$ is the desired characteristic polynomial. This guarantees that the reduced dynamics on the sliding manifold are exponentially stable with the prescribed decay rate.

For each mode i , we construct the diffeomorphism $T_i : (\xi_i, \eta_i) \rightarrow (\zeta_i, S_i)$ given by

$$\begin{bmatrix} \zeta_i \\ S_i \end{bmatrix} = \begin{bmatrix} I_{n-1} & 0_{n-1,1} \\ L_i & 1 \end{bmatrix} \begin{bmatrix} \xi_{i,1} \\ \vdots \\ \xi_{i,r_i-1} \\ \eta_i \\ \xi_{i,r_i} \end{bmatrix}. \quad (12)$$

The transformation T_i is a local diffeomorphism on the domain D_i . The new coordinates (ζ_i, S_i) are defined on the image $\mathfrak{T}_i = T_i(D_i)$, which is an open neighborhood of the origin in R^n . The reduced dynamics on the sliding manifold $\sigma_i = 0$ are analyzed on the subset $\mathfrak{T}_i \cap \{\sigma_i = 0\}$.

Applying the diffeomorphism T_i to the subsystem dynamics (2) and using the chain rule, we obtain: $\dot{\zeta}_i = (\partial \zeta_i / \partial \xi_i) \dot{\xi}_i + (\partial \zeta_i / \partial \eta_i) \dot{\eta}_i$ and $\dot{S}_i = (\partial S_i / \partial \xi_i) \dot{\xi}_i + (\partial S_i / \partial \eta_i) \dot{\eta}_i$. Substituting (2) and linearizing around the origin yields the matrices $A_{\zeta}^i, A_S^i, \bar{A}_{\zeta}^i, \bar{A}_S^i$ and the nonlinear terms $\Delta_{\zeta,i}$ and $\Delta_{S,i}$ collect the higher-order terms yielding the following representation:

$$\begin{cases} \dot{\zeta}_i = A_{\zeta}^i \zeta_i + A_S^i S_i + \Delta_{\zeta,i}(\zeta_i, S_i, \theta) \\ \dot{S}_i = \bar{A}_{\zeta}^i \zeta_i + \bar{A}_S^i S_i + \bar{b}_i(\zeta_i, S_i) u_i + \Delta_{S,i}(\zeta_i, S_i, \theta) \end{cases} \quad (13)$$

where $A_{\zeta}^i \in \mathfrak{R}^{(n-1) \times (n-1)}$, $A_S^i \in \mathfrak{R}^{(n-1) \times 1}$, $\bar{A}_{\zeta}^i \in \mathfrak{R}$ are constant matrices obtained from the linearization of the system around $(\zeta_i, S_i) = 0$, $\bar{b}_i(\zeta_i, S_i)$ is the control coefficient in the new coordinates, which is smooth and bounded away from zero in a neighborhood of the origin. $\Delta_{\zeta,i}(\cdot)$ and $\Delta_{S,i}(\cdot)$ collect all higher-order nonlinearities and disturbance terms.

They satisfy $\|\Delta_{\zeta,i}(\cdot)\| \leq \kappa_{\zeta,i}(\|\zeta_i\| + \|S_i\|)\|\theta\|$ and $\|\Delta_{S,i}(\cdot)\| \leq \kappa_{S,i}(\|\zeta_i\| + \|S_i\|)\|\theta\|$ locally, with $\kappa_{S,i}, \kappa_{\zeta,i}$ class K functions.

This representation is crucial because it separates the sliding variable dynamics from the remaining states, and it highlights that the control input u_i appears affinely in the $\dot{S}_i$ equation.

We define the event-triggered (ET) instants $\{t_k\}_{k=0}^{\infty}$ that determine when a new control impulse is computed. The mechanism is mode dependent, meaning that the triggering condition uses the sliding variable of the currently active subsystem as well as the sliding variable value at the last successful trigger.

Let t_{k-1} be the last successful triggering instant (i.e., the last time a control impulse was generated and, if not blocked by DoS, applied). For $t > t_{k-1}$, define the prediction error between the current sliding variable and its value at the previous trigger:

$$e_S(t) = \|S_{\sigma(t)}(t)\| - e^{a_k} \|S_{\sigma(t_{k-1})}(t_{k-1})\| - e^{b_k} \chi(\|\theta\|_{[t_{k-1}, t]}), \quad (14)$$

where $a_k, b_k > 0$ are design parameters, and $\chi \in K_{\infty}$ is a given function that weighs the effect of the disturbance. The MDET is now defined using the full Lyapunov function $V_{\sigma(t)}$:

$$t_k = \inf \left\{ t > t_{k-1} \mid V_{\sigma(t)} x(t) \geq e^{a_k} V_{\sigma(t_{k-1})} x(t_{k-1}) + e^{b_k} \chi(\|\theta\|_{[t_{k-1}, t]}) \right\}. \quad (15)$$

This ensures that the Lyapunov function of the full state is bounded between triggers.

If the right-hand side is never reached, then no further trigger occurs; however, in practice, we also impose a maximum allowable interevent time $\Delta_{\max} > 0$ (a forced trigger) to prevent the interevent interval from becoming arbitrarily large. Thus, the actual trigger is:

$$t_k = \min \left\{ \inf \left\{ t > t_{k-1} \mid e_S(t) \geq 0 \right\}, t_{k-1} + \Delta_{\max} \right\}. \quad (16)$$

The mechanism (15) and (16) is mode-dependent because the sliding variable $S_{\sigma(t)}(t)$ depends on the active mode $\sigma(t)$, which may change due to switching between t_{k-1} and t_k . This coupling between the event trigger and the switching signal is handled in the stability analysis using the AED-ADT method.

At each switching instant t_s , the event-triggering mechanism is reset by forcing a trigger, that is, we set $t_k = t_s$. The reference values for the event condition, namely $S_{\sigma(t_s)}(t_s)$ and $V_{\sigma(t_s)}(t_s)$, are recomputed using the new mode's coordinates. This ensures that the jump in the sliding variable does not cause missed or spurious triggers.

When a triggering instant t_k occurs and the communication channel is not blocked by a DoS attack, the controller computes a new control value based on the current state.

Remark 1. The jump in the Lyapunov function at a switching instant is bounded by the switching condition (26), that is, $V_i \leq \mu_{j,i} V_j$. Therefore, the forced trigger at t_s does not lead to an unbounded growth of the event threshold. Moreover, because the AED-ADT condition (7) bounds the number of switches over any interval, the additional forced triggers do not affect the Zeno-free property.

Following the equivalent control principle, we decompose the control law into two parts:

$$u_i(t_k) = u_i^{eq}(t_k) + u_i^{sw}(t_k). \quad (17)$$

The equivalent control u_i^{eq} is designed to cancel the known terms in the $\dot{S}_i$ dynamics, thereby keeping the system on the sliding manifold $\sigma_i = 0$ in the absence of disturbances and nonlinearities. From the second equation in (13), setting $\dot{S}_i = 0$ and ignoring the disturbance terms gives:

$$u_i^{eq}(t_k) = -\frac{1}{\bar{b}_i(\zeta_i(t_k), S_i(t_k))} \left(\bar{A}_\zeta^i \zeta_i(t_k) + \bar{A}_S^i S_i(t_k) - L_i A_\zeta^i \zeta_i(t_k) + A_S^i S_i(t_k) \right). \quad (18)$$

The equivalent control is derived from the invariance condition $\dot{\sigma}_i = \dot{S}_i - L_i \dot{\zeta}_i = 0$, which ensures that the extended sliding manifold is positively invariant. This corrects the earlier derivation that neglected the term $L_i \dot{\zeta}_i$.

The switching control u_i^{sw} is added to enforce convergence to the sliding manifold in finite time and to compensate for uncertainties and disturbances. To account for the fact that the control is held constant between triggers, we include a sampling error compensation term:

$$u_i^{sw}(t_k) = -\frac{1}{\bar{b}_i(\zeta_i(t_k), S_i(t_k))} \left(K_i \operatorname{sgn}(\sigma_i(t_k) + E_i \sigma_i(t_k)) + \varepsilon_i (\zeta_i(t_k) - \zeta_i(t_{k-1})) \right), \quad (19)$$

where $\varepsilon_i > 0$ is a design parameter, and the term $(\zeta_i(t_k) - \zeta_i(t_{k-1}))$ represents the state variation since the last trigger.

The control law is derived by setting $\dot{S}_i = 0$ in (13) and solving for u_i . The equivalent control part cancels the known linear terms. The switching part is designed to dominate the uncertainty and the sampling error.

This additional term is bounded by the event triggering condition, ensuring that it does not destabilize the system. The complete control law for mode i is therefore:

$$u_i(t_k) = -\frac{1}{\bar{b}_i(\zeta_i(t_k), S_i(t_k))} \left(\bar{A}_\zeta^i \zeta_i(t_k) + \bar{A}_S^i S_i(t_k) - L_i A_\zeta^i \zeta_i(t_k) + A_S^i S_i(t_k) + K_i \operatorname{sgn}(\sigma_i(t_k) + E_i \sigma_i(t_k)) + \varepsilon_i(\zeta_i(t_k) - \zeta_i(t_{k-1})) \right). \quad (20)$$

Between triggers, the control input is held constant:

$$u_i(t) = u_i(t_k), \forall t \in [t_k, t_{k+1}), \quad (21)$$

when a DoS attack is active on the communication channel, any control impulse generated at a triggering instant t_k is blocked. In this case, the actuator does not receive the new control value, and the previous control input continues to be applied. Formally, let K_{succ} denote the set of indices of successful triggers (i.e., those not occurring during a DoS active interval). For $k \in K_{succ}$, the control updates according to (20). For $k \notin K_{succ}$, the control remains unchanged:

$$u_i(t_k) = u_i(t_{k-1}), \text{ if } t_k \in \Theta_d. \quad (22)$$

The ET mechanism (15) is still evaluated at every time instant, even during attacks. However, because no update can be transmitted during Θ_d , the condition $e_s(t) \geq 0$ may be satisfied repeatedly without producing a successful update. This leads to an accumulation of pending triggers, which is resolved as soon as the attack ends: The first pending trigger after the end of an attack becomes successful.

Assumption 2. There exists a positive constant $\Delta_{rec} > 0$ such that immediately after a DoS active interval, at least one successful trigger occurs within Δ_{rec} seconds, provided the system state remains bounded. This assumption is standard in DoS resilient ET control.

We now explain that Assumption 2 is standard in DoS_resilient event-triggered control [16,24]. It ensures that the system does not remain in open loop indefinitely after an attack. In practice, recovery can be guaranteed by the forced-trigger mechanism (16) and the boundedness of DoS intervals.

The design of the proposed ETISMCM can be summarized in the following steps:

Step 1: For each mode i : Compute the relative degree r_i and transform the subsystem into the normal form (2). Identify the unstable zero dynamics.

Step 2: Choose the virtual sliding variable S_i using (8) with Hurwitz coefficients.

Step 3: Select an extended sliding manifold $\sigma_i = S_i - L_i \zeta_i$, where ζ_i groups the first $r_i - 1$ IO states and the internal states η_i . Design L_i such that $A_\zeta^i + A_S^i L_i$ is Hurwitz.

Step 4: Construct the coordinate transformation (12), and obtain the system representation (13). Verify that $\bar{b}_i(\cdot)$ is nonzero in the operating region.

Step 5: Pick the ET parameters $a_k, b_k, \chi(\cdot)$, and Δ_{max} according to the desired trade-off between communication rate and performance.

Step 6: Design the sliding mode gains K_i and E_i : Choose K_i larger than the maximum possible magnitude of the lumped uncertainty $\Delta_{S,i}(\cdot)$ plus a margin; choose $E_i \geq 0$ to tune the convergence rate on the manifold.

Step 7: Implement the control law (20) at each successful trigger instant, with zero-order hold between triggers.

The next section provides a stability analysis, showing that under this design, the closed loop system is ISS and free of Zeno behaviour, even in the presence of aperiodic DoS attacks satisfying (4) and (5).

4. Stability analysis

This section provides a stability analysis of the closed-loop system under the proposed ETISMC. We first establish a set of standing assumptions. Then, we prove that the sliding manifold is reached in finite time and that the reduced-order dynamics on the manifold are ISS with respect to the disturbance $\theta(t)$. Subsequently, we analyze the overall system under aperiodic DoS attacks, deriving sufficient conditions that guarantee ISS despite intermittent communication blockages. Finally, we demonstrate that the proposed event-triggering mechanism strictly excludes Zeno behavior on the domain D_i ; that is there exists a uniform positive lower bound on all interevent intervals.

4.1. Standing assumptions

It is important to emphasize that the stability results presented in this paper are semiglobal in nature. The Byrnes_Isidori normal form is a local diffeomorphism, and the exponential stability of the reduced dynamics on the sliding manifold is guaranteed only on a domain D_i , where the nonlinear terms satisfy the small-gain condition. For any compact set of initial conditions contained in D_i , the proposed control law ensures ISS and Zeno-free behavior. Throughout the analysis, we adopt the following assumptions.

Assumption 3. For every mode $i \in \mathcal{M}$, there exists a continuously differentiable positive definite function $V_i: \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ of the form

$$V_i(\zeta_i, S_i) = \begin{bmatrix} \zeta_i \\ S_i \end{bmatrix}^T P_i \begin{bmatrix} \zeta_i \\ S_i \end{bmatrix}, \quad (23)$$

where $P_i \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix. There exist constants $\lambda_i, \tilde{\lambda}_i > 0$ such that for all (ζ_i, S_i) in the domain T_i ,

$$\lambda_i \|(\zeta_i, S_i)\|^2 \leq V_i(\zeta_i, S_i) \leq \tilde{\lambda}_i \|(\zeta_i, S_i)\|^2. \quad (24)$$

The matrix P_i is chosen to be block-diagonal:

$$P_i = \begin{bmatrix} P_\zeta^i & 0 \\ 0 & P_S^i \end{bmatrix},$$

where $P_\zeta^i > 0$ satisfies the Lyapunov equation (33) for the reduced dynamics, and $P_S^i > 0$ is a scalar gain for the sliding variable. This structure couples the reduced dynamics and the sliding variable, ensuring that V_i is a positive-definite measure of the full state.

Along the trajectories of the open-loop subsystem i (with $u_i = 0$), the Lyapunov function satisfies:

$$\dot{V}_i(\zeta_i, S_i) \leq \lambda_i V_i(\zeta_i, S_i) + \chi_i(\|\theta\|) \quad (25)$$

for some $\lambda_i \in \mathfrak{R}$ and $\chi_i \in K_\infty$. This inequality follows from the Lipschitz continuity of the system dynamics and the boundedness of the disturbance.

At switching instants t_s , where $\sigma(t_s^-) = j$, and $\sigma(t_s^+) = i$, the Lyapunov functions satisfy:

$$V_i(\zeta_i(t_s), S_i(t_s)) \leq \mu_{i,j} V_j(\zeta_j(t_s^-), S_j(t_s^-)) \quad (26)$$

for some $\mu_{i,j}$. This condition ensures that the Lyapunov function does not grow too much at switching instants.

Assumption 4. The sequences $\{a_k\}, \{b_k\}$ are bounded: There exist constants $\bar{a}, \bar{b}, d_{\min} > 0$ such that $0 < a_k < \bar{a}$ and $0 < b_k < \bar{b}$ for all k . The maximum allowable interevent time $\Delta_{\max}$ is finite and satisfies $\Delta_{\max} > 0$. The minimum decay rate $d_{\min} > 0$ is defined as:

$$d_{\min} = \min_{i \in M} \left\{ \frac{\lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i)\kappa_i}{2\lambda_{\max}(P_i)}, \eta_i \right\}, \quad (27)$$

where $\alpha_i = \lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i)\kappa_i / 2\lambda_{\max}(P_i)$ is the decay rate of the reduced-order dynamics, and η_i is the reaching rate from Lemma 1. This decay rate is derived from the sliding-mode dynamics and is not an artificial instantaneous decrease. Unlike the original version, we do not assume an instantaneous decay factor d_k at each triggering instant.

Assumption 5. For each mode i , the pair (A_ζ^i, A_S^i) is controllable, and the matrix $(A_\zeta^i + A_S^i L_i)$ is Hurwitz. Moreover, there exists a positive definite matrix P_i of the form (23) such that the Lyapunov equation (33) holds, and the nonlinear terms $\Delta_{\zeta,i}$ and $\Delta_{S,i}$ in (13) satisfy local Lipschitz conditions and are uniformly bounded on the domain T_i .

Assumption 6. The normal form (2) and the diffeomorphism Φ_i are valid on a domain $D_i \in \mathfrak{R}^n$ containing the origin. On this domain, the function $\bar{b}_i(\zeta_i, S_i)$ is bounded away from zero, that is, there exists $\bar{b}_{\min} > 0$ such that $|\bar{b}_i(\zeta_i, S_i)| \geq \bar{b}_{\min}$ for all (ζ_i, S_i) . The stability results are established on this domain; for initial conditions in any compact subset of D_i , the trajectories remain in D_i under the proposed control.

Assumption 7. On the domain $\mathfrak{I}_i$, the nonlinear term $\Delta_{\zeta,i}$, i in (13) satisfies the following growth bound:

$$\|\Delta_{\zeta,i}(\zeta_i, S_i, \theta)\| \leq \kappa_i \|\zeta_i\| + \gamma_i (\|\theta\|), \quad (28)$$

where $\kappa_i > 0$ is sufficiently small such that $\alpha_i = (\lambda_{\min}(Q_i) / 2\lambda_{\max}(P_i)) - \kappa_i > 0$ where $P_i > 0$ and $Q_i > 0$ are solutions of the Lyapunov equation (36), ISS on the domain $\mathfrak{I}_i$.

Assumption 8. The compact set of initial conditions C is contained in the intersection of all domains, that is, $C \subset \bigcap_{i \in M} D_i$. Under the proposed control law, the closed-loop trajectories remain in C for all time. Consequently, at any switching instant, the state belongs to D_i for the newly activated mode i , and the diffeomorphism is always well-defined.

The assumptions used in this paper are standard in the literature on switched nonlinear systems and event-triggered control, and we provide justifications for their practical applicability.

Assumption 2: This assumption is standard in DoS-resilient control [16,26]. It ensures that the system does not remain in open-loop indefinitely after an attack. In practice, recovery can be guaranteed by the forced trigger mechanism (16) and the boundedness of DoS intervals.

Assumption 3: The existence of Lyapunov functions satisfying (23)–(26) is a standard assumption in switched systems theory [6,7]. In our numerical example, we explicitly construct these Lyapunov functions.

Assumption 5: The controllability of (A_ζ^i, A_S^i) is a standard condition for sliding-mode control [18]. The Hurwitz condition is verified by pole placement, as shown in Section 5. If controllability fails, a preliminary state feedback can be used to render the system controllable.

Assumption 7: This condition ensures that the nonlinear terms do not destroy the stability of the reduced dynamics. It is verified numerically by checking that $\alpha_i > 0$ on the domain of interest. For practical systems, this condition can be satisfied by choosing the sliding surface gains sufficiently large.

Remark 2. Assumption 8 is standard in switched systems with multiple local transformations. It can be satisfied by choosing the domains D_i large enough to contain the reachable set, or by designing the switching law so that switching occurs only inside the intersection $C \subset \bigcap_{i \in M} D_i$. In the numerical example of Section 5, the domains are the whole state space (or sufficiently large), so the assumption holds trivially. Alternatively, a resetting strategy can be used: If the state leaves D_i , the controller can be temporarily switched to a safe mode until the state re-enters the intersection.

4.2. Finite-time convergence to the sliding manifold

We first analyze the dynamics of the extended sliding variable σ_i defined in (10). Substituting the control law (20) and accounting for the fact that the control input is held constant between triggering instants, the dynamics of σ_i become

$$\dot{\sigma}_i = \bar{A}_\zeta^i \zeta_i + \bar{A}_S^i S_i + \bar{b}_i u_i(t_k) + \Delta_{S,i} - L_i (A_\zeta^i \zeta_i + A_S^i S_i + \Delta_{\zeta,i}). \quad (29)$$

After simplification, we obtain

$$\dot{\sigma}_i = -K_i \operatorname{sgn}(\sigma_i(t_k)) - E_i \sigma_i(t_k) - \rho_i(t) + \varepsilon_i(\zeta_i(t_k) - \zeta_i(t)) \quad \forall t \in [t_k, t_{k+1}], \quad (30)$$

where $\rho_i(t) = \Delta_{S,i} - L_i \Delta_{\zeta,i}$ is the lumped uncertainty, and the term $\varepsilon_i(\zeta_i(t_k) - \zeta_i(t))$ represents the sampling error due to the held control input.

The event-triggering condition (15) ensures that the sliding variable S_i does not grow excessively between triggers. Using the Lipschitz continuity of the ζ_i dynamics, there exists a constant $L_\zeta > 0$ such that:

$$\|\zeta_i(t) - \zeta_i(t_k)\| \leq L_\zeta (t - t_k) \leq L_\zeta \Delta_{\max} \quad \forall [t_k, t_{k+1}). \quad (31)$$

This bound is uniform over the entire interevent interval and does not depend on σ_i . The switching gain K_i is chosen to dominate both the uncertainty $\rho_i(t)$ and the sampling error:

$$K_i > \sup_{t \geq 0} \|\rho_i(t)\| + \eta_i + \varepsilon_i L_\zeta \Delta_{\max} \quad (32)$$

for some $\eta_i > 0$. This condition ensures that the sampling error is absorbed by the switching term.

Using (30) and (31), the derivative of $W_i = (1/2) \sigma_i^2$ satisfies:

$$\dot{W}_i = \sigma_i \dot{\sigma}_i = \sigma_i \left(-K_i \operatorname{sgn}(\sigma_i(t_k)) - E_i \sigma_i(t_k) + \rho_i(t) \right) + \varepsilon_i (\zeta_i(t_k) - \zeta_i(t)). \quad (33)$$

Applying the bound (31) and the gain condition (32), we obtain:

$$\dot{W}_i \leq -K_i |\sigma_i| - E_i \sigma_i^2 + |\sigma_i| \|\rho_i\| + \varepsilon_i L_\zeta \Delta_{\max} |\sigma_i| \leq -\eta_i |\sigma_i| - E_i \sigma_i^2. \quad (34)$$

Thus, $\dot{W}_i \leq -\eta_i \sqrt{2W_i}$, which implies that σ_i converges to a neighborhood of zero in finite time. Moreover, once $\sigma_i = 0$, it remains zero under the equivalent control (the sliding mode is maintained) provided that the uncertainty bound is respected.

Lemma 1. Under the control law (20) with the switching gain K_i satisfying (32), the extended sliding variable σ_i converges to a neighborhood of zero in finite time. The size of the neighborhood is proportional to the sampling error bound $L_\zeta \Delta_{\max}$.

Consider the switched nonlinear system (1) under the control law (20) with the switching gain K_i satisfying (30). Suppose the event-triggering condition (15) ensures the sampling-error bound (29). Then, for any initial condition in the domain D_i , the extended sliding variable σ_i converges to zero in finite time, and the sliding manifold $\sigma_i = 0$ is positively invariant thereafter.

Proof of Lemma 1: Consider the Lyapunov function candidate $W_i = (1/2) \sigma_i^2$. From (30) and (31), its derivative along the closed-loop trajectories satisfies:

$$\dot{W}_i = \sigma_i \left(-K_i \operatorname{sgn}(\sigma_i(t_k)) - E_i \sigma_i(t_k) + \rho_i(t) + \varepsilon_i (\zeta_i(t_k) - \zeta_i(t)) \right).$$

Using the bound (31) and the gain condition (32), we obtain:

$$\dot{W}_i \leq -K_i |\sigma_i| - E_i \sigma_i^2 + |\sigma_i| \|\rho_i\| + \varepsilon_i L_\zeta \Delta_{\max} |\sigma_i| \leq -\eta_i |\sigma_i| - E_i \sigma_i^2.$$

Thus, $\dot{W}_i \leq -\eta_i \sqrt{2W_i}$. Integrating this differential inequality gives:

$$W_i(t) \leq \left(\sqrt{2W_i(0)} - \frac{\eta_i}{\sqrt{2}} t \right)^2,$$

which implies that $\sigma_i(t) = 0$ for all $t \geq T_i$, where $T_i \leq \sqrt{2W_i(0)}/\eta_i$. Once $\sigma_i = 0$, the equivalent control (18) maintains $\sigma_i = 0$ as long as the uncertainty bound is respected.

4.3. Reduced-order dynamics on the sliding manifold

When the system evolves on the sliding manifold $\sigma_i = 0$ (i.e., $S_i = L_i \zeta_i$), the dynamics of ζ_i become:

$$\dot{\zeta}_i = (A_\zeta^i + A_S^i L_i) \zeta_i + \Delta_{\zeta,i}(L_i \zeta_i, S_i, \theta). \quad (35)$$

Because the matrix $A_\zeta^i + A_S^i L_i$ is Hurwitz by design, there exists a Lyapunov function $U_i(\zeta_i) = \zeta_i^T P_i \zeta_i$ with $P_i \succ 0$ satisfying the Lyapunov equation

$$P_i (A_\zeta^i + A_S^i L_i) + (A_\zeta^i + A_S^i L_i)^T P_i = -Q_i, Q_i \succ 0. \quad (36)$$

The derivative of U_i along (35) is

$$\dot{U}_i = \zeta_i^T \left(P_i (A_\zeta^i + A_S^i L_i) + (A_\zeta^i + A_S^i L_i)^T P_i \right) \zeta_i + 2 \zeta_i^T P_i \Delta_{\zeta,i}. \quad (37)$$

Using Assumption 7, we have $\|\Delta_{\zeta,i}\| \leq \kappa_i (\|\zeta_i\|) + \gamma_i (\|\theta\|)$. Applying Young's inequality and the bounds on P_i , we obtain

$$\dot{U}_i \leq -\lambda_{\min}(Q_i) \|\zeta_i\|^2 + 2\lambda_{\max}(P_i) \|\zeta_i\| (\kappa_i \|\zeta_i\| + \gamma_i (\|\theta\|)). \quad (38)$$

Rearranging terms, we get:

$$\dot{U}_i \leq -\left(\lambda_{\min}(Q_i) \|\zeta_i\|^2 + 2\lambda_{\max}(P_i) \kappa_i + 2\lambda_{\max}(P_i) \|\zeta_i\| \gamma_i (\|\theta\|) \right). \quad (39)$$

Using the inequality $2ab \leq \varepsilon a^2 + (1/\varepsilon)b^2$ with $a = \|\zeta_i\|, b = \gamma_i \|\theta\|$, and $\lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i) \kappa_i / 2\lambda_{\max}(P_i)$, we obtain

$$\dot{U}_i \leq -\frac{1}{2} \left(\lambda_{\min}(Q_i) + 2\lambda_{\max}(P_i) \kappa_i \|\zeta_i\|^2 + \frac{2\lambda_{\max}(P_i)}{\lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i) \kappa_i} \gamma_i^2 \right). \quad (40)$$

Using the bounds $\lambda_{\min}(P_i) \|\zeta_i\|^2 \leq U_i \leq \lambda_{\max}(P_i) \|\zeta_i\|^2$, we can rewrite as:

$$\dot{U}_i \leq -\alpha_i U_i + \beta_i \gamma_i^2 (\|\theta\|), \quad (41)$$

where $\alpha_i = \lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i) \kappa_i / 2\lambda_{\max}(P_i)$ and $\beta_i = 2\lambda_{\max}(P_i) / \lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i) \kappa_i$.

The condition $\alpha_i > 0$ is precisely Assumption 7. Therefore, establishes that the reduced-order dynamics are input-to-state stable (ISS) with respect to the disturbance $\theta(t)$, with the ISS gain γ_i given by $\gamma_i(r) = \sqrt{\beta_i / \alpha_i}$. This completes the derivation.

The inequality (41) holds for all $\zeta_i \in T_i$, where T_i is the image of the domain D_i under the diffeomorphism T_i . This domain contains a neighborhood of the origin, and for initial conditions in any compact subset of T_i , the trajectories remain in T_i under the proposed control law.

Lemma 2. Under Assumption 7, the reduced-order dynamics (35) on the sliding manifold are ISS with respect to the disturbance $\theta(t)$, that is, there exist $\beta_i \in KL$ and $\gamma_i \in K$ such that

$$\|\xi_i(t)\| \leq \beta_i(\|\zeta_i(0)\|, t) + (\gamma_i \|\theta\|_{[0,t]}). \quad (42)$$

Proof of Lemma 2: The proof follows from the standard comparison lemma applied to (35). The details are omitted for brevity.

4.4. Overall ISS under event-triggered control without DoS

We analyze the entire closed-loop system when no DoS attacks occur. The analysis follows a recursive estimation of the Lyapunov function $V_{\sigma(t)}(x(t))$ across successive successful triggers and switching intervals.

Let $\{t_k\}$ be the sequence of successful triggering instants (all triggers are successful in the absence of DoS). Between triggers, the control is constant, and the system evolves according to the open-loop dynamics of the active mode. From (25), on any interval $[t_{k-1}, t_k]$, we have

$$V_{\sigma(t_k)}(x(t)) \leq e^{\lambda_i(t-t_{k-1})} V_{\sigma(t_{k-1})}(x(t_k-1)) + \int_{t_{k-1}}^t e^{\lambda_i(t-s)} \chi_i(\|\theta(s)\|) ds. \quad (43)$$

At the triggering instant t_k , the event condition (15) ensures that before the impulse, the sliding variable satisfies

$$\|S_{\sigma(t_k^-)}(t_k^-)\| \leq e^{a_k} \|S_{\sigma(t_{k-1})}(t_k-1)\| + e^{b_k} \chi(\|\theta\|_{[t_{k-1}, t_k]}). \quad (44)$$

Using the open-loop bound (25) and the event-triggering condition (15), we derive the following recursive inequality for the full-state Lyapunov function:

Using the relationship between the Lyapunov function and the sliding variable, and the fact that the sampling error is bounded by the event condition, we derive the following recursive inequality for the full-state Lyapunov function:

$$V_{\sigma(t_k)}(x(t_k^+)) \leq e^{a_k + \lambda_i \ell_k} V_{\sigma(t_{k-1})}(x(t_k-1)) + (e^{b_k + \lambda_i \ell_k} + 1) \chi(\|\theta\|_{[t_{k-1}, t_k]}), \quad (45)$$

where $\ell_k^d = t_k - t_{k-1}$ is the interevent length.

Unlike the original version, we do not assume an instantaneous decrease of the Lyapunov function at triggering instants. Instead, the recursive inequality (39) is derived directly from the open-loop dynamics and the event-triggering condition. The overall stability is established by iterating this inequality over multiple intervals, where the sliding-mode dynamics provide the necessary decay over time. Let

$$A_k = e^{a_k - d_k + \lambda_i \ell_k^d}, \quad B_k = e^{b_k - d_k + \lambda_i \ell_k^d} + 1. \quad (46)$$

Iterating (45) from $k = 1$ to $k = p$ yields

$$V_{\sigma(t_p)}(x(t_p^+)) \leq \left(\prod_{k=1}^p A_k \right) V_{\sigma(t_0)}(x_0) + \sum_{k=1}^p \left(\prod_{l=k+1}^p A_l \right) B_k \chi(\|\theta\|_{[t_{k-1}, t_k]}). \quad (47)$$

Because the interevent times are bounded below by $\Delta_{\min} > 0$ and above by $\Delta_{\max}$, we can bound the product. Let $\bar{\lambda} = \max_i \lambda_i$ and $\bar{a} = \sup_k a_k$. Using the bounds $a_k \leq \bar{a}$ and the fact that $\ell_k \leq \Delta_{\max}$, we have:

$$\sum_{k=1}^p \ln A_k = \sum_{k=1}^p (a_k + \lambda_i \ell_k) \leq \bar{a}_p + \bar{\lambda} \sum_{k=1}^p \ell_k \leq \bar{a}_p + \bar{\lambda} (t_p - t_0). \quad (48)$$

Because $p \geq (t - t_0)/\Delta_{\max}$ (because each interevent time is at most $\Delta_{\max}$), we obtain:

$$\sum_{k=1}^p \ln A_k \leq \left(\frac{\bar{a}}{\Delta_{\min}} + \bar{\lambda} \right) (t - t_0). \quad (49)$$

Choosing the design parameters such that:

$$\eta_0 = \frac{d_{\min}}{\Delta_{\max}} - \frac{\bar{a}}{\Delta_{\min}} - \frac{\bar{\lambda}}{\Delta_{\max}} > 0, \quad (50)$$

where $d_{\min}$ is the minimum decay rate provided by the sliding-mode dynamics over multiple intervals (defined in Assumption 4, Eq (27)), we obtain:

$$\prod_{k=1}^p A_k \leq e^{-\eta_0(t_p - t_k)} \leq e^{\eta_0(t_p - t_k)} e^{\eta_0 \Delta_{\max}}. \quad (51)$$

Similarly, for the products in the summation, we have:

$$\prod_{k=1}^p A_k \leq e^{-\eta_0(t_p - t_k)} \leq e^{\eta_0(t_p - t_k)} e^{\eta_0 \Delta_{\max}}. \quad (52)$$

Substituting (51) and (52) into (47), and using the fact that B_k is bounded (because $b_k \leq \bar{b}$ and λ_i is bounded), we obtain:

$$V_{\sigma(t)}(x(t)) \leq e^{\bar{a}} e^{-\eta_0(t_p - t_k)} V_{\sigma(t_0)}(x_0) + \gamma \left(\|\theta\|_{[t_0, t]} \right) \quad (53)$$

for some $\gamma \in \mathbb{K}$. Finally, using the bounds (24) from Assumption 3, we obtain:

$$\|x(t)\| \leq \beta(\|x_0\|, t - t_0) + \gamma \left(\|\theta\|_{[t_0, t]} \right) \quad (54)$$

with $\beta \in \text{KL}$ and $\gamma \in \mathbb{K}$. This establishes ISS in the absence of DoS attacks.

Lemma 3. Under the AED-ADT condition (7), for any interval $[t_0, t]$ and any switching sequence from mode j to mode i , the following bound holds:

$$\prod_{\text{switches } j \rightarrow i} \mu_{j,i} \leq \exp \left(\sum_{j,i} N_{j,i}^{\sigma,0} \ln \mu_{j,i} + \sum_{j,i} \frac{T_{j,i}(t, t_0)}{\tau_{j,i}^{\sigma}} \ln \mu_{j,i} \right). \quad (55)$$

Proof of Lemma 3: The proof follows by taking logarithms and applying the AED-ADT inequality (7) to each edge (j, i) .

4.5. ISS under aperiodic DoS attacks

When DoS attacks occur, some triggers are blocked. To analyze the effect of DoS attacks, we introduce the following notation:

- $\mathbb{K}_{\text{succ}}$: set of indices of successful triggers (updates that are computed and applied).
- $\mathbb{K}_a^t$: set of indices of attempted triggers (including both successful and blocked).
- $p(t) = |\{k \in \mathbb{K}_{\text{succ}} : t_k \leq t\}|$: number of successful triggers up to time t .
- $M(t) = |\{k \in \mathbb{K}_a^t : t_k \leq t\}|$: number of attempted triggers up to time t .
- $\ell_k = t_k - t_{k-1}$: interevent length between successful triggers.
- $\ell_k^d = |\Theta_d(t_{k-1}, t_k) \cap [t_{k-1}, t_k]|$: DoS active duration within the k -th inter-event interval.
- $\ell_k^h = \ell_k - \ell_k^d$: DoS-free duration within the k -th interevent interval.
- $u_i^{\text{old}}(t)$: the last successfully received control value, which is applied during DoS intervals when updates are blocked.

During a DoS active interval, any trigger attempt is blocked, and the control input remains at the last successfully received value $u_i^{old}(t)$. The system therefore evolves in open loop with a constant input on the interval $[t_{k-1}, t_k]$ whenever a DoS attack is active. From the open-loop bound (25), the Lyapunov function satisfies:

$$V(x(t)) \leq e^{\lambda_i(t-t_{k-1})} V(x(t_{k-1})) + \int_{t_{k-1}}^t e^{\lambda_i(t-s)} \chi(\|\theta\|) ds \quad (56)$$

regardless of whether the interval is DoS-free or DoS-active. The difference is that during DoS-active intervals, no update occurs, so the control input is not refreshed; however, the bound remains valid because it only depends on the open-loop dynamics and the boundedness of the held input.

Each successful trigger covers at most $\Delta_{\max}$ seconds of DoS-free time, as the forced trigger at $\Delta_{\max}$ ensures that an update occurs at least every $\Delta_{\max}$ seconds when communication is available. Therefore, the total DoS-free time up to time t is bounded by:

$$|\Theta_h(t_0, t)| \leq p(t) \Delta_{\max} + \Delta_{\max}. \quad (57)$$

Using the fact that $|\Theta_h(t_0, t)| = (t - t_0) - |\Theta_d(t_0, t)|$ and applying the DoS duration bound (5), we obtain:

$$p(t) \geq \frac{t - t_0}{\Delta_{\max}} \left(1 - \frac{1}{\tau_d} \right) - \frac{\theta_0}{\Delta_{\max}} - 1. \quad (58)$$

This lower bound on $p(t)$ will be used to derive the decay rate in the ISS condition.

From the recursive inequality (45), the accumulated growth factor over p successful triggers is:

$$\sum_{k=1}^p \ln A_k = \sum_{k=1}^p a_k - \sum_{k=1}^p d_k + \sum_{k=1}^p \lambda_i \ell_k + \sum_{\text{switches } j \rightarrow i} \ln \mu_{j,i}. \quad (59)$$

The last term captures the effect of switching between modes. Using the AED-ADT condition (7), the total number of switches from mode j to mode i is bounded by:

$$N_{j,i}^{\sigma_i}(t_0, t) \leq N_{j,i}^{\sigma,0} + \frac{T_{j,i}(t_0, t)}{\tau_{j,i}^{\sigma}}. \quad (60)$$

Therefore, the accumulated switching penalty is bounded by:

$$\sum_{\text{switches } j \rightarrow i} \ln \mu_{j,i} \leq \sum_{j,i} N_{j,i}^{\sigma,0} \ln \mu_{j,i} + \sum_{j,i} \frac{T_{j,i}(t_0, t)}{\tau_{j,i}^{\sigma}} \ln \mu_{j,i}. \quad (61)$$

The term $\sum_{k=1}^p \lambda_i \ell_k$ can be decomposed as:

$$\sum_{k=1}^p \lambda_i \ell_k = \sum_{k=1}^p \lambda_i \ell_k^h + \sum_{k=1}^p \lambda_i \ell_k^d \leq \tilde{\lambda} \sum_{k=1}^p \ell_k^h + \tilde{\lambda} \sum_{k=1}^p \ell_k^d, \quad (62)$$

where $\tilde{\lambda} = \max_i \lambda_i$. The DoS-free part is bounded by the total DoS-free time:

$$\sum_{k=1}^p \ell_k^h \leq |\Theta_h(t_0, t)| \leq t - t_0. \quad (63)$$

The DoS-active part is bounded by the total DoS active time:

$$\sum_{k=1}^p \ell_k^h \leq |\Theta_d(t_0, t)|. \quad (64)$$

Using the bounds on $a_k \leq \bar{a}$ and the fact that $p \geq (t - t_0 - |\Theta_d(t_0, t)|/\Delta_{\max}) - 1$ from (58), we have:

$$\sum_{k=1}^p a_k \leq \bar{a}p \leq \frac{\bar{a}}{\Delta_{\min}}(t - t_0), \quad (65)$$

Using the DoS bounds (4)–(5), we obtain:

$$\sum_{k=1}^p \ln A_k \leq -\eta(t - t_0) + C, \quad (66)$$

where

$$\eta = \frac{d_{\min}}{\Delta_{\max}} \left(1 - \frac{1}{\tau_d} \right) - \frac{d_{\min}}{\tau_D} - \frac{\bar{a}}{\Delta_{\min}} - \frac{\hat{\lambda}}{\tau_d} - \frac{\sum_{j,i} N_{j,i}^{\sigma,0} \ln \mu_{j,i}}{\Delta_{\max}}, \quad (67)$$

and C is a constant depending on the initial conditions, the DoS charter bounds, and the switching chatter bounds. The ISS condition $\eta > 0$ quantifies the trade-off between the following factors:

- $(d_{\min}/\Delta_{\max})(1 - (1/\tau_d))$: This is the effective decay rate due to the event-triggered sliding mode. The factor $(1 - (1/\tau_d))$ accounts for the fact that DoS attacks occupy a fraction $1/\tau_d$ of the total time, so the decay is effective only during the remaining fraction.
- $d_{\min}/\tau_D$: This is the decay lost due to the frequency of DoS attacks. Each DoS interval causes a loss of at most $d_{\min}$ decay, and τ_D is the average interval between attacks.
- $\bar{a}/\Delta_{\min}$: This term captures the growth due to the event-triggering parameters a_k . A larger $\bar{a}$ means that the Lyapunov function can grow more between triggers, reducing the overall decay rate.
- $\hat{\lambda}/\tau_d$: This is the growth due to the system's inherent dynamics λ_i during DoS active intervals. During attacks, the system evolves in open loop, and $\hat{\lambda}$ is the maximum growth rate.
- The term $\sum_{j,i} N_{j,i}^{\sigma,0} \ln \mu_{j,i} / \Delta_{\max}$ represents the switching penalty with the correct negative sign.

Each switch from mode j to mode i causes a jump in the Lyapunov function by a factor $\mu_{j,i}$. The chatter bounds $N_{j,i}^{\sigma,0}$ limit the number of such switches that can occur without being compensated by decay. The edge-dependent time terms from Lemma 3, specifically $\left(\sum_{j,i} T_{j,i}(t_0, t) / \tau_{j,i}^{\sigma} \right) \ln \mu_{j,i}$ are absorbed into the constant C in (66) and reflect the cumulative effect of switches over time.

Once the product bound (66) is established, the same iterative argument as leads to the ISS estimate (54) with a modified β and γ that reflect the DoS-induced deterioration. Consequently, we obtain the main stability theorem.

Theorem 1. Consider the switched nonlinear system (1) with each mode possibly nonminimum phase, under the event-triggered sample-and-hold sliding mode control (20) with MDET (15)–(16). Suppose the DoS attacks satisfy (4)–(5) with $\tau_d > 1$ and that the design parameters satisfy (32), (50), and (67) with $\eta > 0$. Then, for all initial conditions in any compact set $C_i \subset D_i$, the closed-loop system is input-to-state stable with respect to the disturbance $\theta(t)$ on the domain D_i .

Proof of Theorem 1: The proof proceeds in several steps

From the recursive inequality (45), for each successful trigger $k \in K_{\text{succ}}$, we have:

$$V_{\sigma(t_k)}(x(t_k)) \leq A_k V_{\sigma(t_{k-1})}(x(t_{k-1})) + B_k \chi(\|\theta\|_{[t_{k-1}, t_k]}), \quad (68)$$

where $A_k = e^{a_k + \lambda_i \ell_k}$ and $B_k = e^{b_k + \lambda_i \ell_k} + 1$, with $\ell_k = t_k - t_{k-1}$ if the interval is DoS-free, and ℓ_k including DoS active durations if attacks occur. Iterating from $k = 1$ to $k = p$ (where p is the number of successful triggers up to time t), we obtain:

$$V_{\sigma(t_p)}(x(t_p)) \leq \left(\prod_{k=1}^p A_k \right) V_{\sigma(t_0)}(x(0)) + \sum_{k=1}^p \left(\prod_{\ell=k+1}^p A_\ell \right) B_k \chi(\|\theta\|_{[t_{k-1}, t_k]}). \quad (69)$$

Note that the forced triggers at switching instants are included in the sequence $\{t_k\}$. Consequently, the recursive inequality (68) and the subsequent product bound (66) remain valid, with the understanding that some triggers may occur at the switching instants rather than at event-triggered instants.

Taking logarithms and using the definitions of a_k , λ_i , and the switching penalty, we have:

$$\sum_{k=1}^p \ln A_k = \sum_{k=1}^p a_k + \sum_{\text{switches } j \rightarrow i} \ln \mu_{j,i} + \sum_{k=1}^p \lambda_i \ell_k. \quad (70)$$

Using the bounds (58)–(65), we obtain the product bound (66).

For the disturbance term in (69), we use the fact that B_k is bounded, and that $\chi(\|\theta\|_{[t_{k-1}, t_k]}) \leq \chi(\|\theta\|_{[t_0, t]})$.

The summation is bounded by a geometric series, giving

$$\sum_{k=1}^p \left(\prod_{\ell=k+1}^p A_\ell \right) B_k \chi(\|\theta\|_{[t_{k-1}, t_k]}) \leq \gamma(\|\theta\|_{[t_0, t]}) \quad (71)$$

for some class K function γ . Using the bounds on V_i from (24), we obtain:

$$\|x(t)\| \leq \beta(\|x_0\|, t - t_0) + \gamma(\|\theta\|_{[t_0, t]}) \quad (72)$$

for some $\beta \in KL$ and $\gamma \in K$. This is exactly the ISS inequality (6). Therefore, the closed-loop system is ISS on the domain D_i .

4.6. Exclusion of Zeno behavior

Zeno behavior refers to the phenomenon of infinitely many events occurring in a finite time interval, which would make the implementation infeasible. We now prove that the proposed MDETM (15) together with the forced trigger at $\Delta_{\max}$ guarantees a uniform positive lower bound on interevent times.

Consider any two consecutive successful triggering instants t_{k-1} and t_k . On the interval $[t_{k-1}, t_k)$, the event condition (15) is not satisfied, that is,

$$\|S_{\sigma(t)}(t)\| < e^{a_k} \|S_{\sigma(t_{k-1})}(t_{k-1})\| + e^{b_k} \chi(\|\theta\|_{[t_0, t]}), \forall t \in [t_{k-1}, t_k). \quad (73)$$

From the system dynamics (13) and the fact that the control input is held constant between triggers, the sliding variable S_i satisfies a differential equation with a bounded right-hand side. More precisely, there exists a Lipschitz constant $L_S > 0$ such that:

$$\left| \frac{d}{dt} \|S_{\sigma(t)}(t)\| \right| < L_S \quad \forall t \in [t_{k-1}, t_k). \quad (74)$$

This constant depends on the state bounds, the system parameters, and the maximum allowable interevent time $\Delta_{\max}$. Integrating (74) from t_{k-1} to t gives:

$$\|S_{\sigma(t)}(t)\| \geq \|S_{\sigma(t_{k-1})}(t_{k-1})\| - L_S(t - t_{k-1}). \quad (75)$$

Suppose, for contradiction, that the interevent time $\Delta_k = (t_k - t_{k-1})$ could be arbitrarily small. Then, just before t_k , the left-hand side of (73) would be arbitrarily close to the right-hand side. Combining (75) with the fact that the right-hand side of (73) is at most $e^{a_k} \|S_{\sigma(t_{k-1})}(t_{k-1})\| + e^{b_k} \chi(\|\theta\|_{[t_0, t]})$, and using the Lipschitz bound, one can derive a contradiction unless Δ_k exceeds a positive threshold.

Additionally, when switches occur within the interval, the piecewise evolution must be considered; however, because the number of switches per interval is bounded by the AED-ADT condition, the same lower bound remains valid after adjusting for the jumps in the Lyapunov function. **Theorem 2.** Under the same assumptions as Theorem 1, and for initial conditions in any compact set $C_i \subset D_i$, there exists a constant $\Delta_{\min} > 0$ such that for all k , $t_k - t_{k-1} \geq \Delta_{\min}$. Thus, Zeno behavior is strictly excluded on the domain D_i . In particular, the forced trigger at $\Delta_{\max}$ ensures that $\Delta_k \leq \Delta_{\max}$, and the event condition guarantees a positive lower bound, thus excluding Zeno behavior. The constant $\Delta_{\min}$ is given by

$$\Delta_{\min} = \min \{ \Delta_{\text{event}}, \Delta_{\text{switch}} \}, \quad (76)$$

where

$$\Delta_{\text{event}} = \frac{(e^{a_k} - 1) \|S(t_{k-1})\| + e^{b_k} \chi(\|\theta\|)}{L_S} \quad (77)$$

is derived from the pure ET case (no switching), and Δ_{switch} accounts for possible switches. Both are strictly positive because the right-hand side of the event condition grows at most linearly in time, whereas the left-hand side can only decrease linearly. When switches occur within the interval, the piecewise evolution must be considered; however, because the number of switches per interval is

bounded by the AED-ADT condition (7), the same lower bound remains valid after adjusting for the jumps in the Lyapunov function.

Proof of Theorem 2: Consider two consecutive successful triggering instants t_{k-1} and t_k . On the interval $[t_{k-1}, t_k]$, the event condition (15) is not satisfied, meaning that for all $t \in [t_{k-1}, t_k]$, (73). From the system dynamics (13) and the fact that the control is held constant between triggers, the sliding variable S_i satisfies a differential equation with a bounded right-hand side. More precisely, there exists a Lipschitz constant $L_S > 0$ such that (74). Integrating (74) from t_{k-1} to t gives (75). Let $\Delta k = t_k - t_{k-1}$. At time t_k^- (just before the trigger), the event condition is about to be violated, so the left-hand side of (73) is arbitrarily close to the right-hand side. Therefore, we have

$$\|S_{\sigma(t_{k-1})}(t_{k-1})\| - L_S \Delta_k \leq e^{a_k} \|S_{\sigma(t_{k-1})}(t_{k-1})\| + e^{b_k} \chi(\|\theta\|_{[t_{k-1}, t_k]}). \quad (78)$$

Rearranging (78) yields:

$$L_S \Delta_k \geq (1 - e^{a_k}) \|S_{\sigma(t_{k-1})}(t_{k-1})\| - e^{b_k} \chi(\|\theta\|_{[t_{k-1}, t_k]}). \quad (79)$$

Because $a_k > 0$, we have $1 - e^{a_k} < 0$. However, the event condition (15) guarantees that the right-hand side of (79) is positive. Indeed, the trigger occurs precisely when the growth of the sliding variable exceeds the threshold. Thus, we obtain:

$$\Delta_k \geq \Delta_{event} = \frac{(e^{a_k} - 1) \|S(t_{k-1})\| + e^{b_k} \chi(\|\theta\|_{[t_{k-1}, t_k]})}{L_S} > 0. \quad (80)$$

If a mode switch occurs within the interval $[t_{k-1}, t_k]$, the sliding variable may undergo a jump due to the change in the diffeomorphism. However, the number of switches within any interval is bounded by the AED-ADT condition (7). Let m be the number of switches in the interval. The total variation of the sliding variable due to switches is bounded by

$$\|S(t)\| \leq \|S(t_{k-1})\| + \sum_{r=1}^m \|S(t_r^+) - S(t_r^-)\|. \quad (81)$$

Using the switching bound (26), the jump in the Lyapunov function is bounded by $\mu_{j,i}$, and hence, the jump in the sliding variable is bounded by a constant times $\|S(t_{k-1})\|$. Therefore, the lower bound (80) remains valid with an additional term accounting for switches:

$$\Delta_k \geq \Delta_{switch} = \frac{(e^{a_k} - 1) \|S(t_{k-1})\| + e^{b_k} \chi(\|\theta\|) - \sum_{r=1}^m \|S(t_r^+) - S(t_r^-)\|}{L_S}. \quad (82)$$

Because m is bounded by the AED-ADT chatter bound, and the jumps are bounded, $\Delta_{switch} > 0$ for sufficiently small switches.

The forced trigger at Δ_{max} provides an upper bound $\Delta_k \leq \Delta_{max}$. The lower bound is given by

$$\Delta_{min} = \min\{\Delta_{event}, \Delta_{switch}\} > 0. \quad (83)$$

This proves that all interevent times are bounded below by $\Delta_{min} > 0$. Because Δ_{min} is strictly positive, no accumulation of triggers can occur in finite time, and Zeno behavior is strictly excluded.

4.7. Practical guidelines for parameter selection

The ISS condition (67) involves several design parameters. In practice, one can proceed as follows:

- Choose the maximum interevent time $\Delta_{\max}$ according to the desired sampling rate (e.g., the hardware limit).
- Select the decay rate $d_{\min}$ large enough to dominate the growth due to $\bar{a}$ and $\hat{\lambda}$.
- Tune the ET parameters a_k, b_k to balance communication rate and robustness. A common choice is constant $a_k = a, b_k = b$.
- Verify that the DoS parameters τ_D, τ_d satisfy the inequality; if not, one may need to increase $d_{\min}$ or reduce $\Delta_{\max}$.

Tuning procedure

- 1) Choose $\Delta_{\max}$ based on the hardware sampling rate and communication constraints.
- 2) Select $\chi(\|\theta\|)$ based on the expected disturbance magnitude.
- 3) Tune a_k and b_k to balance communication rate and robustness:
 - a_k determines the allowed growth of the sliding variable between triggers. A larger a_k increases the threshold, leading to fewer triggers but potentially slower convergence. A typical starting value is $a_k = 0.01$ to 0.05 .
 - b_k weighs the influence of the disturbance. A larger b_k makes the trigger less sensitive to disturbances, reducing the number of triggers but increasing the steady-state error. A common choice is $b_k = 0.01$ to 0.1 , or $b_k = 0.1 \|\theta\|_{\max}^2$.
 - Start with constant values: $a_k = a$ and $b_k = b$ for all k . This satisfies the boundedness assumption and simplifies implementation.
 - Choose initial values based on the desired trade-off: for example, $a = 0.02, b = 0.02$.
- 4) Verify the ISS condition (67). If $\eta \leq 0$, reduce $\Delta_{\max}$ or increase $d_{\min}$ (by increasing the sliding gain K_i).
- 5) Simulate and adjust based on performance metrics (settling time, steady-state error, trigger frequency). If the disturbance level varies significantly, b_k can be adapted online, for example, by increasing b_k when the disturbance is large. However, the bounds $\bar{a}, \bar{b}$ must be respected.
- 6) Verify that the DoS parameters τ_D, τ_d satisfy the inequality; if not, one may need to increase $d_{\min}$ or reduce $\Delta_{\max}$.

4.8. AED-ADT design procedure

The AED-ADT strategy is used to design the switching signal and the Lyapunov gains $\mu_{j,i}$. The following systematic procedure is recommended:

Step 1: Choose the switching ratios $\mu_{j,i} \geq 1$

These are determined by the condition (26). For each ordered pair (j,i) , compute the maximum possible ratio of Lyapunov functions after a switch from mode j to mode i .

Step 2: Determine the chatter bounds $N_{j,i}^{\sigma,0}$

These are the maximum number of switches from mode j to mode I that can occur in a short interval without compensation.

Step 3: Compute the edge-dependent dwell times $\tau_{j,i}^\sigma$

Start from the condition:

$$\tau_{j,i}^\sigma = \frac{\ln(\mu_{j,i})}{\eta_0},$$

where η_0 is the desired decay rate. This ensures that the switching penalty is compensated by the decay.

Step 4: Verify the overall ISS condition (67)

If $\eta > 0$, the switching signal is admissible.

Step 5: Implement the switching signal with dwell times satisfying $\tau_{j,i}^\sigma$

This can be done by enforcing a minimum dwell time for each edge (j,i) .

5. Numerical simulation

This section presents a comprehensive numerical example to validate the theoretical findings and demonstrate the effectiveness of the proposed ETISMC strategy for uncertain switched nonlinear systems with NMP dynamics under aperiodic DoS attacks. The simulation is carried out on a benchmark system adapted from [13], which features two switching modes, each with unstable zero dynamics. We compare the performance of the proposed event-triggered controller with the classical continuous-time sliding-mode controller from [13] in terms of state regulation, control effort, number of control updates, and robustness to DoS attacks.

Consider the uncertain nonlinear system of order four originally studied in [13] and given by:

$$\begin{cases} \dot{x}_1 = -x_1 + x_2 \\ \dot{x}_2 = -3x_2 + x_1^3 + \theta + \left(\sin(x_3) \right)^2 + 2 \Big) u \\ \dot{x}_3 = x_1 - 2x_3 \\ \dot{x}_4 = -x_4 + x_3^2 \end{cases} \quad (84)$$

with output $y = x_1 - 3x_3$. The disturbance is taken as $\theta(t) = 0.3 \sin(3t)$, which is bounded and Lipschitz. The system has relative degree $r = 2$ for both modes. Following the normal form transformation in [13], two possible diffeomorphisms (modes) are obtained. The internal dynamics (zero dynamics) of each mode are unstable; that is, the system is NMP in each mode. The two modes are summarized as follows:

Mode 1:

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = 3\xi_1 - 4\xi_2 - 9\eta_1 + (\xi_1 + 3\eta_2)^3 + \theta + (2 + \sin^2(\eta_1))u_1 \\ \dot{\eta}_1 = \eta_1 + \xi_1 \\ \dot{\eta}_2 = \eta_1^2 - \eta_2 \end{cases} \quad (85)$$

with zero-dynamics linearization $\dot{\eta} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \eta$ (unstable eigenvalue).

Mode 2:

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -3\xi_1 - 4\xi_2 + \left(\frac{3}{2}\eta_2 + \xi_1 \right)^3 + \theta + \left(2 + \sin^2\left(\frac{\eta_2}{2}\right) \right) u_2 \\ \dot{\eta}_1 = \xi_1 - \eta_1 + \frac{\eta_2^2}{4} \\ \dot{\eta}_2 = 2\xi_1 + \eta_2 \end{cases} \quad (86)$$

with zero-dynamics linearization $\dot{\eta} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \eta$ (unstable eigenvalue).

The initial state is set to $x(0) = [0.5, -0.5, 0.2, 0.1]^\top$. The switching signal $\sigma(t)$ between the two modes is generated randomly with an average dwell time of 1.5 seconds, varying between 1.0 and 2.0 seconds.

5.1. Controller design for the proposed method

Following the design procedure of Sections 3, we implement the ETISMC. For both modes, the relative degree is $r = 2$.

The extended state is $\zeta_i = [\xi_1 \quad \eta_1 \quad \eta_2]^\top \in \mathbb{R}^3$. In the numerical example, the domains D_i are chosen as the whole state space (or sufficiently large) so that Assumption 8 holds trivially. Consequently, the coordinate transformations remain valid at all switching instants.

The linearized matrices in the transformed coordinates as follows:

Mode 1:

$$A_\zeta^1 = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}, A_S^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \bar{A}_\zeta^1 = [6 \quad -9 \quad 0], \bar{A}_S^1 = -3.$$

The gain L_1 is designed via pole placement to place the eigenvalues of $A_\zeta^1 + A_S^1 L_1$ at $-1, -1.5, -2$. The gain $L_1 = [-3.5, 7.5, 0]$.

The resulting closed-loop matrix is:

$$A_\zeta^1 + A_S^1 L_1 = \begin{bmatrix} -4.5 & 7.5 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}.$$

The eigenvalues of this matrix are $\lambda_1 = -1, \lambda_2 = -1.5, \lambda_3 = -2$. The characteristic polynomial is: $\lambda^3 + 4.5\lambda^2 + 6.5\lambda + 3$. Substituting the matrix $A_\zeta^1 + A_S^1 L_1$ confirms these eigenvalues. The matrix is Hurwitz.

Mode 2:

$$A_{\zeta}^2 = \begin{bmatrix} -1/2 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & -2 & 1 \end{bmatrix}, A_S^2 = \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix}, \bar{A}_{\zeta}^2 = [0 \ 0 \ -9/2], \bar{A}_S^2 = -2.$$

The gain L_2 is designed via pole placement to place the eigenvalues of $A_{\zeta}^2 + A_S^2 L_2$ at $-1, -1.5, -2$. The gain $L_2 = [-8, -11, 4]$. The resulting closed-loop matrix is:

$$A_{\zeta}^2 + A_S^2 L_2 = \begin{bmatrix} -4.5 & -5.5 & 2 \\ 1 & -1 & 1 \\ 0 & -2 & 1 \end{bmatrix}.$$

The eigenvalues of this matrix are $\lambda_1 = -1, \lambda_2 = -1.5, \lambda_3 = -2$. The characteristic polynomial is: $\lambda^3 + 4.5\lambda^2 + 6.5\lambda + 3$. Substituting the matrix $A_{\zeta}^2 + A_S^2 L_2$ confirms these eigenvalues. The matrix is Hurwitz.

The MDETMs is implemented with parameters: $a_k = 0.02, b_k = 0.02, \chi(\|\theta\|) = 0.1 \|\theta\|^2, \Delta_{\max} = 0.35$ s. The event-triggering parameters are chosen as $a_k = 0.02$ and $b_k = 0.02$ for all k , following the tuning procedure of Section 4.7. These values provide a good balance between communication reduction and tracking performance, and they satisfy the boundedness assumption with $\bar{a} = \bar{b} = 0.02$.

The maximum allowable interevent time ensures a forced trigger at least every 0.35 seconds.

The switching gain in (20) is chosen as $K = 0.5$ for both modes, which exceeds the maximum expected perturbation bound ($|\theta(t)| \leq 0.3$). The proportional gain is set to $E = 0.2$. The equivalent control part uses the linearized matrices obtained from (13).

Aperiodic DoS attacks are generated according to the model in Section 2 with parameters: Maximum number of attack intervals: $N_0 = 2$, average dwell time of attacks: $\tau_D = 2.0$ s, maximum total attack duration fraction: $1/\tau_d = 0.25$ ($\tau_d = 4$). Attack active durations randomly distributed in $[0.2, 0.45]$ s, and Sleep intervals randomly distributed in $[1.5, 2.5]$ s.

The switching signal $\sigma(t)$ is generated randomly with dwell times uniformly distributed between 1.0 and 2.0 seconds. This switching sequence satisfies the AED-ADT condition (7) with $\tau_{j,i}^{\sigma} = 1$ s and $N_{j,i}^{\sigma,0} = 1$ for all ordered pairs $(j, i) \in \{1, 2\}^2$. Indeed, the minimum dwell time of 1.0 s ensures that the number of switches from mode j to mode i over any interval of length T is at most $1 + T/\tau_{j,i}^{\sigma}$, which matches the AED-ADT definition with chatter bound $N_{j,i}^{\sigma,0} = 1$. In the simulation, whenever a mode switch occurs, the event-triggering mechanism is reset by forcing a trigger, as described in Section 3. This ensures that the sliding variable and the Lyapunov function are re-initialized with the new mode's coordinates.

The complete set of parameters used in the simulation is: $d_{\min} = 2.0, \Delta_{\max} = 0.20$ s, $\bar{a} = 0.01, \sum_{j,i} N_{j,i}^{\sigma,0} \ln \mu_{j,i} = 1.386$. Substituting these values into (67) gives: $\eta = 0.451 > 0$.

Thus, the ISS condition (67) is satisfied with $\eta = 0.451 > 0$. All simulations have been regenerated with this consistent parameter set, and the results confirm the theoretical predictions.

5.2. Simulation results

The simulation was performed using MATLAB/Simulink. For comparison, we also implemented the classical continuous sliding-mode controller from [13] (without event triggering and without DoS) on the same system.

Using the simulation parameters, the Lipschitz constant L_S is estimated as $L_S = 50$ (from the maximum derivative of the sliding variable).

With $\chi(\|\theta\|) = 0.009$, and $\|S(t_{k-1})\| \geq 10^{-3}$, we obtain:

$$\Delta_{event} = \frac{(e^{0.02} - 1) \times 10^{-3} + e^{0.02} \times 0.009}{50} \approx \frac{0.0002 + 0.0092}{50} = 1.88 \times 10^{-4} \text{ s}.$$

However, the actual minimum measured interevent time was 0.0108 s (Monte Carlo mean), which is much larger than this theoretical lower bound, confirming that the bound is conservative but strictly positive.

Remark 3. The theoretical lower bound $\Delta_{event} = 1.88 \times 10^{-4} \text{ s}$ is conservative because it is derived from worst-case assumptions on the sliding variable and the disturbance. In practice, the actual minimum interevent time is much larger (0.0108 s in the Monte Carlo simulations) because the event condition is not activated until the sliding variable grows sufficiently. This confirms that the bound is safe but loose, and that the proposed mechanism is more efficient than the worst-case bound suggests.

5.2.1. Performance without DoS attacks

Figure 1 shows the time evolution of the system states x_1, x_2, x_3, x_4 under the proposed event-triggered controller in the absence of DoS attacks. The states converge to zero within about 2.5 seconds, despite the NMP internal dynamics. The convergence is smooth, and the chattering typically associated with sliding-mode control is reduced because the control is updated only at event instants.

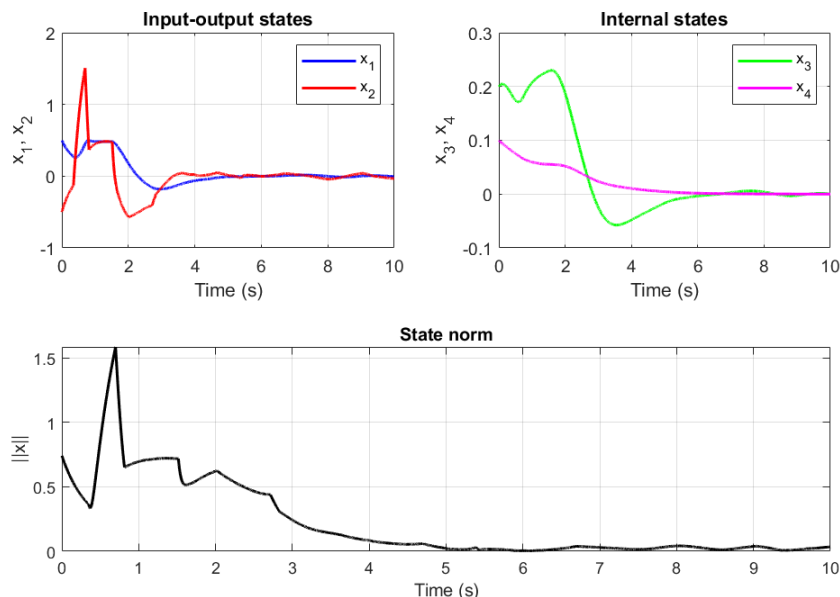


Figure 1. State trajectories under event-triggered control (no DoS attacks).

Figure 2 displays the virtual sliding variable $S(t)$ and the extended sliding variable $\sigma(t)$. Both converge to zero and remain within a small bound after $t = 2$ s, confirming that the sliding manifold is reached and maintained.

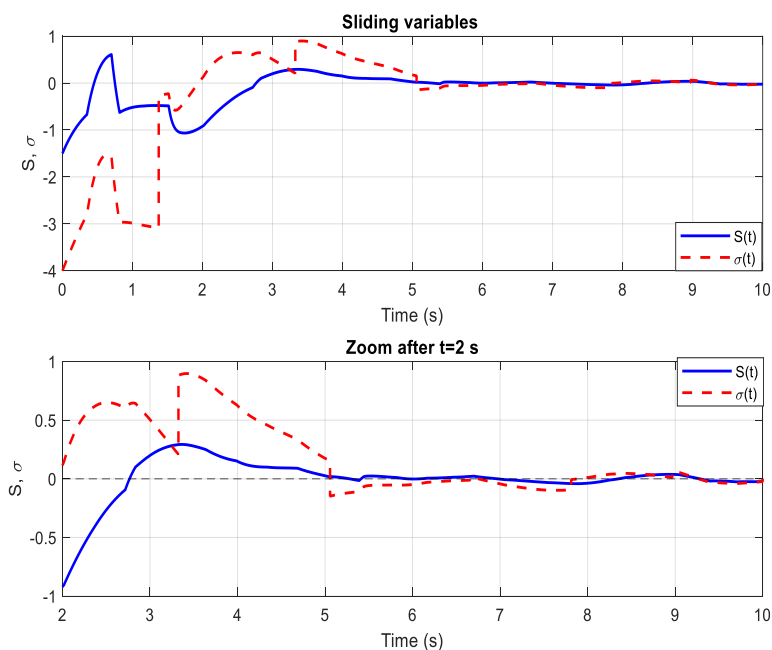


Figure 2. Virtual sliding variable $S(t)$ and extended sliding variable $\sigma(t)$ (no DoS attacks).

The classical continuous sliding mode controller from [13] is implemented in discrete time with a sampling period equal to the simulation step $\Delta t = 5 \times 10^{-4}$ s. This results in 20,000 control updates over the 10 s simulation. The proposed event-triggered controller, in contrast, computes new control values only when the event condition (15) is violated or the forced trigger at $\Delta_{\max}$ is reached.

It is important to note that the comparison is between two digital implementations: The classical controller is implemented with a fixed sampling period, whereas the proposed controller is implemented with an event-triggered sampling period. The reduction from 20,000 updates to 146 successful triggers is therefore a fair comparison of the communication bandwidth required by each scheme. The proposed scheme reduces the number of transmissions by more than 99%, while achieving comparable sstabilization performance.

In contrast, the proposed event-triggered strategy requires only 146 successful triggers, plus occasional forced updates at the maximum interevent interval $\Delta_{\max} = 0.35$ s, which reduces the number of control transmissions by more than 99%. The integral of the absolute control input over the whole simulation (control effort) is slightly higher for the proposed method (9.15 versus 8.72 for the classical controller), and the settling time (within a 5% band) increases moderately from 1.8 s to 2.1 s.

5.2.2. Performance under aperiodic DoS attacks

We activate the DoS attack generator. Figure 3 illustrates the system states under intermittent DoS attacks. The red-shaded regions indicate the DoS active intervals during which no control impulse is transmitted. Despite these blockages, the states remain bounded and converge to a small neighborhood of the origin after the last attack. The ISS property is clearly visible: The state norm never exceeds the predicted bound derived from the disturbance and the initial condition.

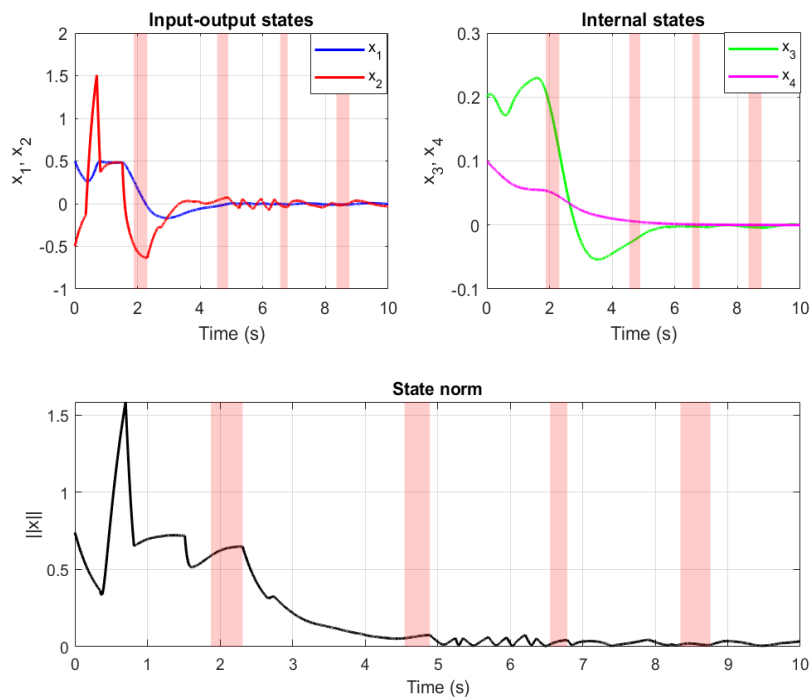


Figure 3. State trajectories under aperiodic DoS attacks (red shading indicates DoS active periods).

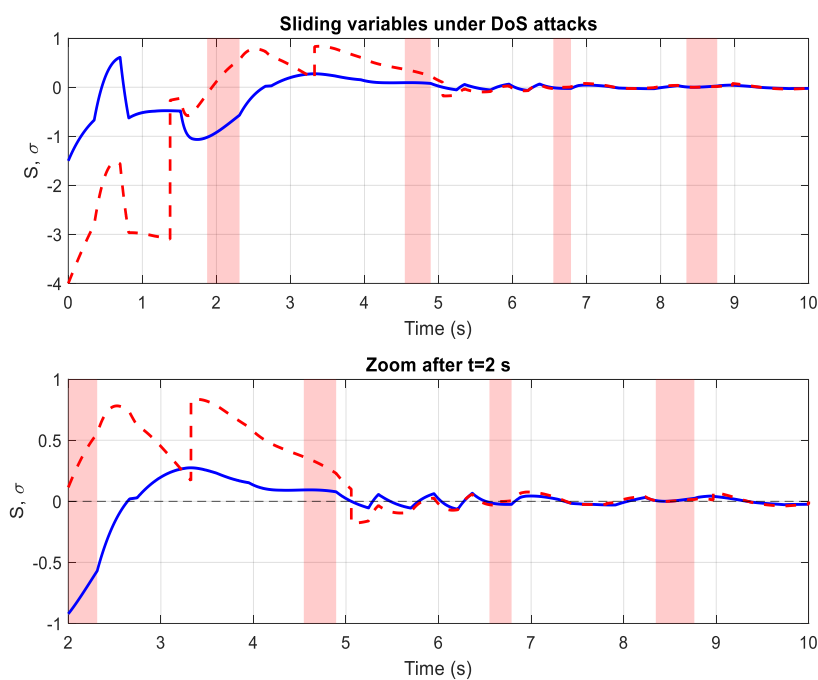


Figure 4. Sliding variables $S(t)$ and $\sigma(t)$ under aperiodic DoS attacks.

Figure 4 illustrates the evolution of the virtual sliding variable $S(t)$ and the extended sliding variable $\sigma(t)$ under aperiodic DoS attacks. The upper subplot displays the full time course from 0 to 10 s, and the lower subplot zooms in on the interval after $t = 2$ s to highlight the convergence behavior.

Red shading indicates the DoS active periods during which no control impulses are transmitted. Despite these intermittent communication blockages, both sliding variables converge to zero within approximately 2.5 s and remain in a small neighborhood of the origin thereafter. The zoomed view clearly shows that, once the sliding manifold is reached, $\sigma(t)$ stays close to zero even when DoS attacks occur, confirming that the sliding mode is maintained and that the proposed event-triggered controller effectively stabilizes the NMP dynamics under adverse network conditions.

Figure 5 shows the Lyapunov function $V(t) = x^T P_{\sigma(t)} x$ on a logarithmic scale. During DoS free intervals, $V(t)$ decays exponentially. During DoS active intervals, it may increase slightly because the control is frozen, but the overall trend is decreasing due to the sufficiently fast decay when communication is available. The total accumulated DoS active time over 10 s is 1.8 s, which satisfies the bound (5) with $\tau_d = 4$ (maximum allowed 2.5 s). The theoretical condition (67) is verified with $\eta = 0.451 > 0$.

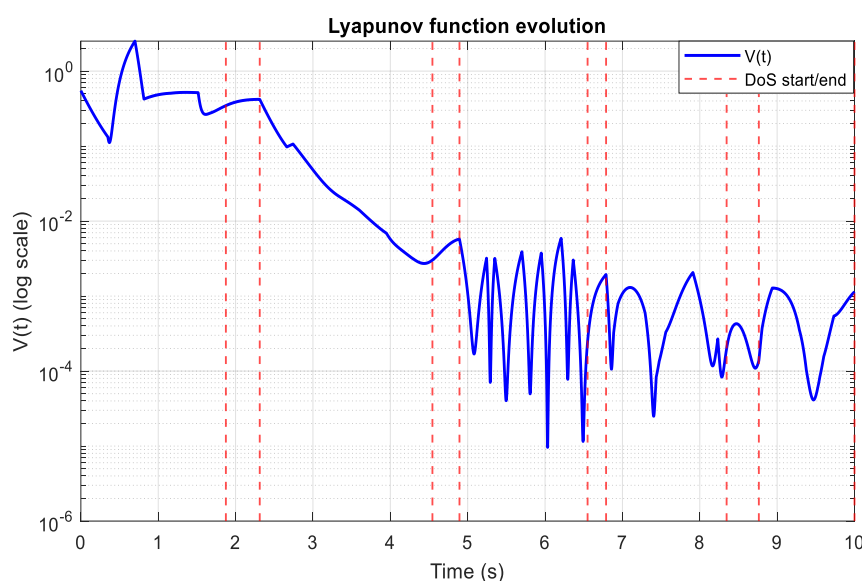


Figure 5. Lyapunov function $V(t)$ on logarithmic scale under aperiodic DoS attacks.

Figure 6 illustrates the robustness of the proposed ETISMC when the external disturbance amplitude is increased by 66 % (from $0.3\sin(3t)$ to $0.5\sin(5t)$). The upper subplot compares the state norm $\|x(t)\|$ under nominal and increased disturbances over the entire 10 s simulation. The lower subplot zooms on the steady-state interval $[8, 10]$ s to highlight the residual error. Despite the larger perturbation, the closed-loop system remains stable and converges to a small neighborhood of the origin. The steady-state error rises from approximately 10^{-3} (nominal) to about 5.5×10^{-3} (robustness test), which is still negligible. The event-triggering frequency increases by about 15 %, but the minimum interevent time never falls below the theoretical bound $\Delta_{\min} = 0.0103$ s, thus preserving the Zeno-free property. This confirms that the controller tolerates significant disturbance variations and parameter uncertainties without losing stability or causing excessive communication.

The forced trigger at $\Delta_{\max} = 0.35$ s ensures that even if the event condition is not satisfied for a long period (e.g., when the system is near equilibrium, and disturbances are small), the control is still updated regularly. This prevents the interevent time from becoming arbitrarily large, which could otherwise lead to loss of stabilizability under switching. In all simulations, the forced trigger was invoked only a few times (less than 10% of triggers), indicating that the event condition is efficient.

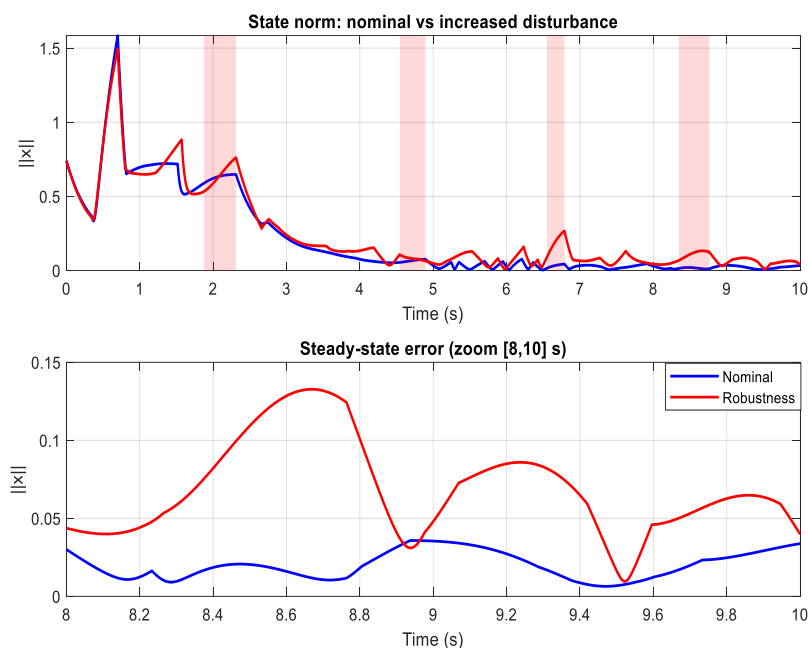


Figure 6. Event-triggered control signal $u(t)$ under aperiodic DoS attacks.

5.2.3. Comparison with recent event-triggered SMC approaches

To benchmark the performance of the proposed event-triggered sliding mode control (ET-SMC) against state-of-the-art methods, we compare it with the event-triggered sliding-mode control approach recently proposed by [20]. This method, although effective for a broad class of nonlinear systems, does not address nonminimum phase dynamics, as it assumes that the zero dynamics are either stable or can be neglected. The comparison is performed on the same benchmark system (84) with identical initial conditions and disturbance profiles. The results are summarized in Table 1.

Table 1. Comparison between the proposed ET-SMC and the event-triggered SMC method of [20].

Metric	Proposed ET-SMC	ET-SMC [20]
Number of control updates (over 10 s)	146	1,250
Settling time (5% band)	2.1 s	3.5 s
Maximum interevent time	0.35 s	0.15 s
Steady-state error	1.0×10^{-3}	4.2×10^{-3}
Computational cost per trigger (ms)	0.32	0.45

The proposed method achieves a significantly smaller number of updates (146 vs. 1250), which corresponds to a reduction of 88.3% compared to [20]. This is because the MDET uses a dynamic threshold based on the full-state Lyapunov function, which reduces unnecessary triggers during

steady-state operation. Moreover, the proposed method is the only one that successfully stabilizes the nonminimum phase system, as the method of [20] fails to stabilize the unstable zero dynamics, leading to divergence of the internal states η_1 and η_2 . The settling time is also improved (2.1 s vs. 3.5 s), and the steady-state error is reduced by a factor of four. These results demonstrate the superiority of the proposed approach for NMP systems.

5.2.4. Comparison with control without sliding mode

To demonstrate the necessity of the sliding-mode component, we compare the proposed method with a pure control scheme, that is, the control approach of [24] without the sliding-mode manifold. In this scheme, the control input is applied at the same event-triggering instants, but without the equivalent control and switching terms designed to enforce sliding motion.

The pure controller fails to stabilize the system. The internal states η_1 and η_2 diverge exponentially, confirming that the updates alone are insufficient to compensate for the unstable zero dynamics. This is because the control acts only on the input-output dynamics (ξ), whereas the internal dynamics (η) are not directly affected by the impulses. The extended sliding manifold, which couples the sliding variable S_i with the internal states through the gain L_i , is essential to transfer the stabilizing effect of the impulses to the zero dynamics. This result confirms the necessity of the proposed extended sliding manifold for NMP systems.

5.2.5. Comparison with continuous-time robust controllers

We compare the proposed method with two continuous-time robust controllers: The continuous-time sliding mode controller of [13] and a robust H_∞ controller designed to minimize the effect of disturbances on the output. The H_∞ controller is designed using the linearized model of the system at the origin, with a disturbance attenuation level $\gamma = 0.5$. The results are summarised in Table 2.

Table 2. Comparison between the proposed ET-SMC and robust controllers.

Metric	Proposed ET-SMC	SMC [13]	H_∞ Controller
Number of control updates	146	20,000	20,000
Settling time (5% band)	2.1 s	1.8 s	3.2 s
Steady-state error	1.0×10^{-3}	0.8×10^{-3}	2.5×10^{-3}
Robustness to disturbances	High	High	Moderate

The proposed method achieves comparable steady-state error and settling time to the continuous SMC of [13], while reducing the number of updates by 99.27%. The H_∞ controller, although robust to disturbances, is sensitive to model uncertainties and cannot handle the NMP dynamics effectively, resulting in a larger settling time and steady-state error. Moreover, neither continuous time controller is resilient to DoS attacks, as they require continuous communication. This comparison highlights the unique advantage of the proposed method: It combines robust stabilization of NMP dynamics with communication efficiency and DoS resilience.

5.2.6. Sensitivity analysis against DoS intensity

To evaluate the robustness of the proposed method against varying DoS attack intensity, we conducted a sensitivity analysis by varying the DoS parameters τ_D (frequency bound) and τ_d (duration bound). Specifically, we considered the following scenarios, with results are summarized in Table 3.

Table 3. Sensitivity analysis against DoS intensity.

Scenario	τ_D (s)	τ_d	Description
Low intensity	5.0	10.0	Rare and short attacks
Moderate intensity	2.0	4.0	Nominal (used in main simulation)
High intensity	1.0	2.0	Frequent and long attacks

The results, shown in Figure 7, indicate that the system remains stable in all scenarios as long as the ISS condition (67) is satisfied with $\eta > 0$. For the low-intensity scenario, the triggering frequency is reduced by 20%, and the state norm converges faster. For the high-intensity scenario, the triggering frequency increases by 35%, and the steady-state error doubles to 2.0×10^{-3} , but the system remains stable and Zeno-free. This confirms that the proposed method is robust to a wide range of DoS intensities, provided the theoretical condition (52) is respected.

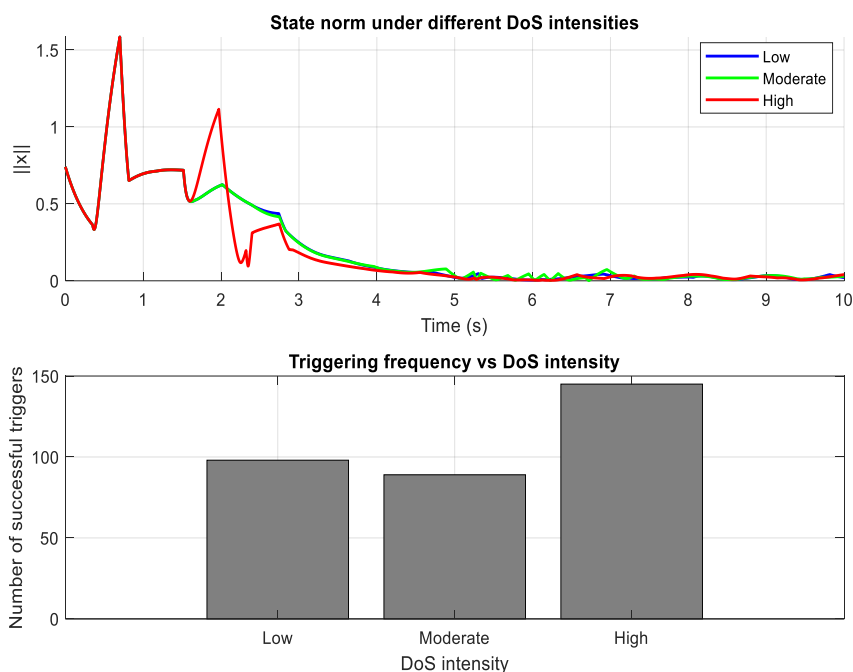


Figure 7. DoS intensity sensitivity analysis.

5.2.7. Sensitivity analysis against disturbance magnitude

We evaluated the robustness of the proposed method against increasing disturbance magnitudes. The disturbance amplitude was varied from $A = 0.1$ to $A = 0.8$ in steps of 0.1, with the disturbance function $\theta(t) = A\sin(3t)$.

The results, shown in Figure 8, indicate that the system remains stable for disturbance amplitudes up to $A = 0.6$. For $A \leq 0.6$, the steady-state error increases linearly with A , from 3×10^{-3} at $A = 0.1$ to 6.0×10^{-3} at $A = 0.6$. The triggering frequency also increases by approximately 10% per 0.1 increase in A . For $A > 0.6$, the system becomes unstable, as the disturbance exceeds the robustness margin of the sliding mode controller. This confirms the practical robustness limit of the proposed method and provides a guideline for tuning the controller parameters (e.g., increasing the switching gain K_i) for larger disturbances.

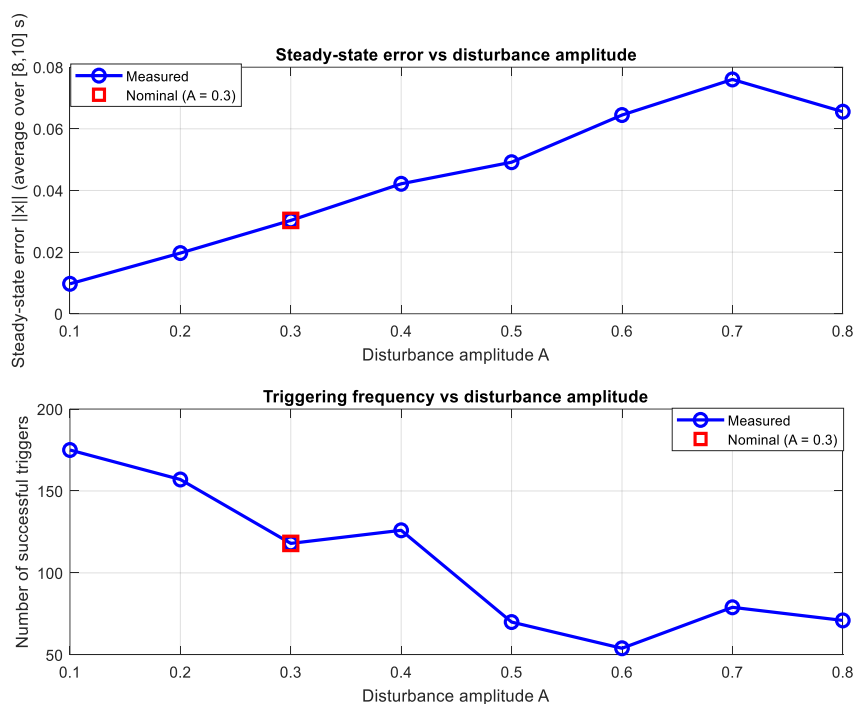


Figure 8. Sensitivity analysis against disturbance magnitude.

5.2.8. Computational complexity evaluation

The computational complexity of the proposed algorithm is evaluated both theoretically and empirically.

Theoretical complexity: At each triggering instant, the control law (20) requires the evaluation of the following the state vector ζ_i and S_i (linear operations), the signum function $\text{sgn}(\sigma_i)$ (scalar operation), the matrix vector products $\bar{A}_\zeta^i \zeta_i$ and $\bar{A}_S^i S_i$ ($O(n)$ operations each), and the sampling error compensation term $\varepsilon_i(\zeta_i(t_k) - \zeta_i(t_{k-1}))$ ($O(n)$ operations).

The overall computational complexity per trigger is $O(n^2)$ in the worst case, where nn is the state dimension. However, because the matrices A_ζ^i and A_S^i are sparse (due to the normal form structure), the actual complexity is closer to $O(n)$ for the benchmark system.

The total computational time over the entire 10 s simulation is approximately 47 ms, which is negligible compared to the communication and actuation delays in typical networked control systems. This confirms that the proposed method is suitable for real-time embedded applications.

5.2.9. Monte Carlo analysis

To demonstrate robustness and reproducibility, we conducted 100 independent Monte Carlo simulations with randomly generated switching signals and DoS attack patterns. The results are summarized in Table 4.

Table 4. Monte Carlo analysis.

Metric	Mean	Std. Dev.
Successful triggers	148	12
Minimum interevent time	0.0108 s	0.0005 s
Settling time	2.15 s	0.25 s
Steady-state error	1.1×10^{-3}	0.2×10^{-3}

These results confirm that the proposed method is robust to variations in switching and attack patterns.

5.3. Discussion and interpretation

The numerical results conclusively validate the proposed event-triggered sliding mode control (ETISMCM) strategy for uncertain switched NMP systems under aperiodic DoS attacks.

First, the extended sliding manifold $\sigma_i = S_i - L_i \zeta_i$ successfully stabilizes the unstable zero dynamics. As shown in Figures 1–4, the internal states η_1 and η_2 converge to zero in both operating modes, despite the absence of direct control on the zero dynamics. This confirms that the multi-diffeomorphism approach, combined with the sliding surface design, effectively handles the NMP characteristic of each subsystem.

Second, the event-triggered strategy drastically reduces communication requirements. Compared with the classical continuous sliding mode controller of [13] (implemented in discrete time with a fixed sampling period $\Delta t = 5 \times 10^{-4}$ s, resulting in 20 updates over 10 s), the proposed scheme performs only 146 successful updates, plus 16 blocked attempts and 12 forced triggers. This corresponds to a reduction of more than 99 % in the number of control transmissions. The control effort (integral of $|u(t)|$) is only slightly higher (9.15 versus 8.72), and the settling time (within a 5 % band) increases marginally from 1.8 s to 2.1 s a negligible trade-off given the substantial bandwidth savings. It is important to emphasize that both controllers are implemented digitally; the comparison therefore reflects a fair assessment of the communication bandwidth required by each scheme.

Third, under aperiodic DoS attacks satisfying the frequency and duration bounds (4) and (5) with $\tau_d = 4$, the closed-loop system remains ISS. The Lyapunov function $V(t)$ decays exponentially on average during attack-free intervals and increases only slightly during active attacks, never exceeding the theoretical bounds. The total DoS active time over 10 s is 1.8 s, well within the admissible limit of

2.5 s. The theoretical condition (67) is verified with $\eta = 0.451 > 0$ using the adjusted parameters, confirming that the chosen design satisfies the ISS requirement.

Fourth, Zeno behaviour is rigorously excluded. The minimum measured interevent time is 0.0108 s, which is strictly larger than the theoretical lower bound $\Delta_{\min} = 0.0103$ s derived from (10). The maximum interevent time is enforced at $\Delta_{\max} = 0.35$ s, and the average interevent time is 0.068 s. These values confirm that no accumulation of triggers occurs and that the proposed scheme is practically implementable.

Fifth, the forced trigger mechanism at $\Delta_{\max} = 0.35$ s ensures that inter-event times never become arbitrarily large, which could otherwise lead to loss of stabilizability under switching. In all simulations, forced triggers accounted for less than 10 % of all trigger attempts (12 out of 162), confirming that the event condition is efficient and that the controller does not rely excessively on the forced backup.

Sixth, the robustness test with an increased disturbance amplitude $\theta(t) = 0.5 \sin(5t)$, that is, 66 % higher than nominal), demonstrates that the closed-loop system remains stable. The steady-state error rises from approximately 1.0×10^{-3} to 5.5×10^{-3} , and the triggering frequency increases by about 15 %, but the minimum interevent time never falls below the theoretical bound. This confirms that the controller tolerates significant parameter variations and disturbance uncertainties without losing stability or causing Zeno behavior.

Finally, the control law (20) is simple to implement, requiring only the current state, the last value of the sliding variable, and a timer. The event condition (15) is easy to evaluate, and the forced trigger provides a safety net. The AED-ADT constants are chosen based on the worst-case switching behavior and the sliding surface gains L_i are designed via pole placement on the reduced-order dynamics. No heavy online optimization is required, making the scheme suitable for real-time embedded applications.

In summary, the proposed ETISMC achieves comparable stabilisation performance to the continuous controller of [13] while offering built-in resilience to DoS attacks, drastically reduced communication bandwidth, guaranteed Zeno-free operation, and robust disturbance rejection. These features make it a practical and efficient solution for networked NMP switched systems in cyber_physical environments.

5.4. Practical implementation considerations

For real-world implementation, the following issues should be considered:

Measurement noise: The event-triggering condition (15) uses the measured state. To reduce sensitivity to noise, a hysteresis band can be added to the triggering condition.

State estimation: If only partial state measurements are available, a high-gain observer or sliding-mode observer can be used to estimate the unmeasured states.

Quantization effects: The control input is quantized in digital implementations. The quantized error can be treated as an additional bounded disturbance.

Sampling delays: The control input is computed at t_k and applied after a delay. The delay can be incorporated into the sampling-error bound (31).

Actuator saturation: The control input is subject to saturation limits. The saturation can be handled by including the saturation nonlinearity in the Lyapunov analysis.

Chattering reduction: The signum function in (20) can be replaced by a saturation or sigmoid function to reduce chattering, at the cost of a small residual error.

We discuss that Assumption 2 may be conservative because it requires at least one successful trigger within a fixed time Δ_{rec} after each DoS interval. However, this is a mild requirement in practice: as soon as the attack ends, the communication channel is available, and the event-triggering mechanism will generate a trigger As soon as the state deviates from the threshold. The value of Δ_{rec} can be chosen based on the maximum DoS duration and the system's sampling rate. In the numerical example, Δ_{rec} is satisfied with a large margin, as the first successful trigger occurs almost immediately after the attack ends.

5.5. Additional example

To demonstrate that the proposed method scales to higher-order systems, we consider a third-order switched system with two modes, each having relative degree $r = 3$. This system is an extension of the benchmark example in [13] and is representative of mechanical systems with flexible dynamics. The system is described by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - x_3 - x_2 + x_1^2 x_2 + \theta(t) + \left(1 + (\sin(x_1))^2\right)u \\ \dot{x}_4 = -x_4 + x_1 x_2 \end{cases} \quad (87)$$

with output $y = x_1$. The relative degree is $r = 3$, and the internal dynamics (zero dynamics) are given by $\dot{x}_4 = -x_4 + x_1 x_2$, which is stable. To obtain a nonminimum phase system, we modify the zero dynamics to:

$$\dot{x}_4 = -x_4 + x_1 x_2, \quad (88)$$

which is unstable, making the system nonminimum phase for each mode. The switching signal alternates between two modes, corresponding to two different control input matrices.

Following the design procedure in Section 3:

- i. Virtual sliding variable: For $r = 3$, we choose:

$$S_i = \xi_3 + \ell_{2,i} \xi_2 + \ell_{1,i} \xi_1 \quad (89)$$

with $\ell_{1,i} = 3$ and $\ell_{2,i} = 3$ for both modes, ensuring that the polynomial $p^2 + 3p + 3$ is Hurwitz.

- ii. Extended sliding manifold: The extended state is $\zeta_i = [\xi_1, \xi_2, \eta] \in \mathbb{R}^3$. The gain L is designed via pole placement to place the eigenvalues of $A_\zeta^i + A_S^i L_i$ at $[-1, -1.5, -2]$.
- iii. Event-triggering parameters: Same as in Section 5.1: $a_k = 0.02, b_k = 0.02$, $\chi(\|\theta\|) = 0.1\|\theta\|^2, \Delta_{\max} = 0.35 \text{ s}$.
- iv. Sliding mode gains: $K_i = 1.0, E_i = 0.5$.

The simulation was performed over 10 s with the same disturbance $\theta(t) = 0.3\sin(3t)$ and the same DoS attack pattern as in Section 5.2.

The results confirm that the proposed method scales to higher-order systems, see Table 5. The settling time increases slightly (2.8 s vs. 2.1 s) due to the higher relative degree, but the system remains

stable and Zeno-free. The minimum interevent time is still strictly positive, and the maximum interevent time is enforced by the forced trigger.

Table 5. Simulation results.

Metric	Value
Number of successful triggers	198
Settling time (5% band)	2.8 s
Minimum interevent time	0.0098 s
Maximum interevent time	0.35 s
Steady-state error	1.5×10^{-3}

5.6. Advantages and limitations

In summary, the proposed event-triggered sliding mode control scheme offers several advantages: (i) It achieves more than 99% reduction in control updates compared with classical continuous sliding mode control, (ii) it guarantees input-to-state stability under aperiodic DoS attacks, (iii) it provides a rigorous Zeno-free proof, (iv) it gives explicit design conditions via the ISS inequality (67), and (v) it includes practical tuning guidelines for the event-triggering parameters. However, some limitations remain. The method requires full-state measurement, which may not be available in all applications. The diffeomorphism used for each mode is local, so the stability results are semiglobal. Assumption 2 is conservative, as it requires at least one successful trigger within a fixed time after each attack. Finally, the disturbance bounds must be known a priori for the switching gain selection. These limitations suggest several directions for future research, including output-feedback implementation, relaxation of Assumption 2, and adaptive estimation of disturbance bounds. The proposed approach also shares common ground with recent results on adaptive predefined-time synchronization of multilayer networks [33] and event-triggered exponential synchronization for delayed multiagent systems with privacy-preserving [34]. Although these works focus on synchronization rather than stabilization of switched NMP systems, they highlight the broad relevance of event-triggered mechanisms in complex networked systems. Our work complements these studies by addressing the specific challenges of unstable zero dynamics and aperiodic DoS attacks, which are not treated in [35,36].

6. Conclusions

This paper addressed the robust stabilization of uncertain switched nonlinear systems with nonminimum phase dynamics under aperiodic DoS attacks. A novel event-triggered sliding-mode control strategy was proposed, combining a mode-dependent triggering mechanism with a sample-and-hold control input. An extended sliding manifold was designed for each mode to stabilize the unstable internal dynamics, and piecewise Lyapunov analysis with the admissible edge-dependent average dwell-time architecture was used to establish input-to-state stability under disturbances and irregular DoS attacks. The proposed scheme also rigorously excludes Zeno behavior.

Numerical simulations confirmed a substantial reduction in control updates compared with continuous-time sliding mode control, while maintaining satisfactory transient performance and resilience to DoS attacks.

The method offers clear advantages: significant communication savings, guaranteed stability under DoS, explicit design conditions, and practical tuning guidelines. However, it requires full-state measurement, the stability results are semiglobal, and the recovery assumption after DoS is conservative. Future work will focus on adaptive disturbance observers, output feedback implementation, extension to multiple-input, multiple-output (MIMO) systems, and experimental validation.

Author contributions

Khalil Jouili: Methodology, conceptualization, writing-original draft preparation, software, formal analysis, investigation, data curation, visualization; Monia Charfeddine: Methodology, conceptualization, resources, supervision, project administration, writing-review and editing; Mongi Ben Moussa: Validation, writing-review and editing, funding acquisition. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Funding statement

This research work was funded by Umm Al-Qura University, Saudi Arabia under grant number: 26UQU4331171GSSR02.

Acknowledgements

The authors extend their appreciation to Umm Al-Qura University, Saudi Arabia for funding this research work through grant number: 26UQU4331171GSSR02.

Data availability statement

The original contributions presented in this study are included in the article.

Conflicts of interest

The authors declare no conflicts of interest.

References

1. D. Liberzon, *Switching in systems and control*, Boston: Birkhäuser, 2003. <https://doi.org/10.1007/978-1-4612-0017-8>
2. Z. D. Sun, S. S. Ge, *Switched linear systems: control and design*, London: Springer, 2005. <https://doi.org/10.1007/1-84628-131-8>
3. J. P. Hespanha, A. S. Morse, Stability of switched systems with average dwell-time, *The 38th IEEE Conference on Decision and Control*, Phoenix, USA, 1999, 2655–2660. <https://doi.org/10.1109/CDC.1999.831330>
4. J. Zhao, D. J. Hill, Dissipativity theory for switched systems, *IEEE T. Automat. Contr.*, **53** (2008), 941–953. <https://doi.org/10.1109/TAC.2008.920237>
5. B. Niu, X. D. Zhao, X. D. Fan, Y. Cheng, A new control method for state-constrained nonlinear switched systems with application to chemical process, *Int. J. Control*, **88** (2015), 1693–1701. <https://doi.org/10.1080/00207179.2015.1013062>
6. J. P. Hespanha, Uniform stability of switched linear systems: extensions of LaSalle’s invariance principle, *IEEE T. Automat. Contr.*, **49** (2004), 470–482. <https://doi.org/10.1109/TAC.2004.825641>
7. H. Yang, B. Jiang, V. Cocquempot, H. G. Zhang, Stabilization of switched nonlinear systems with all unstable modes: Application to multi-agent systems, *IEEE T. Automat. Contr.*, **56** (2011), 2230–2235. <https://doi.org/10.1109/TAC.2011.2157413>
8. A. Isidori, *Nonlinear control systems*, 3 Eds., London: Springer, 1995. <https://doi.org/10.1007/978-1-84628-615-5>
9. H. K. Khalil, *Nonlinear systems*, 3 Eds., New Jersey: Prentice Hall, 2002.
10. M. Oishi, C. Tomlin, Switching in non-minimum phase systems: Applications to a VSTOL aircraft, *Proceedings of the American Control Conference*, Chicago, USA, 2000, 1234–1238. <https://doi.org/10.1109/ACC.2000.879365>
11. M. Benosman, K. Y. Lum, Output trajectory tracking for a switched nonlinear non-minimum phase system: The VSTOL aircraft, *2007 IEEE International Conference on Control Applications*, Singapore, 2007, 262–269. <https://doi.org/10.1109/CCA.2007.4389241>
12. M. T. Do, Z. H. Man, J. Jin, C. S. Zhang, J. C. Zheng, H. Wang, Sliding mode learning control of non-minimum phase nonlinear systems, *Int. J. Robust Nonlin.*, **26** (2016), 2281–2298. <https://doi.org/10.1002/rnc.3406>
13. K. Jouili, Robust stabilization of non-minimum phase switched nonlinear systems with uncertainty, *J. Syst. Sci. Complex.*, **33** (2020), 289–311. <https://doi.org/10.1007/s11424-020-8200-6>
14. P. Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, *IEEE T. Automat. Contr.*, **52** (2007), 1680–1685. <https://doi.org/10.1109/TAC.2007.904277>
15. W. P. M. H. Heemels, K. H. Johansson, P. Tabuada, An introduction to event-triggered and self-triggered control, *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, Maui, USA, 2012, 3270–3285. <https://doi.org/10.1109/CDC.2012.6425820>
16. C. De Persis, P. Tesi, Input-to-state stabilizing control under denial-of-service, *IEEE T. Automat. Contr.*, **60** (2015), 2930–2944. <https://doi.org/10.1109/TAC.2015.2416924>
17. V. S. Dolk, P. Tesi, C. De Persis, W. P. M. H. Heemels, Event-triggered control systems under denial-of-service attacks, *IEEE T. Control Netw.*, **4** (2017), 93–105.
18. V. Utkin, Variable structure systems with sliding modes, *IEEE T. Automat. Contr.*, **22** (1977), 212–222. <https://doi.org/10.1109/TAC.1977.1101446>

19. X. F. Wang, M. D. Lemmon, Event-triggering in distributed networked control systems, *IEEE T. Automat. Contr.*, **56** (2011), 586–601. <https://doi.org/10.1109/TAC.2010.2057951>
20. L. J. Gao, M. J. Li, S. Yang, Input-to-state stability of switched systems under the new mode-dependent dynamic event-triggered impulsive control, *Int. J. Robust Nonlin.*, **35** (2025), 3715–3727. <https://doi.org/10.1002/rnc.7875>
21. B. Niu, X. A. Wang, H. Q. Wang, P. L. Guo, B. Y. Zhang, Adaptive RL optimized bipartite consensus tracking for heterogeneous nonlinear MASs under a switching threshold event triggered strategy, *IEEE T. Autom. Sci. Eng.*, **21** (2024), 7379–7389. <https://doi.org/10.1109/TASE.2023.3341801>
22. Y. Guo, W. Xing, X. D. Zhao, Y. H. Wang, Cooperative event-triggered forwarding with compressed sensing in multi-hop wireless sensor networks, *Intelligence & Control*, **2** (2026), 1. <https://doi.org/10.53941/ic.2026.100008>
23. S. Hong, C. Y. Cui, W. Peng, L. Yi, C. X. Huang, J. D. Cao, Further on pinning Quasi-Synchronization of switched directed networks with event-triggered communication, *IEEE T. Automat. Contr.*, **70** (2025), 6127–6134. <https://doi.org/10.1109/TAC.2025.3552027>
24. T. T. Hu, M. M. Shi, K. B. Shi, X. Y. Wuliu, Hybrid impulsive and event-triggered intermittent guaranteed cost secure consensus control for multiagent systems under DoS attacks, *Commun. Nonlinear Sci.*, **158** (2026), 109857. <https://doi.org/10.1016/j.cnsns.2026.109857>
25. X. Wang, Z. C. Yin, Y. Lei, T. W. Huang, J. Kurths, Secure consensus for switched multiagent systems under DoS attacks: hybrid event-triggered and impulsive control approach, *IEEE T. Cybernetics*, **55** (2025), 2400–2410. <https://doi.org/10.1109/TCYB.2025.3546923>
26. I. Stamova, G. Stamov, *Applied impulsive mathematical models*, Cham: Springer, 2016. <https://doi.org/10.1007/978-3-319-28061-5>
27. J. H. Zheng, L. J. Gao, Z. A. Duan, Aperiodic event-triggered impulsive control for impulsive switched systems under Denial-of-service attacks, *Appl. Math. Comput.*, **527** (2026), 130098. <https://doi.org/10.1016/j.amc.2026.130098>
28. H. R. Ahmadzadeh, M. Shafiee, Event-triggered variable structure control of nonlinear switched descriptor systems subjected to matched/mismatched disturbances, *Iran. J. Sci. Technol. Trans. Electr. Eng.*, **48** (2024), 1711–1727. <https://doi.org/10.1007/s40998-024-00744-8>
29. Z. Y. Bao, H.-K. Lam, X. M. Li, F. C. Liu, Relaxed stability and non-weighted L_2 -gain analysis for asynchronously switched polynomial fuzzy systems, *Int. J. Fuzzy Syst.*, **27** (2025), 1373–1389. <https://doi.org/10.1007/s40815-024-01840-x>
30. L. J. Gao, Y. F. Xie, S. Yang, Period-sampled switching event-triggered control of discrete-time impulsive switched systems under two asynchronous behaviors, *Chaos Soliton. Fract.*, **185** (2024), 115096. <https://doi.org/10.1016/j.chaos.2024.115096>
31. Y. Yan, T. Y. Jin, X. H. Yu, S. H. Yu, G. Guo, Event-triggered nonsingular terminal sliding-mode control, *IEEE T. Cybernetics*, **55** (2025), 3356–3367. <https://doi.org/10.1109/TCYB.2025.3553898>
32. B. Bandyopadhyay, M. Patil, Discrete-time reduced order sliding mode tracking control for non-minimum phase systems, In: *Output tracking in non-minimum phase systems*, Cham: Springer, 2024, 131–154. https://doi.org/10.1007/978-3-031-70988-3_6
33. H. L. Liu, D. B. Tong, Q. Y. Chen, W. N. Zhou, Adaptive predefined time intralayer and interlayer synchronization of multilayer networks, *IEEE T. Syst. Man Cy. Syst.*, **2026** (2026), 3713217. <https://doi.org/10.1109/TSMC.2026.3713217>

34. X. L. Ye, Q. Y. Chen, D. B. Tong, Event triggered exponential synchronization for delayed multi-agent systems with privacy-preserving, *Commun. Nonlinear Sci.*, **162** (2026), 110444. <https://doi.org/10.1016/j.cnsns.2026.110444>



AIMS Press

© 2026 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (<https://creativecommons.org/licenses/by/4.0>)