



*Research article***Event-triggered topology identification of stochastic delayed hypernetworks via output coupling****Xueyi Zhao^{1,2}, Na Li³, and Anjing Fan^{4,*}**¹ School of Mathematics and Statistics, Hanjiang Normal University, Shiyan 442000, China² Hubei Key Laboratory of Applied Mathematics, Faculty of Mathematics and Statistics, Hubei University, Wuhan 430062, China³ School of Mathematics and Statistics, Henan University, Kaifeng 475000, China⁴ School of Computer and Information Engineering, Anyang Normal University, Anyang 455000, China*** Correspondence:** Email: anjingfan2013@163.com.

Abstract: This paper investigated the topology identification of stochastic delayed hypernetworks using only output variables. To overcome the limitation that full state information is often unmeasurable in practice, an adaptive drive-response framework was constructed. Considering the complexity of realistic network environments, multiple interaction delays and stochastic perturbations were explicitly incorporated into the system model. We designed robust output-feedback controllers and parameter updating laws under both continuous-time and event-triggered communication schemes. Particularly, the proposed event-triggered mechanism (ETM) effectively reduced the communication frequency among network nodes while strictly excluding Zeno behavior. Based on the stochastic Lyapunov stability theory, we rigorously proved that the synchronization and identification errors were uniformly ultimately bounded almost surely, ensuring the accurate reconstruction of unknown multilayer topological matrices. Finally, numerical simulations were provided to verify the effectiveness and communication efficiency of the proposed theoretical results.

Keywords: topology identification; event-triggered mechanism; output coupling; stochastic delayed hypernetworks; multiple time delays

Mathematics Subject Classification: 93A16, 93C65, 34K50, 60H10, 93C40

1. Introduction

Multiplex and hypernetworks are widely used to model complex interacting systems across various domains [1–3]. In practical applications, however, the exact network topologies are often unknown,

making structural identification an essential step for system analysis and control [4–6]. Moreover, real-world networks are inevitably subject to communication delays and stochastic noise. These factors can severely degrade synchronization performance and must be explicitly considered in system modeling [7].

Most existing topology identification methods assume that the full state of all network nodes is measurable. In many practical scenarios, measuring the complete state variables is physically difficult or costly, restricting observations to partial output variables [8–10], as well as advanced control strategies [11–13]. Therefore, developing output-based identification schemes is of great practical significance. Furthermore, few studies have simultaneously addressed the identification problem in networks featuring multi-layer structures, stochastic perturbations, and multiple time delays. Additionally, continuous communication in large-scale hypernetworks consumes significant bandwidth and energy. To alleviate this issue, the event-triggered mechanism (ETM) has been introduced to schedule data transmissions only when specific conditions are met [14, 15]. Although ETM has been widely investigated for network synchronization, achieving simultaneous topology identification and synchronization via event-triggered output feedback in stochastic delayed environments remains a challenging problem. This difficulty primarily arises from the complex interactions among discrete triggering errors, multiple temporal delays, and persistent stochastic fluctuations.

In particular, existing topology identification schemes either rely on full-state feedback [16, 17], require continuous communication without delays [18], or focus solely on synchronization with known structures [19, 20], leaving the problem of simultaneous output-based identification and synchronization under stochastic delays largely open. To address these issues, this paper proposes an output-coupling-based adaptive strategy for simultaneous topology identification and synchronization in stochastic delayed hypernetworks. The main contributions are summarized as follows:

First, an adaptive drive-response framework is established for stochastic delayed hypernetworks relying solely on output couplings. This completely removes the restrictive assumption of full-state observability, making the proposed method more applicable to real-world observation-limited scenarios.

Second, robust adaptive controllers and parameter updating laws are designed based on continuous-time output measurements. Through stochastic Lyapunov analysis, the proposed method guarantees precise synchronization and the exact reconstruction of unknown multilayer topological matrices, effectively overcoming the impacts of multiple time delays and noise.

Third, an event-triggered output-feedback identification scheme is further developed to reduce communication frequency. By designing a dynamic triggering condition with a constant lower bound, the proposed ETM ensures that all closed-loop signals are uniformly ultimately bounded almost surely, and rigorously excludes Zeno behavior.

The remainder of this paper is organized as follows. Section 2 provides the preliminaries. Section 3 introduces the network model. Sections 4 and 5 present the main theoretical results and numerical simulations, respectively. Finally, Section 6 concludes the paper.

2. Preliminaries

The mathematical notations adopted in this study are standardized as follows. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ be a complete probability space with a right-continuous filtration $\{\mathcal{F}_t\}_{t \geq 0}$ such that $\mathcal{F}_0$ contains all $\mathcal{P}$ -null sets. The sets of nonnegative real numbers, n -dimensional real vectors, and $n \times m$ real matrices are indicated by $\mathbb{R}_+$, $\mathbb{R}^n$, and $\mathbb{R}^{n \times m}$, respectively. For an arbitrary real symmetric matrix, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ respectively designate its maximum and minimum eigenvalues. The transpose and the trace of a matrix are symbolized by the superscript T and $\text{Tr}(\cdot)$. The operator $\mathbb{E}[\cdot]$ is used to signify the mathematical expectation. Furthermore, $\|\cdot\|$ stands for the standard Euclidean vector norm or the corresponding induced 2-norm for matrices. We denote by $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}_+)$ the family of all nonnegative functions $V(\mathbf{x}, t)$ that are continuously twice differentiable in $\mathbf{x}$ and continuously once differentiable in t .

Definition 2.1. [21] For a general n -dimensional Itô-type stochastic system described by:

$$d\mathbf{x}(t) = f(\mathbf{x}(t), t)dt + g(\mathbf{x}(t), t)d\mathbf{w}(t), \quad (2.1)$$

where an m -dimensional Brownian motion is given by $\mathbf{w}(t)$, the associated infinitesimal operator $\mathcal{L}$ acting on any function $V(\mathbf{x}, t) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}_+)$ is formulated as:

$$\mathcal{L}V(\mathbf{x}, t) = \frac{\partial V(\mathbf{x}, t)}{\partial t} + (\nabla_{\mathbf{x}} V)^T f(\mathbf{x}, t) + \frac{1}{2} \text{Tr} \left[g^T(\mathbf{x}, t) \nabla_{\mathbf{x}}^2 V g(\mathbf{x}, t) \right]. \quad (2.2)$$

Here, $\nabla_{\mathbf{x}}^2 V$ and $\nabla_{\mathbf{x}} V$ refer to the Hessian matrix and the gradient vector of $V(\mathbf{x}, t)$ with respect to $\mathbf{x}$, respectively.

Lemma 2.1. [22] Suppose there exist a scalar continuous function $\omega \in C(\mathbb{R}^n, \mathbb{R}_+)$, a function $\gamma \in L^1(\mathbb{R}_+, \mathbb{R}_+)$, and a function $\mathbf{V} \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}_+)$ satisfying:

$$\lim_{\|\mathbf{x}\| \rightarrow \infty} \inf_{t \geq 0} \mathbf{V}(\mathbf{x}, t) = \infty \quad (2.3)$$

and

$$\mathcal{L}\mathbf{V}(\mathbf{x}, t) \leq \gamma(t) - \omega(\mathbf{x}), \quad \forall (\mathbf{x}, t) \in \mathbb{R}^n \times \mathbb{R}_+. \quad (2.4)$$

If there also exists a constant $p > 2$ such that $\sup_{t \geq 0} \mathbb{E}|\mathbf{x}(t; x_0)|^p < \infty$ for any initial state $x_0 \in \mathbb{R}^n$, then the limit $\lim_{t \rightarrow \infty} \mathbf{V}(\mathbf{x}(t; x_0), t)$ exists and is finite almost surely (a.s.). Furthermore, the following holds:

$$\lim_{t \rightarrow \infty} \omega(\mathbf{x}(t; x_0)) = 0 \quad \text{a.s.} \quad (2.5)$$

Lemma 2.2. [23] Given arbitrary column vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and any symmetric positive definite matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$, the following matrix inequality is unconditionally satisfied:

$$2\mathbf{x}^T \mathbf{y} \leq \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{y}^T \mathbf{Q}^{-1} \mathbf{y}. \quad (2.6)$$

3. Hypernetwork model

Consider a model with N identical systems coupled through the connections of m different networks and output variables. Each node is an n -dimensional nonlinear system, which is described by:

$$\begin{aligned} d\mathbf{x}_i(t) = & (\mathbf{F}(\mathbf{x}_i(t)) + \sum_{k=1}^m \sum_{j=1}^N c^{(k)} a_{ij}^{(k)} \mathbf{B}^{(k)} \mathbf{y}_j(t - \tau_k)) dt \\ & + \sum_{k=1}^m \varphi_i^{(k)}(t, \mathbf{x}_i(t), \mathbf{x}_i(t - \tau_k)) d\mathbf{w}_i^{(1)}(t), \\ \mathbf{y}_i(t) = & \mathbf{H}\mathbf{x}_i(t), i = 1, 2, \dots, N, \end{aligned} \quad (3.1)$$

where $i = 1, 2, \dots, N$, $\mathbf{x}_i(t) = [x_i^1(t), x_i^2(t), \dots, x_i^n(t)]^T$ denotes the state of i -th node; $\mathbf{y}_i(t) \in \mathbb{R}^m$ is the output vector of the i th node; $\mathbf{H} \in \mathbb{R}^{m \times n}$ is the output matrix with full row rank, i.e., $\mathbf{H}\mathbf{H}^T > 0$. The coupling matrix $\mathbf{A}^{(k)} = (a_{ij}^{(k)})_{N \times N}$ ($k = 1, 2, \dots, m$) represents the topology of layer k and $\mathbf{A}^{(k)}$ is an irreducible matrix satisfying $a_{ij}^{(k)} = a_{ji}^{(k)} > 0, i \neq j, a_{ii}^{(k)} = -\sum_{j=1, j \neq i}^N a_{ij}^{(k)}$ for $i = 1, 2, \dots, N$. The matrix $\mathbf{B}_k, k = 1, 2, \dots, m$ is the k th inner coupling matrix. The denotations $\tau_k, k = 1, 2, \dots, m$ are different interaction delays and $c_k, k = 1, 2, \dots, m$ are different coupling strength. Besides, $\mathbf{w}_i^{(1)}(t)$ is an n -dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ where $E(d\mathbf{w}_i^{(1)}) = 0, E(d\mathbf{w}_i^{(1)})^2 = dt, E(d\mathbf{w}_i^{(1)} d\mathbf{w}_j^{(1)}) = 0, i \neq j, \varphi_i^{(k)}(t, \mathbf{x}_i(t), \mathbf{x}_i(t - \tau_k))$ is the noise intensity matrix at layer k .

Unlike existing single-layer or full-state-observable formulations, model (3.1) concurrently incorporates multi-layer topologies with heterogeneous delays, state-dependent stochastic perturbations, and observation constraints restricted strictly to output channels $y_i(t) = Hx_i(t)$, rendering it more reflective of realistic networked systems.

To estimate the unknown topology $\mathbf{A}^{(k)}$, construct a response network by receiving the output variable $\mathbf{y}_i$:

$$\begin{aligned} d\hat{\mathbf{x}}_i(t) = & (\mathbf{F}(\hat{\mathbf{x}}_i(t)) + \sum_{k=1}^m \sum_{j=1}^N c^{(k)} \hat{a}_{ij}^{(k)} \mathbf{B}^{(k)} \hat{\mathbf{y}}_j(t - \tau_k) \\ & + u_i(t)) dt + \sum_{k=1}^m \varphi_i^{(k)}(t, \mathbf{x}_i(t), \mathbf{x}_i(t - \tau_k)) d\mathbf{w}_i^{(2)}(t), \\ \hat{\mathbf{y}}_i(t) = & \mathbf{H}\hat{\mathbf{x}}_i(t), \end{aligned} \quad (3.2)$$

where $i = 1, 2, \dots, N$, $u_i(t)$ is the controller imposed on the i th node, $\hat{\mathbf{A}}^{(k)} = (\hat{a}_{ij}^{(k)})_{N \times N}$ is the estimation of $\mathbf{A}^{(k)}$, and $\mathbf{w}_i^{(2)}(t)$ is an n -dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ with $E(d\mathbf{w}_i^{(2)}) = 0, E(d\mathbf{w}_i^{(2)})^2 = dt, E(d\mathbf{w}_i^{(2)} d\mathbf{w}_j^{(2)}) = 0, i \neq j$.

Let $\tau_{\max} = \max_{1 \leq k \leq m} \{\tau_k\}$ denote the maximum interaction delay. The initial conditions of the drive system (3.1) and the response observer (3.2) are respectively defined as

$$x_i(s) = \phi_i(s) \in C([- \tau_{\max}, 0], \mathbb{R}^n), \quad \hat{x}_i(s) = \hat{\phi}_i(s) \in C([- \tau_{\max}, 0], \mathbb{R}^n), \quad \forall s \in [- \tau_{\max}, 0],$$

where $C([- \tau_{\max}, 0], \mathbb{R}^n)$ denotes the Banach space of continuous vector-valued functions defined on $[- \tau_{\max}, 0]$ equipped with the standard supremum norm $\|\phi\| = \sup_{- \tau_{\max} \leq s \leq 0} \|\phi(s)\|$.

Defining $\mathbf{e}_i(t) = \hat{\mathbf{x}}_i(t) - \mathbf{x}_i(t)$, $\mathbf{e}_{yi}(t) = \hat{\mathbf{y}}_i(t) - \mathbf{y}_i(t)$, $\tilde{a}_{ij}^{(k)} = \hat{a}_{ij}^{(k)} - a_{ij}^{(k)}$, $\sigma_i^{(k)}(t, \mathbf{e}_i(t), \mathbf{e}_i(t - \tau_k))d\mathbf{w}_i(t) = \varphi_i^{(k)}(t, \mathbf{y}_i(t), \mathbf{y}_i(t - \tau_k))d\mathbf{w}_i^{(2)}(t) - \varphi_i^{(k)}(t, \mathbf{x}_i(t), \mathbf{x}_i(t - \tau_k))d\mathbf{w}_i^{(1)}(t)$. One has

$$\begin{aligned} d\mathbf{e}_i(t) &= (\mathbf{F}(\hat{\mathbf{x}}_i(t)) - \mathbf{F}(\mathbf{x}_i(t))) + \sum_{k=1}^m \sum_{j=1}^N c^{(k)} \hat{a}_{ij}^{(k)} \mathbf{B}^{(k)} \hat{\mathbf{y}}_j(t - \tau_k) \\ &\quad - \sum_{k=1}^m \sum_{j=1}^N c^{(k)} a_{ij}^{(k)} \mathbf{B}^{(k)} \mathbf{y}_j(t - \tau_k) + u_i(t) dt \\ &\quad + \sum_{k=1}^m \sigma_i^{(k)}(t, \mathbf{e}_i(t), \mathbf{e}_i(t - \tau_k)) d\mathbf{w}_i(t) \\ &= (\mathbf{F}(\hat{\mathbf{x}}_i(t)) - \mathbf{F}(\mathbf{x}_i(t))) + \sum_{k=1}^m \sum_{j=1}^N c^{(k)} \tilde{a}_{ij}^{(k)} \mathbf{B}^{(k)} \hat{\mathbf{y}}_j(t - \tau_k) \\ &\quad + \sum_{k=1}^m \sum_{j=1}^N c^{(k)} a_{ij}^{(k)} \mathbf{B}^{(k)} \mathbf{e}_{yj}(t - \tau_k) + u_i(t) dt \\ &\quad + \sum_{k=1}^m \sigma_i^{(k)}(t, \mathbf{e}_i(t), \mathbf{e}_i(t - \tau_k)) d\mathbf{w}_i(t), \\ d\mathbf{e}_{yi}(t) &= d\hat{\mathbf{y}}_i(t) - d\mathbf{y}_i(t) = \mathbf{H}d\hat{\mathbf{x}}_i(t) - \mathbf{H}d\mathbf{x}_i(t) = \mathbf{H}d\mathbf{e}_i(t). \end{aligned} \quad (3.3)$$

4. Main results

To begin, some hypotheses are introduced.

Hypothesis 4.1. Assume that the intrinsic nonlinear dynamics $\mathbf{F}(\cdot)$ satisfy the Lipschitz continuity condition. Specifically, there exist strictly positive constants α_i corresponding to each node $i \in \{1, 2, \dots, N\}$, ensuring that the following inequality holds for any vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$:

$$\|\mathbf{F}(\mathbf{y}) - \mathbf{F}(\mathbf{x})\| \leq \alpha_i \|\mathbf{y} - \mathbf{x}\|.$$

Hypothesis 4.2. For each layer $k \in \{1, 2, \dots, m\}$, the time-varying transmission delay $\tau_k(t)$ is a continuously differentiable function with respect to time t . Furthermore, its rate of change is strictly upper-bounded by a known scalar δ , satisfying:

$$0 \leq \dot{\tau}_k(t) \leq \delta < 1.$$

Here, $\delta < 1$ ensures signal causality, while a larger δ requires higher control authority to counteract delay effects.

Hypothesis 4.3. The noise intensity matrix $\sigma_i^{(k)}(t)$ is assumed to be uniformly bounded. Additionally, there exist nonnegative scalars $\mu_i^{(k)}$ and $\nu_i^{(k)}$ such that the trace of its covariance is constrained by the following condition for all $i = 1, \dots, N$ and $k = 1, \dots, m$:

$$\text{Tr}((\sigma_i^{(k)}(t))^T \sigma_i^{(k)}(t)) \leq 2\mu_i^{(k)} \|\mathbf{e}_i(t)\|^2 + 2\nu_i^{(k)} \|\mathbf{e}_i(t - \tau_k)\|^2.$$

Hypothesis 4.4. To guarantee the successful reconstruction of the unknown network topologies, the delayed coupled state trajectories, denoted by the set $\{\{\mathbf{B}^{(k)}\mathbf{y}_j(t - \tau_k)\}_{j=1}^N\}_{k=1}^m\}$, are assumed to be persistently linearly independent when the hypernetwork converges to the synchronization manifold $\mathcal{S} = \{\mathbf{y}_i = \hat{\mathbf{y}}_i \mid i = 1, \dots, N\}$.

Under these hypotheses, the main result is deduced.

Theorem 4.1. Suppose that Hypotheses 4.1–4.4 hold. Then, the uncertain coupling configuration matrix $\mathbf{A}^{(k)}$ in network (3.1) can be identified by the estimated value $\hat{\mathbf{A}}^{(k)}$ via the following adaptive controllers (4.1) and updating laws. Meanwhile, the response system (3.2) reaches complete synchronization with the drive system (3.1).

$$\begin{aligned} u_i(t) &= -d_i(t)\mathbf{H}^T\mathbf{e}_{yi}(t) \\ \dot{d}_i(t) &= h_i\mathbf{e}_{yi}^T(t)\mathbf{H}\mathbf{H}^T\mathbf{e}_{yi}(t) \\ \dot{\hat{a}}_{ij}^{(k)} &= -\mathbf{e}_{yi}^T(t)\mathbf{H}\mathbf{B}^{(k)}\hat{\mathbf{y}}_j(t - \tau_k) \end{aligned} \quad (4.1)$$

where $h_i > 0$ is arbitrary constant and $i, j = 1, 2, \dots, N, k = 1, 2, \dots, m$.

Proof. Consider the following Lyapunov candidate:

$$\begin{aligned} \mathbf{V}(t) &= \frac{1}{2} \sum_{i=1}^N \mathbf{e}_{yi}^T(t)\mathbf{e}_{yi}(t) + \frac{1}{2} \sum_{k=1}^m \sum_{i=1}^N \sum_{j=1}^N c^{(k)}(\tilde{a}_{ij}^{(k)})^2 + \frac{1}{2} \sum_{i=1}^N \frac{1}{h_i} (d_i(t) - d)^2 \\ &\quad + \sum_{k=1}^m \frac{r}{1-\delta} \int_{t-\tau_k(t)}^t \sum_{i=1}^N \mathbf{e}_{yi}^T(\theta)\mathbf{e}_{yi}(\theta)d\theta, \quad d > 0, r > 0 \\ \mathcal{L}\mathbf{V} &= \sum_{i=1}^N \mathbf{e}_{yi}^T(t)\mathbf{H} \left[\mathbf{F}(\hat{\mathbf{x}}_i(t)) - \mathbf{F}(\mathbf{x}_i(t)) + \sum_{k=1}^m \sum_{j=1}^N c^{(k)}\tilde{a}_{ij}^{(k)}\mathbf{B}^{(k)}\hat{\mathbf{y}}_j(t - \tau_k) \right. \\ &\quad \left. + \sum_{k=1}^m \sum_{j=1}^N c^{(k)}a_{ij}^{(k)}\mathbf{B}^{(k)}\mathbf{e}_{yj}(t - \tau_k) - d_i\mathbf{H}^T\mathbf{e}_{yi}(t) \right] \\ &\quad + \sum_{k=1}^m \sum_{i=1}^N \sum_{j=1}^N c^{(k)}\tilde{a}_{ij}^{(k)}\dot{\tilde{a}}_{ij}^{(k)} + \sum_{i=1}^N \frac{1}{h_i} (d_i(t) - d)\dot{d}_i(t) \\ &\quad + \frac{1}{2} \sum_{k=1}^m \sum_{i=1}^N \text{trace}((\mathbf{H}\sigma_i^{(k)})^T\mathbf{H}\sigma_i^{(k)}) + \sum_{k=1}^m \sum_{i=1}^N \frac{r}{1-\delta} \mathbf{e}_{yi}^T(t)\mathbf{e}_{yi}(t) \\ &\quad - \sum_{k=1}^m \sum_{i=1}^N \frac{r}{1-\delta} (1 - \dot{\tau}_k(t))\mathbf{e}_{yi}^T(t - \tau_k)\mathbf{e}_{yi}(t - \tau_k) \\ &= \sum_{i=1}^N \mathbf{e}_{yi}^T(t)\mathbf{H} \left[\mathbf{F}(\hat{\mathbf{x}}_i(t)) - \mathbf{F}(\mathbf{x}_i(t)) \right] \\ &\quad + \sum_{k=1}^m \sum_{i=1}^N \sum_{j=1}^N c^{(k)}\mathbf{e}_{yi}^T(t)a_{ij}^{(k)}\mathbf{H}\mathbf{B}^{(k)}\mathbf{e}_{yj}(t - \tau_k) - \sum_{i=1}^N d\mathbf{e}_{yi}^T(t)\mathbf{H}\mathbf{H}^T\mathbf{e}_{yi}(t) \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \sum_{k=1}^m \sum_{i=1}^N \text{trace}((\mathbf{H}\sigma_i^{(k)})^T \mathbf{H}\sigma_i^{(k)}) + \sum_{k=1}^m \sum_{i=1}^N \frac{r}{1-\delta} \mathbf{e}_{yi}^T(t) \mathbf{e}_{yi}(t) \\
& - \sum_{k=1}^m \sum_{i=1}^N \frac{r}{1-\delta} (1 - \dot{\tau}_k(t)) \mathbf{e}_{yi}^T(t - \tau_k) \mathbf{e}_{yi}(t - \tau_k) \\
& \leq \sum_{i=1}^N \alpha \mathbf{e}_{yi}^T(t) \mathbf{e}_{yi}(t) + \sum_{k=1}^m \sum_{i=1}^N \sum_{j=1}^N c^{(k)} \mathbf{e}_{yi}^T(t) a_{ij}^{(k)} \mathbf{H}\mathbf{B}^{(k)} \mathbf{e}_{yj}(t - \tau_k) - \sum_{i=1}^N d \mathbf{e}_{yi}^T(t) \mathbf{H}\mathbf{H}^T \mathbf{e}_{yi}(t) \\
& + \sum_{k=1}^m \sum_{i=1}^N \mu_i^{(k)} \mathbf{e}_{yi}^T(t) \mathbf{e}_{yi}(t) + \sum_{k=1}^m \sum_{i=1}^N v_i^{(k)} \mathbf{e}_{yi}^T(t - \tau_k) \mathbf{e}_{yi}(t - \tau_k) \\
& + \sum_{k=1}^m \sum_{i=1}^N \frac{r}{1-\delta} \mathbf{e}_{yi}^T(t) \mathbf{e}_{yi}(t) - r \sum_{k=1}^m \sum_{i=1}^N \mathbf{e}_{yi}^T(t - \tau_k) \mathbf{e}_{yi}(t - \tau_k).
\end{aligned}$$

Denote $\mathbf{e}_y(t) = (\mathbf{e}_{y1}(t)^T, \mathbf{e}_{y2}(t)^T, \dots, \mathbf{e}_{yN}(t)^T)^T$, $\bar{\mathbf{B}}^{(k)} = \mathbf{H}\mathbf{B}^{(k)}$, $\mathbf{G}^{(k)} = \mathbf{A}^{(k)} \otimes \bar{\mathbf{B}}^{(k)}$, $\varrho_1 = \lambda_{\min}(\mathbf{H}\mathbf{H}^T)$, $\varrho_2 = \lambda_{\max}(\mathbf{G}^{(k)}(\mathbf{G}^{(k)})^T)$, $\mu_k = \max_{1 \leq i \leq N} \{\mu_i^{(k)}\}$, $v_k = \max_{1 \leq i \leq N} \{v_i^{(k)}\}$. According to Lemma 2.2, one has

$$\begin{aligned}
\mathcal{L}\mathbf{V} & \leq \sum_{i=1}^N (\alpha + \sum_{k=1}^m \frac{r}{1-\delta} + \sum_{k=1}^m \mu_k - d\varrho_1) \mathbf{e}_{yi}^T(t) \mathbf{e}_{yi}(t) + \frac{1}{2} \sum_{k=1}^m c^{(k)} \mathbf{e}_y^T(t) \mathbf{G}^{(k)} (\mathbf{G}^{(k)})^T \mathbf{e}_y(t) \\
& + \frac{1}{2} \sum_{k=1}^m c^{(k)} \mathbf{e}_y^T(t - \tau_k) \mathbf{e}_y(t - \tau_k) + \sum_{k=1}^m \sum_{i=1}^N (v_k - r) \mathbf{e}_{yi}^T(t - \tau_k) \mathbf{e}_{yi}(t - \tau_k) \\
& \leq (\alpha + \sum_{k=1}^m \frac{r}{1-\delta} + \sum_{k=1}^m \mu_k - d\varrho_1 + \frac{1}{2} \sum_{k=1}^m c^{(k)} \varrho_2) \mathbf{e}_y^T(t) \mathbf{e}_y(t) + \sum_{k=1}^m (v_k + \frac{1}{2} c^{(k)} - r) \mathbf{e}_y^T(t - \tau_k) \mathbf{e}_y(t - \tau_k) \\
& = (\mathbf{e}_y^T(t), \mathbf{e}_y^T(t - \tau_1), \dots, \mathbf{e}_y^T(t - \tau_m)) \Xi (\mathbf{e}_y(t), \mathbf{e}_y(t - \tau_1), \dots, \mathbf{e}_y(t - \tau_m))^T,
\end{aligned}$$

where,

$$\Xi = \begin{pmatrix} \Phi_1 & 0 & \cdots & 0 \\ 0 & \Phi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_{m+1} \end{pmatrix},$$

$$\Phi_1 = (\alpha + \sum_{k=1}^m \frac{r}{1-\delta} + \sum_{k=1}^m \mu_k - d\varrho_1 + \frac{1}{2} \sum_{k=1}^m c^{(k)} \varrho_2) \mathbf{I}_{nN}, \Phi_2 = (v_1 + \frac{1}{2} c^{(1)} - r) \mathbf{I}_{nN}, \dots, \Phi_{m+1} = (v_m + \frac{1}{2} c^{(m)} - r) \mathbf{I}_{nN}.$$

Taking $r > v_k + \frac{1}{2} c^{(k)}$, $k = 1, 2, \dots, m$, one has $\Phi_2 < 0, \dots, \Phi_{m+1} < 0$. Then the proper positive constant d can be selected,

$$d > \frac{1}{\varrho_1} [\alpha + \sum_{k=1}^m \frac{r}{1-\delta} + \sum_{k=1}^m \mu_k + \frac{1}{2} \sum_{k=1}^m c^{(k)} \varrho_2],$$

such that $\Phi_1 < 0$, brings about $\Xi < 0$.

Let $\lambda = \lambda_{\max}(\Xi)$, then $\mathcal{L}\mathbf{V} \leq \lambda \mathbf{q}^T \mathbf{q} \triangleq -\omega(\mathbf{q})$, where $\mathbf{q} = (\mathbf{e}_y^T(t), \mathbf{e}_y^T(t - \tau_1), \dots, \mathbf{e}_y^T(t - \tau_m))^T$. In addition, $\lim_{\|\mathbf{x}\| \rightarrow \infty} \inf_{0 \leq t < \infty} \mathbf{V} = \infty$ and σ_i is bounded, and one obtains $\lim_{t \rightarrow \infty} \omega(\mathbf{q}) = 0$ a.s. based on Lemma 2.1.

Also, from (3.3), one gets

$$\lim_{t \rightarrow +\infty} \left(\sum_{k=1}^m \sum_{j=1}^N c^{(k)} \tilde{a}_{ij}^{(k)} \mathbf{B}^{(k)} \hat{\mathbf{y}}_j(t - \tau_k) \right) = 0.$$

When $t \rightarrow \infty$, $\hat{\mathbf{y}}_i(t) \rightarrow \mathbf{y}_i(t)$, $\hat{\mathbf{x}}_i(t) \rightarrow \mathbf{x}_i(t)$, one has $\lim_{t \rightarrow +\infty} (\sum_{k=1}^m \sum_{j=1}^N c^{(k)} \mathbf{B}^{(k)} \mathbf{y}_j(t - \tau_k) (\hat{a}_{ij}^{(k)} - a_{ij}^{(k)})) = 0$. According to Hypothesis 4.4, $\hat{a}_{ij}^{(k)} = a_{ij}^{(k)}$, and one has $\Omega = \{\mathbf{e} = 0, \tilde{a}_{ij}^{(k)} = 0, d = \text{constants}\}$ for $i, j = 1, 2, \dots, N, k = 1, 2, \dots, m$. That is, with the controllers and updating laws (4.1), the response network (3.2) and the drive network (3.1) reach complete synchronization, and the unknown coupling configuration matrix $\mathbf{A}^{(k)}$ can be successfully identified. This completes the proof. $\square$

Remark 1. Unlike existing works that require full-state information, measuring complete internal variables of high-dimensional nodes is often physically infeasible in practice. Notably, the controller $u_i(t)$ and updating laws in (3.3) depend solely on the output measurement error. This output-based strategy significantly reduces measurement costs and enhances the practical applicability of the proposed method.

Theorem 4.2. Suppose that Hypotheses 4.1–4.4 hold. In addition, we impose the following two structural conditions:

(A1) Matching condition: For each layer $k = 1, \dots, m$, there exists a matrix $\bar{\mathbf{B}}^{(k)}$ such that

$$\mathbf{B}^{(k)} = \mathbf{H}^T \bar{\mathbf{B}}^{(k)}. \quad (4.2)$$

(A2) Output injectivity: The output matrix $\mathbf{H}$ is of full column rank, i.e., there exists a constant $\lambda_H > 0$ such that

$$\|\mathbf{H}\mathbf{x}\|^2 \geq \lambda_H \|\mathbf{x}\|^2 \quad \forall \mathbf{x} \in \mathbb{R}^n. \quad (4.3)$$

Consider the event-triggered output feedback controller

$$u_i(t) = -d_i(t) \mathbf{H}^T \mathbf{e}_{yi}(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \quad (4.4)$$

where the triggering instants are determined by

$$t_{k+1}^i = \inf \{t > t_k^i \mid \|\mathbf{e}_{yi}(t_k^i) - \mathbf{e}_{yi}(t)\| \geq \rho_i \|\mathbf{e}_{yi}(t)\| + \epsilon_i\} \quad (4.5)$$

with $0 < \rho_i < \frac{1}{\sqrt{2}}$ and $\epsilon_i > 0$. The adaptive gains are updated through

$$\dot{\hat{a}}_{ij}^{(k)}(t) = -\eta_a c^{(k)} \mathbf{e}_{yi}^T(t) \bar{\mathbf{B}}^{(k)} \hat{\mathbf{y}}_j(t - \tau_k) - \kappa_a \hat{a}_{ij}^{(k)}(t), \quad (4.6)$$

$$\dot{d}_i(t) = \text{Proj}_{[d_m, d_M]} \left(\eta_d \|\mathbf{e}_{yi}(t)\|^2 - \kappa_d d_i(t) \right), \quad (4.7)$$

with $\eta_a, \eta_d, \kappa_a, \kappa_d > 0$, and Proj denotes the standard projection operator that keeps $d_i(t) \in [d_m, d_M]$ with $d_m > 0$ and $d_M > \max_i \{(\alpha_i + \sum_k \mu_i^{(k)}) / (\xi_i \lambda_H)\}$. Then, all closed-loop signals are uniformly ultimately bounded almost surely, and Zeno behavior is almost surely excluded.

Proof. Define the measurement error induced by the event-triggered mechanism as $\mathbf{E}_i(t) = \mathbf{e}_{yi}(t_k^i) - \mathbf{e}_{yi}(t)$. From the triggering condition (4.5), for all $t \in [t_k^i, t_{k+1}^i)$, it unconditionally holds that

$$\|\mathbf{E}_i(t)\| \leq \rho_i \|\mathbf{e}_{yi}(t)\| + \epsilon_i \implies \|\mathbf{E}_i(t)\|^2 \leq 2\rho_i^2 \|\mathbf{e}_{yi}(t)\|^2 + 2\epsilon_i^2. \quad (4.8)$$

By denoting the state synchronization error as $\mathbf{e}_i = \hat{\mathbf{x}}_i - \mathbf{x}_i$ and the output error as $\mathbf{e}_{yi} = \mathbf{H}\mathbf{e}_i$, utilizing the matching condition (4.2), we have the relationship $\mathbf{e}_i^T \mathbf{B}^{(k)} = \mathbf{e}_i^T \mathbf{H}^T \bar{\mathbf{B}}^{(k)} = \mathbf{e}_{yi}^T \bar{\mathbf{B}}^{(k)}$.

To analyze the stability, we construct the following positive definite Lyapunov–Krasovskii functional:

$$V = V_1 + V_2 + V_3 + V_4, \quad (4.9)$$

with

$$V_1 = \frac{1}{2} \sum_{i=1}^N \|\mathbf{e}_i\|^2, \quad V_2 = \frac{1}{2\eta_a} \sum_{k,i,j} (\tilde{a}_{ij}^{(k)})^2, \quad (4.10)$$

$$V_3 = \sum_{i=1}^N \frac{\xi_i}{2\eta_d} (d_i - d_i^*)^2, \quad (4.11)$$

$$V_4 = \sum_{k=1}^m \frac{1}{2(1-\delta_k)} \sum_{j=1}^N \int_{t-\tau_k}^t \mathbf{e}_{yj}^T \mathbf{Q}^{(k)} \mathbf{e}_{yj} ds, \quad (4.12)$$

where $\xi_i = \frac{1}{2} - \rho_i^2 > 0$, $\mathbf{Q}^{(k)} > 0$ are symmetric positive definite matrices to be selected, and $d_i^* \in [d_m, d_M]$ is a sufficiently large target gain whose existence will be guaranteed subsequently.

Applying Itô's formula to V_1 ,

$$\begin{aligned} \mathcal{L}V_1 = & \sum_i \left[\mathbf{e}_i^T (\mathbf{F}(\hat{\mathbf{x}}_i) - \mathbf{F}(\mathbf{x}_i)) + \sum_{k,j} c^{(k)} \tilde{a}_{ij}^{(k)} \mathbf{e}_i^T \mathbf{B}^{(k)} \hat{\mathbf{y}}_j(t - \tau_k) \right. \\ & \left. + \sum_{k,j} c^{(k)} a_{ij}^{(k)} \mathbf{e}_i^T \mathbf{B}^{(k)} \mathbf{e}_{yj}(t - \tau_k) + \mathbf{e}_i^T u_i \right] + \frac{1}{2} \sum_{i,k} \text{Tr}((\sigma_i^{(k)})^T \sigma_i^{(k)}). \end{aligned} \quad (4.13)$$

By the matching condition, the terms involving $\tilde{a}_{ij}^{(k)}$ directly become $\sum_{k,j} c^{(k)} \tilde{a}_{ij}^{(k)} \mathbf{e}_{yi}^T \bar{\mathbf{B}}^{(k)} \hat{\mathbf{y}}_j$. Regarding the control input, an application of Young's inequality provides

$$\begin{aligned} \mathbf{e}_i^T u_i &= -d_i \mathbf{e}_{yi}^T \mathbf{e}_{yi} - d_i \mathbf{e}_{yi}^T \mathbf{E}_i \\ &\leq -d_i \left(1 - \frac{1}{2}\right) \|\mathbf{e}_{yi}\|^2 + \frac{d_i}{2} \|\mathbf{E}_i\|^2. \end{aligned}$$

Substituting the event-triggered bound (4.8) and recalling the projection bound $d_i \leq d_M$, we obtain

$$\mathbf{e}_i^T u_i \leq -d_i \left(\frac{1}{2} - \rho_i^2\right) \|\mathbf{e}_{yi}\|^2 + d_M \epsilon_i^2 = -d_i \xi_i \|\mathbf{e}_{yi}\|^2 + d_M \epsilon_i^2. \quad (4.14)$$

Furthermore, the Lipschitz condition (Hyp. 4.1) yields $\mathbf{e}_i^T (\mathbf{F}(\hat{\mathbf{x}}_i) - \mathbf{F}(\mathbf{x}_i)) \leq \alpha_i \|\mathbf{e}_i\|^2$. Hypothesis 4.3 on the noise intensity implies

$$\frac{1}{2} \sum_k \text{Tr}((\sigma_i^{(k)})^T \sigma_i^{(k)}) \leq \sum_k \mu_i^{(k)} \|\mathbf{e}_i\|^2 + \sum_k \nu_i^{(k)} \|\mathbf{e}_i(t - \tau_k)\|^2. \quad (4.15)$$

Now we compute $\mathcal{L}V_2$. From the adaptive updating law (4.6), it follows that

$$\mathcal{L}V_2 = \sum_{k,i,j} \tilde{a}_{ij}^{(k)} \left(-c^{(k)} \mathbf{e}_{yi}^T \bar{\mathbf{B}}^{(k)} \hat{\mathbf{y}}_j(t - \tau_k) - \frac{\kappa_a}{\eta_a} \hat{a}_{ij}^{(k)} \right). \quad (4.16)$$

Thanks to the matching condition, the cross terms involving $\tilde{a}_{ij}^{(k)}$ in $\mathcal{L}V_1$ and $\mathcal{L}V_2$ cancel each other exactly. For V_3 , since $d_i^* \in [d_m, d_M]$, the projection operator rigorously ensures $(d_i - d_i^*) \text{Proj}(\cdot) \leq (d_i - d_i^*)(\cdot)$. Hence, using the property $-\tilde{d}_i d_i = -\tilde{d}_i(\tilde{d}_i + d_i^*) \leq -\frac{1}{2}\tilde{d}_i^2 + \frac{1}{2}(d_i^*)^2$, we have

$$\begin{aligned} \mathcal{L}V_3 &\leq \frac{\xi_i}{\eta_d} (d_i - d_i^*) (\eta_d \|\mathbf{e}_{yi}\|^2 - \kappa_d d_i) \\ &= \xi_i \tilde{d}_i \|\mathbf{e}_{yi}\|^2 - \frac{\kappa_d \xi_i}{\eta_d} \tilde{d}_i d_i \\ &\leq \xi_i \tilde{d}_i \|\mathbf{e}_{yi}\|^2 - \frac{\kappa_d \xi_i}{2\eta_d} \tilde{d}_i^2 + \frac{\kappa_d \xi_i}{2\eta_d} (d_i^*)^2. \end{aligned} \quad (4.17)$$

Notice that the term $+\xi_i \tilde{d}_i \|\mathbf{e}_{yi}\|^2$ originating from $\mathcal{L}V_3$ perfectly cancels the $-\xi_i \tilde{d}_i \|\mathbf{e}_{yi}\|^2$ implicitly present in the control bound (4.14) (since $d_i = d_i^* + \tilde{d}_i$).

We now handle the delayed coupling terms. The relevant sum involving $a_{ij}^{(k)}$ in $\mathcal{L}V_1$ reads

$$\sum_{i,j,k} c^{(k)} a_{ij}^{(k)} \mathbf{e}_{yi}^T \bar{\mathbf{B}}^{(k)} \mathbf{e}_{yj}(t - \tau_k).$$

Together with the derivative of V_4 , by choosing $\mathbf{Q}^{(k)} = \gamma \mathbf{I}$ with a sufficiently small $\gamma > 0$ and applying standard integral inequalities, we can find a constant $\beta > 0$, which can be made arbitrarily small, such that

$$\sum_{i,j,k} c^{(k)} a_{ij}^{(k)} \mathbf{e}_{yi}^T \bar{\mathbf{B}}^{(k)} \mathbf{e}_{yj}(t - \tau_k) + \dot{V}_4 \leq \beta \sum_{j=1}^N \|\mathbf{e}_{yj}\|^2 + C_\tau, \quad (4.18)$$

where C_τ collects bounded terms stemming from the delay constraints.

Collecting all the estimates and utilizing the output injectivity condition (4.3), which provides $\|\mathbf{e}_{yi}\|^2 \geq \lambda_H \|\mathbf{e}_i\|^2$, we obtain

$$\begin{aligned} \mathcal{L}V &\leq \sum_{i=1}^N \left(\frac{\alpha_i + \sum_k \mu_i^{(k)}}{\lambda_H} + \beta - \xi_i d_i^* \right) \|\mathbf{e}_{yi}\|^2 - \frac{\kappa_a}{2\eta_a} \sum_{k,i,j} (\tilde{a}_{ij}^{(k)})^2 \\ &\quad - \sum_{i=1}^N \frac{\kappa_d \xi_i}{2\eta_d} \tilde{d}_i^2 + \underbrace{\sum_{i=1}^N d_M \epsilon_i^2 + \sum_{i=1}^N \frac{\kappa_d \xi_i}{2\eta_d} (d_i^*)^2}_{=: C_0} + C_\tau. \end{aligned} \quad (4.19)$$

Because $\xi_i > 0$ and $\lambda_H > 0$, we can purposefully choose a large target gain d_i^* (and consequently set d_M sufficiently large to accommodate it) such that $\xi_i d_i^* > \frac{\alpha_i + \sum_k \mu_i^{(k)}}{\lambda_H} + \beta$ for all $i = 1, \dots, N$. Define

$$\mu := \min \left\{ 2(\xi_i d_i^* \lambda_H - \alpha_i - \sum_k \mu_i^{(k)} - \beta), \kappa_a \eta_a, \frac{\kappa_d \xi_i}{\eta_d} \right\} > 0. \quad (4.20)$$

Then there exists a positive constant C_0 (independent of t) such that

$$\mathcal{L}V(t) \leq -\mu V(t) + C_0. \quad (4.21)$$

The inequality (4.21) is a standard stochastic Lyapunov differential inequality. By the generalized Itô formula (Dynkin's formula), taking the expectation yields $\mathbb{E}[V(t)] \leq V(0)e^{-\mu t} + \frac{C_0}{\mu}(1 - e^{-\mu t})$. Hence, $\limsup_{t \rightarrow \infty} \mathbb{E}[V(t)] \leq C_0/\mu$, which indicates that all signals are uniformly ultimately bounded in mean square. To rigorously establish almost-sure uniform ultimate boundedness, we apply Itô's differential formula to $e^{\mu t}V(t)$:

$$d(e^{\mu t}V(t)) = e^{\mu t}(\mathcal{L}V(t) + \mu V(t))dt + e^{\mu t}dM(t) \leq C_0e^{\mu t}dt + e^{\mu t}dM(t),$$

where $M(t) = \int_0^t \sum_{i=1}^N \sum_{k=1}^m e_i^T(s) \sigma_i^{(k)}(s) dw_i(s)$ is a continuous local martingale. Integrating both sides over $[0, t]$ gives

$$V(t) \leq V(0)e^{-\mu t} + \frac{C_0}{\mu}(1 - e^{-\mu t}) + e^{-\mu t} \int_0^t e^{\mu s} dM(s).$$

By virtue of the exponential martingale inequality along with the Borel–Cantelli lemma, it follows that $\lim_{t \rightarrow \infty} e^{-\mu t} \int_0^t e^{\mu s} dM(s) = 0$ almost surely (a.s.). Therefore, taking the superior limit on both sides yields

$$\limsup_{t \rightarrow \infty} V(t) \leq \frac{C_0}{\mu} \quad \text{a.s.},$$

which rigorously proves that all closed-loop error trajectories are uniformly ultimately bounded almost surely and remain confined within the compact set $\mathcal{D} = \{V(t) \leq C_0/\mu\}$ with probability one.

Finally, to rigorously exclude Zeno behavior, we examine the inter-event intervals. For $t \in [t_k^i, t_{k+1}^i)$, utilizing the integral form of the error dynamics yields

$$\mathbf{E}_i(t) = -\mathbf{H} \int_{t_k^i}^t \Psi_i(s) ds - \mathbf{H} \int_{t_k^i}^t \sigma_{ei}(s) d\mathbf{w}_i(s), \quad (4.22)$$

where $\Psi_i(s)$ represents the drift part of $\dot{\mathbf{e}}_{yi}$ and $\sigma_{ei} = \sum_k \sigma_i^{(k)}$. Taking the squared norm and mathematical expectation, and applying the inequality $(a + b)^2 \leq 2a^2 + 2b^2$ together with Itô's isometry, gives

$$\begin{aligned} \mathbb{E}[\|\mathbf{E}_i(t)\|^2] &\leq 2\|\mathbf{H}\|^2 \mathbb{E}\left[\left\|\int_{t_k^i}^t \Psi_i(s) ds\right\|^2\right] + 2\|\mathbf{H}\|^2 \mathbb{E}\left[\left\|\int_{t_k^i}^t \sigma_{ei}(s) d\mathbf{w}_i(s)\right\|^2\right] \\ &\leq 2\|\mathbf{H}\|^2(t - t_k^i) \int_{t_k^i}^t \mathbb{E}[\|\Psi_i(s)\|^2] ds + 2\|\mathbf{H}\|^2 \int_{t_k^i}^t \mathbb{E}[\|\sigma_{ei}(s)\|^2] ds. \end{aligned} \quad (4.23)$$

The almost-sure boundedness proven above guarantees that both $\mathbb{E}[\|\Psi_i\|^2]$ and $\mathbb{E}[\|\sigma_{ei}\|^2]$ are uniformly bounded by a positive constant $M_0 > 0$. Thus,

$$\mathbb{E}[\|\mathbf{E}_i(t)\|^2] \leq 2\|\mathbf{H}\|^2 M_0(t - t_k^i)^2 + 2\|\mathbf{H}\|^2 M_0(t - t_k^i). \quad (4.24)$$

Let $\Delta_k = t_{k+1}^i - t_k^i$. To obtain a linear bound with respect to Δ_k , we distinguish two cases. If $\Delta_k \geq 1$, then the inter-event interval explicitly possesses a strictly positive lower bound of 1, and Zeno behavior is inherently impossible. Conversely, if $\Delta_k < 1$, we have $\Delta_k^2 \leq \Delta_k$, which simplifies the bound to

$$\mathbb{E}[\|\mathbf{E}_i(t)\|^2] \leq M(t - t_k^i), \quad \text{with } M = 4\|\mathbf{H}\|^2 M_0. \quad (4.25)$$

At the triggering instant $t = t_{k+1}^i$, the event condition (4.5) enforces $\|\mathbf{E}_i(t_{k+1}^i)\| \geq \rho_i \|\mathbf{e}_{yi}(t_{k+1}^i)\| + \epsilon_i \geq \epsilon_i$. Taking the expectation of the squared norm yields $\mathbb{E}[\|\mathbf{E}_i(t_{k+1}^i)\|^2] \geq \epsilon_i^2$. Combining this with the upper bound, we have

$$\epsilon_i^2 \leq \mathbb{E}[\|\mathbf{E}_i(t_{k+1}^i)\|^2] \leq M \mathbb{E}[t_{k+1}^i - t_k^i]. \quad (4.26)$$

Consequently, for the case where $\Delta_k < 1$, the expected inter-event interval satisfies $\mathbb{E}[t_{k+1}^i - t_k^i] \geq \frac{\epsilon_i^2}{M} > 0$. To rigorously demonstrate that this positive lower bound in expectation precludes Zeno behavior almost surely (a.s.), let $N_i(T) = \sup\{k \geq 0 : t_k^i \leq T\}$ denote the cumulative number of event triggers generated by the i -th node over any finite time horizon $[0, T]$ ($T > 0$). The total elapsed time of these triggering events satisfies

$$\sum_{k=0}^{N_i(T)-1} (t_{k+1}^i - t_k^i) \leq T.$$

Taking the mathematical expectation on both sides and utilizing the fact that $\mathbb{E}[t_{k+1}^i - t_k^i] \geq \tau_{\min}^* \triangleq \min\left\{1, \frac{\epsilon_i^2}{M}\right\} > 0$, by Wald's identity (or renewal theory), we obtain

$$\mathbb{E}[N_i(T)] \cdot \tau_{\min}^* \leq \mathbb{E}\left[\sum_{k=0}^{N_i(T)-1} (t_{k+1}^i - t_k^i)\right] \leq T \implies \mathbb{E}[N_i(T)] \leq \frac{T}{\tau_{\min}^*} < \infty.$$

Since $N_i(T)$ is a non-negative integer-valued random variable with finite expectation $\mathbb{E}[N_i(T)] < \infty$, an application of Markov's inequality yields

$$\mathcal{P}(N_i(T) = \infty) = \lim_{K \rightarrow \infty} \mathcal{P}(N_i(T) \geq K) \leq \lim_{K \rightarrow \infty} \frac{\mathbb{E}[N_i(T)]}{K} = 0.$$

Consequently, the probability of encountering an infinite number of triggering events in any finite duration $[0, T]$ is strictly zero, confirming that Zeno behavior is almost surely ruled out. This completes the proof. $\square$

Remark 2. In practical implementations, the threshold parameter ϵ_i can be extended to a time-decaying form $\epsilon_i(t) = \epsilon_0 e^{-\kappa t} + \epsilon_\infty$ ($\epsilon_\infty > 0$). Because $\epsilon_\infty \leq \epsilon_i(t) \leq \epsilon_0 + \epsilon_\infty$, the uniform ultimate boundedness is preserved under the upper bound, while the positive infimum ϵ_∞ rigorously preserves the analytical lower bound in (4.26), completely precluding Zeno behavior.

Remark 3. Condition (A1) is a structural matching constraint indicating that coupling interactions between dynamical subsystems occur through measurable output ports. This condition is naturally satisfied in numerous engineering applications, such as coupled multi-agent systems with relative position output coupling or smart power grids communicating through terminal bus voltages.

5. Numerical simulations

5.1. Topology identification via output coupling

To verify the effectiveness and theoretical correctness of the proposed adaptive control and topology identification scheme, numerical simulations are conducted. Consider the two-layer network consisting of 20 nodes of the Lorenz system [24]. Take the first layer a fully connected network, the second layer a star network, and the first node as the central node.

The coupling strengths are set as $c_1 = 0.1, c_2 = 0.2$, the time delays are $\tau_1 = 0.1, \tau_2 = 0.2$, the noise strengths are $\sigma_1 = 0.1, \sigma_2 = 0.2$, the inner coupling matrices are $\mathbf{B}_1 = \mathbf{B}_2 = (1, 0, 0)^T$, and the output matrix is $\mathbf{H} = (1, 0, 0)$. These coupling strengths ensure effective topology identification without suppressing the intrinsic nodal chaos. The initial states of the drive and response networks, as well as the initial values of the adaptive identification matrices, are randomly assigned from $(0, 1)$.

The synchronization behavior between the drive network and the response network is illustrated in Figures 1 and 2. Figure 1 shows the evolution of the global synchronization error over time. The synchronization error drops sharply at the initial stage and converges asymptotically to zero, remaining stable thereafter. This confirms that the proposed adaptive controller effectively achieves global synchronization between the multiplex networks. Figure 2 depicts the time evolution of the state trajectories of the first node in both the drive and response networks. It is evident that despite the chaotic nature of the intrinsic node dynamics and the random initial conditions, the state trajectory of the response network (red dashed line) quickly tracks and perfectly aligns with that of the drive network (blue solid line).

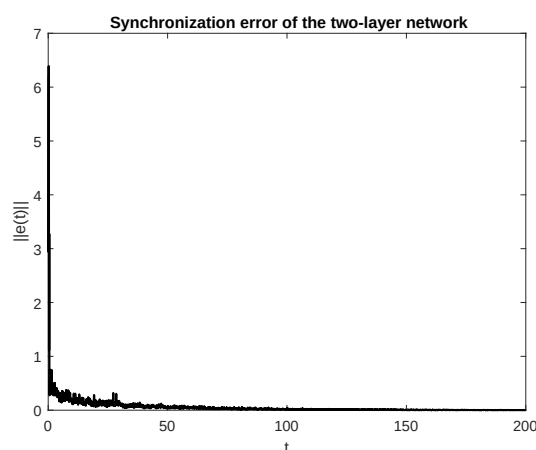


Figure 1. Synchronization error of the two-layer network.

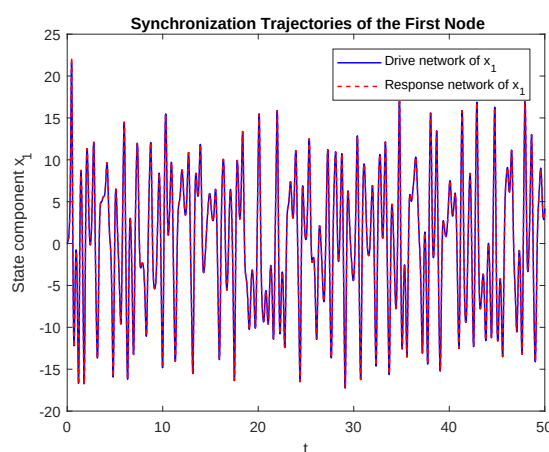


Figure 2. Synchronization trajectories of the first node.

Simultaneous to achieving synchronization, the adaptive update laws are designed to dynamically identify the unknown topological structures of both network layers. Figure 3 displays the time evolution of partial elements within the identified Laplacian matrices of Layer 1 ($a_{lj}^{(1)}, j = 1, 2, \dots, 20$) and Layer 2 ($a_{lj}^{(2)}, j = 1, 2, \dots, 20$), respectively. Starting from random initial estimations, the coupling weights undergo transient oscillations and eventually converge to stable constant values, which perfectly match the true theoretical parameters of the preset network topologies.

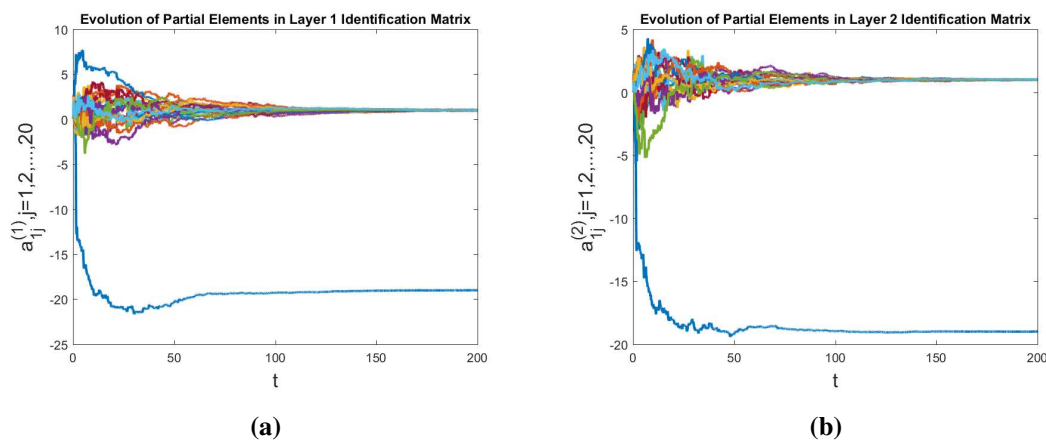


Figure 3. Topology identification of $a_{lj}^{(k)}, j = 1, 2, \dots, 20$.

To rigorously quantify the identification performance, the relative topology identification errors for both layers are plotted in Figure 4. As the time t increases, the relative errors for Layer 1 (blue solid line) and Layer 2 (red dashed line) both smoothly converge to zero. This mathematically demonstrates that the estimated matrices have perfectly reconstructed the actual topology matrices.

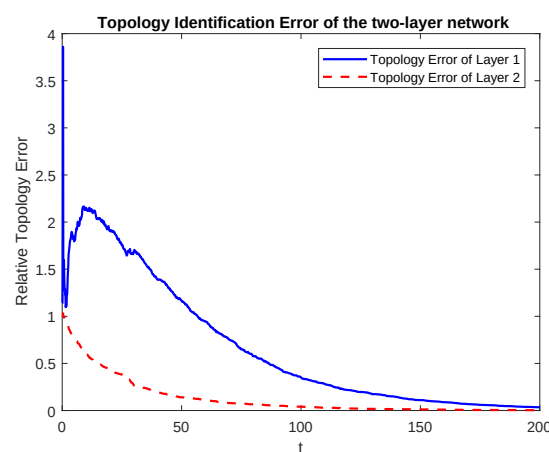


Figure 4. Topology identification error of the two-layer network.

Finally, Figure 5 provides a highly intuitive visual validation through heatmap comparisons of the Layer 1 topology matrix at different time instants. At $t = 0$, the identified matrix is completely random. At $t = 100$, the underlying structural characteristics begin to emerge clearly. By $t = T_{end}$, the heatmap of the identified matrix becomes visually and numerically identical to the true topology matrix.

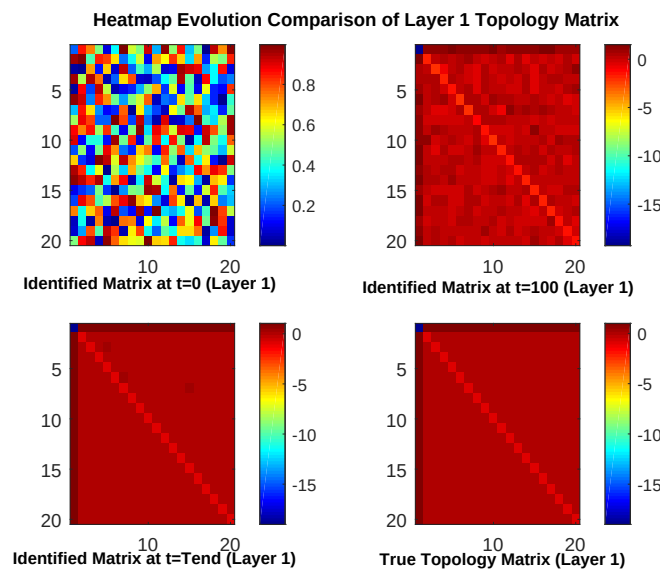


Figure 5. Heatmap evolution c of layer 1 topology matrix.

In summary, the simulation results strongly validate the theoretical analysis. The proposed adaptive control strategy exhibits excellent robustness, successfully achieving simultaneous generalized synchronization and exact topology identification in two-layer complex networks under random initial conditions.

5.2. Event-triggered topology identification via output coupling

To verify the effectiveness of the proposed event-triggered output-feedback strategy, a two-layer hypernetwork ($m = 2$) consisting of $N = 4$ nodes is constructed. The node dynamics are governed by the Lorenz chaotic system. The network topologies are set as follows: Layer 1 is a fully connected network, and Layer 2 is a star network with Node 1 as the central hub. The true connection weights for existing edges in both layers are set to $a_{ij}^{(k)} = 1$. The interaction delays are selected as $\tau_1 = 0.4$ and $\tau_2 = 0.6$, and the stochastic perturbation intensity is set to $\sigma_i = 0.01$.

For the ETM, the parameters are set as follows: the relative threshold is $\rho_i = 0.05$, and the time-varying threshold is defined as $\epsilon(t) = \epsilon_0 e^{-0.3t} + \epsilon_\infty$, where $\epsilon_0 = 0.5$ and $\epsilon_\infty = 0.001$ serves as a small positive lower bound to exclude Zeno behavior. The initial adaptive feedback gains are set to $d_i(s) = 5.0$ for $s \in [-\max(\tau_k), 0]$.

The dynamic evolution of the two-layer stochastic hypernetwork under the event-triggered scheme is illustrated in Figure 6.

- (1) **Synchronization Performance:** As shown in Figure 6a, the synchronization errors $\|\mathbf{e}_i(t)\|$ initially exhibit fluctuations due to the chaotic nature of the Lorenz system and time delays. Under the designed adaptive controllers, all errors rapidly converge to a small neighborhood of zero, achieving uniformly ultimately bounded synchronization despite stochastic perturbations.
- (2) **Topology Identification:** Figure 6c and Figure 6d present the identification results for Layer 1 and Layer 2, respectively. Driven solely by output measurements, the estimated parameters $\hat{a}_{ij}^{(1)}$

and $\hat{a}_{ij}^{(2)}$ overcome initial oscillations and converge accurately to their true values (indicated by the dashed lines at 1.0), verifying the effectiveness of the updating laws.

- (3) **Adaptive Gains:** Figure 6b shows the evolution of the adaptive gains $d_i(t)$. These gains increase during the initial transient phase to provide sufficient control effort and remain stable once the network reaches synchronization.
- (4) **Communication Efficiency and Zeno-Freeness:** Figure 6e evaluates the ETM. The bottom-left global scatter plot indicates that data transmissions are frequent initially but become sparse in the steady state, thereby reducing communication overhead. Furthermore, the zoomed-in view in Figure 6f ($t \in [15, 17]$) confirms that the triggering intervals are strictly positive, verifying that continuous communication is avoided and Zeno behavior is successfully excluded.

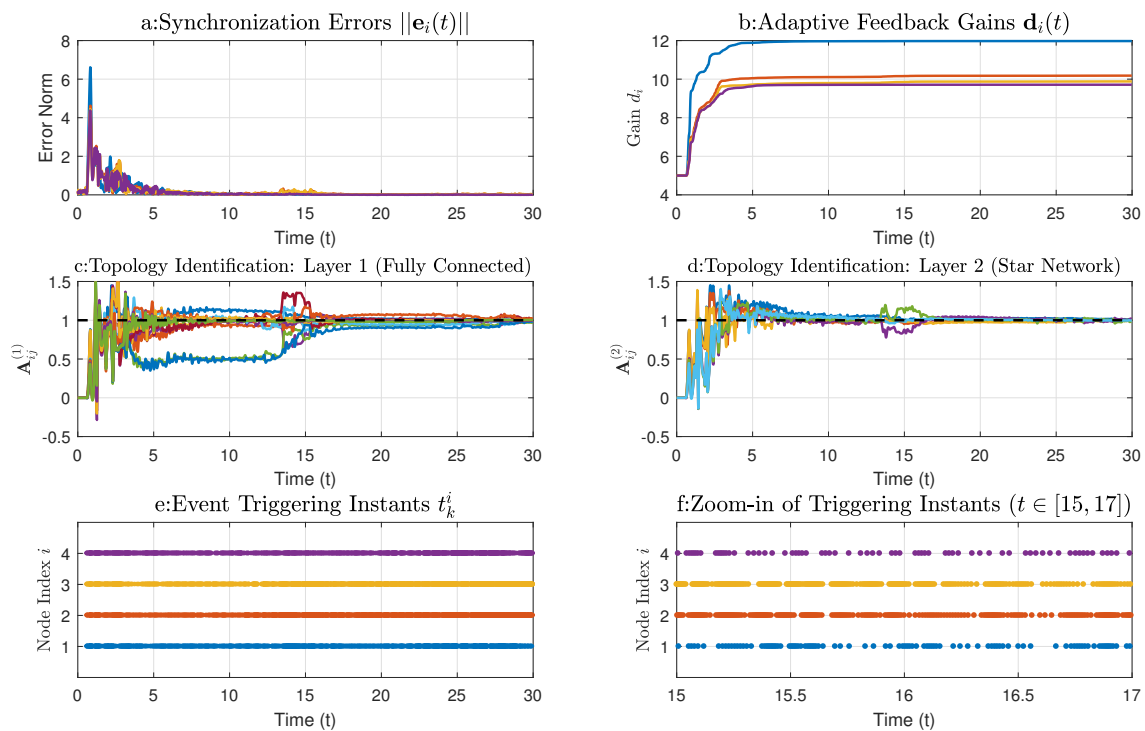


Figure 6. Dynamical evolution of the two-layer event-triggered stochastic hypernetwork. a: Synchronization errors. b: Adaptive feedback gains. c: Identified weights of Layer 1 (fully connected). d: Identified weights of Layer 2 (star network). e: Global event-triggering instants. f: Zoom-in view of the discrete triggering instants.

6. Conclusions

This paper has investigated the simultaneous synchronization and topology identification of stochastic delayed hypernetworks using only output measurements. Based on stochastic Lyapunov stability theory, continuous-time and event-triggered output-feedback control strategies were

proposed. Theoretical analysis proves that the proposed adaptive updating laws guarantee all closed-loop signals are uniformly ultimately bounded almost surely, and Zeno behavior is strictly excluded under the ETM. Consequently, the unknown multilayer topological matrices can be accurately reconstructed.

Numerical simulations verify the effectiveness of the theoretical results. The proposed method achieves robust synchronization and exact structural identification despite multiple interaction delays and stochastic perturbations. Meanwhile, the introduced ETM significantly reduces the communication burden among network nodes. This approach provides a practical and resource-efficient tool for the structural inference of complex systems. While the current framework assumes undirected interconnections, future research will explore extending the output-feedback event-triggered identification architecture to directed hypernetworks with asymmetric Laplacians and switching topologies.

Author contributions

Xueyi Zhao: Conceptualization, Methodology, Software, Investigation, Formal analysis, Visualization, Writing—original draft; Na Li: Methodology, Supervision, Writing—review and editing; Anjing Fan: Conceptualization, Supervision, Writing—review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant 62303161, the Hubei Provincial Natural Science Foundation Innovation and Development Joint Fund Project under Grant No. 2026AFC0294, the Outstanding Young and Middle-aged Scientific and Technological Innovation Team in Colleges and Universities of Hubei Province under Grant T2022035, and the Open Foundation of Hubei Key Laboratory of Applied Mathematics (Hubei University) under Grant HBAM202103.

Conflict of interest

The authors declare no conflict of interest.

References

1. D. Irving, F. Sorrentino, Synchronization of dynamical hypernetworks: Dimensionality reduction through simultaneous block-diagonalization of matrices, *Phys. Rev. E*, **86** (2012), 056102. <http://doi.org/10.1103/PhysRevE.86.056102>

2. S. Boccaletti, G. Bianconi, R. Criado, C. I. Del Genio, J. Gómez-Gardeñes, M. Romance, et al., The structure and dynamics of multilayer networks, *Phys. Rep.*, **544** (2014), 1–122. <http://doi.org/10.1016/j.physrep.2014.07.001>
3. A. Aleta, G. F. de Arruda, Y. Moreno, Multilayer network science: Theory, methods, and applications, *J. Complex Netw.*, **13** (2025), cnae024. <http://doi.org/10.1093/comnet/cnae024>
4. S. Zhu, J. Zhou, J. A. Lu, Identifying partial topology of complex dynamical networks with distributed delay, *Commun. Nonlinear Sci. Numer. Simul.*, **126** (2023), 107504. <http://doi.org/10.1016/j.cnsns.2023.107504>
5. H. Liu, Z. C. Shang, Z. Y. Ren, J. A. Lu, Recovering unknown topology in a two-layer multiplex network: One layer infers the other layer, *Sci. China Technol. Sci.*, **65** (2022), 1493–1505. <http://doi.org/10.1007/s11431-021-1977-8>
6. X. Y. Zhao, J. Zhou, J. A. Lu, Pinning synchronization of multiplex delayed networks with stochastic perturbations, *IEEE Trans. Cybern.*, **49** (2019), 4262–4270. <http://doi.org/10.1109/TCYB.2018.2861426>
7. C. I. del Genio, J. Gómez-Gardeñes, I. Bonamassa, S. Boccaletti, Synchronization in networks with multiple interaction layers, *Sci. Adv.*, **2** (2016), e1601679. <http://doi.org/10.1126/sciadv.1601679>
8. J. L. Wang, H. N. Wu, T. Huang, Synchronization of complex dynamical networks with output coupling, *IEEE Trans. Circuits Syst. I, Reg. Papers*, **58** (2011), 1920–1930. <http://doi.org/10.1109/TCSI.2010.2097740>
9. J. L. Wang, H. N. Wu, T. Huang, Passivity and output synchronization of complex dynamical networks with multiple output couplings, *IEEE Trans. Neural Netw. Learn. Syst.*, **31** (2020), 4106–4118. <http://doi.org/10.1109/TNNLS.2019.2949019>
10. J. L. Wang, H. N. Wu, T. Huang, Synchronization of complex dynamical networks with output coupling and time-varying delays, *IEEE Trans. Cybern.*, **51** (2021), 867–877. <http://doi.org/10.1109/TCYB.2019.2925565>
11. Y. Yao, Z. Liu, Y. Kang, Y. Zhao, J. Tan, L. Gu, et al., Sliding flexible prescribed performance boundary-guided reinforcement learning control for input-constrained nonlinear systems, *IEEE Trans. Cybern.*, **56** (2026), 968–979. <http://doi.org/10.1109/TCYB.2025.3610913>
12. Y. Yao, X. Liang, Y. Kang, Y. Zhao, J. Tan, L. Gu, Dual flexible prescribed performance control of input saturated high-order nonlinear systems, *IEEE Trans. Cybern.*, **55** (2025), 1147–1158. <http://doi.org/10.1109/TCYB.2024.3524242>
13. Y. Yao, Y. Kang, Y. Zhao, P. Li, J. Tan, Flexible prescribed performance output feedback control for nonlinear systems with input saturation, *IEEE Trans. Fuzzy Syst.*, **32** (2024), 6012–6022. <http://doi.org/10.1109/TFUZZ.2024.3418772>
14. P. Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, *IEEE Trans. Autom. Control*, **52** (2007), 1680–1685. <http://doi.org/10.1109/TAC.2007.904277>
15. D. V. Dimarogonas, E. Frazzoli, K. H. Johansson, Distributed event-triggered control for multi-agent systems, *IEEE Trans. Autom. Control*, **57** (2012), 1291–1297. <http://doi.org/10.1109/TAC.2011.2174666>

16. H. Liu, Y. Li, Z. Li, J. Lü, J. A. Lu, Topology identification of multilink complex dynamical networks via adaptive observers incorporating chaotic exosignals, *IEEE Trans. Cybern.*, **52** (2022), 6255–6268. <http://doi.org/10.1109/TCYB.2020.3031647>
17. F. Sorrentino, A. N. Gorban, I. Y. Tyukin, Discovering the topology of complex networks via adaptive estimators, *Chaos*, **29** (2019), 083121. <http://doi.org/10.1063/1.5097931>
18. Y. Li, X. Wang, Z. Chen, Topology identification of complex dynamical networks with stochastic perturbations and output coupling, *Chaos Solitons Fract.*, **164** (2022), 112664. <http://doi.org/10.1016/j.chaos.2022.112664>
19. D. Ding, Z. Wang, D. W. C. Ho, G. Wei, Event-triggered synchronization of stochastic neural networks with time-varying delays, *IEEE Trans. Neural Netw. Learn. Syst.*, **29** (2018), 161–172. <http://doi.org/10.1109/TNNLS.2016.2616494>
20. X. Ge, Q. L. Han, D. Ding, Event-triggered consensus control for multi-agent systems: A survey, *IEEE Trans. Syst. Man Cybern. Syst.*, **51** (2021), 4–19. <http://doi.org/10.1109/TSMC.2019.2956191>
21. X. Mao, *Stochastic Differential Equations and Applications*, 2 Eds., Chichester: Horwood Publishing, 2007. <http://doi.org/10.1533/9780857099402>
22. X. R. Mao, Stochastic versions of the LaSalle theorem, *J. Differ. Equ.*, **153** (1999), 175–195. <http://doi.org/10.1006/jdeq.1998.3535>
23. J. Q. Lu, J. D. Cao, Synchronization-based approach for parameters identification in delayed chaotic neural networks, *Phys. A*, **382** (2007), 672–682. <http://doi.org/10.1016/j.physa.2007.04.019>
24. E. N. Lorenz, Deterministic nonperiodic flow, *J. Atmos. Sci.*, **20** (1963), 130–141. [https://doi.org/10.1175/1520-0469\(1963\)020<0130:DNF>2.0.CO;2](https://doi.org/10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2)



AIMS Press

© 2026 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (<https://creativecommons.org/licenses/by/4.0>)